Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.1-Hyperbolic-sine/163-6.1.5-Hyperbolic-sine-functions

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 3:31am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [369]. This is test number [163].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (369)	0.00 (0)
Mathematica	100.00 (369)	0.00 (0)
Fricas	95.66 (353)	4.34 (16)
Maple	86.45 (319)	13.55 (50)
Giac	75.34 (278)	24.66 (91)
Maxima	72.09 (266)	27.91 (103)
Mupad	59.89 (221)	40.11 (148)
Sympy	32.52 (120)	67.48 (249)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

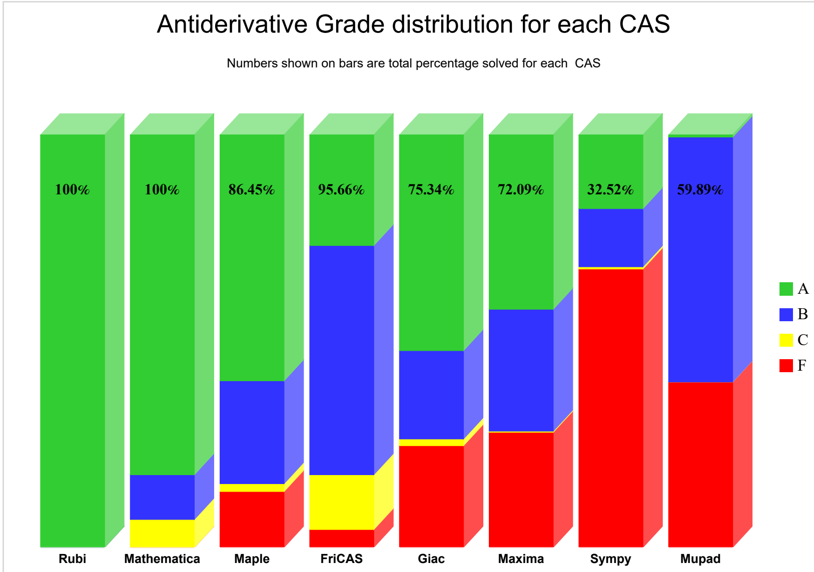
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

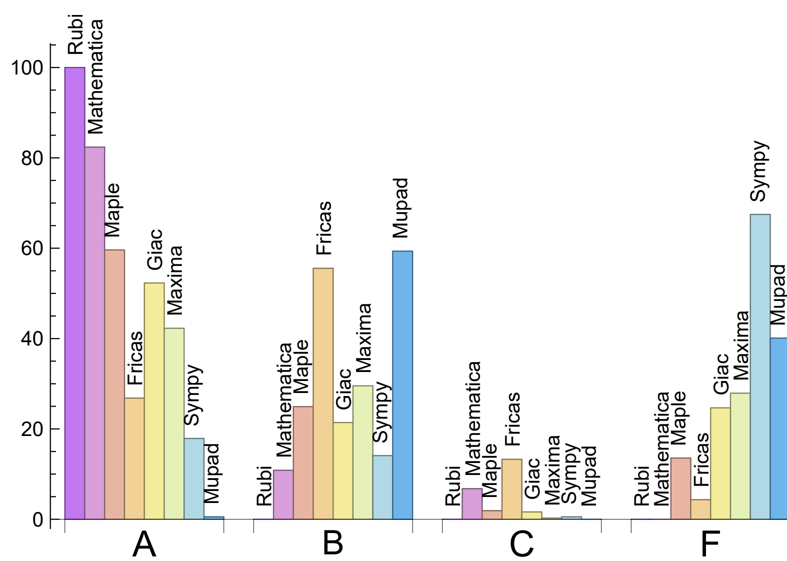
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	82.38	10.84	6.78	0.00
Maple	59.62	24.93	1.90	13.55
Giac	52.30	21.41	1.63	24.66
Maxima	42.28	29.54	0.27	27.91
Fricas	26.83	55.56	13.28	4.34
Sympy	17.89	14.09	0.54	67.48
Mupad	N/A	59.35	0.00	40.11

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	50	100.00 %	0.00 %	0.00 %
Fricas	16	100.00 %	0.00 %	0.00 %
Giac	91	97.80 %	2.20 %	0.00 %
Maxima	103	98.06 %	0.00 %	1.94 %
Sympy	249	79.12 %	17.67 %	3.21 %
Mupad	148	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

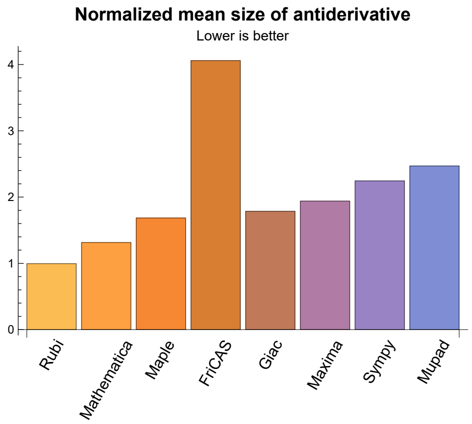
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

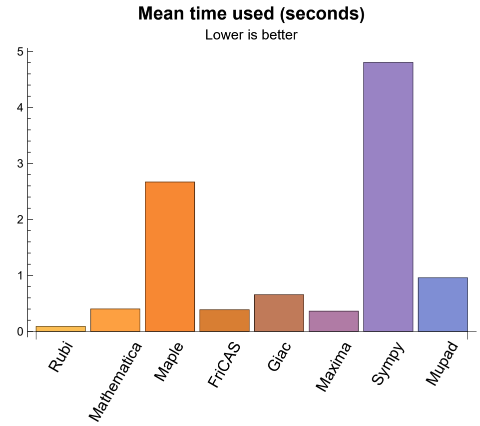
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	86.12	0.99	66.00	1.00
Mathematica	0.40	110.21	1.31	72.00	1.00
Maple	2.67	147.97	1.69	93.00	1.41
Maxima	0.36	136.73	1.94	93.50	1.49
Fricas	0.39	443.95	4.06	156.00	2.17
Sympy	4.80	115.35	2.24	58.00	1.60
Giac	0.66	185.51	1.79	78.50	1.37
Mupad	0.96	173.07	2.47	74.00	1.58

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{264, 265\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{356, 362, 363, 364\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

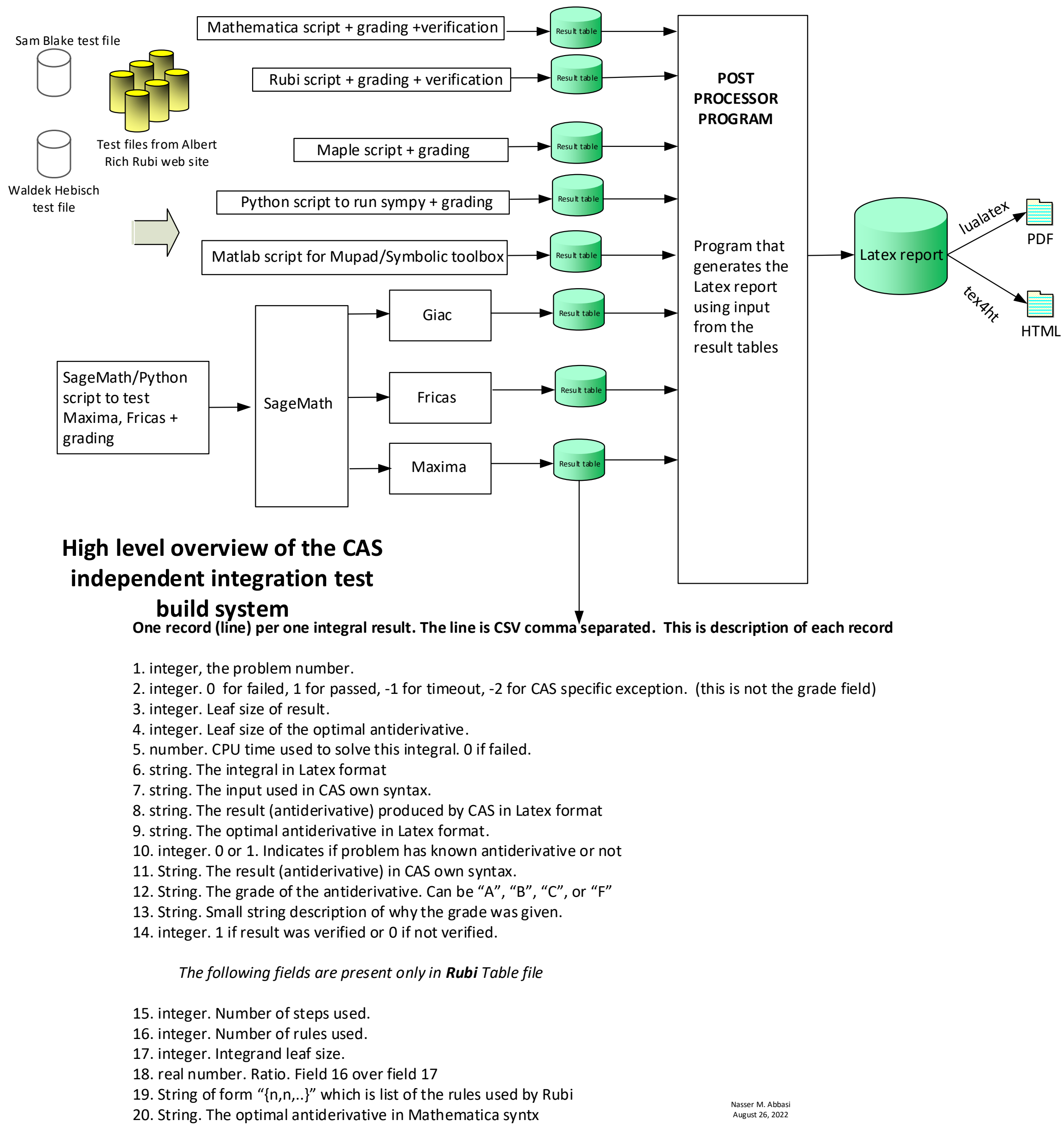
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 48, 49, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 214, 216, 219, 221, 222, 224, 226, 228, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367 }

B grade: { 1, 40, 42, 43, 50, 52, 53, 54, 55, 68, 92, 93, 94, 95, 119, 158, 160, 162, 164, 171, 175, 194, 208, 210, 213, 215, 217, 218, 220, 223, 225, 227, 248, 274, 295, 297, 298, 300, 327, 365 }

C grade: { 9, 13, 17, 21, 25, 29, 148, 188, 190, 192, 200, 202, 229, 237, 239, 259, 280, 284, 285, 316, 317, 320, 321, 368, 369 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 44, 45, 46, 47, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 137, 140, 141, 142, 145, 152, 153, 156, 157, 163, 165, 176, 183, 186, 193, 194, 195, 196, 202, 203, 204, 205, 206, 207, 212, 213, 222, 223, 224, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 260, 264, 265, 274, 275, 279, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 318, 319, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade: { 24, 40, 41, 42, 43, 48, 49, 50, 68, 72, 73, 103, 104, 105, 106, 107, 109, 126, 127, 128, 131, 132, 135, 138, 139, 143, 144, 154, 155, 158, 159, 160, 161, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 187, 188, 189, 190, 191, 192, 197, 198, 199, 200, 201, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 234, 246, 255, 256, 257, 258, 259, 295, 296, 297, 298, 299, 300 }

C grade: { 312, 313, 316, 317, 320, 321, 339 }

F grade: { 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 112, 113, 114, 123, 124, 125, 146, 147, 148, 149, 150, 151, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 285, 286, 287, 288, 325, 326, 327, 328 }

2.1.4 Maxima

A grade: { 1, 2, 4, 6, 40, 41, 43, 44, 48, 49, 50, 56, 57, 60, 61, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 84, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 119, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 165, 171, 173, 175, 176, 181, 183, 184, 186, 190, 192, 193, 194, 195, 196, 202, 203, 204, 229, 230, 231, 233, 238, 239, 246, 247, 248, 249, 250, 251, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade: { 3, 5, 42, 45, 46, 47, 51, 52, 53, 54, 55, 58, 59, 62, 63, 78, 83, 85, 86, 87, 103, 104, 116, 117, 118, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 172, 174, 177, 178, 179, 180, 182, 185, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 234, 235, 236, 237, 240, 241, 242, 243, 252, 253, 254, 255, 256, 276, 278, 307, 309, 335 }

C grade: { 339 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 336, 368, 369 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 40, 41, 43, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 88, 89, 90, 92, 93, 96, 97, 98, 99, 100, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 163, 167, 171, 173, 175, 176, 178, 182, 185, 194, 212, 231, 232, 247, 248, 260, 264, 265, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 285, 286, 287, 288, 289, 290, 295, 298, 301, 303, 305, 311, 316, 320, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 366, 367 }

B grade: { 5, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 95, 101, 102, 103, 104, 123, 124, 125, 129, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 180, 181, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 272, 273, 278, 291, 292, 293, 294, 296, 297, 299, 300, 302, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 324, 333, 334, 335, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

C grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 105, 106, 107, 108, 109, 110, 111, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 279, 280, 281, 282, 283, 284 }

F grade: { 31, 32, 33, 34, 35, 36, 37, 38, 39, 261, 262, 263, 325, 326, 327, 328 }

2.1.6 Sympy

A grade: { 1, 3, 5, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 142, 165, 171, 173, 175, 181, 184, 185, 193, 203, 211, 212, 220, 221, 224, 247, 248, 264, 265, 311, 315, 319, 366, 367 }

B grade: { 2, 4, 6, 74, 75, 101, 129, 133, 158, 159, 160, 161, 162, 163, 164, 172, 174, 176, 182, 183, 186, 192, 201, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 225, 226, 227, 246, 274, 276, 278, 301, 302, 303, 304, 310, 314, 318, 323, 331, 336 }

C grade: { 253, 324 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 130, 131, 132, 135, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 166, 167, 168, 169, 170, 177, 178, 179, 180, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 202, 204, 205, 206, 207, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 305, 306, 307, 308, 309, 312, 313, 316, 317, 320, 321, 322, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

2.1.7 Giac

A grade: { 2, 4, 6, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 143, 144, 145, 152, 153, 154, 155, 156, 157, 167, 171, 173, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 197, 202, 203, 205, 210, 212, 220, 222, 226, 228, 230, 231, 232, 233, 236, 238, 241, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 264, 265, 266, 267, 276, 277, 278, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade: { 1, 3, 5, 45, 100, 104, 131, 132, 135, 140, 141, 142, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 188, 196, 198, 199, 200, 201, 204, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 227, 229, 234, 235, 237, 239, 240, 242, 255, 256, 268, 269, 270, 271, 272, 273, 274, 275, 289, 290, 291, 295, 296, 297, 298, 299, 300, 312 }

C grade: { 322, 323, 324, 339, 343, 346 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 292, 293, 294, 325, 326, 327, 328, 368, 369 }

2.1.8 Mupad

A grade: { 264, 265 }

B grade: { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 135, 136, 142, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 331, 333, 334, 335, 336, 366, 367 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 131, 132, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 244, 245, 255, 256, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 329, 330, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.004	0.009	0.149	0.263	0.390	0.047	0.396	0.045

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	23	46	32	18
N.S.	1	1.00	0.92	1.08	1.28	0.92	1.84	1.28	0.72
time (sec)	N/A	0.006	0.019	0.408	0.255	0.458	0.075	0.405	0.373

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	54	38	36	54	24
N.S.	1	1.00	1.07	1.00	2.00	1.41	1.33	2.00	0.89
time (sec)	N/A	0.010	0.011	0.421	0.258	0.408	0.103	0.411	0.055

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	33	60	49	95	60	32
N.S.	1	1.00	0.72	0.72	1.30	1.07	2.07	1.30	0.70
time (sec)	N/A	0.014	0.031	0.787	0.264	0.358	0.171	0.431	0.080

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	82	79	58	82	31
N.S.	1	1.00	1.07	1.00	2.00	1.93	1.41	2.00	0.76
time (sec)	N/A	0.014	0.013	0.442	0.272	0.429	0.256	0.410	0.409

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	47	86	90	139	88	42
N.S.	1	1.00	0.64	0.70	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.026	0.030	0.845	0.290	0.431	0.408	0.426	0.133

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	116	0	326	0	0	-1
N.S.	1	1.00	0.73	1.13	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.107	0.657	0.000	0.080	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	164	0	202	0	0	-1
N.S.	1	1.00	0.85	2.05	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.058	0.655	0.000	0.091	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	83	100	0	103	0	0	-1
N.S.	1	1.00	1.04	1.25	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.068	0.619	0.000	0.103	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	37	0	0	-1
N.S.	1	1.00	0.93	2.00	0.00	0.69	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.077	0.753	0.000	0.090	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	-1
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.090	0.391	0.000	0.098	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	152	0	0	-1
N.S.	1	1.00	0.75	2.03	0.00	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.039	0.668	0.000	0.126	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	101	0	314	0	0	-1
N.S.	1	1.00	1.08	1.26	0.00	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.059	0.609	0.000	0.136	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	192	0	621	0	0	-1
N.S.	1	1.00	0.71	1.86	0.00	6.03	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.121	0.664	0.000	0.104	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	122	0	394	0	0	-1
N.S.	1	1.00	0.66	1.05	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.199	0.770	0.000	0.090	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	170	0	249	0	0	-1
N.S.	1	1.00	0.77	1.93	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.088	0.711	0.000	0.102	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	106	0	115	0	0	-1
N.S.	1	1.00	1.00	1.20	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.092	0.674	0.000	0.128	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	52	111	0	42	0	0	-1
N.S.	1	1.00	0.93	1.98	0.00	0.75	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.034	0.874	0.000	0.111	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	89	0	27	0	0	-1
N.S.	1	1.00	0.96	1.59	0.00	0.48	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.026	0.516	0.000	0.095	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	62	159	0	169	0	0	-1
N.S.	1	1.00	0.72	1.85	0.00	1.97	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.045	0.731	0.000	0.078	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	84	114	0	347	0	0	-1
N.S.	1	1.00	0.93	1.27	0.00	3.86	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.085	0.674	0.000	0.089	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	79	205	0	675	0	0	-1
N.S.	1	1.00	0.67	1.74	0.00	5.72	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.123	0.734	0.000	0.079	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	65	122	0	104	0	0	-1
N.S.	1	1.00	0.71	1.34	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.121	0.682	0.000	0.089	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	169	0	94	0	0	-1
N.S.	1	1.00	0.89	2.73	0.00	1.52	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.046	0.694	0.000	0.100	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	94	104	0	79	0	0	-1
N.S.	1	1.00	1.52	1.68	0.00	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.099	0.637	0.000	0.083	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	91	0	53	0	0	-1
N.S.	1	1.00	0.93	3.03	0.00	1.77	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.018	0.805	0.000	0.085	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	68	0	20	0	0	-1
N.S.	1	1.00	0.93	2.27	0.00	0.67	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.019	0.516	0.000	0.114	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	159	0	87	0	0	-1
N.S.	1	1.00	0.86	2.74	0.00	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.069	0.821	0.000	0.085	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	83	113	0	132	0	0	-1
N.S.	1	1.00	1.34	1.82	0.00	2.13	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.045	0.673	0.000	0.080	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	80	204	0	178	0	0	-1
N.S.	1	1.00	0.88	2.24	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.095	0.740	0.000	0.098	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.042	0.254	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.027	0.211	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.027	0.207	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.031	0.263	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.031	0.257	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.030	0.259	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.033	0.360	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.030	0.274	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.032	0.271	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	134	102	59	67	58	50	50
N.S.	1	1.00	2.91	2.22	1.28	1.46	1.26	1.09	1.09
time (sec)	N/A	0.050	0.135	0.621	0.276	0.454	0.089	0.430	0.490

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	75	45	55	41	38	38
N.S.	1	1.00	1.14	2.08	1.25	1.53	1.14	1.06	1.06
time (sec)	N/A	0.034	0.081	0.547	0.289	0.501	0.071	0.426	0.430

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	79	52	33	37	20	26	24
N.S.	1	1.00	3.59	2.36	1.50	1.68	0.91	1.18	1.09
time (sec)	N/A	0.042	0.081	0.610	0.282	0.571	0.051	0.416	0.425

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	29	29	12	16	8	10	12
N.S.	1	1.00	2.07	2.07	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.021	0.031	0.357	0.289	0.695	0.029	0.420	0.407

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	21	29	33	0	24	35
N.S.	1	1.00	1.58	1.11	1.53	1.74	0.00	1.26	1.84
time (sec)	N/A	0.029	0.017	0.476	0.278	0.459	0.000	0.435	0.490

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	36	35	51	77	0	44	51
N.S.	1	1.00	1.57	1.52	2.22	3.35	0.00	1.91	2.22
time (sec)	N/A	0.042	0.032	0.531	0.271	0.438	0.000	0.421	0.636

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	49	53	75	126	0	51	70
N.S.	1	1.00	1.32	1.43	2.03	3.41	0.00	1.38	1.89
time (sec)	N/A	0.053	0.143	0.579	0.273	0.440	0.000	0.418	0.665

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	71	103	174	0	58	85
N.S.	1	1.00	1.13	1.51	2.19	3.70	0.00	1.23	1.81
time (sec)	N/A	0.053	0.171	0.540	0.328	0.394	0.000	0.426	0.719

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	65	98	71	89	70	50	97
N.S.	1	1.00	1.12	1.69	1.22	1.53	1.21	0.86	1.67
time (sec)	N/A	0.080	0.196	0.764	0.284	0.504	0.095	0.428	0.566

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	75	59	74	54	38	79
N.S.	1	1.00	1.02	1.70	1.34	1.68	1.23	0.86	1.80
time (sec)	N/A	0.091	0.094	0.825	0.279	0.377	0.080	0.416	0.576

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	74	52	40	50	41	22	71
N.S.	1	1.00	2.31	1.62	1.25	1.56	1.28	0.69	2.22
time (sec)	N/A	0.045	0.064	0.586	0.300	0.389	0.057	0.481	0.567

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	81	32	37	20	25
N.S.	1	1.00	0.71	0.81	2.61	1.03	1.19	0.65	0.81
time (sec)	N/A	0.022	0.008	0.691	0.298	0.425	0.052	0.434	0.522

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	91	44	55	78	0	34	41
N.S.	1	1.00	2.68	1.29	1.62	2.29	0.00	1.00	1.21
time (sec)	N/A	0.059	0.059	0.605	0.300	0.345	0.000	0.451	0.283

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	88	58	79	130	0	46	85
N.S.	1	1.00	2.10	1.38	1.88	3.10	0.00	1.10	2.02
time (sec)	N/A	0.080	0.287	0.723	0.298	0.425	0.000	0.430	0.709

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	131	76	105	174	0	59	79
N.S.	1	1.00	2.26	1.31	1.81	3.00	0.00	1.02	1.36
time (sec)	N/A	0.101	0.283	0.694	0.277	0.477	0.000	0.434	0.774

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	143	92	127	226	0	84	189
N.S.	1	1.00	2.23	1.44	1.98	3.53	0.00	1.31	2.95
time (sec)	N/A	0.091	1.425	0.741	0.293	0.423	0.000	0.416	1.095

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	20	16	15	15	17
N.S.	1	1.00	1.56	0.74	0.74	0.59	0.56	0.56	0.63
time (sec)	N/A	0.010	0.048	0.814	0.265	0.335	0.052	0.415	0.199

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	55	94	50	61	25	29
N.S.	1	1.00	1.03	0.93	1.59	0.85	1.03	0.42	0.49
time (sec)	N/A	0.021	0.078	1.273	0.287	0.440	0.105	0.419	0.530

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	211	85	109	36	40
N.S.	1	1.00	0.92	1.00	2.40	0.97	1.24	0.41	0.45
time (sec)	N/A	0.032	0.097	1.424	0.290	0.402	0.182	0.431	0.691

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	372	120	155	47	53
N.S.	1	1.00	0.74	1.03	3.18	1.03	1.32	0.40	0.45
time (sec)	N/A	0.044	0.119	1.411	0.286	0.354	0.268	0.422	0.959

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	20	20	16	17	15	17
N.S.	1	1.00	1.56	0.74	0.74	0.59	0.63	0.56	0.63
time (sec)	N/A	0.009	0.048	0.860	0.286	0.349	0.053	0.422	0.146

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	55	94	50	61	25	29
N.S.	1	1.00	1.00	0.93	1.59	0.85	1.03	0.42	0.49
time (sec)	N/A	0.019	0.057	1.332	0.295	0.500	0.106	0.413	0.536

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	88	211	85	109	36	40
N.S.	1	1.00	0.92	1.00	2.40	0.97	1.24	0.41	0.45
time (sec)	N/A	0.030	0.093	1.168	0.318	0.424	0.180	0.408	0.635

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	87	121	372	120	155	47	52
N.S.	1	1.00	0.74	1.03	3.18	1.03	1.32	0.40	0.44
time (sec)	N/A	0.047	0.109	1.165	0.304	0.422	0.291	0.411	0.906

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	75	0	0	76	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.058	0.644	0.000	0.381	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	0	0	76	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.056	0.520	0.000	0.404	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	145	0	0	101	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.326	1.947	0.000	0.397	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	0	0	63	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.150	1.839	0.000	0.429	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	74	89	0	27	0	0	53
N.S.	1	1.00	2.39	2.87	0.00	0.87	0.00	0.00	1.71
time (sec)	N/A	0.011	0.029	1.658	0.000	0.385	0.000	0.000	0.754

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	84	0	0	93	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	1.79	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.066	1.732	0.000	0.392	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	156	0	0	235	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.167	1.665	0.000	0.372	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	210	0	0	348	0	0	-1
N.S.	1	1.00	1.72	0.00	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.137	1.700	0.000	0.419	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	202	158	799	0	156	199
N.S.	1	1.00	0.97	1.87	1.46	7.40	0.00	1.44	1.84
time (sec)	N/A	0.231	0.325	0.369	0.501	0.359	0.000	0.418	0.757

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	152	118	459	0	117	159
N.S.	1	1.00	1.00	1.85	1.44	5.60	0.00	1.43	1.94
time (sec)	N/A	0.138	0.106	0.362	0.498	0.507	0.000	0.413	0.612

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	1253	86	129
N.S.	1	1.00	1.07	1.61	1.47	4.18	21.98	1.51	2.26
time (sec)	N/A	0.083	0.078	0.350	0.491	0.549	175.163	0.434	0.542

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	63	65	134	252	67	99
N.S.	1	1.00	1.11	1.34	1.38	2.85	5.36	1.43	2.11
time (sec)	N/A	0.044	0.040	0.326	0.486	0.442	47.382	0.421	0.534

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	49	83	156	0	82	287
N.S.	1	1.00	1.16	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.053	0.039	0.440	0.493	0.372	0.000	0.419	0.638

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	73	100	345	0	98	292
N.S.	1	1.00	1.37	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.092	0.298	0.524	0.494	0.489	0.000	0.441	0.697

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	118	108	154	929	0	137	617
N.S.	1	1.00	1.46	1.33	1.90	11.47	0.00	1.69	7.62
time (sec)	N/A	0.229	0.391	0.591	0.495	0.443	0.000	0.432	0.998

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	151	194	1676	0	171	694
N.S.	1	1.00	1.71	1.39	1.78	15.38	0.00	1.57	6.37
time (sec)	N/A	0.347	0.659	0.625	0.497	0.557	0.000	0.439	0.879

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	118	218	256	1769	0	235	305
N.S.	1	1.00	0.73	1.35	1.58	10.92	0.00	1.45	1.88
time (sec)	N/A	0.303	0.318	0.665	0.508	0.565	0.000	0.419	0.862

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	161	208	1053	0	184	274
N.S.	1	1.00	0.83	1.40	1.81	9.16	0.00	1.60	2.38
time (sec)	N/A	0.198	0.264	0.511	0.499	0.502	0.000	0.422	0.781

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	127	149	521	0	131	228
N.S.	1	1.00	1.04	1.53	1.80	6.28	0.00	1.58	2.75
time (sec)	N/A	0.100	0.143	0.499	0.500	0.442	0.000	0.425	0.791

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	97	117	341	0	99	142
N.S.	1	1.00	1.13	1.62	1.95	5.68	0.00	1.65	2.37
time (sec)	N/A	0.054	0.076	0.449	0.489	0.439	0.000	0.418	0.701

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	91	115	162	672	0	142	1001
N.S.	1	1.00	1.07	1.35	1.91	7.91	0.00	1.67	11.78
time (sec)	N/A	0.161	0.135	0.658	0.549	0.802	0.000	0.425	2.700

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	141	251	1740	0	205	1017
N.S.	1	1.00	1.03	1.23	2.18	15.13	0.00	1.78	8.84
time (sec)	N/A	0.270	0.484	0.716	0.499	0.526	0.000	0.434	3.393

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	156	175	363	3754	0	203	977
N.S.	1	1.00	0.99	1.11	2.30	23.76	0.00	1.28	6.18
time (sec)	N/A	0.475	0.496	0.806	0.521	0.811	0.000	0.436	3.400

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	214	219	477	6430	0	236	975
N.S.	1	1.00	1.08	1.11	2.41	32.47	0.00	1.19	4.92
time (sec)	N/A	0.618	0.654	0.782	0.499	0.590	0.000	0.435	3.301

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	81	40	19	28	31	32	39
N.S.	1	1.00	1.11	0.55	0.26	0.38	0.42	0.44	0.53
time (sec)	N/A	0.021	0.028	0.971	0.474	0.363	0.166	0.430	0.351

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	142	74	79	103	75	67	106
N.S.	1	1.00	1.39	0.73	0.77	1.01	0.74	0.66	1.04
time (sec)	N/A	0.036	0.197	1.278	0.536	0.424	0.216	0.391	0.979

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	204	110	124	193	138	89	147
N.S.	1	1.00	1.56	0.84	0.95	1.47	1.05	0.68	1.12
time (sec)	N/A	0.062	0.377	1.404	0.510	0.630	0.286	0.413	1.056

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	265	144	167	283	197	111	237
N.S.	1	1.00	1.66	0.90	1.04	1.77	1.23	0.69	1.48
time (sec)	N/A	0.085	0.466	1.404	0.490	0.487	0.367	0.429	1.261

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	171	42	36	26	31	28	32
N.S.	1	1.00	4.62	1.14	0.97	0.70	0.84	0.76	0.86
time (sec)	N/A	0.010	0.026	0.868	0.534	0.348	0.116	0.422	0.615

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	183	84	64	103	82	65	102
N.S.	1	1.00	2.77	1.27	0.97	1.56	1.24	0.98	1.55
time (sec)	N/A	0.026	0.192	1.174	0.495	0.452	0.162	0.414	1.068

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	277	126	108	193	141	87	143
N.S.	1	1.00	2.92	1.33	1.14	2.03	1.48	0.92	1.51
time (sec)	N/A	0.047	0.513	1.305	0.598	0.393	0.231	0.417	1.617

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	308	168	152	283	202	109	232
N.S.	1	1.00	2.48	1.35	1.23	2.28	1.63	0.88	1.87
time (sec)	N/A	0.072	1.622	1.319	0.481	0.382	0.301	0.423	2.112

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	138	145	272	223	314	269	160
N.S.	1	1.00	0.75	0.79	1.49	1.22	1.72	1.47	0.87
time (sec)	N/A	0.193	0.473	1.043	0.288	0.472	0.321	0.428	0.603

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	108	108	182	146	240	200	114
N.S.	1	1.00	0.79	0.79	1.33	1.07	1.75	1.46	0.83
time (sec)	N/A	0.113	0.261	0.860	0.335	0.343	0.205	0.451	0.336

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	71	71	115	91	128	135	75
N.S.	1	1.00	0.77	0.77	1.25	0.99	1.39	1.47	0.82
time (sec)	N/A	0.052	0.127	0.769	0.284	0.374	0.131	0.429	0.502

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	51	55	46	78	76	41
N.S.	1	1.00	0.92	0.98	1.06	0.88	1.50	1.46	0.79
time (sec)	N/A	0.013	0.067	0.626	0.266	0.416	0.085	0.424	0.491

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	31	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.07	1.00
time (sec)	N/A	0.007	0.009	0.261	0.286	0.387	0.043	0.433	0.428

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	43	67	162	196	67	55
N.S.	1	1.00	1.18	0.98	1.52	3.68	4.45	1.52	1.25
time (sec)	N/A	0.029	0.031	0.665	0.488	0.441	4.928	0.430	0.760

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	118	138	423	0	119	200
N.S.	1	1.00	1.08	1.49	1.75	5.35	0.00	1.51	2.53
time (sec)	N/A	0.051	0.197	1.131	0.498	0.370	0.000	0.431	0.876

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	117	280	315	1347	0	231	-1
N.S.	1	1.00	0.92	2.20	2.48	10.61	0.00	1.82	-0.01
time (sec)	N/A	0.093	0.198	1.177	0.498	0.406	0.000	0.420	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	159	494	551	2934	0	357	-1
N.S.	1	1.00	0.91	2.84	3.17	16.86	0.00	2.05	-0.01
time (sec)	N/A	0.156	0.497	1.201	0.513	0.363	0.000	0.438	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	178	917	0	464	0	0	-1
N.S.	1	1.00	0.99	5.12	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.314	1.155	0.000	0.094	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	676	0	263	0	0	-1
N.S.	1	1.00	0.93	4.51	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.255	0.964	0.000	0.132	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	262	0	173	0	0	-1
N.S.	1	1.00	1.08	4.37	0.00	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.141	1.098	0.000	0.107	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	125	0	61	0	0	-1
N.S.	1	1.00	1.00	2.08	0.00	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.139	0.704	0.000	0.092	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	456	0	407	0	0	-1
N.S.	1	1.00	0.86	4.85	0.00	4.33	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.110	0.980	0.000	0.144	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	166	438	0	1291	0	0	-1
N.S.	1	1.00	0.84	2.22	0.00	6.55	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.455	1.147	0.000	0.103	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	101	218	0	174	0	0	-1
N.S.	1	1.00	0.79	1.70	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.260	1.011	0.000	0.109	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	0	0	126	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.258	2.312	0.000	0.359	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	82	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.158	2.254	0.000	0.413	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	66	0	0	49	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	1.02	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.055	2.240	0.000	0.338	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	39	26	23	15	17	21
N.S.	1	1.00	1.65	1.70	1.13	1.00	0.65	0.74	0.91
time (sec)	N/A	0.027	0.072	0.400	0.305	0.516	0.057	0.423	0.117

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	52	141	43	53	32	39
N.S.	1	1.00	0.74	1.21	3.28	1.00	1.23	0.74	0.91
time (sec)	N/A	0.030	0.023	0.510	0.285	0.390	0.119	0.396	0.607

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	91	267	70	82	46	52
N.S.	1	1.00	0.74	1.34	3.93	1.03	1.21	0.68	0.76
time (sec)	N/A	0.039	0.030	0.530	0.331	0.414	0.213	0.427	0.835

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	128	469	95	110	60	66
N.S.	1	1.00	0.74	1.41	5.15	1.04	1.21	0.66	0.73
time (sec)	N/A	0.048	0.039	0.557	0.287	0.374	0.373	0.410	1.039

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	59	39	27	24	15	18	21
N.S.	1	1.00	2.19	1.44	1.00	0.89	0.56	0.67	0.78
time (sec)	N/A	0.033	0.058	0.440	0.273	0.420	0.061	0.408	0.109

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	32	52	141	43	51	32	37
N.S.	1	1.00	0.65	1.06	2.88	0.88	1.04	0.65	0.76
time (sec)	N/A	0.030	0.021	0.530	0.319	0.427	0.119	0.414	0.595

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	92	91	267	70	82	46	52
N.S.	1	1.00	1.21	1.20	3.51	0.92	1.08	0.61	0.68
time (sec)	N/A	0.045	0.162	0.566	0.317	0.467	0.213	0.394	0.804

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	63	128	469	95	109	60	68
N.S.	1	1.00	0.62	1.27	4.64	0.94	1.08	0.59	0.67
time (sec)	N/A	0.053	0.038	0.607	0.299	0.402	0.374	0.406	1.004

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	85	0	0	188	0	0	-1
N.S.	1	1.00	1.29	0.00	0.00	2.85	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.090	1.851	0.000	0.413	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	105	0	0	264	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	3.34	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.179	1.853	0.000	0.457	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	184	0	0	347	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	3.15	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.161	1.809	0.000	0.389	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	241	1893	0	1139	0	0	-1
N.S.	1	1.00	0.93	7.31	0.00	4.40	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.606	1.189	0.000	0.147	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	196	1037	0	635	0	0	-1
N.S.	1	1.00	0.95	5.01	0.00	3.07	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.458	1.290	0.000	0.116	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	151	897	0	324	0	0	-1
N.S.	1	1.00	0.92	5.47	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.256	1.122	0.000	0.093	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	72	124	147	422	75	269
N.S.	1	1.00	1.11	1.31	2.25	2.67	7.67	1.36	4.89
time (sec)	N/A	0.051	0.075	0.358	0.503	0.451	48.919	0.428	0.930

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	113	229	444	0	119	223
N.S.	1	1.00	1.11	1.53	3.09	6.00	0.00	1.61	3.01
time (sec)	N/A	0.062	0.117	0.464	0.513	0.374	0.000	0.429	0.879

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	131	314	537	1614	0	279	-1
N.S.	1	1.00	1.02	2.45	4.20	12.61	0.00	2.18	-0.01
time (sec)	N/A	0.131	0.204	0.517	0.538	0.452	0.000	0.439	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	189	633	982	3870	0	477	-1
N.S.	1	1.00	1.01	3.39	5.25	20.70	0.00	2.55	-0.01
time (sec)	N/A	0.232	0.334	0.584	0.572	0.402	0.000	0.434	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	80	128	154	340	82	331
N.S.	1	1.00	1.10	1.33	2.13	2.57	5.67	1.37	5.52
time (sec)	N/A	0.062	0.051	0.410	0.522	0.409	44.709	0.420	1.202

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	128	6	3	6	6
N.S.	1	1.00	1.00	1.17	21.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.265	0.507	0.377	0.149	0.425	0.021

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	36	230	58	0	30	49
N.S.	1	1.00	1.00	3.00	19.17	4.83	0.00	2.50	4.08
time (sec)	N/A	0.023	0.026	0.419	0.528	0.360	0.000	0.429	0.571

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	37	34	42	51	33	48
N.S.	1	1.00	0.82	1.09	1.00	1.24	1.50	0.97	1.41
time (sec)	N/A	0.027	0.068	0.267	0.497	0.462	0.958	0.424	0.589

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	109	266	0	183	0	0	-1
N.S.	1	1.00	0.80	1.96	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.375	1.125	0.000	0.107	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	159	517	0	633	0	0	-1
N.S.	1	1.00	0.90	2.94	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.467	1.296	0.000	0.106	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	236	806	0	2167	0	0	-1
N.S.	1	1.00	0.94	3.21	0.00	8.63	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.606	1.444	0.000	0.116	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	53	511	0	120	-1
N.S.	1	1.00	0.68	0.60	1.00	9.64	0.00	2.26	-0.02
time (sec)	N/A	0.025	0.029	0.631	0.491	0.392	0.000	0.412	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	226	0	70	-1
N.S.	1	1.00	0.76	0.71	1.03	6.65	0.00	2.06	-0.03
time (sec)	N/A	0.018	0.027	0.586	0.533	0.434	0.000	0.422	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	71	15	34	21
N.S.	1	1.00	1.00	1.15	1.31	5.46	1.15	2.62	1.62
time (sec)	N/A	0.010	0.005	0.641	0.511	0.380	0.112	0.396	0.459

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	49	24	110	0	1	-1
N.S.	1	1.00	1.18	2.88	1.41	6.47	0.00	0.06	-0.06
time (sec)	N/A	0.011	0.005	0.866	0.501	0.494	0.000	0.408	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	71	62	327	0	37	-1
N.S.	1	1.00	1.05	1.69	1.48	7.79	0.00	0.88	-0.02
time (sec)	N/A	0.018	0.030	0.800	0.529	0.397	0.000	0.413	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	67	89	96	875	0	52	-1
N.S.	1	1.00	1.10	1.46	1.57	14.34	0.00	0.85	-0.02
time (sec)	N/A	0.025	0.070	0.798	0.557	0.385	0.000	0.419	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	67	0	0	823	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	6.10	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.136	0.806	0.000	0.133	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	317	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.051	1.058	0.000	0.093	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	60	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.97	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.069	1.000	0.000	0.126	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	0	0	97	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	1.62	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.024	0.790	0.000	0.084	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	53	0	0	639	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	7.34	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.062	0.777	0.000	0.112	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	69	0	0	1676	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	12.41	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.186	0.773	0.000	0.161	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	171	100	1597	0	114	-1
N.S.	1	1.00	0.40	1.30	0.76	12.10	0.00	0.86	-0.01
time (sec)	N/A	0.034	0.123	1.507	0.489	0.444	0.000	0.417	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	125	63	659	0	50	-1
N.S.	1	1.00	0.49	1.60	0.81	8.45	0.00	0.64	-0.01
time (sec)	N/A	0.021	0.073	1.428	0.525	0.415	0.000	0.421	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	84	27	180	0	26	-1
N.S.	1	1.00	0.67	2.33	0.75	5.00	0.00	0.72	-0.03
time (sec)	N/A	0.011	0.031	1.502	0.502	0.390	0.000	0.396	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	50	18	122	0	13	38
N.S.	1	1.00	1.00	3.12	1.12	7.62	0.00	0.81	2.38
time (sec)	N/A	0.009	0.006	1.286	0.513	0.366	0.000	0.447	0.466

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	74	171	1163	0	27	48
N.S.	1	1.00	0.50	1.09	2.51	17.10	0.00	0.40	0.71
time (sec)	N/A	0.016	0.031	1.230	0.517	0.397	0.000	0.406	0.528

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	90	467	3093	0	39	256
N.S.	1	1.00	0.40	0.76	3.96	26.21	0.00	0.33	2.17
time (sec)	N/A	0.023	0.048	1.189	0.493	0.480	0.000	0.427	0.542

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	219	186	90	91	124	86	93
N.S.	1	1.00	4.38	3.72	1.80	1.82	2.48	1.72	1.86
time (sec)	N/A	0.042	0.126	0.596	0.285	0.360	0.140	0.427	0.777

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	142	75	72	100	71	77
N.S.	1	1.00	0.98	3.30	1.74	1.67	2.33	1.65	1.79
time (sec)	N/A	0.032	0.039	0.488	0.288	0.402	0.123	0.441	0.670

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	131	138	66	67	82	62	67
N.S.	1	1.00	3.45	3.63	1.74	1.76	2.16	1.63	1.76
time (sec)	N/A	0.032	0.183	0.566	0.268	0.369	0.109	0.413	0.622

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	94	51	48	63	47	51
N.S.	1	1.00	0.85	2.85	1.55	1.45	1.91	1.42	1.55
time (sec)	N/A	0.027	0.016	0.461	0.268	0.431	0.091	0.413	0.559

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	93	90	42	41	48	38	41
N.S.	1	1.00	3.58	3.46	1.62	1.58	1.85	1.46	1.58
time (sec)	N/A	0.026	0.114	0.542	0.290	0.390	0.076	0.404	0.123

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	13	27	22	27	23	31
N.S.	1	1.00	0.80	0.87	1.80	1.47	1.80	1.53	2.07
time (sec)	N/A	0.021	0.009	0.441	0.278	0.384	0.060	0.412	0.475

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	40	14	17	14	14	7
N.S.	1	1.00	4.25	5.00	1.75	2.12	1.75	1.75	0.88
time (sec)	N/A	0.020	0.035	0.541	0.273	0.495	0.042	0.420	0.462

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	5	11	8	11	10
N.S.	1	1.00	1.00	1.00	0.71	1.57	1.14	1.57	1.43
time (sec)	N/A	0.013	0.006	0.306	0.276	0.367	0.035	0.410	0.466

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	18	43	41	52	0	51	46
N.S.	1	1.00	0.75	1.79	1.71	2.17	0.00	2.12	1.92
time (sec)	N/A	0.025	0.018	0.478	0.278	0.399	0.000	0.416	0.198

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	49	53	26	0	29	63
N.S.	1	1.00	0.88	1.96	2.12	1.04	0.00	1.16	2.52
time (sec)	N/A	0.028	0.025	0.503	0.268	0.360	0.000	0.411	0.581

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	61	91	92	143	0	92	115
N.S.	1	1.00	1.17	1.75	1.77	2.75	0.00	1.77	2.21
time (sec)	N/A	0.037	0.033	0.533	0.270	0.357	0.000	0.409	0.899

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	93	205	62	0	53	231
N.S.	1	1.00	0.95	2.51	5.54	1.68	0.00	1.43	6.24
time (sec)	N/A	0.033	0.051	0.534	0.273	0.422	0.000	0.425	1.012

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	94	137	140	245	0	118	249
N.S.	1	1.00	1.18	1.71	1.75	3.06	0.00	1.48	3.11
time (sec)	N/A	0.049	0.051	0.573	0.270	0.380	0.000	0.410	1.911

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	121	112	54	55	65	50	54
N.S.	1	1.00	3.02	2.80	1.35	1.38	1.62	1.25	1.35
time (sec)	N/A	0.051	0.135	0.561	0.315	0.493	0.100	0.422	0.143

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	70	39	34	44	35	37
N.S.	1	1.00	1.79	5.00	2.79	2.43	3.14	2.50	2.64
time (sec)	N/A	0.023	0.010	0.556	0.276	0.374	0.082	0.418	0.095

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	64	30	31	29	26	28
N.S.	1	1.00	1.53	2.13	1.00	1.03	0.97	0.87	0.93
time (sec)	N/A	0.045	0.058	0.675	0.275	0.390	0.068	0.411	0.477

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	53	23	26	26	21	24
N.S.	1	1.00	1.00	3.79	1.64	1.86	1.86	1.50	1.71
time (sec)	N/A	0.027	0.011	0.792	0.271	0.406	0.083	0.430	0.549

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	69	29	12	16	8	10	12
N.S.	1	1.00	4.93	2.07	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.021	0.042	0.825	0.281	0.384	0.043	0.419	0.648

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	8	16	19	10	12
N.S.	1	1.00	1.00	1.00	0.80	1.60	1.90	1.00	1.20
time (sec)	N/A	0.014	0.009	0.333	0.274	0.430	0.043	0.409	0.578

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	70	70	103	0	70	86
N.S.	1	1.00	0.76	2.06	2.06	3.03	0.00	2.06	2.53
time (sec)	N/A	0.027	0.033	0.672	0.307	0.429	0.000	0.415	0.754

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	70	117	44	0	41	109
N.S.	1	1.00	0.84	1.89	3.16	1.19	0.00	1.11	2.95
time (sec)	N/A	0.049	0.017	0.605	0.307	0.336	0.000	0.419	0.732

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	116	120	201	0	105	198
N.S.	1	1.00	1.13	1.93	2.00	3.35	0.00	1.75	3.30
time (sec)	N/A	0.039	0.035	0.754	0.285	0.462	0.000	0.409	1.196

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	116	317	80	0	65	139
N.S.	1	1.00	0.96	2.37	6.47	1.63	0.00	1.33	2.84
time (sec)	N/A	0.052	0.029	0.704	0.312	0.353	0.000	0.415	0.533

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	45	56	33	50	31	27	41
N.S.	1	1.00	1.61	2.00	1.18	1.79	1.11	0.96	1.46
time (sec)	N/A	0.028	0.064	0.922	0.290	0.393	0.086	0.404	0.186

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	40	36	53	28	34	16	19
N.S.	1	1.00	2.00	1.80	2.65	1.40	1.70	0.80	0.95
time (sec)	N/A	0.021	0.049	0.887	0.277	0.354	0.060	0.412	0.658

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	37	12	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.31	0.75	1.00
time (sec)	N/A	0.015	0.020	0.334	0.280	0.356	0.062	0.400	0.652

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	45	56	33	50	31	27	39
N.S.	1	1.00	1.73	2.15	1.27	1.92	1.19	1.04	1.50
time (sec)	N/A	0.026	0.062	0.916	0.277	0.379	0.083	0.401	0.157

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	36	53	28	34	16	19
N.S.	1	1.00	1.90	1.80	2.65	1.40	1.70	0.80	0.95
time (sec)	N/A	0.021	0.047	0.659	0.268	0.358	0.067	0.407	0.592

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	36	12	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.25	0.75	1.00
time (sec)	N/A	0.015	0.021	0.361	0.278	0.370	0.063	0.415	0.200

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	137	516	308	2105	0	254	287
N.S.	1	1.00	0.99	3.74	2.23	15.25	0.00	1.84	2.08
time (sec)	N/A	0.093	0.142	0.378	0.277	0.484	0.000	0.428	1.318

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	463	410	283	1486	0	288	302
N.S.	1	1.00	3.19	2.83	1.95	10.25	0.00	1.99	2.08
time (sec)	N/A	0.285	4.065	0.421	0.487	0.411	0.000	0.423	1.241

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	82	299	180	865	0	139	169
N.S.	1	1.00	1.01	3.69	2.22	10.68	0.00	1.72	2.09
time (sec)	N/A	0.062	0.091	0.360	0.270	0.543	0.000	0.435	0.875

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	553	220	170	569	0	168	200
N.S.	1	1.00	5.70	2.27	1.75	5.87	0.00	1.73	2.06
time (sec)	N/A	0.172	1.501	0.395	0.499	0.429	0.000	0.419	0.856

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	146	81	221	0	61	77
N.S.	1	1.00	1.00	3.84	2.13	5.82	0.00	1.61	2.03
time (sec)	N/A	0.042	0.045	0.346	0.281	0.482	0.000	0.424	0.617

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	396	100	81	171	377	83	87
N.S.	1	1.00	7.33	1.85	1.50	3.17	6.98	1.54	1.61
time (sec)	N/A	0.077	0.583	0.368	0.498	0.369	130.494	0.421	0.632

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	22	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	2.00	1.00
time (sec)	N/A	0.018	0.005	0.212	0.274	0.390	0.174	0.416	0.063

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	99	64	66	57	0	89	93
N.S.	1	1.00	2.06	1.33	1.38	1.19	0.00	1.85	1.94
time (sec)	N/A	0.043	0.068	0.442	0.505	0.438	0.000	0.415	1.352

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	71	89	259	0	87	321
N.S.	1	1.00	1.14	1.20	1.51	4.39	0.00	1.47	5.44
time (sec)	N/A	0.058	0.127	0.454	0.500	0.375	0.000	0.413	1.044

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	161	159	652	0	214	291
N.S.	1	1.00	0.89	1.85	1.83	7.49	0.00	2.46	3.34
time (sec)	N/A	0.092	0.101	0.566	0.561	0.488	0.000	0.414	2.237

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	182	230	1142	0	180	634
N.S.	1	1.00	1.02	1.82	2.30	11.42	0.00	1.80	6.34
time (sec)	N/A	0.158	0.253	0.505	0.536	0.391	0.000	0.421	1.607

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	313	345	2707	0	369	548
N.S.	1	1.00	1.00	2.32	2.56	20.05	0.00	2.73	4.06
time (sec)	N/A	0.135	0.166	0.698	0.547	0.516	0.000	0.426	4.894

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	350	438	3175	0	323	1010
N.S.	1	1.00	1.00	2.40	3.00	21.75	0.00	2.21	6.92
time (sec)	N/A	0.297	0.326	0.549	0.490	0.412	0.000	0.441	2.197

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	660	205	176	833	0	178	256
N.S.	1	1.00	7.02	2.18	1.87	8.86	0.00	1.89	2.72
time (sec)	N/A	0.156	3.262	0.631	0.485	0.436	0.000	0.411	0.880

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	119	102	370	133	82	60
N.S.	1	1.00	0.90	2.98	2.55	9.25	3.32	2.05	1.50
time (sec)	N/A	0.043	0.079	0.588	0.280	0.372	0.375	0.402	0.706

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	502	101	100	362	0	97	132
N.S.	1	1.00	8.10	1.63	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.077	1.350	0.569	0.473	0.391	0.000	0.403	0.671

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	51	19	22	14
N.S.	1	1.00	1.00	1.08	1.00	3.92	1.46	1.69	1.08
time (sec)	N/A	0.018	0.011	0.296	0.260	0.416	0.261	0.411	0.532

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	121	123	149	383	0	186	186
N.S.	1	1.00	1.53	1.56	1.89	4.85	0.00	2.35	2.35
time (sec)	N/A	0.073	0.325	0.635	0.490	0.387	0.000	0.412	1.795

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	138	215	802	0	167	302
N.S.	1	1.00	1.01	1.48	2.31	8.62	0.00	1.80	3.25
time (sec)	N/A	0.114	0.173	0.577	0.492	0.383	0.000	0.426	0.971

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	260	212	375	2615	0	295	519
N.S.	1	1.00	1.91	1.56	2.76	19.23	0.00	2.17	3.82
time (sec)	N/A	0.121	1.580	0.812	0.520	0.440	0.000	0.449	5.073

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	137	266	490	3044	0	287	476
N.S.	1	1.00	0.95	1.85	3.40	21.14	0.00	1.99	3.31
time (sec)	N/A	0.225	0.309	0.716	0.506	0.519	0.000	0.417	1.058

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	96	93	413	86	107	53	231
N.S.	1	1.00	3.10	3.00	13.32	2.77	3.45	1.71	7.45
time (sec)	N/A	0.052	0.103	0.701	0.287	0.364	0.102	0.411	1.172

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	79	95	151	99	92	113
N.S.	1	1.00	1.17	2.19	2.64	4.19	2.75	2.56	3.14
time (sec)	N/A	0.058	0.059	0.715	0.259	0.434	0.128	0.429	0.446

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	67	47	109	38	46	29	80
N.S.	1	1.00	2.91	2.04	4.74	1.65	2.00	1.26	3.48
time (sec)	N/A	0.050	0.044	0.586	0.261	0.367	0.061	0.396	0.672

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	45	42	55	32	53	29
N.S.	1	1.00	0.77	1.73	1.62	2.12	1.23	2.04	1.12
time (sec)	N/A	0.037	0.017	0.593	0.270	0.435	0.080	0.414	0.171

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	28	17	19	23	24
N.S.	1	1.00	1.00	0.89	1.47	0.89	1.00	1.21	1.26
time (sec)	N/A	0.021	0.008	0.540	0.269	0.368	0.089	0.411	0.150

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	32	23	27	37	22	24	28
N.S.	1	1.00	2.67	1.92	2.25	3.08	1.83	2.00	2.33
time (sec)	N/A	0.029	0.028	0.444	0.278	0.425	0.063	0.424	0.181

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	67	31	32	23	25
N.S.	1	1.00	1.00	2.27	4.47	2.07	2.13	1.53	1.67
time (sec)	N/A	0.042	0.009	0.609	0.276	0.413	0.056	0.422	0.574

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	100	59	61	90	61	44	74
N.S.	1	1.00	3.85	2.27	2.35	3.46	2.35	1.69	2.85
time (sec)	N/A	0.055	0.027	0.746	0.279	0.366	0.092	0.404	0.307

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	33	68	205	63	70	51	44
N.S.	1	1.00	1.43	2.96	8.91	2.74	3.04	2.22	1.91
time (sec)	N/A	0.056	0.009	0.691	0.259	0.439	0.092	0.411	0.614

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	164	93	91	144	100	62	124
N.S.	1	1.00	4.56	2.58	2.53	4.00	2.78	1.72	3.44
time (sec)	N/A	0.068	0.033	0.763	0.270	0.360	0.124	0.424	0.973

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	112	116	573	104	128	65	395
N.S.	1	1.00	2.38	2.47	12.19	2.21	2.72	1.38	8.40
time (sec)	N/A	0.092	0.113	0.822	0.270	0.398	0.129	0.415	4.070

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	52	114	115	197	129	102	209
N.S.	1	1.00	0.79	1.73	1.74	2.98	1.95	1.55	3.17
time (sec)	N/A	0.046	0.075	0.813	0.296	0.368	0.132	0.420	1.503

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	84	70	197	56	66	41	139
N.S.	1	1.00	2.27	1.89	5.32	1.51	1.78	1.11	3.76
time (sec)	N/A	0.077	0.069	0.771	0.301	0.377	0.086	0.416	0.296

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	66	61	94	58	66	99
N.S.	1	1.00	0.81	1.83	1.69	2.61	1.61	1.83	2.75
time (sec)	N/A	0.026	0.024	0.693	0.270	0.375	0.101	0.438	0.873

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	48	54	36	33	49
N.S.	1	1.00	1.00	0.92	1.92	2.16	1.44	1.32	1.96
time (sec)	N/A	0.027	0.021	0.578	0.274	0.402	0.088	0.411	0.304

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	35	53	78	49	48	60
N.S.	1	1.00	2.54	1.35	2.04	3.00	1.88	1.85	2.31
time (sec)	N/A	0.045	0.108	0.747	0.265	0.460	0.101	0.428	0.746

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	51	63	70	53	54	56
N.S.	1	1.00	1.00	1.76	2.17	2.41	1.83	1.86	1.93
time (sec)	N/A	0.031	0.013	0.974	0.288	0.419	0.115	0.409	0.260

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	107	58	67	100	66	50	111
N.S.	1	1.00	3.82	2.07	2.39	3.57	2.36	1.79	3.96
time (sec)	N/A	0.072	0.039	0.959	0.279	0.405	0.117	0.400	0.208

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	68	171	59	65	38	40
N.S.	1	1.00	1.00	2.52	6.33	2.19	2.41	1.41	1.48
time (sec)	N/A	0.027	0.010	0.960	0.270	0.380	0.097	0.415	0.606

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	175	74	103	160	114	74	246
N.S.	1	1.00	3.65	1.54	2.15	3.33	2.38	1.54	5.12
time (sec)	N/A	0.061	0.044	1.034	0.279	0.435	0.148	0.439	0.770

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	108	163	241	1199	0	197	654
N.S.	1	1.00	0.87	1.31	1.94	9.67	0.00	1.59	5.27
time (sec)	N/A	0.135	0.277	0.532	0.488	0.347	0.000	0.428	1.475

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	166	160	655	0	211	291
N.S.	1	1.00	1.74	1.89	1.82	7.44	0.00	2.40	3.31
time (sec)	N/A	0.136	0.142	0.579	0.501	0.454	0.000	0.416	2.278

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	84	89	257	0	87	330
N.S.	1	1.00	1.00	1.22	1.29	3.72	0.00	1.26	4.78
time (sec)	N/A	0.068	0.142	0.500	0.477	0.345	0.000	0.413	0.938

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	73	66	57	0	89	95
N.S.	1	1.00	0.75	1.52	1.38	1.19	0.00	1.85	1.98
time (sec)	N/A	0.050	0.042	0.513	0.470	0.397	0.000	0.420	1.404

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	46	40	0	39	195
N.S.	1	1.00	1.00	1.05	2.30	2.00	0.00	1.95	9.75
time (sec)	N/A	0.030	0.009	0.465	0.282	0.379	0.000	0.418	0.887

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	81	97	228	0	95	304
N.S.	1	1.00	1.46	1.45	1.73	4.07	0.00	1.70	5.43
time (sec)	N/A	0.166	0.134	0.487	0.474	0.432	0.000	0.406	0.835

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	104	116	427	0	125	1163
N.S.	1	1.00	0.87	2.00	2.23	8.21	0.00	2.40	22.37
time (sec)	N/A	0.067	0.043	0.450	0.257	0.411	0.000	0.431	1.236

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	176	169	212	1303	0	194	778
N.S.	1	1.00	1.63	1.56	1.96	12.06	0.00	1.80	7.20
time (sec)	N/A	0.279	0.314	0.585	0.479	0.454	0.000	0.412	1.414

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	144	262	523	3534	0	292	543
N.S.	1	1.00	0.64	1.17	2.33	15.78	0.00	1.30	2.42
time (sec)	N/A	0.328	0.302	0.681	0.511	0.391	0.000	0.441	1.297

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	150	221	375	2850	0	307	501
N.S.	1	1.00	1.11	1.64	2.78	21.11	0.00	2.27	3.71
time (sec)	N/A	0.250	0.522	0.750	0.498	0.464	0.000	0.441	3.890

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	100	142	223	900	0	181	377
N.S.	1	1.00	0.69	0.99	1.55	6.25	0.00	1.26	2.62
time (sec)	N/A	0.184	0.214	0.609	0.475	0.421	0.000	0.424	1.149

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	146	136	155	423	0	199	190
N.S.	1	1.00	1.72	1.60	1.82	4.98	0.00	2.34	2.24
time (sec)	N/A	0.093	0.178	0.665	0.483	0.457	0.000	0.408	1.755

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	27	33	75	158	0	75	240
N.S.	1	1.00	0.84	1.03	2.34	4.94	0.00	2.34	7.50
time (sec)	N/A	0.035	0.038	0.432	0.275	0.499	0.000	0.420	0.974

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	102	118	165	1257	0	148	897
N.S.	1	1.00	1.28	1.48	2.06	15.71	0.00	1.85	11.21
time (sec)	N/A	0.289	0.375	0.648	0.490	0.484	0.000	0.426	1.776

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	142	202	1463	0	190	1375
N.S.	1	1.00	0.96	1.87	2.66	19.25	0.00	2.50	18.09
time (sec)	N/A	0.077	0.173	0.562	0.287	0.455	0.000	0.449	1.633

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	214	221	339	3648	0	242	1450
N.S.	1	1.00	1.35	1.39	2.13	22.94	0.00	1.52	9.12
time (sec)	N/A	0.484	0.641	0.687	0.488	0.527	0.000	0.429	1.534

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	0	356	0	0	-1
N.S.	1	1.00	1.00	0.81	0.00	9.62	0.00	0.00	-0.03
time (sec)	N/A	0.042	0.017	0.712	0.000	0.578	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	370	0	0	-1
N.S.	1	1.00	1.00	0.79	0.00	15.42	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.011	0.802	0.000	0.463	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	92	68	170	745	87	198
N.S.	1	1.00	1.16	1.80	1.33	3.33	14.61	1.71	3.88
time (sec)	N/A	0.100	0.054	0.465	0.486	0.422	52.416	0.431	2.643

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	48	46	19	36	20	22	24
N.S.	1	1.00	1.92	1.84	0.76	1.44	0.80	0.88	0.96
time (sec)	N/A	0.059	0.053	0.590	0.264	0.408	0.076	0.407	0.135

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	81	44	20	35	20	21	23
N.S.	1	1.00	3.00	1.63	0.74	1.30	0.74	0.78	0.85
time (sec)	N/A	0.067	0.075	0.592	0.262	0.461	0.081	0.400	0.118

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	132	117	125	172	0	123	914
N.S.	1	1.00	1.48	1.31	1.40	1.93	0.00	1.38	10.27
time (sec)	N/A	0.146	0.276	0.716	0.489	1.351	0.000	0.424	8.673

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	76	106	183	0	102	164
N.S.	1	1.00	1.08	1.27	1.77	3.05	0.00	1.70	2.73
time (sec)	N/A	0.118	0.128	0.633	0.490	0.433	0.000	0.430	11.212

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	93	117	125	172	0	123	864
N.S.	1	1.00	1.04	1.31	1.40	1.93	0.00	1.38	9.71
time (sec)	N/A	0.180	0.186	0.641	0.483	2.111	0.000	0.439	10.644

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	58	141	172	0	90	539
N.S.	1	1.00	1.16	1.00	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.120	0.084	0.559	0.489	0.687	0.000	0.420	2.182

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	85	136	178	361	1318	127	656
N.S.	1	1.00	1.05	1.68	2.20	4.46	16.27	1.57	8.10
time (sec)	N/A	0.124	0.224	3.592	0.481	0.500	21.374	0.430	1.849

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	151	346	837	0	170	279
N.S.	1	1.00	1.00	1.34	3.06	7.41	0.00	1.50	2.47
time (sec)	N/A	0.139	0.381	5.997	0.508	0.427	0.000	0.430	1.196

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	170	416	743	2652	0	405	-1
N.S.	1	1.00	0.94	2.31	4.13	14.73	0.00	2.25	-0.01
time (sec)	N/A	0.204	0.483	6.006	0.516	0.422	0.000	0.454	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	235	844	1290	5966	0	685	-1
N.S.	1	1.00	0.94	3.38	5.16	23.86	0.00	2.74	-0.00
time (sec)	N/A	0.327	1.132	6.221	0.530	0.508	0.000	0.491	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	319	919	0	1655	0	0	-1
N.S.	1	1.00	0.73	2.09	0.00	3.77	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.567	0.862	0.000	0.416	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	240	710	0	1247	0	0	-1
N.S.	1	1.00	0.73	2.17	0.00	3.81	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.496	0.849	0.000	0.439	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	576	505	0	837	0	0	-1
N.S.	1	1.00	2.68	2.35	0.00	3.89	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.639	0.858	0.000	0.499	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	-1
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	-0.02
time (sec)	N/A	0.331	0.071	9.793	0.353	0.420	0.000	0.412	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.056	180.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.031	180.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	0.023	180.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	6.940	180.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	20.769	180.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	0	52	44	0	47	43
N.S.	1	1.00	0.76	0.00	0.96	0.81	0.00	0.87	0.80
time (sec)	N/A	0.009	0.045	0.063	0.271	0.491	0.000	0.415	0.646

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	0	67	91	0	169	53
N.S.	1	1.00	0.62	0.00	0.76	1.03	0.00	1.92	0.60
time (sec)	N/A	0.015	0.088	4.326	0.277	0.490	0.000	0.450	0.646

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	0	114	200	0	665	93
N.S.	1	1.00	0.81	0.00	0.77	1.34	0.00	4.46	0.62
time (sec)	N/A	0.030	0.402	2.837	0.309	0.449	0.000	0.468	0.720

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	129	294	0	777	102
N.S.	1	1.00	0.87	0.00	0.68	1.54	0.00	4.07	0.53
time (sec)	N/A	0.042	0.341	4.898	0.308	0.457	0.000	0.454	0.689

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	54	0	64	98	0	235	56
N.S.	1	1.01	0.74	0.00	0.88	1.34	0.00	3.22	0.77
time (sec)	N/A	0.021	0.100	0.069	0.281	0.366	0.000	0.432	0.704

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	89	0	87	248	0	758	74
N.S.	1	1.02	0.74	0.00	0.72	2.07	0.00	6.32	0.62
time (sec)	N/A	0.034	0.225	2.997	0.275	0.405	0.000	0.460	0.754

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	292	0	138	585	0	3225	118
N.S.	1	1.00	1.44	0.00	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.073	0.967	2.149	0.294	0.391	0.000	0.498	0.849

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	267	311	0	161	1125	0	6884	136
N.S.	1	1.00	1.17	0.00	0.61	4.23	0.00	25.88	0.51
time (sec)	N/A	0.098	2.402	4.731	0.291	0.488	0.000	0.535	0.867

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	37	40	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	2.06	2.22	1.00
time (sec)	N/A	0.014	0.012	1.438	0.256	0.394	0.263	0.405	0.658

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	49	40	0	81	32
N.S.	1	1.00	0.92	1.15	1.26	1.03	0.00	2.08	0.82
time (sec)	N/A	0.024	0.022	1.866	0.257	0.503	0.000	0.415	0.693

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	0	86	65	76	81	37
N.S.	1	1.00	1.05	0.00	2.00	1.51	1.77	1.88	0.86
time (sec)	N/A	0.024	0.015	180.000	0.257	0.388	1.783	0.424	0.724

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	0	93	84	0	114	51
N.S.	1	1.00	0.70	0.00	1.27	1.15	0.00	1.56	0.70
time (sec)	N/A	0.033	0.036	180.000	0.265	0.461	0.000	0.442	0.785

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	0	130	130	110	115	49
N.S.	1	1.00	1.05	0.00	2.00	2.00	1.69	1.77	0.75
time (sec)	N/A	0.040	0.017	180.000	0.264	0.393	10.682	0.432	0.839

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	96	227	0	331	0	0	-1
N.S.	1	1.00	0.86	2.05	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.068	12.600	0.000	0.173	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	114	143	0	171	0	0	-1
N.S.	1	1.00	1.03	1.29	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.104	7.128	0.000	0.087	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	58	0	0	-1
N.S.	1	1.00	0.94	2.03	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.032	6.961	0.000	0.134	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	39	0	0	-1
N.S.	1	1.00	0.92	1.67	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.035	6.570	0.000	0.091	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	246	0	0	-1
N.S.	1	1.00	0.75	1.98	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.049	8.804	0.000	0.103	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	122	144	0	504	0	0	-1
N.S.	1	1.00	1.10	1.30	0.00	4.54	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.094	7.406	0.000	0.121	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	86	0	0	162	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.209	2.213	0.000	0.496	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	74	0	0	117	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.182	2.296	0.000	0.511	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	0	0	68	0	41	-1
N.S.	1	1.00	1.42	0.00	0.00	1.58	0.00	0.95	-0.02
time (sec)	N/A	0.037	0.086	2.251	0.000	0.454	0.000	0.604	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	128	0	81	-1
N.S.	1	1.00	1.17	0.00	0.00	1.24	0.00	0.79	-0.01
time (sec)	N/A	0.058	0.153	2.342	0.000	0.443	0.000	0.640	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	48	0	102	-1
N.S.	1	1.00	1.00	1.06	0.00	1.33	0.00	2.83	-0.03
time (sec)	N/A	0.033	0.016	0.889	0.000	0.438	0.000	0.419	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	50	0	73	0	97	-1
N.S.	1	1.00	0.95	1.28	0.00	1.87	0.00	2.49	-0.03
time (sec)	N/A	0.045	0.030	1.336	0.000	0.430	0.000	0.435	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	74	0	118	0	167	-1
N.S.	1	1.00	0.92	1.25	0.00	2.00	0.00	2.83	-0.02
time (sec)	N/A	0.061	0.052	1.295	0.000	0.501	0.000	0.441	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	113	0	253	0	0	-1
N.S.	1	1.00	0.95	1.53	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.226	1.135	0.000	0.530	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	85	120	0	277	0	0	-1
N.S.	1	1.00	1.06	1.50	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.287	18.348	0.000	0.524	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	172	228	0	701	0	0	-1
N.S.	1	1.00	1.20	1.59	0.00	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.347	8.986	0.000	0.473	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	373	347	0	171	0	764	-1
N.S.	1	1.00	3.69	3.44	0.00	1.69	0.00	7.56	-0.01
time (sec)	N/A	0.136	0.512	1.192	0.000	0.458	0.000	1.822	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	358	0	370	0	749	-1
N.S.	1	1.00	1.05	3.35	0.00	3.46	0.00	7.00	-0.01
time (sec)	N/A	0.148	0.606	22.786	0.000	0.431	0.000	7.221	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	599	700	0	717	0	1383	-1
N.S.	1	1.00	3.09	3.61	0.00	3.70	0.00	7.13	-0.01
time (sec)	N/A	0.247	0.927	8.939	0.000	0.487	0.000	8.618	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	449	459	0	220	0	1624	-1
N.S.	1	1.00	3.71	3.79	0.00	1.82	0.00	13.42	-0.01
time (sec)	N/A	0.204	1.030	1.445	0.000	0.488	0.000	4.565	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	136	468	0	548	0	1596	-1
N.S.	1	1.00	1.05	3.63	0.00	4.25	0.00	12.37	-0.01
time (sec)	N/A	0.224	1.455	31.032	0.000	0.514	0.000	20.373	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	671	922	0	1092	0	3021	-1
N.S.	1	1.00	2.97	4.08	0.00	4.83	0.00	13.37	-0.00
time (sec)	N/A	0.344	4.213	21.255	0.000	0.519	0.000	25.073	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	80	68	113	139	60	58
N.S.	1	1.00	0.75	0.96	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.028	0.034	0.913	0.257	0.464	3.248	0.426	0.532

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	61	53	95	207	57	42
N.S.	1	1.00	0.79	1.07	0.93	1.67	3.63	1.00	0.74
time (sec)	N/A	0.026	0.025	0.727	0.260	0.533	1.235	0.431	0.799

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	52	40	54	78	34	36
N.S.	1	1.00	0.80	1.06	0.82	1.10	1.59	0.69	0.73
time (sec)	N/A	0.020	0.017	0.523	0.265	0.506	0.485	0.404	0.615

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	24	50	63	24	18
N.S.	1	1.00	1.00	1.61	1.04	2.17	2.74	1.04	0.78
time (sec)	N/A	0.011	0.010	0.342	0.266	0.403	0.214	0.417	0.069

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	27	30	0	24	16
N.S.	1	1.00	1.00	0.89	1.42	1.58	0.00	1.26	0.84
time (sec)	N/A	0.013	0.010	0.398	0.264	0.479	0.000	0.411	0.070

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	25	52	157	0	48	52
N.S.	1	1.00	0.90	0.60	1.24	3.74	0.00	1.14	1.24
time (sec)	N/A	0.022	0.052	0.346	0.261	0.513	0.000	0.416	0.617

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	32	68	88	0	31	31
N.S.	1	1.00	0.94	1.03	2.19	2.84	0.00	1.00	1.00
time (sec)	N/A	0.020	0.015	1.126	0.274	0.446	0.000	0.429	0.582

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	78	100	705	0	75	135
N.S.	1	1.00	0.74	0.77	0.99	6.98	0.00	0.74	1.34
time (sec)	N/A	0.036	0.050	0.963	0.268	0.572	0.000	0.421	0.602

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	43	172	233	0	42	42
N.S.	1	1.00	0.67	0.65	2.61	3.53	0.00	0.64	0.64
time (sec)	N/A	0.039	0.027	0.987	0.256	0.390	0.000	0.414	0.601

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	47	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.013	0.011	0.508	0.268	0.555	0.191	0.411	0.085

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	22	13	26	20	13	12
N.S.	1	1.00	0.84	1.16	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.008	0.008	0.247	0.264	0.432	0.095	0.400	0.562

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	34	18	25	0	19	26
N.S.	1	1.00	1.00	3.09	1.64	2.27	0.00	1.73	2.36
time (sec)	N/A	0.009	0.008	0.579	0.485	0.394	0.000	0.398	0.170

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	46	32	182	0	33	36
N.S.	1	1.00	1.00	1.44	1.00	5.69	0.00	1.03	1.12
time (sec)	N/A	0.016	0.029	0.654	0.466	0.508	0.000	0.414	0.715

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	67	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.014	0.014	0.429	0.267	0.511	0.186	0.415	0.580

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	26	13	40	20	13	12
N.S.	1	1.00	0.84	1.37	0.68	2.11	1.05	0.68	0.63
time (sec)	N/A	0.008	0.008	0.499	0.333	0.378	0.092	0.403	0.053

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	22	79	73	83	0	43	65
N.S.	1	1.00	0.41	1.46	1.35	1.54	0.00	0.80	1.20
time (sec)	N/A	0.039	0.010	0.566	0.468	0.488	0.000	0.414	0.178

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	34	148	85	560	0	86	91
N.S.	1	1.00	0.32	1.41	0.81	5.33	0.00	0.82	0.87
time (sec)	N/A	0.099	0.022	0.665	0.493	0.478	0.000	0.422	0.401

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	34	17	87	42	17	17
N.S.	1	1.00	1.00	1.31	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.013	0.016	0.418	0.282	0.426	0.173	0.405	0.085

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	26	13	46	20	13	13
N.S.	1	1.00	1.00	1.37	0.68	2.42	1.05	0.68	0.68
time (sec)	N/A	0.008	0.010	0.419	0.257	0.447	0.096	0.421	0.055

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	22	56	95	132	0	96	106
N.S.	1	1.00	0.19	0.50	0.84	1.17	0.00	0.85	0.94
time (sec)	N/A	0.052	0.013	0.668	0.482	0.463	0.000	0.443	0.371

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	34	68	107	207	0	108	120
N.S.	1	1.00	0.26	0.52	0.82	1.58	0.00	0.82	0.92
time (sec)	N/A	0.064	0.020	0.770	0.482	0.399	0.000	0.428	0.921

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	157	326	142	4705	0	1211	166
N.S.	1	1.00	0.78	1.61	0.70	23.29	0.00	6.00	0.82
time (sec)	N/A	0.059	0.447	1.177	0.274	0.396	0.000	0.467	1.565

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	143	98	1115	1052	890	97
N.S.	1	1.00	0.65	1.08	0.74	8.45	7.97	6.74	0.73
time (sec)	N/A	0.038	0.124	1.080	0.277	0.389	14.038	0.459	0.916

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	77	67	376	416	598	73
N.S.	1	1.00	0.67	1.03	0.89	5.01	5.55	7.97	0.97
time (sec)	N/A	0.014	0.073	0.176	0.272	0.392	2.473	0.429	0.657

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	3.434	0.226	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	87	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	1.793	0.237	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	299	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	12.953	0.410	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	202	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	5.206	0.322	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	106	184	90	218	0	269	-1
N.S.	1	1.00	0.42	0.74	0.36	0.87	0.00	1.08	-0.00
time (sec)	N/A	0.185	0.080	3.368	0.489	0.406	0.000	0.450	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	76	128	62	126	0	195	-1
N.S.	1	1.00	0.47	0.79	0.38	0.78	0.00	1.20	-0.01
time (sec)	N/A	0.103	0.042	1.954	0.492	0.377	0.000	0.428	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	48	89	36	66	275	71	77
N.S.	1	1.00	0.65	1.20	0.49	0.89	3.72	0.96	1.04
time (sec)	N/A	0.077	0.029	1.877	0.495	0.346	2.825	0.426	0.632

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	68	39	42	0	85	-1
N.S.	1	1.00	0.96	1.48	0.85	0.91	0.00	1.85	-0.02
time (sec)	N/A	0.091	0.039	9.832	0.487	0.377	0.000	0.427	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	69	84	121	0	87	76
N.S.	1	1.00	0.79	1.19	1.45	2.09	0.00	1.50	1.31
time (sec)	N/A	0.105	0.054	9.776	0.476	0.349	0.000	0.427	0.588

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	72	80	209	315	0	122	89
N.S.	1	1.00	0.49	0.54	1.42	2.14	0.00	0.83	0.61
time (sec)	N/A	0.149	0.049	12.625	0.496	0.359	0.000	0.446	0.616

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	84	91	386	592	0	161	353
N.S.	1	1.00	0.42	0.46	1.94	2.97	0.00	0.81	1.77
time (sec)	N/A	0.196	0.055	10.586	0.490	0.372	0.000	0.449	0.622

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	62	0	42	99	32	45
N.S.	1	1.00	0.68	1.51	0.00	1.02	2.41	0.78	1.10
time (sec)	N/A	0.010	0.033	0.510	0.000	0.365	0.204	0.405	0.095

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	65	103	0	73	-1
N.S.	1	1.00	0.94	0.85	0.76	1.21	0.00	0.86	-0.01
time (sec)	N/A	0.056	0.064	1.198	0.267	0.363	0.000	0.414	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	97	81	129	0	91	-1
N.S.	1	1.00	0.91	0.96	0.80	1.28	0.00	0.90	-0.01
time (sec)	N/A	0.106	0.120	3.780	0.266	0.377	0.000	0.416	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	45	0	45	-1
N.S.	1	1.00	0.78	0.80	0.69	0.69	0.00	0.69	-0.02
time (sec)	N/A	0.049	0.047	1.151	0.270	0.390	0.000	0.413	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	48	47	75	0	49	-1
N.S.	1	1.00	1.11	0.74	0.72	1.15	0.00	0.75	-0.02
time (sec)	N/A	0.062	0.081	6.681	0.283	0.352	0.000	0.418	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	105	89	164	0	101	-1
N.S.	1	1.00	1.07	0.91	0.77	1.43	0.00	0.88	-0.01
time (sec)	N/A	0.126	0.287	6.529	0.264	0.409	0.000	0.410	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	103	100	90	213	0	106	-1
N.S.	1	1.00	0.94	0.91	0.82	1.94	0.00	0.96	-0.01
time (sec)	N/A	0.114	0.102	0.777	0.304	0.369	0.000	0.413	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	356	-1
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.41	-0.01
time (sec)	N/A	0.142	0.545	2.939	0.478	0.369	0.000	0.425	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	287	207	200	445	0	223	-1
N.S.	1	1.00	1.20	0.87	0.84	1.86	0.00	0.93	-0.00
time (sec)	N/A	0.215	0.300	3.410	0.494	0.371	0.000	0.420	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	124	126	106	337	0	132	-1
N.S.	1	1.00	1.08	1.10	0.92	2.93	0.00	1.15	-0.01
time (sec)	N/A	0.165	0.223	0.765	0.269	0.424	0.000	0.440	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	147	434	0	388	-1
N.S.	1	1.00	1.37	0.98	0.91	2.70	0.00	2.41	-0.01
time (sec)	N/A	0.209	0.448	3.007	0.483	0.382	0.000	0.432	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	354	265	236	725	0	281	-1
N.S.	1	1.00	1.38	1.03	0.92	2.82	0.00	1.09	-0.00
time (sec)	N/A	0.361	0.541	3.314	0.512	0.408	0.000	0.449	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	277	0	132	-1
N.S.	1	1.00	0.78	0.88	0.79	2.08	0.00	0.99	-0.01
time (sec)	N/A	0.143	0.119	0.814	0.270	0.359	0.000	0.421	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	304	0	150	-1
N.S.	1	1.00	0.81	0.86	0.81	1.89	0.00	0.93	-0.01
time (sec)	N/A	0.168	0.193	1.312	0.281	0.350	0.000	0.417	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	549	0	264	-1
N.S.	1	1.00	0.79	0.86	0.78	2.03	0.00	0.97	-0.00
time (sec)	N/A	0.256	0.334	1.699	0.285	0.373	0.000	0.403	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	70	69	146	0	75	-1
N.S.	1	1.00	0.94	0.86	0.85	1.80	0.00	0.93	-0.01
time (sec)	N/A	0.136	0.246	0.735	0.273	0.385	0.000	0.412	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	-1
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	-0.01
time (sec)	N/A	0.164	0.378	1.432	0.276	0.353	0.000	0.437	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	272	144	143	492	0	155	-1
N.S.	1	1.00	1.59	0.84	0.84	2.88	0.00	0.91	-0.01
time (sec)	N/A	0.238	0.843	1.823	0.270	0.373	0.000	0.428	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	166	147	127	382	0	172	-1
N.S.	1	1.00	1.19	1.05	0.91	2.73	0.00	1.23	-0.01
time (sec)	N/A	0.274	0.516	0.814	0.283	0.379	0.000	0.425	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	482	0	198	-1
N.S.	1	1.00	1.41	0.97	0.88	2.63	0.00	1.08	-0.01
time (sec)	N/A	0.245	1.016	1.563	0.283	0.411	0.000	0.439	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	480	302	263	970	0	352	-1
N.S.	1	1.00	1.60	1.01	0.88	3.23	0.00	1.17	-0.00
time (sec)	N/A	0.482	4.024	1.983	0.285	0.388	0.000	0.451	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	135	156	133	339	0	167	-1
N.S.	1	1.00	0.88	1.02	0.87	2.22	0.00	1.09	-0.01
time (sec)	N/A	0.220	0.209	0.875	0.283	0.385	0.000	0.418	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	211	189	429	0	223	-1
N.S.	1	1.00	0.84	0.96	0.86	1.96	0.00	1.02	-0.00
time (sec)	N/A	0.285	0.391	1.458	0.278	0.383	0.000	0.430	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	263	316	271	689	0	339	-1
N.S.	1	1.00	0.83	1.00	0.86	2.19	0.00	1.08	-0.00
time (sec)	N/A	0.369	0.673	1.846	0.279	0.384	0.000	0.444	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	179	160	139	325	0	181	-1
N.S.	1	1.00	1.16	1.04	0.90	2.11	0.00	1.18	-0.01
time (sec)	N/A	0.265	0.586	0.873	0.265	0.396	0.000	0.435	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	-1
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	-0.00
time (sec)	N/A	0.286	1.589	1.491	0.282	0.375	0.000	0.424	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	503	326	287	852	0	369	-1
N.S.	1	1.00	1.56	1.01	0.89	2.64	0.00	1.14	-0.00
time (sec)	N/A	0.428	4.739	3.148	0.296	0.386	0.000	0.454	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	252	186	155	437	0	207	-1
N.S.	1	1.00	1.57	1.16	0.96	2.71	0.00	1.29	-0.01
time (sec)	N/A	0.366	1.065	0.922	0.273	0.398	0.000	0.412	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	219	602	0	271	-1
N.S.	1	1.00	1.42	1.04	0.92	2.52	0.00	1.13	-0.00
time (sec)	N/A	0.408	4.080	1.763	0.281	0.402	0.000	0.465	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	323	1100	0	427	-1
N.S.	1	1.00	8.69	1.12	0.94	3.20	0.00	1.24	-0.00
time (sec)	N/A	0.649	6.428	2.084	0.295	0.471	0.000	0.449	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	22	41	35	24
N.S.	1	1.00	1.00	0.83	1.17	0.73	1.37	1.17	0.80
time (sec)	N/A	0.026	0.031	0.293	0.268	0.398	0.061	0.411	0.598

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	73	81	58	85	75	46
N.S.	1	1.00	0.86	1.30	1.45	1.04	1.52	1.34	0.82
time (sec)	N/A	0.051	0.046	0.424	0.272	0.370	0.093	0.407	0.082

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	316	0	0	-1
N.S.	1	1.00	0.85	1.00	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.222	0.500	0.000	0.373	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	248	376	0	1346	0	0	-1
N.S.	1	1.00	0.92	1.39	0.00	4.97	0.00	0.00	-0.00
time (sec)	N/A	0.608	0.249	0.511	0.000	0.414	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [261] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	4	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	3	3	1.00	10	0.300
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	3	3	1.00	10	0.300
13	A	3	3	1.00	10	0.300
14	A	4	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	3	2	1.00	14	0.143
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	1	1	1.00	14	0.071
27	A	1	1	1.00	14	0.071
28	A	2	2	1.00	14	0.143
29	A	2	2	1.00	14	0.143
30	A	3	2	1.00	14	0.143
31	A	1	1	1.00	12	0.083
32	A	1	1	1.00	12	0.083
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	A	1	1	1.00	12	0.083
36	A	1	1	1.00	12	0.083
37	A	1	1	1.00	10	0.100
38	A	1	1	1.00	12	0.083
39	A	1	1	1.00	12	0.083
40	A	6	5	1.00	13	0.385
41	A	2	2	1.00	13	0.154
42	A	3	3	1.00	13	0.231
43	A	2	2	1.00	11	0.182
44	A	3	3	1.00	11	0.273
45	A	5	5	1.00	13	0.385
46	A	6	6	1.00	13	0.462
47	A	6	5	1.00	13	0.385
48	A	3	3	1.00	13	0.231
49	A	6	6	1.00	13	0.462
50	A	3	3	1.00	13	0.231
51	A	2	2	1.00	11	0.182
52	A	4	4	1.00	11	0.364
53	A	6	6	1.00	13	0.462
54	A	7	7	1.00	13	0.538
55	A	7	6	1.00	13	0.462
56	A	1	1	1.00	14	0.071
57	A	2	2	1.00	14	0.143
58	A	3	2	1.00	14	0.143
59	A	4	2	1.00	14	0.143
60	A	1	1	1.00	14	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	14	0.143
62	A	3	2	1.00	14	0.143
63	A	4	2	1.00	14	0.143
64	A	3	3	1.00	16	0.188
65	A	3	3	1.00	16	0.188
66	A	3	2	1.00	17	0.118
67	A	2	2	1.00	17	0.118
68	A	1	1	1.00	17	0.059
69	A	2	2	1.00	17	0.118
70	A	3	3	1.00	17	0.176
71	A	4	3	1.00	17	0.176
72	A	7	7	1.00	13	0.538
73	A	6	6	1.00	13	0.462
74	A	6	6	1.00	13	0.462
75	A	4	4	1.00	11	0.364
76	A	5	5	1.00	11	0.454
77	A	7	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	8	7	1.00	13	0.538
80	A	7	7	1.00	13	0.538
81	A	6	6	1.00	13	0.462
82	A	5	5	1.00	13	0.385
83	A	5	5	1.00	11	0.454
84	A	6	6	1.00	11	0.546
85	A	7	7	1.00	13	0.538
86	A	8	7	1.00	13	0.538
87	A	9	7	1.00	13	0.538
88	A	4	3	1.00	14	0.214
89	A	6	5	1.00	14	0.357
90	A	7	6	1.00	14	0.429
91	A	8	6	1.00	14	0.429
92	A	1	1	1.00	14	0.071
93	A	3	3	1.00	14	0.214
94	A	4	4	1.00	14	0.286
95	A	5	4	1.00	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	3	1.00	12	0.250
97	A	3	3	1.00	12	0.250
98	A	2	2	1.00	12	0.167
99	A	1	1	1.00	12	0.083
100	A	2	1	1.00	10	0.100
101	A	3	3	1.00	12	0.250
102	A	5	5	1.00	12	0.417
103	A	6	6	1.00	12	0.500
104	A	7	6	1.00	12	0.500
105	A	7	7	1.00	10	0.700
106	A	6	6	1.00	10	0.600
107	A	2	2	1.00	10	0.200
108	A	2	2	1.00	10	0.200
109	A	4	4	1.00	10	0.400
110	A	7	7	1.00	10	0.700
111	A	5	5	1.00	13	0.385
112	A	4	3	1.00	20	0.150
113	A	3	3	1.00	20	0.150
114	A	2	2	1.00	20	0.100
115	A	2	2	1.00	15	0.133
116	A	2	2	1.00	15	0.133
117	A	3	3	1.00	15	0.200
118	A	4	3	1.00	15	0.200
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	17	0.118
121	A	3	3	1.00	17	0.176
122	A	4	3	1.00	17	0.176
123	A	3	3	1.00	20	0.150
124	A	3	3	1.00	20	0.150
125	A	4	4	1.00	20	0.200
126	A	8	6	1.00	17	0.353
127	A	7	6	1.00	17	0.353
128	A	6	6	1.00	17	0.353
129	A	4	4	1.00	15	0.267
130	A	5	5	1.00	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	5	1.00	15	0.333
132	A	7	5	1.00	15	0.333
133	A	4	4	1.00	20	0.200
134	A	2	2	1.00	20	0.100
135	A	2	2	1.00	16	0.125
136	A	2	2	1.00	13	0.154
137	A	5	5	1.00	17	0.294
138	A	6	6	1.00	17	0.353
139	A	7	6	1.00	17	0.353
140	A	4	3	1.00	10	0.300
141	A	3	3	1.00	10	0.300
142	A	2	2	1.00	10	0.200
143	A	2	2	1.00	10	0.200
144	A	3	3	1.00	10	0.300
145	A	4	3	1.00	10	0.300
146	A	7	4	1.00	10	0.400
147	A	5	4	1.00	10	0.400
148	A	4	4	1.00	10	0.400
149	A	4	4	1.00	10	0.400
150	A	5	4	1.00	10	0.400
151	A	7	4	1.00	10	0.400
152	A	7	3	1.00	10	0.300
153	A	5	3	1.00	10	0.300
154	A	3	3	1.00	10	0.300
155	A	3	3	1.00	10	0.300
156	A	3	2	1.00	10	0.200
157	A	3	2	1.00	10	0.200
158	A	5	3	1.00	13	0.231
159	A	3	2	1.00	13	0.154
160	A	4	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	3	3	1.00	13	0.231
163	A	2	1	1.00	13	0.077
164	A	2	2	1.00	13	0.154
165	A	2	2	1.00	11	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	11	0.273
167	A	3	3	1.00	13	0.231
168	A	4	3	1.00	13	0.231
169	A	3	2	1.00	13	0.154
170	A	4	3	1.00	13	0.231
171	A	4	4	1.00	13	0.308
172	A	2	2	1.00	13	0.154
173	A	3	3	1.00	13	0.231
174	A	3	2	1.00	13	0.154
175	A	2	2	1.00	13	0.154
176	A	2	2	1.00	11	0.182
177	A	4	3	1.00	11	0.273
178	A	4	3	1.00	13	0.231
179	A	4	3	1.00	13	0.231
180	A	4	2	1.00	13	0.154
181	A	3	2	1.00	15	0.133
182	A	1	1	1.00	15	0.067
183	A	2	2	1.00	13	0.154
184	A	3	2	1.00	15	0.133
185	A	1	1	1.00	15	0.067
186	A	2	2	1.00	13	0.154
187	A	3	2	1.00	13	0.154
188	A	7	6	1.00	13	0.462
189	A	3	2	1.00	13	0.154
190	A	6	6	1.00	13	0.462
191	A	3	2	1.00	13	0.154
192	A	5	5	1.00	13	0.385
193	A	2	2	1.00	11	0.182
194	A	6	6	1.00	11	0.546
195	A	5	5	1.00	13	0.385
196	A	7	6	1.00	13	0.462
197	A	6	6	1.00	13	0.462
198	A	8	7	1.00	13	0.538
199	A	7	6	1.00	13	0.462
200	A	6	6	1.00	13	0.462

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	2	1.00	13	0.154
202	A	5	5	1.00	13	0.385
203	A	2	2	1.00	11	0.182
204	A	7	6	1.00	11	0.546
205	A	6	6	1.00	13	0.462
206	A	7	6	1.00	13	0.462
207	A	7	6	1.00	13	0.462
208	A	6	5	1.00	13	0.385
209	A	6	5	1.00	13	0.385
210	A	5	4	1.00	13	0.308
211	A	5	5	1.00	11	0.454
212	A	4	4	1.00	11	0.364
213	A	4	4	1.00	13	0.308
214	A	5	4	1.00	13	0.308
215	A	5	5	1.00	13	0.385
216	A	5	4	1.00	13	0.308
217	A	6	5	1.00	13	0.385
218	A	10	6	1.00	13	0.462
219	A	4	3	1.00	13	0.231
220	A	10	5	1.00	13	0.385
221	A	4	3	1.00	11	0.273
222	A	3	2	1.00	11	0.182
223	A	7	5	1.00	13	0.385
224	A	3	2	1.00	13	0.154
225	A	9	6	1.00	13	0.462
226	A	3	2	1.00	13	0.154
227	A	11	5	1.00	13	0.385
228	A	13	9	1.00	13	0.692
229	A	7	6	1.00	13	0.462
230	A	8	7	1.00	13	0.538
231	A	6	5	1.00	11	0.454
232	A	4	4	1.00	11	0.364
233	A	7	7	1.00	13	0.538
234	A	3	2	1.00	13	0.154
235	A	7	7	1.00	13	0.538

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	16	9	1.00	13	0.692
237	A	7	6	1.00	13	0.462
238	A	13	9	1.00	13	0.692
239	A	6	5	1.00	11	0.454
240	A	3	2	1.00	11	0.182
241	A	8	7	1.00	13	0.538
242	A	3	2	1.00	13	0.154
243	A	8	7	1.00	13	0.538
244	A	4	4	1.00	13	0.308
245	A	3	3	1.00	13	0.231
246	A	7	6	1.00	15	0.400
247	A	5	4	1.00	15	0.267
248	A	5	4	1.00	17	0.235
249	A	11	9	1.00	15	0.600
250	A	9	8	1.00	15	0.533
251	A	12	11	1.00	15	0.733
252	A	6	6	1.00	15	0.400
253	A	7	7	1.00	31	0.226
254	A	8	8	1.00	31	0.258
255	A	9	8	1.00	31	0.258
256	A	10	8	1.00	31	0.258
257	A	13	8	1.00	14	0.571
258	A	11	7	1.00	14	0.500
259	A	9	6	1.00	12	0.500
260	A	13	5	1.00	20	0.250
261	A	5	3	1.00	36	0.083
262	A	4	3	1.00	36	0.083
263	A	2	2	1.00	34	0.059
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	1	1	1.00	11	0.091
267	A	2	2	1.00	13	0.154
268	A	2	2	1.00	13	0.154
269	A	3	2	1.00	13	0.154
270	A	1	1	1.01	15	0.067

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	2	1.02	17	0.118
272	A	2	2	1.00	17	0.118
273	A	3	2	1.00	17	0.118
274	A	2	1	1.00	15	0.067
275	A	3	2	1.00	17	0.118
276	A	3	1	1.00	17	0.059
277	A	4	2	1.00	17	0.118
278	A	3	1	1.00	17	0.059
279	A	4	3	1.00	19	0.158
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	19	0.105
282	A	3	2	1.00	19	0.105
283	A	4	3	1.00	19	0.158
284	A	4	3	1.00	19	0.158
285	A	8	8	1.00	18	0.444
286	A	6	6	1.00	18	0.333
287	A	3	3	1.00	18	0.167
288	A	4	4	1.00	18	0.222
289	A	4	4	1.00	10	0.400
290	A	5	5	1.00	12	0.417
291	A	6	4	1.00	12	0.333
292	A	5	5	1.00	11	0.454
293	A	6	6	1.00	13	0.462
294	A	9	5	1.00	13	0.385
295	A	5	5	1.00	14	0.357
296	A	6	6	1.00	16	0.375
297	A	9	5	1.00	16	0.312
298	A	6	6	1.00	17	0.353
299	A	7	7	1.00	19	0.368
300	A	10	6	1.00	19	0.316
301	A	4	3	1.00	16	0.188
302	A	5	4	1.00	16	0.250
303	A	4	3	1.00	16	0.188
304	A	4	3	1.00	14	0.214
305	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	4	4	1.00	16	0.250
307	A	3	3	1.00	16	0.188
308	A	6	5	1.00	16	0.312
309	A	5	4	1.00	16	0.250
310	A	4	3	1.00	10	0.300
311	A	4	3	1.00	8	0.375
312	A	5	5	1.00	8	0.625
313	A	6	6	1.00	10	0.600
314	A	4	3	1.00	10	0.300
315	A	4	3	1.00	8	0.375
316	A	9	9	1.00	8	1.125
317	A	13	9	1.00	10	0.900
318	A	4	3	1.00	10	0.300
319	A	4	3	1.00	8	0.375
320	A	15	12	1.00	8	1.500
321	A	16	13	1.00	10	1.300
322	A	2	2	1.00	18	0.111
323	A	2	2	1.00	18	0.111
324	A	1	1	1.00	16	0.062
325	A	1	1	1.00	16	0.062
326	A	1	1	1.00	18	0.056
327	A	2	2	1.00	18	0.111
328	A	2	2	1.00	18	0.111
329	A	6	5	1.00	25	0.200
330	A	6	5	1.00	25	0.200
331	A	5	4	1.00	25	0.160
332	A	4	4	1.00	25	0.160
333	A	4	4	1.00	25	0.160
334	A	6	5	1.00	25	0.200
335	A	6	5	1.00	25	0.200
336	A	1	1	1.00	10	0.100
337	A	6	4	1.00	12	0.333
338	A	6	4	1.00	15	0.267
339	A	6	3	1.00	12	0.250
340	A	4	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	6	3	1.00	17	0.176
342	A	8	5	1.00	16	0.312
343	A	9	6	1.00	18	0.333
344	A	14	5	1.00	18	0.278
345	A	8	5	1.00	19	0.263
346	A	9	6	1.00	21	0.286
347	A	14	5	1.00	21	0.238
348	A	8	4	1.00	16	0.250
349	A	9	4	1.00	18	0.222
350	A	14	4	1.00	18	0.222
351	A	6	4	1.00	18	0.222
352	A	7	4	1.00	20	0.200
353	A	10	4	1.00	20	0.200
354	A	8	5	1.00	21	0.238
355	A	9	5	1.00	23	0.217
356	A	14	5	1.00	23	0.217
357	A	8	4	1.00	19	0.210
358	A	10	4	1.00	21	0.190
359	A	14	4	1.00	21	0.190
360	A	8	5	1.00	21	0.238
361	A	10	5	1.00	23	0.217
362	A	14	5	1.00	23	0.217
363	A	8	5	1.00	24	0.208
364	A	10	5	1.00	26	0.192
365	A	14	5	1.00	26	0.192
366	A	6	5	1.00	6	0.833
367	A	9	6	1.00	6	1.000
368	A	8	4	1.00	16	0.250
369	A	8	4	1.00	19	0.210

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sinh(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\cosh(a+bx)}{b}$$

[Out] cosh(b*x+a)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2718\}$

$$\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b*x],x]

[Out] Cosh[a + b*x]/b

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.01, size = 21, normalized size = 2.10

$$\frac{\cosh(a)\cosh(bx)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b*x],x]

[Out] (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b

Maple [A]
time = 0.15, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\cosh(bx+a)}{b}$	11
default	$\frac{\cosh(bx+a)}{b}$	11
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}$	27
meijerg	$\frac{\sinh(a)\sinh(bx)}{b}-\frac{\cosh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] cosh(b*x+a)/b
```

Maxima [A]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)/b
```

Fricas [A]
time = 0.39, size = 10, normalized size = 1.00

$$\frac{\cosh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] cosh(b*x + a)/b
```

Sympy [A]
time = 0.05, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\cosh(a+bx)}{b} & \text{for } b \neq 0 \\ x\sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x)
```

```
[Out] Piecewise((cosh(a + b*x)/b, Ne(b, 0)), (x*sinh(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.40, size = 26, normalized size = 2.60

$$\frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b
```

Mupad [B]
time = 0.04, size = 10, normalized size = 1.00

$$\frac{\cosh(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x),x)
```

```
[Out] cosh(a + b*x)/b
```

3.2 $\int \sinh^2(a+bx)\,dx$

Optimal. Leaf size=25

$$-\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

```
[Out] -1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b}-\frac{\int 1\,dx}{2}\\ &= -\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$\frac{-2(a+bx)+\sinh(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2,x]
```

```
[Out] (-2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A]
time = 0.41, size = 27, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
risch	$-\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cosh(b*x+a)*sinh(b*x+a)-1/2*b*x-1/2*a)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 1.28

$$-\frac{1}{2}\,x+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Fricas [A]
time = 0.46, size = 23, normalized size = 0.92

$$-\frac{bx-\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.07, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sinh^{2}\left(a+bx\right)}{2}-\frac{x\cosh^{2}\left(a+bx\right)}{2}+\frac{\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b} & \text{for}\ b\neq 0\\ x\sinh^{2}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh
(a + b*x)/(2*b), Ne(b, 0)), (x*sinh(a)**2, True))
```

Giac [A]
time = 0.40, size = 32, normalized size = 1.28

$$-\frac{1}{2}\,x+\frac{e^{(2\,bx+2\,a)}}{8\,b}-\frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B]
time = 0.37, size = 18, normalized size = 0.72

$$\frac{\sinh(2\,a+2\,b\,x)}{4\,b}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2,x)
```

```
[Out] sinh(2*a + 2*b*x)/(4*b) - x/2
```

3.3 $\int \sinh^3(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] -cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = $\{2713\}$

$$\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]/b) + Cosh[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{b}\\ &= -\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.07

$$-\frac{3\cosh(a+bx)}{4b}+\frac{\cosh(3(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3,x]
```

```
[Out] (-3*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b)
```

Maple [A]
time = 0.42, size = 27, normalized size = 1.00

method	result	size
default	$-\frac{3\cosh(bx+a)}{4b}+\frac{\cosh(3bx+3a)}{12b}$	27
risch	$\frac{e^{3bx+3a}}{24b}-\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*cosh(b*x+a)/b+1/12/b*cosh(3*b*x+3*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 0.26, size = 54, normalized size = 2.00

$$\frac{e^{(3\,bx+3\,a)}}{24\,b}-\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}+\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Fricas [A]
time = 0.41, size = 38, normalized size = 1.41

$$\frac{\cosh\left(bx+a\right)^{3}+3\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-9\,\cosh\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 9*cosh(b*x + a))/
b
```

Sympy [A]
time = 0.10, size = 36, normalized size = 1.33

$$\begin{cases}\frac{\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{b}-\frac{2\cosh^{3}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ x\sinh^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)/b - 2*cosh(a + b*x)**3/(3*b), Ne(
b, 0)), (x*sinh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.
time = 0.41, size = 54, normalized size = 2.00

$$\frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Mupad [B]
time = 0.06, size = 24, normalized size = 0.89

$$-\frac{3\cosh(a+b\,x)-\cosh(a+b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3,x)
```

```
[Out] -(3*cosh(a + b*x) - cosh(a + b*x)^3)/(3*b)
```

3.4 $\int \sinh^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{8}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}$$

```
[Out] 3/8*x-3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)*sinh(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]*Sinh[a + b
*x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sinh^4(a+bx)\,dx&=\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}-\frac{3}{4}\int \sinh^2(a+bx)\,dx\\&=-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\&=\frac{3x}{8}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^4,x]
[Out] (12*(a + b*x) - 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A]
time = 0.79, size = 33, normalized size = 0.72

method	result	size
default	$\frac{3x}{8}-\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+4a)}{32b}$	33
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}-\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*x-1/4*sinh(2*b*x+2*a)/b+1/32*sinh(4*b*x+4*a)/b
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.30

$$\frac{3}{8}x+\frac{e^{(4\,bx+4\,a)}}{64\,b}-\frac{e^{(2\,bx+2\,a)}}{8\,b}+\frac{e^{(-2\,bx-2\,a)}}{8\,b}-\frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.07

$$\frac{\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+3\,bx+\left(\cosh\left(bx+a\right)^{3}-4\,\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="fricas")
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.
time = 0.17, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} + \frac{5\sinh^3(a+bx)\cosh(a+bx)}{8b} - \frac{3\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sinh^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 + 5*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) - 3*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**4, True))
```

Giac [A]
time = 0.43, size = 60, normalized size = 1.30

$$\frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B]
time = 0.08, size = 32, normalized size = 0.70

$$\frac{3\,x}{8} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} - \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^4,x)
```

```
[Out] (3*x)/8 - (sinh(2*a + 2*b*x)/4 - sinh(4*a + 4*b*x)/32)/b
```

3.5 $\int \sinh^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

```
[Out] cosh(b*x+a)/b-2/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]/b - (2*Cosh[a + b*x]^3)/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cosh(a+bx)\right)}{b}\\ &= \frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.07

$$\frac{5\cosh(a+bx)}{8b}-\frac{5\cosh(3(a+bx))}{48b}+\frac{\cosh(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^5,x]
```

```
[Out] (5*Cosh[a + b*x])/(8*b) - (5*Cosh[3*(a + b*x)])/(48*b) + Cosh[5*(a + b*x)]/
(80*b)
```

Maple [A]
time = 0.44, size = 41, normalized size = 1.00

method	result	size
default	$\frac{5\cosh(bx+a)}{8b}-\frac{5\cosh(3bx+3a)}{48b}+\frac{\cosh(5bx+5a)}{80b}$	41
risch	$\frac{e^{5bx+5a}}{160b}-\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}+\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}+\frac{e^{-5bx-5a}}{160b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 5/8*cosh(b*x+a)/b-5/48/b*cosh(3*b*x+3*a)+1/80/b*cosh(5*b*x+5*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.
time = 0.27, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b}-\frac{5\,e^{(3\,bx+3\,a)}}{96\,b}+\frac{5\,e^{(bx+a)}}{16\,b}+\frac{5\,e^{(-bx-a)}}{16\,b}-\frac{5\,e^{(-3\,bx-3\,a)}}{96\,b}+\frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="maxima")
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(37) = 74$.
time = 0.43, size = 79, normalized size = 1.93

$$\frac{3\cosh\left(bx+a\right)^{5}+15\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{4}-25\cosh\left(bx+a\right)^{3}+15\left(2\cosh\left(bx+a\right)^{3}-5\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{2}+150\cosh\left(bx+a\right)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="fricas")
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 25*cosh(b*x +
 a)^3 + 15*(2*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^2 + 150*cosh
(b*x + a))/b
```

Sympy [A]
time = 0.26, size = 58, normalized size = 1.41

$$\begin{cases}\frac{\sinh^{4}\left(a+bx\right)\cosh\left(a+bx\right)}{b}-\frac{4\sinh^{2}\left(a+bx\right)\cosh^{3}\left(a+bx\right)}{3b}+\frac{8\cosh^{5}\left(a+bx\right)}{15b} & \text{for}\ b\neq 0\\ x\sinh^{5}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((sinh(a + b*x)**4*cosh(a + b*x)/b - 4*sinh(a + b*x)**2*cosh(a + b
*x)**3/(3*b) + 8*cosh(a + b*x)**5/(15*b), Ne(b, 0)), (x*sinh(a)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.
time = 0.41, size = 82, normalized size = 2.00

$$\frac{e^{(5\,bx+5\,a)}}{160\,b} - \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} + \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} + \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B]
time = 0.41, size = 31, normalized size = 0.76

$$\frac{\frac{\cosh(a+b\,x)^5}{5} - \frac{2\cosh(a+b\,x)^3}{3} + \cosh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^5,x)
```

```
[Out] (cosh(a + b*x) - (2*cosh(a + b*x)^3)/3 + cosh(a + b*x)^5/5)/b
```

3.6 $\int \sinh^6(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}$$

```
[Out] -5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b-5/24*cosh(b*x+a)*sinh(b*x+a)^3/b+1/6
*cosh(b*x+a)*sinh(b*x+a)^5/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b}+\frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (5*Cosh[a + b*x]*Sinh[
a + b*x]^3)/(24*b) + (Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \sinh^6(a+bx)\,dx &= \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}-\frac{5}{6}\int \sinh^4(a+bx)\,dx\\
&= -\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}+\frac{5}{8}\int \sinh^2(a+bx)\,dx\\
&= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}\\
&= -\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b}-\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.64

$$\frac{-60a-60bx+45\sinh(2(a+bx))-9\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^6,x]
```

```
[Out] (-60*a - 60*b*x + 45*Sinh[2*(a + b*x)] - 9*Sinh[4*(a + b*x)] + Sinh[6*(a + b*x)])/(192*b)
```

Maple [A]
time = 0.84, size = 47, normalized size = 0.70

method	result	size
default	$-\frac{5x}{16}+\frac{15\sinh(2bx+2a)}{64b}-\frac{3\sinh(4bx+4a)}{64b}+\frac{\sinh(6bx+6a)}{192b}$	47
risch	$-\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{3\,e^{4bx+4a}}{128b}+\frac{15\,e^{2bx+2a}}{128b}-\frac{15\,e^{-2bx-2a}}{128b}+\frac{3\,e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*x+15/64*sinh(2*b*x+2*a)/b-3/64*sinh(4*b*x+4*a)/b+1/192*sinh(6*b*x+6*a)/b
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.28

$$-\frac{\left(9\,e^{(-2\,bx-2\,a)}-45\,e^{(-4\,bx-4\,a)}-1\right)e^{(6\,bx+6\,a)}}{384\,b}-\frac{5\,(bx+a)}{16\,b}-\frac{45\,e^{(-2\,bx-2\,a)}-9\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*b*x - 2*a) - 45*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 5/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) - 9*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/b
```

Fricas [A]
time = 0.43, size = 90, normalized size = 1.34

$$\frac{3\,\cosh(bx+a)\sinh(bx+a)^5+2\left(5\,\cosh(bx+a)^3-9\,\cosh(bx+a)\right)\sinh(bx+a)^3-30\,bx+3\left(\cosh(bx+a)^5-6\,\cosh(bx+a)^3+15\,\cosh(bx+a)\right)\sinh(bx+a)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 9*cosh(b*x +
 a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(61) = 122$.
time = 0.41, size = 139, normalized size = 2.07

$$\begin{cases} \frac{5x\sinh^6(a+bx)}{16} - \frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16} + \frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16} - \frac{5x\cosh^6(a+bx)}{16} + \frac{11\sinh^5(a+bx)\cosh(a+bx)}{16b} - \frac{5\sinh^3(a+bx)\cosh^3(a+bx)}{6b} + \frac{5\sinh(a+bx)\cosh^5(a+bx)}{16b} & \text{for } b \neq 0 \\ x\sinh^6(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**6/16 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**2
/16 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - 5*x*cosh(a + b*x)**6/16 +
 11*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 5*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)
**6, True))
```

Giac [A]
time = 0.43, size = 88, normalized size = 1.31

$$-\frac{5}{16}x + \frac{e^{(6bx+6a)}}{384b} - \frac{3e^{(4bx+4a)}}{128b} + \frac{15e^{(2bx+2a)}}{128b} - \frac{15e^{(-2bx-2a)}}{128b} + \frac{3e^{(-4bx-4a)}}{128b} - \frac{e^{(-6bx-6a)}}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/16*x + 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b
*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b + 3/128*e^(-4*b*x - 4*a)/b - 1/384*
e^(-6*b*x - 6*a)/b
```

Mupad [B]
time = 0.13, size = 42, normalized size = 0.63

$$\frac{\frac{15\sinh(2a+2bx)}{64} - \frac{3\sinh(4a+4bx)}{64} + \frac{\sinh(6a+6bx)}{192}}{b} - \frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^6,x)
```

```
[Out] ((15*sinh(2*a + 2*b*x))/64 - (3*sinh(4*a + 4*b*x))/64 + sinh(6*a + 6*b*x)/1
92)/b - (5*x)/16
```

3.7 $\int \sinh^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{10iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}}-\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b}+\frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b}$$

```
[Out] 2/7*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b+10/21*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^
2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b
*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)-10/21*cosh(b*x+a)*si
nh(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2715, 2721, 2720}

$$\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\cosh(a+bx)}{21b}-\frac{10i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b*Sqrt[Sinh[a + b*x]]) - (10*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(21*b) + (2
*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} - \frac{5}{7}\int \sinh^{\frac{3}{2}}(a+bx)\,dx$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sinh(a+}}$$
$$= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{\left(5\sqrt{i\sinh(a+b}\right.}{21\sqrt{}}$$
$$= -\frac{10iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}} - \frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2}{}$$

Mathematica [A]
time = 0.11, size = 75, normalized size = 0.73

$$\frac{40iF\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)} - 26\sinh(2(a+bx)) + 3\sinh(4(a+bx))}{84b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(7/2),x]
[Out] ((40*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] -
 26*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(84*b*Sqrt[Sinh[a + b*x]])
```

Maple [A]
time = 0.66, size = 116, normalized size = 1.13

method	result
default	$\frac{\frac{5i\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\right.}{21}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] (5/21*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/7*cosh(b*x+a)^4
*sinh(b*x+a)-16/21*cosh(b*x+a)^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)
/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 326, normalized size = 3.17

$$\frac{40 \left(\sqrt{2} \cosh\left(bx+a\right)^{3}+3 \sqrt{2} \cosh\left(bx+a\right)^{2} \sinh\left(bx+a\right)+3 \sqrt{2} \cosh\left(bx+a\right) \sinh\left(bx+a\right)^{2}+\sqrt{2} \sinh\left(bx+a\right)^{3}\right) \mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+\left(3 \cosh\left(bx+a\right)^{6}+18 \cosh\left(bx+a\right) \sinh\left(bx+a\right)^{5}+3 \sinh\left(bx+a\right)^{6}+\left(45 \cosh\left(bx+a\right)^{2}-23\right) \sinh\left(bx+a\right)^{4}-23 \cosh\left(bx+a\right)^{4}+4 \left(15 \cosh\left(bx+a\right)^{3}-23 \cosh\left(bx+a\right)\right) \sinh\left(bx+a\right)^{3}+\left(45 \cosh\left(bx+a\right)^{4}-138 \cosh\left(bx+a\right)^{2}-23\right) \sinh\left(bx+a\right)^{2}-23 \cosh\left(bx+a\right)^{2}+2 \left(9 \cosh\left(bx+a\right)^{5}-46 \cosh\left(bx+a\right)^{3}-23 \cosh\left(bx+a\right)\right) \sinh\left(bx+a\right)+3\right) \sqrt{\sinh\left(bx+a\right)}}{84 \left(b \cosh\left(bx+a\right)^{3}+3 b \cosh\left(bx+a\right)^{2} \sinh\left(bx+a\right)+3 b \cosh\left(bx+a\right) \sinh\left(bx+a\right)^{2}+b \sinh\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a)
 + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*weier
strassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^6 +
18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2
- 23)*sinh(b*x + a)^4 - 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 23*cos
h(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 - 138*cosh(b*x + a)^2 - 2
3)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 46*cosh(b*
x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) + 3)*sqrt(sinh(b*x + a)))/(b*cos
h(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b
*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(7/2),x)
[Out] int(sinh(a + b*x)^(7/2), x)
```

3.8 $\int \sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}+\frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}$$

```
[Out] 2/5*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2715, 2721, 2719}

$$\frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx$$
$$= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}}$$
$$= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}$$

Mathematica [**A**]
time = 0.06, size = 68, normalized size = 0.85

$$\frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}+\sinh(a+bx)\sinh(2(a+bx))}{5b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(5/2),x]
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Sin
h[a + b*x]*Sinh[2*(a + b*x)])/(5*b*Sqrt[Sinh[a + b*x]])
```

Maple [**A**]
time = 0.66, size = 164, normalized size = 2.05

method	result
default	$-\frac{6\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 202, normalized size = 2.52

$$\frac{12\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+\left(\cosh\left(bx+a\right)^{4}+4\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sinh\left(bx+a\right)^{4}+6\left(\cosh\left(bx+a\right)^{2}+2\right)\sinh\left(bx+a\right)^{2}+12\cosh\left(bx+a\right)^{2}+4\left(\cosh\left(bx+a\right)^{3}+6\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)-1\right)\sqrt{\sinh\left(bx+a\right)}}{10\left(b\cosh\left(bx+a\right)^{2}+2b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
osh(b*x + a) + sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*c
osh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) - 1)*s
qrt(sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) +
b*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{5}{2}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{sinh}(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(5/2),x)
```

```
[Out] int(sinh(a + b*x)^(5/2), x)
```

3.9 $\int \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}+\frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x
)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/
sinh(b*x+a)^(1/2)+2/3*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2715, 2721, 2720}

$$\frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b}+\frac{2i\sqrt{i\sinh(a+bx)}F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*S
qrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx \\
&= \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}} \\
&= \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 83, normalized size = 1.04

$$
\frac{\sinh(2(a+bx)) - 2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sqrt{1-\cosh(2a+2bx)-\sinh(2a+2bx)}}{3b\sqrt{\sinh(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (Sinh[2*(a + b*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] +
 Sinh[2*(a + b*x)]]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b*x]])/(3*b*S
qrt[Sinh[a + b*x]])
```

Maple [**A**]
time = 0.62, size = 100, normalized size = 1.25

method	result
default	$-\frac{\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a)}\right.}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maximа [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 1.29

$$-\frac{2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\sqrt{\sinh\left(bx+a\right)}}{3\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInverse
(4, 0, cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)
+ b*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{3}{2}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(3/2),x)
```

```
[Out] int(sinh(a + b*x)^(3/2), x)
```

3.10 $\int \sqrt{\sinh(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2721, 2719}

$$-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sinh(a+bx)}\,dx &= \frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.93

$$\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh
[a + b*x]])
```

Maple [A]
time = 0.75, size = 108, normalized size = 2.00

method	result
default	$\frac{\sqrt{-i\left(\sinh\left(bx+a\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(bx+a\right)+i\right)}\,\sqrt{i\sinh\left(bx+a\right)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\right.\right.}{\cosh(bx+a)\sqrt{\sinh\left(bx+a\right)}\,b}$
risch	$\frac{\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{-bx-a}}}{b}-\frac{\left(\frac{2\,\mathrm{e}^{2bx+2a}-2}{\sqrt{\left(\mathrm{e}^{2bx+2a}-1\right)\mathrm{e}^{bx+a}}}-\frac{\sqrt{\mathrm{e}^{bx+a}+1}\,\sqrt{-2\,\mathrm{e}^{bx+a}+2}\,\sqrt{-\mathrm{e}^{bx+a}}\,\left(-2\right.}{\sqrt{\mathrm{e}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 37, normalized size = 0.69

$$-\frac{2\left(\sqrt{2}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+\sqrt{\sinh\left(bx+a\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a) +
 sinh(b*x + a))) + sqrt(sinh(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh{\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\mathrm{sinh}\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^(1/2),x)
```

```
[Out] int(sinh(a + b*x)^(1/2), x)
```

3.11 $\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal. Leaf size=54

$$-\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sin
h(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2721, 2720}

$$-\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|\,2\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt
[Sinh[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx &= \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{\sqrt{\sinh(a+bx)}}\\ &= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 0.89

$$-\frac{2F\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sinh[a + b*x]],x]
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]])
```

Maple [A]
time = 0.39, size = 87, normalized size = 1.61

method	result
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\sqrt{2}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{-i(\sinh(\right.}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 24, normalized size = 0.44

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(sinh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(1/2),x)
[Out] int(1/sinh(a + b*x)^(1/2), x)
```

3.12 $\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2716, 2721, 2719}

$$-\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}}-\frac{2i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2 +
 I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} + \int \sqrt{\sinh(a+bx)}\,dx$$
$$= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} + \frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}$$
$$= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.75

$$-\frac{2\left(\cosh(a+bx)-E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}\right)}{b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sin
h[a + b*x]]))/(b*Sqrt[Sinh[a + b*x]])
```

Maple [A]
time = 0.67, size = 154, normalized size = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\sqrt{2}\sqrt{1+i\sinh(bx+a)}\sqrt{i\sinh(bx+a)}\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 152, normalized size = 2.00

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}-\sqrt{2}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+2\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}\right)\sqrt{\sinh\left(bx+a\right)}\right)}{b\cosh\left(bx+a\right)^{2}+2b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt
(2)*sinh(b*x + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sinh^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\mathrm{sinh}\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(3/2),x)
```

```
[Out] int(1/sinh(a + b*x)^(3/2), x)
```

3.13 $\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] -2/3*cosh(b*x+a)/b/sinh(b*x+a)^(3/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2716, 2721, 2720}

$$-\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)}+\frac{2i\sqrt{i\sinh(a+bx)}\,F\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$
$$= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.06, size = 86, normalized size = 1.08

$$-\frac{2\left(\cosh(a+bx)+\,_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sinh(a+bx)\sqrt{1-\cosh(2a+2bx)-\sinh(2a+2bx)}\right)}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] + S
inh[2*(a + b*x)]]*Sinh[a + b*x]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b
*x]]))/(3*b*Sinh[a + b*x]^(3/2))
```

Maple [A]
time = 0.61, size = 101, normalized size = 1.26

method	result
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(bx+a}\right.}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 314, normalized size = 3.92

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{4}+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sqrt{2}\sinh\left(bx+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bx+a\right)^{2}-\sqrt{2}\right)\sinh\left(bx+a\right)^{2}-2\sqrt{2}\cosh\left(bx+a\right)^{2}+4\left(\sqrt{2}\cosh\left(bx+a\right)^{3}-\sqrt{2}\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+2\left(\cosh\left(bx+a\right)^{3}+3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+\sinh\left(bx+a\right)^{3}+\left(3\cosh\left(bx+a\right)^{2}+1\right)\sinh\left(bx+a\right)+\cosh\left(bx+a\right)\right)\sqrt{\sinh\left(bx+a\right)}\right)}{3\left(b\cosh\left(bx+a\right)^{4}+4b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+b\sinh\left(bx+a\right)^{4}-2b\cosh\left(bx+a\right)^{2}+2\left(3b\cosh\left(bx+a\right)^{2}-b\right)\sinh\left(bx+a\right)^{2}+4\left(b\cosh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 +
sqrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b*x
+ a)^2 - 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 - sqrt(2)*c
osh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(4, 0, cosh(b*x +
 a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2
 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))
*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sinh\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(5/2),x)
[Out] int(1/sinh(a + b*x)^(5/2), x)
```

3.14 $\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=103

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/5*cosh(b*x+a)/b/sinh(b*x+a)^(5/2)+6/5*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)-6/
5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2716, 2721, 2719}

$$-\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}}+\frac{6i\sqrt{\sinh(a+bx)}\,E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2)) + (6*Cosh[a + b*x])/(5*b*Sqrt[
Sinh[a + b*x]]) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh
[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,d}{5\sqrt{i\sinh(a+bx)}} \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.12, size = 73, normalized size = 0.71

$$
\frac{-2\coth(a+bx)+6iE\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)(i\sinh(a+bx))^{3/2}+3\sinh(2(a+bx))}{5b\sinh^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Coth[a + b*x] + (6*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Si
nh[a + b*x])^(3/2) + 3*Sinh[2*(a + b*x)])/(5*b*Sinh[a + b*x]^(3/2))
```

Maple [**A**]
time = 0.66, size = 192, normalized size = 1.86

method	result
default	$-\frac{6\sqrt{-i(\sinh(bx+a)+i)}\sqrt{2}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}(\sinh^2(bx+a))\,\mathrm{EllipticE}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/sinh(b*x+a)^(5/2)*(6*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x
+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticE((-I*(sinh(b*x+a
)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*
x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticF((-I*(sinh(b*x+
a)+I))^(1/2),1/2*2^(1/2))-6*sinh(b*x+a)^4-4*sinh(b*x+a)^2+2)/cosh(b*x+a)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 621, normalized size = 6.03

$$\frac{2\left(3\left(\sqrt{2}\cosh(bx+a)^6+6\sqrt{2}\cosh(bx+a)\sinh(bx+a)^5+\sqrt{2}\sinh(bx+a)^6+3\left(5\sqrt{2}\cosh(bx+a)^2-\sqrt{2}\right)\sinh(bx+a)^4-3\sqrt{2}\cosh(bx+a)^4+4\left(5\sqrt{2}\cosh(bx+a)^3-3\sqrt{2}\cosh(bx+a)\right)\sinh(bx+a)^3+3\left(5\sqrt{2}\cosh(bx+a)^4-6\sqrt{2}\cosh(bx+a)^2+\sqrt{2}\right)\sinh(bx+a)^2+3\sqrt{2}\cosh(bx+a)^2+6\left(\sqrt{2}\cosh(bx+a)^5-2\sqrt{2}\cosh(bx+a)^3+\sqrt{2}\cosh(bx+a)\right)\sinh(bx+a)-\sqrt{2}\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cosh(bx+a)+\sinh(bx+a)\right)\right)+2\left(3\cosh(bx+a)^6+18\cosh(bx+a)\sinh(bx+a)^5+3\sinh(bx+a)^6+\left(45\cosh(bx+a)^2-8\right)\sinh(bx+a)^4-8\cosh(bx+a)^4+4\left(15\cosh(bx+a)^3-8\cosh(bx+a)\right)\sinh(bx+a)^3+\left(45\cosh(bx+a)^4-48\cosh(bx+a)^2+1\right)\sinh(bx+a)^2+\cosh(bx+a)^2+2\left(9\cosh(bx+a)^5-16\cosh(bx+a)^3+\cosh(bx+a)\right)\sinh(bx+a)\right)\sqrt{\sinh(bx+a)}\right)}{5\left(b\cosh(bx+a)^6+6b\cosh(bx+a)\sinh(bx+a)^5+b\sinh(bx+a)^6-3b\cosh(bx+a)^4+3\left(5b\cosh(bx+a)^2-b\right)\sinh(bx+a)^4+4\left(5b\cosh(bx+a)^3-3b\cosh(bx+a)\right)\sinh(bx+a)^3+3b\cosh(bx+a)^2+3\left(5b\cosh(bx+a)^4-6b\cosh(bx+a)^2+b\right)\sinh(bx+a)^2+6\left(b\cosh(bx+a)^5-2b\cosh(bx+a)^3+b\cosh(bx+a)\right)\sinh(bx+a)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5 +
 sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b*x
 + a)^4 - 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 - 3*sqrt
(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 - 6*sqrt(
2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a)^2 +
 6*(sqrt(2)*cosh(b*x + a)^5 - 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cosh(b*x
+ a))*sinh(b*x + a) - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*cosh(b*x +
a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 - 8)*sinh(b*x
+ a)^4 - 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 8*cosh(b*x + a))*sinh(
b*x + a)^3 + (45*cosh(b*x + a)^4 - 48*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2
+ cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 16*cosh(b*x + a)^3 + cosh(b*x +
a))*sinh(b*x + a))*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x +
 a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh
(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x +
a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*co
sh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)**(7/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + b*x)^(7/2),x)
[Out] int(1/sinh(a + b*x)^(7/2), x)
```

3.15 $\int (b\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$-\frac{10ib^4F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}}-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}+\frac{2b\cosh(c+dx)(b\sinh}{7d}$$

```
[Out] 2/7*b*cosh(d*x+c)*(b*sinh(d*x+c))^(5/2)/d+10/21*I*b^4*(sin(1/2*I*c+1/4*Pi+1
/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*
Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*sinh(d*x+c))^(1/2)-10/21*
b^3*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2720}

$$-\frac{10ib^4\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{b\sinh(c+dx)}}-\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d}+\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*b^4*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]
])/(d*Sqrt[b*Sinh[c + d*x]]) - (10*b^3*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])
/(21*d) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} - \frac{1}{7}(5b^2)\int (b\sinh(c+dx))^{3/2}\,dx$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{1}{21}$$
$$= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{(5}{}$$
$$= -\frac{10ib^4F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+}}{21d}$$

Mathematica [A]
time = 0.20, size = 76, normalized size = 0.66

$$\frac{b^3\left(-23\cosh(c+dx)+3\cosh(3(c+dx))-\frac{20F\left(\frac{1}{4}(-2ic+\pi-2idx)|2\right)}{\sqrt{i\sinh(c+dx)}}\right)\sqrt{b\sinh(c+dx)}}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(7/2),x]
[Out] (b^3*(-23*Cosh[c + d*x] + 3*Cosh[3*(c + d*x)] - (20*EllipticF[((-2*I)*c + P
i - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]])*Sqrt[b*Sinh[c + d*x]])/(42*d)
```

Maple [A]
time = 0.77, size = 122, normalized size = 1.05

method	result
default	$\frac{b^4\left(5i\sqrt{1-i\sinh(dx+c)}\sqrt{2}\sqrt{1+i\sinh(dx+c)}\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx-\right.\right.}{21\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/21*b^4*(5*I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+6*cosh(d*x+
c)^4*sinh(d*x+c)-16*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 394, normalized size = 3.40

$$\frac{40\left(\sqrt{2}\,b^{3}\cosh\left(dx+c\right)^{3}+3\sqrt{2}\,b^{3}\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\sqrt{2}\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+\sqrt{2}\,b^{3}\sinh\left(dx+c\right)^{3}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))+\left(3\,b^{3}\cosh\left(dx+c\right)^{6}+18\,b^{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+3\,b^{3}\sinh\left(dx+c\right)^{6}-23\,b^{3}\cosh\left(dx+c\right)^{4}-23\,b^{3}\cosh\left(dx+c\right)^{2}+\left(45\,b^{3}\cosh\left(dx+c\right)^{2}-23\,b^{3}\right)\sinh\left(dx+c\right)^{4}+4\left(15\,b^{3}\cosh\left(dx+c\right)^{3}-23\,b^{3}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+3\,b^{3}+\left(45\,b^{3}\cosh\left(dx+c\right)^{4}-138\,b^{3}\cosh\left(dx+c\right)^{2}-23\,b^{3}\right)\sinh\left(dx+c\right)^{2}+2\left(9\,b^{3}\cosh\left(dx+c\right)^{5}-46\,b^{3}\cosh\left(dx+c\right)^{3}-23\,b^{3}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\sqrt{b\sinh\left(dx+c\right)}}{84\left(d\cosh\left(dx+c\right)^{3}+3\,d\cosh\left(dx+c\right)^{2}\sinh\left(dx+c\right)+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+d\sinh\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(40*(sqrt(2)*b^3*cosh(d*x + c)^3 + 3*sqrt(2)*b^3*cosh(d*x + c)^2*sinh(
d*x + c) + 3*sqrt(2)*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + sqrt(2)*b^3*sinh(d
*x + c)^3)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))
 + (3*b^3*cosh(d*x + c)^6 + 18*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 3*b^3*si
nh(d*x + c)^6 - 23*b^3*cosh(d*x + c)^4 - 23*b^3*cosh(d*x + c)^2 + (45*b^3*c
osh(d*x + c)^2 - 23*b^3)*sinh(d*x + c)^4 + 4*(15*b^3*cosh(d*x + c)^3 - 23*b
^3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b^3 + (45*b^3*cosh(d*x + c)^4 - 138*b
^3*cosh(d*x + c)^2 - 23*b^3)*sinh(d*x + c)^2 + 2*(9*b^3*cosh(d*x + c)^5 - 4
6*b^3*cosh(d*x + c)^3 - 23*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*
x + c)))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\sinh(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(7/2),x)
[Out] int((b*sinh(c + d*x))^(7/2), x)
```

3.16 $\int (b\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$\frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}+\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 2/5*b*cosh(d*x+c)*(b*sinh(d*x+c))^(3/2)/d-6/5*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2
*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi
+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2719}

$$\frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}+\frac{6ib^2E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(5*
d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{5/2}\,dx = \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{1}{5}\left(3b^2\right)\int \sqrt{b\sinh(c+dx)}\,dx$$
$$= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{\left(3b^2\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+dx)}}{5\sqrt{i\sinh(c+dx)}}$$
$$= \frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3}}{5d}$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.77

$$\frac{b^2\sqrt{b\sinh(c+dx)}\left(-\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)|2\right)}{\sqrt{i\sinh(c+dx)}}+\sinh(2(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(5/2),x]
[Out] (b^2*Sqrt[b*Sinh[c + d*x]]*(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4
, 2])/Sqrt[I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(5*d)
```

Maple [A]
time = 0.71, size = 170, normalized size = 1.93

method	result
default	$-\frac{b^3\left(6\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/5*b^3*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d
*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*Elliptic
F((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cos
h(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 249, normalized size = 2.83

$$\frac{12\left(\sqrt{2}\,b^{2}\cosh\left(dx+c\right)^{2}+2\sqrt{2}\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{2}\,b^{2}\sinh\left(dx+c\right)^{2}\right)\sqrt{b}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\left(b^{2}\cosh\left(dx+c\right)^{4}+4\,b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+b^{2}\sinh\left(dx+c\right)^{4}+12\,b^{2}\cosh\left(dx+c\right)^{2}+6\left(b^{2}\cosh\left(dx+c\right)^{2}+2\,b^{2}\right)\sinh\left(dx+c\right)^{2}-b^{2}+4\left(b^{2}\cosh\left(dx+c\right)^{3}+6\,b^{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)\sqrt{b\sinh\left(dx+c\right)}}{10\left(d\cosh\left(dx+c\right)^{2}+2\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+d\sinh\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*b^2*cosh(d*x + c)^2 + 2*sqrt(2)*b^2*cosh(d*x + c)*sinh(d*
x + c) + sqrt(2)*b^2*sinh(d*x + c)^2)*sqrt(b)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + (b^2*cosh(d*x + c)^4 +
 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 12*b^2*cosh(d*
x + c)^2 + 6*(b^2*cosh(d*x + c)^2 + 2*b^2)*sinh(d*x + c)^2 - b^2 + 4*(b^2*c
osh(d*x + c)^3 + 6*b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))
/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(5/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(5/2), x)
```

3.17 $\int (b\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=88

$$\frac{2ib^2F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}}+\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

```
[Out] -2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I
*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2
)/d/(b*sinh(d*x+c))^(1/2)+2/3*b*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}+\frac{2ib^2\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{1}{3}b^2\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$$
$$= \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{\left(b^2\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3\sqrt{b\sinh(c+dx)}}$$
$$= \frac{2ib^2F\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}} + \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 88, normalized size = 1.00

$$\frac{b^2\left(\sinh(2(c+dx)) - 2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{1-\cosh(2c+2dx)-\sinh(2c+2dx)}\right)}{3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b^2*(Sinh[2*(c + d*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cosh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/
(3*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A]
time = 0.67, size = 106, normalized size = 1.20

method	result
default	$-\frac{b^2\left(i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+\right.\right.}{3\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)
^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 115, normalized size = 1.31

$$-\frac{2\left(\sqrt{2}\,b\cosh\left(dx+c\right)+\sqrt{2}\,b\sinh\left(dx+c\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))-\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+b\right)\sqrt{b\sinh\left(dx+c\right)}}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*(sqrt(2)*b*cosh(d*x + c) + sqrt(2)*b*sinh(d*x + c))*sqrt(b)*weierst
rassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) - (b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(b*sinh(d*x + c)
))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh{(c+dx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(3/2),x)
[Out] Integral((b*sinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\mathrm{sinh}(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(3/2),x)
[Out] int((b*sinh(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b\sinh(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2719}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sinh(c+dx)}\,dx &= \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[b*Sinh[c + d*x]])/(
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A]
time = 0.87, size = 111, normalized size = 1.98

method	result
default	$\frac{b\sqrt{-i(\sinh(dx+c)+i)}\sqrt{2}\sqrt{-i(i-\sinh(dx+c))}\sqrt{i\sinh(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\right.\right.}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$
risch	$\frac{\sqrt{2}\sqrt{b\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}}{d}-\frac{\left(\frac{2b\,\mathrm{e}^{2dx+2c}-2b}{b\sqrt{\mathrm{e}^{dx+c}\left(b\,\mathrm{e}^{2dx+2c}-b\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\sqrt{-2\,\mathrm{e}^{dx+c}+2}\sqrt{-\mathrm{e}^{dx+c}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*(2*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I
*sinh(d*x+c))^(1/2),1/2*2^(1/2)))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 42, normalized size = 0.75

$$-\frac{2\left(\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh(dx+c)+\sinh(dx+c)))+\sqrt{b\sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(b*sinh(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \sinh (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(1/2),x)
```

```
[Out] int((b*sinh(c + d*x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*
sinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2720}

$$-\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx &= \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{\sqrt{b\sinh(c+dx)}}\\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.96

$$\frac{2iF\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sinh[c + d*x]],x]
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[
b*Sinh[c + d*x]])
```

Maple [A]
time = 0.52, size = 89, normalized size = 1.59

method	result
default	$\frac{i\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\sinh\left(dx-\right.\right.}\right.}{\cosh(dx+c)\sqrt{b\sinh\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*EllipticF((-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/(b
*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 27, normalized size = 0.48

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))/(sqrt(b)
*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sinh{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,\mathrm{sinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(1/2), x)
```

3.20 $\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(1/2)+2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*
d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/b^2/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2719}

$$-\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}}-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi
/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\int \sqrt{b\sinh(c+dx)}\,dx}{b^2}$$
$$= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{b^2\sqrt{i\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

Mathematica [**A**]
time = 0.05, size = 62, normalized size = 0.72

$$-\frac{2\left(\cosh(c+dx)-E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{bd\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sin
h[c + d*x]]))/(b*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [**A**]
time = 0.73, size = 159, normalized size = 1.85

method	result
default	$\frac{2\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\left(dx+c\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^
(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(1/2
)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh
(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/b/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 169, normalized size = 1.97

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(dx+c\right)^{2}+2\sqrt{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{2}\sinh\left(dx+c\right)^{2}-\sqrt{2}\right)\sqrt{b}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+2\left(\cosh\left(dx+c\right)^{2}+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}\right)\sqrt{b\sinh\left(dx+c\right)}\right)}{b^{2}d\cosh\left(dx+c\right)^{2}+2\,b^{2}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b^{2}d\sinh\left(dx+c\right)^{2}-b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(d*x + c)^2 + 2*sqrt(2)*cosh(d*x + c)*sinh(d*x + c) + sqrt
(2)*sinh(d*x + c)^2 - sqrt(2))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(b*sinh(d*x + c)))/(b^2*d*cosh
(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 -
 b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\sinh\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(3/2), x)
```

3.21 $\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[Out] -2/3*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(3/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^2/d/(b*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2720}

$$-\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}}+\frac{2i\sqrt{i\sinh(c+dx)}\,F\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[(
I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d*x
]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx}{3b^2}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2\sqrt{b\sinh(c+dx)}}$$
$$= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 84, normalized size = 0.93

$$-\frac{2\left(\coth(c+dx)+\sqrt{2}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{-\left((1+\coth(c+dx))\sinh^2(c+dx)\right)}\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3
*b^2*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A]
time = 0.67, size = 114, normalized size = 1.27

method	result
default	$-\frac{i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)}\right.}{3b^2\sinh(dx+c)\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^2/sinh(d*x+c)*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(
1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*s
inh(d*x+c)+2*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 347, normalized size = 3.86

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(dx+c\right)^{4}+4\sqrt{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\sqrt{2}\sinh\left(dx+c\right)^{4}+2\left(3\sqrt{2}\cosh\left(dx+c\right)^{2}-\sqrt{2}\right)\sinh\left(dx+c\right)^{2}-2\sqrt{2}\cosh\left(dx+c\right)^{2}+4\left(\sqrt{2}\cosh\left(dx+c\right)^{3}-\sqrt{2}\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)+\sqrt{2}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))+2\left(\cosh\left(dx+c\right)^{3}+3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+\sinh\left(dx+c\right)^{3}+\left(3\cosh\left(dx+c\right)^{2}+1\right)\sinh\left(dx+c\right)+\cosh\left(dx+c\right)\right)\sqrt{b\sinh\left(dx+c\right)}\right)}{3\left(b^{3}d\cosh\left(dx+c\right)^{4}+4b^{3}d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+b^{3}d\sinh\left(dx+c\right)^{4}-2b^{3}d\cosh\left(dx+c\right)^{2}+b^{3}d+2\left(3b^{3}d\cosh\left(dx+c\right)^{2}-b^{3}d\right)\sinh\left(dx+c\right)^{2}+4\left(b^{3}d\cosh\left(dx+c\right)^{3}-b^{3}d\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*cosh(d*x + c)^4 + 4*sqrt(2)*cosh(d*x + c)*sinh(d*x + c)^3 +
sqrt(2)*sinh(d*x + c)^4 + 2*(3*sqrt(2)*cosh(d*x + c)^2 - sqrt(2))*sinh(d*x
+ c)^2 - 2*sqrt(2)*cosh(d*x + c)^2 + 4*(sqrt(2)*cosh(d*x + c)^3 - sqrt(2)*c
osh(d*x + c))*sinh(d*x + c) + sqrt(2))*sqrt(b)*weierstrassPInverse(4, 0, co
sh(d*x + c) + sinh(d*x + c)) + 2*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d
*x + c))*sqrt(b*sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 - 2*b^3*d*cosh(d*x + c)^2 + b^3
*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*sinh(d*x + c)^2 + 4*(b^3*d*cosh(d*
x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(5/2),x)
[Out] int(1/(b*sinh(c + d*x))^(5/2), x)
```

3.22 $\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}+\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}+\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2/5*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(5/2)+6/5*cosh(d*x+c)/b^3/d/(b*sinh(d*
x+c))^(1/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*P
i+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c
))^(1/2)/b^4/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2719}

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}+\frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}}-\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) + (6*Cosh[c + d*x])/(5*b
^3*d*Sqrt[b*Sinh[c + d*x]]) + (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2,
2]*Sqrt[b*Sinh[c + d*x]])/(b^4*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} - \frac{3\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{3\int \sqrt{b\sinh(c+dx)}\,dx}{5b^4}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{\left(3\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\,\mathrm{s}}}{5b^4\sqrt{i\sinh(c+}}$$
$$= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)\sqrt{b}}{5b^4d\sqrt{i\sinh(c+}}$$

Mathematica [**A**]
time = 0.12, size = 79, normalized size = 0.67

$$-\frac{2\left(-3\cosh(c+dx)+\coth(c+dx)\mathrm{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)\sqrt{i\sinh(c+dx)}\right)}{5b^3d\sqrt{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[((-2*I)*c
 + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(5*b^3*d*Sqrt[b*Sinh[c + d
*x]])
```

Maple [**A**]
time = 0.73, size = 205, normalized size = 1.74

method	result
default	$-\frac{6\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,(\sinh^2(dx+c))\,\mathrm{EllipticE}\left(\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b^3/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)
```

```
+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(b*s
inh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 675, normalized size = 5.72

$$\frac{2\left(3\left(\sqrt{2}\cosh(dx+c)^6+6\sqrt{2}\cosh(dx+c)\sinh(dx+c)^5+\sqrt{2}\sinh(dx+c)^6+3\left(5\sqrt{2}\cosh(dx+c)^2-\sqrt{2}\right)\sinh(dx+c)^4-3\sqrt{2}\cosh(dx+c)^4+4\left(5\sqrt{2}\cosh(dx+c)^3-3\sqrt{2}\cosh(dx+c)\right)\sinh(dx+c)^3+3\left(5\sqrt{2}\cosh(dx+c)^4-6\sqrt{2}\cosh(dx+c)^2+\sqrt{2}\right)\sinh(dx+c)^2+3\sqrt{2}\cosh(dx+c)^2+6\left(\sqrt{2}\cosh(dx+c)^5-2\sqrt{2}\cosh(dx+c)^3+\sqrt{2}\cosh(dx+c)\right)\sinh(dx+c)-\sqrt{2}\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cosh(dx+c)+\sinh(dx+c)\right)\right)+2\left(3\cosh(dx+c)^6+18\cosh(dx+c)\sinh(dx+c)^5+3\sinh(dx+c)^6+\left(45\cosh(dx+c)^2-8\right)\sinh(dx+c)^4-8\cosh(dx+c)^4+4\left(15\cosh(dx+c)^3-8\cosh(dx+c)\right)\sinh(dx+c)^3+\left(45\cosh(dx+c)^4-48\cosh(dx+c)^2+1\right)\sinh(dx+c)^2+\cosh(dx+c)^2+2\left(9\cosh(dx+c)^5-16\cosh(dx+c)^3+\cosh(dx+c)\right)\sinh(dx+c)\right)\sqrt{b\sinh(dx+c)}\right)}{5\left(b^4d\cosh(dx+c)^6+6b^4d\cosh(dx+c)\sinh(dx+c)^5+b^4d\sinh(dx+c)^6-3b^4d\cosh(dx+c)^4+3b^4d\cosh(dx+c)^2-b^4d+3\left(5b^4d\cosh(dx+c)^2-b^4d\right)\sinh(dx+c)^4+4\left(5b^4d\cosh(dx+c)^3-3b^4d\cosh(dx+c)\right)\sinh(dx+c)^3+3\left(5b^4d\cosh(dx+c)^4-6b^4d\cosh(dx+c)^2+b^4d\right)\sinh(dx+c)^2+6\left(b^4d\cosh(dx+c)^5-2b^4d\cosh(dx+c)^3+b^4d\cosh(dx+c)\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(d*x + c)^6 + 6*sqrt(2)*cosh(d*x + c)*sinh(d*x + c)^5 +
 sqrt(2)*sinh(d*x + c)^6 + 3*(5*sqrt(2)*cosh(d*x + c)^2 - sqrt(2))*sinh(d*x
 + c)^4 - 3*sqrt(2)*cosh(d*x + c)^4 + 4*(5*sqrt(2)*cosh(d*x + c)^3 - 3*sqrt
(2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*sqrt(2)*cosh(d*x + c)^4 - 6*sqrt(
2)*cosh(d*x + c)^2 + sqrt(2))*sinh(d*x + c)^2 + 3*sqrt(2)*cosh(d*x + c)^2 +
 6*(sqrt(2)*cosh(d*x + c)^5 - 2*sqrt(2)*cosh(d*x + c)^3 + sqrt(2)*cosh(d*x
+ c))*sinh(d*x + c) - sqrt(2))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + 2*(3*cosh(d*x + c)^6 + 18*cos
h(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x + c)^6 + (45*cosh(d*x + c)^2 - 8)*s
inh(d*x + c)^4 - 8*cosh(d*x + c)^4 + 4*(15*cosh(d*x + c)^3 - 8*cosh(d*x + c
))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 - 48*cosh(d*x + c)^2 + 1)*sinh(d*x
 + c)^2 + cosh(d*x + c)^2 + 2*(9*cosh(d*x + c)^5 - 16*cosh(d*x + c)^3 + cos
h(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 +
6*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cos
h(d*x + c)^4 + 3*b^4*d*cosh(d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2
 - b^4*d)*sinh(d*x + c)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 3*(5*b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2
 + b^4*d)*sinh(d*x + c)^2 + 6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(7/2), x)
```

3.23 $\int (i\sinh(c+dx))^{7/2}\,dx$

Optimal. Leaf size=91

$$-\frac{10iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{21d}+\frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}$$

```
[Out] 10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*
x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/7*I*cosh(d*x+c)*(I*
sinh(d*x+c))^(5/2)/d+10/21*I*cosh(d*x+c)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2715, 2720}

$$-\frac{10iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{21d}+\frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d}+\frac{10i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((10*I)/21)*Cosh[c
 + d*x]*Sqrt[I*Sinh[c + d*x]])/d + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c + d*x
])^(5/2))/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{7/2}\,dx &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}+\frac{5}{7}\int (i\sinh(c+dx))^{3/2}\,dx\\ &= \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}+\frac{5}{21}\int\\ &= -\frac{10iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{21d}+\frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d}+\frac{2i\cosh(c+\end{aligned}$$

Mathematica [A]
time = 0.12, size = 65, normalized size = 0.71

$$\frac{i\left(20F\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)+(23\cosh(c+dx)-3\cosh(3(c+dx)))\sqrt{i\sinh(c+dx)}\right)}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] ((I/42)*(20*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + (23*Cosh[c + d*x]
 - 3*Cosh[3*(c + d*x)])*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [A]
time = 0.68, size = 122, normalized size = 1.34

method	result
default	$-\frac{i\left(6i(\cosh^4(dx+c))\sinh(dx+c)-5\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \text{Ellipt}\right.}{21\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*I*(6*I*cosh(d*x+c)^4*sinh(d*x+c)-5*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1
+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/
2),1/2*2^(1/2))-16*I*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 104, normalized size = 1.14

$$\frac{\left(\sqrt{\frac{1}{2}}\left(-3i\,e^{(6\,dx+6\,c)}+23i\,e^{(4\,dx+4\,c)}+23i\,e^{(2\,dx+2\,c)}-3i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-40i\,\sqrt{2}\,\sqrt{i}\,e^{(3\,dx+3\,c)}\text{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)e^{(-3\,dx-3\,c)}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(sqrt(1/2)*(-3*I*e^(6*d*x + 6*c) + 23*I*e^(4*d*x + 4*c) + 23*I*e^(2*d*
x + 2*c) - 3*I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) - 40*I*sqr
t(2)*sqrt(I)*e^(3*d*x + 3*c)*weierstrassPInverse(4, 0, e^(d*x + c)))*e^(-3*
d*x - 3*c)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (\sinh(c+d\,x)\ 1\mathrm{i})^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(7/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(7/2), x)
```

3.24 $\int (i \sinh(c+dx))^{5/2} \, dx$

Optimal. Leaf size=62

$$-\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2715, 2719}

$$\frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d
*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{5/2}\,dx &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}+\frac{3}{5}\int\sqrt{i\sinh(c+dx)}\,dx\\ &= -\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.89

$$\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)-\sqrt{i\sinh(c+dx)}\,\sinh(2(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] ((6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] - Sqrt[I*Sinh[c + d*x]]*
Sinh[2*(c + d*x)])/(5*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(82) = 164.
time = 0.69, size = 169, normalized size = 2.73

method	result
default	$-\frac{i\left(3\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh\left(dx+\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I*(3*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d
*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-6*(1-I*sinh(d*x
+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticE(
(1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*cosh(d*x+c)^4-2*cosh(d*x+c)^2)/cosh(
d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 94, normalized size = 1.52

$$-\frac{\left(\sqrt{\frac{1}{2}}\left(e^{(4\,dx+4\,c)}+12\,e^{(2\,dx+2\,c)}-1\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+12\,\sqrt{2}\,\sqrt{i}\,e^{(2\,dx+2\,c)}\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)e^{(-2\,dx-2\,c)}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/10*(sqrt(1/2)*(e^(4*d*x + 4*c) + 12*e^(2*d*x + 2*c) - 1)*sqrt(I*e^(2*d*x
 + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 12*sqrt(2)*sqrt(I)*e^(2*d*x + 2*c)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))*e^(-2*d*x - 2*c)
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (i \sinh (c + dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (\sinh(c + d\,x)\ 1\mathrm{i})^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(5/2), x)
```

3.25 $\int (i\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=62

$$-\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d}+\frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2715, 2720}

$$\frac{2i\sqrt{i\sinh(c+dx)}\,\cosh(c+dx)}{3d}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (i\sinh(c+dx))^{3/2}\,dx &= \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}+\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d}+\frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 94, normalized size = 1.52

$$-\frac{2i\sqrt{i\sinh(c+dx)}\left(-\cosh(c+dx)+\operatorname{csch}(c+dx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{1-\cosh(2c+2dx)-\sinh(2c+2dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[I*Sinh[c + d*x]]*(-Cosh[c + d*x] + Csch[c + d*x]*Hypergeom
etric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cos
h[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/d
```

Maple [A]
time = 0.64, size = 104, normalized size = 1.68

method	result
default	$\frac{i\left(\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh\left(dx+c\right)}\right.\right.}{3\cosh(dx+c)\sqrt{i\sinh\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*((1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+
c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*I*cosh(d*x+c)^2*
sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 79, normalized size = 1.27

$$\frac{\left(\sqrt{\frac{1}{2}}\left(i\,e^{(2\,dx+2\,c)}+i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-2i\,\sqrt{2}\,\sqrt{i}\,e^{(dx+c)}\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)e^{(-dx-c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(I*e^(2*d*x + 2*c) + I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*
d*x - 1/2*c) - 2*I*sqrt(2)*sqrt(I)*e^(d*x + c)*weierstrassPInverse(4, 0, e^
(d*x + c)))*e^(-d*x - c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (i \sinh (c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (\mathrm{sinh}(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(3/2), x)
```

3.26 $\int \sqrt{i \sinh(c+dx)}\, dx$

Optimal. Leaf size=30

$$-\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2719}

$$-\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{i \sinh(c+dx)}\, dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.93

$$\frac{2iE\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticE[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Maple [A]
time = 0.80, size = 91, normalized size = 3.03

method	result
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\mathrm{E}\right.}{\cosh(dx+c)d}$
risch	$\frac{\sqrt{2}\,\sqrt{i\,(\mathrm{e}^{2dx+2c}-1)\,\mathrm{e}^{-dx-c}}}{d}-\frac{\left(-\frac{2i\left(-i+i\mathrm{e}^{2dx+2c}\right)}{\sqrt{\mathrm{e}^{dx+c}\left(-i+i\mathrm{e}^{2dx+2c}\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\,\sqrt{-2\,\mathrm{e}^{dx+c}+2}\,\sqrt{-\mathrm{e}^{dx}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(2*Elliptic E((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(d*x+c))^(1/2),1/ 2*2^(1/2)))/cosh(d*x+c)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(I*sinh(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 53, normalized size = 1.77

$$-\frac{2\left(\sqrt{\frac{1}{2}}\,\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\sqrt{2}\,\sqrt{i}\,\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -2*(sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + sqrt(2)*sq rt(I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{i\sinh(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*sinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((sinh(c + d*x)*1i)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2720}

$$-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.93

$$\frac{2iF\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Maple [A]
time = 0.52, size = 68, normalized size = 2.27

method	result
default	$\frac{i\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)},\frac{\sqrt{2}}{2}\right)}{\cosh\left(dx+c\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*EllipticF((
-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 20, normalized size = 0.67

$$-\frac{2i\sqrt{2}\sqrt{i}\,\text{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*I*sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, e^(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{i\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(I*sinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(1/2), x)
```

3.28 $\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*
EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2*I*cosh(d*x+c)/d/(I*sin
h(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2716, 2719}

$$\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*S
qrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx &= \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}-\int \sqrt{i\sinh(c+dx)}\,dx\\ &= \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.86

$$\frac{2\left(-iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)+\coth(c+dx)\sqrt{i\sinh(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*((-I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + Coth[c + d*x]*Sqrt[I
*Sinh[c + d*x]]))/d
```

Maple [A]
time = 0.82, size = 159, normalized size = 2.74

method	result
default	$-\frac{i\left(2\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\left(dx\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c
))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(
1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*s
inh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 87, normalized size = 1.50

$$\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{ie^{(2dx+2c)}-i}\,e^{\left(\frac{3}{2}dx+\frac{3}{2}c\right)}+\left(\sqrt{2}\sqrt{i}\,e^{(2dx+2c)}-\sqrt{2}\sqrt{i}\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)}{de^{(2dx+2c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(3/2*d*x + 3/2*c) + (sqrt(2)*s
qrt(I)*e^(2*d*x + 2*c) - sqrt(2)*sqrt(I))*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, e^(d*x + c))))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i \sinh (c+dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(\sinh (c+d\,x)\ 1\mathrm{i})^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(3/2), x)
```

3.29 $\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)/d/(I*
sinh(d*x+c))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2716, 2720}

$$\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2iF\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx &= \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2iF\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.05, size = 83, normalized size = 1.34

$$\frac{2\left(\coth(c+dx)+\sqrt{2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{-\left((1+\coth(c+dx))\sinh^2(c+dx)\right)}\right)}{3d\sqrt{i\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3*
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A]
time = 0.67, size = 113, normalized size = 1.82

method	result
default	$-\frac{i\left(-\sqrt{1-i\sinh\left(dx+c\right)}\,\sqrt{2}\,\sqrt{1+i\sinh\left(dx+c\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\mathrm{EllipticF}\left(\sqrt{1-i\sinh\left(dx+\right.\right.\right.}}{3\sinh(dx+c)\cosh(dx+c)\sqrt{i\sinh\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/sinh(d*x+c)*(-(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2
)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*sinh
(d*x+c)+2*I*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 132, normalized size = 2.13

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(i\,e^{(3\,dx+3\,c)}+i\,e^{(dx+c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\left(i\,\sqrt{2}\,\sqrt{i}\,e^{(4\,dx+4\,c)}-2i\,\sqrt{2}\,\sqrt{i}\,e^{(2\,dx+2\,c)}+i\,\sqrt{2}\,\sqrt{i}\right)\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)}{3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(1/2)*(I*e^(3*d*x + 3*c) + I*e^(d*x + c))*sqrt(I*e^(2*d*x + 2*c
) - I)*e^(-1/2*d*x - 1/2*c) + (I*sqrt(2)*sqrt(I)*e^(4*d*x + 4*c) - 2*I*sqrt
(2)*sqrt(I)*e^(2*d*x + 2*c) + I*sqrt(2)*sqrt(I))*weierstrassPInverse(4, 0,
e^(d*x + c)))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\sinh(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(5/2), x)
```

3.30 $\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x
)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)/d/(I
*sinh(d*x+c))^(5/2)+6/5*I*cosh(d*x+c)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2716, 2719}

$$\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d*
x])/(d*(I*Sinh[c + d*x])^(5/2)) + (((6*I)/5)*Cosh[c + d*x])/(d*Sqrt[I*Sinh[
c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx &= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{3}{5}\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx \\
&= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}} - \frac{3}{5}\int \sqrt{i\sinh(c+dx)}\,dx \\
&= \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d} + \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 80, normalized size = 0.88

$$
-\frac{2i\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5d\sqrt{i\sinh(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((-2*I)/5)*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[(
(-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(d*Sqrt[I*Sinh[c +
 d*x]])
```

Maple [A]
time = 0.74, size = 204, normalized size = 2.24

method	result
default	$-\frac{i\left(6\sqrt{-i\left(\sinh\left(dx+c\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(i-\sinh\left(dx+c\right)\right)}\,\sqrt{i\sinh\left(dx+c\right)}\,\left(\sinh^2(dx+c)\right)\operatorname{EllipticE}\left(\right.\right.}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(I*sin
h(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 178, normalized size = 1.96

$$\frac{2\left(2\sqrt{\frac{1}{2}}\left(3\,e^{(6\,dx+6\,c)}-8\,e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+3\left(\sqrt{2}\,\sqrt{i}\,e^{(6\,dx+6\,c)}-3\,\sqrt{2}\,\sqrt{i}\,e^{(4\,dx+4\,c)}+3\,\sqrt{2}\,\sqrt{i}\,e^{(2\,dx+2\,c)}-\sqrt{2}\,\sqrt{i}\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)}{5\left(de^{(6\,dx+6\,c)}-3\,de^{(4\,dx+4\,c)}+3\,de^{(2\,dx+2\,c)}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(2*sqrt(1/2)*(3*e^(6*d*x + 6*c) - 8*e^(4*d*x + 4*c) + e^(2*d*x + 2*c))*
sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 3*(sqrt(2)*sqrt(I)*e^(6*
d*x + 6*c) - 3*sqrt(2)*sqrt(I)*e^(4*d*x + 4*c) + 3*sqrt(2)*sqrt(I)*e^(2*d*x
 + 2*c) - sqrt(2)*sqrt(I))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
e^(d*x + c))))/(d*e^(6*d*x + 6*c) - 3*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*
c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\sinh(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(\sinh(c+d\,x)\,1\mathrm{i})^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(1/(sinh(c + d*x)*1i)^(7/2), x)
```

3.31 $\int (b\sinh(c+dx))^{4/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/7*cosh(d*x+c)*hypergeom([1/2, 7/6],[13/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(7/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{4/3}\,dx = \frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}\tanh(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(4/3)*Tanh[c + d*x])/(7*d)
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (b \sinh (dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)*b*sinh(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sinh (c+dx))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b \sinh(c + d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(4/3),x)
```

```
[Out] int((b*sinh(c + d*x))^(4/3), x)
```

3.32 $\int (b\sinh(c+dx))^{2/3}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/5*cosh(d*x+c)*hypergeom([1/2, 5/6],[11/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(5/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^{2/3}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}\tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(2/3)*Tanh[c + d*x])/(5*d)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b\sinh (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh (c+dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\sinh(c + d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(2/3),x)
[Out] int((b*sinh(c + d*x))^(2/3), x)
```

3.33 $\int \sqrt[3]{b\sinh(c+dx)}\, dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/4*cosh(d*x+c)*hypergeom([1/2, 2/3],[5/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(4/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{4/3}\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)}{4bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\sinh(c+dx)}\, dx = \frac{3\cosh(c+dx)\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}\tanh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]
*(b*Sinh[c + d*x])^(1/3)*Tanh[c + d*x])/(4*d)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b \sinh (dx+c))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int((b*sinh(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sinh (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\sinh(c+d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^(1/3),x)
```

```
[Out] int((b*sinh(c + d*x))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/2*cosh(d*x+c)*hypergeom([1/3, 1/2],[4/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(2/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2722\}$

$$\frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx=\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\sinh^2(c+dx)\right)\tanh(c+dx)}{2d\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(2*d*(b*Sinh[c + d*x])^(1/3))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sinh \left(d x + c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b*sinh(d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b \sinh \left(c + d x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\sinh\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(1/3),x)
[Out] int(1/(b*sinh(c + d*x))^(1/3), x)
```

3.35 $\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3*cosh(d*x+c)*hypergeom([1/6, 1/2],[7/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^(1
/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2722\}$

$$\frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{3\cosh(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(2/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(2/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(1/3)/(b*sinh(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sinh(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,\sinh\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(2/3),x)
```

```
[Out] int(1/(b*sinh(c + d*x))^(2/3), x)
```

3.36 $\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

```
[Out] -3*cosh(d*x+c)*hypergeom([-1/6, 1/2],[5/6],-sinh(d*x+c)^2)/b/d/(b*sinh(d*x+
c))^(1/3)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*d
*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^
2]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(4/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*sinh(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^(2/3)/(b^2*sinh(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\sinh(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sinh(c + d*x))^(4/3),x)
[Out] int(1/(b*sinh(c + d*x))^(4/3), x)
```

3.37 $\int (b\sinh(c+dx))^n\,dx$

Optimal. Leaf size=70

$$\frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(b*sinh(
d*x+c))^(1+n)/b/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$\frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sinh(c+dx))^n\,dx = \frac{\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(b\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(b*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(b\sinh \left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(d*x+c))^n,x)
```

```
[Out] int((b*sinh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh \left(c+dx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))**n,x)
```

```
[Out] Integral((b*sinh(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\sinh(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x))^n,x)
[Out] int((b*sinh(c + d*x))^n, x)
```

3.38 $\int (i \sinh(c+dx))^n\, dx$

Optimal. Leaf size=72

$$-\frac{i \cosh(c+dx)\, {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; -\sinh^2(c+dx)\right) (i \sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] -I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{i \cosh(c+dx)(i \sinh(c+dx))^{n+1}\, {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; -\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(I*Sinh[c + d*x])^n,x]
```

```
[Out] ((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (i \sinh(c+dx))^n\, dx = -\frac{i \cosh(c+dx)\, {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; -\sinh^2(c+dx)\right) (i \sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\, {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; -\sinh^2(c+dx)\right) (i \sinh(c+dx))^n \tanh(c+dx)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(I*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(I*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [**F**]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(i \sinh \left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((I*sinh(d*x+c))^n,x)
```

```
[Out] int((I*sinh(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(I*e^(2*d*x + 2*c) - I)*e^(-d*x - c))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(i \sinh \left(c+dx\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((I*sinh(c + d*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (\sinh(c+d\,x)\ 1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(c + d*x)*1i)^n,x)
[Out] int((sinh(c + d*x)*1i)^n, x)
```

3.39 $\int (-i\sinh(c+dx))^n \, dx$

Optimal. Leaf size=72

$$\frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(-I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x
]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (-i\sinh(c+dx))^n \, dx = \frac{i\cosh(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.93

$$\frac{\sqrt{\cosh^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*((-I)*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (-i\sinh(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*sinh(d*x+c))^n,x)
```

```
[Out] int((-I*sinh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(-I*e^(2*d*x + 2*c) + I)*e^(-d*x - c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-i\sinh(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))**n,x)
```

```
[Out] Integral((-I*sinh(c + d*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (-\sinh(c + d\,x)\ 1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sinh(c + d*x)*1i)^n,x)
[Out] int((-sinh(c + d*x)*1i)^n, x)
```

3.40 $\int \frac{\sinh^4(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=46

$$\frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}$$

```
[Out] 3/2*I*x-4*cosh(x)+4/3*cosh(x)^3-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)^3/(I+
sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2846, 2827, 2715, 8, 2713}

$$\frac{3ix}{2} + \frac{4\cosh^3(x)}{3} - 4\cosh(x) - \frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i} - \frac{3}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((3*I)/2)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 - ((3*I)/2)*Cosh[x]*Sinh[x] - (Co
sh[x]*Sinh[x]^3)/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}+\int \sinh^2(x)(-3i+4\sinh(x))\,dx\\ &= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}-3i\int \sinh^2(x)\,dx+4\int \sinh^3(x)\,dx\\ &= -\frac{3}{2}i\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}+\frac{3}{2}i\int 1\,dx-4\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh\right.\\ &= \frac{3ix}{2}-4\cosh(x)+\frac{4\cosh^3(x)}{3}-\frac{3}{2}i\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.
time = 0.14, size = 134, normalized size = 2.91

$$\frac{\cosh(x)\left(-16i\left(\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+\sqrt{\cosh^2(x)}\right)-\left(16\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+7\sqrt{\cosh^2(x)}\right)\sinh(x)-i\sqrt{\cosh^2(x)}\sinh^2(x)+2\sqrt{\cosh^2(x)}\sinh^3(x)+i\sinh^{-1}(\sinh(x))(i+\sinh(x))\right)}{6\sqrt{\cosh^2(x)}\,(i+\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-16*I)*(ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + Sqrt[Cosh[x]^2]) -
 (16*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + 7*Sqrt[Cosh[x]^2])*Sinh[x] - I*S
qrt[Cosh[x]^2]*Sinh[x]^2 + 2*Sqrt[Cosh[x]^2]*Sinh[x]^3 + I*ArcSinh[Sinh[x]]
*(I + Sinh[x])))/(6*Sqrt[Cosh[x]^2]*(I + Sinh[x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(37) = 74.
time = 0.62, size = 102, normalized size = 2.22

method	result

risch	$\frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7\,e^{x}}{8}-\frac{7\,e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}-\frac{2}{e^{x}+i}$
default	$\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{-\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{-\frac{3}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{\frac{3}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*ln(tanh(1/2*x)+1)+(-1/2+1/2*I)/(tanh(1/2*x)+1)^2-(3/2+1/2*I)/(tanh(1/
2*x)+1)+1/3/(tanh(1/2*x)+1)^3-3/2*I*ln(tanh(1/2*x)-1)+(3/2-1/2*I)/(tanh(1/2
*x)-1)-(1/2+1/2*I)/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3-2*I/(tanh(1/2*x)
+I)
```

Maxima [A]
time = 0.28, size = 59, normalized size = 1.28

$$\frac{3}{2}i\,x-\frac{2\,e^{(-x)}-18i\,e^{(-2\,x)}+69\,e^{(-3\,x)}+i}{8\left(-3i\,e^{(-3\,x)}+3\,e^{(-4\,x)}\right)}-\frac{7}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x - 1/8*(2*e^(-x) - 18*I*e^(-2*x) + 69*e^(-3*x) + I)/(-3*I*e^(-3*x) +
 3*e^(-4*x)) - 7/8*e^(-x) + 1/8*I*e^(-2*x) + 1/24*e^(-3*x)
```

Fricas [A]
time = 0.45, size = 67, normalized size = 1.46

$$-\frac{3\left(-12i\,x+7i\right)e^{(4\,x)}+3\left(12\,x+23\right)e^{(3\,x)}-e^{(7\,x)}+2i\,e^{(6\,x)}+18\,e^{(5\,x)}+18i\,e^{(2\,x)}+2\,e^{x}-i}{24\left(e^{(4\,x)}+i\,e^{(3\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(-12*I*x + 7*I)*e^(4*x) + 3*(12*x + 23)*e^(3*x) - e^(7*x) + 2*I*e^
(6*x) + 18*e^(5*x) + 18*I*e^(2*x) + 2*e^x - I)/(e^(4*x) + I*e^(3*x))
```

Sympy [A]
time = 0.09, size = 58, normalized size = 1.26

$$\frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7e^{x}}{8}-\frac{7e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}-\frac{2}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x)),x)
```

[Out] 3*I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 - 7*exp(x)/8 - 7*exp(-x)/8 + I*exp(-2*x)/8 + exp(-3*x)/24 - 2/(exp(x) + I)

Giac [A]
time = 0.43, size = 50, normalized size = 1.09

$$\frac{3}{2}i\,x - \frac{\left(69\,e^{(3\,x)} + 18i\,e^{(2\,x)} + 2\,e^{x} - i\right)e^{(-3\,x)}}{24\,(e^{x} + i)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="giac")

[Out] 3/2*I*x - 1/24*(69*e^(3*x) + 18*I*e^(2*x) + 2*e^x - I)*e^(-3*x)/(e^x + I) + 1/24*e^(3*x) - 1/8*I*e^(2*x) - 7/8*e^x

Mupad [B]
time = 0.49, size = 50, normalized size = 1.09

$$\frac{x\,3\mathrm{i}}{2} - \frac{7\,\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{7\,\mathrm{e}^{x}}{8} - \frac{2}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^4/(sinh(x) + 1i),x)

[Out] (x*3i)/2 - (7*exp(-x))/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/24 + exp(3*x)/24 - (7*exp(x))/8 - 2/(exp(x) + 1i)

3.41 $\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{3x}{2}-2i\cosh(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}$$

```
[Out] -3/2*x-2*I*cosh(x)+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2846, 2813}

$$-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I +
 Sinh[x])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}+\int \sinh(x)(-2i+3\sinh(x))\,dx\\ &= -\frac{3x}{2}-2i\cosh(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 41, normalized size = 1.14

$$\frac{1}{2}\cosh(x)\left(-\frac{3\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}+\frac{4-i\sinh(x)+\sinh^2(x)}{i+\sinh(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-3*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (4 - I*Sinh[x] + Sinh[x]^
2)/(I + Sinh[x])))/2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(30) = 60.
time = 0.55, size = 75, normalized size = 2.08

method	result
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{i e^{x}}{2}-\frac{i e^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$
default	$\frac{\frac{1}{2}-i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{\frac{1}{2}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-I)/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+(1/2+I)
/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*ln(tanh(1/2*x)-1)+2/(tanh(1/2*x)
+I)
```

Maxima [**A**]
time = 0.29, size = 45, normalized size = 1.25

$$-\frac{3}{2}x-\frac{3e^{(-x)}+20i\,e^{(-2x)}+i}{8\left(-i\,e^{(-2x)}+e^{(-3x)}\right)}-\frac{1}{2}i\,e^{(-x)}-\frac{1}{8}\,e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/8*(3*e^(-x) + 20*I*e^(-2*x) + I)/(-I*e^(-2*x) + e^(-3*x)) - 1/2*
I*e^(-x) - 1/8*e^(-2*x)
```

Fricas [**A**]
time = 0.50, size = 55, normalized size = 1.53

$$-\frac{4\,(3\,x-1)e^{(3\,x)}+4\,(3i\,x+5i)e^{(2\,x)}-e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{x}+i}{8\,\left(e^{(3\,x)}+i\,e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*x - 1)*e^(3*x) + 4*(3*I*x + 5*I)*e^(2*x) - e^(5*x) + 3*I*e^(4*x)
 - 3*e^x + I)/(e^(3*x) + I*e^(2*x))
```

Sympy [A]
time = 0.07, size = 41, normalized size = 1.14

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x)),x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x)/2 - I*exp(-x)/2 - exp(-2*x)/8 - 2*I/(exp(x)
+ I)
```

Giac [A]
time = 0.43, size = 38, normalized size = 1.06

$$-\frac{3}{2}x-\frac{\left(20i\,e^{(2\,x)}-3\,e^{x}+i\right)e^{(-2\,x)}}{8\,(e^{x}+i)}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/8*(20*I*e^(2*x) - 3*e^x + I)*e^(-2*x)/(e^x + I) + 1/8*e^(2*x) -
1/2*I*e^x
```

Mupad [B]
time = 0.43, size = 38, normalized size = 1.06

$$\frac{\mathrm{e}^{2\,x}}{8}-\frac{\mathrm{e}^{-x}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{-2\,x}}{8}-\frac{3\,x}{2}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}-\frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] exp(2*x)/8 - (exp(-x)*1i)/2 - exp(-2*x)/8 - (3*x)/2 - (exp(x)*1i)/2 - 2i/(e
xp(x) + 1i)
```

3.42 $\int \frac{\sinh^2(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=22

$$-ix + \cosh(x) + \frac{i\cosh(x)}{i + \sinh(x)}$$

```
[Out] -I*x+cosh(x)+I*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2825, 2814, 2727}

$$-ix + \cosh(x) + \frac{i\cosh(x)}{\sinh(x) + i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x] + (I*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx &= \cosh(x) - i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx \\ &= -ix + \cosh(x) - \int \frac{1}{i+\sinh(x)}\,dx \\ &= -ix + \cosh(x) + \frac{i\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 79 vs. $2(22) = 44$.
time = 0.08, size = 79, normalized size = 3.59

$$\frac{\cosh(x)\left(2i + \frac{2i\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} + \sinh(x) + \frac{2\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sinh(x)}{\sqrt{\cosh^2(x)}}\right)}{i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*(2*I + ((2*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
 + Sinh[x] + (2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sinh[x])/Sqrt[Cosh[x]^2
]))/(I + Sinh[x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.
time = 0.61, size = 52, normalized size = 2.36

method	result	size
risch	$-ix + \frac{e^{x}}{2} + \frac{e^{-x}}{2} + \frac{2}{e^{x}+i}$	25
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{1}{\tanh\left(\frac{x}{2}\right)-1} - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{\tanh\left(\frac{x}{2}\right)+1} + \frac{2i}{\tanh\left(\frac{x}{2}\right)+i}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)-I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)
+2*I/(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

time = 0.28, size = 33, normalized size = 1.50

$$-i\,x+\frac{5\,e^{(-x)}-i}{2\,\left(-i\,e^{(-x)}+e^{(-2\,x)}\right)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*x + 1/2*(5*e^(-x) - I)/(-I*e^(-x) + e^(-2*x)) + 1/2*e^(-x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.
time = 0.57, size = 37, normalized size = 1.68

$$\frac{(-2i\,x+i)e^{(2\,x)}+(2\,x+5)e^{x}+e^{(3\,x)}+i}{2\,\left(e^{(2\,x)}+i\,e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/2*((-2*I*x + I)*e^(2*x) + (2*x + 5)*e^x + e^(3*x) + I)/(e^(2*x) + I*e^x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.91

$$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{2}{e^{x}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x)),x)
[Out] -I*x + exp(x)/2 + exp(-x)/2 + 2/(exp(x) + I)
```

Giac [A]
time = 0.42, size = 26, normalized size = 1.18

$$-i\,x+\frac{(5\,e^{x}+i)e^{(-x)}}{2\,(e^{x}+i)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] -I*x + 1/2*(5*e^x + I)*e^(-x)/(e^x + I) + 1/2*e^x
```

Mupad [B]
time = 0.43, size = 24, normalized size = 1.09

$$\frac{\mathrm{e}^{-x}}{2}-x\,1\mathrm{i}+\frac{\mathrm{e}^{x}}{2}+\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(sinh(x) + 1i),x)
[Out] exp(-x)/2 - x*1i + exp(x)/2 + 2/(exp(x) + 1i)
```

3.43 $\int \frac{\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=14

$$x-\frac{\cosh(x)}{i+\sinh(x)}$$

```
[Out] x-cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2814, 2727}

$$x-\frac{\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x]),x]
[Out] x - Cosh[x]/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{i+\sinh(x)}\,dx &= x-i\int \frac{1}{i+\sinh(x)}\,dx\\ &= x-\frac{\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.
time = 0.03, size = 29, normalized size = 2.07

$$x-\frac{2\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] x - (2*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.36, size = 29, normalized size = 2.07

method	result	size
risch	$x + \frac{2i}{e^x + i}$	13
default	$-\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) - \frac{2}{\tanh\left(\frac{x}{2}\right) + i}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-2/(tanh(1/2*x)+I)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.86

$$x + \frac{2i}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] x + 2*I/(e^(-x) - I)
```

Fricas [A]
time = 0.70, size = 16, normalized size = 1.14

$$\frac{xe^x + i\,x + 2i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (x*e^x + I*x + 2*I)/(e^x + I)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.57

$$x + \frac{2i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x)
```

```
[Out] x + 2*I/(exp(x) + I)
```

Giac [A]
time = 0.42, size = 10, normalized size = 0.71

$$x + \frac{2i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] x + 2*I/(e^x + I)
```

Mupad [B]
time = 0.41, size = 12, normalized size = 0.86

$$x + \frac{2\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(sinh(x) + 1i),x)
```

```
[Out] x + 2i/(exp(x) + 1i)
```

3.44 $\int \frac{\operatorname{csch}(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=19

$$i \tanh^{-1}(\cosh(x)) + \frac{\cosh(x)}{i + \sinh(x)}$$

```
[Out] I*arctanh(cosh(x))+cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2826, 2727, 3855}

$$\frac{\cosh(x)}{\sinh(x) + i} + i \tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] + Cosh[x]/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(x)}{i + \sinh(x)} \, dx &= -\left(i \int \operatorname{csch}(x) \, dx\right) + i \int \frac{1}{i + \sinh(x)} \, dx \\ &= i \tanh^{-1}(\cosh(x)) + \frac{\cosh(x)}{i + \sinh(x)} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.58

$$\operatorname{sech}(x)\left(-i+i\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}+\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] Sech[x]*(-I + I*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + Sinh[x])
```

Maple [A]
time = 0.48, size = 21, normalized size = 1.11

method	result	size
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$	21
risch	$-\frac{2i}{e^x+i}+i\ln\left(e^x+1\right)-i\ln\left(e^x-1\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(tanh(1/2*x))+2/(tanh(1/2*x)+I)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.53

$$-\frac{2i}{e^{(-x)}-i}+i\log\left(e^{(-x)}+1\right)-i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*I/(e^(-x) - I) + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.46, size = 33, normalized size = 1.74

$$\frac{(i\,e^x-1)\log\left(e^x+1\right)+(-i\,e^x+1)\log\left(e^x-1\right)-2i}{e^x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] ((I*e^x - 1)*log(e^x + 1) + (-I*e^x + 1)*log(e^x - 1) - 2*I)/(e^x + I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{\sinh(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I), x)
```

Giac [A]
time = 0.44, size = 24, normalized size = 1.26

$$-\frac{2i}{e^{x} + i} + i\,\log\left(e^{x} + 1\right) - i\,\log\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -2*I/(e^x + I) + I*log(e^x + 1) - I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.49, size = 35, normalized size = 1.84

$$-\ln\left(\mathrm{e}^{x}\,2\mathrm{i} - 2\mathrm{i}\right)\,1\mathrm{i} + \ln\left(\mathrm{e}^{x}\,2\mathrm{i} + 2\mathrm{i}\right)\,1\mathrm{i} - \frac{2\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i - 2i/(exp(x) + 1i)
```

3.45 $\int \frac{\mathbf{csch}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$-\tanh^{-1}(\cosh(x))+2i\coth(x)+\frac{\coth(x)}{i+\sinh(x)}$$

```
[Out] -arctanh(cosh(x))+2*I*coth(x)+coth(x)/(I+sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2847, 2827, 3852, 8, 3855}

$$2i\coth(x)-\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + (2*I)*Coth[x] + Coth[x]/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)}{i+\sinh(x)}+\int \operatorname{csch}^2(x)(-2i+\sinh(x))\,dx\\
&= \frac{\coth(x)}{i+\sinh(x)}-2i\int \operatorname{csch}^2(x)\,dx+\int \operatorname{csch}(x)\,dx\\
&= -\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{i+\sinh(x)}-2\text{Subst}(\int 1\,dx,x,-i\coth(x))\\
&= -\tanh^{-1}(\cosh(x))+2i\coth(x)+\frac{\coth(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 1.57

$$\operatorname{sech}(x)\left(1-\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}+i\operatorname{csch}(x)+2i\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x]),x]
[Out] Sech[x]*(1 - ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + I*Csch[x] + (2*I)*S
inh[x])
```

Maple [A]
time = 0.53, size = 35, normalized size = 1.52

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	35
risch	$\frac{-4+2i\mathrm{e}^{x}+2\,\mathrm{e}^{2x}}{(\mathrm{e}^{2x}-1)(\mathrm{e}^{x}+i)}-\ln\left(\mathrm{e}^{x}+1\right)+\ln\left(\mathrm{e}^{x}-1\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*tanh(1/2*x)+2*I/(tanh(1/2*x)+I)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.
time = 0.27, size = 51, normalized size = 2.22

$$-\frac{2\left(-i\,e^{(-x)} + e^{(-2\,x)} - 2\right)}{e^{(-x)} + i\,e^{(-2\,x)} - e^{(-3\,x)} - i} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(-I*e^(-x) + e^(-2*x) - 2)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) - log(e^
(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(19) = 38$.
time = 0.44, size = 77, normalized size = 3.35

$$-\frac{\left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i\right)\log\left(e^{x} + 1\right) - \left(e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i\right)\log\left(e^{x} - 1\right) - 2\,e^{(2\,x)} - 2i\,e^{x} + 4}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -((e^(3*x) + I*e^(2*x) - e^x - I)*log(e^x + 1) - (e^(3*x) + I*e^(2*x) - e^x
 - I)*log(e^x - 1) - 2*e^(2*x) - 2*I*e^x + 4)/(e^(3*x) + I*e^(2*x) - e^x -
I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{\sinh(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(sinh(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.
time = 0.42, size = 44, normalized size = 1.91

$$\frac{2\left(e^{(2\,x)} + i\,e^{x} - 2\right)}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i} - \log\left(e^{x} + 1\right) + \log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) + I*e^x - 2)/(e^(3*x) + I*e^(2*x) - e^x - I) - log(e^x + 1) + lo
g(abs(e^x - 1))
```

Mupad [B]
time = 0.64, size = 51, normalized size = 2.22

$$\ln(2-2\,\mathrm{e}^{x})-\ln(-2\,\mathrm{e}^{x}-2)+\frac{2\,\mathrm{e}^{2\,x}-4+\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i}+\mathrm{e}^{3\,x}-\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)),x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + (2*exp(2*x) + exp(x)*2i - 4)/(exp
(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.46 $\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{3}{2}i\tanh^{-1}(\cosh(x))-2\coth(x)+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}$$

```
[Out] -3/2*I*arctanh(cosh(x))-2*coth(x)+3/2*I*coth(x)*csch(x)+coth(x)*csch(x)/(I+
sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$-2\coth(x)-\frac{3}{2}i\tanh^{-1}(\cosh(x))+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/2)*ArcTanh[Cosh[x]] - 2*Coth[x] + ((3*I)/2)*Coth[x]*Csch[x] + (Coth
[x]*Csch[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}+\int \operatorname{csch}^3(x)(-3i+2\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}-3i\int \operatorname{csch}^3(x)\,dx+2\int \operatorname{csch}^2(x)\,dx\\ &= \frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}+\frac{3}{2}i\int \operatorname{csch}(x)\,dx-2i\mathrm{Subst}(\int 1\,dx,x,-i\coth(x)\\ &= -\frac{3}{2}i\tanh^{-1}(\cosh(x))-2\coth(x)+\frac{3}{2}i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 49, normalized size = 1.32

$$\frac{1}{2}i\left(4i+3\operatorname{csch}(x)-3\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}\operatorname{csch}(x)+2i\operatorname{csch}^2(x)+\operatorname{csch}^3(x)\right)\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] (I/2)*(4*I + 3*Csch[x] - 3*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2]*Csch[x]
 + (2*I)*Csch[x]^2 + Csch[x]^3)*Tanh[x]
```

Maple [A]
time = 0.58, size = 53, normalized size = 1.43

method	result	size

default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{i\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{8\tanh\left(\frac{x}{2}\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	53
risch	$\frac{i(3\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+3i\mathrm{e}^{3x}+4-i\mathrm{e}^{x})}{(\mathrm{e}^{2x}-1)^2(\mathrm{e}^{x}+i)}+\frac{3i\ln(\mathrm{e}^{x}-1)}{2}-\frac{3i\ln(\mathrm{e}^{x}+1)}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*tanh(1/2*x)-1/8*I*tanh(1/2*x)^2-2/(tanh(1/2*x)+I)+1/8*I/tanh(1/2*x)^2+
3/2*I*ln(tanh(1/2*x))-1/2/tanh(1/2*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.
time = 0.27, size = 75, normalized size = 2.03

$$-\frac{e^{(-x)}+5i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-4i}{e^{(-x)}+2i\,e^{(-2\,x)}-2\,e^{(-3\,x)}-i\,e^{(-4\,x)}+e^{(-5\,x)}-i}-\frac{3}{2}i\,\log\left(e^{(-x)}+1\right)+\frac{3}{2}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) - 4*I)/(e^(-x) + 2*I*e^
(-2*x) - 2*e^(-3*x) - I*e^(-4*x) + e^(-5*x) - I) - 3/2*I*log(e^(-x) + 1) +
3/2*I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(27) = 54$.
time = 0.44, size = 126, normalized size = 3.41

$$-\frac{3\left(i\,e^{(5\,x)}-e^{(4\,x)}-2i\,e^{(3\,x)}+2\,e^{(2\,x)}+i\,e^{x}-1\right)\log\left(e^{x}+1\right)+3\left(-i\,e^{(5\,x)}+e^{(4\,x)}+2i\,e^{(3\,x)}-2\,e^{(2\,x)}-i\,e^{x}+1\right)\log\left(e^{x}-1\right)-6i\,e^{(4\,x)}+6\,e^{(3\,x)}+10i\,e^{(2\,x)}-2\,e^{x}-8i}{2\left(e^{(5\,x)}+i\,e^{(4\,x)}-2\,e^{(3\,x)}-2i\,e^{(2\,x)}+e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(3*(I*e^(5*x) - e^(4*x) - 2*I*e^(3*x) + 2*e^(2*x) + I*e^x - 1)*log(e^x
 + 1) + 3*(-I*e^(5*x) + e^(4*x) + 2*I*e^(3*x) - 2*e^(2*x) - I*e^x + 1)*log(
e^x - 1) - 6*I*e^(4*x) + 6*e^(3*x) + 10*I*e^(2*x) - 2*e^x - 8*I)/(e^(5*x) +
 I*e^(4*x) - 2*e^(3*x) - 2*I*e^(2*x) + e^x + I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^3(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**3/(sinh(x) + I), x)
```

Giac [A]
time = 0.42, size = 51, normalized size = 1.38

$$\frac{i\,e^{(3\,x)}-2\,e^{(2\,x)}+i\,e^{x}+2}{\left(e^{(2\,x)}-1\right)^{2}}+\frac{2i}{e^{x}+i}-\frac{3}{2}i\,\log\left(e^{x}+1\right)+\frac{3}{2}i\,\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] (I*e^(3*x) - 2*e^(2*x) + I*e^x + 2)/(e^(2*x) - 1)^2 + 2*I/(e^x + I) - 3/2*I
*log(e^x + 1) + 3/2*I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.66, size = 70, normalized size = 1.89

$$-\frac{\ln\left(-\mathrm{e}^{x}\,3\mathrm{i}-3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{\ln\left(-\mathrm{e}^{x}\,3\mathrm{i}+3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}+\frac{\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{-2+\mathrm{e}^{x}\,1\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (log(3i - exp(x)*3i)*3i)/2 - (log(- exp(x)*3i - 3i)*3i)/2 + 2i/(exp(x) + 1i
) + (exp(x)*2i)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*1i - 2)/(exp(2*x) - 1
)
```

3.47 $\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{2}\tanh^{-1}(\cosh(x)) - 4i\coth(x) + \frac{4}{3}i\coth^3(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}$$

```
[Out] 3/2*arctanh(cosh(x))-4*I*coth(x)+4/3*I*coth(x)^3-3/2*coth(x)*csch(x)+coth(x
)*csch(x)^2/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2847, 2827, 3852, 3853, 3855}

$$\frac{4}{3}i\coth^3(x) - 4i\coth(x) + \frac{3}{2}\tanh^{-1}(\cosh(x)) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 - (4*I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - (3*Coth[x]*C
sch[x])/2 + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} + \int \operatorname{csch}^4(x)(-4i+3\sinh(x))\,dx \\
&= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} - 4i\int \operatorname{csch}^4(x)\,dx + 3\int \operatorname{csch}^3(x)\,dx \\
&= -\frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} - \frac{3}{2}\int \operatorname{csch}(x)\,dx + 4\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\right. \\
&= \frac{3}{2}\tanh^{-1}(\cosh(x)) - 4i\coth(x) + \frac{4}{3}i\coth^3(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 53, normalized size = 1.13

$$\frac{1}{6}\operatorname{sech}(x)\left(-9+9\tanh^{-1}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)} - 8i\operatorname{csch}(x) - 3\operatorname{csch}^2(x) + 2i\operatorname{csch}^3(x) - 16i\sinh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x]),x]
[Out] (Sech[x]*(-9 + 9*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] - (8*I)*Csch[x] -
 3*Csch[x]^2 + (2*I)*Csch[x]^3 - (16*I)*Sinh[x]))/6
```

Maple [A]
time = 0.54, size = 71, normalized size = 1.51

method	result
default	$-\frac{7i\tanh(\frac{x}{2})}{8} + \frac{i(\tanh^3(\frac{x}{2}))}{24} + \frac{(\tanh^2(\frac{x}{2}))}{8} - \frac{2i}{\tanh(\frac{x}{2})+i} + \frac{i}{24\tanh(\frac{x}{2})^3} - \frac{7i}{8\tanh(\frac{x}{2})} - \frac{1}{8\tanh(\frac{x}{2})^2} - \frac{3\ln(\tanh(\frac{x}{2}))}{2}$

risch	$-\frac{9i e^{5x}-24 e^{4x}+9 e^{6x}-24i e^{3x}+39 e^{2x}+7i e^{x}-16}{3(e^{2x}-1)^{3}(e^{x}+i)}-\frac{3\ln(e^{x}-1)}{2}+\frac{3\ln(e^{x}+1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -7/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-2*I/(tanh(1/2*x)+
I)+1/24*I/tanh(1/2*x)^3-7/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2-3/2*ln(tanh(1/2
*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(35) = 70$.
time = 0.33, size = 103, normalized size = 2.19

$$\frac{-7i\,e^{(-x)}+39\,e^{(-2\,x)}+24i\,e^{(-3\,x)}-24\,e^{(-4\,x)}-9i\,e^{(-5\,x)}+9\,e^{(-6\,x)}-16}{3\left(e^{(-x)}+3i\,e^{(-2\,x)}-3\,e^{(-3\,x)}-3i\,e^{(-4\,x)}+3\,e^{(-5\,x)}+i\,e^{(-6\,x)}-e^{(-7\,x)}-i\right)}+\frac{3}{2}\,\log\left(e^{(-x)}+1\right)-\frac{3}{2}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(-7*I*e^(-x) + 39*e^(-2*x) + 24*I*e^(-3*x) - 24*e^(-4*x) - 9*I*e^(-5*x)
 + 9*e^(-6*x) - 16)/(e^(-x) + 3*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) + 3*
e^(-5*x) + I*e^(-6*x) - e^(-7*x) - I) + 3/2*log(e^(-x) + 1) - 3/2*log(e^(-x
) - 1)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(35) = 70$.
time = 0.39, size = 174, normalized size = 3.70

$$\frac{9\left(e^{(7\,x)}+i\,e^{(6\,x)}-3\,e^{(5\,x)}-3i\,e^{(4\,x)}+3\,e^{(3\,x)}+3i\,e^{(2\,x)}-e^{x}-i\right)\log\left(e^{x}+1\right)-9\left(e^{(7\,x)}+i\,e^{(6\,x)}-3\,e^{(5\,x)}-3i\,e^{(4\,x)}+3\,e^{(3\,x)}+3i\,e^{(2\,x)}-e^{x}-i\right)\log\left(e^{x}-1\right)-18\,e^{(6\,x)}-18i\,e^{(5\,x)}+48\,e^{(4\,x)}+48i\,e^{(3\,x)}-78\,e^{(2\,x)}-14i\,e^{x}+32}{6\left(e^{(7\,x)}+i\,e^{(6\,x)}-3\,e^{(5\,x)}-3i\,e^{(4\,x)}+3\,e^{(3\,x)}+3i\,e^{(2\,x)}-e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^(
2*x) - e^x - I)*log(e^x + 1) - 9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(
4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x - I)*log(e^x - 1) - 18*e^(6*x) - 18*I*
e^(5*x) + 48*e^(4*x) + 48*I*e^(3*x) - 78*e^(2*x) - 14*I*e^x + 32)/(e^(7*x)
+ I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x - I)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{4}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**4/(sinh(x) + I), x)
```

Giac [A]
time = 0.43, size = 58, normalized size = 1.23

$$-\frac{2}{e^x+i}-\frac{3\,e^{(5\,x)}+6i\,e^{(4\,x)}-24i\,e^{(2\,x)}-3\,e^x+10i}{3\,\left(e^{(2\,x)}-1\right)^3}+\frac{3}{2}\,\log\left(e^x+1\right)-\frac{3}{2}\,\log\left(\left|e^x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -2/(e^x + I) - 1/3*(3*e^(5*x) + 6*I*e^(4*x) - 24*I*e^(2*x) - 3*e^x + 10*I)/
(e^(2*x) - 1)^3 + 3/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.72, size = 85, normalized size = 1.81

$$\frac{3\,\ln\left(3\,\mathrm{e}^x+3\right)}{2}-\frac{3\,\ln\left(3\,\mathrm{e}^x-3\right)}{2}-\frac{\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}-\frac{2\,\mathrm{e}^x}{\left(\mathrm{e}^{2\,x}-1\right)^2}-\frac{2}{\mathrm{e}^x+1\mathrm{i}}-\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}+\frac{4\mathrm{i}}{\left(\mathrm{e}^{2\,x}-1\right)^2}+\frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)),x)
```

```
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) + 1i) - 2i/(exp(2*x) - 1) + 4i/(ex
p(2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

3.48 $\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=58

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)+\frac{7}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}$$

```
[Out] -7/2*x-16/3*I*cosh(x)+7/2*cosh(x)*sinh(x)-1/3*cosh(x)*sinh(x)^3/(I+sinh(x))
^2-8/3*cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2844, 3056, 2813}

$$-\frac{7x}{2}-\frac{16}{3}i\cosh(x)-\frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2}-\frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)}+\frac{7}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-7*x)/2 - ((16*I)/3)*Cosh[x] + (7*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^3)
/(3*(I + Sinh[x])^2) - (8*Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int \frac{\sinh^2(x)(-3i+5\sinh(x))}{i+\sinh(x)}\,dx\\ &= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}-\frac{1}{3}i\int (16+21i\sinh(x))\sinh(x)\,dx\\ &= -\frac{7x}{2}-\frac{16}{3}i\cosh(x)+\frac{7}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 65, normalized size = 1.12

$$\frac{1}{12}\left(-42x-24i\cosh(x)-\frac{4}{i+\sinh(x)}-\frac{8\sinh\left(\frac{x}{2}\right)(10i+11\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}+3\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-42*x - (24*I)*Cosh[x] - 4/(I + Sinh[x]) - (8*Sinh[x/2]*(10*I + 11*Sinh[x]
))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Sinh[2*x])/12
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(45) = 90.
time = 0.76, size = 98, normalized size = 1.69

method	result
risch	$-\frac{7x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}-\frac{2i(21ie^{x}+12e^{2x}-11)}{3(e^{x}+i)^3}$
default	$\frac{2i}{(\tanh(\frac{x}{2})+i)^2}+\frac{4}{3(\tanh(\frac{x}{2})+i)^3}+\frac{6}{\tanh(\frac{x}{2})+i}+\frac{\frac{1}{2}-2i}{\tanh(\frac{x}{2})+1}-\frac{1}{2(\tanh(\frac{x}{2})+1)^2}-\frac{7\ln(\tanh(\frac{x}{2})+1)}{2}+\frac{\frac{1}{2}+2i}{\tanh(\frac{x}{2})-1}+$ 2(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^2+4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+I)+(1/2-2*I)/(ta
nh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-7/2*ln(tanh(1/2*x)+1)+(1/2+2*I)/(tanh(1/
2*x)-1)+1/2/(tanh(1/2*x)-1)^2+7/2*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.28, size = 71, normalized size = 1.22

$$-\frac{7}{2}x+\frac{15\,e^{(-x)}+239i\,e^{(-2\,x)}-405\,e^{(-3\,x)}-216i\,e^{(-4\,x)}+3i}{8\left(3i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-9i\,e^{(-4\,x)}+3\,e^{(-5\,x)}\right)}-i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -7/2*x + 1/8*(15*e^(-x) + 239*I*e^(-2*x) - 405*e^(-3*x) - 216*I*e^(-4*x) +
3*I)/(3*I*e^(-2*x) - 9*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x)) - I*e^(-x) - 1
/8*e^(-2*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(42) = 84$.
time = 0.50, size = 89, normalized size = 1.53

$$-\frac{21\,(4\,x-3)e^{(5\,x)}+21\,(12i\,x+7i)e^{(4\,x)}-3\,(84\,x+127)e^{(3\,x)}-(84i\,x+239i)e^{(2\,x)}-3\,e^{(7\,x)}+15i\,e^{(6\,x)}+15\,e^{x}-3i}{24\left(e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{(3\,x)}-i\,e^{(2\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(21*(4*x - 3)*e^(5*x) + 21*(12*I*x + 7*I)*e^(4*x) - 3*(84*x + 127)*e^
(3*x) - (84*I*x + 239*I)*e^(2*x) - 3*e^(7*x) + 15*I*e^(6*x) + 15*e^x - 3*I)
/(e^(5*x) + 3*I*e^(4*x) - 3*e^(3*x) - I*e^(2*x))
```

Sympy [A]
time = 0.09, size = 70, normalized size = 1.21

$$-\frac{7x}{2}+\frac{-24ie^{2x}+42e^{x}+22i}{3e^{3x}+9ie^{2x}-9e^{x}-3i}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -7*x/2 + (-24*I*exp(2*x) + 42*exp(x) + 22*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9
*exp(x) - 3*I) + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Giac [A]
time = 0.43, size = 50, normalized size = 0.86

$$-\frac{7}{2}x-\frac{\left(216i\,e^{(4\,x)}-405\,e^{(3\,x)}-239i\,e^{(2\,x)}+15\,e^{x}-3i\right)e^{(-2\,x)}}{24\,(e^{x}+i)^{3}}+\frac{1}{8}\,e^{(2\,x)}-i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -7/2*x - 1/24*(216*I*e^(4*x) - 405*e^(3*x) - 239*I*e^(2*x) + 15*e^x - 3*I)*
e^(-2*x)/(e^x + I)^3 + 1/8*e^(2*x) - I*e^x
```

Mupad [B]
time = 0.57, size = 97, normalized size = 1.67

$$\frac{e^{2x}}{8} - e^{-x}\,1\mathrm{i} - \frac{e^{-2x}}{8} - \frac{7x}{2} - e^{x}\,1\mathrm{i} - \frac{-2 + \frac{e^{x}\,8\mathrm{i}}{3}}{e^{2x} - 1 + e^{x}\,2\mathrm{i}} + \frac{4\,e^{x} - \frac{e^{2x}\,8\mathrm{i}}{3} + \frac{8}{3}\mathrm{i}}{e^{2x}\,3\mathrm{i} + e^{3x} - 3\,e^{x} - \mathrm{i}} - \frac{8\mathrm{i}}{3\,(e^{x} + 1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (7*x)/2 - exp(x)*1i - ((exp(x)*8i)/
3 - 2)/(exp(2*x) + exp(x)*2i - 1) + (4*exp(x) - (exp(2*x)*8i)/3 + 8i/3)/(ex
p(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 8i/(3*(exp(x) + 1i))
```

3.49 $\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=44

$$-2ix + \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} + \frac{2i\cosh(x)}{i+\sinh(x)}$$

```
[Out] -2*I*x+4/3*cosh(x)-1/3*cosh(x)*sinh(x)^2/(I+sinh(x))^2+2*I*cosh(x)/(I+sinh(
x))
```

Rubi [A]
time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2844, 3047, 3102, 12, 2814, 2727}

$$-2ix + \frac{4\cosh(x)}{3} - \frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2} + \frac{2i\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*x + (4*Cosh[x])/3 - (Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x])^2) + ((2*I)
*Cosh[x])/(I + Sinh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
```

```
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int \frac{\sinh(x)(-2i+4\sinh(x))}{i+\sinh(x)}\,dx\\
&= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}i\int \frac{2\sinh(x)+4i\sinh^2(x)}{i+\sinh(x)}\,dx\\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int -\frac{6i\sinh(x)}{i+\sinh(x)}\,dx\\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx\\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-2\int \frac{1}{i+\sinh(x)}\,dx\\
&= -2ix+\frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{2i\cosh(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 45, normalized size = 1.02

$$\frac{1}{3}\cosh(x)\left(-\frac{6i\sinh^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}+\frac{-10+14i\sinh(x)+3\sinh^2(x)}{(i+\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(((-6*I)*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (-10 + (14*I)*Sinh[x]
 + 3*Sinh[x]^2)/(I + Sinh[x])^2))/3
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.
time = 0.82, size = 75, normalized size = 1.70

method	result
risch	$-2ix+\frac{e^x}{2}+\frac{e^{-x}}{2}+\frac{10ie^x+6e^{2x}-\frac{16}{3}}{(e^x+i)^3}$
default	$\frac{4i}{3(\tanh(\frac{x}{2})+i)^3}+\frac{4i}{\tanh(\frac{x}{2})+i}-\frac{2}{(\tanh(\frac{x}{2})+i)^2}+2i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh(\frac{x}{2})-1}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I/(tanh(1/2*x)+I)^3+4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)+I)^2+2*I*ln(tanh
(1/2*x)-1)-1/(tanh(1/2*x)-1)-2*I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)
```

Maxima [**A**]
time = 0.28, size = 59, normalized size = 1.34

$$-2i\,x-\frac{41\,e^{(-x)}+69i\,e^{(-2\,x)}-39\,e^{(-3\,x)}-3i}{2\,(3i\,e^{(-x)}-9\,e^{(-2\,x)}-9i\,e^{(-3\,x)}+3\,e^{(-4\,x)})}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/2*(41*e^(-x) + 69*I*e^(-2*x) - 39*e^(-3*x) - 3*I)/(3*I*e^(-x) -
9*e^(-2*x) - 9*I*e^(-3*x) + 3*e^(-4*x)) + 1/2*e^(-x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.
time = 0.38, size = 74, normalized size = 1.68

$$-\frac{3\,(4i\,x-3i)e^{(4\,x)}-6\,(6\,x+5)e^{(3\,x)}+6\,(-6i\,x-11i)e^{(2\,x)}+(12\,x+41)e^{x}-3\,e^{(5\,x)}+3i}{6\,(e^{(4\,x)}+3i\,e^{(3\,x)}-3\,e^{(2\,x)}-i\,e^{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(4*I*x - 3*I)*e^(4*x) - 6*(6*x + 5)*e^(3*x) + 6*(-6*I*x - 11*I)*e^(
2*x) + (12*x + 41)*e^x - 3*e^(5*x) + 3*I)/(e^(4*x) + 3*I*e^(3*x) - 3*e^(2*x
) - I*e^x)
```

Sympy [A]
time = 0.08, size = 54, normalized size = 1.23

$$-2ix + \frac{18e^{2x} + 30ie^{x} - 16}{3e^{3x} + 9ie^{2x} - 9e^{x} - 3i} + \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] -2*I*x + (18*exp(2*x) + 30*I*exp(x) - 16)/(3*exp(3*x) + 9*I*exp(2*x) - 9*ex
p(x) - 3*I) + exp(x)/2 + exp(-x)/2
```

Giac [A]
time = 0.42, size = 38, normalized size = 0.86

$$-2i\,x + \frac{\left(39\,e^{(3\,x)} + 69i\,e^{(2\,x)} - 41\,e^{x} - 3i\right)e^{(-x)}}{6\,\left(e^{x} + i\right)^{3}} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*x + 1/6*(39*e^(3*x) + 69*I*e^(2*x) - 41*e^x - 3*I)*e^(-x)/(e^x + I)^3
+ 1/2*e^x
```

Mupad [B]
time = 0.58, size = 79, normalized size = 1.80

$$\frac{e^{-x}}{2} - x\,2i + \frac{e^{x}}{2} + \frac{2\,e^{x} + \frac{4}{3}i}{e^{2\,x} - 1 + e^{x}\,2i} + \frac{2\,e^{2\,x} - 2 + \frac{e^{x}\,8i}{3}}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i} + \frac{2}{e^{x} + 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-x)/2 - x*2i + exp(x)/2 + (2*exp(x) + 4i/3)/(exp(2*x) + exp(x)*2i - 1)
+ (2*exp(2*x) + (exp(x)*8i)/3 - 2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
 + 2/(exp(x) + 1i)
```

3.50 $\int \frac{\sinh^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=32

$$x+\frac{i\cosh(x)}{3(i+\sinh(x))^2}-\frac{5\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] x+1/3*I*cosh(x)/(I+sinh(x))^2-5/3*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2837, 2814, 2727}

$$x-\frac{5\cosh(x)}{3(\sinh(x)+i)}+\frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x + ((I/3)*Cosh[x])/(I + Sinh[x])^2 - (5*Cosh[x])/(3*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{i\cosh(x)}{3(i+\sinh(x))^2} + \frac{1}{3}\int \frac{-2i+3\sinh(x)}{i+\sinh(x)}\,dx$$
$$= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5}{3}i\int \frac{1}{i+\sinh(x)}\,dx$$
$$= x + \frac{i\cosh(x)}{3(i+\sinh(x))^2} - \frac{5\cosh(x)}{3(i+\sinh(x))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.
time = 0.06, size = 74, normalized size = 2.31

$$\frac{3(-4i+3x)\cosh\left(\frac{x}{2}\right) + (10i-3x)\cosh\left(\frac{3x}{2}\right) - 6i(-3i+2x+x\cosh(x))\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (3*(-4*I + 3*x)*Cosh[x/2] + (10*I - 3*x)*Cosh[(3*x)/2] - (6*I)*(-3*I + 2*x
+ x*Cosh[x])*Sinh[x/2])/(6*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.59, size = 52, normalized size = 1.62

method	result	size
risch	$x+\frac{2i(9i\mathrm{e}^{x}+6\,\mathrm{e}^{2x}-5)}{3(\mathrm{e}^{x}+i)^{3}}$	26
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tanh(1/2*x)-1)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)+I)^3-2/(tanh(1/2*
x)+I)+ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.30, size = 40, normalized size = 1.25

$$x - \frac{2\left(9\,e^{(-x)} + 6i\,e^{(-2\,x)} - 5i\right)}{3\left(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - e^{(-3\,x)} - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] x - 2/3*(9*e^(-x) + 6*I*e^(-2*x) - 5*I)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x)
 - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.
time = 0.39, size = 50, normalized size = 1.56

$$\frac{3\,xe^{(3\,x)} - 3\,(-3i\,x - 4i)e^{(2\,x)} - 9\,(x+2)e^{x} - 3i\,x - 10i}{3\,\left(e^{(3\,x)} + 3i\,e^{(2\,x)} - 3\,e^{x} - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*e^(3*x) - 3*(-3*I*x - 4*I)*e^(2*x) - 9*(x + 2)*e^x - 3*I*x - 10*I)
/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.28

$$x + \frac{12ie^{2x} - 18e^{x} - 10i}{3e^{3x} + 9ie^{2x} - 9e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] x + (12*I*exp(2*x) - 18*exp(x) - 10*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x
) - 3*I)
```

Giac [A]
time = 0.48, size = 22, normalized size = 0.69

$$x - \frac{2\left(-6i\,e^{(2\,x)} + 9\,e^{x} + 5i\right)}{3\,\left(e^{x} + i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] x - 2/3*(-6*I*e^(2*x) + 9*e^x + 5*I)/(e^x + I)^3
```

Mupad [B]
time = 0.57, size = 71, normalized size = 2.22

$$x + \frac{-\frac{2}{3} + \frac{e^{x}\,4i}{3}}{e^{2\,x} - 1 + e^{x}\,2i} - \frac{\frac{4\,e^{x}}{3} - \frac{e^{2\,x}\,4i}{3} + \frac{4}{3}i}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i} + \frac{4i}{3\,\left(e^{x} + 1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] x + ((exp(x)*4i)/3 - 2/3)/(exp(2*x) + exp(x)*2i - 1) - ((4*exp(x))/3 - (exp
(2*x)*4i)/3 + 4i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 4i/(3*(exp(x
) + 1i))
```

3.51 $\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=31

$$-\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{2i\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] -1/3*cosh(x)/(I+sinh(x))^2-2/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2829, 2727}

$$-\frac{2i\cosh(x)}{3(\sinh(x)+i)}-\frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*Cosh[x]/(I + Sinh[x])^2 - (((2*I)/3)*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}+\frac{2}{3}\int \frac{1}{i+\sinh(x)}\,dx\\ &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{2i\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.71

$$\frac{\cosh(x)(1-2i\sinh(x))}{3(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(I + Sinh[x])^2,x]
[Out] (Cosh[x]*(1 - (2*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A]
time = 0.69, size = 25, normalized size = 0.81

method	result	size
risch	$-\frac{2(3i e^{x}+3 e^{2x}-2)}{3(e^{x}+i)^{3}}$	23
default	$-\frac{4i}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{2}{(\tanh(\frac{x}{2})+i)^{2}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -4/3*I/(tanh(1/2*x)+I)^3+2/(tanh(1/2*x)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(21) = 42.
time = 0.30, size = 81, normalized size = 2.61

$$-\frac{2i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{4}{3\,(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -2*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2*e^(-2*x)/(3*e^(-x)
 + 3*I*e^(-2*x) - e^(-3*x) - I) - 4/3/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) -
 I)
```

Fricas [A]
time = 0.43, size = 32, normalized size = 1.03

$$-\frac{2\,\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\,\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 1.19

$$\frac{-6e^{2x}-6ie^{x}+4}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))**2,x)
[Out] (-6*exp(2*x) - 6*I*exp(x) + 4)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

Giac [A]
time = 0.43, size = 20, normalized size = 0.65

$$-\frac{2\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\left(e^{x}+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^x + I)^3
```

Mupad [B]
time = 0.52, size = 25, normalized size = 0.81

$$-\frac{2\left(3\,\mathrm{e}^{x}-\mathrm{e}^{2\,x}\,3\mathrm{i}+2\mathrm{i}\right)}{3\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(sinh(x) + 1i)^2,x)
[Out] -(2*(3*exp(x) - exp(2*x)*3i + 2i))/(3*(exp(x)*1i - 1)^3)
```

3.52 $\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=34

$$\tanh^{-1}(\cosh(x))+\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] arctanh(cosh(x))+1/3*cosh(x)/(I+sinh(x))^2-4/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2845, 3057, 12, 3855}

$$-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}+\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] ArcTanh[Cosh[x]] + Cosh[x]/(3*(I + Sinh[x])^2) - (((4*I)/3)*Cosh[x])/(I + S
inh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx &= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}(x)(3i-\sinh(x))}{i+\sinh(x)}\,dx\\ &= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}+\frac{1}{3}i\int 3i\operatorname{csch}(x)\,dx\\ &= \frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}-\int \operatorname{csch}(x)\,dx\\ &= \tanh^{-1}(\cosh(x))+\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 91 vs. $2(34) = 68$.
time = 0.06, size = 91, normalized size = 2.68

$$\frac{\cosh\left(\frac{x}{2}\right)\left(6-9\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\cosh\left(\frac{3x}{2}\right)\left(-8+3\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+6i\left(-3+2\log\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(I + Sinh[x])^2,x]
[Out] (Cosh[x/2]*(6 - 9*Log[Tanh[x/2]]) + Cosh[(3*x)/2]*(-8 + 3*Log[Tanh[x/2]]) +
 (6*I)*(-3 + 2*Log[Tanh[x/2]] + Cosh[x]*Log[Tanh[x/2]])*Sinh[x/2])/(6*(Cosh
[x/2] - I*Sinh[x/2])^3)
```

Maple [A]
time = 0.60, size = 44, normalized size = 1.29

method	result	size
risch	$-\frac{2(9i e^{x}+3\,e^{2x}-4)}{3(e^{x}+i)^3}+\ln(e^{x}+1)-\ln(e^{x}-1)$	36

default	$-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tanh(1/2*x))+4/3*I/(tanh(1/2*x)+I)^3-4*I/(tanh(1/2*x)+I)-2/(tanh(1/2*x)
+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.
time = 0.30, size = 55, normalized size = 1.62

$$\frac{2\left(-9i\,e^{(-x)}+3\,e^{(-2\,x)}-4\right)}{3\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i\right)}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(-9*I*e^(-x) + 3*e^(-2*x) - 4)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I)
 + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(24) = 48$.
time = 0.34, size = 78, normalized size = 2.29

$$\frac{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)\log\left(e^{x}-1\right)-6\,e^{(2\,x)}-18i\,e^{x}+8}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 3*(e^(3*x) + 3*I*
e^(2*x) - 3*e^x - I)*log(e^x - 1) - 6*e^(2*x) - 18*I*e^x + 8)/(e^(3*x) + 3*
I*e^(2*x) - 3*e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}(x)}{\left(\sinh(x)+i\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I)**2, x)
```

<u>Giac</u> [A]
time = 0.45, size = 34, normalized size = 1.00

$$-\frac{2\left(3\,e^{(2\,x)}+9i\,e^{x}-4\right)}{3\left(e^{x}+i\right)^{3}}+\log\left(e^{x}+1\right)-\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 9*I*e^x - 4)/(e^x + I)^3 + log(e^x + 1) - log(abs(e^x - 1
))
```

<u>Mupad</u> [B]
time = 0.28, size = 41, normalized size = 1.21

$$\ln\left(\mathrm{e}^{x}+1\right)-\ln\left(\mathrm{e}^{x}-1\right)-\frac{2}{\mathrm{e}^{x}+1\mathrm{i}}-\frac{2\mathrm{i}}{\left(\mathrm{e}^{x}+1\mathrm{i}\right)^{2}}-\frac{4}{3\left(\mathrm{e}^{x}+1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x) + 1) - log(exp(x) - 1) - 2/(exp(x) + 1i) - 2i/(exp(x) + 1i)^2 -
4/(3*(exp(x) + 1i)^3)
```

3.53 $\int \frac{\text{csch}^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=42

$$2 i \tanh^{-1}(\cosh(x))+\frac{10 \coth(x)}{3}+\frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2 i \coth(x)}{i+\sinh(x)}$$

```
[Out] 2*I*arctanh(cosh(x))+10/3*coth(x)+1/3*coth(x)/(I+sinh(x))^2-2*I*coth(x)/(I+
sinh(x))
```

Rubi [A]

time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$\frac{10 \coth(x)}{3}+2 i \tanh^{-1}(\cosh(x))-\frac{2 i \coth(x)}{\sinh(x)+i}+\frac{\coth(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + (10*Coth[x])/3 + Coth[x]/(3*(I + Sinh[x])^2) - ((2
*I)*Coth[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}^2(x)(4i-2\sinh(x))}{i+\sinh(x)}\,dx\\
&= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}+\frac{1}{3}\int \operatorname{csch}^2(x)(-10-6i\sinh(x))\,dx\\
&= \frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}-2i\int \operatorname{csch}(x)\,dx-\frac{10}{3}\int \operatorname{csch}^2(x)\,dx\\
&= 2i\tanh^{-1}(\cosh(x))+\frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}+\frac{10}{3}i\text{Subst}(\int 1\,dx,x,-i\coth(\\
&= 2i\tanh^{-1}(\cosh(x))+\frac{10\coth(x)}{3}+\frac{\coth(x)}{3(i+\sinh(x))^2}-\frac{2i\coth(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.
time = 0.29, size = 88, normalized size = 2.10

$$\frac{1}{6}\left(3\coth\left(\frac{x}{2}\right)+12i\log\left(\cosh\left(\frac{x}{2}\right)\right)-12i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{2}{i+\sinh(x)}-\frac{4\sinh\left(\frac{x}{2}\right)(8i+7\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}+3\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (3*Coth[x/2] + (12*I)*Log[Cosh[x/2]] - (12*I)*Log[Sinh[x/2]] + 2/(I + Sinh[
x]) - (4*Sinh[x/2]*(8*I + 7*Sinh[x]))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Tanh[
x/2])/6
```

Maple [A]
time = 0.72, size = 58, normalized size = 1.38

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{6}{\tanh\left(\frac{x}{2}\right)+i}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	58
risch	$-\frac{4i\left(9i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}-12i\mathrm{e}^{x}-11\,\mathrm{e}^{2x}+5\right)}{3\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)^{3}}+2i\ln\left(\mathrm{e}^{x}+1\right)-2i\ln\left(\mathrm{e}^{x}-1\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+
I)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(30) = 60$.
time = 0.30, size = 79, normalized size = 1.88

$$\frac{4\left(12\,e^{(-x)}+11i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-5i\right)}{3\left(3\,e^{(-x)}+4i\,e^{(-2\,x)}-4\,e^{(-3\,x)}-3i\,e^{(-4\,x)}+e^{(-5\,x)}-i\right)}+2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4/3*(12*e^(-x) + 11*I*e^(-2*x) - 9*e^(-3*x) - 3*I*e^(-4*x) - 5*I)/(3*e^(-x)
 + 4*I*e^(-2*x) - 4*e^(-3*x) - 3*I*e^(-4*x) + e^(-5*x) - I) + 2*I*log(e^(-x
) + 1) - 2*I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(30) = 60$.
time = 0.43, size = 130, normalized size = 3.10

$$-\frac{2\left(3\left(-i\,e^{(5\,x)}+3\,e^{(4\,x)}+4i\,e^{(3\,x)}-4\,e^{(2\,x)}-3i\,e^{x}+1\right)\log\left(e^{x}+1\right)+3\left(i\,e^{(5\,x)}-3\,e^{(4\,x)}-4i\,e^{(3\,x)}+4\,e^{(2\,x)}+3i\,e^{x}-1\right)\log\left(e^{x}-1\right)+6i\,e^{(4\,x)}-18\,e^{(3\,x)}-22i\,e^{(2\,x)}+24\,e^{x}+10i\right)}{3\left(e^{(5\,x)}+3i\,e^{(4\,x)}-4\,e^{(3\,x)}-4i\,e^{(2\,x)}+3\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*(-I*e^(5*x) + 3*e^(4*x) + 4*I*e^(3*x) - 4*e^(2*x) - 3*I*e^x + 1)*lo
g(e^x + 1) + 3*(I*e^(5*x) - 3*e^(4*x) - 4*I*e^(3*x) + 4*e^(2*x) + 3*I*e^x -
```

```
 1)*log(e^x - 1) + 6*I*e^(4*x) - 18*e^(3*x) - 22*I*e^(2*x) + 24*e^x + 10*I)
/(e^(5*x) + 3*I*e^(4*x) - 4*e^(3*x) - 4*I*e^(2*x) + 3*e^x + I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(sinh(x) + I)**2, x)
```

Giac [A]
time = 0.43, size = 46, normalized size = 1.10

$$\frac{2}{e^{(2x)}-1} - \frac{2\left(6i\,e^{(2x)} - 15\,e^{x} - 7i\right)}{3\left(e^{x}+i\right)^{3}} + 2i\,\log\left(e^{x}+1\right) - 2i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/(e^(2*x) - 1) - 2/3*(6*I*e^(2*x) - 15*e^x - 7*I)/(e^x + I)^3 + 2*I*log(e^
x + 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.71, size = 85, normalized size = 2.02

$$\frac{2}{e^{2x}-1+e^{x}\,2i} + \frac{2}{e^{2x}-1} - \ln\left(e^{x}\,4i-4i\right)\,2i + \ln\left(e^{x}\,4i+4i\right)\,2i - \frac{4i}{e^{x}+1i} - \frac{4i}{3\left(e^{2x}\,3i+e^{3x}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + 2/(exp(2*x) + exp(x)*2i -
 1) - 4i/(exp(x) + 1i) - 4i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)) +
2/(exp(2*x) - 1)
```

3.54 $\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=58

$$-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{16}{3}i\coth(x)+\frac{7}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}$$

```
[Out] -7/2*arctanh(cosh(x))+16/3*I*coth(x)+7/2*coth(x)*csch(x)+1/3*coth(x)*csch(x
)/(I+sinh(x))^2-8/3*I*coth(x)*csch(x)/(I+sinh(x))
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\frac{16}{3}i\coth(x)-\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{7}{2}\coth(x)\operatorname{csch}(x)-\frac{8i\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-7*ArcTanh[Cosh[x]])/2 + ((16*I)/3)*Coth[x] + (7*Coth[x]*Csch[x])/2 + (Cot
h[x]*Csch[x])/(3*(I + Sinh[x])^2) - (((8*I)/3)*Coth[x]*Csch[x])/(I + Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}^3(x)(5i-3\sinh(x))}{i+\sinh(x)}\,dx\\ &= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}+\frac{1}{3}\int \operatorname{csch}^3(x)(-21-16i\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}-\frac{16}{3}i\int \operatorname{csch}^2(x)\,dx-7\int \operatorname{csch}^3(x)\,dx\\ &= \frac{7}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}+\frac{7}{2}\int \operatorname{csch}(x)\,dx-\frac{16}{3}\mathrm{Sub}\\ &= -\frac{7}{2}\tanh^{-1}(\cosh(x))+\frac{16}{3}i\coth(x)+\frac{7}{2}\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2}-\frac{8i\coth(}{3(i+}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 131 vs. 2(58) = 116.
time = 0.28, size = 131, normalized size = 2.26

$$\frac{1}{24}\left(24i\coth\left(\frac{x}{2}\right)+3\text{csch}^2\left(\frac{x}{2}\right)+84\log\left(\tanh\left(\frac{x}{2}\right)\right)+3\text{sech}^2\left(\frac{x}{2}\right)+\frac{8}{\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^2}+\frac{160i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}+\frac{16\sinh\left(\frac{x}{2}\right)}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}+24i\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((24*I)*Coth[x/2] + 3*Csch[x/2]^2 + 84*Log[Tanh[x/2]] + 3*Sech[x/2]^2 + 8/(
Cosh[x/2] - I*Sinh[x/2])^2 + ((160*I)*Sinh[x/2])/(Cosh[x/2] - I*Sinh[x/2])
+ (16*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])^3 + (24*I)*Tanh[x/2])/24
```

Maple [A]
time = 0.69, size = 76, normalized size = 1.31

method	result
risch	$\frac{-98\,e^{4x}+63ie^{5x}+97\,e^{2x}-126ie^{3x}+21\,e^{6x}-32+75ie^{x}}{3\left(e^{2x}-1\right)^2\left(e^{x}+i\right)^3}+\frac{7\ln\left(e^{x}-1\right)}{2}-\frac{7\ln\left(e^{x}+1\right)}{2}$
default	$i\tanh\left(\frac{x}{2}\right)-\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{\tanh\left(\frac{x}{2}\right)}+\frac{1}{8\tanh\left(\frac{x}{2}\right)^2}+\frac{7\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}+\frac{8i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}+\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I*tanh(1/2*x)-1/8*tanh(1/2*x)^2+I/tanh(1/2*x)+1/8/tanh(1/2*x)^2+7/2*ln(tanh
(1/2*x))+8*I/(tanh(1/2*x)+I)-4/3*I/(tanh(1/2*x)+I)^3+2/(tanh(1/2*x)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(40) = 80.
time = 0.28, size = 105, normalized size = 1.81

$$-\frac{-75i\,e^{(-x)}+97\,e^{(-2\,x)}+126i\,e^{(-3\,x)}-98\,e^{(-4\,x)}-63i\,e^{(-5\,x)}+21\,e^{(-6\,x)}-32}{3\left(3\,e^{(-x)}+5i\,e^{(-2\,x)}-7\,e^{(-3\,x)}-7i\,e^{(-4\,x)}+5\,e^{(-5\,x)}+3i\,e^{(-6\,x)}-e^{(-7\,x)}-i\right)}-\frac{7}{2}\log\left(e^{(-x)}+1\right)+\frac{7}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(-75*I*e^(-x) + 97*e^(-2*x) + 126*I*e^(-3*x) - 98*e^(-4*x) - 63*I*e^(-
5*x) + 21*e^(-6*x) - 32)/(3*e^(-x) + 5*I*e^(-2*x) - 7*e^(-3*x) - 7*I*e^(-4*
x) + 5*e^(-5*x) + 3*I*e^(-6*x) - e^(-7*x) - I) - 7/2*log(e^(-x) + 1) + 7/2*
log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(40) = 80.
time = 0.48, size = 174, normalized size = 3.00

$$-\frac{21\left(e^{(7\,x)}+3i\,e^{(6\,x)}-5\,e^{(5\,x)}-7i\,e^{(4\,x)}+7\,e^{(3\,x)}+5i\,e^{(2\,x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-21\left(e^{(7\,x)}+3i\,e^{(6\,x)}-5\,e^{(5\,x)}-7i\,e^{(4\,x)}+7\,e^{(3\,x)}+5i\,e^{(2\,x)}-3\,e^{x}-i\right)\log\left(e^{x}-1\right)-42\,e^{(6\,x)}-126i\,e^{(5\,x)}+196\,e^{(4\,x)}+252i\,e^{(3\,x)}-194\,e^{(2\,x)}-150i\,e^{x}+64}{6\left(e^{(7\,x)}+3i\,e^{(6\,x)}-5\,e^{(5\,x)}-7i\,e^{(4\,x)}+7\,e^{(3\,x)}+5i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) + 5*I
*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x)
- 7*I*e^(4*x) + 7*e^(3*x) + 5*I*e^(2*x) - 3*e^x - I)*log(e^x - 1) - 42*e^(6
*x) - 126*I*e^(5*x) + 196*e^(4*x) + 252*I*e^(3*x) - 194*e^(2*x) - 150*I*e^x
 + 64)/(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) + 5*I*e
^(2*x) - 3*e^x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**3/(sinh(x) + I)**2, x)
```

Giac [A]
time = 0.43, size = 59, normalized size = 1.02

$$\frac{e^{(3x)}+4i\,e^{(2x)}+e^x-4i}{\left(e^{(2x)}-1\right)^2}+\frac{2\left(9\,e^{(2x)}+21i\,e^x-10\right)}{3\,(e^x+i)^3}-\frac{7}{2}\log\left(e^x+1\right)+\frac{7}{2}\log\left(|e^x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] (e^(3*x) + 4*I*e^(2*x) + e^x - 4*I)/(e^(2*x) - 1)^2 + 2/3*(9*e^(2*x) + 21*I
*e^x - 10)/(e^x + I)^3 - 7/2*log(e^x + 1) + 7/2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.77, size = 79, normalized size = 1.36

$$\frac{e^x}{e^{2x}-1}-\frac{7\ln\left(e^x+1\right)}{2}-\frac{7\ln\left(\frac{1}{e^x-1}\right)}{2}+\frac{2\,e^x}{\left(e^{2x}-1\right)^2}+\frac{6}{e^x+1i}+\frac{2i}{\left(e^x+1i\right)^2}+\frac{4}{3\,\left(e^x+1i\right)^3}+\frac{4i}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] exp(x)/(exp(2*x) - 1) - (7*log(exp(x) + 1))/2 - (7*log(1/(exp(x) - 1)))/2 +
 (2*exp(x))/(exp(2*x) - 1)^2 + 6/(exp(x) + 1i) + 2i/(exp(x) + 1i)^2 + 4/(3*
(exp(x) + 1i)^3) + 4i/(exp(2*x) - 1)
```

3.55 $\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=64

$$-5i\tanh^{-1}(\cosh(x))-12\coth(x)+4\coth^3(x)+5i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}$$

```
[Out] -5*I*arctanh(cosh(x))-12*coth(x)+4*coth(x)^3+5*I*coth(x)*csch(x)+1/3*coth(x
)*csch(x)^2/(I+sinh(x))^2-10/3*I*coth(x)*csch(x)^2/(I+sinh(x))
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2845, 3057, 2827, 3852, 3853, 3855}

$$4\coth^3(x)-12\coth(x)-5i\tanh^{-1}(\cosh(x))+5i\coth(x)\operatorname{csch}(x)-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)}+\frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*I)*ArcTanh[Cosh[x]] - 12*Coth[x] + 4*Coth[x]^3 + (5*I)*Coth[x]*Csch[x]
+ (Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (((10*I)/3)*Coth[x]*Csch[x]^2)/
(I + Sinh[x])
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}\int \frac{\operatorname{csch}^4(x)(6i-4\sinh(x))}{i+\sinh(x)}\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}+\frac{1}{3}\int \operatorname{csch}^4(x)(-36-30i\sinh(x))\,dx\\ &= \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}-10i\int \operatorname{csch}^3(x)\,dx-12\int \operatorname{csch}^4(x)\,dx\\ &= 5i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2}-\frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}+5i\int \operatorname{csch}(x)\,dx-12\\ &= -5i\tanh^{-1}(\cosh(x))-12\coth(x)+4\coth^3(x)+5i\coth(x)\operatorname{csch}(x)+\frac{\coth(x)\operatorname{csch}^2(}{3(i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(64) = 128.

time = 1.42, size = 143, normalized size = 2.23

$$\frac{1}{24}\left(-44\coth\left(\frac{x}{2}\right)+6i\mathrm{csch}^2\left(\frac{x}{2}\right)+\frac{1}{2}\mathrm{csch}^4\left(\frac{x}{2}\right)\sinh(x)+2\left(-60i\log\left(\cosh\left(\frac{x}{2}\right)\right)+60i\log\left(\sinh\left(\frac{x}{2}\right)\right)+3i\mathrm{sech}^2\left(\frac{x}{2}\right)-4\mathrm{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)-\frac{4}{i+\sinh(x)}+\frac{8\sinh\left(\frac{x}{2}\right)(14i+13\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}-22\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-44*Coth[x/2] + (6*I)*Csch[x/2]^2 + (Csch[x/2]^4*Sinh[x])/2 + 2*((-60*I)*L
og[Cosh[x/2]] + (60*I)*Log[Sinh[x/2]] + (3*I)*Sech[x/2]^2 - 4*Csch[x]^3*Sin
h[x/2]^4 - 4/(I + Sinh[x]) + (8*Sinh[x/2]*(14*I + 13*Sinh[x]))/(I*Cosh[x/2]
 + Sinh[x/2])^3 - 22*Tanh[x/2]))/24
```

Maple [A]
time = 0.74, size = 92, normalized size = 1.44

method	result
risch	$\frac{2i\left(45i e^{7x}+15 e^{8x}-135i e^{5x}-85 e^{6x}+155i e^{3x}+153 e^{4x}-57i e^{x}-99 e^{2x}+24\right)}{3\left(e^{2x}-1\right)^3\left(e^{x}+i\right)^3}+5i\ln\left(e^{x}-1\right)-5i\ln\left(e^{x}+1\right)$
default	$-\frac{15\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}-\frac{i\left(\tanh^2\left(\frac{x}{2}\right)\right)}{4}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^2}+5i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{24\tanh\left(\frac{x}{2}\right)^3}-\frac{15}{8\tanh\left(\frac{x}{2}\right)}+\frac{2}{(\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -15/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+1/4*I/tanh(1/2*x)^
2+5*I*ln(tanh(1/2*x))+1/24/tanh(1/2*x)^3-15/8/tanh(1/2*x)+2*I/(tanh(1/2*x)+
I)^2+4/3/(tanh(1/2*x)+I)^3-10/(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.
time = 0.29, size = 127, normalized size = 1.98

$$-\frac{2\left(57\,e^{(-x)}+99i\,e^{(-2x)}-155\,e^{(-3x)}-153i\,e^{(-4x)}+135\,e^{(-5x)}+85i\,e^{(-6x)}-45\,e^{(-7x)}-15i\,e^{(-8x)}-24i\right)}{3\left(3\,e^{(-x)}+6i\,e^{(-2x)}-10\,e^{(-3x)}-12i\,e^{(-4x)}+12\,e^{(-5x)}+10i\,e^{(-6x)}-6\,e^{(-7x)}-3i\,e^{(-8x)}+e^{(-9x)}-i\right)}-5i\log\left(e^{(-x)}+1\right)+5i\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(57*e^(-x) + 99*I*e^(-2*x) - 155*e^(-3*x) - 153*I*e^(-4*x) + 135*e^(-5
*x) + 85*I*e^(-6*x) - 45*e^(-7*x) - 15*I*e^(-8*x) - 24*I)/(3*e^(-x) + 6*I*e
^(-2*x) - 10*e^(-3*x) - 12*I*e^(-4*x) + 12*e^(-5*x) + 10*I*e^(-6*x) - 6*e^(
-7*x) - 3*I*e^(-8*x) + e^(-9*x) - I) - 5*I*log(e^(-x) + 1) + 5*I*log(e^(-x)
 - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(50) = 100.
time = 0.42, size = 226, normalized size = 3.53

$$-\frac{15\left(i\,e^{(9x)}-3\,e^{(8x)}-6i\,e^{(7x)}+10\,e^{(6x)}+12i\,e^{(5x)}-12\,e^{(4x)}-10i\,e^{(3x)}+6\,e^{(2x)}+3i\,e^{x}-1\right)\log\left(e^{x}+1\right)+15\left(-i\,e^{(9x)}+3\,e^{(8x)}+6i\,e^{(7x)}-10\,e^{(6x)}-12i\,e^{(5x)}+12\,e^{(4x)}+10i\,e^{(3x)}-6\,e^{(2x)}-3i\,e^{x}+1\right)\log\left(e^{x}-1\right)-30i\,e^{(8x)}+90\,e^{(7x)}+170i\,e^{(6x)}-270\,e^{(5x)}-306i\,e^{(4x)}+310\,e^{(3x)}+198i\,e^{(2x)}-114\,e^{x}-48i}{3\left(e^{(9x)}+3i\,e^{(8x)}-6\,e^{(7x)}-10i\,e^{(6x)}+12\,e^{(5x)}+12i\,e^{(4x)}-10\,e^{(3x)}-6i\,e^{(2x)}+3\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*(I*e^(9*x) - 3*e^(8*x) - 6*I*e^(7*x) + 10*e^(6*x) + 12*I*e^(5*x) -
 12*e^(4*x) - 10*I*e^(3*x) + 6*e^(2*x) + 3*I*e^x - 1)*log(e^x + 1) + 15*(-I
*e^(9*x) + 3*e^(8*x) + 6*I*e^(7*x) - 10*e^(6*x) - 12*I*e^(5*x) + 12*e^(4*x)
 + 10*I*e^(3*x) - 6*e^(2*x) - 3*I*e^x + 1)*log(e^x - 1) - 30*I*e^(8*x) + 90
*e^(7*x) + 170*I*e^(6*x) - 270*e^(5*x) - 306*I*e^(4*x) + 310*e^(3*x) + 198*
I*e^(2*x) - 114*e^x - 48*I)/(e^(9*x) + 3*I*e^(8*x) - 6*e^(7*x) - 10*I*e^(6*
x) + 12*e^(5*x) + 12*I*e^(4*x) - 10*e^(3*x) - 6*I*e^(2*x) + 3*e^x + I)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 84, normalized size = 1.31

$$-\frac{2\left(-15i\,e^{(8x)}+45\,e^{(7x)}+85i\,e^{(6x)}-135\,e^{(5x)}-153i\,e^{(4x)}+155\,e^{(3x)}+99i\,e^{(2x)}-57\,e^{x}-24i\right)}{3\left(e^{(3x)}+i\,e^{(2x)}-e^{x}-i\right)^{3}}-5i\,\log\left(e^{x}+1\right)+5i\,\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-15*I*e^(8*x) + 45*e^(7*x) + 85*I*e^(6*x) - 135*e^(5*x) - 153*I*e^(4*
x) + 155*e^(3*x) + 99*I*e^(2*x) - 57*e^x - 24*I)/(e^(3*x) + I*e^(2*x) - e^x
 - I)^3 - 5*I*log(e^x + 1) + 5*I*log(abs(e^x - 1))
```

Mupad [**B**]
time = 1.09, size = 189, normalized size = 2.95

$$-\ln\left(-e^{x}\,10i-10i\right)\,5i+\ln\left(-e^{x}\,10i+10i\right)\,5i-\frac{\frac{16\,e^{x}}{3}-\frac{e^{2x}\,32i}{3}+\frac{16}{3}i}{12\,e^{5x}-10\,e^{3x}+e^{4x}\,12i-e^{2x}\,6i-e^{6x}\,10i-6\,e^{7x}+e^{8x}\,3i+e^{9x}+3\,e^{x}+1i}+\frac{\frac{20\,e^{2x}}{3}-\frac{44}{3}+\frac{e^{x}\,16i}{3}}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1-e^{3x}\,4i+e^{5x}\,2i+e^{x}\,2i}-\frac{10\,e^{x}-e^{2x}\,10i+\frac{20}{3}i}{e^{2x}\,1i+e^{3x}-e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] log(10i - exp(x)*10i)*5i - log(- exp(x)*10i - 10i)*5i - ((16*exp(x))/3 - (e
xp(2*x)*32i)/3 + 16i/3)/(exp(4*x)*12i - 10*exp(3*x) - exp(2*x)*6i + 12*exp(
5*x) - exp(6*x)*10i - 6*exp(7*x) + exp(8*x)*3i + exp(9*x) + 3*exp(x) + 1i)
+ ((20*exp(2*x))/3 + (exp(x)*16i)/3 - 44/3)/(3*exp(2*x) - exp(3*x)*4i - 3*e
xp(4*x) + exp(5*x)*2i + exp(6*x) + exp(x)*2i - 1) - (10*exp(x) - exp(2*x)*1
0i + 20i/3)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.56 $\int \frac{1}{1+i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

```
[Out] I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2727}

$$\frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [A]
time = 0.81, size = 20, normalized size = 0.74

method	result	size
risch	$\frac{2i}{d(e^{dx+c}-i)}$	18
derivativedivides	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(-I+tanh(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.74

$$-\frac{2}{d(i\,e^{(-dx-c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(I*e^(-d*x - c) - 1))
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.59

$$\frac{2i}{de^{(dx+c)}-i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(d*e^(d*x + c) - I*d)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.56

$$\frac{2i}{de^{c}e^{dx}-id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x)
```

```
[Out] 2*I/(d*exp(c)*exp(d*x) - I*d)
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.56

$$\frac{2i}{d(e^{(dx+c)} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*I/(d*(e^(d*x + c) - I))
```

Mupad [B]
time = 0.20, size = 17, normalized size = 0.63

$$\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1),x)
```

```
[Out] 2i/(d*(exp(c + d*x) - 1i))
```

3.57 $\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}$$

```
[Out] 1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c
))
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2729, 2727}

$$\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/(
d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1+i\sinh(c+dx)}\,dx\\ &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 1.03

$$\frac{3i-4i\cosh(c+dx)-i\cosh(2(c+dx))-4\sinh(c+dx)+\sinh(2(c+dx))}{6d(-i+\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] (3*I - (4*I)*Cosh[c + d*x] - I*Cosh[2*(c + d*x)] - 4*Sinh[c + d*x] + Sinh[2
*(c + d*x)])/(6*d*(-I + Sinh[c + d*x])^2)
```

Maple [A]
time = 1.27, size = 55, normalized size = 0.93

method	result	size
risch	$\frac{-\frac{2i}{3}+2\,\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}-i\right)^{3}d}$	28
derivativedivides	$\frac{\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
default	$\frac{\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*I/(-I+tanh(1/2*d*x+1/2*c))^2+2/(-I+tanh(1/2*d*x+1/2*c))-4/3/(-I+tanh
(1/2*d*x+1/2*c))^3)
```

Maxima [A]
time = 0.29, size = 94, normalized size = 1.59

$$\frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)}+\frac{2i}{3\,d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 + I)) + 2/3*I/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 + I))
```

Fricas [A]
time = 0.44, size = 50, normalized size = 0.85

$$\frac{2\left(3\,e^{(dx+c)}-i\right)}{3\left(de^{(3\,dx+3\,c)}-3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}+i\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*e^(d*x + c) - I)/(d*e^(3*d*x + 3*c) - 3*I*d*e^(2*d*x + 2*c) - 3*d*e^
(d*x + c) + I*d)
```

Sympy [A]
time = 0.10, size = 61, normalized size = 1.03

$$\frac{6e^{c}e^{dx}-2i}{3de^{3c}e^{3dx}-9ide^{2c}e^{2dx}-9de^{c}e^{dx}+3id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**2,x)
```

```
[Out] (6*exp(c)*exp(d*x) - 2*I)/(3*d*exp(3*c)*exp(3*d*x) - 9*I*d*exp(2*c)*exp(2*d
*x) - 9*d*exp(c)*exp(d*x) + 3*I*d)
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.42

$$\frac{2\left(3\,e^{(dx+c)}-i\right)}{3\,d(e^{(dx+c)}-i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*e^(d*x + c) - I)/(d*(e^(d*x + c) - I)^3)
```

Mupad [B]
time = 0.53, size = 29, normalized size = 0.49

$$-\frac{\frac{2}{3}+\mathrm{e}^{c+d\,x}\,2\mathrm{i}}{d\,(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i})^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^2,x)
```

```
[Out] -(exp(c + d*x)*2i + 2/3)/(d*(exp(c + d*x)*1i + 1)^3)
```

3.58 $\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}$$

```
[Out] 1/5*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^3+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^2+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/15)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^2) + (((2*I)/15)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx\\ &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1+i\sinh(c+dx)}\\ &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.92

$$\frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))+15i\sinh(c+dx)-6i\sinh(2(c+dx))-i\sinh(3(c+dx))}{30d(-i+\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] + (15*I)*S
inh[c + d*x] - (6*I)*Sinh[2*(c + d*x)] - I*Sinh[3*(c + d*x)])/(30*d*(-I + S
inh[c + d*x])^3)
```

Maple [A]
time = 1.42, size = 88, normalized size = 1.00

method	result	size
risch	$-\frac{4i\left(-5i\,{\mathrm e}^{dx+c}+10\,{\mathrm e}^{2dx+2c}-1\right)}{15d\left({\mathrm e}^{dx+c}-i\right)^{5}}$	40
derivativedivides	$\frac{-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{d}$	88
default	$\frac{-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4*I/(-I+tanh(1/2*d*x+1/2*c))^4-16/3/(-I+tanh(1/2*d*x+1/2*c))^3+8/5/(-
I+tanh(1/2*d*x+1/2*c))^5+2/(-I+tanh(1/2*d*x+1/2*c))+4*I/(-I+tanh(1/2*d*x+1/
2*c))^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(70) = 140.
time = 0.29, size = 211, normalized size = 2.40

$$\frac{20i\,e^{(-dx-c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}+\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}-\frac{4}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) + 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) - 4/(d*(75*I*e
^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) - 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) - 15))
```

Fricas [A]
time = 0.40, size = 85, normalized size = 0.97

$$-\frac{4\left(10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}-i\right)}{15\left(de^{(5\,dx+5\,c)}-5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}+10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}-i\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) - I)/(d*e^(5*d*x + 5*c) - 5*I*d
*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) + 10*I*d*e^(2*d*x + 2*c) + 5*d*e^(d
*x + c) - I*d)
```

Sympy [A]
time = 0.18, size = 109, normalized size = 1.24

$$\frac{-40ie^{2c}e^{2dx}-20e^{c}e^{dx}+4i}{15de^{5c}e^{5dx}-75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}+150ide^{2c}e^{2dx}+75de^{c}e^{dx}-15id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**3,x)
```

```
[Out] (-40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) + 4*I)/(15*d*exp(5*c)*exp(5
*d*x) - 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) + 150*I*d*ex
p(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) - 15*I*d)
```

Giac [A]
time = 0.43, size = 36, normalized size = 0.41

$$-\frac{4i\left(10\,e^{(2\,dx+2\,c)}-5i\,e^{(dx+c)}-1\right)}{15\,d\left(e^{(dx+c)}-i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -4/15*I*(10*e^(2*d*x + 2*c) - 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) - I)^5)
```

Mupad [B]
time = 0.69, size = 40, normalized size = 0.45

$$-\frac{\frac{4}{15}-\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{3}+\frac{\mathrm{e}^{c+d\,x}\,4\mathrm{i}}{3}}{d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^3,x)
```

```
[Out] -((exp(c + d*x)*4i)/3 - (8*exp(2*c + 2*d*x))/3 + 4/15)/(d*(exp(c + d*x)*1i
+ 1)^5)
```

3.59 $\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}$$

```
[Out] 1/7*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^4+3/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^3+2/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+2/35*I*cosh(d*x+c)/d/(1+I*si
nh(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2729, 2727}

$$\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4) + (((3*I)/35)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x])^2) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1+i\sinh(c+dx)}$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}$$
$$= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.74

$$\frac{21i\cosh\left(\frac{3}{2}(c+dx)\right) - i\cosh\left(\frac{7}{2}(c+dx)\right) + 35\sinh\left(\frac{1}{2}(c+dx)\right) - 7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((21*I)*Cosh[(3*(c + d*x))/2] - I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x)
/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^7)
```

Maple [A]
time = 1.41, size = 121, normalized size = 1.03

method	result
risch	$-\frac{4\left(-7\,e^{dx+c}-21i e^{2dx+2c}+35\,e^{3dx+3c}+i\right)}{35\left(e^{dx+c}-i\right)^7 d}$
derivativedivides	$\frac{-\frac{16i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{6i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{}{(-i+\tanh}}{d}$
default	$\frac{-\frac{16i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{6i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{}{(-i+\tanh}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-16*I/(-I+tanh(1/2*d*x+1/2*c))^4-16/7/(-I+tanh(1/2*d*x+1/2*c))^7+6*I/(
-I+tanh(1/2*d*x+1/2*c))^2+2/(-I+tanh(1/2*d*x+1/2*c))+72/5/(-I+tanh(1/2*d*x+
1/2*c))^5-12/(-I+tanh(1/2*d*x+1/2*c))^3+8*I/(-I+tanh(1/2*d*x+1/2*c))^6)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(93) = 186$.
time = 0.29, size = 372, normalized size = 3.18

$$\frac{4 e^{(-dx-c)}}{5 d\left(7 e^{(-dx-c)} - 21 i e^{(-2 dx-2 c)} - 35 e^{(-3 dx-3 c)} + 35 i e^{(-4 dx-4 c)} + 21 e^{(-5 dx-5 c)} - 7 i e^{(-6 dx-6 c)} - e^{(-7 dx-7 c)} + i\right)} - \frac{12 i e^{(-2 dx-2 c)}}{5 d\left(7 e^{(-dx-c)} - 21 i e^{(-2 dx-2 c)} - 35 e^{(-3 dx-3 c)} + 35 i e^{(-4 dx-4 c)} + 21 e^{(-5 dx-5 c)} - 7 i e^{(-6 dx-6 c)} - e^{(-7 dx-7 c)} + i\right)} - \frac{4 e^{(-3 dx-3 c)}}{d\left(7 e^{(-dx-c)} - 21 i e^{(-2 dx-2 c)} - 35 e^{(-3 dx-3 c)} + 35 i e^{(-4 dx-4 c)} + 21 e^{(-5 dx-5 c)} - 7 i e^{(-6 dx-6 c)} - e^{(-7 dx-7 c)} + i\right)} + \frac{4 i}{35 d\left(7 e^{(-dx-c)} - 21 i e^{(-2 dx-2 c)} - 35 e^{(-3 dx-3 c)} + 35 i e^{(-4 dx-4 c)} + 21 e^{(-5 dx-5 c)} - 7 i e^{(-6 dx-6 c)} - e^{(-7 dx-7 c)} + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
- 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c)
 - e^(-7*d*x - 7*c) + I)) - 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21
*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-
5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) + I)) - 4*e^(-3*d*x
- 3*c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) + 3
5*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) - e^(-7*d
*x - 7*c) + I)) + 4/35*I/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^
(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*
x - 6*c) - e^(-7*d*x - 7*c) + I))
```

Fricas [A]
time = 0.35, size = 120, normalized size = 1.03

$$-\frac{4\left(35 e^{(3 dx+3 c)} - 21 i e^{(2 dx+2 c)} - 7 e^{(dx+c)} + i\right)}{35\left(d e^{(7 dx+7 c)} - 7 i d e^{(6 dx+6 c)} - 21 d e^{(5 dx+5 c)} + 35 i d e^{(4 dx+4 c)} + 35 d e^{(3 dx+3 c)} - 21 i d e^{(2 dx+2 c)} - 7 d e^{(dx+c)} + i d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*e^
(7*d*x + 7*c) - 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) + 35*I*d*e^(4*
d*x + 4*c) + 35*d*e^(3*d*x + 3*c) - 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x + c
) + I*d)
```

Sympy [A]
time = 0.27, size = 155, normalized size = 1.32

$$\frac{-140 e^{3c} e^{3dx} + 84 i e^{2c} e^{2dx} + 28 e^{c} e^{dx} - 4i}{35 d e^{7c} e^{7dx} - 245 i d e^{6c} e^{6dx} - 735 d e^{5c} e^{5dx} + 1225 i d e^{4c} e^{4dx} + 1225 d e^{3c} e^{3dx} - 735 i d e^{2c} e^{2dx} - 245 d e^{c} e^{dx} + 35 i d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))**4,x)
```

```
[Out] (-140*exp(3*c)*exp(3*d*x) + 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x) -
 4*I)/(35*d*exp(7*c)*exp(7*d*x) - 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp(5
*c)*exp(5*d*x) + 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*x)
- 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) + 35*I*d)
```

Giac [A]
time = 0.42, size = 47, normalized size = 0.40

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}-21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}+i\right)}{35\,d(e^{(dx+c)}-i)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*(e
^(d*x + c) - I)^7)
```

Mupad [B]
time = 0.96, size = 53, normalized size = 0.45

$$-\frac{\left(7\,\mathrm{e}^{c+d\,x}+\mathrm{e}^{2\,c+2\,d\,x}\,21\mathrm{i}-35\,\mathrm{e}^{3\,c+3\,d\,x}-\mathrm{i}\right)\,4\mathrm{i}}{35\,d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i + 1)^4,x)
```

```
[Out] -((7*exp(c + d*x) + exp(2*c + 2*d*x)*21i - 35*exp(3*c + 3*d*x) - 1i)*4i)/(3
5*d*(exp(c + d*x)*1i + 1)^7)
```

3.60 $\int \frac{1}{1-i\sinh(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

```
[Out] -I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2727}

$$-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx = -\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.56

$$\frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]))
```

Maple [A]
time = 0.86, size = 20, normalized size = 0.74

method	result	size
risch	$-\frac{2i}{d(e^{dx+c}+i)}$	18
derivativedivides	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/(I+tanh(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.74

$$\frac{2}{d(i\,e^{(-dx-c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="maxima")
[Out] 2/(d*(I*e^(-d*x - c) + 1))
```

Fricas [A]
time = 0.35, size = 16, normalized size = 0.59

$$-\frac{2i}{de^{(dx+c)}+i\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="fricas")
[Out] -2*I/(d*e^(d*x + c) + I*d)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.63

$$-\frac{2i}{de^{c}e^{dx}+id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x)
```

```
[Out] -2*I/(d*exp(c)*exp(d*x) + I*d)
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.56

$$-\frac{2i}{d(e^{(dx+c)} + i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="giac")
[Out] -2*I/(d*(e^(d*x + c) + I))
```

Mupad [B]
time = 0.15, size = 17, normalized size = 0.63

$$-\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(c + d*x)*1i - 1),x)
[Out] -2i/(d*(exp(c + d*x) + 1i))
```

3.61 $\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}$$

```
[Out] -1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+
c))
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2729, 2727}

$$-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((-1/3*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x]
)/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-i\sinh(c+dx)}\,dx\\ &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 1.00

$$-\frac{\cosh\left(\frac{3}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}{3d\left(i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] -1/3*(Cosh[(3*(c + d*x))/2] + (3*I)*Sinh[(c + d*x)/2])/(d*(I*Cosh[(c + d*x)
/2] + Sinh[(c + d*x)/2])^3)
```

Maple [A]
time = 1.33, size = 55, normalized size = 0.93

method	result	size
risch	$\frac{\frac{2i}{3}+2\,\mathrm{e}^{dx+c}}{d\left(\mathrm{e}^{dx+c}+i\right)^3}$	28
derivativedivides	$\frac{-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	55
default	$\frac{-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/3/(I+tanh(1/2*d*x+1/2*c))^3-2*I/(I+tanh(1/2*d*x+1/2*c))^2+2/(I+tanh
(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.30, size = 94, normalized size = 1.59

$$\frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)}-\frac{2i}{3\,d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 - I)) - 2/3*I/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 - I))
```

Fricas [A]
time = 0.50, size = 50, normalized size = 0.85

$$\frac{2\left(3\,e^{(dx+c)}+i\right)}{3\left(de^{(3\,dx+3\,c)}+3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}-i\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*e^(d*x + c) + I)/(d*e^(3*d*x + 3*c) + 3*I*d*e^(2*d*x + 2*c) - 3*d*e^
(d*x + c) - I*d)
```

Sympy [A]
time = 0.11, size = 61, normalized size = 1.03

$$\frac{6e^{c}e^{dx} + 2i}{3de^{3c}e^{3dx} + 9ide^{2c}e^{2dx} - 9de^{c}e^{dx} - 3id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**2,x)
```

```
[Out] (6*exp(c)*exp(d*x) + 2*I)/(3*d*exp(3*c)*exp(3*d*x) + 9*I*d*exp(2*c)*exp(2*d
*x) - 9*d*exp(c)*exp(d*x) - 3*I*d)
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.42

$$\frac{2\left(3\,e^{(dx+c)} + i\right)}{3\,d\left(e^{(dx+c)} + i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*e^(d*x + c) + I)/(d*(e^(d*x + c) + I)^3)
```

Mupad [B]
time = 0.54, size = 29, normalized size = 0.49

$$-\frac{2\left(-1 + \mathrm{e}^{c+d\,x}\,3\mathrm{i}\right)}{3\,d\left(-1 + \mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i - 1)^2,x)
```

```
[Out] -(2*(exp(c + d*x)*3i - 1))/(3*d*(exp(c + d*x)*1i - 1)^3)
```

3.62 $\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}$$

```
[Out] -1/5*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^3-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^2-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2729, 2727}

$$-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-1/5*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/15)*Cosh[c +
 d*x])/(d*(1 - I*Sinh[c + d*x])^2) - (((2*I)/15)*Cosh[c + d*x])/(d*(1 - I*S
inh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}+\frac{2}{5}\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx\\ &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}+\frac{2}{15}\int \frac{1}{1-i\sinh(c+dx)}\,dx\\ &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 0.92

$$\frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))-15i\sinh(c+dx)+6i\sinh(2(c+dx))+i\sinh(3(c+dx))}{30d(i+\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] - (15*I)*S
inh[c + d*x] + (6*I)*Sinh[2*(c + d*x)] + I*Sinh[3*(c + d*x)])/(30*d*(I + Si
nh[c + d*x])^3)
```

Maple [A]
time = 1.17, size = 88, normalized size = 1.00

method	result	size
risch	$\frac{4i\left(5i\,e^{dx+c}+10\,e^{2dx+2c}-1\right)}{15d\left(e^{dx+c}+i\right)^5}$	40
derivativedivides	$\frac{\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$	88
default	$\frac{\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{4i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/(I+tanh(1/2*d*x+1/2*c))+4*I/(I+tanh(1/2*d*x+1/2*c))^4+8/5/(I+tanh(1/
2*d*x+1/2*c))^5-4*I/(I+tanh(1/2*d*x+1/2*c))^2-16/3/(I+tanh(1/2*d*x+1/2*c))^
3)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(70) = 140.
time = 0.32, size = 211, normalized size = 2.40

$$\frac{20i\,e^{(-dx-c)}}{-15\,d\left(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1\right)}-\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d\left(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1\right)}+\frac{4}{-15\,d\left(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) - 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) + 4/(d*(75*I*e
^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) + 75*e^(-4*d*x
 - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15))
```

Fricas [A]
time = 0.42, size = 85, normalized size = 0.97

$$-\frac{4\left(-10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+i\right)}{15\left(de^{(5\,dx+5\,c)}+5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}-10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}+i\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4/15*(-10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) + I)/(d*e^(5*d*x + 5*c) + 5*I*
d*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) - 10*I*d*e^(2*d*x + 2*c) + 5*d*e^(
d*x + c) + I*d)
```

Sympy [A]
time = 0.18, size = 109, normalized size = 1.24

$$\frac{40ie^{2c}e^{2dx}-20e^{c}e^{dx}-4i}{15de^{5c}e^{5dx}+75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}-150ide^{2c}e^{2dx}+75de^{c}e^{dx}+15id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**3,x)
```

```
[Out] (40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) - 4*I)/(15*d*exp(5*c)*exp(5*
d*x) + 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) - 150*I*d*exp
(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) + 15*I*d)
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.41

$$\frac{4i\left(10\,e^{(2\,dx+2\,c)}+5i\,e^{(dx+c)}-1\right)}{15\,d\left(e^{(dx+c)}+i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/15*I*(10*e^(2*d*x + 2*c) + 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) + I)^5)
```

Mupad [B]
time = 0.63, size = 40, normalized size = 0.45

$$-\frac{4\left(10\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,5\mathrm{i}\right)}{15\,d\,\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(c + d*x)*1i - 1)^3,x)
```

```
[Out] -(4*(exp(c + d*x)*5i + 10*exp(2*c + 2*d*x) - 1))/(15*d*(exp(c + d*x)*1i - 1
)^5)
```

3.63 $\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=117

$$-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}$$

```
[Out] -1/7*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^4-3/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^3-2/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-2/35*I*cosh(d*x+c)/d/(1-I*s
inh(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {2729, 2727}

$$-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}-\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-1/7*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4) - (((3*I)/35)*Cosh[c +
 d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*S
inh[c + d*x])^2) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} + \frac{3}{7}\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} + \frac{6}{35}\int \frac{1}{(1-i\sinh(c+$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx$$
$$= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx$$

Mathematica [A]
time = 0.11, size = 87, normalized size = 0.74

$$\frac{-21i\cosh\left(\frac{3}{2}(c+dx)\right)+i\cosh\left(\frac{7}{2}(c+dx)\right)+35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-21*I)*Cosh[(3*(c + d*x))/2] + I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x
)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)
/2])^7)
```

Maple [A]
time = 1.16, size = 121, normalized size = 1.03

method	result
risch	$-\frac{4\left(-7\,e^{dx+c}+21ie^{2dx+2c}+35\,e^{3dx+3c}-i\right)}{35d\left(e^{dx+c}+i\right)^7}$
derivativedivides	$\frac{\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{6i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{6i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(72/5/(I+tanh(1/2*d*x+1/2*c))^5+16*I/(I+tanh(1/2*d*x+1/2*c))^4+2/(I+tan
h(1/2*d*x+1/2*c))-8*I/(I+tanh(1/2*d*x+1/2*c))^6-16/7/(I+tanh(1/2*d*x+1/2*c)
)^7-6*I/(I+tanh(1/2*d*x+1/2*c))^2-12/(I+tanh(1/2*d*x+1/2*c))^3)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(93) = 186$.
time = 0.30, size = 372, normalized size = 3.18

$$\frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)}+21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}-35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-7\,c)}-i)}+\frac{12i\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)}+21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}-35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-7\,c)}-i)}-\frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)}+21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}-35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-7\,c)}-i)}-\frac{4i}{35\,d(7\,e^{(-dx-c)}+21i\,e^{(-2\,dx-2\,c)}-35\,e^{(-3\,dx-3\,c)}-35i\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7i\,e^{(-6\,dx-6\,c)}-e^{(-7\,dx-7\,c)}-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
- 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c)
 - e^(-7*d*x - 7*c) - I)) + 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21
*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-
5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) - I)) - 4*e^(-3*d*x
- 3*c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) - 3
5*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) - e^(-7*d
*x - 7*c) - I)) - 4/35*I/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^
(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*
x - 6*c) - e^(-7*d*x - 7*c) - I))
```

Fricas [**A**]
time = 0.42, size = 120, normalized size = 1.03

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\left(de^{(7\,dx+7\,c)}+7i\,de^{(6\,dx+6\,c)}-21\,de^{(5\,dx+5\,c)}-35i\,de^{(4\,dx+4\,c)}+35\,de^{(3\,dx+3\,c)}+21i\,de^{(2\,dx+2\,c)}-7\,de^{(dx+c)}-i\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*e^
(7*d*x + 7*c) + 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) - 35*I*d*e^(4*
d*x + 4*c) + 35*d*e^(3*d*x + 3*c) + 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x + c
) - I*d)
```

Sympy [**A**]
time = 0.29, size = 155, normalized size = 1.32

$$\frac{-140e^{3c}e^{3dx}-84ie^{2c}e^{2dx}+28e^{c}e^{dx}+4i}{35de^{7c}e^{7dx}+245ide^{6c}e^{6dx}-735de^{5c}e^{5dx}-1225ide^{4c}e^{4dx}+1225de^{3c}e^{3dx}+735ide^{2c}e^{2dx}-245de^{c}e^{dx}-35id}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))**4,x)
```

```
[Out] (-140*exp(3*c)*exp(3*d*x) - 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x) +
 4*I)/(35*d*exp(7*c)*exp(7*d*x) + 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp(5
*c)*exp(5*d*x) - 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*x)
+ 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) - 35*I*d)
```

Giac [A]
time = 0.41, size = 47, normalized size = 0.40

$$-\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\,d(e^{(dx+c)}+i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*(e
^(d*x + c) + I)^7)
```

Mupad [B]
time = 0.91, size = 52, normalized size = 0.44

$$-\frac{4\left(21\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,7\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}\,35\mathrm{i}\right)}{35\,d\,\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*1i - 1)^4,x)
```

```
[Out] -(4*(exp(c + d*x)*7i + 21*exp(2*c + 2*d*x) - exp(3*c + 3*d*x)*35i - 1))/(35
*d*(exp(c + d*x)*1i - 1)^7)
```

3.64 $\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2830, 2728, 212}

$$\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx &= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} + i\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx \\
&= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} - 2\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right) \\
&= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 1.32

$$\frac{2\left((1+i)\sqrt[4]{-1}\,\mathrm{ArcTan}\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
[Out] (2*((1 + I)*(-1)^(1/4)*ArcTan[(I + Tanh[x/4])/Sqrt[2]] + Cosh[x/2] - I*Sinh
[x/2])*(Cosh[x/2] + I*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(x\right)}{\sqrt{a+ia\sinh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
[Out] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Fricas [A]
time = 0.38, size = 76, normalized size = 1.33

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)+2\,\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,e^{x}-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) - sqrt(2)
*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) + 2*sqrt(1/2*I*a*
e^(-x))*(I*e^x - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh{(x)}}{\sqrt{ia\,(\sinh{(x)}-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(I*a*(sinh(x) - I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sinh(x)}{\sqrt{a+a\sinh{(x)}\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2),x)
```

```
[Out] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2), x)
```

3.65 $\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}$$

```
[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a-I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a-I*a*sinh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2830, 2728, 212}

$$\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a - I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx &= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} - i\int \frac{1}{\sqrt{a-ia\sinh(x)}}\,dx\\ &= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} + 2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 1.33

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\left((1+i)(-1)^{3/4}\text{ArcTan}\left(\frac{-i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\sinh\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a-ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
[Out] (2*(Cosh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*((1 + I)*(-1)^(3/4)*ArcTan[(-I
+ Tanh[x/4])/Sqrt[2]] + Sinh[x/2])))/Sqrt[a - I*a*Sinh[x]]
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
[Out] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Fricas [A]
time = 0.40, size = 76, normalized size = 1.33

$$-\frac{\sqrt{2}\,\sqrt{a}\,\log\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)+2\,\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\,(-i\,e^{x}-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) - sqrt(2
)*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) + 2*sqrt(-1/2*I
*a*e^(-x))*(-I*e^x - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh(x)}{\sqrt{-ia\,(\sinh(x)+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(-I*a*(sinh(x) + I)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sinh(x)}{\sqrt{a-a\sinh(x)\ 1\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2),x)
```

```
[Out] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2), x)
```

3.66 $\int (a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 2/5*I*a*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(3/2)/d+64/15*I*a^3*cosh(d*x+c)/d/(
a+I*a*sinh(d*x+c))^(1/2)+16/15*I*a^2*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2726, 2725}

$$\frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((16*I)/
15)*a^2*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d + (((2*I)/5)*a*Cosh[c
+ d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\sinh(c+dx))^{5/2}\,dx &= \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int (a+ia\sinh(c+dx))^{3/2} \\ &= \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2ia\cosh(c+dx)(a+ia\sinh(c}{5d} \\ &= \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}+\frac{2}{}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 145, normalized size = 1.39

$$\frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(-150i\cosh\left(\frac{1}{2}(c+dx)\right)-25i\cosh\left(\frac{3}{2}(c+dx)\right)+3i\cosh\left(\frac{5}{2}(c+dx)\right)-150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sinh\left(\frac{5}{2}(c+dx)\right)\right)}{30d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*((-150*I)*Cosh[(c +
 d*x)/2] - (25*I)*Cosh[(3*(c + d*x))/2] + (3*I)*Cosh[(5*(c + d*x))/2] - 150
*Sinh[(c + d*x)/2] + 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(
30*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F]
time = 1.95, size = 0, normalized size = 0.00

$$\int \left(a+ia\sinh\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Fricas [**A**]
time = 0.40, size = 101, normalized size = 0.97

$$-\frac{\left(3\,a^{2}e^{(5\,dx+5\,c)}-25i\,a^{2}e^{(4\,dx+4\,c)}-150\,a^{2}e^{(3\,dx+3\,c)}-150i\,a^{2}e^{(2\,dx+2\,c)}-25\,a^{2}e^{(dx+c)}+3i\,a^{2}\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\,e^{(-2\,dx-2\,c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*a^2*e^(5*d*x + 5*c) - 25*I*a^2*e^(4*d*x + 4*c) - 150*a^2*e^(3*d*x
+ 3*c) - 150*I*a^2*e^(2*d*x + 2*c) - 25*a^2*e^(d*x + c) + 3*I*a^2)*sqrt(1/2
*I*a*e^(-d*x - c))*e^(-2*d*x - 2*c)/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\sinh(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2),x)
```

```
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.67 $\int (a+ia\sinh(c+dx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

```
[Out] 8/3*I*a^2*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)+2/3*I*a*cosh(d*x+c)*(a+I*
a*sinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2726, 2725}

$$\frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*
a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ia\sinh(c+dx))^{3/2}\,dx &= \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}+\frac{1}{3}(4a)\int \sqrt{a+ia\sinh(c+dx)}\,d\\ &= \frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}}+\frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 113, normalized size = 1.64

$$-\frac{a(-i+\sinh(c+dx))\sqrt{a+ia\sinh(c+dx)}\left(9\cosh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{3}{2}(c+dx)\right)-9i\sinh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*(-I + Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]]*(9*Cosh[(c + d*x)/
2] + Cosh[(3*(c + d*x))/2] - (9*I)*Sinh[(c + d*x)/2] + I*Sinh[(3*(c + d*x))
/2]))/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3)
```

Maple [F]
time = 1.84, size = 0, normalized size = 0.00

$$\int (a+ia\sinh(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.43, size = 63, normalized size = 0.91

$$\frac{\left(i\,ae^{(3\,dx+3\,c)}+9\,ae^{(2\,dx+2\,c)}+9i\,ae^{(dx+c)}+a\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\,e^{(-dx-c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*e^(3*d*x + 3*c) + 9*a*e^(2*d*x + 2*c) + 9*I*a*e^(d*x + c) + a)*sqr
t(1/2*I*a*e^(-d*x - c))*e^(-d*x - c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia \sinh (c + dx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sinh(c + d*x) + a)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sinh(c + d\,x)\ 1\mathrm{i})^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((a + a*sinh(c + d*x)*1i)^(3/2), x)
```

3.68 $\int \sqrt{a+ia\sinh(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[Out] 2*I*a*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2725}

$$\frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.
time = 0.03, size = 74, normalized size = 2.39

$$\frac{2\left(i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a+ia\sinh(c+dx)}}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (2*(I*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*Sqrt[a + I*a*Sinh[c + d*x]])/(
d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.
time = 1.66, size = 89, normalized size = 2.87

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)}{\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*d*x+2*c)+2*exp(d*x+c)-I)*exp(-d*x-c))^(1/2)/(I*exp(2*
d*x+2*c)+2*exp(d*x+c)-I)*(exp(d*x+c)+I)*(exp(d*x+c)-I)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Fricas [A]
time = 0.38, size = 27, normalized size = 0.87

$$\frac{2\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\,\left(e^{(dx+c)}+i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(1/2*I*a*e^(-d*x - c))*(e^(d*x + c) + I)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\sinh\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*sinh(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [B]
time = 0.75, size = 53, normalized size = 1.71

$$\frac{\sqrt{2}\,\left(\mathrm{e}^{c+d\,x}+1{}\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-c-d\,x}\,{\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}^2\,1{}\mathrm{i}}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2^(1/2)*(exp(c + d*x) + 1i)*(a*exp(- c - d*x)*(exp(c + d*x) - 1i)^2*1i)^(1
/2))/(d*(exp(c + d*x) - 1i))
```

3.69 $\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$

Optimal. Leaf size=52

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2728, 212}

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d
*x]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \frac{(2i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{d}$$
$$= \frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 1.62

$$\frac{(2+2i)\sqrt[4]{-1}\,\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
[Out] ((2 + 2*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4
])]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]))/(d*Sqrt[a + I*a*Sinh[c +
d*x]])
```

Maple [F]
time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+ia\sinh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
[Out] int(1/(a+I*a*sinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(39) = 78.
time = 0.39, size = 93, normalized size = 1.79

$$i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{1}{2}\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)-i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{1}{2}\sqrt{2}\,ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] I*sqrt(2)*sqrt(1/(a*d^2))*log(1/2*sqrt(2)*a*d*sqrt(1/(a*d^2)) + sqrt(1/2*I*
a*e^(-d*x - c))) - I*sqrt(2)*sqrt(1/(a*d^2))*log(-1/2*sqrt(2)*a*d*sqrt(1/(a
*d^2)) + sqrt(1/2*I*a*e^(-d*x - c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ia\sinh{(c+dx)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sinh(c + d*x) + a), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{a+a\sinh\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(1/2), x)
```

3.70 $\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{2}\,\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] 1/2*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(3/2)+1/4*I*arctanh(1/2*cosh(d*x+c)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2729, 2728, 212}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(c+dx)}{\sqrt{2}\,\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]]
)])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^
(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx &= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{4a}\\ &= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}+\frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{2ad}\\ &= \frac{i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.17, size = 156, normalized size = 1.79

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\left((1-i)\sqrt[4]{-1}\,\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2ad(-i+\sinh(c+dx))\sqrt{a+ia\sinh(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] - I*((1 - I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sinh[(c + d*x)/2])))/(2*a*d*(-I + Si
nh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]])
```

Maple [**F**]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+ia\sinh\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(64) = 128$.
time = 0.37, size = 235, normalized size = 2.70

$$\frac{\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2\,dx+2\,c)}+2\,a^2de^{(dx+c)}-i\,a^2d\right)\sqrt{\frac{1}{a^3d^2}}\log\left(\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)+\sqrt{\frac{1}{2}}\left(-i\,a^2de^{(2\,dx+2\,c)}-2\,a^2de^{(dx+c)}+i\,a^2d\right)\sqrt{\frac{1}{a^3d^2}}\log\left(-\sqrt{\frac{1}{2}}\,a^2d\sqrt{\frac{1}{a^3d^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)-2\,\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\left(i\,e^{(2\,dx+2\,c)}-e^{(dx+c)}\right)}{2\left(a^2de^{(2\,dx+2\,c)}-2i\,a^2de^{(dx+c)}-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(I*a^2*d*e^(2*d*x + 2*c) + 2*a^2*d*e^(d*x + c) - I*a^2*d)*sq
rt(1/(a^3*d^2))*log(sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^(-d*
x - c))) + sqrt(1/2)*(-I*a^2*d*e^(2*d*x + 2*c) - 2*a^2*d*e^(d*x + c) + I*a^
2*d)*sqrt(1/(a^3*d^2))*log(-sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*
a*e^(-d*x - c))) - 2*sqrt(1/2*I*a*e^(-d*x - c))*(I*e^(2*d*x + 2*c) - e^(d*x
 + c)))/(a^2*d*e^(2*d*x + 2*c) - 2*I*a^2*d*e^(d*x + c) - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\sinh\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sinh(c + d*x) + a)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(3/2), x)
```

3.71 $\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] 1/4*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(5/2)+3/16*I*cosh(d*x+c)/a/d/(a+I*a
*sinh(d*x+c))^(3/2)+3/32*I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*s
inh(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2729, 2728, 212}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}+\frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d
*x])^(5/2)) + (((3*I)/16)*Cosh[c + d*x])/(a*d*(a + I*a*Sinh[c + d*x])^(3/2)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{32a^2}$$

$$= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{(3i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{ia\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{16a^2d}$$

$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.14, size = 210, normalized size = 1.72

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4i\cosh\left(\frac{1}{2}(c+dx)\right)+(3-3i)\sqrt[4]{-1}\,\text{ArcTan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^4+4\sinh\left(\frac{1}{2}(c+dx)\right)+6\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2\sinh\left(\frac{1}{2}(c+dx)\right)+3\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3\right)}{16d(a+ia\sinh(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((4*I)*Cosh[(c + d*x)/2] + (3 -
3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 4*Sinh[(c + d*x)/2] + 6*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2] + 3*((-I)*Cosh[(c +
d*x)/2] + Sinh[(c + d*x)/2])^3))/(16*d*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+ia\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(91) = 182$.
time = 0.42, size = 348, normalized size = 2.85

$$-\frac{3\sqrt{\frac{1}{2}}\left(-i\,a^3de^{(4\,dx+4\,c)}-4\,a^3de^{(3\,dx+3\,c)}+6i\,a^3de^{(2\,dx+2\,c)}+4\,a^3de^{(dx+c)}-i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)+3\sqrt{\frac{1}{2}}\left(i\,a^3de^{(4\,dx+4\,c)}+4\,a^3de^{(3\,dx+3\,c)}-6i\,a^3de^{(2\,dx+2\,c)}-4\,a^3de^{(dx+c)}+i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)-2\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\left(-3i\,e^{(4\,dx+4\,c)}-11\,e^{(3\,dx+3\,c)}-11i\,e^{(2\,dx+2\,c)}-3\,e^{(dx+c)}\right)}{16\left(a^3de^{(4\,dx+4\,c)}-4i\,a^3de^{(3\,dx+3\,c)}-6\,a^3de^{(2\,dx+2\,c)}+4i\,a^3de^{(dx+c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(1/2)*(-I*a^3*d*e^(4*d*x + 4*c) - 4*a^3*d*e^(3*d*x + 3*c) + 6*
I*a^3*d*e^(2*d*x + 2*c) + 4*a^3*d*e^(d*x + c) - I*a^3*d)*sqrt(1/(a^5*d^2))*
log(sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + 3*sqr
t(1/2)*(I*a^3*d*e^(4*d*x + 4*c) + 4*a^3*d*e^(3*d*x + 3*c) - 6*I*a^3*d*e^(2*
d*x + 2*c) - 4*a^3*d*e^(d*x + c) + I*a^3*d)*sqrt(1/(a^5*d^2))*log(-sqrt(1/2
)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) - 2*sqrt(1/2*I*a*e^
(-d*x - c))*(-3*I*e^(4*d*x + 4*c) - 11*e^(3*d*x + 3*c) - 11*I*e^(2*d*x + 2*
c) - 3*e^(d*x + c)))/(a^3*d*e^(4*d*x + 4*c) - 4*I*a^3*d*e^(3*d*x + 3*c) - 6
*a^3*d*e^(2*d*x + 2*c) + 4*I*a^3*d*e^(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\,\mathrm{sinh}\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(5/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.72 $\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$-\frac{a(2a^2-b^2)\,x}{2b^4}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{a\cosh(x)\sinh(x)}{2b^2}+\frac{\cosh(x)\sinh^2(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-b^2)*x/b^4-1/3*(2-3*a^2/b^2)*cosh(x)/b-1/2*a*cosh(x)*sinh(x)/
b^2+1/3*cosh(x)*sinh(x)^2/b-2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2872, 3128, 3102, 2814, 2739, 632, 212}

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b}-\frac{ax(2a^2-b^2)}{2b^4}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}}-\frac{a\sinh(x)\cosh(x)}{2b^2}+\frac{\sinh^2(x)\cosh(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - b^2)*x)/b^4 - (2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^4*Sqrt[a^2 + b^2]) - ((2 - (3*a^2)/b^2)*Cosh[x])/(3*b) - (a*Cosh[
x]*Sinh[x])/(2*b^2) + (Cosh[x]*Sinh[x]^2)/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{\sinh(x)(2a+2b\sinh(x)+3a\sinh^2(x))}{a+b\sinh(x)}\,dx}{3b}$$

$$= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{-3a^2+ab\sinh(x)-2(3a^2-2b^2)\sinh^2(x)}{a+b\sinh(x)}\,dx}{6b^2}$$

$$= \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{i\int \frac{3ia^2b-3ia(2a^2-b^2)\sinh(}{a+b\sinh(x)}}{6b^3}$$

$$= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{a^4 \int}{}$$

$$= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{(2a}{}$$

$$= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{(4a}{}$$

$$= -\frac{a(2a^2-b^2)\,x}{2b^4} - \frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(}{2b^2}$$

Mathematica [A]
time = 0.33, size = 105, normalized size = 0.97

$$\frac{3b(4a^2-3b^2)\cosh(x)+b^3\cosh(3x)+3a\left(-4a^2x+2b^2x+\frac{8a^3\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-b^2\sinh(2x)\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (3*b*(4*a^2 - 3*b^2)*Cosh[x] + b^3*Cosh[3*x] + 3*a*(-4*a^2*x + 2*b^2*x + (8
*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - b^2*Sin
h[2*x]))/(12*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(94) = 188$.
time = 0.37, size = 202, normalized size = 1.87

method	result
risch	$-\frac{a^3x}{b^4}+\frac{ax}{2b^2}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^2}+\frac{e^{x}a^2}{2b^3}-\frac{3\,e^{x}}{8b}+\frac{e^{-x}a^2}{2b^3}-\frac{3\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^2}+\frac{e^{-3x}}{24b}+\frac{a^4\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^4}$

default	$-\frac{1}{3b(\tanh(\frac{x}{2})-1)^3}-\frac{a+b}{2b^2(\tanh(\frac{x}{2})-1)^2}-\frac{2a^2+ab-b^2}{2b^3(\tanh(\frac{x}{2})-1)}+\frac{a(2a^2-b^2)\ln(\tanh(\frac{x}{2})-1)}{2b^4}+\frac{1}{3b(\tanh(\frac{x}{2})+1)^3}-\frac{-a+}{2b^2(\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b-b^2
)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2-b^2)/b^4*ln(tanh(1/2*x)-1)+1/3/b/(tanh(1
/2*x)+1)^3-1/2*(-a+b)/b^2/(tanh(1/2*x)+1)^2-1/2*(-2*a^2+a*b+b^2)/b^3/(tanh(
1/2*x)+1)-1/2*a*(2*a^2-b^2)/b^4*ln(tanh(1/2*x)+1)+2*a^4/b^4/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.50, size = 158, normalized size = 1.46

$$\frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}-\frac{(3\,abe^{(-x)}-b^2-3\,(4\,a^2-3\,b^2)e^{(-2\,x)})e^{(3\,x)}}{24\,b^3}+\frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\,(4\,a^2-3\,b^2)e^{(-x)}}{24\,b^3}-\frac{(2\,a^3-ab^2)x}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^4) - 1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 - 3*b^2)*e^(-2*
x))*e^(3*x)/b^3 + 1/24*(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 - 3*b^2)*e
^(-x))/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(96) = 192.
time = 0.36, size = 799, normalized size = 7.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*b^3 + b^5)*cosh(x)^6 + (a^2*b^3 + b^5)*sinh(x)^6 - 3*(a^3*b^2 +
a*b^4)*cosh(x)^5 - 3*(a^3*b^2 + a*b^4 - 2*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^
5 + a^2*b^3 + b^5 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b +
 a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b
^5)*cosh(x)^2 - 5*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 + b
^5)*cosh(x)^3 - 15*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4
)*x + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(4*a^4*b + a^2*b
^3 - 3*b^5)*cosh(x)^2 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b^5)*co
sh(x)^4 - 10*(a^3*b^2 + a*b^4)*cosh(x)^3 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*c
```

```
osh(x) + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh(
x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sin
h(x) - b)) + 3*(a^3*b^2 + a*b^4)*cosh(x) + 3*(2*(a^2*b^3 + b^5)*cosh(x)^5 +
 a^3*b^2 + a*b^4 - 5*(a^3*b^2 + a*b^4)*cosh(x)^4 - 12*(2*a^5 + a^3*b^2 - a*
b^4)*x*cosh(x)^2 + 4*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^3 + 2*(4*a^4*b + a
^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^2*b^4 +
 b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 + b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 +
b^6)*sinh(x)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 156, normalized size = 1.44

$$\frac{a^4 \log\left(\left|\frac{2\,be^x+2\,a-2\,\sqrt{a^2+b^2}}{2\,be^x+2\,a+2\,\sqrt{a^2+b^2}}\right|\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x-9\,b^2e^x}{24\,b^3}-\frac{(2\,a^3-ab^2)x}{2\,b^4}+\frac{\left(3\,ab^2e^x+b^3+3\,(4\,a^2b-3\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*
a^2*e^x - 9*b^2*e^x)/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x +
b^3 + 3*(4*a^2*b - 3*b^3)*e^(2*x))*e^(-3*x)/b^4
```

Mupad [**B**]
time = 0.76, size = 199, normalized size = 1.84

$$\frac{\mathrm{e}^{-3\,x}}{24\,b}+\frac{\mathrm{e}^{3\,x}}{24\,b}+\frac{x\,\left(a\,b^2-2\,a^3\right)}{2\,b^4}+\frac{\mathrm{e}^x\,\left(4\,a^2-3\,b^2\right)}{8\,b^3}+\frac{a\,\mathrm{e}^{-2\,x}}{8\,b^2}-\frac{a\,\mathrm{e}^{2\,x}}{8\,b^2}+\frac{\mathrm{e}^{-x}\,\left(4\,a^2-3\,b^2\right)}{8\,b^3}-\frac{a^4\,\ln\left(-\frac{2\,a^4\,\mathrm{e}^x}{b^5}-\frac{2\,a^4\,\left(b-a\,\mathrm{e}^x\right)}{b^5\,\sqrt{a^2+b^2}}\right)}{b^4\,\sqrt{a^2+b^2}}+\frac{a^4\,\ln\left(\frac{2\,a^4\,\left(b-a\,\mathrm{e}^x\right)}{b^5\,\sqrt{a^2+b^2}}-\frac{2\,a^4\,\mathrm{e}^x}{b^5}\right)}{b^4\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) + (x*(a*b^2 - 2*a^3))/(2*b^4) + (exp(x)*
(4*a^2 - 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (
exp(-x)*(4*a^2 - 3*b^2))/(8*b^3) - (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^4*(
b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(b^4*(a^2 + b^2)^(1/2)) + (a^4*log
((2*a^4*(b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2)) - (2*a^4*exp(x))/b^5))/(b^4
*(a^2 + b^2)^(1/2))
```

3.73 $\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=82

$$\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}$$

```
[Out] 1/2*(2*a^2-b^2)*x/b^3-a*cosh(x)/b^2+1/2*cosh(x)*sinh(x)/b+2*a^3*arctanh((b-
a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2872, 3102, 2814, 2739, 632, 212}

$$\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2
]])/(b^3*Sqrt[a^2 + b^2]) - (a*Cosh[x])/b^2 + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)\sinh(x)}{2b} - \frac{\int \frac{a+b\sinh(x)+2a\sinh^2(x)}{a+b\sinh(x)}\,dx}{2b} \\
&= -\frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^2} \\
&= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{a^3\int \frac{1}{a+b\sinh(x)}\,dx}{b^3} \\
&= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3} \\
&= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} + \frac{(4a^3)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\right.}{b^3} \\
&= \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 82, normalized size = 1.00

$$\frac{4a^2x-2b^2x-\frac{8a^3\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(x)+b^2\sinh(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (4*a^2*x - 2*b^2*x - (8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqr
t[-a^2 - b^2] - 4*a*b*Cosh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(72) = 144$.
time = 0.36, size = 152, normalized size = 1.85

method	result
default	$\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-2a^2+b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^3}-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\left(2a^2-b^2\right)\ln}{}$
risch	$\frac{x\,a^2}{b^3}-\frac{x}{2b}+\frac{\mathrm{e}^{2x}}{8b}-\frac{a\,\mathrm{e}^{x}}{2b^2}-\frac{a\,\mathrm{e}^{-x}}{2b^2}-\frac{\mathrm{e}^{-2x}}{8b}+\frac{a^3\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^3}-\frac{a^3\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)-1)+1/2/b^3*(-2*a^2+b^
2)*ln(tanh(1/2*x)-1)-1/2/b/(tanh(1/2*x)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1/2*x)+
1)+1/2*(2*a^2-b^2)/b^3*ln(tanh(1/2*x)+1)-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [**A**]
time = 0.50, size = 118, normalized size = 1.44

$$-\frac{a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^3}-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}-\frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2}+\frac{\left(2\,a^2-b^2\right)x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) - 1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x)
 + b*e^(-2*x))/b^2 + 1/2*(2*a^2 - b^2)*x/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(74) = 148$.
time = 0.51, size = 459, normalized size = 5.60

$$\frac{(a^2b^2+b^4)\cosh(x)^4+(a^2b^2+b^4)\sinh(x)^4-a^2b^2-b^4+4\,(2\,a^4+a^2b^2-b^4)x\cosh(x)^2-4\,(a^3b+ab^3)\cosh(x)^3-4\,(a^3b+ab^3-(a^2b^2+b^4)\cosh(x))\sinh(x)^3+2\,(3\,(a^2b^2+b^4)\cosh(x)^2+2\,(2\,a^4+a^2b^2-b^4)x-6\,(a^3b+ab^3)\cosh(x))\sinh(x)^2+8\,(a^3\cosh(x)^2+2\,a^3\cosh(x)\sinh(x)+a^3\sinh(x)^2)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-4\,(a^3b+ab^3)\cosh(x)-4\,(a^3b+ab^3-(a^2b^2+b^4)\cosh(x)^3-2\,(2\,a^4+a^2b^2-b^4)x\cosh(x)+3\,(a^3b+ab^3)\cosh(x)^2)\sinh(x)}{8\,((a^2b^3+b^5)\cosh(x)^2+2\,(a^2b^3+b^5)\cosh(x)\sinh(x)+(a^2b^3+b^5)\sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 - a^2*b^2 - b^4
+ 4*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 + b^4)*c
osh(x)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/
(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x)^3
- 2*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^2*b^3 + b^5)*cosh(x)*sinh(x) + (a^2*b^
3 + b^5)*sinh(x)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 117, normalized size = 1.43

$$-\frac{a^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^3}+\frac{be^{(2\,x)}-4\,ae^x}{8\,b^2}+\frac{(2\,a^2-b^2)x}{2\,b^3}-\frac{(4\,abe^x+b^2)e^{(-2\,x)}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) + 1/8*(b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2
*a^2 - b^2)*x/b^3 - 1/8*(4*a*b*e^x + b^2)*e^(-2*x)/b^3
```

Mupad [B]
time = 0.61, size = 159, normalized size = 1.94

$$\frac{e^{2x}}{8b} - \frac{e^{-2x}}{8b} + \frac{x(2a^2 - b^2)}{2b^3} - \frac{a e^x}{2b^2} - \frac{a e^{-x}}{2b^2} - \frac{a^3 \ln\left(\frac{2a^3 e^x}{b^4} - \frac{2a^3(b - a e^x)}{b^4\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} + \frac{a^3 \ln\left(\frac{2a^3 e^x}{b^4} + \frac{2a^3(b - a e^x)}{b^4\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) + (x*(2*a^2 - b^2))/(2*b^3) - (a*exp(x))/(
2*b^2) - (a*exp(-x))/(2*b^2) - (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b - a*
exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^
3*exp(x))/b^4 + (2*a^3*(b - a*exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2
+ b^2)^(1/2))
```

3.74 $\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{ax}{b^2}-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^
2+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2825, 12, 2814, 2739, 632, 212}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt
[a^2 + b^2]) + Cosh[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b} - \frac{\int \frac{a\sinh(x)}{a+b\sinh(x)}\,dx}{b} \\
&= \frac{\cosh(x)}{b} - \frac{a\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} - \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} - \frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{\cosh(x)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 1.07

$$
\frac{a\left(-x+\frac{2a\,\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (a*(-x + (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + b*Cosh[x])/b^2
```

Maple [A]
time = 0.35, size = 92, normalized size = 1.61

method	result	size
default	$\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}+\frac{2a^{2}\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}$	92
risch	$-\frac{ax}{b^{2}}+\frac{e^{x}}{2b}+\frac{e^{-x}}{2b}+\frac{a^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{a^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)+2*a^2/b^2/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b/(tanh(1/2*x)-1)+a/b^2*ln(
tanh(1/2*x)-1)
```

Maxima [A]
time = 0.49, size = 84, normalized size = 1.47

$$\frac{a^{2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{ax}{b^{2}}+\frac{e^{(-x)}}{2\,b}+\frac{e^{x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.
time = 0.55, size = 238, normalized size = 4.18

$$\frac{a^{2}b+b^{3}-2\left(a^{3}+ab^{2}\right)x\cosh\left(x\right)+\left(a^{2}b+b^{3}\right)\cosh\left(x\right)^{2}+\left(a^{2}b+b^{3}\right)\sinh\left(x\right)^{2}+2\left(a^{2}\cosh\left(x\right)+a^{2}\sinh\left(x\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}+b^{2}+2\left(b^{2}\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^{2}+b^{2}}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)-b}\right)-2\left(\left(a^{3}+ab^{2}\right)x-\left(a^{2}b+b^{3}\right)\cosh\left(x\right)\right)\sinh\left(x\right)}{2\left(\left(a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)+\left(a^{2}b^{2}+b^{4}\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b + b^3 - 2*(a^3 + a*b^2)*x*cosh(x) + (a^2*b + b^3)*cosh(x)^2 + (a
^2*b + b^3)*sinh(x)^2 + 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 + b^2)*log((
```

```
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*((a^3
 + a*b^2)*x - (a^2*b + b^3)*cosh(x))*sinh(x))/((a^2*b^2 + b^4)*cosh(x) + (a
^2*b^2 + b^4)*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(49) = 98$.

time = 175.16, size = 1253, normalized size = 21.98

$$\begin{cases} \tilde{\infty} \cosh(x) & \text{for } a = 0 \wedge b = 0 \\ \frac{\cosh(x)}{b} & \text{for } a = 0 \\ \frac{bx\tanh^2(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} - \frac{bx}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} - \frac{2b\tanh(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} + \frac{x\sqrt{-b^2}\tanh^3(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} - \frac{x\sqrt{-b^2}\tanh(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} - \frac{2\sqrt{-b^2}\tanh^2(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} + \frac{4\sqrt{-b^2}}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) - b\sqrt{-b^2}\tanh^2(\frac{x}{2}) + b\sqrt{-b^2}} & \text{for } a = -\sqrt{-b^2} \\ \frac{bx\tanh^2(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} - \frac{bx}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} - \frac{2b\tanh(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} - \frac{x\sqrt{-b^2}\tanh^3(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} + \frac{x\sqrt{-b^2}\tanh(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} + \frac{2\sqrt{-b^2}\tanh^2(\frac{x}{2})}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} - \frac{4\sqrt{-b^2}}{b^2\tanh^3(\frac{x}{2}) - b^2\tanh(\frac{x}{2}) + b\sqrt{-b^2}\tanh^2(\frac{x}{2}) - b\sqrt{-b^2}} & \text{for } a = \sqrt{-b^2} \\ \frac{\frac{x\sinh^2(x)}{2} - \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}}{a} & \text{for } b = 0 \\ -\frac{a^2\log\left(\tanh(\frac{x}{2}) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2(\frac{x}{2})}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} + \frac{a^2\log\left(\tanh(\frac{x}{2}) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} + \frac{a^2\log\left(\tanh(\frac{x}{2}) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2(\frac{x}{2})}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} - \frac{a^2\log\left(\tanh(\frac{x}{2}) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} - \frac{ax\sqrt{a^2+b^2}\tanh^2(\frac{x}{2})}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} + \frac{ax\sqrt{a^2+b^2}}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} - \frac{2b\sqrt{a^2+b^2}}{b^2\sqrt{a^2+b^2}\tanh^2(\frac{x}{2}) - b^2\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*cosh(x), Eq(a, 0) & Eq(b, 0)), (cosh(x)/b, Eq(a, 0)), (b*x*t
anh(x/2)**2/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**
2 + b*sqrt(-b**2)) - b*x/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2
)*tanh(x/2)**2 + b*sqrt(-b**2)) - 2*b*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*t
anh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqrt(-b**2)) + x*sqrt(-b**2)*tanh
(x/2)**3/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 +
 b*sqrt(-b**2)) - x*sqrt(-b**2)*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/
2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqrt(-b**2)) - 2*sqrt(-b**2)*tanh(x/2)*
*2/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqr
t(-b**2)) + 4*sqrt(-b**2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**
2)*tanh(x/2)**2 + b*sqrt(-b**2)), Eq(a, -sqrt(-b**2))), (b*x*tanh(x/2)**2/(
b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b
**2)) - b*x/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**
2 - b*sqrt(-b**2)) - 2*b*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*
sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) - x*sqrt(-b**2)*tanh(x/2)**3/(b**
2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2
)) + x*sqrt(-b**2)*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-
b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) + 2*sqrt(-b**2)*tanh(x/2)**2/(b**2*tanh
(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) - 4
*sqrt(-b**2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)*
*2 - b*sqrt(-b**2)), Eq(a, sqrt(-b**2))), ((x*sinh(x)**2/2 - x*cosh(x)**2/2
 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-a**2*log(tanh(x/2) - b/a - sqrt(a**2
+ b**2)/a)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a*
*2 + b**2)) + a**2*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b**2*sqrt(a*
*2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a**2 + b**2)) + a**2*log(tanh(x/2) - b/
a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2
- b**2*sqrt(a**2 + b**2)) - a**2*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)
/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a**2 + b**2)) - a*x*sqrt(
```

```
a**2 + b**2)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(
a**2 + b**2)) + a*x*sqrt(a**2 + b**2)/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2
- b**2*sqrt(a**2 + b**2)) - 2*b*sqrt(a**2 + b**2)/(b**2*sqrt(a**2 + b**2)*t
anh(x/2)**2 - b**2*sqrt(a**2 + b**2)), True))
```

Giac [A]
time = 0.43, size = 86, normalized size = 1.51

$$\frac{a^2 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Mupad [B]
time = 0.54, size = 129, normalized size = 2.26

$$\frac{e^{-x}}{2\,b} + \frac{e^x}{2\,b} - \frac{a\,x}{b^2} - \frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3} - \frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}} - \frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (a*x)/b^2 - (a^2*log(- (2*a^2*exp(x))/b^3 -
(2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2)) +
(a^2*log((2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(x))/b^
3))/(b^2*(a^2 + b^2)^(1/2))
```

3.75 $\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=47

$$\frac{x}{b}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] x/b+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2814, 2739, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\ &= \frac{x}{b}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{x}{b}+\frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{x}{b}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.11

$$\frac{x-\frac{2a\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x]),x]
[Out] (x - (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/b
```

Maple [A]
time = 0.33, size = 63, normalized size = 1.34

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$	63
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}-\frac{a\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tanh(1/2*x)+1)-1/b*ln(tanh(1/2*x)-1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(1
/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.49, size = 65, normalized size = 1.38

$$-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) + x/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.
time = 0.44, size = 134, normalized size = 2.85

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+(a^2+b^2)x}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) + (a^2 + b^2)*x)/(a^2*b + b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(37) = 74$.
time = 47.38, size = 252, normalized size = 5.36

$$\begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{\cosh(x)}{a} & \text{for } b=0 \\ \frac{bx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}}-\frac{2b}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}}-\frac{x\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}} & \text{for } a=-\sqrt{-b^2} \\ \frac{bx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}}-\frac{2b}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}}+\frac{x\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}} & \text{for } a=\sqrt{-b^2} \\ \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}-\frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (cosh(x)/a, Eq(b,
0)), (b*x*tanh(x/2)/(b**2*tanh(x/2) - b*sqrt(-b**2)) - 2*b/(b**2*tanh(x/2)
- b*sqrt(-b**2)) - x*sqrt(-b**2)/(b**2*tanh(x/2) - b*sqrt(-b**2)), Eq(a, -s
qrt(-b**2))), (b*x*tanh(x/2)/(b**2*tanh(x/2) + b*sqrt(-b**2)) - 2*b/(b**2*t
anh(x/2) + b*sqrt(-b**2)) + x*sqrt(-b**2)/(b**2*tanh(x/2) + b*sqrt(-b**2)),
 Eq(a, sqrt(-b**2))), (a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*sqrt
(a**2 + b**2)) - a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqrt(a**2
+ b**2)) + x/b, True))
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.43

$$-\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) + x/b
```

Mupad [B]
time = 0.53, size = 99, normalized size = 2.11

$$\frac{x}{b}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}-\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{2}\,\sqrt{a^{2}+b^{2}}}\right)}{b\,\sqrt{a^{2}+b^{2}}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}+\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{2}\,\sqrt{a^{2}+b^{2}}}\right)}{b\,\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x)),x)
```

```
[Out] x/b - (a*log((2*a*exp(x))/b^2 - (2*a*(b - a*exp(x)))/(b^2*(a^2 + b^2)^(1/2)
)))/(b*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(x))/b^2 + (2*a*(b - a*exp(x)))/
(b^2*(a^2 + b^2)^(1/2))))/(b*(a^2 + b^2)^(1/2))
```

3.76 $\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] -arctanh(cosh(x))/a+2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b
^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2826, 3855, 2739, 632, 212}

$$\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a) + (2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx &= \frac{\int \operatorname{csch}(x)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{(4b)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\ &= -\frac{\tanh^{-1}(\cosh(x))}{a}+\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.16

$$\frac{-\frac{2b\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x]),x]
[Out] ((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[T
anh[x/2]])/a
```

Maple [A]
time = 0.44, size = 49, normalized size = 0.98

method	result	size

default	$-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	49
risch	$\frac{b\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{b\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\ln(\mathrm{e}^x+1)}{a}+\frac{\ln(\mathrm{e}^x-1)}{a}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1
/a*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.49, size = 83, normalized size = 1.66

$$-\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*a) - log(e^(-x) + 1)/a + log(e^(-x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(46) = 92$.
time = 0.37, size = 156, normalized size = 3.12

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(a^2+b^2)\log\left(\cosh(x)+\sinh(x)+1\right)+(a^2+b^2)\log\left(\cosh(x)+\sinh(x)-1\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.42, size = 82, normalized size = 1.64

$$-\frac{b\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a}-\frac{\log\left(e^{x}+1\right)}{a}+\frac{\log\left(\left|e^{x}-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a) - log(e^x + 1)/a + log(abs(e^x - 1))/a
```

Mupad [B]
time = 0.64, size = 287, normalized size = 5.74

$$\frac{\ln\left(32\,a-32\,a\,\mathrm{e}^{x}\right)}{a}-\frac{\ln\left(32\,a+32\,a\,\mathrm{e}^{x}\right)}{a}+\frac{b\,\ln\left(128\,a^{5}\,\mathrm{e}^{x}-64\,a^{2}\,b^{3}-64\,a^{4}\,b-128\,a^{4}\,\mathrm{e}^{x}\,\sqrt{a^{2}+b^{2}}+32\,a\,b^{4}\,\mathrm{e}^{x}+160\,a^{3}\,b^{2}\,\mathrm{e}^{x}+32\,a\,b^{3}\,\sqrt{a^{2}+b^{2}}+64\,a^{3}\,b\,\sqrt{a^{2}+b^{2}}-96\,a^{2}\,b^{2}\,\mathrm{e}^{x}\,\sqrt{a^{2}+b^{2}}\right)\,\sqrt{a^{2}+b^{2}}}{a^{3}+a\,b^{2}}-\frac{b\,\ln\left(64\,a^{4}\,b+64\,a^{2}\,b^{3}-128\,a^{5}\,\mathrm{e}^{x}-128\,a^{4}\,\mathrm{e}^{x}\,\sqrt{a^{2}+b^{2}}-32\,a\,b^{4}\,\mathrm{e}^{x}-160\,a^{3}\,b^{2}\,\mathrm{e}^{x}+32\,a\,b^{3}\,\sqrt{a^{2}+b^{2}}+64\,a^{3}\,b\,\sqrt{a^{2}+b^{2}}-96\,a^{2}\,b^{2}\,\mathrm{e}^{x}\,\sqrt{a^{2}+b^{2}}\right)\,\sqrt{a^{2}+b^{2}}}{a^{3}+a\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*sinh(x))),x)
```

```
[Out] log(32*a - 32*a*exp(x))/a - log(32*a + 32*a*exp(x))/a + (b*log(128*a^5*exp(
x) - 64*a^2*b^3 - 64*a^4*b - 128*a^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a*b^4*ex
p(x) + 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b^2 +
 a^3) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(x) - 128*a^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a*b^4*exp(x) - 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)
^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a*b^2 + a^3)
```

3.77 $\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^
(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 12, 2826, 3855, 2739, 632, 212}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(a^2*Sqrt[a^2 + b^2]) - Coth[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)}{a} - \frac{\int \frac{b\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}$$
$$= -\frac{\coth(x)}{a} - \frac{b\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}$$
$$= -\frac{\coth(x)}{a} - \frac{b\int \operatorname{csch}(x)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} - \frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{a}$$

Mathematica [A]
time = 0.30, size = 81, normalized size = 1.37

$$-\frac{a\coth\left(\frac{x}{2}\right)+2b\left(-\frac{2b\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)+a\tanh\left(\frac{x}{2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x]),x]
[Out] -1/2*(a*Coth[x/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] + Log[Tanh[x/2]]) + a*Tanh[x/2])/a^2
```

Maple [A]
time = 0.52, size = 73, normalized size = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}$	73
risch	$-\frac{2}{a(\mathrm{e}^{2x}-1)}+\frac{b\ln(\mathrm{e}^x+1)}{a^2}+\frac{b^2\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^2}-\frac{b^2\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^2}-\frac{b\ln(\mathrm{e}^x-1)}{a^2}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))+2*b^2/a^2/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.49, size = 100, normalized size = 1.69

$$\frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^2}+\frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{2}{ae^{(-2\,x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) + b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2/(
a*e^(-2*x) - a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(55) = 110.
time = 0.49, size = 345, normalized size = 5.85

$$\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+\left(a^2b+b^3-\left(a^2b+b^3\right)\cosh\left(x\right)^2-2\left(a^2b+b^3\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^2b+b^3\right)\sinh\left(x\right)^2\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(a^2b+b^3-\left(a^2b+b^3\right)\cosh\left(x\right)^2-2\left(a^2b+b^3\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^2b+b^3\right)\sinh\left(x\right)^2\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^4+a^2b^2-\left(a^4+a^2b^2\right)\cosh\left(x\right)^2-2\left(a^4+a^2b^2\right)\cosh\left(x\right)\sinh\left(x\right)-\left(a^4+a^2b^2\right)\sinh\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 -
 b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3
)*cosh(x)*sinh(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3)*cosh(x)*sinh(x) - (
a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^4 + a^2*b^2 - (a^4 +
 a^2*b^2)*cosh(x)^2 - 2*(a^4 + a^2*b^2)*cosh(x)*sinh(x) - (a^4 + a^2*b^2)*s
inh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.44, size = 98, normalized size = 1.66

$$\frac{b^2 \log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^2}+\frac{b\log\left(e^x+1\right)}{a^2}-\frac{b\log\left(\left|e^x-1\right|\right)}{a^2}-\frac{2}{a(e^{(2\,x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1)
)/a^2 - 2/(a*(e^(2*x) - 1))
```

Mupad [B]
time = 0.70, size = 292, normalized size = 4.95

$$\frac{2}{a-a\,e^{2\,x}}-\frac{b\ln\left(32\,e^x-32\right)}{a^2}+\frac{b\ln\left(32\,e^x+32\right)}{a^2}+\frac{b^2\ln\left(128\,a^4\,e^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,e^x+128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x-64\,a^2\,b\,\sqrt{a^2+b^2}+96\,a\,b^2\,e^x\,\sqrt{a^2+b^2}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}-\frac{b^2\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,e^x+32\,b^4\,e^x-128\,a^3\,e^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,e^x+64\,a^2\,b\,\sqrt{a^2+b^2}-96\,a\,b^2\,e^x\,\sqrt{a^2+b^2}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))),x)
```

```
[Out] 2/(a - a*exp(2*x)) - (b*log(32*exp(x) - 32))/a^2 + (b*log(32*exp(x) + 32))/
a^2 + (b^2*log(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1
/2) + 32*b^4*exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a^2*b*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(a^4 + a^2*b^2) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 128*a^4*exp(x) + 32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)
```

3.78 $\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cosh(x))/a^3+b*coth(x)/a^2-1/2*coth(x)*csch(x)/a+2*
b^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\frac{b\coth(x)}{a^2}+\frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3}+\frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^3) + (2*b^3*ArcTanh[(b - a*Tanh[x/2])
/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) + (b*Coth[x])/a^2 - (Coth[x]*Csch[
x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{i\int \frac{\operatorname{csch}^2(x)\left(2ib+ia\sinh(x)+ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{2a}$$

$$= \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{\int \frac{\operatorname{csch}(x)\left(a^2-2b^2+ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{2a^2}$$

$$= \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{b^3\int \frac{1}{a+b\sinh(x)}\,dx}{a^3} - \frac{(a^2-2b^2)\int \operatorname{csch}(x)\,dx}{2a^3}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,d\right.}{a^3}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{(4b^3)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\right.}{a}$$

$$= \frac{(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^3} + \frac{2b^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Mathematica [A]
time = 0.39, size = 118, normalized size = 1.46

$$-\frac{\frac{16b^3\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{x}{2}\right) + a^2\operatorname{csch}^2\left(\frac{x}{2}\right) + 4(a^2-2b^2)\log\left(\tanh\left(\frac{x}{2}\right)\right) + a^2\operatorname{sech}^2\left(\frac{x}{2}\right) - 4ab\tanh\left(\frac{x}{2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/8*((16*b^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 + 4*(a^2 - 2*b^2)*Log[Tanh[x/2]] + a^2*
Sech[x/2]^2 - 4*a*b*Tanh[x/2])/a^3
```

Maple [A]
time = 0.59, size = 108, normalized size = 1.33

method	result
default	$\frac{\frac{a\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+2b\tanh\left(\frac{x}{2}\right)}{4a^2} - \frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{1}{8a\tanh\left(\frac{x}{2}\right)^2} + \frac{(-2a^2+4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^3} + \frac{b}{2a^2\tanh\left(\frac{x}{2}\right)}$
risch	$-\frac{a\,e^{3x}-2b\,e^{2x}+a\,e^{x}+2b}{\left(e^{2x}-1\right)^2a^2} + \frac{\ln(e^x+1)}{2a} - \frac{\ln(e^x+1)b^2}{a^3} + \frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right.}{\sqrt{a^2+b^2}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/2*a*tanh(1/2*x)^2+2*b*tanh(1/2*x))-2/a^3*b^3/(a^2+b^2)^(1/2)*arc
tanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/8/a/tanh(1/2*x)^2+1/4/a^3
*(-2*a^2+4*b^2)*ln(tanh(1/2*x))+1/2/a^2*b/tanh(1/2*x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(73) = 146$.
time = 0.49, size = 154, normalized size = 1.90

$$-\frac{b^3 \log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{ae^{(-x)}+2\,be^{(-2\,x)}+ae^{(-3\,x)}-2\,b}{2\,a^2e^{(-2\,x)}-a^2e^{(-4\,x)}-a^2}+\frac{(a^2-2\,b^2)\log\left(e^{(-x)}+1\right)}{2\,a^3}-\frac{(a^2-2\,b^2)\log\left(e^{(-x)}-1\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) + (a*e^(-x) + 2*b*e^(-2*x) + a*e^(-3*x) - 2*b)/(2*a^
2*e^(-2*x) - a^2*e^(-4*x) - a^2) + 1/2*(a^2 - 2*b^2)*log(e^(-x) + 1)/a^3 -
1/2*(a^2 - 2*b^2)*log(e^(-x) - 1)/a^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(73) = 146$.
time = 0.44, size = 929, normalized size = 11.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^
2*b^2)*cosh(x))*sinh(x)^2 - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^
3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^
2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(a^4 + a^2*b^2)*c
osh(x) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cos
h(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4
- 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 -
 a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)
```

```
^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
 ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)*sin
h(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4
 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2
 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^3 - (a^
4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^4
+ a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^5 + a^3*b^2 + (a^5 + a^3*b^2)*cosh(x)^4 + 4*(a^5 + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + a^3*b^2)*sinh(x)^4 - 2*(a^5 + a^3*b^2)*cosh(x)^2 - 2*(a^5
 + a^3*b^2 - 3*(a^5 + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + a^3*b^2)*co
sh(x)^3 - (a^5 + a^3*b^2)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.43, size = 137, normalized size = 1.69

$$-\frac{b^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^3}+\frac{(a^2-2\,b^2)\log(e^x+1)}{2\,a^3}-\frac{(a^2-2\,b^2)\log(|e^x-1|)}{2\,a^3}-\frac{ae^{(3\,x)}-2\,be^{(2\,x)}+ae^x+2\,b}{a^2\left(e^{(2\,x)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1
/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^3 - (a*e^(3*x) - 2*b*e^(2*x) + a*e^x +
 2*b)/(a^2*(e^(2*x) - 1)^2)
```

Mupad [B]
time = 1.00, size = 617, normalized size = 7.62

$$\frac{e^x}{a-a\,e^{2\,x}}-\frac{2\,e^x}{a-2\,a\,e^{2\,x}+a\,e^{4\,x}}-\frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,a}+\frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{2\,a}+\frac{2\,b}{a^2\,e^{2\,x}-a^2}+\frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{a^3}-\frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{a^3}-\frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2+b^2}-32\,a^3\,b^3-32\,a^6\,e^x+24\,b^6\,e^x-40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^5\,e^x\,\sqrt{a^2+b^2}+112\,a^2\,b^4\,e^x+56\,a^4\,b^2\,e^x+16\,a^4\,b\,\sqrt{a^2+b^2}+72\,a\,b^4\,e^x\,\sqrt{a^2+b^2}+72\,a^3\,b^2\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^5+a^3\,b^2}+\frac{b^3\,\ln\left(24\,b^5\,\sqrt{a^2+b^2}-48\,a\,b^5+16\,a^5\,b-32\,a^3\,b^3-32\,a^6\,e^x+24\,b^6\,e^x+40\,a^2\,b^3\,\sqrt{a^2+b^2}+32\,a^5\,e^x\,\sqrt{a^2+b^2}+112\,a^2\,b^4\,e^x+56\,a^4\,b^2\,e^x-16\,a^4\,b\,\sqrt{a^2+b^2}-72\,a\,b^4\,e^x\,\sqrt{a^2+b^2}-72\,a^3\,b^2\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^5+a^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] exp(x)/(a - a*exp(2*x)) - (2*exp(x))/(a - 2*a*exp(2*x) + a*exp(4*x)) - log(
4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*a) + log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*a) + (2*b)/(a^2*exp(2*x) - a^2) + (b^2*log(4*a^4 +
 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/a
^3 - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) -
20*a^2*b^2*exp(x)))/a^3 - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 + b^2)
^(1/2) - 32*a^3*b^3 - 32*a^6*exp(x) + 24*b^6*exp(x) - 40*a^2*b^3*(a^2 + b^2
)^(1/2) - 32*a^5*exp(x)*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2
*exp(x) + 16*a^4*b*(a^2 + b^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) +
72*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (
b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 - 32*a^
6*exp(x) + 24*b^6*exp(x) + 40*a^2*b^3*(a^2 + b^2)^(1/2) + 32*a^5*exp(x)*(a^
2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 16*a^4*b*(a^2 + b
^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

3.79 $\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=109

$$-\frac{b(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{(2a^2-3b^2)\coth(x)}{3a^3}+\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{\coth(x)\operatorname{c}}{3a}$$

```
[Out] -1/2*b*(a^2-2*b^2)*arctanh(cosh(x))/a^4+1/3*(2*a^2-3*b^2)*coth(x)/a^3+1/2*b
*coth(x)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a-2*b^4*arctanh((b-a*tanh(1/2*x)
)/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{b(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{(2a^2-3b^2)\coth(x)}{3a^3}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/a^4 - (2*b^4*ArcTanh[(b - a*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a^4*Sqrt[a^2 + b^2]) + ((2*a^2 - 3*b^2)*Coth[x])/(3*
a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = -\frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{i\int \frac{\operatorname{csch}^3(x)(3ib+2ia\sinh(x)+2ib\sinh^2(x))}{a+b\sinh(x)}\,dx}{3a}$$

$$= \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{\int \frac{\operatorname{csch}^2(x)(2(2a^2-3b^2)+ab\sinh(x)-3b^2\sinh^2(x))}{a+b\sinh(x)}\,dx}{6a^2}$$

$$= \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} - \frac{i\int \frac{\operatorname{csch}(x)(3ib(a^2-2b^2)+3iab}{a+b\sinh(x)}}{6a^3}$$

$$= \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} + \frac{(b(a^2}{}$$

$$= -\frac{b(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csc}}{3a}$$

$$= -\frac{b(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csc}}{3a}$$

$$= -\frac{b(a^2-2b^2)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\,c}{}$$

Mathematica [A]
time = 0.66, size = 186, normalized size = 1.71

$$\frac{\frac{48b^4\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 4a(2a^2-3b^2)\coth\left(\frac{x}{2}\right) + 3a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 12a^2b\log\left(\tanh\left(\frac{x}{2}\right)\right) - 24b^3\log\left(\tanh\left(\frac{x}{2}\right)\right) + 3a^2b\operatorname{sech}^2\left(\frac{x}{2}\right) + 8a^3\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{1}{2}a^3\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x) + 8a^3\tanh\left(\frac{x}{2}\right) - 12ab^2\tanh\left(\frac{x}{2}\right)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((48*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 4*a
*(2*a^2 - 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 + 12*a^2*b*Log[Tanh[x/2]]
- 24*b^3*Log[Tanh[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4
 - (a^3*Csch[x/2]^4*Sinh[x])/2 + 8*a^3*Tanh[x/2] - 12*a*b^2*Tanh[x/2])/(24*
a^4)
```

Maple [A]
time = 0.62, size = 151, normalized size = 1.39

method	result
default	$-\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^2}{3}+ab\left(\tanh^2\left(\frac{x}{2}\right)\right)-3a^2\tanh\left(\frac{x}{2}\right)+4b^2\tanh\left(\frac{x}{2}\right)}{8a^3} + \frac{2b^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{1}{24a\tanh\left(\frac{x}{2}\right)^3} - \frac{-3a^2+4b^2}{8a^3\tanh\left(\frac{x}{2}\right)}$

risch	$-\frac{-3ab\,e^{5x}+6b^2e^{4x}+12a^2e^{2x}-12b^2e^{2x}+3b\,e^{x}a-4a^2+6b^2}{3a^3(e^{2x}-1)^3}-\frac{b\ln(e^x+1)}{2a^2}+\frac{b^3\ln(e^x+1)}{a^4}+\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^3*(1/3*tanh(1/2*x)^3*a^2+a*b*tanh(1/2*x)^2-3*a^2*tanh(1/2*x)+4*b^2*t
anh(1/2*x))+2/a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^
2+b^2)^(1/2))-1/24/a/tanh(1/2*x)^3-1/8/a^3*(-3*a^2+4*b^2)/tanh(1/2*x)+1/8/a
^2*b/tanh(1/2*x)^2+1/2/a^4*b*(a^2-2*b^2)*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.50, size = 194, normalized size = 1.78

$$\frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^4}-\frac{3\,abe^{(-x)}-6\,b^2e^{(-4\,x)}-3\,abe^{(-5\,x)}+4\,a^2-6\,b^2-12\,(a^2-b^2)e^{(-2\,x)}}{3\,(3\,a^3e^{(-2\,x)}-3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}-a^3)}-\frac{(a^2b-2\,b^3)\log\left(e^{(-x)}+1\right)}{2\,a^4}+\frac{(a^2b-2\,b^3)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(-x) - 6*b^2*e^(-4*x) - 3*a*b*e^(-5*x)
 + 4*a^2 - 6*b^2 - 12*(a^2 - b^2)*e^(-2*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x
) + a^3*e^(-6*x) - a^3) - 1/2*(a^2*b - 2*b^3)*log(e^(-x) + 1)/a^4 + 1/2*(a^
2*b - 2*b^3)*log(e^(-x) - 1)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. 2(97) = 194.
time = 0.56, size = 1676, normalized size = 15.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 8*a^5
- 4*a^3*b^2 - 12*a*b^4 - 12*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(2*a^3*b^2 + 2*
a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 12*(5*(a^4*b + a^2*b^3)*co
sh(x)^2 - 4*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 24*(a^5 - a*b^4)*cosh(x)
^2 - 12*(2*a^5 - 2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 + a*b
^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4
*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*
sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*
cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cos
```

```
h(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b + a^2*b^3)*cosh(x) -
3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2*b^5
 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 - 5*(
a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 - 2*b
^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b -
 a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a^2*b
^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 +
6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1
) + 3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2
*b^5 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 -
 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 -
 2*b^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4
*b - a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a
^2*b^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^
2 + 6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 - 1) - 6*(a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 8*(a^3*b^2 + a
*b^4)*cosh(x)^3 + 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x
)^6 + 6*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 + a^4*b^2)*sinh(x)^6 - a^6
 - a^4*b^2 - 3*(a^6 + a^4*b^2)*cosh(x)^4 - 3*(a^6 + a^4*b^2 - 5*(a^6 + a^4*
b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + a^4*b^2)*cosh(x)^3 - 3*(a^6 + a^4*b
^2)*cosh(x))*sinh(x)^3 + 3*(a^6 + a^4*b^2)*cosh(x)^2 + 3*(a^6 + a^4*b^2 + 5
*(a^6 + a^4*b^2)*cosh(x)^4 - 6*(a^6 + a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a
^6 + a^4*b^2)*cosh(x)^5 - 2*(a^6 + a^4*b^2)*cosh(x)^3 + (a^6 + a^4*b^2)*cos
h(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*sinh(x)), x)
```

Giac [A]

time = 0.44, size = 171, normalized size = 1.57

$$\frac{b^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4}-\frac{(a^2b-2\,b^3)\log\left(e^x+1\right)}{2\,a^4}+\frac{(a^2b-2\,b^3)\log\left(|e^x-1|\right)}{2\,a^4}+\frac{3\,abe^{(5\,x)}-6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2-6\,b^2}{3\,a^3\left(e^{(2\,x)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2*(a^2*b - 2*b^3)*log(e^x + 1)/a^4 +
1/2*(a^2*b - 2*b^3)*log(abs(e^x - 1))/a^4 + 1/3*(3*a*b*e^(5*x) - 6*b^2*e^(4
*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 - 6*b^2)/(a^3*(e^
(2*x) - 1)^3)
```

Mupad [B]
time = 0.88, size = 694, normalized size = 6.37

$$\frac{8}{3\,\left(a-3\,a\,e^{2\,x}+3\,a\,e^{4\,x}-a\,e^{6\,x}\right)}-\frac{4}{a-2\,a\,e^{2\,x}+a\,e^{4\,x}}-\frac{2\,b^2}{a^3\,e^{2\,x}-a^3}+\frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{2\,a^2}-\frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{2\,a^2}-\frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,e^x-24\,b^4\,e^x+20\,a^2\,b^2\,e^x\right)}{a^4}+\frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,e^x+24\,b^4\,e^x-20\,a^2\,b^2\,e^x\right)}{a^4}+\frac{2\,b\,e^x}{a^2\,e^{4\,x}-2\,a^2\,e^{2\,x}+a^2}+\frac{b\,e^x}{a^2\,e^{2\,x}-a^2}+\frac{b^4\,\ln\left(16\,a^5\,b^2-48\,a\,b^6-32\,a^3\,b^4-24\,b^6\,\sqrt{a^2+b^2}+24\,b^7\,e^x-40\,a^2\,b^4\,\sqrt{a^2+b^2}+16\,a^4\,b^2\,\sqrt{a^2+b^2}-32\,a^6\,b\,e^x+112\,a^2\,b^5\,e^x+56\,a^4\,b^3\,e^x+72\,a\,b^5\,e^x\,\sqrt{a^2+b^2}-32\,a^5\,b\,e^x\,\sqrt{a^2+b^2}+72\,a^3\,b^3\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^6+a^4\,b^2}-\frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2+b^2}-48\,a\,b^6-32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,e^x+40\,a^2\,b^4\,\sqrt{a^2+b^2}-16\,a^4\,b^2\,\sqrt{a^2+b^2}-32\,a^6\,b\,e^x+112\,a^2\,b^5\,e^x+56\,a^4\,b^3\,e^x-72\,a\,b^5\,e^x\,\sqrt{a^2+b^2}+32\,a^5\,b\,e^x\,\sqrt{a^2+b^2}-72\,a^3\,b^3\,e^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^6+a^4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] 8/(3*(a - 3*a*exp(2*x) + 3*a*exp(4*x) - a*exp(6*x))) - 4/(a - 2*a*exp(2*x)
+ a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) - a^3) + (b*log(4*a^4 + 24*b^4 - 20*a
^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/(2*a^2) - (b*lo
g(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*e
xp(x)))/(2*a^2) - (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*
b^4*exp(x) + 20*a^2*b^2*exp(x)))/a^4 + (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2
 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*exp(x)))/a^4 + (2*b*exp(x))/(a
^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) + (b*exp(x))/(a^2*exp(2*x) - a^2) + (b^
4*log(16*a^5*b^2 - 48*a*b^6 - 32*a^3*b^4 - 24*b^6*(a^2 + b^2)^(1/2) + 24*b^
7*exp(x) - 40*a^2*b^4*(a^2 + b^2)^(1/2) + 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32
*a^6*b*exp(x) + 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) + 72*a*b^5*exp(x)*(a
^2 + b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 + b^2)^(1/2) + 72*a^3*b^3*exp(x)*(a^
2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^6 + a^4*b^2) - (b^4*log(24*b^6*(a^2 +
 b^2)^(1/2) - 48*a*b^6 - 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*exp(x) + 40*a^2*b
^4*(a^2 + b^2)^(1/2) - 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32*a^6*b*exp(x) + 112
*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a*b^5*exp(x)*(a^2 + b^2)^(1/2) + 3
2*a^5*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^
2 + b^2)^(1/2))/(a^6 + a^4*b^2)
```

3.80 $\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=162

$$\frac{(6a^2-b^2)\,x}{2b^4}+\frac{2a^3(3a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}}-\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)}+\frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)}-\frac{a}{b\,(a^2}$$

```
[Out] 1/2*(6*a^2-b^2)*x/b^4+2*a^3*(3*a^2+4*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^
2)^(1/2))/b^4/(a^2+b^2)^(3/2)-a*(3*a^2+2*b^2)*cosh(x)/b^3/(a^2+b^2)+1/2*(3*
a^2+b^2)*cosh(x)*sinh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)^2/b/(a^2+b^2)/(a
+b*sinh(x))
```

Rubi [A]
time = 0.30, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2871, 3128, 3102, 2814, 2739, 632, 212}

$$-\frac{a^2\sinh^2(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}+\frac{(3a^2+b^2)\sinh(x)\cosh(x)}{2b^2\,(a^2+b^2)}+\frac{x(6a^2-b^2)}{2b^4}-\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)}+\frac{2a^3(3a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*x)/(2*b^4) + (2*a^3*(3*a^2 + 4*b^2)*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^4*(a^2 + b^2)^(3/2)) - (a*(3*a^2 + 2*b^2)*Cosh[x])/(
b^3*(a^2 + b^2)) + ((3*a^2 + b^2)*Cosh[x]*Sinh[x])/(2*b^2*(a^2 + b^2)) - (a
^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{\int \frac{\sinh(x)(2a^2-ab\sinh(x)+(3a^2+b^2)\sinh^2(x))}{a+b\sinh(x)}\,dx}{b\,(a^2+b^2)}$$

$$= \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{\int \frac{-a(3a^2+b^2)+b(a^2-b^2)\sinh(x)}{a+b\sinh(}}{2b^2\,(a^2+}$$

$$= -\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} +$$

$$= \frac{(6a^2-b^2)\,x}{2b^4} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\,\mathrm{s}}{b\,(a^2+b^2)\,(a+}$$

$$= \frac{(6a^2-b^2)\,x}{2b^4} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\,\mathrm{s}}{b\,(a^2+b^2)\,(a+}$$

$$= \frac{(6a^2-b^2)\,x}{2b^4} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\,\mathrm{s}}{b\,(a^2+b^2)\,(a+}$$

$$= \frac{(6a^2-b^2)\,x}{2b^4} + \frac{2a^3(3a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2}{}$$

<u>**Mathematica**</u> [A]
time = 0.32, size = 118, normalized size = 0.73

$$\frac{-2(-6a^2+b^2)\,x + \frac{8a^3(3a^2+4b^2)\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} - 8ab\cosh(x) - \frac{4a^4b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + b^2\sinh(2x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(-6*a^2 + b^2)*x + (8*a^3*(3*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2]])/Sqrt
[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - 8*a*b*Cosh[x] - (4*a^4*b*Cosh[x])/((a^2
 + b^2)*(a + b*Sinh[x])) + b^2*Sinh[2*x])/(4*b^4)
```

<u>**Maple**</u> [A]
time = 0.66, size = 218, normalized size = 1.35

method	result

default	$-\frac{1}{2b^2(\tanh(\frac{x}{2})+1)^2}-\frac{-b+4a}{2b^3(\tanh(\frac{x}{2})+1)}+\frac{(6a^2-b^2)\ln(\tanh(\frac{x}{2})+1)}{2b^4}+\frac{1}{2b^2(\tanh(\frac{x}{2})-1)^2}-\frac{-b-4a}{2b^3(\tanh(\frac{x}{2})-1)}+\frac{(-6a^2+}{}$
risch	$\frac{3x\,a^2}{b^4}-\frac{x}{2b^2}+\frac{e^{2x}}{8b^2}-\frac{a\,e^{x}}{b^3}-\frac{a\,e^{-x}}{b^3}-\frac{e^{-2x}}{8b^2}+\frac{2a^4(a\,e^{x}-b)}{b^4(a^2+b^2)(b\,e^{2x}+2a\,e^{x}-b)}+\frac{3a^5\ln\left(e^{x}+\frac{a(a^2+b^2)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}b^4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2/(tanh(1/2*x)+1)^2-1/2*(-b+4*a)/b^3/(tanh(1/2*x)+1)+1/2*(6*a^2-b^2)
/b^4*ln(tanh(1/2*x)+1)+1/2/b^2/(tanh(1/2*x)-1)^2-1/2*(-b-4*a)/b^3/(tanh(1/2
*x)-1)+1/2/b^4*(-6*a^2+b^2)*ln(tanh(1/2*x)-1)+2/b^4*a^3*((b^2/(a^2+b^2)*tan
h(1/2*x)+a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(3*a^2+4*b^2)/(
a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.51, size = 256, normalized size = 1.58

$$-\frac{(3a^2+4b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{a^2b^3+b^5-6(a^3b^2+ab^4)e^{(-x)}-(32a^4b+17a^2b^3+b^5)e^{(-2x)}-8(2a^5-a^3b^2-ab^4)e^{(-3x)}}{8((a^2b^5+b^7)e^{(-2x)}+2(a^3b^4+ab^6)e^{(-3x)}-(a^2b^5+b^7)e^{(-4x)})}-\frac{8ae^{(-x)}+be^{(-2x)}}{8b^3}+\frac{(6a^2-b^2)x}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(3*a^2 + 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/8*(a^2*b^3 + b^5 - 6
*(a^3*b^2 + a*b^4)*e^(-x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(-2*x) - 8*(2*a
^5 - a^3*b^2 - a*b^4)*e^(-3*x))/((a^2*b^5 + b^7)*e^(-2*x) + 2*(a^3*b^4 + a*
b^6)*e^(-3*x) - (a^2*b^5 + b^7)*e^(-4*x)) - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b
^3 + 1/2*(6*a^2 - b^2)*x/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(154) = 308$.
time = 0.56, size = 1769, normalized size = 10.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^4*b^3 + 2*a^2*b^5 + b^7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^6 + (a
^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^6 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(
```

```
x)^5 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)
)*sinh(x)^5 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a
^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^4 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^
5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^2 - 4*(6*a^6*b + 11*a^4*b^
3 + 4*a^2*b^5 - b^7)*x + 30*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^
4 + 8*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh
(x)^3 + 4*(4*a^7 + 4*a^5*b^2 + 5*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 15
*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^2 + 2*(6*a^7 + 11*a^5*b^2 + 4*a^3*b^
4 - a*b^6)*x - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*
a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - (32*a^6*b + 49*a^4*b^3 +
 18*a^2*b^5 + b^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^2
 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x)^4 + 60*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 6*(16*a^6*b + 3
3*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x
)*cosh(x)^2 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x - 24*(2*a^7 + 2*
a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh(x))*sinh(x)^2 +
8*((3*a^5*b + 4*a^3*b^3)*cosh(x)^4 + (3*a^5*b + 4*a^3*b^3)*sinh(x)^4 + 2*(3
*a^6 + 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 + 4*a^4*b^2 + 2*(3*a^5*b + 4*a^3*b^3
)*cosh(x))*sinh(x)^3 - (3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - (3*a^5*b + 4*a^3*b
^3 - 6*(3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - 6*(3*a^6 + 4*a^4*b^2)*cosh(x))*sin
h(x)^2 + 2*(2*(3*a^5*b + 4*a^3*b^3)*cosh(x)^3 + 3*(3*a^6 + 4*a^4*b^2)*cosh(
x)^2 - (3*a^5*b + 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cos
h(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b
)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 6*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*cosh(x) + 2*(3*a^5*b^2 + 6*a^3*b^4 + 3*a*b^6 + 3*(a^4*b^3
 + 2*a^2*b^5 + b^7)*cosh(x)^5 - 15*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^4
- 2*(16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4
*a^2*b^5 - b^7)*x)*cosh(x)^3 + 12*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2
+ 4*a^3*b^4 - a*b^6)*x)*cosh(x)^2 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b
^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x))/((a^4*
b^5 + 2*a^2*b^7 + b^9)*cosh(x)^4 + (a^4*b^5 + 2*a^2*b^7 + b^9)*sinh(x)^4 +
2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x)^3 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8
+ 2*(a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 - (a^4*b^5 + 2*a^2*b^7 +
 b^9)*cosh(x)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9 - 6*(a^4*b^5 + 2*a^2*b^7 + b^9
)*cosh(x)^2 - 6*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 + 2*(2*(a^
4*b^5 + 2*a^2*b^7 + b^9)*cosh(x)^3 + 3*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x
)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 235, normalized size = 1.45

$$-\frac{\left(3\,a^{5}+4\,a^{3}b^{2}\right)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{\left(6\,a^{2}-b^{2}\right)x}{2\,b^{4}}+\frac{b^{2}e^{(2\,x)}-8\,abe^{x}}{8\,b^{4}}+\frac{\left(a^{2}b^{3}+b^{5}+8\,\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)e^{(3\,x)}-\left(32\,a^{4}b+17\,a^{2}b^{3}+b^{5}\right)e^{(2\,x)}+6\,\left(a^{3}b^{2}+ab^{4}\right)e^{x}\right)e^{(-2\,x)}}{8\,\left(a^{2}+b^{2}\right)\left(be^{(2\,x)}+2\,ae^{x}-b\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^5 + 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/2*(6*a^2
 - b^2)*x/b^4 + 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(a^2*b^3 + b^5 + 8*
(2*a^5 - a^3*b^2 - a*b^4)*e^(3*x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(2*x) +
 6*(a^3*b^2 + a*b^4)*e^x)*e^(-2*x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b
^4)
```

Mupad [B]
time = 0.86, size = 305, normalized size = 1.88

$$\frac{e^{2\,x}}{8\,b^{2}}-\frac{e^{-2\,x}}{8\,b^{2}}-\frac{\frac{2\,a^{4}}{b^{2}\,\left(a^{2}\,b+b^{3}\right)}-\frac{2\,a^{5}\,e^{x}}{b^{3}\,\left(a^{2}\,b+b^{3}\right)}}{2\,a\,e^{x}-b+b\,e^{2\,x}}+\frac{x\,\left(6\,a^{2}-b^{2}\right)}{2\,b^{4}}-\frac{a\,e^{x}}{b^{3}}-\frac{a\,e^{-x}}{b^{3}}-\frac{a^{3}\,\ln\left(\frac{2\,e^{x}\,\left(3\,a^{5}+4\,a^{3}\,b^{2}\right)}{a^{2}\,b^{5}+b^{7}}-\frac{2\,a^{3}\,\left(b-a\,e^{x}\right)\,\left(3\,a^{2}+4\,b^{2}\right)}{b^{5}\,{\left(a^{2}+b^{2}\right)}^{3/2}}\right)\,\left(3\,a^{2}+4\,b^{2}\right)}{b^{4}\,{\left(a^{2}+b^{2}\right)}^{3/2}}+\frac{a^{3}\,\ln\left(\frac{2\,e^{x}\,\left(3\,a^{5}+4\,a^{3}\,b^{2}\right)}{a^{2}\,b^{5}+b^{7}}+\frac{2\,a^{3}\,\left(b-a\,e^{x}\right)\,\left(3\,a^{2}+4\,b^{2}\right)}{b^{5}\,{\left(a^{2}+b^{2}\right)}^{3/2}}\right)\,\left(3\,a^{2}+4\,b^{2}\right)}{b^{4}\,{\left(a^{2}+b^{2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*a^4)/(b^2*(a^2*b + b^3)) - (2*a^
5*exp(x))/(b^3*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 -
b^2))/(2*b^4) - (a*exp(x))/b^3 - (a*exp(-x))/b^3 - (a^3*log((2*exp(x)*(3*a^
5 + 4*a^3*b^2))/(b^7 + a^2*b^5) - (2*a^3*(b - a*exp(x))*(3*a^2 + 4*b^2))/(b
^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/2)) + (a^3*log(
(2*exp(x)*(3*a^5 + 4*a^3*b^2))/(b^7 + a^2*b^5) + (2*a^3*(b - a*exp(x))*(3*a
^2 + 4*b^2))/(b^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/
2))
```

3.81 $\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2ax}{b^3}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}}+\frac{(2a^2+b^2)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[Out] -2*a*x/b^3-2*a^2*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b
^3/(a^2+b^2)^(3/2)+(2*a^2+b^2)*cosh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)/b/
(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2871, 3102, 2814, 2739, 632, 212}

$$\frac{(2a^2+b^2)\cosh(x)}{b^2\left(a^2+b^2\right)}-\frac{a^2\sinh(x)\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^3*(a^2 + b^2)^(3/2)) + ((2*a^2 + b^2)*Cosh[x])/(b^2*(a^2 + b^2))
- (a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}+\frac{\int \frac{a^2-ab\sinh(x)+(2a^2+b^2)\sinh^2(x)}{a+b\sinh(x)}\,dx}{b(a^2+b^2)}$$

$$=\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}+\frac{i\int \frac{-ia^2b+2ia(a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{b^2(a^2+b^2)}$$

$$=-\frac{2ax}{b^3}+\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}+\frac{(a^2(2a^2+3b^2))\int \frac{1}{a+b\sinh}}{b^3(a^2+b^2)}$$

$$=-\frac{2ax}{b^3}+\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}+\frac{(2a^2(2a^2+3b^2))\operatorname{Subst}(}{b^3(a}$$

$$=-\frac{2ax}{b^3}+\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}-\frac{(4a^2(2a^2+3b^2))\operatorname{Subst}\Big(}{}$$

$$=-\frac{2ax}{b^3}-\frac{2a^2(2a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}}+\frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)}-\frac{a^2\cosh(x}{b(a^2+b^2)(a}$$

Mathematica [A]
time = 0.26, size = 95, normalized size = 0.83

$$\frac{-2ax-\frac{2a^2(2a^2+3b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\cosh(x)\left(b+\frac{a^3b}{(a^2+b^2)(a+b\sinh(x))}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x - (2*a^2*(2*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]]
)/(-a^2 - b^2)^(3/2) + Cosh[x]*(b + (a^3*b)/((a^2 + b^2)*(a + b*Sinh[x]))))
/b^3
```

Maple [A]
time = 0.51, size = 161, normalized size = 1.40

method	result
default	$-\frac{1}{b^2(\tanh(\frac{x}{2})-1)}+\frac{2a\ln(\tanh(\frac{x}{2})-1)}{b^3}-\frac{4a^2\left(\frac{\frac{b^2\tanh(\frac{x}{2})}{2a^2+2b^2}+\frac{ab}{2a^2+2b^2}}{a(\tanh^2(\frac{x}{2}))-2b\tanh(\frac{x}{2})-a}-\frac{(2a^2+3b^2)\arctan\left(\frac{2a\tanh(\frac{x}{2})-2b}{2\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}\right)}{b^3}+\frac{1}{b^2(\tanh}$

risch	$-\frac{2ax}{b^3}+\frac{e^x}{2b^2}+\frac{e^{-x}}{2b^2}-\frac{2a^3(a\,e^x-b)}{b^3(a^2+b^2)(b\,e^{2x}+2a\,e^x-b)}+\frac{2a^4\ln\left(e^x+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}b^3}+\frac{3a^2\ln\left(e^x+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4}{b(a^2+b^2}\right.}{(a^2+b^2)^{\frac{3}{2}}b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/(tanh(1/2*x)-1)+2/b^3*a*ln(tanh(1/2*x)-1)-4/b^3*a^2*((1/2*b^2/(a^2+b
^2)*tanh(1/2*x)+1/2*a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-1/2*
(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(
1/2)))+1/b^2/(tanh(1/2*x)+1)-2/b^3*a*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.50, size = 208, normalized size = 1.81

$$\frac{(2\,a^2+3\,b^2)a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^3+b^5)\sqrt{a^2+b^2}}+\frac{a^2b^2+b^4+2\,(3\,a^3b+ab^3)e^{(-x)}+(4\,a^4-a^2b^2-b^4)e^{(-2\,x)}}{2\,((a^2b^4+b^6)e^{(-x)}+2\,(a^3b^3+ab^5)e^{(-2\,x)}-(a^2b^4+b^6)e^{(-3\,x)})}-\frac{2\,ax}{b^3}+\frac{e^{(-x)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2 + 3*b^2)*a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) + 1/2*(a^2*b^2 + b^4 + 2*
(3*a^3*b + a*b^3)*e^(-x) + (4*a^4 - a^2*b^2 - b^4)*e^(-2*x))/((a^2*b^4 + b^
6)*e^(-x) + 2*(a^3*b^3 + a*b^5)*e^(-2*x) - (a^2*b^4 + b^6)*e^(-3*x)) - 2*a*
x/b^3 + 1/2*e^(-x)/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(111) = 222$.
time = 0.50, size = 1053, normalized size = 9.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 - (
a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*x + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x)^2 + 2*(2*
a^6 + 2*a^4*b^2 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 + 4*(a^6 + 2*a^4*
b^2 + a^2*b^4)*x - 3*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b^3 + a*
b^5)*x)*cosh(x))*sinh(x)^2 - 2*((2*a^4*b + 3*a^2*b^3)*cosh(x)^3 + (2*a^4*b
```

```
+ 3*a^2*b^3)*sinh(x)^3 + 2*(2*a^5 + 3*a^3*b^2)*cosh(x)^2 + (4*a^5 + 6*a^3*b
^2 + 3*(2*a^4*b + 3*a^2*b^3)*cosh(x))*sinh(x)^2 - (2*a^4*b + 3*a^2*b^3)*cos
h(x) - (2*a^4*b + 3*a^2*b^3 - 3*(2*a^4*b + 3*a^2*b^3)*cosh(x)^2 - 4*(2*a^5
+ 3*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^5
 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^
5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5
- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5
)*x - 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x))*sinh(x))
/((a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^3 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(
x)^3 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 + (2*a^5*b^3 + 4*a^3*b^5 +
 2*a*b^7 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x))*sinh(x)^2 - (a^4*b^4 + 2*
a^2*b^6 + b^8)*cosh(x) - (a^4*b^4 + 2*a^2*b^6 + b^8 - 3*(a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^2 - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 184, normalized size = 1.60

$$\frac{\left(2\,a^4+3\,a^2b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^3+b^5\right)\sqrt{a^2+b^2}}-\frac{2\,ax}{b^3}+\frac{e^x}{2\,b^2}-\frac{\left(a^2b^2+b^4+\left(4\,a^4-a^2b^2-b^4\right)e^{(2\,x)}-2\,\left(3\,a^3b+ab^3\right)e^x\right)e^{(-x)}}{2\,\left(a^2+b^2\right)\left(be^{(2\,x)}+2\,ae^x-b\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4 + 3*a^2*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) - 2*a*x/b^3 +
 1/2*e^x/b^2 - 1/2*(a^2*b^2 + b^4 + (4*a^4 - a^2*b^2 - b^4)*e^(2*x) - 2*(3*
a^3*b + a*b^3)*e^x)*e^(-x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b^3)
```

Mupad [**B**]
time = 0.78, size = 274, normalized size = 2.38

$$\frac{\mathrm{e}^{-x}}{2\,b^2}+\frac{\frac{2\,a^3}{b\,\left(a^2\,b+b^3\right)}-\frac{2\,a^4\,\mathrm{e}^x}{b^2\,\left(a^2\,b+b^3\right)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}+\frac{\mathrm{e}^x}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{a^2\,\ln\left(-\frac{2\,\mathrm{e}^x\,\left(2\,a^4+3\,a^2\,b^2\right)}{a^2\,b^4+b^6}-\frac{2\,a^2\,\left(b-a\,\mathrm{e}^x\right)\,\left(2\,a^2+3\,b^2\right)}{b^4\,{\left(a^2+b^2\right)}^{3/2}}\right)\,\left(2\,a^2+3\,b^2\right)}{b^3\,{\left(a^2+b^2\right)}^{3/2}}+\frac{a^2\,\ln\left(\frac{2\,a^2\,\left(b-a\,\mathrm{e}^x\right)\,\left(2\,a^2+3\,b^2\right)}{b^4\,{\left(a^2+b^2\right)}^{3/2}}-\frac{2\,\mathrm{e}^x\,\left(2\,a^4+3\,a^2\,b^2\right)}{a^2\,b^4+b^6}\right)\,\left(2\,a^2+3\,b^2\right)}{b^3\,{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*b^2) + ((2*a^3)/(b*(a^2*b + b^3)) - (2*a^4*exp(x))/(b^2*(a^2*b +
 b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + exp(x)/(2*b^2) - (2*a*x)/b^3 - (a^2
*log(- (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4) - (2*a^2*(b - a*exp(x
))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^2)^(3/2)))*(2*a^2 + 3*b^2))/(b^3*(a^2 + b
^2)^(3/2)) + (a^2*log((2*a^2*(b - a*exp(x))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^
2)^(3/2)) - (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4))*(2*a^2 + 3*b^2)
)/(b^3*(a^2 + b^2)^(3/2))
```

3.82 $\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=83

$$\frac{x}{b^2}+\frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[Out] x/b^2+2*a*(a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+b
^2)^(3/2)-a^2*cosh(x)/b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2869, 2814, 2739, 632, 212}

$$\frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2
*(a^2 + b^2)^(3/2)) - (a^2*Cosh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} - \frac{i\int \frac{-iab+i(a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{b\,(a^2+b^2)} \\
&= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} - \frac{(a(a^2+2b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{b^2\,(a^2+b^2)} \\
&= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} - \frac{(2a(a^2+2b^2))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right.}{b^2\,(a^2+b^2)} \\
&= \frac{x}{b^2} - \frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{(4a(a^2+2b^2))\,\mathrm{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\right.}{b^2\,(a^2+b^2)} \\
&= \frac{x}{b^2} + \frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\,(a^2+b^2)^{3/2}} - \frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 86, normalized size = 1.04

$$\frac{x+\frac{2a(a^2+2b^2)\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}-\frac{a^2b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

[Out] (x + (2*a*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - (a^2*b*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/b^2

Maple [A]
time = 0.50, size = 127, normalized size = 1.53

method	result
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{2a\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{ab}{a^2+b^2}}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{b^2}$
risch	$\frac{x}{b^2}+\frac{2a^2\left(a\,\mathrm{e}^{x}-b\right)}{b^2\left(a^2+b^2\right)\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)}+\frac{a^3\ln\left(\mathrm{e}^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}b^2}+\frac{2a\ln\left(\mathrm{e}^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{a^3\ln\left(\mathrm{e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] 1/b^2*ln(tanh(1/2*x)+1)-1/b^2*ln(tanh(1/2*x)-1)+2/b^2*a*((b^2/(a^2+b^2)*tanh(1/2*x)+a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(a^2+2*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))

Maxima [A]
time = 0.50, size = 149, normalized size = 1.80

$$-\frac{(a^2+2\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^3e^{(-x)}+a^2b\right)}{a^2b^3+b^5+2\,(a^3b^2+ab^4)e^{(-x)}-(a^2b^3+b^5)e^{(-2\,x)}}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")

[Out] -(a^2 + 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^3*e^(-x) + a^2*b)/(a^2*b^3 + b^5 + 2*(a^3*b^2 + a*b^4)*e^(-x) - (a^2*b^3 + b^5)*e^(-2*x)) + x/b^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(79) = 158.
time = 0.44, size = 521, normalized size = 6.28

$$\frac{2\,a^4b+2\,a^2b^3-(a^4b+2\,a^2b^3+b^5)x\cosh(x)^2-(a^4b+2\,a^2b^3+b^5)x\sinh(x)^2+\left(a^3b+2\,ab^3-(a^3b+2\,ab^3)\cosh(x)^2-(a^3b+2\,ab^3)\sinh(x)^2-2\,(a^4+2\,a^2b^2)\cosh(x)-2\,(a^4+2\,a^2b^2+(a^3b+2\,ab^3)\cosh(x))\sinh(x)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+(a^4b+2\,a^2b^3+b^5)x-2\,(a^5+a^3b^2+(a^5+2\,a^3b^2+ab^4)x)\cosh(x)-2\,(a^5+a^3b^2+(a^4b+2\,a^2b^3+b^5)x\cosh(x)+(a^5+2\,a^3b^2+ab^4)x)\sinh(x)}{a^4b^3+2\,a^2b^5+b^7-(a^4b^3+2\,a^2b^5+b^7)\cosh(x)^2-(a^4b^3+2\,a^2b^5+b^7)\sinh(x)^2-2\,(a^5b^2+2\,a^3b^4+ab^6)\cosh(x)-2\,(a^5b^2+2\,a^3b^4+ab^6+(a^4b^3+2\,a^2b^5+b^7)\cosh(x))\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")

```
[Out] (2*a^4*b + 2*a^2*b^3 - (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^2 - (a^4*b + 2*a
^2*b^3 + b^5)*x*sinh(x)^2 + (a^3*b + 2*a*b^3 - (a^3*b + 2*a*b^3)*cosh(x)^2
- (a^3*b + 2*a*b^3)*sinh(x)^2 - 2*(a^4 + 2*a^2*b^2)*cosh(x) - 2*(a^4 + 2*a^
2*b^2 + (a^3*b + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x
)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (a^4*b + 2*a^2*b^
3 + b^5)*x - 2*(a^5 + a^3*b^2 + (a^5 + 2*a^3*b^2 + a*b^4)*x)*cosh(x) - 2*(a
^5 + a^3*b^2 + (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + (a^5 + 2*a^3*b^2 + a*b
^4)*x)*sinh(x))/(a^4*b^3 + 2*a^2*b^5 + b^7 - (a^4*b^3 + 2*a^2*b^5 + b^7)*co
sh(x)^2 - (a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^2 - 2*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*cosh(x) - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 131, normalized size = 1.58

$$-\frac{(a^3+2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\,(a^3e^x-a^2b)}{(a^2b^2+b^4)(be^{(2\,x)}+2\,ae^x-b)}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*e^x - a
^2*b)/((a^2*b^2 + b^4)*(b*e^(2*x) + 2*a*e^x - b)) + x/b^2
```

Mupad [**B**]
time = 0.79, size = 228, normalized size = 2.75

$$\frac{x}{b^2}-\frac{\frac{2\,a^2}{a^2\,b+b^3}-\frac{2\,a^3\,\mathrm{e}^x}{b\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)}-\frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}}+\frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)}+\frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] x/b^2 - ((2*a^2)/(a^2*b + b^3) - (2*a^3*exp(x))/(b*(a^2*b + b^3)))/(2*a*exp
(x) - b + b*exp(2*x)) - (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 + b^2))
 - (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 + 2*b^2
))/(b^2*(a^2 + b^2)^(3/2)) + (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 +
b^2)) + (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 +
2*b^2))/(b^2*(a^2 + b^2)^(3/2))
```

3.83 $\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\cosh(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[Out] -2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a*cosh(x)/(
a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2833, 12, 2739, 632, 212}

$$\frac{a\cosh(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}-\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*Co
sh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{\int \frac{b}{a+b\sinh(x)}\,dx}{a^2+b^2}\\ &= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\ &= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\ &= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}-\frac{(4b)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\ &= -\frac{2b\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 1.13

$$-\frac{2b\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (a*C
osh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Maple [A]
time = 0.45, size = 97, normalized size = 1.62

method	result	size
default	$\frac{8b\tanh\left(\frac{x}{2}\right)+8a}{(-4a^2-4b^2)\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}-\frac{8b\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}}$	97
risch	$-\frac{2a(a\,\mathrm{e}^{x}-b)}{b(a^2+b^2)(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(2*b*tanh(1/2*x)+2*a)/(-4*a^2-4*b^2)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-
8*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.
time = 0.49, size = 117, normalized size = 1.95

$$\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\left(a^3b+ab^3\right)e^{(-x)}-(a^2b^2+b^4)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(a
^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e
^(-x) - (a^2*b^2 + b^4)*e^(-2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(56) = 112$.
time = 0.44, size = 341, normalized size = 5.68

$$-\frac{2a^3b+2ab^3+\left(b^3\cosh(x)^2+b^3\sinh(x)^2+2ab^2\cosh(x)-b^3+2\left(b^3\cosh(x)+ab^2\right)\sinh(x)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^2+b^2}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)-b}\right)-2\left(a^4+a^2b^2\right)\cosh(x)-2\left(a^4+a^2b^2\right)\sinh(x)}{a^4b^2+2a^2b^4+b^6-\left(a^4b^2+2a^2b^4+b^6\right)\cosh(x)^2-\left(a^4b^2+2a^2b^4+b^6\right)\sinh(x)^2-2\left(a^5b+2a^3b^3+ab^5\right)\cosh(x)-2\left(a^5b+2a^3b^3+ab^5+\left(a^4b^2+2a^2b^4+b^6\right)\cosh(x)\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b + 2*a*b^3 + (b^3*cosh(x)^2 + b^3*sinh(x)^2 + 2*a*b^2*cosh(x) - b^
3 + 2*(b^3*cosh(x) + a*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
- 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
```

```
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4 + a^2*b^2)*cosh(x
) - 2*(a^4 + a^2*b^2)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^
2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2 + 2*a
^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 99, normalized size = 1.65

$$\frac{b\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\,\left(a^{2}e^{x}-ab\right)}{\left(a^{2}b+b^{3}\right)\left(be^{(2\,x)}+2\,ae^{x}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^2*e^x - a*b)/((a^2*b + b^3)*(b*e^(2*x) +
 2*a*e^x - b))
```

Mupad [**B**]
time = 0.70, size = 142, normalized size = 2.37

$$\frac{\frac{2\,a\,b}{a^2\,b+b^3}-\frac{2\,a^2\,\mathrm{e}^x}{a^2\,b+b^3}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}}-\frac{b\,\ln\left(-\frac{2\,\mathrm{e}^x}{a^2+b^2}-\frac{2\,\left(b-a\,\mathrm{e}^x\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}+\frac{b\,\ln\left(\frac{2\,\left(b-a\,\mathrm{e}^x\right)}{{\left(a^2+b^2\right)}^{3/2}}-\frac{2\,\mathrm{e}^x}{a^2+b^2}\right)}{{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*a*b)/(a^2*b + b^3) - (2*a^2*exp(x))/(a^2*b + b^3))/(2*a*exp(x) - b + b*
exp(2*x)) - (b*log(- (2*exp(x))/(a^2 + b^2) - (2*(b - a*exp(x)))/(a^2 + b^2
)^(3/2)))/(a^2 + b^2)^(3/2) + (b*log((2*(b - a*exp(x)))/(a^2 + b^2)^(3/2) -
 (2*exp(x))/(a^2 + b^2)))/(a^2 + b^2)^(3/2)
```

3.84 $\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}(\cosh(x))}{a^2}+\frac{2b(2a^2+b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[Out] -arctanh(cosh(x))/a^2+2*b*(2*a^2+b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a^2/(a^2+b^2)^(3/2)+b^2*cosh(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2881, 3080, 3855, 2739, 632, 212}

$$\frac{2b(2a^2+b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}-\frac{\tanh^{-1}(\cosh(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a^2) + (2*b*(2*a^2 + b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)) + (b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b
*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx &= \frac{b^2\cosh(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}(x)(a^2+b^2-ab\sinh(x))}{a+b\sinh(x)}\,dx}{a\,(a^2+b^2)} \\
&= \frac{b^2\cosh(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{\int \operatorname{csch}(x)\,dx}{a^2} - \frac{(b(2a^2+b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{a^2\,(a^2+b^2)} \\
&= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} - \frac{(2b(2a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2} \cdots\right.}{a^2\,(a^2+b^2)} \\
&= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{(4b(2a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x \cdots}\right.}{a^2\,(a^2+ \cdots} \\
&= -\frac{\tanh^{-1}(\cosh(x))}{a^2} + \frac{2b(2a^2+b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\,(a^2+b^2)^{3/2}} + \frac{b^2\cosh(x)}{a\,(a^2+b^2)\,(a+b\sinh(x \cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 91, normalized size = 1.07

$$\frac{\frac{2b(2a^2+b^2)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{ab^2\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2
)^(3/2) + Log[Tanh[x/2]] + (a*b^2*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/a
^2
```

Maple [A]
time = 0.66, size = 115, normalized size = 1.35

method	result
default	$\frac{4b\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)}-\frac{ab}{2\left(a^2+b^2\right)}}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$
risch	$-\frac{2b(a\,\mathrm{e}^x-b)}{a(a^2+b^2)(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^x-b)}+\frac{\ln(\mathrm{e}^x-1)}{a^2}+\frac{2b\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{b^3\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*b*((-1/2*b^2/(a^2+b^2)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(a*tanh(1/2*x)^
2-2*b*tanh(1/2*x)-a)-1/2*(2*a^2+b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(
1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/a^2*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.55, size = 162, normalized size = 1.91

$$-\frac{(2\,a^2b+b^3)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}+\frac{2\left(abe^{(-x)}+b^2\right)}{a^3b+ab^3+2\left(a^4+a^2b^2\right)e^{(-x)}-(a^3b+ab^3)e^{(-2\,x)}}-\frac{\log\left(e^{(-x)}+1\right)}{a^2}+\frac{\log\left(e^{(-x)}-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(2*a^2*b + b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + 2*(a*b*e^(-x) + b^2)/(a^3*
```

b + a*b^3 + 2*(a^4 + a^2*b^2)*e^(-x) - (a^3*b + a*b^3)*e^(-2*x)) - log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(81) = 162$.
time = 0.80, size = 672, normalized size = 7.91

$$-\frac{2\,a^{3}b^{2}+2\,ab^{4}-\left(2\,a^{2}b^{2}+b^{4}-\left(2\,a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{2}-\left(2\,a^{2}b^{2}+b^{4}\right)\sinh\left(x\right)^{2}-2\left(2\,a^{3}b+ab^{3}\right)\cosh\left(x\right)-2\left(2\,a^{3}b+ab^{3}+\left(2\,a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2ab\cosh(x)+2a^{2}+b^{2}+2\left(b^{2}\cosh(x)+ab\right)\sinh(x)+2\sqrt{a^{2}+b^{2}}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)-b}\right)-2\left(a^{4}b+a^{2}b^{3}\right)\cosh\left(x\right)+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)^{2}-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sinh\left(x\right)^{2}-2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cosh\left(x\right)-2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)^{2}-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sinh\left(x\right)^{2}-2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cosh\left(x\right)-2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-2\left(a^{4}b+a^{2}b^{3}\right)\sinh\left(x\right)}{a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5}-\left(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5}\right)\cosh\left(x\right)^{2}-\left(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5}\right)\sinh\left(x\right)^{2}-2\left(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4}\right)\cosh\left(x\right)-2\left(a^{7}+2\,a^{5}b^{2}+a^{3}b^{4}+\left(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5}\right)\cosh\left(x\right)\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] -(2*a^3*b^2 + 2*a*b^4 - (2*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cosh(x)^2 - (2*a^2*b^2 + b^4)*sinh(x)^2 - 2*(2*a^3*b + a*b^3)*cosh(x) - 2*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4*b + a^2*b^3)*cosh(x) + (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(a^4*b + a^2*b^3)*sinh(x))/(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x)^2 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*sinh(x)^2 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*cosh(x) - 2*(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x))*sinh(x))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(x)/(a+b*sinh(x))**2,x)

[Out] Integral(csch(x)/(a + b*sinh(x))**2, x)

Giac [A]
time = 0.43, size = 142, normalized size = 1.67

$$-\frac{\left(2\,a^{2}b+b^{3}\right)\log\left(\left|\frac{2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}}{2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}}\right|\right)}{\left(a^{4}+a^{2}b^{2}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(abe^{x}-b^{2}\right)}{\left(a^{3}+ab^{2}\right)\left(be^{(2\,x)}+2\,ae^{x}-b\right)}-\frac{\log\left(e^{x}+1\right)}{a^{2}}+\frac{\log\left(\left|e^{x}-1\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b + b^3)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) - 2*(a*b*e^x - b
^2)/((a^3 + a*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - log(e^x + 1)/a^2 + log(abs(
e^x - 1))/a^2
```

Mupad [B]
time = 2.70, size = 1001, normalized size = 11.78

$$\frac{\frac{2b^5}{a(a^2b^3+b^5)}-\frac{2b^4e^x}{a^2b^3+b^5}}{2ae^x-b+be^{2x}}+\frac{\ln(e^x-1)}{a^2}-\frac{\ln(e^x+1)}{a^2}-\frac{b\ln\left(\frac{32(-8e^xa^5+4a^4b-10e^xa^3b^2+6a^2b^3-3e^xab^4+2b^5)}{a(a^4b^3+a^2b^5)(a^2b+b^3)}+\frac{b\left(\frac{32(-4e^xa^7+2a^6b-11e^xa^5b^2+9a^4b^3-10e^xa^3b^4+8a^2b^5-3e^xab^6+2b^7)}{ab^5(a^5+2a^3b^2+ab^4)}-\frac{b(2a^2+b^2)\sqrt{(a^2+b^2)^3}\left(\frac{32(-7e^xa^4+4a^3b-4e^xa^2b^2+2ab^3)}{b(a^2b^3+b^5)}+\frac{32(2a^2+b^2)\sqrt{(a^2+b^2)^3}(-4e^xa^5+3a^4b-3e^xa^3b^2+2a^2b^3)}{b^4(a^8+3a^6b^2+3a^4b^4+a^2b^6)}\right)}{a^8+3a^6b^2+3a^4b^4+a^2b^6}\right)(2a^2+b^2)\sqrt{(a^2+b^2)^3}}{a^8+3a^6b^2+3a^4b^4+a^2b^6}\right)(2a^2+b^2)\sqrt{(a^2+b^2)^3}}{a^8+3a^6b^2+3a^4b^4+a^2b^6}+\frac{b\ln\left(\frac{32(-8e^xa^5+4a^4b-10e^xa^3b^2+6a^2b^3-3e^xab^4+2b^5)}{a(a^4b^3+a^2b^5)(a^2b+b^3)}-\frac{b\left(\frac{32(-4e^xa^7+2a^6b-11e^xa^5b^2+9a^4b^3-10e^xa^3b^4+8a^2b^5-3e^xab^6+2b^7)}{ab^5(a^5+2a^3b^2+ab^4)}+\frac{b(2a^2+b^2)\sqrt{(a^2+b^2)^3}\left(\frac{32(-7e^xa^4+4a^3b-4e^xa^2b^2+2ab^3)}{b(a^2b^3+b^5)}-\frac{32(2a^2+b^2)\sqrt{(a^2+b^2)^3}(-4e^xa^5+3a^4b-3e^xa^3b^2+2a^2b^3)}{b^4(a^8+3a^6b^2+3a^4b^4+a^2b^6)}\right)}{a^8+3a^6b^2+3a^4b^4+a^2b^6}\right)(2a^2+b^2)\sqrt{(a^2+b^2)^3}}{a^8+3a^6b^2+3a^4b^4+a^2b^6}\right)(2a^2+b^2)\sqrt{(a^2+b^2)^3}}{a^8+3a^6b^2+3a^4b^4+a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*b^5)/(a*(b^5 + a^2*b^3)) - (2*b^4*exp(x))/(b^5 + a^2*b^3))/(2*a*exp(x)
- b + b*exp(2*x)) + log(exp(x) - 1)/a^2 - log(exp(x) + 1)/a^2 - (b*log((32*
(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^3*b^2*e
xp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) + (b*((32*(2*a^6*b + 2*b^7 +
8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*exp(x) -
 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) - (b*(2*a^2 + b^2)*(
(a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*b^2*exp
(x)))/(b*(b^5 + a^2*b^3)) + (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*(3*a^4*
b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^6 + 3*a
^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 + b^
2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 +
 b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2) + (b*l
og((32*(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^
3*b^2*exp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) - (b*((32*(2*a^6*b + 2
*b^7 + 8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*e
xp(x) - 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) + (b*(2*a^2 +
 b^2)*((a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*
b^2*exp(x)))/(b*(b^5 + a^2*b^3)) - (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*
(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^
6 + 3*a^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a
^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(
2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2)
```

3.85 $\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=115

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2b^2(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}}-\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)}+\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*b^2*(3*a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)-(a^2+2*b^2)*coth(x)/a^2/(a^2+b^2)+b^2*coth
(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.27, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)}+\frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}-\frac{2b^2(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*b^2*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2
])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)) - ((a^2 + 2*b^2)*Coth[x])/(a^2
*(a^2 + b^2)) + (b^2*Coth[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^2(x)(a^2+2b^2-ab\sinh(x)+b^2\sinh^2(x))}{a+b\sinh(x)}\,dx}{a(a^2+b^2)}$$

$$= -\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}(x)(2ib(a^2+b^2)-iab^2\sinh(x))}{a+b\sinh(x)}}{a^2(a^2+b^2)}$$

$$= -\frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{(2b)\int \operatorname{csch}(x)\,dx}{a^3} + \frac{(b^2(3a^2+2}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{(2b^2(3a^2+}{}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{(4b^2(3a^2+}{}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2b^2(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh(\frac{x}{2})}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} +$$

Mathematica [A]
time = 0.48, size = 118, normalized size = 1.03

$$-\frac{\frac{4b^2(3a^2+2b^2)\text{ArcTan}\left(\frac{b-a\tanh(\frac{x}{2})}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + a\coth\left(\frac{x}{2}\right) + 4b\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{2ab^3\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + a\tanh\left(\frac{x}{2}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((4*b^2*(3*a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-
a^2 - b^2)^(3/2) + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b^3*Cosh[x])/((a
^2 + b^2)*(a + b*Sinh[x])) + a*Tanh[x/2])/a^3
```

Maple [A]
time = 0.72, size = 141, normalized size = 1.23

method	result
default	$-\frac{\tanh(\frac{x}{2})}{2a^2} - \frac{1}{2a^2\tanh(\frac{x}{2})} - \frac{2b\ln(\tanh(\frac{x}{2}))}{a^3} - \frac{2b^2\left(\frac{-\frac{b^2\tanh(\frac{x}{2})}{a^2+b^2}-\frac{ab}{a^2+b^2}}{a\left(\tanh^2(\frac{x}{2})\right)-2b\tanh(\frac{x}{2})-a} - \frac{(3a^2+2b^2)\operatorname{arctanh}\left(\frac{2a\tanh(\frac{x}{2})-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}\right)}{a^3}$

risch	$-\frac{2(-a\,b^2\mathrm{e}^{3x}+a^2b\,\mathrm{e}^{2x}+2b^3\mathrm{e}^{2x}+2a^3\mathrm{e}^{x}+3a\,b^2\mathrm{e}^{x}-a^2b-2b^3)}{a^2(\mathrm{e}^{2x}-1)(a^2+b^2)(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}-\frac{2b\ln(\mathrm{e}^x-1)}{a^3}+\frac{2b\ln(\mathrm{e}^x+1)}{a^3}+\frac{3b^2\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right.}{(a^2+b^2)^{\frac{3}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))-2*b^2/a^3*
((-b^2/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x
)-a)-(3*a^2+2*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(111) = 222$.
time = 0.50, size = 251, normalized size = 2.18

$$\frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}}+\frac{2\left(ab^2e^{(-3\,x)}-a^2b-2\,b^3-(2\,a^3+3\,ab^2)e^{(-x)}+(a^2b+2\,b^3)e^{(-2\,x)}\right)}{a^4b+a^2b^3+2\,(a^5+a^3b^2)e^{(-x)}-2\,(a^4b+a^2b^3)e^{(-2\,x)}-2\,(a^5+a^3b^2)e^{(-3\,x)}+(a^4b+a^2b^3)e^{(-4\,x)}}+\frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3}-\frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^(-3*x) - a^2
*b - 2*b^3 - (2*a^3 + 3*a*b^2)*e^(-x) + (a^2*b + 2*b^3)*e^(-2*x))/(a^4*b +
a^2*b^3 + 2*(a^5 + a^3*b^2)*e^(-x) - 2*(a^4*b + a^2*b^3)*e^(-2*x) - 2*(a^5
+ a^3*b^2)*e^(-3*x) + (a^4*b + a^2*b^3)*e^(-4*x)) + 2*b*log(e^(-x) + 1)/a^3
 - 2*b*log(e^(-x) - 1)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(111) = 222$.
time = 0.53, size = 1740, normalized size = 15.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^5*b + 6*a^3*b^3 + 4*a*b^5 + 2*(a^4*b^2 + a^2*b^4)*cosh(x)^3 + 2*(a^4*b
^2 + a^2*b^4)*sinh(x)^3 - 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 - 2*(a^
5*b + 3*a^3*b^3 + 2*a*b^5 - 3*(a^4*b^2 + a^2*b^4)*cosh(x))*sinh(x)^2 + (3*a
^2*b^3 + 2*b^5 + (3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (3*a^2*b^3 + 2*b^5)*sinh(x
)^4 + 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 2*(3*a^3*b^2 + 2*a*b^4 + 2*(3*a^2
*b^3 + 2*b^5)*cosh(x))*sinh(x)^3 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 2*(3*a
^2*b^3 + 2*b^5 - 3*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(3*a^3*b^2 + 2*a*b^4)*
```

```
cosh(x))*sinh(x)^2 - 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x) - 2*(3*a^3*b^2 + 2*a*b
^4 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 +
2*(3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4)*cosh(x) + 2*(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4
+ b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2
*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 -
2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*
(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b
^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*
cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(a^4*b^2 + 2*a^2*
b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 +
b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^
2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(
x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5
 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*
cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) - 2*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 3*(a^4*b^2 + a^2*b^4)*cosh
(x)^2 + 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x))/(a^7*b + 2*a^5*b^
3 + a^3*b^5 + (a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^4 + (a^7*b + 2*a^5*b^3
+ a^3*b^5)*sinh(x)^4 + 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x)^3 + 2*(a^8 + 2
*a^6*b^2 + a^4*b^4 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x)^3 - 2
*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5 -
 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 3*(a^8 + 2*a^6*b^2 + a^4*b^4)*
cosh(x))*sinh(x)^2 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x) - 2*(a^8 + 2*a^6
*b^2 + a^4*b^4 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^3 - 3*(a^8 + 2*a^6
*b^2 + a^4*b^4)*cosh(x)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.43, size = 205, normalized size = 1.78

$$\frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}}+\frac{2\,(ab^2e^{(3\,x)}-a^2be^{(2\,x)}-2\,b^3e^{(2\,x)}-2\,a^3e^x-3\,ab^2e^x+a^2b+2\,b^3)}{(a^4+a^2b^2)(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^x+b)}+\frac{2\,b\log(e^x+1)}{a^3}-\frac{2\,b\log(|e^x-1|)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^
(3*x) - a^2*b*e^(2*x) - 2*b^3*e^(2*x) - 2*a^3*e^x - 3*a*b^2*e^x + a^2*b + 2
*b^3)/((a^4 + a^2*b^2)*(b*e^(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b
)) + 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3
```

Mupad [B]
time = 3.39, size = 1017, normalized size = 8.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 + 25*a^8*b^6))/(a^4*b^2*(16*b^9
 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (2*exp(x)*(48*a^3*b^12 + 152*a^
5*b^10 + 155*a^7*b^8 + 50*a^9*b^6))/(a^4*b^3*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (2*exp(2*x)*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 +
25*a^8*b^6))/(a^4*b^2*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (2
*exp(3*x)*(16*a^3*b^12 + 40*a^5*b^10 + 25*a^7*b^8))/(a^4*b^3*(16*b^9 + 56*a
^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*exp(3*x) - 2*b*ex
p(2*x) + b*exp(4*x)) - (2*b*log(exp(x) - 1))/a^3 + (2*b*log(exp(x) + 1))/a^
3 + (b^2*log(- (64*(3*a^2 + 2*b^2)*(4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^
2*exp(x)))/(a^6*b*(a^2 + b^2)^2) - (32*(3*a^2 + 2*b^2)*(8*a^9*b - 8*b^7*((a
^2 + b^2)^3)^(1/2) + 3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) -
 24*a^2*b^5*((a^2 + b^2)^3)^(1/2) - 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^
4*b^6*exp(x) - 33*a^6*b^4*exp(x) - 40*a^8*b^2*exp(x) + 41*a^3*b^4*exp(x)*((
a^2 + b^2)^3)^(1/2) + 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) + 14*a*b^6*ex
p(x)*((a^2 + b^2)^3)^(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*(
(a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^
2) - (b^2*log((32*(3*a^2 + 2*b^2)*(8*a^9*b + 8*b^7*((a^2 + b^2)^3)^(1/2) +
3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) + 24*a^2*b^5*((a^2 + b
^2)^3)^(1/2) + 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^4*b^6*exp(x) - 33*a^6
*b^4*exp(x) - 40*a^8*b^2*exp(x) - 41*a^3*b^4*exp(x)*((a^2 + b^2)^3)^(1/2) -
 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) - 14*a*b^6*exp(x)*((a^2 + b^2)^3)^
(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (64*(3*a^2 + 2*b^2)*(
```

```
4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^6*b*(a^2 + b^2)^2))*((
a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^2
)
```

3.86 $\int \frac{\text{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=158

$$\frac{(a^2-6b^2)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{2b^3(4a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}}+\frac{b(2a^2+3b^2)\coth(x)}{a^3\left(a^2+b^2\right)}-\frac{(a^2+3b^2)\coth(x)}{2a^2\left(a^2+b^2\right)}$$

```
[Out] 1/2*(a^2-6*b^2)*arctanh(cosh(x))/a^4+2*b^3*(4*a^2+3*b^2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(3/2)+b*(2*a^2+3*b^2)*coth(x)/a^3/(a
^2+b^2)-1/2*(a^2+3*b^2)*coth(x)*csch(x)/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)/a
/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.48, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$-\frac{(a^2+3b^2)\coth(x)\text{csch}(x)}{2a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\text{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{(a^2-6b^2)\tanh^{-1}(\cosh(x))}{2a^4}+\frac{2b^3(4a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\left(a^2+b^2\right)^{3/2}}+\frac{b(2a^2+3b^2)\coth(x)}{a^3\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*ArcTanh[Cosh[x]])/(2*a^4) + (2*b^3*(4*a^2 + 3*b^2)*ArcTanh[(
b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^4*(a^2 + b^2)^(3/2)) + (b*(2*a^2 + 3*
b^2)*Coth[x])/(a^3*(a^2 + b^2)) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*a^2*(a
^2 + b^2)) + (b^2*Coth[x]*Csch[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^3(x)(a^2+3b^2-ab\sinh(x)+2b^2\sinh^2(x))}{a+b\sinh(x)}dx}{a(a^2+b^2)}$$

$$= -\frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}^2(x)(2ib(2a^2+3b^2)-}{}}{}$$

$$= \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} -$$

$$= \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} -$$

$$= \frac{(a^2-6b^2)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} +$$

$$= \frac{(a^2-6b^2)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} +$$

$$= \frac{(a^2-6b^2)\tanh^{-1}(\cosh(x))}{2a^4} + \frac{2b^3(4a^2+3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4(a^2+b^2)^{3/2}} + \frac{b(2a^2+3b^2)}{a^3(a^2+}$$

Mathematica [A]
time = 0.50, size = 156, normalized size = 0.99

$$\frac{\frac{16b^3(4a^2+3b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + 8ab\coth\left(\frac{x}{2}\right) - a^2\operatorname{csch}^2\left(\frac{x}{2}\right) - 4(a^2-6b^2)\log\left(\tanh\left(\frac{x}{2}\right)\right) - a^2\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{8ab^4\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + 8ab\tanh\left(\frac{x}{2}\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((16*b^3*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) + 8*a*b*Coth[x/2] - a^2*Csch[x/2]^2 - 4*(a^2 - 6*b^2)*Log[Tanh
[x/2]] - a^2*Sech[x/2]^2 + (8*a*b^4*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
+ 8*a*b*Tanh[x/2])/(8*a^4)
```

Maple [A]
time = 0.81, size = 175, normalized size = 1.11

method	result

default	$\frac{\frac{a\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+4b\tanh\left(\frac{x}{2}\right)}{4a^3}-\frac{1}{8a^2\tanh\left(\frac{x}{2}\right)^2}+\frac{\left(-2a^2+12b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^4}+\frac{b}{a^3\tanh\left(\frac{x}{2}\right)}+\frac{4b^3\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)}-\frac{ab}{2\left(a^2+b^2\right)}}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\cdots\right.}{\cdots}$
risch	$-\frac{a^3b\,\mathrm{e}^{5x}+3a\,b^3\mathrm{e}^{5x}+2a^4\mathrm{e}^{4x}-2a^2b^2\mathrm{e}^{4x}-6b^4\mathrm{e}^{4x}-8a^3b\,\mathrm{e}^{3x}-12a\,b^3\mathrm{e}^{3x}+2a^4\mathrm{e}^{2x}+10a^2b^2\mathrm{e}^{2x}+12b^4\mathrm{e}^{2x}+7a^3b\,\mathrm{e}^{x}+9b^3\mathrm{e}^{x}a-4a^2b^2-6b^4}{\left(a^2+b^2\right)a^3\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)\left(\mathrm{e}^{2x}-1\right)^2}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(1/2*a*tanh(1/2*x)^2+4*b*tanh(1/2*x))-1/8/a^2/tanh(1/2*x)^2+1/4/a^4
*(-2*a^2+12*b^2)*ln(tanh(1/2*x))+1/a^3*b/tanh(1/2*x)+4/a^4*b^3*((-1/2*b^2/(
a^2+b^2)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
-1/2*(4*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(150) = 300$.
time = 0.52, size = 363, normalized size = 2.30

$$-\frac{\left(4\,a^2b^3+3\,b^5\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^6+a^4b^2\right)\sqrt{a^2+b^2}}+\frac{4\,a^2b^2+6\,b^4+\left(7\,a^3b+9\,ab^3\right)e^{(-x)}-2\left(a^4+5\,a^2b^2+6\,b^4\right)e^{(-2\,x)}-4\left(2\,a^3b+3\,ab^3\right)e^{(-3\,x)}-2\left(a^4-a^2b^2-3\,b^4\right)e^{(-4\,x)}+\left(a^3b+3\,ab^3\right)e^{(-5\,x)}}{a^5b+a^3b^3+2\left(a^6+a^4b^2\right)e^{(-x)}-3\left(a^5b+a^3b^3\right)e^{(-2\,x)}-4\left(a^6+a^4b^2\right)e^{(-3\,x)}+3\left(a^5b+a^3b^3\right)e^{(-4\,x)}+2\left(a^6+a^4b^2\right)e^{(-5\,x)}-\left(a^5b+a^3b^3\right)e^{(-6\,x)}}+\frac{\left(a^2-6\,b^2\right)\log\left(e^{(-x)}+1\right)}{2\,a^4}-\frac{\left(a^2-6\,b^2\right)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) + (4*a^2*b^2 + 6*b^4 + (
7*a^3*b + 9*a*b^3)*e^(-x) - 2*(a^4 + 5*a^2*b^2 + 6*b^4)*e^(-2*x) - 4*(2*a^3
*b + 3*a*b^3)*e^(-3*x) - 2*(a^4 - a^2*b^2 - 3*b^4)*e^(-4*x) + (a^3*b + 3*a*
b^3)*e^(-5*x))/(a^5*b + a^3*b^3 + 2*(a^6 + a^4*b^2)*e^(-x) - 3*(a^5*b + a^3
*b^3)*e^(-2*x) - 4*(a^6 + a^4*b^2)*e^(-3*x) + 3*(a^5*b + a^3*b^3)*e^(-4*x)
+ 2*(a^6 + a^4*b^2)*e^(-5*x) - (a^5*b + a^3*b^3)*e^(-6*x)) + 1/2*(a^2 - 6*b
^2)*log(e^(-x) + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(e^(-x) - 1)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3754 vs. $2(150) = 300$.
time = 0.81, size = 3754, normalized size = 23.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^5*b^2 + 20*a^3*b^4 + 12*a*b^6 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)
*cosh(x)^5 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*sinh(x)^5 - 4*(a^7 - 4*a^3*b
^4 - 3*a*b^6)*cosh(x)^4 - 2*(2*a^7 - 8*a^3*b^4 - 6*a*b^6 + 5*(a^6*b + 4*a^4
*b^3 + 3*a^2*b^5)*cosh(x))*sinh(x)^4 + 8*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*
cosh(x)^3 + 4*(4*a^6*b + 10*a^4*b^3 + 6*a^2*b^5 - 5*(a^6*b + 4*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 - 4*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x))*sinh(x)^3 - 4*(
a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x)^2 - 4*(a^7 + 6*a^5*b^2 + 11
*a^3*b^4 + 6*a*b^6 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)^3 + 6*(a^7 -
 4*a^3*b^4 - 3*a*b^6)*cosh(x)^2 - 6*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cosh(
x))*sinh(x)^2 + 2*((4*a^2*b^4 + 3*b^6)*cosh(x)^6 + (4*a^2*b^4 + 3*b^6)*sinh
(x)^6 - 4*a^2*b^4 - 3*b^6 + 2*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 2*(4*a^3*b^
3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^5 - 3*(4*a^2*b^4 + 3*b
^6)*cosh(x)^4 - (12*a^2*b^4 + 9*b^6 - 15*(4*a^2*b^4 + 3*b^6)*cosh(x)^2 - 10
*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^4 - 4*(4*a^3*b^3 + 3*a*b^5)*cosh(x)
^3 - 4*(4*a^3*b^3 + 3*a*b^5 - 5*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 5*(4*a^3*b^
3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^3 + 3*(4*a^
2*b^4 + 3*b^6)*cosh(x)^2 + (12*a^2*b^4 + 9*b^6 + 15*(4*a^2*b^4 + 3*b^6)*cos
h(x)^4 + 20*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^3 - 18*(4*a^2*b^4 + 3*b^6)*cosh(x
)^2 - 12*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^2 + 2*(4*a^3*b^3 + 3*a*b^5)
*cosh(x) + 2*(4*a^3*b^3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x)^5 + 5*(4*
a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 6*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 6*(4*a^3*b
^3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x))*sqrt(a^2
+ b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2
*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a
))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b
)) - 2*(7*a^6*b + 16*a^4*b^3 + 9*a^2*b^5)*cosh(x) - (a^6*b - 4*a^4*b^3 - 11
*a^2*b^5 - 6*b^7 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^
6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x)^5 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3
*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4
*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b
^5 - 18*b^7 - 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a
^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*
b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6
*a*b^6 - 5*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*
a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b
^5 - 6*b^7)*cosh(x))*sinh(x)^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)
*cosh(x)^2 - (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^
4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^3 - 18*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2
- 12*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 -
 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^
4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7
 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*
a^2*b^5 - 6*b^7)*cosh(x)^3 - 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cos
```

```
h(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(c
osh(x) + sinh(x) + 1) + (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7 - (a^6*b -
4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5
 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^5
- 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2
*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^
7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 - 15*(a^6*b - 4*
a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a^7 - 4*a^5*b^2 - 11*a^3*b^4
- 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*
cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 - 5*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b
^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)
^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - (3*a^6*b - 12*a
^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*
cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 - 18*(a^6
*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 12*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*
a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a
^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 -
 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(c...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**3/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.44, size = 203, normalized size = 1.28

$$-\frac{(4a^2b^3+3b^5)\log\left(\frac{|2be^x+2a-2\sqrt{a^2+b^2}|}{|2be^x+2a+2\sqrt{a^2+b^2}|}\right)}{(a^6+a^4b^2)\sqrt{a^2+b^2}}-\frac{2(ab^3e^x-b^4)}{(a^5+a^3b^2)(be^{(2x)}+2ae^x-b)}+\frac{(a^2-6b^2)\log(e^x+1)}{2a^4}-\frac{(a^2-6b^2)\log(|e^x-1|)}{2a^4}-\frac{ae^{(3x)}-4be^{(2x)}+ae^x+4b}{a^3(e^{(2x)}-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) - 2*(a*b^3*e
```

```
^x - b^4)/((a^5 + a^3*b^2)*(b*e^(2*x) + 2*a*e^x - b)) + 1/2*(a^2 - 6*b^2)*l
og(e^x + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(abs(e^x - 1))/a^4 - (a*e^(3*x) - 4*
b*e^(2*x) + a*e^x + 4*b)/(a^3*(e^(2*x) - 1)^2)
```

Mupad [B]
time = 3.40, size = 977, normalized size = 6.18

$$\frac{\frac{4b}{a^3}-\frac{e^x}{a^2}}{e^{2x}-1}+\frac{\frac{2b^7}{a^3(b^5+a^2b^3)}-\frac{2b^6e^x}{a^2(b^5+a^2b^3)}}{2ae^x-b+be^{2x}}-\frac{\ln(e^x-1)(a^2-6b^2)}{2a^4}+\frac{\ln(e^x+1)(a^2-6b^2)}{2a^4}-\frac{2e^x}{a^2(e^{4x}-2e^{2x}+1)}+\frac{b^3\ln\left(\frac{8(4a^2+3b^2)\left(20a^9b^5-72b^{11}\sqrt{(a^2+b^2)^3}-9a^3b^{11}-30a^5b^9-18a^7b^7-2a^{13}b+15a^{11}b^3+4a^{14}e^x-192a^2b^9\sqrt{(a^2+b^2)^3}-128a^4b^7\sqrt{(a^2+b^2)^3}+27a^4b^{10}e^x+72a^6b^8e^x+30a^8b^6e^x-48a^{10}b^4e^x-29a^{12}b^2e^x+312a^3b^8e^x\sqrt{(a^2+b^2)^3}+206a^5b^6e^x\sqrt{(a^2+b^2)^3}+8ab^4e^x\left((a^2+b^2)^3\right)^{3/2}+118ab^{10}e^x\sqrt{(a^2+b^2)^3}\right)}{a^9b^2\sqrt{(a^2+b^2)^3}(a^2+b^2)^4}-\frac{8(-4a^4+21a^2b^2+18b^4)(-4e^xa^5+2a^4b+19e^xa^3b^2-10a^2b^3+21e^xab^4-12b^5)}{a^9b^2(a^2+b^2)^2}\right)\sqrt{(a^2+b^2)^3}(4a^2+3b^2)}{a^{10}+3a^8b^2+3a^6b^4+a^4b^6}-\frac{b^3\ln\left(\frac{8(4a^2+3b^2)\left(2a^{13}b-72b^{11}\sqrt{(a^2+b^2)^3}+9a^3b^{11}+30a^5b^9+18a^7b^7-20a^9b^5-15a^{11}b^3-4a^{14}e^x-192a^2b^9\sqrt{(a^2+b^2)^3}-128a^4b^7\sqrt{(a^2+b^2)^3}-27a^4b^{10}e^x-72a^6b^8e^x-30a^8b^6e^x+48a^{10}b^4e^x+29a^{12}b^2e^x+312a^3b^8e^x\sqrt{(a^2+b^2)^3}+206a^5b^6e^x\sqrt{(a^2+b^2)^3}+8ab^4e^x\left((a^2+b^2)^3\right)^{3/2}+118ab^{10}e^x\sqrt{(a^2+b^2)^3}\right)}{a^9b^2\sqrt{(a^2+b^2)^3}(a^2+b^2)^4}-\frac{8(-4a^4+21a^2b^2+18b^4)(-4e^xa^5+2a^4b+19e^xa^3b^2-10a^2b^3+21e^xab^4-12b^5)}{a^9b^2(a^2+b^2)^2}\right)\sqrt{(a^2+b^2)^3}(4a^2+3b^2)}{a^{10}+3a^8b^2+3a^6b^4+a^4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*b)/a^3 - exp(x)/a^2)/(exp(2*x) - 1) + ((2*b^7)/(a^3*(b^5 + a^2*b^3)) -
(2*b^6*exp(x))/(a^2*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x)) - (log(
exp(x) - 1)*(a^2 - 6*b^2))/(2*a^4) + (log(exp(x) + 1)*(a^2 - 6*b^2))/(2*a^4
) - (2*exp(x))/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) + (b^3*log((8*(4*a^2 + 3*b
^2)*(20*a^9*b^5 - 72*b^11*((a^2 + b^2)^3)^(1/2) - 9*a^3*b^11 - 30*a^5*b^9 -
 18*a^7*b^7 - 2*a^13*b + 15*a^11*b^3 + 4*a^14*exp(x) - 192*a^2*b^9*((a^2 +
b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) + 27*a^4*b^10*exp(x) + 72
*a^6*b^8*exp(x) + 30*a^8*b^6*exp(x) - 48*a^10*b^4*exp(x) - 29*a^12*b^2*exp(
x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5*b^6*exp(x)*((a^2 +
b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 118*a*b^10*exp(x)*((
a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (8*(1
8*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*b^3 - 4*a^5*exp(x) +
 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 + b^2)^2))*((a^2 + b^2
)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4 + 3*a^8*b^2) - (b^3
*log((8*(4*a^2 + 3*b^2)*(2*a^13*b - 72*b^11*((a^2 + b^2)^3)^(1/2) + 9*a^3*b
^11 + 30*a^5*b^9 + 18*a^7*b^7 - 20*a^9*b^5 - 15*a^11*b^3 - 4*a^14*exp(x) -
192*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) - 27*
a^4*b^10*exp(x) - 72*a^6*b^8*exp(x) - 30*a^8*b^6*exp(x) + 48*a^10*b^4*exp(x
) + 29*a^12*b^2*exp(x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5
*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) +
118*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4) - (8*(18*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*
b^3 - 4*a^5*exp(x) + 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 +
b^2)^2))*((a^2 + b^2)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4
 + 3*a^8*b^2)
```

3.87 $\int \frac{\text{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=198

$$-\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{2b^4(5a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}}+\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\left(a^2+b^2\right)}+\frac{b(a^2+$$

```
[Out] -b*(a^2-4*b^2)*arctanh(cosh(x))/a^5-2*b^4*(5*a^2+4*b^2)*arctanh((b-a*tanh(1
/2*x))/(a^2+b^2)^(1/2))/a^5/(a^2+b^2)^(3/2)+1/3*(2*a^4-7*a^2*b^2-12*b^4)*co
th(x)/a^4/(a^2+b^2)+b*(a^2+2*b^2)*coth(x)*csch(x)/a^3/(a^2+b^2)-1/3*(a^2+4*
b^2)*coth(x)*csch(x)^2/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)^2/a/(a^2+b^2)/(a+b
*sinh(x))
```

Rubi [A]
time = 0.62, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$,
Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$-\frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\text{csch}^2(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{2b^4(5a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}}+\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\left(a^2+b^2\right)}+\frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] -((b*(a^2 - 4*b^2)*ArcTanh[Cosh[x]])/a^5) - (2*b^4*(5*a^2 + 4*b^2)*ArcTanh[
(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^5*(a^2 + b^2)^(3/2)) + ((2*a^4 - 7*a
^2*b^2 - 12*b^4)*Coth[x])/(3*a^4*(a^2 + b^2)) + (b*(a^2 + 2*b^2)*Coth[x]*Cs
ch[x])/(a^3*(a^2 + b^2)) - ((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/(3*a^2*(a^2 +
b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^4(x)(a^2+4b^2-ab\sinh(x)+3b^2\sinh^2(x))}{a+b\sinh(x)}\,dx}{a\,(a^2+b^2)}$$

$$= -\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a^2\,(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{i\int \operatorname{csch}^3(x)\,(6ib(a^2+2b^2)+i$$

$$= \frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3\,(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a^2\,(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\,(a^2+b^2)\,(a+b\sinh($$

$$= \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\,(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3\,(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\operatorname{csc}}{3a^2\,(a^2+b^2)}$$

$$= \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\,(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3\,(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\operatorname{csc}}{3a^2\,(a^2+b^2)}$$

$$= -\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\,(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x}{a^3\,(a^2+b^2}$$

$$= -\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\,(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x}{a^3\,(a^2+b^2}$$

$$= -\frac{b(a^2-4b^2)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2b^4(5a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\,(a^2+b^2)^{3/2}} + \frac{(2a^4-7a^2}{3a}$$

Mathematica [A]

time = 0.65, size = 214, normalized size = 1.08

$$\frac{-\frac{48b^4(5a^2+4b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + 4a(2a^2-9b^2)\coth\left(\frac{x}{2}\right) + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 24(a-2b)b(a+2b)\log\left(\tanh\left(\frac{x}{2}\right)\right) + 6a^2b\operatorname{sech}^2\left(\frac{x}{2}\right) + 8a^3\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{1}{2}a^3\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x) - \frac{24ab^5\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + 4a(2a^2-9b^2)\tanh\left(\frac{x}{2}\right)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((-48*b^4*(5*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
 - b^2)^(3/2) + 4*a*(2*a^2 - 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 + 24*(a
 - 2*b)*b*(a + 2*b)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*
Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b^5*Cosh[x])/((a^2 + b^2)
*(a + b*Sinh[x])) + 4*a*(2*a^2 - 9*b^2)*Tanh[x/2])/(24*a^5)
```

Maple [A]

time = 0.78, size = 219, normalized size = 1.11

method	result

default	$-\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^2}{3}+2ab\left(\tanh^2\left(\frac{x}{2}\right)\right)-3a^2\tanh\left(\frac{x}{2}\right)+12b^2\tanh\left(\frac{x}{2}\right)}{8a^4}-\frac{1}{24a^2\tanh\left(\frac{x}{2}\right)^3}-\frac{-3a^2+12b^2}{8a^4\tanh\left(\frac{x}{2}\right)}+\frac{b}{4a^3\tanh\left(\frac{x}{2}\right)^2}+\frac{b(a^2-4}{}$
risch	$-\frac{2(-3a^3b^2e^{7x}-6a\,b^4e^{7x}-6a^4b\,e^{6x}+3a^2b^3e^{6x}+12b^5e^{6x}+21a^3b^2e^{5x}+30a\,b^4e^{5x}+6a^4b\,e^{4x}-21a^2b^3e^{4x}-36b^5e^{4x}+12a^5e^{3x}-21a^3b^2e}{3a^4(e^{2x}-1)^3(a^2+b^2)(b\,e^{2x}+2a\,e^x-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^4*(1/3*tanh(1/2*x)^3*a^2+2*a*b*tanh(1/2*x)^2-3*a^2*tanh(1/2*x)+12*b^
2*tanh(1/2*x))-1/24/a^2/tanh(1/2*x)^3-1/8/a^4*(-3*a^2+12*b^2)/tanh(1/2*x)+1
/4/a^3*b/tanh(1/2*x)^2+1/a^5*b*(a^2-4*b^2)*ln(tanh(1/2*x))-2*b^4/a^5*((-b^2
/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(
5*a^2+4*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1
/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(190) = 380$.
time = 0.50, size = 477, normalized size = 2.41

$$\frac{(5a^2b^4+4b^6)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^7+a^5b^2)\sqrt{a^2+b^2}}+\frac{2\left(2a^4b-7a^2b^3-12b^5+(4a^5-11a^3b^2-18ab^4)e^{(-x)}-(2a^4b-25a^2b^3-36b^5)e^{(-2x)}-3(4a^5-7a^3b^2-14ab^4)e^{(-3x)}+3(2a^4b-7a^2b^3-12b^5)e^{(-4x)}-3(7a^3b^2+10ab^4)e^{(-5x)}-3(2a^4b-a^2b^3-4b^5)e^{(-6x)}+3(a^3b^2+2ab^4)e^{(-7x)}\right)}{3\left(a^6b+a^4b^3+2(a^7+a^5b^2)e^{(-x)}-4(a^6b+a^4b^3)e^{(-2x)}-6(a^7+a^5b^2)e^{(-3x)}+6(a^6b+a^4b^3)e^{(-4x)}+6(a^7+a^5b^2)e^{(-5x)}-4(a^6b+a^4b^3)e^{(-6x)}-2(a^7+a^5b^2)e^{(-7x)}+(a^6b+a^4b^3)e^{(-8x)}\right)}-\frac{(a^2b-4b^3)\log(e^{(-x)}+1)}{a^5}+\frac{(a^2b-4b^3)\log(e^{(-x)}-1)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2/3*(2*a^4*b - 7*a^2*b^
3 - 12*b^5 + (4*a^5 - 11*a^3*b^2 - 18*a*b^4)*e^(-x) - (2*a^4*b - 25*a^2*b^3
 - 36*b^5)*e^(-2*x) - 3*(4*a^5 - 7*a^3*b^2 - 14*a*b^4)*e^(-3*x) + 3*(2*a^4*
b - 7*a^2*b^3 - 12*b^5)*e^(-4*x) - 3*(7*a^3*b^2 + 10*a*b^4)*e^(-5*x) - 3*(2
*a^4*b - a^2*b^3 - 4*b^5)*e^(-6*x) + 3*(a^3*b^2 + 2*a*b^4)*e^(-7*x))/(a^6*b
 + a^4*b^3 + 2*(a^7 + a^5*b^2)*e^(-x) - 4*(a^6*b + a^4*b^3)*e^(-2*x) - 6*(a
^7 + a^5*b^2)*e^(-3*x) + 6*(a^6*b + a^4*b^3)*e^(-4*x) + 6*(a^7 + a^5*b^2)*e
^(-5*x) - 4*(a^6*b + a^4*b^3)*e^(-6*x) - 2*(a^7 + a^5*b^2)*e^(-7*x) + (a^6*
b + a^4*b^3)*e^(-8*x)) - (a^2*b - 4*b^3)*log(e^(-x) + 1)/a^5 + (a^2*b - 4*b
^3)*log(e^(-x) - 1)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6430 vs. $2(190) = 380$.
time = 0.59, size = 6430, normalized size = 32.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*a^7*b - 10*a^5*b^3 - 38*a^3*b^5 - 24*a*b^7 - 6*(a^6*b^2 + 3*a^4*b^4
 + 2*a^2*b^6)*cosh(x)^7 - 6*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*sinh(x)^7 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^6 - 6*(2*a^7*b + a^5*b^3
 - 5*a^3*b^5 - 4*a*b^7 + 7*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x))*sinh(
x)^6 + 6*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^5 + 6*(7*a^6*b^2 + 1
7*a^4*b^4 + 10*a^2*b^6 - 21*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^2 - 6
*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^5 + 6*(2*a^7*b
- 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x)^4 + 6*(2*a^7*b - 5*a^5*b^3 - 1
9*a^3*b^5 - 12*a*b^7 - 35*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^3 - 15*
(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^2 + 5*(7*a^6*b^2 + 17*a^4
*b^4 + 10*a^2*b^6)*cosh(x))*sinh(x)^4 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 -
 14*a^2*b^6)*cosh(x)^3 + 6*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2*b^6 - 3
5*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*cosh(x)^4 - 20*(2*a^7*b + a^5*b^3 - 5*a
^3*b^5 - 4*a*b^7)*cosh(x)^3 + 10*(7*a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh
(x)^2 + 4*(2*a^7*b - 5*a^5*b^3 - 19*a^3*b^5 - 12*a*b^7)*cosh(x))*sinh(x)^3
- 2*(2*a^7*b - 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7)*cosh(x)^2 - 2*(2*a^7*b -
 23*a^5*b^3 - 61*a^3*b^5 - 36*a*b^7 + 63*(a^6*b^2 + 3*a^4*b^4 + 2*a^2*b^6)*
cosh(x)^5 + 45*(2*a^7*b + a^5*b^3 - 5*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 30*(7*
a^6*b^2 + 17*a^4*b^4 + 10*a^2*b^6)*cosh(x)^3 - 18*(2*a^7*b - 5*a^5*b^3 - 19
*a^3*b^5 - 12*a*b^7)*cosh(x)^2 - 9*(4*a^8 - 3*a^6*b^2 - 21*a^4*b^4 - 14*a^2
*b^6)*cosh(x))*sinh(x)^2 - 3*((5*a^2*b^5 + 4*b^7)*cosh(x)^8 + (5*a^2*b^5 +
4*b^7)*sinh(x)^8 + 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^7 + 2*(5*a^3*b^4 + 4*a*b
^6 + 4*(5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^7 + 5*a^2*b^5 + 4*b^7 - 4*(5*a^
2*b^5 + 4*b^7)*cosh(x)^6 - 2*(10*a^2*b^5 + 8*b^7 - 14*(5*a^2*b^5 + 4*b^7)*c
osh(x)^2 - 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^6 - 6*(5*a^3*b^4 + 4*a*
b^6)*cosh(x)^5 - 2*(15*a^3*b^4 + 12*a*b^6 - 28*(5*a^2*b^5 + 4*b^7)*cosh(x)^
3 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7)*cosh(x))*si
nh(x)^5 + 6*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 2*(15*a^2*b^5 + 12*b^7 + 35*(5*
a^2*b^5 + 4*b^7)*cosh(x)^4 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 - 30*(5*a^2
*b^5 + 4*b^7)*cosh(x)^2 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)^4 + 6*(
5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 + 2*(15*a^3*b^4 + 12*a*b^6 + 28*(5*a^2*b^5 +
 4*b^7)*cosh(x)^5 + 35*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 - 40*(5*a^2*b^5 + 4*
b^7)*cosh(x)^3 - 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 + 12*(5*a^2*b^5 + 4*b^7
)*cosh(x))*sinh(x)^3 - 4*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 2*(10*a^2*b^5 + 8*
b^7 - 14*(5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 21*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^5
 + 30*(5*a^2*b^5 + 4*b^7)*cosh(x)^4 + 30*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^3 -
18*(5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x))*sinh(x)
^2 - 2*(5*a^3*b^4 + 4*a*b^6)*cosh(x) + 2*(4*(5*a^2*b^5 + 4*b^7)*cosh(x)^7 -
 5*a^3*b^4 - 4*a*b^6 + 7*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^6 - 12*(5*a^2*b^5 +
4*b^7)*cosh(x)^5 - 15*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^4 + 12*(5*a^2*b^5 + 4*b
^7)*cosh(x)^3 + 9*(5*a^3*b^4 + 4*a*b^6)*cosh(x)^2 - 4*(5*a^2*b^5 + 4*b^7)*c
```

```
osh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b
*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) - b)) - 2*(4*a^8 - 7*a^6*b^2 - 29*a^4*b^4 - 18*a^2*b
^6)*cosh(x) + 3*((a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^8 + (a^6
*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*sinh(x)^8 + a^6*b^2 - 2*a^4*b^4 - 7*a
^2*b^6 - 4*b^8 + 2*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^7 + 2*
(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7 + 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b
^6 - 4*b^8)*cosh(x))*sinh(x)^7 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8
)*cosh(x)^6 - 2*(2*a^6*b^2 - 4*a^4*b^4 - 14*a^2*b^6 - 8*b^8 - 14*(a^6*b^2 -
 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 7*(a^7*b - 2*a^5*b^3 - 7*a^3*b^
5 - 4*a*b^7)*cosh(x))*sinh(x)^6 - 6*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^
7)*cosh(x)^5 - 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b^5 - 12*a*b^7 - 28*(a^6*b^2
 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 21*(a^7*b - 2*a^5*b^3 - 7*a^3
*b^5 - 4*a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*co
sh(x))*sinh(x)^5 + 6*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^4 +
2*(3*a^6*b^2 - 6*a^4*b^4 - 21*a^2*b^6 - 12*b^8 + 35*(a^6*b^2 - 2*a^4*b^4 -
7*a^2*b^6 - 4*b^8)*cosh(x)^4 + 35*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)
*cosh(x)^3 - 30*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^2 - 15*(a
^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x))*sinh(x)^4 + 6*(a^7*b - 2*a
^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^3 + 2*(3*a^7*b - 6*a^5*b^3 - 21*a^3*b
^5 - 12*a*b^7 + 28*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^5 + 35
*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*a*b^7)*cosh(x)^4 - 40*(a^6*b^2 - 2*a^4*
b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x)^3 - 30*(a^7*b - 2*a^5*b^3 - 7*a^3*b^5 - 4*
a*b^7)*cosh(x)^2 + 12*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - 4*b^8)*cosh(x))*si
nh(x)^3 - 4*(a^6*b^2 - 2*a^4*b^4 - 7*a^2*b^6 - ...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**4/(a + b*sinh(x))**2, x)
```

Giac [**A**]
time = 0.44, size = 236, normalized size = 1.19

$$\frac{(5a^2b^4 + 4b^6)\log\left(\frac{\left|2be^x + 2a - 2\sqrt{a^2+b^2}\right|}{\left|2be^x + 2a + 2\sqrt{a^2+b^2}\right|}\right)}{(a^7 + a^5b^2)\sqrt{a^2+b^2}} + \frac{2(ab^4e^x - b^5)}{(a^6 + a^4b^2)(be^{(2x)} + 2ae^x - b)} - \frac{(a^2b - 4b^3)\log(e^x + 1)}{a^5} + \frac{(a^2b - 4b^3)\log(|e^x - 1|)}{a^5} + \frac{2(3abe^{(5x)} - 9b^2e^{(4x)} - 6a^2e^{(2x)} + 18b^2e^{(2x)} - 3abe^x + 2a^2 - 9b^2)}{3a^4(e^{(2x)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^4*e^
x - b^5)/((a^6 + a^4*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - (a^2*b - 4*b^3)*log(
e^x + 1)/a^5 + (a^2*b - 4*b^3)*log(abs(e^x - 1))/a^5 + 2/3*(3*a*b*e^(5*x) -
 9*b^2*e^(4*x) - 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b*e^x + 2*a^2 - 9*b^2
)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 3.30, size = 975, normalized size = 4.92

$$\frac{\ln(e^x-1)\,(a^2 b-4b^3)}{a^5}-\frac{8}{3a^2\,(3e^{2x}-3e^{4x}+e^{6x}-1)}-\frac{\frac{4}{a^2}-\frac{4\,b\,e^x}{a^3}}{e^{4x}-2e^{2x}+1}-\frac{\frac{6b^2}{a^4}-\frac{2b\,e^x}{a^3}}{e^{2x}-1}-\frac{\frac{2b^8}{a^4(b^5+a^2b^3)}-\frac{2b^7e^x}{a^3(b^5+a^2b^3)}}{2a\,e^x-b+b\,e^{2x}}-\frac{\ln(e^x+1)\,(a^2 b-4b^3)}{a^5}-\frac{b^4\ln\left(\cdots\right)\sqrt{(a^2+b^2)^3}\,\cdots}{a^{11}+3a^9b^2+3a^7b^4+a^5b^6}\ [illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] (log(exp(x) - 1)*(a^2*b - 4*b^3))/a^5 - 8/(3*a^2*(3*exp(2*x) - 3*exp(4*x) +
 exp(6*x) - 1)) - (4/a^2 - (4*b*exp(x))/a^3)/(exp(4*x) - 2*exp(2*x) + 1) -
((6*b^2)/a^4 - (2*b*exp(x))/a^3)/(exp(2*x) - 1) - ((2*b^8)/(a^4*(b^5 + a^2*
b^3)) - (2*b^7*exp(x))/(a^3*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x))
 - (log(exp(x) + 1)*(a^2*b - 4*b^3))/a^5 - (b^4*log((32*b*(16*b^4 - 5*a^4 +
 16*a^2*b^2)*(2*a^4*b - 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x)
+ 11*a^3*b^2*exp(x)))/(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(5*a^5*b
^9 - 32*b^11*((a^2 + b^2)^3)^(1/2) - 2*a^13*b + 20*a^7*b^7 + 24*a^9*b^5 + 7
*a^11*b^3 + 4*a^14*exp(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*(
(a^2 + b^2)^3)^(1/2) - 15*a^6*b^8*exp(x) - 50*a^8*b^6*exp(x) - 52*a^10*b^4*
exp(x) - 13*a^12*b^2*exp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79
*a^5*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2
) + 51*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4))*((a^2 + b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^
7*b^4 + 3*a^9*b^2) + (b^4*log((32*b*(16*b^4 - 5*a^4 + 16*a^2*b^2)*(2*a^4*b
- 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x) + 11*a^3*b^2*exp(x)))/
(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(2*a^13*b - 32*b^11*((a^2 + b^
2)^3)^(1/2) - 5*a^5*b^9 - 20*a^7*b^7 - 24*a^9*b^5 - 7*a^11*b^3 - 4*a^14*exp
(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*((a^2 + b^2)^3)^(1/2) +
 15*a^6*b^8*exp(x) + 50*a^8*b^6*exp(x) + 52*a^10*b^4*exp(x) + 13*a^12*b^2*e
xp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79*a^5*b^6*exp(x)*((a^2
+ b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 51*a*b^10*exp(x)*(
(a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*((a^2 +
b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^7*b^4 + 3*a^9*b^2)
```

3.88 $\int \frac{1}{3+5i\sinh(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] 1/4*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-1/4*I*ln(cosh(1/2*d
*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2739, 630, 31}

$$\frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((I/4)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - ((I/4)*Log[Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{(2i)\text{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= -\frac{(3i)\text{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4d} + \frac{(3i)\text{Subst}\left(\int \frac{1}{9+3x}\,dx, x, \tan\left(\frac{1}{2}(ic\right.\right.}{4d}$$
$$= \frac{i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.11

$$\frac{\text{ArcTan}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\text{ArcTan}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log(4-5\cosh(c+dx))}{8d} + \frac{i\log(4+5\cosh(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ArcTan[3*Coth[(c + d*x)/2]]/(4*d) + ArcTan[3*Tanh[(c + d*x)/2]]/(4*d) - ((I
/8)*Log[4 - 5*Cosh[c + d*x]])/d + ((I/8)*Log[4 + 5*Cosh[c + d*x]])/d
```

Maple [A]
time = 0.97, size = 40, normalized size = 0.55

method	result	size
risch	$\frac{i\ln\left(e^{dx+c}+\frac{4}{5}-\frac{3i}{5}\right)}{4d} - \frac{i\ln\left(e^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right)}{4d}$	36
derivativedivides	$\frac{\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{4} - \frac{i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{4}}{d}$	40
default	$\frac{\frac{i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{4} - \frac{i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{4}}{d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*I*ln(tanh(1/2*d*x+1/2*c)-3*I)-1/4*I*ln(3*tanh(1/2*d*x+1/2*c)-I))
```

Maxima [A]
time = 0.47, size = 19, normalized size = 0.26

$$\frac{\arctan\left(\frac{5}{4}i\,e^{(-dx-c)} - \frac{3}{4}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(5/4*I*e^(-d*x - c) - 3/4)/d
```

Fricas [A]
time = 0.36, size = 28, normalized size = 0.38

$$\frac{i\,\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)-i\,\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*log(e^(d*x + c) - 3/5*I + 4/5) - I*log(e^(d*x + c) - 3/5*I - 4/5))/d
```

Sympy [A]
time = 0.17, size = 31, normalized size = 0.42

$$\frac{\text{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(\frac{(-16ii-3i)e^{-c}}{5}+e^{dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log((-16*_i*I - 3*I)*exp(-c)/5 + exp(d*
x))))/d
```

Giac [A]
time = 0.43, size = 32, normalized size = 0.44

$$-\frac{-i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) + I*log(-(2*I - 1)*e^(d*x + c)
 + I - 2))/d
```

Mupad [B]
time = 0.35, size = 39, normalized size = 0.53

$$-\frac{\ln\left(-\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2-\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}+\frac{\ln\left(\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2+\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3),x)
```

```
[Out] (log(exp(d*x)*exp(c)*(2 + 3i/2) + 5/2)*1i)/(4*d) - (log(exp(d*x)*exp(c)*(2
- 3i/2) - 5/2)*1i)/(4*d)
```

3.89 $\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{5i\,\mathrm{co}}{16d(3+}$$

```
[Out] -3/64*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+3/64*I*ln(cosh(1/
2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/16*I*cosh(d*x+c)/d/(3+5*I*sinh(d*
x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2743, 12, 2739, 630, 31}

$$\frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (((-3*I)/64)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((3*I)/64
)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/16)*Cosh[c
+ d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}+\frac{1}{16}\int -\frac{3}{3+5i\sinh(c+dx)}\,dx \\
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}-\frac{3}{16}\int \frac{1}{3+5i\sinh(c+dx)}\,dx \\
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}+\frac{(3i)\text{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d} \\
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}+\frac{(9i)\text{Subst}\left(\int \frac{1}{1+3x}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{64d}-\frac{(9i
\\
&= -\frac{3i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{3i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}+\frac{5}{16d(3
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 142, normalized size = 1.39

$$\frac{-9\left(2\text{ArcTan}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)+2\text{ArcTan}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-i\log(4-5\cosh(c+dx))+i\log(4+5\cosh(c+dx))\right)+40\left(\frac{1}{3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)}+\frac{3}{\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}\right)\sinh\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-9*(2*ArcTan[3*Coth[(c + d*x)/2]] + 2*ArcTan[3*Tanh[(c + d*x)/2]] - I*Log[
4 - 5*Cosh[c + d*x]] + I*Log[4 + 5*Cosh[c + d*x]]) + 40*((3*Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^(-1) + 3/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x
)/2]))*Sinh[(c + d*x)/2])/(384*d)
```

Maple [A]
time = 1.28, size = 74, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}}{d}$	74
default	$\frac{\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}}{d}$	74
risch	$\frac{i(3\,\mathrm{e}^{dx+c}-5i)}{8d(5\,\mathrm{e}^{2dx+2c}-5-6i\mathrm{e}^{dx+c})}-\frac{3i\ln(\mathrm{e}^{dx+c}+\frac{4}{5}-\frac{3i}{5})}{64d}+\frac{3i\ln(\mathrm{e}^{dx+c}-\frac{4}{5}-\frac{3i}{5})}{64d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3/64*I*ln(3*tanh(1/2*d*x+1/2*c)-I)+5/48/(3*tanh(1/2*d*x+1/2*c)-I)-3/64
*I*ln(tanh(1/2*d*x+1/2*c)-3*I)+5/16/(tanh(1/2*d*x+1/2*c)-3*I))
```

Maxima [A]
time = 0.54, size = 79, normalized size = 0.77

$$\frac{3i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{64\,d}+\frac{3i\,e^{(-dx-c)}-5}{-8\,d(-6i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/64*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d + (3*I*
e^(-d*x - c) - 5)/(d*(48*I*e^(-d*x - c) + 40*e^(-2*d*x - 2*c) - 40))
```

Fricas [A]
time = 0.42, size = 103, normalized size = 1.01

$$-\frac{3\left(5i\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}-5i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+3\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)-24i\,e^{(dx+c)}-40}{64\left(5\,de^{(2\,dx+2\,c)}-6i\,de^{(dx+c)}-5\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/64*(3*(5*I*e^(2*d*x + 2*c) + 6*e^(d*x + c) - 5*I)*log(e^(d*x + c) - 3/5*
I + 4/5) + 3*(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)*log(e^(d*x + c) -
 3/5*I - 4/5) - 24*I*e^(d*x + c) - 40)/(5*d*e^(2*d*x + 2*c) - 6*I*d*e^(d*x
+ c) - 5*d)
```

Sympy [A]
time = 0.22, size = 75, normalized size = 0.74

$$\frac{3ie^{c}e^{dx}+5}{40de^{2c}e^{2dx}-48ide^{c}e^{dx}-40d}+\frac{\operatorname{RootSum}\left(4096z^{2}+9,\left(i\mapsto i\log\left(\frac{(256ii-9i)e^{-c}}{15}+e^{dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**2,x)
```

```
[Out] (3*I*exp(c)*exp(d*x) + 5)/(40*d*exp(2*c)*exp(2*d*x) - 48*I*d*exp(c)*exp(d*x
) - 40*d) + RootSum(4096*_z**2 + 9, Lambda(_i, _i*log((256*_i*I - 9*I)*exp(
-c)/15 + exp(d*x))))/d
```

Giac [A]
time = 0.39, size = 67, normalized size = 0.66

$$-\frac{\frac{8\left(-3i\,e^{(dx+c)}-5\right)}{5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5}+3i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)-3i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(-3*I*e^(d*x + c) - 5)/(5*e^(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5) +
 3*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) - 3*I*log(-(2*I - 1)*e^(d*x + c) +
 I - 2))/d
```

Mupad [B]
time = 0.98, size = 106, normalized size = 1.04

$$-\frac{5}{8\,\left(5\,d-5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{c+d\,x}\,6\mathrm{i}\right)}-\frac{\ln\left(-\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3-\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d}+\frac{\ln\left(\frac{15}{4}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(-3+\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d}-\frac{\mathrm{e}^{c+d\,x}\,3\mathrm{i}}{8\,\left(5\,d-5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{c+d\,x}\,6\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^2,x)
```

```
[Out] (log(15/4 - exp(d*x)*exp(c)*(3 - 9i/4))*3i)/(64*d) - (log(- exp(d*x)*exp(c)
*(3 + 9i/4) - 15/4)*3i)/(64*d) - 5/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*
c + 2*d*x))) - (exp(c + d*x)*3i)/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*c
+ 2*d*x)))
```

3.90 $\int \frac{1}{(3+5i\sinh(c+dx))^3} dx$

Optimal. Leaf size=131

$$\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}+\frac{5i\,c}{32d(3+}$$

```
[Out] 43/2048*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-43/2048*I*ln(co
sh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/32*I*cosh(d*x+c)/d/(3+5*I*si
nh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2743, 2833, 12, 2739, 630, 31}

$$-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (((43*I)/2048)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - (((43*I)
/2048)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/32)*Co
sh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) - (((45*I)/512)*Cosh[c + d*x])
/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx &= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{1}{32}\int \frac{-6+5i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{1}{512}\int \frac{43}{3+5i\sinh\cdots}\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{43}{512}\int \frac{1}{3+5i\sinh\cdots}\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}-\frac{(43i)\mathrm{Subst}\left(\int \frac{\cdots}{3+10\cdots}\right.}{\cdots}\\
&= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}-\frac{(129i)\mathrm{Subst}\left(\int \frac{1}{1+3\cdots}\right.}{\cdots}\\
&= \frac{43i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}+\frac{\cdots}{32d(\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 204, normalized size = 1.56

$$\frac{86\mathrm{ArcTan}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)+86\mathrm{ArcTan}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-43i\log(4-5\cosh(c+dx))+43i\log(4+5\cosh(c+dx))-\frac{80i}{\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{80i}{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\left(-\frac{120}{3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)}-\frac{360}{\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}\right)\sinh\left(\frac{1}{2}(c+dx)\right)}{4096d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (86*ArcTan[3*Coth[(c + d*x)/2]] + 86*ArcTan[3*Tanh[(c + d*x)/2]] - (43*I)*L
og[4 - 5*Cosh[c + d*x]] + (43*I)*Log[4 + 5*Cosh[c + d*x]] - (80*I)/(3*Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (80*I)/(Cosh[(c + d*x)/2] + (3*I)*S
inh[(c + d*x)/2])^2 + (-120/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) - 3
60/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(4096*
d)
```

Maple [A]
time = 1.40, size = 110, normalized size = 0.84

method	result
risch	$-\frac{i\left(-387 i \mathrm{e}^{2 d x+2 c}+215 \,\mathrm{e}^{3 d x+3 c}+225 i-325 \,\mathrm{e}^{d x+c}\right)}{256 d \left(5 \,\mathrm{e}^{2 d x+2 c}-5-6 i \mathrm{e}^{d x+c}\right)^{2}}-\frac{43 i \ln \left(\mathrm{e}^{d x+c}-\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}+\frac{43 i \ln \left(\mathrm{e}^{d x+c}+\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}$
derivativedivides	$\frac{-\frac{43 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152 \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{2}}-\frac{155}{4608 \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}+\frac{25 i}{128 \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right.\right.}{2048}}{d}$
default	$\frac{-\frac{43 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152 \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{2}}-\frac{155}{4608 \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}+\frac{25 i}{128 \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right.\right.}{2048}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-43/2048*I*ln(3*tanh(1/2*d*x+1/2*c)-I)-25/1152*I/(3*tanh(1/2*d*x+1/2*c
)-I)^2-155/4608/(3*tanh(1/2*d*x+1/2*c)-I)+25/128*I/(tanh(1/2*d*x+1/2*c)-3*I
)^2+43/2048*I*ln(tanh(1/2*d*x+1/2*c)-3*I)+15/512/(tanh(1/2*d*x+1/2*c)-3*I))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 0.95

$$-\frac{43i \log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{2048\,d} - \frac{-325i\,e^{(-dx-c)} - 387\,e^{(-2\,dx-2\,c)} + 215i\,e^{(-3\,dx-3\,c)} + 225}{-256\,d(60i\,e^{(-dx-c)} + 86\,e^{(-2\,dx-2\,c)} - 60i\,e^{(-3\,dx-3\,c)} - 25\,e^{(-4\,dx-4\,c)} - 25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/2048*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d - (
-325*I*e^(-d*x - c) - 387*e^(-2*d*x - 2*c) + 215*I*e^(-3*d*x - 3*c) + 225)/
(d*(-15360*I*e^(-d*x - c) - 22016*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*
c) + 6400*e^(-4*d*x - 4*c) + 6400))
```

Fricas [A]
time = 0.63, size = 193, normalized size = 1.47

$$-\frac{43\,(-25i\,e^{(4\,dx+4\,c)} - 60\,e^{(3\,dx+3\,c)} + 86i\,e^{(2\,dx+2\,c)} + 60\,e^{(dx+c)} - 25i)\log\left(e^{(dx+c)} - \frac{3}{5}i + \frac{4}{5}\right) + 43\,(25i\,e^{(4\,dx+4\,c)} + 60\,e^{(3\,dx+3\,c)} - 86i\,e^{(2\,dx+2\,c)} - 60\,e^{(dx+c)} + 25i)\log\left(e^{(dx+c)} - \frac{3}{5}i - \frac{4}{5}\right) + 1720i\,e^{(3\,dx+3\,c)} + 3096\,e^{(2\,dx+2\,c)} - 2600i\,e^{(dx+c)} - 1800}{2048\,(25\,de^{(4\,dx+4\,c)} - 60i\,de^{(3\,dx+3\,c)} - 86\,de^{(2\,dx+2\,c)} + 60i\,de^{(dx+c)} + 25\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2048*(43*(-25*I*e^(4*d*x + 4*c) - 60*e^(3*d*x + 3*c) + 86*I*e^(2*d*x + 2
*c) + 60*e^(d*x + c) - 25*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 43*(25*I*e^(4
*d*x + 4*c) + 60*e^(3*d*x + 3*c) - 86*I*e^(2*d*x + 2*c) - 60*e^(d*x + c) +
25*I)*log(e^(d*x + c) - 3/5*I - 4/5) + 1720*I*e^(3*d*x + 3*c) + 3096*e^(2*d
*x + 2*c) - 2600*I*e^(d*x + c) - 1800)/(25*d*e^(4*d*x + 4*c) - 60*I*d*e^(3*
d*x + 3*c) - 86*d*e^(2*d*x + 2*c) + 60*I*d*e^(d*x + c) + 25*d)
```

Sympy [A]
time = 0.29, size = 138, normalized size = 1.05

$$\frac{-215ie^{3c}e^{3dx} - 387e^{2c}e^{2dx} + 325ie^{c}e^{dx} + 225}{6400de^{4c}e^{4dx} - 15360ide^{3c}e^{3dx} - 22016de^{2c}e^{2dx} + 15360ide^{c}e^{dx} + 6400d} + \frac{\operatorname{RootSum}\left(4194304z^{2} + 1849, \left(i \mapsto i\log\left(\frac{(-8192ii - 129i)e^{-c}}{215} + e^{dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**3,x)
```

```
[Out] (-215*I*exp(3*c)*exp(3*d*x) - 387*exp(2*c)*exp(2*d*x) + 325*I*exp(c)*exp(d*
x) + 225)/(6400*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 220
16*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 6400*d) + RootSum(41
94304*_z**2 + 1849, Lambda(_i, _i*log((-8192*_i*I - 129*I)*exp(-c)/215 + ex
p(d*x))))/d
```

Giac [A]
time = 0.41, size = 89, normalized size = 0.68

$$-\frac{\frac{8\left(-215i\,e^{(3\,dx+3\,c)} - 387\,e^{(2\,dx+2\,c)} + 325i\,e^{(dx+c)} + 225\right)}{\left(-5i\,e^{(2\,dx+2\,c)} - 6\,e^{(dx+c)} + 5i\right)^{2}} - 43i\,\log\left(-(i-2)\,e^{(dx+c)} - 2i + 1\right) + 43i\,\log\left(-(2i-1)\,e^{(dx+c)} + i - 2\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(8*(-215*I*e^(3*d*x + 3*c) - 387*e^(2*d*x + 2*c) + 325*I*e^(d*x + c
) + 225)/(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)^2 - 43*I*log(-(I - 2)
*e^(d*x + c) - 2*I + 1) + 43*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

Mupad [B]
time = 1.06, size = 147, normalized size = 1.12

$$\frac{\frac{129}{6400\,d} + \frac{\mathrm{e}^{c+d\,x}\,43\mathrm{i}}{1280\,d}}{1 - \mathrm{e}^{2\,c+2\,d\,x} + \frac{\mathrm{e}^{c+d\,x}\,6\mathrm{i}}{5}} - \frac{\ln\left(-\frac{215}{4} + \mathrm{e}^{c+d\,x}\,\left(43 - \frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} + \frac{\ln\left(\frac{215}{4} + \mathrm{e}^{c+d\,x}\,\left(43 + \frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} - \frac{-\frac{3}{200\,d} + \frac{\mathrm{e}^{c+d\,x}\,7\mathrm{i}}{1000\,d}}{\mathrm{e}^{4\,c+4\,d\,x} - \frac{86\,\mathrm{e}^{2\,c+2\,d\,x}}{25} + 1 + \frac{\mathrm{e}^{c+d\,x}\,12\mathrm{i}}{5} - \frac{\mathrm{e}^{3\,c+3\,d\,x}\,12\mathrm{i}}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^3,x)
```

```
[Out] ((exp(c + d*x)*43i)/(1280*d) + 129/(6400*d))/((exp(c + d*x)*6i)/5 - exp(2*c
 + 2*d*x) + 1) - (log(exp(c + d*x)*(43 - 129i/4) - 215/4)*43i)/(2048*d) + (
log(exp(c + d*x)*(43 + 129i/4) + 215/4)*43i)/(2048*d) - ((exp(c + d*x)*7i)/
(1000*d) - 3/(200*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

3.91 $\int \frac{1}{(3+5i\sinh(c+dx))^4} dx$

Optimal. Leaf size=160

$$-\frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{279i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{}{48}$$

```
[Out] -279/32768*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+279/32768*I*
ln(cosh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/48*I*cosh(d*x+c)/d/(3+5
*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^2+995/24576*I*
cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2743, 2833, 12, 2739, 630, 31}

$$\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{279i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (((-279*I)/32768)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((27
9*I)/32768)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/4
8)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3) - (((25*I)/512)*Cosh[c +
d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) + (((995*I)/24576)*Cosh[c + d*x])/(d*
(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx &= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}+\frac{1}{48}\int \frac{-9+10i\sinh(c+dx)}{(3+5i\sinh(c+dx))^3}\,dx \\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{\int \frac{154-75i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,d}{1536} \\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{995i\cosh(c+}{24576d(3+5i\sinh} \\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{995i\cosh(c+}{24576d(3+5i\sinh} \\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{995i\cosh(c+}{24576d(3+5i\sinh} \\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{995i\cosh(c+}{24576d(3+5i\sinh} \\
&= -\frac{279i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{279i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}+\frac{}{48}
\end{aligned}$$

Mathematica [A]

time = 0.47, size = 265, normalized size = 1.66

$$\frac{-5022\text{ArcTan}\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)-5022\text{ArcTan}\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2511i\log(4-5\cosh(c+dx))-2511i\log(4+5\cosh(c+dx))+\frac{4640i}{\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1440i}{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+40\left(\frac{80}{\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{199}{3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)}+\frac{240}{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{597}{\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}\right)\sinh\left(\frac{1}{2}(c+dx)\right)}{589824d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (-5022*ArcTan[3*Coth[(c + d*x)/2]] - 5022*ArcTan[3*Tanh[(c + d*x)/2]] + (25
11*I)*Log[4 - 5*Cosh[c + d*x]] - (2511*I)*Log[4 + 5*Cosh[c + d*x]] + (4640*
I)/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (1440*I)/(Cosh[(c + d*x)
/2] + (3*I)*Sinh[(c + d*x)/2])^2 + 40*(80/(3*Cosh[(c + d*x)/2] + I*Sinh[(c
+ d*x)/2])^3 + 199/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + 240/(Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2])^3 + 597/(Cosh[(c + d*x)/2] + (3*I)*
Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(589824*d)
```

Maple [A]
time = 1.40, size = 144, normalized size = 0.90

method	result
risch	$\frac{i\left(-62775 i e^{4dx+4c}+20925\, e^{5dx+5c}+119310 i e^{2dx+2c}-111042\, e^{3dx+3c}-24875 i+68625\, e^{dx+c}\right)}{12288 d\left(5\, e^{2dx+2c}-5-6 i e^{dx+c}\right)^3}+\frac{279 i \ln\left(e^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right.}{32768 d}$
derivativedivides	$\frac{\frac{275 i}{27648\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{279 i \ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{32768}-\frac{125}{20736\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{3505}{221184\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{279 i \ln\left(\tan\right.}{3}}{d}$
default	$\frac{\frac{275 i}{27648\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{279 i \ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{32768}-\frac{125}{20736\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{3505}{221184\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{279 i \ln\left(\tan\right.}{3}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(275/27648*I/(3*tanh(1/2*d*x+1/2*c)-I)^2+279/32768*I*ln(3*tanh(1/2*d*x+
1/2*c)-I)-125/20736/(3*tanh(1/2*d*x+1/2*c)-I)^3+3505/221184/(3*tanh(1/2*d*x
+1/2*c)-I)-279/32768*I*ln(tanh(1/2*d*x+1/2*c)-3*I)+75/1024*I/(tanh(1/2*d*x+
1/2*c)-3*I)^2-125/768/(tanh(1/2*d*x+1/2*c)-3*I)^3+345/8192/(tanh(1/2*d*x+1/
2*c)-3*I))
```

Maxima [A]
time = 0.49, size = 167, normalized size = 1.04

$$\frac{279i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{32768\,d}+\frac{68625i\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}-111042i\,e^{(-3\,dx-3\,c)}-62775\,e^{(-4\,dx-4\,c)}+20925i\,e^{(-5\,dx-5\,c)}-24875}{-12288\,d(-450i\,e^{(-dx-c)}-915\,e^{(-2\,dx-2\,c)}+1116i\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}-450i\,e^{(-5\,dx-5\,c)}-125\,e^{(-6\,dx-6\,c)}+125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/32768*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d +
(68625*I*e^(-d*x - c) + 119310*e^(-2*d*x - 2*c) - 111042*I*e^(-3*d*x - 3*c)
 - 62775*e^(-4*d*x - 4*c) + 20925*I*e^(-5*d*x - 5*c) - 24875)/(d*(5529600*I
*e^(-d*x - c) + 11243520*e^(-2*d*x - 2*c) - 13713408*I*e^(-3*d*x - 3*c) - 1
1243520*e^(-4*d*x - 4*c) + 5529600*I*e^(-5*d*x - 5*c) + 1536000*e^(-6*d*x -
 6*c) - 1536000))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(126) = 252$.
time = 0.49, size = 283, normalized size = 1.77

$$-\frac{837\left(125i\,e^{(6\,dx+6\,c)}+450\,e^{(5\,dx+5\,c)}-915i\,e^{(4\,dx+4\,c)}-1116\,e^{(3\,dx+3\,c)}+915i\,e^{(2\,dx+2\,c)}+450\,e^{(dx+c)}-125i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+837\left(-125i\,e^{(6\,dx+6\,c)}-450\,e^{(5\,dx+5\,c)}+915i\,e^{(4\,dx+4\,c)}+1116\,e^{(3\,dx+3\,c)}-915i\,e^{(2\,dx+2\,c)}-450\,e^{(dx+c)}+125i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i-\frac{4}{5}\right)-167400i\,e^{(5\,dx+5\,c)}-502200\,e^{(4\,dx+4\,c)}+888336i\,e^{(3\,dx+3\,c)}+954480\,e^{(2\,dx+2\,c)}-549000i\,e^{(dx+c)}-199000}{98304\left(125\,de^{(6\,dx+6\,c)}-450i\,de^{(5\,dx+5\,c)}-915\,de^{(4\,dx+4\,c)}+1116i\,de^{(3\,dx+3\,c)}+915\,de^{(2\,dx+2\,c)}-450i\,de^{(dx+c)}-125\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/98304*(837*(125*I*e^(6*d*x + 6*c) + 450*e^(5*d*x + 5*c) - 915*I*e^(4*d*x
 + 4*c) - 1116*e^(3*d*x + 3*c) + 915*I*e^(2*d*x + 2*c) + 450*e^(d*x + c) -
125*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 837*(-125*I*e^(6*d*x + 6*c) - 450*e
^(5*d*x + 5*c) + 915*I*e^(4*d*x + 4*c) + 1116*e^(3*d*x + 3*c) - 915*I*e^(2*
d*x + 2*c) - 450*e^(d*x + c) + 125*I)*log(e^(d*x + c) - 3/5*I - 4/5) - 1674
00*I*e^(5*d*x + 5*c) - 502200*e^(4*d*x + 4*c) + 888336*I*e^(3*d*x + 3*c) +
954480*e^(2*d*x + 2*c) - 549000*I*e^(d*x + c) - 199000)/(125*d*e^(6*d*x + 6
*c) - 450*I*d*e^(5*d*x + 5*c) - 915*d*e^(4*d*x + 4*c) + 1116*I*d*e^(3*d*x +
 3*c) + 915*d*e^(2*d*x + 2*c) - 450*I*d*e^(d*x + c) - 125*d)
```

Sympy [A]
time = 0.37, size = 197, normalized size = 1.23

$$\frac{20925ie^{5c}e^{5dx}+62775e^{4c}e^{4dx}-111042ie^{3c}e^{3dx}-119310e^{2c}e^{2dx}+68625ie^{c}e^{dx}+24875}{1536000de^{6c}e^{6dx}-5529600ide^{5c}e^{5dx}-11243520de^{4c}e^{4dx}+13713408ide^{3c}e^{3dx}+11243520de^{2c}e^{2dx}-5529600ide^{c}e^{dx}-1536000d}+\frac{\operatorname{RootSum}\left(1073741824z^{2}+77841,\left(i\mapsto i\log\left(\frac{(131072ii-837i)e^{-c}}{1395}+e^{dx}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**4,x)
```

```
[Out] (20925*I*exp(5*c)*exp(5*d*x) + 62775*exp(4*c)*exp(4*d*x) - 111042*I*exp(3*c
)*exp(3*d*x) - 119310*exp(2*c)*exp(2*d*x) + 68625*I*exp(c)*exp(d*x) + 24875
)/(1536000*d*exp(6*c)*exp(6*d*x) - 5529600*I*d*exp(5*c)*exp(5*d*x) - 112435
20*d*exp(4*c)*exp(4*d*x) + 13713408*I*d*exp(3*c)*exp(3*d*x) + 11243520*d*ex
p(2*c)*exp(2*d*x) - 5529600*I*d*exp(c)*exp(d*x) - 1536000*d) + RootSum(1073
741824*_z**2 + 77841, Lambda(_i, _i*log((131072*_i*I - 837*I)*exp(-c)/1395
+ exp(d*x))))/d
```

Giac [A]
time = 0.43, size = 111, normalized size = 0.69

$$\frac{\frac{8\left(20925i\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}-111042i\,e^{(3\,dx+3\,c)}-119310\,e^{(2\,dx+2\,c)}+68625i\,e^{(dx+c)}+24875\right)}{\left(5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5\right)^{3}}-837i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+837i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/98304*(8*(20925*I*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) - 111042*I*e^(3
*d*x + 3*c) - 119310*e^(2*d*x + 2*c) + 68625*I*e^(d*x + c) + 24875)/(5*e^(2
*d*x + 2*c) - 6*I*e^(d*x + c) - 5)^3 - 837*I*log(-(I - 2)*e^(d*x + c) - 2*I
 + 1) + 837*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

Mupad [B]
time = 1.26, size = 237, normalized size = 1.48

$$-\frac{\frac{837}{102400\,d}+\frac{{\mathrm{e}}^{c+d\,x}\,279{}\mathrm{i}}{20480\,d}}{1-{\mathrm{e}}^{2\,c+2\,d\,x}+\frac{{\mathrm{e}}^{c+d\,x}\,6{}\mathrm{i}}{5}}+\frac{\frac{7}{3750\,d}+\frac{{\mathrm{e}}^{c+d\,x}\,39{}\mathrm{i}}{6250\,d}}{\frac{183\,{\mathrm{e}}^{4\,c+4\,d\,x}}{25}-\frac{183\,{\mathrm{e}}^{2\,c+2\,d\,x}}{25}-{\mathrm{e}}^{6\,c+6\,d\,x}+1+\frac{{\mathrm{e}}^{c+d\,x}\,18{}\mathrm{i}}{5}-\frac{{\mathrm{e}}^{3\,c+3\,d\,x}\,1116{}\mathrm{i}}{125}+\frac{{\mathrm{e}}^{5\,c+5\,d\,x}\,18{}\mathrm{i}}{5}}-\frac{\ln\left(-\frac{1395}{4}+{\mathrm{e}}^{c+d\,x}\,\left(-279-\frac{837}{4}{}\mathrm{i}\right)\right)\,279{}\mathrm{i}}{32768\,d}+\frac{\ln\left(\frac{1395}{4}+{\mathrm{e}}^{c+d\,x}\,\left(-279+\frac{837}{4}{}\mathrm{i}\right)\right)\,279{}\mathrm{i}}{32768\,d}-\frac{\frac{791}{80000\,d}+\frac{{\mathrm{e}}^{c+d\,x}\,93{}\mathrm{i}}{16000\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-\frac{86\,{\mathrm{e}}^{2\,c+2\,d\,x}}{25}+1+\frac{{\mathrm{e}}^{c+d\,x}\,12{}\mathrm{i}}{5}-\frac{{\mathrm{e}}^{3\,c+3\,d\,x}\,12{}\mathrm{i}}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*5i + 3)^4,x)
```

```
[Out] ((exp(c + d*x)*39i)/(6250*d) + 7/(3750*d))/((exp(c + d*x)*18i)/5 - (183*exp
(2*c + 2*d*x))/25 - (exp(3*c + 3*d*x)*1116i)/125 + (183*exp(4*c + 4*d*x))/2
5 + (exp(5*c + 5*d*x)*18i)/5 - exp(6*c + 6*d*x) + 1) - ((exp(c + d*x)*279i)
/(20480*d) + 837/(102400*d))/((exp(c + d*x)*6i)/5 - exp(2*c + 2*d*x) + 1) -
 (log(- exp(c + d*x)*(279 + 837i/4) - 1395/4)*279i)/(32768*d) + (log(1395/4
 - exp(c + d*x)*(279 - 837i/4))*279i)/(32768*d) - ((exp(c + d*x)*93i)/(1600
0*d) + 791/(80000*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

3.92 $\int \frac{1}{5+3i\sinh(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{4} - \frac{i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x-1/2*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{2736\}$

$$\frac{x}{4} - \frac{i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{x}{4} - \frac{i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 171 vs. 2(37) = 74.
time = 0.03, size = 171, normalized size = 4.62

$$-\frac{i\text{ArcTan}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} + \frac{i\text{ArcTan}\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} - \frac{\log(5\cosh(c+dx)-4\sinh(c+dx))}{8d} + \frac{\log(5\cosh(c+dx)+4\sinh(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-1/4*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])]])/d + ((I/4)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])]])/d - Log[5*Cosh[c +
 d*x] - 4*Sinh[c + d*x]]/(8*d) + Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]/(8*
d)
```

Maple [A]
time = 0.87, size = 42, normalized size = 1.14

method	result	size
risch	$-\frac{\ln\left(-3i+{\mathrm e}^{d x +c}\right)}{4 d}+\frac{\ln\left({\mathrm e}^{d x +c}-\frac{i}{3}\right)}{4 d}$	32
derivativedivides	$\frac{-\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{4}+\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{4}}{d}$	42
default	$\frac{-\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{4}+\frac{\ln\left(5 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{4}}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)+1/4*ln(5*tanh(1/2*d*x+1/2*c)+4-3*
I))
```

Maxima [A]
time = 0.53, size = 36, normalized size = 0.97

$$\frac{\log\left(-\frac{6\left(-i\,e^{(-dx-c)}+3\right)}{6i\,e^{(-dx-c)}-2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*log(-6*(-I*e^(-d*x - c) + 3)/(6*I*e^(-d*x - c) - 2))/d
```

Fricas [A]
time = 0.35, size = 26, normalized size = 0.70

$$\frac{\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(e^(d*x + c) - 1/3*I) - log(e^(d*x + c) - 3*I))/d
```

Sympy [A]
time = 0.12, size = 31, normalized size = 0.84

$$\frac{-\frac{\log\left(e^{dx}-3ie^{-c}\right)}{4}+\frac{\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x)
[Out] (-log(exp(d*x) - 3*I*exp(-c))/4 + log(exp(d*x) - I*exp(-c)/3)/4)/d
```

Giac [A]
time = 0.42, size = 28, normalized size = 0.76

$$\frac{\log\left(3\,e^{(dx+c)}-i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="giac")
[Out] 1/4*(log(3*e^(d*x + c) - I) - log(e^(d*x + c) - 3*I))/d
```

Mupad [B]
time = 0.61, size = 32, normalized size = 0.86

$$-\frac{\ln\left(-\frac{e^{d\,x}\,e^{c}}{2}+\frac{3}{2}i\right)-\ln\left(\frac{9\,e^{d\,x}\,e^{c}}{2}-\frac{3}{2}i\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5),x)
[Out] -(log(3i/2 - (exp(d*x)*exp(c))/2) - log((9*exp(d*x)*exp(c))/2 - 3i/2))/(4*d
)
```

3.93 $\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{5x}{64}-\frac{5i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

```
[Out] 5/64*x-5/32*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/16*I*cosh(d*x+c)/d/
(5+3*I*sinh(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2743, 12, 2736}

$$-\frac{5i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}+\frac{5x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (((5*I)/32)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))} - \frac{1}{16}\int -\frac{5}{5+3i\sinh(c+dx)}\,dx$$
$$= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))} + \frac{5}{16}\int \frac{1}{5+3i\sinh(c+dx)}\,dx$$
$$= \frac{5x}{64} - \frac{5i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d} - \frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 183 vs. $2(66) = 132$.
time = 0.19, size = 183, normalized size = 2.77

$$\frac{24i - 50i\text{ArcTan}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right) + 50i\text{ArcTan}\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right) - 25\log(5\cosh(c+dx)-4\sinh(c+dx)) + 25\log(5\cosh(c+dx)+4\sinh(c+dx)) - \frac{120\cosh(c+dx)}{-5i+3\sinh(c+dx)}}{640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (24*I - (50*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c +
d*x)/2] - 2*Sinh[(c + d*x)/2])] + (50*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 25*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 25*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] - (1
20*Cosh[c + d*x])/(-5*I + 3*Sinh[c + d*x]))/(640*d)
```

Maple [A]
time = 1.17, size = 84, normalized size = 1.27

method	result	size
risch	$-\frac{i\left(5\,\mathrm{e}^{dx+c}-3i\right)}{8d\left(3\,\mathrm{e}^{2dx+2c}-3-10i\mathrm{e}^{dx+c}\right)}+\frac{5\ln\left(\mathrm{e}^{dx+c}-\frac{i}{3}\right)}{64d}-\frac{5\ln\left(-3i+\mathrm{e}^{dx+c}\right)}{64d}$	73
derivativedivides	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i}-\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{64}}{d}$	84
default	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i}-\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{64}}{d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-9/80-3/20*I)/(5*tanh(1/2*d*x+1/2*c)+4-3*I)+5/64*ln(5*tanh(1/2*d*x+1/
2*c)+4-3*I)+(-9/80+3/20*I)/(5*tanh(1/2*d*x+1/2*c)-4-3*I)-5/64*ln(5*tanh(1/2
*d*x+1/2*c)-4-3*I))
```

Maxima [A]
time = 0.49, size = 64, normalized size = 0.97

$$-\frac{5i\,\arctan\left(\frac{3}{4}\,e^{(-dx-c)}+\frac{5}{4}i\right)}{32\,d}-\frac{5i\,e^{(-dx-c)}-3}{-8\,d(-10i\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/32*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (5*I*e^(-d*x - c) - 3)/(d*(80*
I*e^(-d*x - c) + 24*e^(-2*d*x - 2*c) - 24))
```

Fricas [A]
time = 0.45, size = 103, normalized size = 1.56

$$\frac{5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-3i\right)-40i\,e^{(dx+c)}-24}{64\,(3\,de^{(2\,dx+2\,c)}-10i\,de^{(dx+c)}-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 1/3*I)
 - 5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 3*I) - 40
*I*e^(d*x + c) - 24)/(3*d*e^(2*d*x + 2*c) - 10*I*d*e^(d*x + c) - 3*d)
```

Sympy [A]
time = 0.16, size = 82, normalized size = 1.24

$$\frac{-5ie^{c}e^{dx}-3}{24de^{2c}e^{2dx}-80ide^{c}e^{dx}-24d}+\frac{-\frac{5\log\left(e^{dx}-3ie^{-c}\right)}{64}+\frac{5\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{64}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**2,x)
```

```
[Out] (-5*I*exp(c)*exp(d*x) - 3)/(24*d*exp(2*c)*exp(2*d*x) - 80*I*d*exp(c)*exp(d*
x) - 24*d) + (-5*log(exp(d*x) - 3*I*exp(-c))/64 + 5*log(exp(d*x) - I*exp(-c
)/3)/64)/d
```

Giac [A]
time = 0.41, size = 65, normalized size = 0.98

$$-\frac{\frac{8\,(5i\,e^{(dx+c)}+3)}{3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3}-5\,\log\left(3\,e^{(dx+c)}-i\right)+5\,\log\left(e^{(dx+c)}-3i\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(5*I*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3) -
 5*log(3*e^(d*x + c) - I) + 5*log(e^(d*x + c) - 3*I))/d
```

Mupad [B]
time = 1.07, size = 102, normalized size = 1.55

$$\frac{3}{8\,\left(3\,d-3\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+d\,{\mathrm{e}}^{c+d\,x}\,10{}\mathrm{i}\right)}-\frac{5\,\ln\left(-\frac{5\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c}{4}+\frac{15}{4}{}\mathrm{i}\right)}{64\,d}+\frac{5\,\ln\left(\frac{45\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c}{4}-\frac{15}{4}{}\mathrm{i}\right)}{64\,d}+\frac{{\mathrm{e}}^{c+d\,x}\,5{}\mathrm{i}}{8\,\left(3\,d-3\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+d\,{\mathrm{e}}^{c+d\,x}\,10{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^2,x)
```

```
[Out] 3/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))) - (5*log(15i/4 - (5
*exp(d*x)*exp(c))/4))/(64*d) + (5*log((45*exp(d*x)*exp(c))/4 - 15i/4))/(64*
d) + (exp(c + d*x)*5i)/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))
)
```

3.94 $\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$\frac{59x}{2048}-\frac{59i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}$$

```
[Out] 59/2048*x-59/1024*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/32*I*cosh(d*x
+c)/d/(5+3*I*sinh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 2833, 12, 2736}

$$-\frac{59i\text{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}+\frac{59x}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (((59*I)/1024)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
 - (((3*I)/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((45*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx &= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{1}{32}\int \frac{-10+3i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx\\ &= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}+\frac{1}{512}\int \frac{59}{5+3i\sinh\cdots}\\ &= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}+\frac{59}{512}\int \frac{1}{5+3i\sinh\cdots}\\ &= \frac{59x}{2048}-\frac{59i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh\cdots}{512d(5+3i\,\mathrm{s}\cdots}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. 2(95) = 190.
time = 0.51, size = 277, normalized size = 2.92

$$\frac{-118i\,\mathrm{ArcTan}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+118i\,\mathrm{ArcTan}\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-59\log(5\cosh(c+dx)-4\sinh(c+dx))+59\log(5\cosh(c+dx)+4\sinh(c+dx))+\frac{48}{\left((1+2i)\cosh\left(\frac{1}{2}(c+dx)\right)-(2+i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{48}{\left((2+i)\cosh\left(\frac{1}{2}(c+dx)\right)+(1+2i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{144\sinh\left(\frac{1}{2}(c+dx)\right)\left(-3i\cosh\left(\frac{1}{2}(c+dx)\right)+5\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{-5i+3\sinh(c+dx)}}{4096d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-118*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])] + (118*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(c
+ d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 59*Log[5*Cosh[c + d
*x] - 4*Sinh[c + d*x]] + 59*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] + 48/((1
 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2 + 48/((2 + I)*Cosh
[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^2 - (144*Sinh[(c + d*x)/2]*((-
3*I)*Cosh[(c + d*x)/2] + 5*Sinh[(c + d*x)/2]))/(-5*I + 3*Sinh[c + d*x]))/(4
096*d)
```

Maple [A]
time = 1.30, size = 126, normalized size = 1.33

method	result
risch	$-\frac{3i(-295i\,\mathrm{e}^{2dx+2c}+59\,\mathrm{e}^{3dx+3c}+45i-241\,\mathrm{e}^{dx+c})}{256d(3\,\mathrm{e}^{2dx+2c}-3-10i\mathrm{e}^{dx+c})^2}-\frac{59\ln(-3i+\mathrm{e}^{dx+c})}{2048d}+\frac{59\ln(\mathrm{e}^{dx+c}-\frac{i}{3})}{2048d}$
derivativedivides	$\frac{\frac{-\frac{63}{3200}-\frac{27i}{400}}{(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)^2}+\frac{-\frac{963}{12800}+\frac{123i}{1600}}{5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i}-\frac{59\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)}{2048}+\frac{\frac{63}{3200}-\frac{27i}{400}}{(5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i)^2}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh(\frac{dx}{2}+\frac{c}{2})+4}}{d}$
default	$\frac{\frac{-\frac{63}{3200}-\frac{27i}{400}}{(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)^2}+\frac{-\frac{963}{12800}+\frac{123i}{1600}}{5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i}-\frac{59\ln(5\tanh(\frac{dx}{2}+\frac{c}{2})-4-3i)}{2048}+\frac{\frac{63}{3200}-\frac{27i}{400}}{(5\tanh(\frac{dx}{2}+\frac{c}{2})+4-3i)^2}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh(\frac{dx}{2}+\frac{c}{2})+4}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-63/3200-27/400*I)/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2+(-963/12800+123/16
00*I)/(5*tanh(1/2*d*x+1/2*c)-4-3*I)-59/2048*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)
+(63/3200-27/400*I)/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-(963/12800+123/1600*I)/
(5*tanh(1/2*d*x+1/2*c)+4-3*I)+59/2048*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I))
```

Maxima [A]
time = 0.60, size = 108, normalized size = 1.14

$$-\frac{59i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{1024\,d}+\frac{3\left(241i\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}-59i\,e^{(-3\,dx-3\,c)}-45\right)}{-256\,d(60i\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}-60i\,e^{(-3\,dx-3\,c)}-9\,e^{(-4\,dx-4\,c)}-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/1024*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d + 3*(241*I*e^(-d*x - c) + 295
*e^(-2*d*x - 2*c) - 59*I*e^(-3*d*x - 3*c) - 45)/(d*(-15360*I*e^(-d*x - c) -
 30208*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*c) + 2304*e^(-4*d*x - 4*c)
+ 2304))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(75) = 150.
time = 0.39, size = 193, normalized size = 2.03

$$\frac{59\left(9\,e^{(4\,dx+4\,c)}-60i\,e^{(3\,dx+3\,c)}-118\,e^{(2\,dx+2\,c)}+60i\,e^{(dx+c)}+9\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-59\left(9\,e^{(4\,dx+4\,c)}-60i\,e^{(3\,dx+3\,c)}-118\,e^{(2\,dx+2\,c)}+60i\,e^{(dx+c)}+9\right)\log\left(e^{(dx+c)}-3i\right)-1416i\,e^{(3\,dx+3\,c)}-7080\,e^{(2\,dx+2\,c)}+5784i\,e^{(dx+c)}+1080}{2048\left(9\,de^{(4\,dx+4\,c)}-60i\,de^{(3\,dx+3\,c)}-118\,de^{(2\,dx+2\,c)}+60i\,de^{(dx+c)}+9\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(59*(9*e^(4*d*x + 4*c) - 60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c)
+ 60*I*e^(d*x + c) + 9)*log(e^(d*x + c) - 1/3*I) - 59*(9*e^(4*d*x + 4*c) -
60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c) + 60*I*e^(d*x + c) + 9)*log(e^(d
*x + c) - 3*I) - 1416*I*e^(3*d*x + 3*c) - 7080*e^(2*d*x + 2*c) + 5784*I*e^(
```

```
d*x + c) + 1080)/(9*d*e^(4*d*x + 4*c) - 60*I*d*e^(3*d*x + 3*c) - 118*d*e^(2
*d*x + 2*c) + 60*I*d*e^(d*x + c) + 9*d)
```

Sympy [A]
time = 0.23, size = 141, normalized size = 1.48

$$\frac{-177ie^{3c}e^{3dx}-885e^{2c}e^{2dx}+723ie^{c}e^{dx}+135}{2304de^{4c}e^{4dx}-15360ide^{3c}e^{3dx}-30208de^{2c}e^{2dx}+15360ide^{c}e^{dx}+2304d}+\frac{-\frac{59\log\left(e^{dx}-3ie^{-c}\right)}{2048}+\frac{59\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{2048}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**3,x)
```

```
[Out] (-177*I*exp(3*c)*exp(3*d*x) - 885*exp(2*c)*exp(2*d*x) + 723*I*exp(c)*exp(d*
x) + 135)/(2304*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 302
08*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 2304*d) + (-59*log(e
xp(d*x) - 3*I*exp(-c))/2048 + 59*log(exp(d*x) - I*exp(-c)/3)/2048)/d
```

Giac [A]
time = 0.42, size = 87, normalized size = 0.92

$$-\frac{\frac{24\left(-59i\,e^{(3\,dx+3\,c)}-295\,e^{(2\,dx+2\,c)}+241i\,e^{(dx+c)}+45\right)}{\left(-3i\,e^{(2\,dx+2\,c)}-10\,e^{(dx+c)}+3i\right)^{2}}-59\,\log\left(3\,e^{(dx+c)}-i\right)+59\,\log\left(e^{(dx+c)}-3i\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(24*(-59*I*e^(3*d*x + 3*c) - 295*e^(2*d*x + 2*c) + 241*I*e^(d*x + c
) + 45)/(-3*I*e^(2*d*x + 2*c) - 10*e^(d*x + c) + 3*I)^2 - 59*log(3*e^(d*x +
 c) - I) + 59*log(e^(d*x + c) - 3*I))/d
```

Mupad [B]
time = 1.62, size = 143, normalized size = 1.51

$$\frac{\frac{295}{2304\,d}+\frac{e^{c+d\,x}\,59i}{768\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10i}{3}}-\frac{59\,\ln\left(-\frac{59\,e^{c+d\,x}}{4}+\frac{177}{4}i\right)}{2048\,d}+\frac{59\,\ln\left(\frac{531\,e^{c+d\,x}}{4}-\frac{177}{4}i\right)}{2048\,d}-\frac{\frac{5}{72\,d}+\frac{e^{c+d\,x}\,41i}{216\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20i}{3}-\frac{e^{3\,c+3\,d\,x}\,20i}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^3,x)
```

```
[Out] ((exp(c + d*x)*59i)/(768*d) + 295/(2304*d))/((exp(c + d*x)*10i)/3 - exp(2*c
 + 2*d*x) + 1) - (59*log(177i/4 - (59*exp(c + d*x))/4))/(2048*d) + (59*log(
(531*exp(c + d*x))/4 - 177i/4))/(2048*d) - ((exp(c + d*x)*41i)/(216*d) + 5/
(72*d))/((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x
)*20i)/3 + exp(4*c + 4*d*x) + 1)
```

3.95 $\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$

Optimal. Leaf size=124

$$\frac{385x}{32768}-\frac{385i\operatorname{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}$$

```
[Out] 385/32768*x-385/16384*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-1/16*I*cosh
(d*x+c)/d/(5+3*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))^
2-311/8192*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 2833, 12, 2736}

$$-\frac{385i\operatorname{ArcTan}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d}-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}+\frac{385x}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (((385*I)/16384)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])
])/d - ((I/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3) - (((25*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((311*I)/8192)*Cosh[c
+ d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx &= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{1}{48}\int \frac{-15+6i\sinh(c+dx)}{(5+3i\sinh(c+dx))^3}\,dx \\
&= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} + \frac{\int \frac{186-75i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}}{1536} \\
&= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c}{8192d(5+3i\sinh} \\
&= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c}{8192d(5+3i\sinh} \\
&= \frac{385x}{32768} - \frac{385i\tan^{-1}\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh}{512d(5+3}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 308 vs. 2(124) = 248.
time = 1.62, size = 308, normalized size = 2.48

$$\frac{-3850i\,\mathrm{ArcTan}\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+3850i\,\mathrm{ArcTan}\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-1925\log(5\cosh(c+dx)-4\sinh(c+dx))+1925\log(5\cosh(c+dx)+4\sinh(c+dx))+\frac{2656-192i}{\left((1+2i)\cosh\left(\frac{1}{2}(c+dx)\right)-(2+i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2656+192i}{\left((2+i)\cosh\left(\frac{1}{2}(c+dx)\right)+(1+2i)\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2(-235150+166615\cosh(c+dx)+82530\cosh(2(c+dx))-13995\cosh(3(c+dx))-298563i\sinh(c+dx)+89364i\sinh(2(c+dx))+8397i\sinh(3(c+dx)))}{(-5i+3\sinh(c+dx))^3}}{327680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-3850*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])] + (3850*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 1925*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 1925*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] +
(2656 - 192*I)/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2
+ (2656 + 192*I)/((2 + I)*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^
2 + (2*(-235150 + 166615*Cosh[c + d*x] + 82530*Cosh[2*(c + d*x)] - 13995*Co
```

```
sh[3*(c + d*x)] - (298563*I)*Sinh[c + d*x] + (89364*I)*Sinh[2*(c + d*x)] +
(8397*I)*Sinh[3*(c + d*x)]))/(-5*I + 3*Sinh[c + d*x])^3)/(327680*d)
```

Maple [A]
time = 1.32, size = 168, normalized size = 1.35

method	result
risch	$-\frac{i\left(-86625 i \mathrm{e}^{4 d x+4 c}+10395 \,\mathrm{e}^{5 d x+5 c}+218466 i \mathrm{e}^{2 d x+2 c}-239470 \,\mathrm{e}^{3 d x+3 c}-8397 i+73575 \,\mathrm{e}^{d x+c}\right)}{12288 d \left(3 \,\mathrm{e}^{2 d x+2 c}-3-10 i \mathrm{e}^{d x+c}\right)^{3}}-\frac{385 \ln \left(-3 i+\mathrm{e}^{d x+c}\right)}{32768 d}$
derivativedivides	$\frac{\frac{\frac{1053}{32000}+\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)^{3}}+\frac{-\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}+\frac{8361 i}{256000}}{5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i}-\frac{385 \ln \left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{32768}+\frac{\frac{1053}{32000}-\frac{9}{80}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{\frac{1053}{32000}+\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)^{3}}+\frac{-\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}+\frac{8361 i}{256000}}{5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i}-\frac{385 \ln \left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)}{32768}+\frac{\frac{1053}{32000}-\frac{9}{80}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)\right.}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((1053/32000+99/8000*I)/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^3-(783/128000+375
3/64000*I)/(5*tanh(1/2*d*x+1/2*c)-4-3*I)^2+(-39933/1024000+8361/256000*I)/(
5*tanh(1/2*d*x+1/2*c)-4-3*I)-385/32768*ln(5*tanh(1/2*d*x+1/2*c)-4-3*I)+(105
3/32000-99/8000*I)/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^3+(783/128000-3753/64000*I
)/(5*tanh(1/2*d*x+1/2*c)+4-3*I)^2-(39933/1024000+8361/256000*I)/(5*tanh(1/2
*d*x+1/2*c)+4-3*I)+385/32768*ln(5*tanh(1/2*d*x+1/2*c)+4-3*I))
```

Maxima [A]
time = 0.48, size = 152, normalized size = 1.23

$$-\frac{385 i \arctan\left(\frac{3}{4} e^{(-dx-c)}+\frac{5}{4} i\right)}{16384\, d}-\frac{73575 i\, e^{(-dx-c)}+218466\, e^{(-2\,dx-2\,c)}-239470 i\, e^{(-3\,dx-3\,c)}-86625\, e^{(-4\,dx-4\,c)}+10395 i\, e^{(-5\,dx-5\,c)}-8397}{-12288\, d\left(-270 i\, e^{(-dx-c)}-981\, e^{(-2\,dx-2\,c)}+1540 i\, e^{(-3\,dx-3\,c)}+981\, e^{(-4\,dx-4\,c)}-270 i\, e^{(-5\,dx-5\,c)}-27\, e^{(-6\,dx-6\,c)}+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/16384*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (73575*I*e^(-d*x - c) + 2
18466*e^(-2*d*x - 2*c) - 239470*I*e^(-3*d*x - 3*c) - 86625*e^(-4*d*x - 4*c)
 + 10395*I*e^(-5*d*x - 5*c) - 8397)/(d*(3317760*I*e^(-d*x - c) + 12054528*e
^(-2*d*x - 2*c) - 18923520*I*e^(-3*d*x - 3*c) - 12054528*e^(-4*d*x - 4*c) +
 3317760*I*e^(-5*d*x - 5*c) + 331776*e^(-6*d*x - 6*c) - 331776))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(98) = 196.
time = 0.38, size = 283, normalized size = 2.28

$$\frac{1155\left(27\, e^{(6\,dx+6\,c)}-270 i\, e^{(5\,dx+5\,c)}-981\, e^{(4\,dx+4\,c)}+1540 i\, e^{(3\,dx+3\,c)}+981\, e^{(2\,dx+2\,c)}-270 i\, e^{(dx+c)}-27\right)\log\left(e^{(dx+c)}-\frac{1}{3} i\right)-1155\left(27\, e^{(6\,dx+6\,c)}-270 i\, e^{(5\,dx+5\,c)}-981\, e^{(4\,dx+4\,c)}+1540 i\, e^{(3\,dx+3\,c)}+981\, e^{(2\,dx+2\,c)}-270 i\, e^{(dx+c)}-27\right)\log\left(e^{(dx+c)}-3 i\right)-83160 i\, e^{(5\,dx+5\,c)}-693000\, e^{(4\,dx+4\,c)}+1915760 i\, e^{(3\,dx+3\,c)}+1747728\, e^{(2\,dx+2\,c)}-588600 i\, e^{(dx+c)}-67176}{98304\left(27\, de^{(6\,dx+6\,c)}-270 i\, de^{(5\,dx+5\,c)}-981\, de^{(4\,dx+4\,c)}+1540 i\, de^{(3\,dx+3\,c)}+981\, de^{(2\,dx+2\,c)}-270 i\, de^{(dx+c)}-27\, d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/98304*(1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x + 5*c) - 981*e^(4*d*x +
4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c) - 270*I*e^(d*x + c) - 2
7)*log(e^(d*x + c) - 1/3*I) - 1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x + 5
*c) - 981*e^(4*d*x + 4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c) -
270*I*e^(d*x + c) - 27)*log(e^(d*x + c) - 3*I) - 83160*I*e^(5*d*x + 5*c) -
693000*e^(4*d*x + 4*c) + 1915760*I*e^(3*d*x + 3*c) + 1747728*e^(2*d*x + 2*c
) - 588600*I*e^(d*x + c) - 67176)/(27*d*e^(6*d*x + 6*c) - 270*I*d*e^(5*d*x
+ 5*c) - 981*d*e^(4*d*x + 4*c) + 1540*I*d*e^(3*d*x + 3*c) + 981*d*e^(2*d*x
+ 2*c) - 270*I*d*e^(d*x + c) - 27*d)
```

Sympy [A]
time = 0.30, size = 202, normalized size = 1.63

$$\frac{-10395ie^{5c}e^{5dx} - 86625e^{4c}e^{4dx} + 239470ie^{3c}e^{3dx} + 218466e^{2c}e^{2dx} - 73575ie^{c}e^{dx} - 8397}{331776de^{6c}e^{6dx} - 3317760ide^{5c}e^{5dx} - 12054528de^{4c}e^{4dx} + 18923520ide^{3c}e^{3dx} + 12054528de^{2c}e^{2dx} - 3317760ide^{c}e^{dx} - 331776d} + \frac{-\frac{385\log\left(e^{dx}-3ie^{-c}\right)}{32768} + \frac{385\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{32768}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**4,x)
```

```
[Out] (-10395*I*exp(5*c)*exp(5*d*x) - 86625*exp(4*c)*exp(4*d*x) + 239470*I*exp(3*
c)*exp(3*d*x) + 218466*exp(2*c)*exp(2*d*x) - 73575*I*exp(c)*exp(d*x) - 8397
)/(331776*d*exp(6*c)*exp(6*d*x) - 3317760*I*d*exp(5*c)*exp(5*d*x) - 1205452
8*d*exp(4*c)*exp(4*d*x) + 18923520*I*d*exp(3*c)*exp(3*d*x) + 12054528*d*exp
(2*c)*exp(2*d*x) - 3317760*I*d*exp(c)*exp(d*x) - 331776*d) + (-385*log(exp(
d*x) - 3*I*exp(-c))/32768 + 385*log(exp(d*x) - I*exp(-c)/3)/32768)/d
```

Giac [A]
time = 0.42, size = 109, normalized size = 0.88

$$-\frac{\frac{8\left(10395i\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}-239470i\,e^{(3\,dx+3\,c)}-218466\,e^{(2\,dx+2\,c)}+73575i\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)^{3}} - 1155\,\log\left(3\,e^{(dx+c)}-i\right) + 1155\,\log\left(e^{(dx+c)}-3i\right)}{98304\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(8*(10395*I*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) - 239470*I*e^(
3*d*x + 3*c) - 218466*e^(2*d*x + 2*c) + 73575*I*e^(d*x + c) + 8397)/(3*e^(2
*d*x + 2*c) - 10*I*e^(d*x + c) - 3)^3 - 1155*log(3*e^(d*x + c) - I) + 1155*
log(e^(d*x + c) - 3*I))/d
```

Mupad [B]
time = 2.11, size = 232, normalized size = 1.87

$$\frac{\frac{1925}{36864\,d}+\frac{e^{c+d\,x}\,385i}{12288\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10i}{3}} + \frac{\frac{41}{486\,d}+\frac{e^{c+d\,x}\,365i}{1458\,d}}{\frac{109\,e^{4\,c+4\,d\,x}}{3}-\frac{109\,e^{2\,c+2\,d\,x}}{3}-e^{6\,c+6\,d\,x}+1+e^{c+d\,x}\,10i-\frac{e^{3\,c+3\,d\,x}\,1540i}{27}+e^{5\,c+5\,d\,x}\,10i} - \frac{385\,\ln\left(-\frac{385\,e^{c+d\,x}}{4}+\frac{1155}{4}i\right)}{32768\,d} + \frac{385\,\ln\left(\frac{3465\,e^{c+d\,x}}{4}-\frac{1155}{4}i\right)}{32768\,d} - \frac{\frac{3461}{31104\,d}+\frac{e^{c+d\,x}\,385i}{10368\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20i}{3}-\frac{e^{3\,c+3\,d\,x}\,20i}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*3i + 5)^4,x)
```

```
[Out] ((exp(c + d*x)*385i)/(12288*d) + 1925/(36864*d))/((exp(c + d*x)*10i)/3 - ex
p(2*c + 2*d*x) + 1) + ((exp(c + d*x)*365i)/(1458*d) + 41/(486*d))/(exp(c +
d*x)*10i - (109*exp(2*c + 2*d*x))/3 - (exp(3*c + 3*d*x)*1540i)/27 + (109*ex
p(4*c + 4*d*x))/3 + exp(5*c + 5*d*x)*10i - exp(6*c + 6*d*x) + 1) - (385*log
(1155i/4 - (385*exp(c + d*x))/4))/(32768*d) + (385*log((3465*exp(c + d*x))/
4 - 1155i/4))/(32768*d) - ((exp(c + d*x)*385i)/(10368*d) + 3461/(31104*d))/
((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x)*20i)/3
 + exp(4*c + 4*d*x) + 1)
```

3.96 $\int (a+b\sinh(c+dx))^5\,dx$

Optimal. Leaf size=183

$$\frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x+\frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d}+\frac{7ab^2(22a^2-23b^2)\cosh(c+dx)\sinh(c$$

```
[Out] 1/8*a*(8*a^4-40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4-192*a^2*b^2+16*b^4)*cosh(
d*x+c)/d+7/120*a*b^2*(22*a^2-23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47*a
^2-16*b^2)*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d+9/20*a*b*cosh(d*x+c)*(a+b*sinh
(d*x+c))^3/d+1/5*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^4/d
```

Rubi [A]
time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2735, 2832, 2813}

$$\frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d}+\frac{7ab^2(22a^2-23b^2)\sinh(c+dx)\cosh(c+dx)}{120d}+\frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d}+\frac{1}{8}ax\left(8a^4-40a^2b^2+15b^4\right)+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 - 192*a^2*b^2 + 16*b^4)
*Cosh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 - 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)
/(60*d) + (9*a*b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(20*d) + (b*Cosh[c
+ d*x]*(a + b*Sinh[c + d*x])^4)/(5*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sinh(c+dx))^5\,dx &= \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\frac{1}{5}\int (a+b\sinh(c+dx))^3\left(5a^2-4b^2+\right.\\ &= \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}+\\ &= \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d}+\frac{9ab\cosh(c+dx)(a+b\sin}{20d}\\ &= \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x+\frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d}+\frac{7ab^2(}{}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 138, normalized size = 0.75

$$\frac{300b(8a^4-12a^2b^2+b^4)\cosh(c+dx)+50(8a^2b^3-b^5)\cosh(3(c+dx))+6b^5\cosh(5(c+dx))+15a(4(8a^4-40a^2b^2+15b^4)(c+dx)+40(2a^2b^2-b^4)\sinh(2(c+dx))+5b^4\sinh(4(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^5,x]

[Out] (300*b*(8*a^4 - 12*a^2*b^2 + b^4)*Cosh[c + d*x] + 50*(8*a^2*b^3 - b^5)*Cosh
[3*(c + d*x)] + 6*b^5*Cosh[5*(c + d*x)] + 15*a*(4*(8*a^4 - 40*a^2*b^2 + 15*
b^4)*(c + d*x) + 40*(2*a^2*b^2 - b^4)*Sinh[2*(c + d*x)] + 5*b^4*Sinh[4*(c +
 d*x)]))/(480*d)
```

Maple [A]
time = 1.04, size = 145, normalized size = 0.79

method	result
default	$a^5x+\frac{\left(-\frac{5}{16}b^5+\frac{5}{2}a^2b^3\right)\cosh(3dx+3c)}{3d}+\frac{\left(-\frac{5}{2}a\,b^4+5a^3b^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(\frac{5}{8}b^5-\frac{15}{2}a^2b^3+5a^4b\right)\cosh(dx+c)}{d}+\frac{15xa\,b^4}{8}-5$
risch	$a^5x-5x\,a^3b^2+\frac{15xa\,b^4}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5a\,b^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}-\frac{5b^5e^{3dx+3c}}{96d}+\frac{5b^2a^3e^{2dx+2c}}{4d}-\frac{5b^4a\,e^{2dx+2}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^5,x,method=_RETURNVERBOSE)

[Out] a^5*x+1/3*(-5/16*b^5+5/2*a^2*b^3)/d*cosh(3*d*x+3*c)+1/2*(-5/2*a*b^4+5*a^3*b
^2)*sinh(2*d*x+2*c)/d+(5/8*b^5-15/2*a^2*b^3+5*a^4*b)/d*cosh(d*x+c)+15/8*x*a
*b^4-5*x*a^3*b^2+1/80*b^5/d*cosh(5*d*x+5*c)+5/32*a*b^4*sinh(4*d*x+4*c)/d
```

Maxima [A]
time = 0.29, size = 272, normalized size = 1.49

$$\frac{5}{64}ab^4\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{5}{4}a^3b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^5x+\frac{1}{480}b^5\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{5}{12}a^2b^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{5a^4b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 5/4*a^3*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 5*a^4*b*cosh(d*x + c)/d
```

Fricas [A]
time = 0.47, size = 223, normalized size = 1.22

$$\frac{3b^5\cosh(dx+c)^5+15b^5\cosh(dx+c)\sinh(dx+c)^4+150ab^4\cosh(dx+c)\sinh(dx+c)^3+25(8a^2b^3-b^5)\cosh(dx+c)^3+30(8a^5-40a^3b^2+15ab^4)dx+15(2b^5\cosh(dx+c)^3+5(8a^2b^3-b^5)\cosh(dx+c))\sinh(dx+c)^2+150(8a^4b-12a^2b^3+b^5)\cosh(dx+c)+150(ab^4\cosh(dx+c)^3+4(2a^3b^2-ab^4)\cosh(dx+c))\sinh(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*cosh(d*x + c)^5 + 15*b^5*cosh(d*x + c)*sinh(d*x + c)^4 + 150*a
*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 25*(8*a^2*b^3 - b^5)*cosh(d*x + c)^3 +
 30*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*d*x + 15*(2*b^5*cosh(d*x + c)^3 + 5*(8*
a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + 150*(8*a^4*b - 12*a^2*b^3 +
 b^5)*cosh(d*x + c) + 150*(a*b^4*cosh(d*x + c)^3 + 4*(2*a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.32, size = 314, normalized size = 1.72

$$\begin{cases} a^5x+\frac{5a^4b\cosh(c+dx)}{d}+5a^3b^2x\sinh^2(c+dx)-5a^3b^2x\cosh^2(c+dx)+\frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d}+\frac{10a^2b^3\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{20a^2b^3\cosh^3(c+dx)}{3d}+\frac{15ab^4x\sinh^4(c+dx)}{8}-\frac{15ab^4x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{15ab^4x\cosh^4(c+dx)}{8}+\frac{25ab^4\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{15ab^4\sinh(c+dx)\cosh^3(c+dx)}{8d}+\frac{b^5\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b^5\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b^5\cosh^5(c+dx)}{15d} & \text{for } d\neq 0 \\ x(a+b\sinh(c))^5 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*cosh(c + d*x)/d + 5*a**3*b**2*x*sinh(c + d*x)*
*2 - 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 10*a**2*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 20*a**2*b**3*cosh(c
+ d*x)**3/(3*d) + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 + 25*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) - 15*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + b**5*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**5*sinh(c + d*x)**2*cosh
```

```
(c + d*x)**3/(3*d) + 8*b**5*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*s
inh(c))**5, True))
```

Giac [A]
time = 0.43, size = 269, normalized size = 1.47

$$\frac{b^5 e^{(5\,dx+5\,c)}}{160\,d}+\frac{5\,ab^4 e^{(4\,dx+4\,c)}}{64\,d}-\frac{5\,ab^4 e^{(-4\,dx-4\,c)}}{64\,d}+\frac{b^5 e^{(-5\,dx-5\,c)}}{160\,d}+\frac{1}{8}\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)x+\frac{5\left(8\,a^2b^3-b^5\right)e^{(3\,dx+3\,c)}}{96\,d}+\frac{5\left(2\,a^3b^2-ab^4\right)e^{(2\,dx+2\,c)}}{8\,d}+\frac{5\left(8\,a^4b-12\,a^2b^3+b^5\right)e^{(dx+c)}}{16\,d}+\frac{5\left(8\,a^4b-12\,a^2b^3+b^5\right)e^{(-dx-c)}}{16\,d}-\frac{5\left(2\,a^3b^2-ab^4\right)e^{(-2\,dx-2\,c)}}{8\,d}+\frac{5\left(8\,a^2b^3-b^5\right)e^{(-3\,dx-3\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^(
-4*d*x - 4*c)/d + 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 - 40*a^3*b^2 +
15*a*b^4)*x + 5/96*(8*a^2*b^3 - b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2 - a
*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(d*x + c)/d +
 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 - a*b^4)
*e^(-2*d*x - 2*c)/d + 5/96*(8*a^2*b^3 - b^5)*e^(-3*d*x - 3*c)/d
```

Mupad [B]
time = 0.60, size = 160, normalized size = 0.87

$$\frac{75\,b^5\cosh(c+d\,x)-\frac{25\,b^5\cosh(3\,c+3\,d\,x)}{2}+\frac{3\,b^5\cosh(5\,c+5\,d\,x)}{2}-900\,a^2\,b^3\cosh(c+d\,x)-150\,a\,b^4\sinh(2\,c+2\,d\,x)+\frac{75\,a\,b^4\sinh(4\,c+4\,d\,x)}{4}+100\,a^2\,b^3\cosh(3\,c+3\,d\,x)+300\,a^3\,b^2\sinh(2\,c+2\,d\,x)+600\,a^4\,b\cosh(c+d\,x)+120\,a^5\,d\,x+225\,a\,b^4\,d\,x-600\,a^3\,b^2\,d\,x}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^5,x)
```

```
[Out] (75*b^5*cosh(c + d*x) - (25*b^5*cosh(3*c + 3*d*x))/2 + (3*b^5*cosh(5*c + 5*
d*x))/2 - 900*a^2*b^3*cosh(c + d*x) - 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b
^4*sinh(4*c + 4*d*x))/4 + 100*a^2*b^3*cosh(3*c + 3*d*x) + 300*a^3*b^2*sinh(
2*c + 2*d*x) + 600*a^4*b*cosh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x - 600*
a^3*b^2*d*x)/(120*d)
```

3.97 $\int (a+b\sinh(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x+\frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d}+\frac{b^2(26a^2-9b^2)\cosh(c+dx)\sinh(c+dx)}{24d}+\frac{7ab\cosh}{}$$

```
[Out] 1/8*(8*a^4-24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2-16*b^2)*cosh(d*x+c)/d+1/24*b
^2*(26*a^2-9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*cosh(d*x+c)*(a+b*sinh(
d*x+c))^2/d+1/4*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2735, 2832, 2813}

$$\frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d}+\frac{b^2(26a^2-9b^2)\sinh(c+dx)\cosh(c+dx)}{24d}+\frac{1}{8}x\left(8a^4-24a^2b^2+3b^4\right)+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/
(6*d) + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(12*d) + (b*Cosh[c + d*x]*(a + b*Sin
h[c + d*x])^3)/(4*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
```

```
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+\frac{1}{4}\int (a+b\sinh(c+dx))^2\left(4a^2-3b^2+\right.$$
$$=\frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}+$$
$$=\frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x+\frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d}+\frac{b^2(26a^2-9b^2)\cos}{}$$

Mathematica [A]
time = 0.26, size = 108, normalized size = 0.79

$$\frac{96ab(4a^2-3b^2)\cosh(c+dx)+32ab^3\cosh(3(c+dx))+3(4(8a^4-24a^2b^2+3b^4)(c+dx)+8(6a^2b^2-b^4)\sinh(2(c+dx))+b^4\sinh(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] (96*a*b*(4*a^2 - 3*b^2)*Cosh[c + d*x] + 32*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(
8*a^4 - 24*a^2*b^2 + 3*b^4)*(c + d*x) + 8*(6*a^2*b^2 - b^4)*Sinh[2*(c + d*x
)] + b^4*Sinh[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.86, size = 108, normalized size = 0.79

method	result
default	$x\,a^4+\frac{\left(-\frac{1}{2}b^4+3a^2b^2\right)\sinh(2dx+2c)}{2d}+\frac{\left(4a^3b-3a\,b^3\right)\cosh(dx+c)}{d}+\frac{a\,b^3\cosh(3dx+3c)}{3d}+\frac{3x\,b^4}{8}-3x\,a^2b^2+\frac{b^4\sinh(4dx}{32d}$
risch	$x\,a^4+\frac{3x\,b^4}{8}-3x\,a^2b^2+\frac{b^4\mathrm{e}^{4dx+4c}}{64d}+\frac{a\,b^3\mathrm{e}^{3dx+3c}}{6d}+\frac{3b^2\mathrm{e}^{2dx+2c}a^2}{4d}-\frac{b^4\mathrm{e}^{2dx+2c}}{8d}+\frac{2a^3b\,\mathrm{e}^{dx+c}}{d}-\frac{3a\,b^3\mathrm{e}^{dx+c}}{2d}+2a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^4+1/2*(-1/2*b^4+3*a^2*b^2)*sinh(2*d*x+2*c)/d+(4*a^3*b-3*a*b^3)/d*cosh(d
*x+c)+1/3*a*b^3/d*cosh(3*d*x+3*c)+3/8*x*b^4-3*x*a^2*b^2+1/32*b^4*sinh(4*d*x
+4*c)/d
```

Maxima [A]
time = 0.34, size = 182, normalized size = 1.33

$$\frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{3}{4}a^2b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^4x+\frac{1}{6}ab^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{4a^3b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/4*a^2*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 4*a^3*b*cosh(d*x + c)/d
```

Fricas [A]
time = 0.34, size = 146, normalized size = 1.07

$$\frac{3 b^4 \cosh (dx+c) \sinh (dx+c)^3 + 8 a b^3 \cosh (dx+c)^3 + 24 a b^3 \cosh (dx+c) \sinh (dx+c)^2 + 3 (8 a^4 - 24 a^2 b^2 + 3 b^4) dx + 24 (4 a^3 b - 3 a b^3) \cosh (dx+c) + 3 (b^4 \cosh (dx+c)^3 + 4 (6 a^2 b^2 - b^4) \cosh (dx+c)) \sinh (dx+c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^3 + 24*a*
b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*d*x + 24
*(4*a^3*b - 3*a*b^3)*cosh(d*x + c) + 3*(b^4*cosh(d*x + c)^3 + 4*(6*a^2*b^2
- b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A]
time = 0.20, size = 240, normalized size = 1.75

$$\begin{cases} a^4 x + \frac{4 a^3 b \cosh (c+dx)}{d} + 3 a^2 b^2 x \sinh^2 (c+dx) - 3 a^2 b^2 x \cosh^2 (c+dx) + \frac{3 a^2 b^2 \sinh (c+dx) \cosh (c+dx)}{d} + \frac{4 a b^3 \sinh^2 (c+dx) \cosh (c+dx)}{d} - \frac{8 a b^3 \cosh^3 (c+dx)}{3d} + \frac{3 b^4 x \sinh^4 (c+dx)}{8} - \frac{3 b^4 x \sinh^2 (c+dx) \cosh^2 (c+dx)}{4} + \frac{3 b^4 x \cosh^4 (c+dx)}{8} + \frac{5 b^4 \sinh^3 (c+dx) \cosh (c+dx)}{8d} - \frac{3 b^4 \sinh (c+dx) \cosh^3 (c+dx)}{8d} & \text{for } d \neq 0 \\ x (a + b \sinh (c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*cosh(c + d*x)/d + 3*a**2*b**2*x*sinh(c + d*x)*
*2 - 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 4*a*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 8*a*b**3*cosh(c + d*x)**
3/(3*d) + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 + 5*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) - 3*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*sinh(c))**4, True))
```

Giac [A]
time = 0.45, size = 200, normalized size = 1.46

$$\frac{b^4 e^{(4 dx+4 c)}}{64 d} + \frac{a b^3 e^{(3 dx+3 c)}}{6 d} + \frac{a b^3 e^{(-3 dx-3 c)}}{6 d} - \frac{b^4 e^{(-4 dx-4 c)}}{64 d} + \frac{1}{8} (8 a^4 - 24 a^2 b^2 + 3 b^4) x + \frac{(6 a^2 b^2 - b^4) e^{(2 dx+2 c)}}{8 d} + \frac{(4 a^3 b - 3 a b^3) e^{(dx+c)}}{2 d} + \frac{(4 a^3 b - 3 a b^3) e^{(-dx-c)}}{2 d} - \frac{(6 a^2 b^2 - b^4) e^{(-2 dx-2 c)}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d + 1/6*a*b^3*e^(-3*
d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 - 24*a^2*b^2 + 3*b^
4)*x + 1/8*(6*a^2*b^2 - b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^
(d*x + c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 - b^4
)*e^(-2*d*x - 2*c)/d
```

Mupad [B]
time = 0.34, size = 114, normalized size = 0.83

$$\frac{\frac{3\,b^4\,\sinh(4\,c+4\,d\,x)}{4}-6\,b^4\,\sinh(2\,c+2\,d\,x)+8\,a\,b^3\,\cosh(3\,c+3\,d\,x)+36\,a^2\,b^2\,\sinh(2\,c+2\,d\,x)-72\,a\,b^3\,\cosh(c+d\,x)+96\,a^3\,b\,\cosh(c+d\,x)+24\,a^4\,d\,x+9\,b^4\,d\,x-72\,a^2\,b^2\,d\,x}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^4,x)
```

```
[Out] ((3*b^4*sinh(4*c + 4*d*x))/4 - 6*b^4*sinh(2*c + 2*d*x) + 8*a*b^3*cosh(3*c +
 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) - 72*a*b^3*cosh(c + d*x) + 96*a^3*b*
cosh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x - 72*a^2*b^2*d*x)/(24*d)
```

3.98 $\int (a+b\sinh (c+dx))^3 \, dx$

Optimal. Leaf size=92

$$\frac{1}{2}a\left(2a^2-3b^2\right)x+\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+$$

```
[Out] 1/2*a*(2*a^2-3*b^2)*x+2/3*b*(4*a^2-b^2)*cosh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+1/3*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2735, 2813}

$$\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*x)/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*Cosh[c + d*x]*(a + b*Sinh[c + d*x
])^2)/(3*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^3\,dx=\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}+\frac{1}{3}\int (a+b\sinh(c+dx))\left(3a^2-2b^2+5a\right.$$

$$=\frac{1}{2}a\left(2a^2-3b^2\right)x+\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}$$

Mathematica [A]
time = 0.13, size = 71, normalized size = 0.77

$$\frac{6a(2a^2-3b^2)(c+dx)-9b(-4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))+9ab^2\sinh(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^3,x]
[Out] (6*a*(2*a^2 - 3*b^2)*(c + d*x) - 9*b*(-4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cos
h[3*(c + d*x)] + 9*a*b^2*Sinh[2*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.77, size = 71, normalized size = 0.77

method	result
default	$a^3x+\frac{\left(-\frac{3}{4}b^3+3a^2b\right)\cosh(dx+c)}{d}-\frac{3a\,b^2x}{2}+\frac{b^3\cosh(3dx+3c)}{12d}+\frac{3a\,b^2\sinh(2dx+2c)}{4d}$
risch	$a^3x-\frac{3a\,b^2x}{2}+\frac{b^3\mathrm{e}^{3dx+3c}}{24d}+\frac{3a\,b^2\mathrm{e}^{2dx+2c}}{8d}+\frac{3b\,\mathrm{e}^{dx+c}a^2}{2d}-\frac{3b^3\mathrm{e}^{dx+c}}{8d}+\frac{3b\,\mathrm{e}^{-dx-c}a^2}{2d}-\frac{3b^3\mathrm{e}^{-dx-c}}{8d}-\frac{3a\,b^2\mathrm{e}^{-2dx-2c}}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3*x+(-3/4*b^3+3*a^2*b)/d*cosh(d*x+c)-3/2*a*b^2*x+1/12*b^3/d*cosh(3*d*x+3*
c)+3/4*a*b^2*sinh(2*d*x+2*c)/d
```

Maxima [A]
time = 0.28, size = 115, normalized size = 1.25

$$-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^3x+\frac{1}{24}b^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{3a^2b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="maxima")
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c
)/d) + 3*a^2*b*cosh(d*x + c)/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 0.99

$$\frac{b^3\cosh(dx+c)^3+3b^3\cosh(dx+c)\sinh(dx+c)^2+18ab^2\cosh(dx+c)\sinh(dx+c)+6(2a^3-3ab^2)dx+9(4a^2b-b^3)\cosh(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

[Out] 1/12*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 18*a*b^2*cosh(d*x + c)*sinh(d*x + c) + 6*(2*a^3 - 3*a*b^2)*d*x + 9*(4*a^2*b - b^3)*cosh(d*x + c))/d

Sympy [A]
time = 0.13, size = 128, normalized size = 1.39

$$\begin{cases} a^3x + \frac{3a^2b\cosh(c+dx)}{d} + \frac{3ab^2x\sinh^2(c+dx)}{2} - \frac{3ab^2x\cosh^2(c+dx)}{2} + \frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{b^3\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2b^3\cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a+b\sinh(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c))**3,x)

[Out] Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)/d + 3*a*b**2*x*sinh(c + d*x)**2/2 - 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b**3*cosh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c))**3, True))

Giac [A]
time = 0.43, size = 135, normalized size = 1.47

$$\frac{b^3e^{(3\,dx+3\,c)}}{24\,d} + \frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d} - \frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d} + \frac{b^3e^{(-3\,dx-3\,c)}}{24\,d} + \frac{1}{2}\left(2\,a^3 - 3\,ab^2\right)x + \frac{3\,(4\,a^2b - b^3)e^{(dx+c)}}{8\,d} + \frac{3\,(4\,a^2b - b^3)e^{(-dx-c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2*d*x - 2*c)/d + 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 - 3*a*b^2)*x + 3/8*(4*a^2*b - b^3)*e^(d*x + c)/d + 3/8*(4*a^2*b - b^3)*e^(-d*x - c)/d

Mupad [B]
time = 0.50, size = 75, normalized size = 0.82

$$\frac{6\,d\,x\,a^3 + 18\,a^2\,b\cosh(c+d\,x) + 9\sinh(c+d\,x)\;a\,b^2\cosh(c+d\,x) - 9\,d\,x\,a\,b^2 + 2\,b^3\cosh(c+d\,x)^3 - 6\,b^3\cosh(c+d\,x)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sinh(c + d*x))^3,x)

[Out] (2*b^3*cosh(c + d*x)^3 - 6*b^3*cosh(c + d*x) + 18*a^2*b*cosh(c + d*x) + 6*a^3*d*x + 9*a*b^2*cosh(c + d*x)*sinh(c + d*x) - 9*a*b^2*d*x)/(6*d)

3.99 $\int (a+b\sinh(c+dx))^2\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}\left(2a^2-b^2\right)x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2-b^2)*x+2*a*b*cosh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2723}

$$\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{1}{2}\left(2a^2-b^2\right)x+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A]
time = 0.07, size = 48, normalized size = 0.92

$$\frac{2(2a^2-b^2)\,(c+dx)+8ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 - b^2)*(c + d*x) + 8*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

Maple [A]
time = 0.63, size = 51, normalized size = 0.98

method	result	size
derivativedivides	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$	51
default	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$	51
risch	$a^2x-\frac{x\,b^2}{2}+\frac{b^2\mathrm{e}^{2dx+2c}}{8d}+\frac{ab\,\mathrm{e}^{dx+c}}{d}+\frac{ab\,\mathrm{e}^{-dx-c}}{d}-\frac{b^2\mathrm{e}^{-2dx-2c}}{8d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*cosh(d*x+c)+a^2*
(d*x+c))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.06

$$-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x+\frac{2\,ab\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*cos
h(d*x + c)/d
```

Fricas [A]
time = 0.42, size = 46, normalized size = 0.88

$$\frac{b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(2\,a^2-b^2\right)dx+4\,ab\cosh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)*sinh(d*x + c) + (2*a^2 - b^2)*d*x + 4*a*b*cosh(d*x +
 c))/d
```

Sympy [A]
time = 0.09, size = 78, normalized size = 1.50

$$\begin{cases} a^2x+\frac{2ab\cosh\left(c+dx\right)}{d}+\frac{b^2x\sinh^2\left(c+dx\right)}{2}-\frac{b^2x\cosh^2\left(c+dx\right)}{2}+\frac{b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x\left(a+b\sinh\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*cosh(c + d*x)/d + b**2*x*sinh(c + d*x)**2/2 - b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*sinh(c))**2, True))
```

Giac [A]
time = 0.42, size = 76, normalized size = 1.46

$$\frac{1}{2}\left(2\,a^{2}-b^{2}\right)x+\frac{b^{2}e^{(2\,dx+2\,c)}}{8\,d}+\frac{abe^{(dx+c)}}{d}+\frac{abe^{(-dx-c)}}{d}-\frac{b^{2}e^{(-2\,dx-2\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 - b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d + a*b*e
^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

Mupad [B]
time = 0.49, size = 41, normalized size = 0.79

$$a^{2}\,x-\frac{b^{2}\,x}{2}+\frac{\frac{\sinh(2\,c+2\,d\,x)\,b^{2}}{4}+2\,a\cosh(c+d\,x)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x))^2,x)
```

```
[Out] a^2*x - (b^2*x)/2 + ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*cosh(c + d*x))/d
```

3.100 $\int (a + b\sinh(c + dx))\,dx$

Optimal. Leaf size=15

$$ax + \frac{b\cosh(c + dx)}{d}$$

```
[Out] a*x+b*cosh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2718\}$

$$ax + \frac{b\cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x],x]
[Out] a*x + (b*Cosh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\sinh(c + dx))\,dx &= ax + b\int \sinh(c + dx)\,dx\\ &= ax + \frac{b\cosh(c + dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.73

$$ax + \frac{b\cosh(c)\cosh(dx)}{d} + \frac{b\sinh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x],x]
[Out] a*x + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A]
time = 0.26, size = 16, normalized size = 1.07

method	result	size
default	$ax + \frac{b\cosh(dx+c)}{d}$	16
derivativedivides	$\frac{(dx+c)a+b\cosh(dx+c)}{d}$	21
risch	$ax + \frac{b\,e^{dx+c}}{2d} + \frac{b\,e^{-dx-c}}{2d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*cosh(d*x+c)/d
```

Maxima [A]
time = 0.29, size = 15, normalized size = 1.00

$$ax + \frac{b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*cosh(d*x + c)/d
```

Fricas [A]
time = 0.39, size = 17, normalized size = 1.13

$$\frac{adx + b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + b*cosh(d*x + c))/d
```

Sympy [A]
time = 0.04, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\cosh(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\sinh(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.43, size = 31, normalized size = 2.07

$$ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
```

Mupad [**B**]
time = 0.43, size = 15, normalized size = 1.00

$$a\,x + \frac{b\cosh(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x),x)
```

```
[Out] a*x + (b*cosh(c + d*x))/d
```

3.101 $\int \frac{1}{a+b\sinh(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

```
[Out] -2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2739, 632, 210}

$$-\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{(2i)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= \frac{(4i)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 1.18

$$\frac{2\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-1),x]
[Out] (2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d)
```

Maple [A]
time = 0.66, size = 43, normalized size = 0.98

method	result	size
derivativedivides	$\frac{2\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(41) = 82$.
time = 0.44, size = 162, normalized size = 3.68

$$\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}\,(b\cosh(dx+c)+b\sinh(dx+c)+a)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^
2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*
cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))/(sqrt(a
^2 + b^2)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(39) = 78$.
time = 4.93, size = 196, normalized size = 4.45

$$\begin{cases} \frac{\tilde{\infty}x}{\sinh(c)} & \text{for}\ a=0\wedge b=0\wedge d=0 \\ \frac{x}{a+b\sinh(c)} & \text{for}\ d=0 \\ \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for}\ a=0 \\ \frac{2\sqrt{-b^2}}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-bd\sqrt{-b^2}} & \text{for}\ a=-\sqrt{-b^2} \\ -\frac{2\sqrt{-b^2}}{b^2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+bd\sqrt{-b^2}} & \text{for}\ a=\sqrt{-b^2} \\ -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/(a + b*sinh(c
)), Eq(d, 0)), (log(tanh(c/2 + d*x/2))/(b*d), Eq(a, 0)), (2*sqrt(-b**2)/(b*
*2*d*tanh(c/2 + d*x/2) - b*d*sqrt(-b**2)), Eq(a, -sqrt(-b**2))), (-2*sqrt(-
b**2)/(b**2*d*tanh(c/2 + d*x/2) + b*d*sqrt(-b**2)), Eq(a, sqrt(-b**2))), (-
log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(d*sqrt(a**2 + b**2)) +
log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(d*sqrt(a**2 + b**2)), T
rue))
```

Giac [A]
time = 0.43, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B]
time = 0.76, size = 55, normalized size = 1.25

$$\frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{-a^2\,d^2-b^2\,d^2}}\right)}{\sqrt{-a^2\,d^2-b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)),x)
```

```
[Out] (2*atan((a*d + b*d*exp(d*x)*exp(c))/(- a^2*d^2 - b^2*d^2)^(1/2)))/(- a^2*d^
2 - b^2*d^2)^(1/2)
```

3.102 $\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}-\frac{b\cosh(c+dx)}{\left(a^2+b^2\right)d(a+b\sinh(c+dx))}$$

```
[Out] -2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))
```

Rubi [A]

time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 12, 2739, 632, 210}

$$-\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b\cosh(c+dx)}{d\left(a^2+b^2\right)(a+b\sinh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2
)*d) - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx &= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}+\frac{\int \frac{a}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}-\frac{(2ia)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+i\right.\right.}{(a^2+b^2)\,d}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}+\frac{(4ia)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\right.}{(a^2+b^2)\,d}\\
&= -\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 85, normalized size = 1.08

$$
-\frac{\frac{2a\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{b\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] -(((2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3
/2) + (b*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])))/d)
```

Maple [A]
time = 1.13, size = 118, normalized size = 1.49

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{2a\,\mathrm{e}^{dx+c}-2b}{d(a^2+b^2)\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)}+\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right.}{(a^2+b^2)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tanh(1/2*d*x+1/2*c)-b/(a^2+b^2))/(a*tanh(1/2*d*x+
1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.50, size = 138, normalized size = 1.75

$$\frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{2\left(ae^{(-dx-c)}+b\right)}{(a^2b+b^3+2\,(a^3+ab^2)e^{(-dx-c)}-(a^2b+b^3)e^{(-2\,dx-2\,c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) + b)/((a^2*b + b^3 + 2*
(a^3 + a*b^2)*e^(-d*x - c) - (a^2*b + b^3)*e^(-2*d*x - 2*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(76) = 152.
time = 0.37, size = 423, normalized size = 5.35

$$-\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)-ab+2\left(ab\cosh\left(dx+c\right)+a^2\right)\sinh\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\sqrt{a^2+b^2}\left(b\cosh(dx+c)+b\sinh(dx+c)+a\right)}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\left(b\cosh(dx+c)+a\right)\sinh(dx+c)-b}\right)-2\left(a^3+ab^2\right)\cosh\left(dx+c\right)-2\left(a^3+ab^2\right)\sinh\left(dx+c\right)}{(a^4b+2\,a^2b^3+b^5)d\cosh\left(dx+c\right)^2+(a^4b+2\,a^2b^3+b^5)d\sinh\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)d\cosh\left(dx+c\right)-(a^4b+2\,a^2b^3+b^5)d+2\left((a^4b+2\,a^2b^3+b^5)d\cosh\left(dx+c\right)+(a^5+2\,a^3b^2+ab^4)d\right)\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) - a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 + b^2
```

```
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(
a^3 + a*b^2)*cosh(d*x + c) - 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) - (a^4*b + 2*a^2*b^3 + b^5)*
d + 2*((a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x))**(-2), x)
```

Giac [A]
time = 0.43, size = 119, normalized size = 1.51

$$\frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(ae^{(dx+c)}-b)}{(a^2+b^2)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) - b)/((a^2
+ b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)))/d
```

Mupad [B]
time = 0.88, size = 200, normalized size = 2.53

$$\frac{a\,\ln\left(\frac{2\,a\,(b-a\,\mathrm{e}^{c+d\,x})}{b\,(a^2+b^2)^{3/2}}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2+b^2)}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2+b^2)}-\frac{2\,a\,(b-a\,\mathrm{e}^{c+d\,x})}{b\,(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\frac{2\,b^2}{d\,(a^2\,b+b^3)}-\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^{c+d\,x}-b+b\,\mathrm{e}^{2\,c+2\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^2,x)
```

```
[Out] (a*log((2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2)) - (2*a*exp(c + d*x)
)/(b*(a^2 + b^2))))/(d*(a^2 + b^2)^(3/2)) - (a*log(- (2*a*exp(c + d*x))/(b*
(a^2 + b^2)) - (2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2))))/(d*(a^2 +
 b^2)^(3/2)) - ((2*b^2)/(d*(a^2*b + b^3)) - (2*a*b*exp(c + d*x))/(d*(a^2*b
+ b^3)))/(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x))
```

3.103 $\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d}-\frac{b\cosh(c+dx)}{2\,(a^2+b^2)\,d(a+b\sinh(c+dx))^2}-\frac{3ab\cosh(c+dx)}{2\,(a^2+b^2)^2\,d(a+b\sinh(c+dx))}$$

```
[Out] -(2*a^2-b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(
5/2)/d-1/2*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^2-3/2*a*b*cosh(d*x+c
)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{5/2}}-\frac{3ab\cosh(c+dx)}{2d\,(a^2+b^2)^2\,(a+b\sinh(c+dx))}-\frac{b\cosh(c+dx)}{2d\,(a^2+b^2)\,(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) - (b*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])
^2) - (3*a*b*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx &= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\frac{\int \frac{-2a+b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{2\left(a^2+b^2\right)}\\ &= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))}+\frac{\int\\ &= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))}+\frac{(2\\ &= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))}-\frac{(i(\\ &= -\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\frac{3ab\cosh(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\sinh(c+dx))}+\frac{(2\\ &= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}-\frac{b\cosh(c+dx)}{2\left(a^2+b^2\right)d(a+b\sinh(c+dx))^2}-\end{aligned}$$

Mathematica [A]

time = 0.20, size = 117, normalized size = 0.92

$$\frac{\frac{2(2a^2-b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{b\cosh(c+dx)(4a^2+b^2+3ab\sinh(c+dx))}{(a+b\sinh(c+dx))^2}}{2\left(a^2+b^2\right)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] - (b*Cosh[c + d*x]*(4*a^2 + b^2 + 3*a*b*Sinh[c + d*x]))/(a + b*
Sinh[c + d*x])^2)/(2*(a^2 + b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(118) = 236$.
time = 1.18, size = 280, normalized size = 2.20

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
risch	$\frac{2a^2b\,e^{3dx+3c}-b^3e^{3dx+3c}+6a^3e^{2dx+2c}-3ab^2e^{2dx+2c}-10a^2b\,e^{dx+c}-e^{dx+c}b^3+3ab^2}{d(a^2+b^2)^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^2}+\frac{\ln\left(e^{dx+c}+\frac{(a^2+b^2)^{\frac{5}{2}}a-a^6-3a^4b}{b(a^2+b^2)}\right.}{(a^2+b^2)^{\frac{5}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tanh(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/
2*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(120) = 240$.
time = 0.50, size = 315, normalized size = 2.48

$$\frac{(2a^2-b^2)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}\,d}-\frac{3ab^2+(10a^2b+b^3)e^{(-dx-c)}+3(2a^3-ab^2)e^{(-2dx-2c)}-(2a^2b-b^3)e^{(-3dx-3c)}}{(a^4b^2+2a^2b^4+b^6+4(a^5b+2a^3b^3+ab^5)e^{(-dx-c)}+2(2a^6+3a^4b^2-b^6)e^{(-2dx-2c)}-4(a^5b+2a^3b^3+ab^5)e^{(-3dx-3c)}+(a^4b^2+2a^2b^4+b^6)e^{(-4dx-4c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c
) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d) - (3*
a*b^2 + (10*a^2*b + b^3)*e^(-d*x - c) + 3*(2*a^3 - a*b^2)*e^(-2*d*x - 2*c)
- (2*a^2*b - b^3)*e^(-3*d*x - 3*c))/((a^4*b^2 + 2*a^2*b^4 + b^6 + 4*(a^5*b
+ 2*a^3*b^3 + a*b^5)*e^(-d*x - c) + 2*(2*a^6 + 3*a^4*b^2 - b^6)*e^(-2*d*x -
 2*c) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*d*x - 3*c) + (a^4*b^2 + 2*a^2*b
^4 + b^6)*e^(-4*d*x - 4*c))*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(120) = 240$.
time = 0.41, size = 1347, normalized size = 10.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 + 2*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2*
(2*a^4*b + a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*cos
h(d*x + c)^2 + 6*(2*a^5 + a^3*b^2 - a*b^4 + (2*a^4*b + a^2*b^3 - b^5)*cosh(
d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 - b^4)*cosh(d*x + c)^4 + (2*a^2*b^2
 - b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 - b^4 + 4*(2*a^3*b - a*b^3)*cosh(d*x +
c)^3 + 4*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 - 4*a^2*b^2 + b^
4 + 3*(2*a^2*b^2 - b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 4*(2*a^3*b - a*b^3)*cosh(d*x + c) - 4*(2*a^3*b - a*b^3
- (2*a^2*b^2 - b^4)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 -
 (4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(10*a
^4*b + 11*a^2*b^3 + b^5)*cosh(d*x + c) - 2*(10*a^4*b + 11*a^2*b^3 + b^5 - 3
*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*sinh(d*x + c)^4 + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 + 3*a^4
*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c) + (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d)*sinh(d*x + c)^3 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos
h(d*x + c) + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c)^2
 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c) + (2*a^8 + 5*a
```

```
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^4*b
^4 + 3*a^2*b^6 + b^8)*d + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^3 + 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 +
 (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c) - (a^7*b +
 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 231, normalized size = 1.82

$$\frac{\frac{(2a^2-b^2)\log\left(\frac{\left|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(2a^2be^{(3dx+3c)}-b^3e^{(3dx+3c)}+6a^3e^{(2dx+2c)}-3ab^2e^{(2dx+2c)}-10a^2be^{(dx+c)}-b^3e^{(dx+c)}+3ab^2\right)}{(a^4+2a^2b^2+b^4)\left(be^{(2dx+2c)}+2ae^{(dx+c)}-b\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2
*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + 2*(2*a^2*b*e^(3*d*x + 3*c) - b^3*e^(3*d*x + 3*c) + 6*a^3*e^(2*d
*x + 2*c) - 3*a*b^2*e^(2*d*x + 2*c) - 10*a^2*b*e^(d*x + c) - b^3*e^(d*x + c
) + 3*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
- b)^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\sinh(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^3, x)
```

3.104 $\int \frac{1}{(a+b\sinh(c+dx))^4} dx$

Optimal. Leaf size=174

$$-\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}d}-\frac{b\cosh(c+dx)}{3(a^2+b^2)d(a+b\sinh(c+dx))^3}-\frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2d(a+b\sinh(c+$$

```
[Out] -a*(2*a^2-3*b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(7/2)/d-1/3*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^3-5/6*a*b*cosh(d
*x+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))^2-1/6*b*(11*a^2-4*b^2)*cosh(d*x+c)/(a
^2+b^2)^3/d/(a+b*sinh(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$-\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{7/2}}-\frac{b(11a^2-4b^2)\cosh(c+dx)}{6d(a^2+b^2)^3(a+b\sinh(c+dx))}-\frac{5ab\cosh(c+dx)}{6d(a^2+b^2)^2(a+b\sinh(c+dx))^2}-\frac{b\cosh(c+dx)}{3d(a^2+b^2)(a+b\sinh(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d)) - (b*Cosh[c + d*x])/(3*(a^2 + b^2)*d*(a + b*Sinh[c + d
*x])^3) - (5*a*b*Cosh[c + d*x])/(6*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x])^2)
 - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*Sinh[c + d*
x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{\int \frac{-3a+2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx}{3(a^2+b^2)}$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} + \int$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} - 6$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} - 6$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} - 6$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} - 6$$

$$= -\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}\,d} - \frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}$$

Mathematica [A]
time = 0.50, size = 159, normalized size = 0.91

$$\frac{\frac{6a(2a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{b\cosh(c+dx)\left(-18a^4-5a^2b^2-2b^4+3ab(-9a^2+b^2)\sinh(c+dx)+(-11a^2b^2+4b^4)\sinh^2(c+dx)\right)}{(a+b\sinh(c+dx))^3}}{6(a^2+b^2)^3\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2] + (b*Cosh[c + d*x]*(-18*a^4 - 5*a^2*b^2 - 2*b^4 + 3*a*b*(-9
*a^2 + b^2)*Sinh[c + d*x] + (-11*a^2*b^2 + 4*b^4)*Sinh[c + d*x]^2))/(a + b*
Sinh[c + d*x])^3)/(6*(a^2 + b^2)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(163) = 326.
time = 1.20, size = 494, normalized size = 2.84

method	result

derivativedivides	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)(\tan}{3a^3\left(a^6+3a^4b^2+3a^2b^4+\right.}}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}$
default	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\left(\tanh^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)(\tan}{3a^3\left(a^6+3a^4b^2+3a^2b^4+\right.}}{\left(a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}$
risch	$\frac{6a^3b^2e^{5dx+5c}-9a\,b^4e^{5dx+5c}+30a^4b\,e^{4dx+4c}-45a^2b^3e^{4dx+4c}+44a^5e^{3dx+3c}-82a^3b^2e^{3dx+3c}+24a\,b^4e^{3dx+3c}-102a^4b\,e^{2dx}}{3d(a^2+b^2)^3\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
anh(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-
4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^3+1/a^2*
b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b
^6)*tanh(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6))/(a*tanh(1/2*d*x+1/2*c)^2-2*b*tanh(1/2*d*x+1/2*c)-a)^3+a*(2*a^2-3
*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(165) = 330$.
time = 0.51, size = 551, normalized size = 3.17

$$\frac{(2a^2-3b^2)a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}\,d}-\frac{11a^2b^3-4b^5+15(4a^3b^2-ab^4)e^{(-dx-c)}+6(17a^4b-6a^2b^3+2b^5)e^{(-2dx-2c)}+2(22a^5-41a^3b^2+12ab^4)e^{(-3dx-3c)}-15(2a^4b-3a^2b^3)e^{(-4dx-4c)}+3(2a^3b^2-3ab^4)e^{(-5dx-5c)}}{3(a^6b^3+3a^4b^5+3a^2b^7+b^9+6(a^7b^2+3a^5b^4+3a^3b^6+ab^8)e^{(-dx-c)}+3(4a^8b+11a^6b^3+9a^4b^5+a^2b^7-b^9)e^{(-2dx-2c)}+4(2a^9+3a^7b^2-3a^5b^4-7a^3b^6-3ab^8)e^{(-3dx-3c)}-3(4a^8b+11a^6b^3+9a^4b^5+a^2b^7-b^9)e^{(-4dx-4c)}+6(a^7b^2+3a^5b^4+3a^3b^6+ab^8)e^{(-5dx-5c)}-(a^6b^3+3a^4b^5+3a^2b^7+b^9)e^{(-6dx-6c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - 3*b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
 - c) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)*d) - 1/3*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-d*x - c)
+ 6*(17*a^4*b - 6*a^2*b^3 + 2*b^5)*e^(-2*d*x - 2*c) + 2*(22*a^5 - 41*a^3*b^
2 + 12*a*b^4)*e^(-3*d*x - 3*c) - 15*(2*a^4*b - 3*a^2*b^3)*e^(-4*d*x - 4*c)
+ 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*d*x - 5*c))/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-d*x - c) + 3*(4*
a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*d*x - 2*c) + 4*(2*a^9
 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*d*x - 3*c) - 3*(4*a^8
```

```
*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*d*x - 4*c) + 6*(a^7*b^2
+ 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*d*x - 5*c) - (a^6*b^3 + 3*a^4*b^5 +
3*a^2*b^7 + b^9)*e^(-6*d*x - 6*c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2934 vs. $2(165) = 330$.
time = 0.36, size = 2934, normalized size = 16.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 - 30*(2*
a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 - 30*(2*a^6*b - a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^3 - 4*(22*a^7 -
 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6 + 15*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c)^2 + 1
2*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7 - 5*(2*a^5*b^2 - a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 - 15*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
- (22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 + 3*((2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 - 3*a*b^5)*sinh
(d*x + c)^6 - 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 - 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 - 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 - 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 - 3*(8*a^5*b - 14*a^3*b^3 +
 3*a*b^5 - 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^4 - 20*(2*a^4*b^2 - 3*a^2*
b^4)*cosh(d*x + c)^3 - 6*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 -
 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 -
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 - 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 - (8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
```

```
+ a)*sinh(d*x + c) - b)) - 30*(4*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
 - 6*(20*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^2 - 4*(17*a^6*b
+ 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 +
 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 + 4
*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 + 4
*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
+ 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2
+ 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^1
1 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(
5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 +
10*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) +
 (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d)*sinh(d*x + c)^
4 - 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x +
 c)^2 + 4*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*
x + c)^3 + 15*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*
cosh(d*x + c) + (2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^
7 - b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b
^8 - 3*a*b^10)*d*cosh(d*x + c) - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a
^4*b^7 - b^11)*d)*sinh(d*x + c)^2 - (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 -
3*a*b^10)*d*cosh(d*x + c)^2 - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*
b^7 - b^11)*d*cosh(d*x + c) + (a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(165) = 330$.

time = 0.44, size = 357, normalized size = 2.05

$$\frac{\frac{3\left(2a^3-3ab^2\right)\log\left(\frac{\left|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(6a^3b^2e^{(5dx+5c)}-9ab^4e^{(5dx+5c)}+30a^4be^{(4dx+4c)}-45a^2b^3e^{(4dx+4c)}+44a^5e^{(3dx+3c)}-82a^3b^2e^{(3dx+3c)}+24ab^4e^{(3dx+3c)}-102a^4be^{(2dx+2c)}+36a^2b^3e^{(2dx+2c)}-12b^5e^{(2dx+2c)}+60a^3b^2e^{(dx+c)}-15ab^4e^{(dx+c)}-11a^2b^3+4b^5\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(be^{(2dx+2c)}+2ae^{(dx+c)}-b\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(6*a^3*b^2*e^(5*d*x + 5*c) - 9*a*b^4*e^(5*d
*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) - 45*a^2*b^3*e^(4*d*x + 4*c) + 44*a^5*
e^(3*d*x + 3*c) - 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4*e^(3*d*x + 3*c) - 1
02*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) - 12*b^5*e^(2*d*x + 2
*c) + 60*a^3*b^2*e^(d*x + c) - 15*a*b^4*e^(d*x + c) - 11*a^2*b^3 + 4*b^5)/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) -
b)^3))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\sinh\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^4, x)
```

3.105 $\int (a+b\sinh(x))^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2i(23a^2-9b^2)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-$$

```
[Out] 2/5*b*cosh(x)*(a+b*sinh(x))^(3/2)+16/15*a*b*cosh(x)*(a+b*sinh(x))^(1/2)+2/1
5*I*(23*a^2-9*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+
b*sinh(x))/(a-I*b))^(1/2)-16/15*I*a*(a^2+b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{16ia(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{15\sqrt{a+b\sinh(x)}}+\frac{2i(23a^2-9b^2)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*b*Cosh[x]*(a + b*Sinh[x])^(3/2
))/5 + (((2*I)/15)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (((16*I)/15)*a*(
a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}(5a^2-3b^2)+4ab\sinh(x)\right)$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{4}{15}\int \frac{\frac{1}{4}a(15a^2-17}{}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{1}{15}(23a^2-9b^2)\int$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{\left((23a^2-9b^2)\sqrt{a-}\right.}{}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(23a^2-9b^2)\,E\left(\frac{\pi}{4}\right.}{15\sqrt{}}$$

Mathematica [A]
time = 0.31, size = 178, normalized size = 0.99

$$\frac{2(23ia^3+23a^2b-9iab^2-9b^3)\,E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}} - 16ia(a^2+b^2)\,F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}} + b\cosh(x)\left(22a^2-3b^2+3b^2\cosh(2x)+28ab\sinh(x)\right)}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2),x]
[Out] (2*((23*I)*a^3 + 23*a^2*b - (9*I)*a*b^2 - 9*b^3)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (16*I)*a*(a^2 + b
^2)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)] + b*Cosh[x]*(22*a^2 - 3*b^2 + 3*b^2*Cosh[2*x] + 28*a*b*Sinh[x]))
/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 916 vs. 2(203) = 406.
time = 1.16, size = 917, normalized size = 5.12

method	result
default	$\frac{\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^3b}{15}+\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-
a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*sinh(x))/
(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*
EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2-
9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/
2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE
((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^4+3*b^4*sinh(x)
^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)^2+14*a*b^3*sinh(x)
+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 464, normalized size = 2.59

$$-\frac{4\left(\sqrt{2}\,(a^3+33\,ab^2)\cosh(x)^2+2\sqrt{2}\,(a^3+33\,ab^2)\cosh(x)\sinh(x)+\sqrt{2}\,(a^3+33\,ab^2)\sinh(x)^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2+3b^2)}{3b^2},-\frac{8\,(8a^3+9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)+12\left(\sqrt{2}\,(23a^2b-9b^3)\cosh(x)^2+2\sqrt{2}\,(23a^2b-9b^3)\cosh(x)\sinh(x)+\sqrt{2}\,(23a^2b-9b^3)\sinh(x)^2\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4a^2+3b^2)}{3b^2},-\frac{8\,(8a^3+9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4a^2+3b^2)}{3b^2},-\frac{8\,(8a^3+9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)-3\left(3b^3\cosh(x)^4+3b^3\sinh(x)^4+22\,ab^2\cosh(x)^3+22\,ab^2\cosh(x)+2\,(6b^3\cosh(x)+11\,ab^2)\sinh(x)^3-3b^3-4\,(23a^2b-9b^3)\cosh(x)^2+2\,(9b^3\cosh(x)^2+33\,ab^2\cosh(x)-46a^2b+18b^3)\sinh(x)^2+2\,(6b^3\cosh(x)^3+33\,ab^2\cosh(x)^2+11\,ab^2-4\,(23a^2b-9b^3)\cosh(x))\sinh(x)\right)\sqrt{b\sinh(x)+a}}{90\,(b\cosh(x)^2+2b\cosh(x)\sinh(x)+b\sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(a^3 + 33*a*b^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 + 33*a*b^2)*c
osh(x)*sinh(x) + sqrt(2)*(a^3 + 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassPIn
```

```
verse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x
) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(23*a^2*b - 9*b^3)*cosh(x)^2 + 2*sq
rt(2)*(23*a^2*b - 9*b^3)*cosh(x)*sinh(x) + sqrt(2)*(23*a^2*b - 9*b^3)*sinh(
x)^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^
2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x)^4 + 3*
b^3*sinh(x)^4 + 22*a*b^2*cosh(x)^3 + 22*a*b^2*cosh(x) + 2*(6*b^3*cosh(x) +
11*a*b^2)*sinh(x)^3 - 3*b^3 - 4*(23*a^2*b - 9*b^3)*cosh(x)^2 + 2*(9*b^3*cos
h(x)^2 + 33*a*b^2*cosh(x) - 46*a^2*b + 18*b^3)*sinh(x)^2 + 2*(6*b^3*cosh(x)
^3 + 33*a*b^2*cosh(x)^2 + 11*a*b^2 - 4*(23*a^2*b - 9*b^3)*cosh(x))*sinh(x))
*sqrt(b*sinh(x) + a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\sinh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(5/2),x)
```

```
[Out] int((a + b*sinh(x))^(5/2), x)
```

3.106 $\int (a+b\sinh(x))^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/3*b*cosh(x)*(a+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin
(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(
a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(x)}}+\frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8ia\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x
, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (
((2*I)/3)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
b*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{3}\int\frac{\frac{1}{2}(3a^2-b^2)+2ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}(4a)\int\sqrt{a+b\sinh(x)}\,dx+\frac{1}{3}(-a^2-b^2)\int\frac{}{\sqrt{a+}}$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+$$

$$= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)F(}{3}$$

Mathematica [A]
time = 0.26, size = 139, normalized size = 0.93

$$\frac{2b\cosh(x)(a+b\sinh(x))+8a(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*(a + b*Sinh[x]) + 8*a*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((
-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*Ell
ipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*
b)])/(3*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(178) = 356.
time = 0.96, size = 676, normalized size = 4.51

method	result
default	$\frac{\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sin
h(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a^2*b+I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a
))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(
1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(
x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3+3*(-(a+b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2-4*(-(a+b*s
inh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a)
)^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*
a^3-4*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+b^3*sinh(x)+a*b^2)/b/cosh
(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 263, normalized size = 1.75

$$\frac{2\left(\sqrt{2}\left(a^{2}-3b^{2}\right)\cosh\left(x\right)+\sqrt{2}\left(a^{2}-3b^{2}\right)\sinh\left(x\right)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}+3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}+9ab^{2}\right)}{27b^{3}},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)-24\left(\sqrt{2}ab\cosh\left(x\right)+\sqrt{2}ab\sinh\left(x\right)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\left(4a^{2}+3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}+9ab^{2}\right)}{27b^{3}},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^{2}+3b^{2}\right)}{3b^{2}},-\frac{8\left(8a^{3}+9ab^{2}\right)}{27b^{3}},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)+3\left(b^{2}\cosh\left(x\right)^{2}+b^{2}\sinh\left(x\right)^{2}-8ab\cosh\left(x\right)+b^{2}+2\left(b^{2}\cosh\left(x\right)-4ab\right)\sinh\left(x\right)\right)\sqrt{b\sinh\left(x\right)+a}}{9\left(b\cosh\left(x\right)+b\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(sqrt(2)*(a^2 - 3*b^2)*cosh(x) + sqrt(2)*(a^2 - 3*b^2)*sinh(x))*sqrt
(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^
3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 24*(sqrt(2)*a*b*cosh(x) + sqr
t(2)*a*b*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8
*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^2*cosh
(x)^2 + b^2*sinh(x)^2 - 8*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) - 4*a*b)*sinh(
x))*sqrt(b*sinh(x) + a))/(b*cosh(x) + b*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{sinh}\left(x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(3/2),x)
[Out] int((a + b*sinh(x))^(3/2), x)
```

3.107 $\int \sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I
*b))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2734, 2732}

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt
[(a + b*Sinh[x])/(a - I*b)]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{a+b\sinh(x)}\,dx=\frac{\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$
$$=\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [**A**]
time = 0.14, size = 65, normalized size = 1.08

$$\frac{2(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(78) = 156.
time = 1.10, size = 262, normalized size = 4.37

method	result
default	$\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(i\,\operatorname{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\operatorname{EllipticF}\left(\sqrt{}\right.\right.}{b\cosh(x)\sqrt{a}}$

risch	$\sqrt{2}\,\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)\mathrm{e}^{-x}}+$	$\frac{\left(\frac{4a\left(a+\sqrt{a^{2}+b^{2}}\right)\sqrt{\frac{\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}+b^{2}}}{b}\right)b}{a+\sqrt{a^{2}+b^{2}}}}\sqrt{\frac{\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}+b^{2}}}{b}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*a-a*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 173, normalized size = 2.88

$$\frac{2\left(\sqrt{2}\,a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\,\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)-3\,\sqrt{2}\,b^{\frac{3}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\,\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)-3\,\sqrt{b\sinh\left(x\right)+a}\,b\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(
8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*sqrt(2)*
b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^
3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3
, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*sqrt(b*sinh(x) + a)*b)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+b\,\mathrm{sinh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x))^(1/2),x)
```

```
[Out] int((a + b*sinh(x))^(1/2), x)
```

3.108 $\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{2iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh
(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2742, 2740}

$$\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{\sqrt{a+b\sinh(x)}}$$
$$= \frac{2iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

Mathematica [A]
time = 0.14, size = 60, normalized size = 1.00

$$\frac{2iF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [A]
time = 0.70, size = 125, normalized size = 2.08

method	result	size
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh(x)\sqrt{a+b\sinh(x)}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 61, normalized size = 1.02

$$\frac{2\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\,\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b
^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a+b\sinh{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(x))^(1/2), x)
```

3.109 $\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] -2*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2
)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/
2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)/((a+b*sinh(x))/(a-I*b))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2743, 21, 2734, 2732}

$$-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 -
(I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*Sin
h[x])/(a - I*b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx &= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}-\frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}\\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\int\sqrt{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.86

$$\frac{-2b\cosh(x)+2(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x] + 2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b
)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(110) = 220$.
time = 0.98, size = 456, normalized size = 4.85

method	result
default	$\frac{2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\ \sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x)
)*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a^2+(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)
*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sin
h(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^2-b^2*sinh(x)^2-b^2)/(a^2+
b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 407, normalized size = 4.33

$$-\frac{2\left(\left(\sqrt{2}\,ab\cosh(x)^2+\sqrt{2}\,ab\sinh(x)^2+2\sqrt{2}\,a^2\cosh(x)-\sqrt{2}\,ab+2\left(\sqrt{2}\,ab\cosh(x)+\sqrt{2}\,a^2\right)\sinh(x)\right)\sqrt{b}\ \mathrm{weierstrassPInverse}\left(\frac{4(4a^2+3b^2)}{3b^2},-\frac{8(8a^3+9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)-3\left(\sqrt{2}\,b^2\cosh(x)^2+\sqrt{2}\,b^2\sinh(x)^2+2\sqrt{2}\,ab\cosh(x)-\sqrt{2}\,b^2+2\left(\sqrt{2}\,b^2\cosh(x)+\sqrt{2}\,ab\right)\sinh(x)\right)\sqrt{b}\ \mathrm{weierstrassZeta}\left(\frac{4(4a^2+3b^2)}{3b^2},-\frac{8(8a^3+9ab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4(4a^2+3b^2)}{3b^2},-\frac{8(8a^3+9ab^2)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)-6\left(b^2\cosh(x)^2+b^2\sinh(x)^2+ab\cosh(x)+(2b^2\cosh(x)+ab)\sinh(x)\right)\sqrt{b\sinh(x)+a}\right)}{3\left(a^2b^2+b^4-(a^2b^2+b^4)\cosh(x)^2-(a^2b^2+b^4)\sinh(x)^2-2(a^3b+ab^3)\cosh(x)-2(a^3b+ab^3+(a^2b^2+b^4)\cosh(x))\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*a*b*cosh(x)^2 + sqrt(2)*a*b*sinh(x)^2 + 2*sqrt(2)*a^2*cosh(x
) - sqrt(2)*a*b + 2*(sqrt(2)*a*b*cosh(x) + sqrt(2)*a^2)*sinh(x))*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3
*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*(sqrt(2)*b^2*cosh(x)^2 + sqrt(2)*
b^2*sinh(x)^2 + 2*sqrt(2)*a*b*cosh(x) - sqrt(2)*b^2 + 2*(sqrt(2)*b^2*cosh(x
) + sqrt(2)*a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2,
-8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -
```

```
8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 6*(
b^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)*sinh(x)
)*sqrt(b*sinh(x) + a))/(a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cosh(x)^2 - (a^2*b^
2 + b^4)*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 + (a^2*b^
2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\sinh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(x))^(3/2), x)
```

3.110 $\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}}-\frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\,(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2iF\left(\frac{\pi}{4}\right.}{3\,(}$$

[Out] -2/3*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-8/3*a*b*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)^2/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a^2+b^2)/(a+b*sinh(x))^(1/2)

Rubi [A]
time = 0.16, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}}-\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}}-\frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\,(a^2+b^2)\,\sqrt{a+b\sinh(x)}}+\frac{8ia\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\,(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (8*a*b*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\,(a^2+b^2)}$$

$$= -\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}(3a^2-b^2)+ab}{\sqrt{a+b\sin}}}{3\,(a^2+b^2}$$

$$= -\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}} + \frac{(4a)\int \sqrt{a+b\,s}}{3\,(a^2+}$$

$$= -\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}} + \frac{\left(4a\sqrt{a+b\sinh}\right.}{3\,(a^2}$$

$$= -\frac{2b\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3\,(a^2+b^2)^2\,\sqrt{a+b\sinh(x)}} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2}{ia}\right.}{3\,(a^2+b^2)^2}$$

Mathematica [A]
time = 0.46, size = 166, normalized size = 0.84

$$\frac{\frac{8iaE\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))^2}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - 2i(a^2+b^2)\,F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))\sqrt{\frac{a+b\sinh(x)}{a-ib}} - 2b\cosh(x)\left(5a^2+b^2+4ab\sinh(x)\right)}{3\,(a^2+b^2)^2\,(a+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (((8*I)*a*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])
^2)/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*EllipticF[(Pi - (2*
I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*
b)] - 2*b*Cosh[x]*(5*a^2 + b^2 + 4*a*b*Sinh[x]))/(3*(a^2 + b^2)^2*(a + b*Si
nh[x])^(3/2))
```

Maple [A]
time = 1.15, size = 438, normalized size = 2.22

method	result

default	$\frac{\sqrt{(\cosh^2(x))(a+b\sinh(x))}\left(-\frac{2\sqrt{(\cosh^2(x))(a+b\sinh(x))}}{3b(a^2+b^2)\left(\sinh(x)+\frac{a}{b}\right)^2}-\frac{8b(\cosh^2(x))a}{3(a^2+b^2)^2\sqrt{(\cosh^2(x))(a+b\sin}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(a+b*sinh(x)))
^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^2*(a+b*sinh(x
)))^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*sinh(x)-a)/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(co
sh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I
*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*
(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a
-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 1291, normalized size = 6.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(a^2*b^2 - 3*b^4)*cosh(x)^4 + sqrt(2)*(a^2*b^2 - 3*b^4)*sinh(
x)^4 + 4*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 - 3*b^4)
*cosh(x) + sqrt(2)*(a^3*b - 3*a*b^3))*sinh(x)^3 + 2*sqrt(2)*(2*a^4 - 7*a^2*
b^2 + 3*b^4)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2*b^2 - 3*b^4)*cosh(x)^2 + 6*sqrt(
2)*(a^3*b - 3*a*b^3)*cosh(x) + sqrt(2)*(2*a^4 - 7*a^2*b^2 + 3*b^4))*sinh(x)
^2 - 4*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 - 3*b^4)*cos
```

```
h(x)^3 + 3*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x)^2 + sqrt(2)*(2*a^4 - 7*a^2*b^2
 + 3*b^4)*cosh(x) - sqrt(2)*(a^3*b - 3*a*b^3))*sinh(x) + sqrt(2)*(a^2*b^2 -
 3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3
+ 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 12*(sqrt(2)*a*b^
3*cosh(x)^4 + sqrt(2)*a*b^3*sinh(x)^4 + 4*sqrt(2)*a^2*b^2*cosh(x)^3 - 4*sqr
t(2)*a^2*b^2*cosh(x) + sqrt(2)*a*b^3 + 4*(sqrt(2)*a*b^3*cosh(x) + sqrt(2)*a
^2*b^2)*sinh(x)^3 + 2*sqrt(2)*(2*a^3*b - a*b^3)*cosh(x)^2 + 2*(3*sqrt(2)*a*
b^3*cosh(x)^2 + 6*sqrt(2)*a^2*b^2*cosh(x) + sqrt(2)*(2*a^3*b - a*b^3))*sinh
(x)^2 + 4*(sqrt(2)*a*b^3*cosh(x)^3 + 3*sqrt(2)*a^2*b^2*cosh(x)^2 - sqrt(2)*
a^2*b^2 + sqrt(2)*(2*a^3*b - a*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) +
3*b*sinh(x) + 2*a)/b)) - 6*(4*a*b^3*cosh(x)^4 + 4*a*b^3*sinh(x)^4 + (13*a^2
*b^2 + b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b^2 + b^4)*sinh(x)^3 + 4
*(2*a^3*b - a*b^3)*cosh(x)^2 + (24*a*b^3*cosh(x)^2 + 8*a^3*b - 4*a*b^3 + 3*
(13*a^2*b^2 + b^4)*cosh(x))*sinh(x)^2 - (3*a^2*b^2 - b^4)*cosh(x) + (16*a*b
^3*cosh(x)^3 - 3*a^2*b^2 + b^4 + 3*(13*a^2*b^2 + b^4)*cosh(x)^2 + 8*(2*a^3*
b - a*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(a^4*b^3 + 2*a^2*b^5 + b^
7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*sin
h(x)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 + 2*a^3*b^4
 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b + 3*
a^4*b^3 - b^7)*cosh(x)^2 + 2*(2*a^6*b + 3*a^4*b^3 - b^7 + 3*(a^4*b^3 + 2*a^
2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2
 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x) - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6
 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
cosh(x)^2 - (2*a^6*b + 3*a^4*b^3 - b^7)*cosh(x))*sinh(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\sinh(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))**(5/2),x)
```

```
[Out] Integral((a + b*sinh(x))**(-5/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x))^(5/2),x)
[Out] int(1/(a + b*sinh(x))^(5/2), x)
```

3.111 $\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=128

$$\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2iaF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a
-I*b))^(1/2)-2*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^
(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2i\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*S
qrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*a*EllipticF[Pi/4 - (I/2)*x, (2*b)/
(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{\int \sqrt{a+b\sinh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left(a\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\cdots}}}{b\sqrt{a+b\sinh(x)}}$$

$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2iaF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A]
time = 0.26, size = 101, normalized size = 0.79

$$\frac{2\left((ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-iaF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - I*a*Ellip
ticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[x])/(a - I*b
)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A]
time = 1.01, size = 218, normalized size = 1.70

method	result
default	$\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\,\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\,\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}}{b^{2}\cosh(x)\sqrt{a+b\sinh(x)}}$
risch	$\frac{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)\sqrt{2}\,\mathrm{e}^{-x}}{b\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)\mathrm{e}^{-x}}}+\frac{\left(-\frac{4\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)}{b\sqrt{\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)\mathrm{e}^{x}}}+\frac{4\left(a+\sqrt{a^{2}+b^{2}}\right)\sqrt{\frac{\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}+b^{2}}}{b}\right)b}{a+\sqrt{a^{2}+b^{2}}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 174, normalized size = 1.36

$$-\frac{2\left(2\sqrt{2}\,a\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+3\sqrt{2}\,b^{\frac{3}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)+3\sqrt{b\sinh(x)+a}\,b\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/2
7*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(
2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*sinh(x) + a)*b)/b
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(a + b*sinh(x))^(1/2),x)
```

```
[Out] int(sinh(x)/(a + b*sinh(x))^(1/2), x)
```

3.112 $\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=112

$$\frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x)$$

```
[Out] 2/35*a*(7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+I*a*sinh(
x))^(5/2)+64/105*a^3*(7*I*A+5*B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+16/105*a^2*(
7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2830, 2726, 2725}

$$\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (64*a^3*((7*I)*A + 5*B)*Cosh[x])/(105*Sqrt[a + I*a*Sinh[x]]) + (16*a^2*((7*
I)*A + 5*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/105 + (2*a*((7*I)*A + 5*B)*Cosh[
x]*(a + I*a*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}+\frac{1}{7}(7A-5iB)\int (a+ia\sinh(x))^{5}$$
$$=\frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\,\mathrm{si}$$
$$=\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(7iA+5B)\cos$$
$$=\frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\,\mathrm{sin}}$$

Mathematica [**A**]
time = 0.26, size = 100, normalized size = 0.89

$$\frac{a^2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}\,(1246iA+1040B+(-42iA-120B)\cosh(2x)+(-392A+505iB)\sinh(x)-15iB\sinh(3x))}{210\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (a^2*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((1246*I)*A + 1040*B +
 ((-42*I)*A - 120*B)*Cosh[2*x] + (-392*A + (505*I)*B)*Sinh[x] - (15*I)*B*Si
nh[3*x]))/(210*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [**F**]
time = 2.31, size = 0, normalized size = 0.00

$$\int (a+ia\sinh(x))^{\frac{5}{2}}(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

```
[Out] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Fricas [A]
time = 0.36, size = 126, normalized size = 1.12

$$-\frac{1}{420}\left(15\,Ba^2e^{(7\,x)}+21\,(2\,A-5i\,B)a^2e^{(6\,x)}+35\,(-10i\,A-11\,B)a^2e^{(5\,x)}-525\,(4\,A-3i\,B)a^2e^{(4\,x)}+525\,(-4i\,A-3\,B)a^2e^{(3\,x)}-35\,(10\,A-11i\,B)a^2e^{(2\,x)}+21\,(2i\,A+5\,B)a^2e^x-15i\,Ba^2\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/420*(15*B*a^2*e^(7*x) + 21*(2*A - 5*I*B)*a^2*e^(6*x) + 35*(-10*I*A - 11*
B)*a^2*e^(5*x) - 525*(4*A - 3*I*B)*a^2*e^(4*x) + 525*(-4*I*A - 3*B)*a^2*e^(
3*x) - 35*(10*A - 11*I*B)*a^2*e^(2*x) + 21*(2*I*A + 5*B)*a^2*e^x - 15*I*B*a
^2)*sqrt(1/2*I*a*e^(-x))*e^(-3*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(5/2)*(A+B*sinh(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\sinh(x))\,\left(a+a\sinh(x)\,1\mathrm{i}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2),x)
```

```
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2), x)
```

3.113 $\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=81

$$\frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

```
[Out] 2/5*B*cosh(x)*(a+I*a*sinh(x))^(3/2)+8/15*a^2*(5*I*A+3*B)*cosh(x)/(a+I*a*sin
h(x))^(1/2)+2/15*a*(5*I*A+3*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2830, 2726, 2725}

$$\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (8*a^2*((5*I)*A + 3*B)*Cosh[x])/(15*Sqrt[a + I*a*Sinh[x]]) + (2*a*((5*I)*A
+ 3*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/15 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(
3/2))/5
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2} + \frac{1}{5}(5A-3iB)\int (a+ia\sinh(x)$$
$$= \frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)} + \frac{2}{5}B\cosh(x)(a+ia\,\mathrm{s}$$
$$= \frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}} + \frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(}$$

Mathematica [A]
time = 0.16, size = 83, normalized size = 1.02

$$-\frac{a\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}\,(-50iA-39B+3B\cosh(2x)+2(5A-9iB)\sinh(x))}{15\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
[Out] -1/15*(a*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((-50*I)*A - 39*B
+ 3*B*Cosh[2*x] + 2*(5*A - (9*I)*B)*Sinh[x]))/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [F]
time = 2.25, size = 0, normalized size = 0.00

$$\int (a+ia\sinh(x))^{\frac{3}{2}}(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
[Out] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Fricas [A]
time = 0.41, size = 82, normalized size = 1.01

$$\frac{1}{30}\left(3i\,Bae^{(5\,x)}-5\,(-2i\,A-3\,B)ae^{(4\,x)}+30\,(3\,A-2i\,B)ae^{(3\,x)}-30\,(-3i\,A-2\,B)ae^{(2\,x)}+5\,(2\,A-3i\,B)ae^{x}-3\,Ba\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(3*I*B*a*e^(5*x) - 5*(-2*I*A - 3*B)*a*e^(4*x) + 30*(3*A - 2*I*B)*a*e^(
3*x) - 30*(-3*I*A - 2*B)*a*e^(2*x) + 5*(2*A - 3*I*B)*a*e^x - 3*B*a)*sqrt(1/
2*I*a*e^(-x))*e^(-2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (ia(\sinh{(x)}-i))^{\frac{3}{2}}\,(A+B\sinh{(x)})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((I*a*(sinh(x) - I))**(3/2)*(A + B*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\sinh(x))\,\left(a+a\sinh(x)\,1\mathrm{i}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2),x)
```

```
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2), x)
```

3.114 $\int \sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx$

Optimal. Leaf size=48

$$\frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

```
[Out] 2/3*a*(3*I*A+B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+2/3*B*cosh(x)*(a+I*a*sinh(x))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {2830, 2725}

$$\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*a*((3*I)*A + B)*Cosh[x])/(3*Sqrt[a + I*a*Sinh[x]]) + (2*B*Cosh[x]*Sqrt[a
 + I*a*Sinh[x]])/3
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx &= \frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{1}{3}(3A-iB)\int \sqrt{a+ia\sinh(x)}\\ &= \frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 1.38

$$\frac{2\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}\,(3A-2iB+B\sinh(x))}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
[Out] (2*(I*Cosh[x/2] + Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*(3*A - (2*I)*B + B*Sinh[
x]))/(3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F]
time = 2.24, size = 0, normalized size = 0.00

$$\int \sqrt{a+ia\sinh(x)}\,(A+B\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
[Out] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Fricas [A]
time = 0.34, size = 49, normalized size = 1.02

$$\frac{1}{3}\left(Be^{(3x)}+3(2A-iB)e^{(2x)}-3(-2iA-B)e^{x}-iB\right)\sqrt{\frac{1}{2}iae^{(-x)}}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
[Out] 1/3*(B*e^(3*x) + 3*(2*A - I*B)*e^(2*x) - 3*(-2*I*A - B)*e^x - I*B)*sqrt(1/2
*I*a*e^(-x))*e^(-x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\left(\sinh\left(x\right)-i\right)}\left(A+B\sinh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))**(1/2)*(A+B*sinh(x)),x)
[Out] Integral(sqrt(I*a*(sinh(x) - I))*(A + B*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A+B\,\mathrm{sinh}(x)\right)\,\sqrt{a+a\,\mathrm{sinh}\left(x\right)\,1{}\mathrm{i}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2), x)
```

3.115 $\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$Bx-\frac{(iA+B)\cosh(x)}{i+\sinh(x)}$$

```
[Out] B*x-(I*A+B)*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2814, 2727}

$$Bx-\frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] B*x - ((I*A + B)*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx &= Bx-(-A+iB)\int \frac{1}{i+\sinh(x)}\,dx\\ &= Bx-\frac{(iA+B)\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 38, normalized size = 1.65

$$Bx+\frac{2(A-iB)\sinh\left(\frac{x}{2}\right)}{i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] B*x + (2*(A - I*B)*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])
```

Maple [A]
time = 0.40, size = 39, normalized size = 1.70

method	result	size
risch	$Bx - \frac{2A}{e^x+i} + \frac{2iB}{e^x+i}$	26
default	$-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i(-iB+A)}{\tanh\left(\frac{x}{2}\right)+i}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)-2*I*(-I*B+A)/(tanh(1/2*x)+I)
```

Maxima [A]
time = 0.30, size = 26, normalized size = 1.13

$$B\left(x + \frac{2i}{e^{(-x)} - i}\right) - \frac{2A}{e^{(-x)} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] B*(x + 2*I/(e^(-x) - I)) - 2*A/(e^(-x) - I)
```

Fricas [A]
time = 0.52, size = 23, normalized size = 1.00

$$\frac{Bxe^x + iBx - 2A + 2iB}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x + I*B*x - 2*A + 2*I*B)/(e^x + I)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.65

$$Bx + \frac{-2A + 2iB}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x)
[Out] B*x + (-2*A + 2*I*B)/(exp(x) + I)
```

Giac [**A**]
time = 0.42, size = 17, normalized size = 0.74

$$Bx - \frac{2(A - iB)}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="giac")
[Out] B*x - 2*(A - I*B)/(e^x + I)
```

Mupad [**B**]
time = 0.12, size = 21, normalized size = 0.91

$$B\,x - \frac{2\,A - B\,2\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i),x)
[Out] B*x - (2*A - B*2i)/(exp(x) + 1i)
```

3.116 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=43

$$-\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] -1/3*(I*A+B)*cosh(x)/(I+sinh(x))^2-1/3*(A+2*I*B)*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2829, 2727}

$$-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*((I*A + B)*Cosh[x])/(I + Sinh[x])^2 - ((A + (2*I)*B)*Cosh[x])/(3*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx&=-\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}+\frac{1}{3}(-iA+2B)\int\frac{1}{i+\sinh(x)}\,dx\\&=-\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.74

$$\frac{\cosh(x)(-2iA+B-(A+2iB)\sinh(x))}{3(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((-2*I)*A + B - (A + (2*I)*B)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A]
time = 0.51, size = 52, normalized size = 1.21

method	result	size
risch	$-\frac{2(3A\,\mathrm{e}^{x}+3iB\,\mathrm{e}^{x}+3B\,\mathrm{e}^{2x}+iA-2B)}{3(\mathrm{e}^{x}+i)^{3}}$	36
default	$-\frac{-2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2(2iB-2A)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-2*I*A-2*B)/(tanh(1/2*x)+I)^2-2/3*(2*I*B-2*A)/(tanh(1/2*x)+I)^3-2*A/(tanh
(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.
time = 0.28, size = 141, normalized size = 3.28

$$-\frac{2}{3}A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{i}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}\right)-\frac{2}{3}B\left(\frac{3i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*A*(3*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) - I/(3*e^(-x) + 3
*I*e^(-2*x) - e^(-3*x) - I)) - 2/3*B*(3*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) -
 e^(-3*x) - I) - 3*e^(-2*x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2/(3
*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I))
```

Fricas [A]
time = 0.39, size = 43, normalized size = 1.00

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,(A+i\,B)e^{x}+i\,A-2\,B\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*(A + I*B)*e^x + I*A - 2*B)/(e^(3*x) + 3*I*e^(2*x) - 3
*e^x - I)
```

Sympy [A]
time = 0.12, size = 53, normalized size = 1.23

$$\frac{-2iA - 6Be^{2x} + 4B + (-6A - 6iB)\,e^{x}}{3e^{3x} + 9ie^{2x} - 9e^{x} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**2,x)
```

```
[Out] (-2*I*A - 6*B*exp(2*x) + 4*B + (-6*A - 6*I*B)*exp(x))/(3*exp(3*x) + 9*I*exp
(2*x) - 9*exp(x) - 3*I)
```

Giac [A]
time = 0.40, size = 32, normalized size = 0.74

$$-\frac{2\left(3\,Be^{(2\,x)} + 3\,Ae^{x} + 3i\,Be^{x} + i\,A - 2\,B\right)}{3\left(e^{x} + i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x + 3*I*B*e^x + I*A - 2*B)/(e^x + I)^3
```

Mupad [B]
time = 0.61, size = 39, normalized size = 0.91

$$-\frac{\frac{2\,A}{3} + \frac{B\,4\mathrm{i}}{3} - \mathrm{e}^{x}\left(-2\,B + A\,2\mathrm{i}\right) - B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{\left(-1 + \mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^2,x)
```

```
[Out] -((2*A)/3 + (B*4i)/3 - exp(x)*(A*2i - 2*B) - B*exp(2*x)*2i)/(exp(x)*1i - 1)
^3
```

3.117 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx$

Optimal. Leaf size=68

$$-\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}$$

```
[Out] -1/5*(I*A+B)*cosh(x)/(I+sinh(x))^3-1/15*(2*A+3*I*B)*cosh(x)/(I+sinh(x))^2+1
/15*(2*I*A-3*B)*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2829, 2729, 2727}

$$\frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)}-\frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2}-\frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] -1/5*((I*A + B)*Cosh[x])/(I + Sinh[x])^3 - ((2*A + (3*I)*B)*Cosh[x])/(15*(I
 + Sinh[x])^2) + (((2*I)*A - 3*B)*Cosh[x])/(15*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx &= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}+\frac{1}{5}(-2iA+3B)\int \frac{1}{(i+\sinh(x))^2}\,dx \\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{1}{15}(-2A-3iB)\int \frac{1}{i+\sinh(x)}\,dx \\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3}-\frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2}+\frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.74

$$
\frac{\cosh(x)\left(-7iA+3B-3(2A+3iB)\sinh(x)+(2iA-3B)\sinh^2(x)\right)}{15(i+\sinh(x))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] (Cosh[x]*((-7*I)*A + 3*B - 3*(2*A + (3*I)*B)*Sinh[x] + ((2*I)*A - 3*B)*Sinh
[x]^2))/(15*(I + Sinh[x])^3)
```

Maple [A]
time = 0.53, size = 91, normalized size = 1.34

method	result	size
risch	$-\frac{2\left(15B\,\mathrm{e}^{3x}-15B\,\mathrm{e}^{x}+15iB\,\mathrm{e}^{2x}-2A+10iA\,\mathrm{e}^{x}-3iB+20A\,\mathrm{e}^{2x}\right)}{15\left(\mathrm{e}^{x}+i\right)^{5}}$	51
default	$-\frac{-8iB+8A}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2iB-4A}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2(-4iA-4B)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2(8iA+6B)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(8*A-8*I*B)/(tanh(1/2*x)+I)^4+2*I*A/(tanh(1/2*x)+I)-(-4*A+2*I*B)/(tanh
(1/2*x)+I)^2-2/5*(-4*I*A-4*B)/(tanh(1/2*x)+I)^5-2/3*(8*I*A+6*B)/(tanh(1/2*x
)+I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(50) = 100.
time = 0.33, size = 267, normalized size = 3.93

$$
-\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}+\frac{5ie^{(-2x)}}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}-\frac{5e^{(-3x)}}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}-\frac{i}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}\right)-\frac{4}{15}A\left(-\frac{5ie^{(-x)}}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}+\frac{10e^{(-2x)}}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}-\frac{1}{5e^{(-x)}+10ie^{(-2x)}-10e^{(-3x)}-5ie^{(-4x)}+e^{(-5x)}-i}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*B*(5*e^(-x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e
^(-5*x) - I) + 5*I*e^(-2*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e
^(-4*x) + e^(-5*x) - I) - 5*e^(-3*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x
) - 5*I*e^(-4*x) + e^(-5*x) - I) - I/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x
) - 5*I*e^(-4*x) + e^(-5*x) - I)) - 4/15*A*(-5*I*e^(-x)/(5*e^(-x) + 10*I*e^
(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) + 10*e^(-2*x)/(5*e^(-x)
 + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) - 1/(5*e^(-x)
 + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I))
```

Fricas [**A**]
time = 0.41, size = 70, normalized size = 1.03

$$-\frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A+3i\,B)e^{(2\,x)}+5\,(2i\,A-3\,B)e^{x}-2\,A-3i\,B\right)}{15\,\left(e^{(5\,x)}+5i\,e^{(4\,x)}-10\,e^{(3\,x)}-10i\,e^{(2\,x)}+5\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*B*e^(3*x) + 5*(4*A + 3*I*B)*e^(2*x) + 5*(2*I*A - 3*B)*e^x - 2*A -
 3*I*B)/(e^(5*x) + 5*I*e^(4*x) - 10*e^(3*x) - 10*I*e^(2*x) + 5*e^x + I)
```

Sympy [**A**]
time = 0.21, size = 82, normalized size = 1.21

$$\frac{4A-30Be^{3x}+6iB+(-40A-30iB)\,e^{2x}+(-20iA+30B)\,e^{x}}{15e^{5x}+75ie^{4x}-150e^{3x}-150ie^{2x}+75e^{x}+15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**3,x)
```

```
[Out] (4*A - 30*B*exp(3*x) + 6*I*B + (-40*A - 30*I*B)*exp(2*x) + (-20*I*A + 30*B)
*exp(x))/(15*exp(5*x) + 75*I*exp(4*x) - 150*exp(3*x) - 150*I*exp(2*x) + 75*
exp(x) + 15*I)
```

Giac [**A**]
time = 0.43, size = 46, normalized size = 0.68

$$-\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}+15i\,Be^{(2\,x)}+10i\,Ae^{x}-15\,Be^{x}-2\,A-3i\,B\right)}{15\,\left(e^{x}+i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*I*B*e^(2*x) + 10*I*A*e^x - 15*B*e^x
 - 2*A - 3*I*B)/(e^x + I)^5
```

Mupad [B]
time = 0.83, size = 52, normalized size = 0.76

$$\frac{\frac{A\,4\mathrm{i}}{15}-\frac{2\,B}{5}-\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^x\,\left(\frac{4\,A}{3}+B\,2\mathrm{i}\right)+2\,B\,\mathrm{e}^{2\,x}-B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{{\left(-1+\mathrm{e}^x\,1\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^3,x)
```

```
[Out] ((A*4i)/15 - (2*B)/5 - (A*exp(2*x)*8i)/3 + exp(x)*((4*A)/3 + B*2i) + 2*B*ex
p(2*x) - B*exp(3*x)*2i)/(exp(x)*1i - 1)^5
```

3.118 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$

Optimal. Leaf size=91

$$-\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}$$

```
[Out] -1/7*(I*A+B)*cosh(x)/(I+sinh(x))^4-1/35*(3*A+4*I*B)*cosh(x)/(I+sinh(x))^3+2
/105*(3*I*A-4*B)*cosh(x)/(I+sinh(x))^2+2/105*(3*A+4*I*B)*cosh(x)/(I+sinh(x)
)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2829, 2729, 2727}

$$\frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)}+\frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2}-\frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3}-\frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] -1/7*((I*A + B)*Cosh[x])/(I + Sinh[x])^4 - ((3*A + (4*I)*B)*Cosh[x])/(35*(I
 + Sinh[x])^3) + (2*((3*I)*A - 4*B)*Cosh[x])/(105*(I + Sinh[x])^2) + (2*(3*
A + (4*I)*B)*Cosh[x])/(105*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}+\frac{1}{7}(-3iA+4B)\int \frac{1}{(i+\sinh(x))^3}\,dx$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}-\frac{1}{35}(2(3A+4iB))\int \frac{1}{(i+\sinh(x))^2}\,dx$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{1}{105}(2(3iA-$$
$$= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{2(3A+4iB)\,\mathrm{c}}{105(i+\mathrm{sin}}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.74

$$\frac{\cosh(x)\left(-36iA+13B-13(3A+4iB)\sinh(x)+8i(3A+4iB)\sinh^2(x)+(6A+8iB)\sinh^3(x)\right)}{105(i+\sinh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Sinh[x] + (8*I)*(3*A + (4*I
)*B)*Sinh[x]^2 + (6*A + (8*I)*B)*Sinh[x]^3))/(105*(I + Sinh[x])^4)
```

Maple [A]
time = 0.56, size = 128, normalized size = 1.41

method	result
risch	$-\frac{4(4B-84B\,\mathrm{e}^{2x}-21A\,\mathrm{e}^{x}+70iB\,\mathrm{e}^{3x}+70B\,\mathrm{e}^{4x}+63iA\,\mathrm{e}^{2x}-28iB\,\mathrm{e}^{x}-3iA+105A\,\mathrm{e}^{3x})}{105(\mathrm{e}^{x}+i)^{7}}$
default	$-\frac{-32iA-24B}{2(\tanh(\frac{x}{2})+i)^4}-\frac{2(-10iB+18A)}{3(\tanh(\frac{x}{2})+i)^3}-\frac{2(32iB-36A)}{5(\tanh(\frac{x}{2})+i)^5}-\frac{24iA+24B}{3(\tanh(\frac{x}{2})+i)^6}-\frac{6iA+2B}{(\tanh(\frac{x}{2})+i)^2}+\frac{2A}{\tanh(\frac{x}{2})+i}-\frac{2(-8iB+8}{7(\tanh(\frac{x}{2})-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I+sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-32*I*A-24*B)/(tanh(1/2*x)+I)^4-2/3*(18*A-10*I*B)/(tanh(1/2*x)+I)^3-2
/5*(-36*A+32*I*B)/(tanh(1/2*x)+I)^5-1/3*(24*I*A+24*B)/(tanh(1/2*x)+I)^6-(6*
I*A+2*B)/(tanh(1/2*x)+I)^2+2*A/(tanh(1/2*x)+I)-2/7*(8*A-8*I*B)/(tanh(1/2*x)
+I)^7
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.
time = 0.29, size = 469, normalized size = 5.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) +
21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 21*I*e^(-2*x)/(7*e^(-x) + 21*I
*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-
7*x) - I) - 35*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-
4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - I/(7*e^(-x) + 21*I*e^(-
2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x)
- I)) - 8/105*B*(-14*I*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*
I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 42*e^(-2*x)/(7*e^
(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-
6*x) - e^(-7*x) - I) + 35*I*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x
) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - 35*e^(-4*x
)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7
*I*e^(-6*x) - e^(-7*x) - I) - 2/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 3
5*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I))
```

Fricas [A]
time = 0.37, size = 95, normalized size = 1.04

$$-\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A+2i\,B)e^{(3\,x)}+21\,(3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A+4i\,B)e^{x}-3i\,A+4\,B\right)}{105\left(e^{(7\,x)}+7i\,e^{(6\,x)}-21\,e^{(5\,x)}-35i\,e^{(4\,x)}+35\,e^{(3\,x)}+21i\,e^{(2\,x)}-7\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -4/105*(70*B*e^(4*x) + 35*(3*A + 2*I*B)*e^(3*x) + 21*(3*I*A - 4*B)*e^(2*x)
- 7*(3*A + 4*I*B)*e^x - 3*I*A + 4*B)/(e^(7*x) + 7*I*e^(6*x) - 21*e^(5*x) -
35*I*e^(4*x) + 35*e^(3*x) + 21*I*e^(2*x) - 7*e^x - I)
```

Sympy [A]
time = 0.37, size = 110, normalized size = 1.21

$$\frac{12iA-280Be^{4x}-16B+(-420A-280iB)\,e^{3x}+(84A+112iB)\,e^{x}+(-252iA+336B)\,e^{2x}}{105e^{7x}+735ie^{6x}-2205e^{5x}-3675ie^{4x}+3675e^{3x}+2205ie^{2x}-735e^{x}-105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**4,x)
```

```
[Out] (12*I*A - 280*B*exp(4*x) - 16*B + (-420*A - 280*I*B)*exp(3*x) + (84*A + 112
*I*B)*exp(x) + (-252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) + 735*I*exp(6*x)
- 2205*exp(5*x) - 3675*I*exp(4*x) + 3675*exp(3*x) + 2205*I*exp(2*x) - 735*e
xp(x) - 105*I)
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.66

$$-\frac{4 \left(70 \, B e^{(4 \, x)} + 105 \, A e^{(3 \, x)} + 70 i \, B e^{(3 \, x)} + 63 i \, A e^{(2 \, x)} - 84 \, B e^{(2 \, x)} - 21 \, A e^{x} - 28 i \, B e^{x} - 3 i \, A + 4 \, B\right)}{105 \, {\left(e^{x} + i\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*I*B*e^(3*x) + 63*I*A*e^(2*x) - 84
*B*e^(2*x) - 21*A*e^x - 28*I*B*e^x - 3*I*A + 4*B)/(e^x + I)^7
```

Mupad [B]
time = 1.04, size = 66, normalized size = 0.73

$$-\frac{\frac{16\,B}{105}+4\,A\,{\mathrm{e}}^{3\,x}-{\mathrm{e}}^x\,\left(\frac{4\,A}{5}+\frac{B\,16{}\mathrm{i}}{15}\right)-\frac{16\,B\,{\mathrm{e}}^{2\,x}}{5}+\frac{8\,B\,{\mathrm{e}}^{4\,x}}{3}-\frac{A\,4{}\mathrm{i}}{35}+\frac{A\,{\mathrm{e}}^{2\,x}\,12{}\mathrm{i}}{5}+\frac{B\,{\mathrm{e}}^{3\,x}\,8{}\mathrm{i}}{3}}{{\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^4,x)
```

```
[Out] -((16*B)/105 - (A*4i)/35 + (A*exp(2*x)*12i)/5 + 4*A*exp(3*x) - exp(x)*((4*A
)/5 + (B*16i)/15) - (16*B*exp(2*x))/5 + (B*exp(3*x)*8i)/3 + (8*B*exp(4*x))/
3)/(exp(x) + 1i)^7
```

3.119 $\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$-Bx+\frac{(iA-B)\cosh(x)}{i-\sinh(x)}$$

```
[Out] -B*x+(I*A-B)*cosh(x)/(I-sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2814, 2727}

$$-Bx+\frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*x) + ((I*A - B)*Cosh[x])/(I - Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx &= -Bx+(A+iB)\int \frac{1}{i-\sinh(x)}\,dx\\ &= -Bx+\frac{(iA-B)\cosh(x)}{i-\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.
time = 0.06, size = 59, normalized size = 2.19

$$\frac{\left(i\cosh\left(\frac{x}{2}\right)-\sinh\left(\frac{x}{2}\right)\right)\left(Bx\cosh\left(\frac{x}{2}\right)+i(2A+B(2i+x))\sinh\left(\frac{x}{2}\right)\right)}{-i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] ((I*Cosh[x/2] - Sinh[x/2])*(B*x*Cosh[x/2] + I*(2*A + B*(2*I + x))*Sinh[x/2]
))/(-I + Sinh[x])
```

Maple [A]
time = 0.44, size = 39, normalized size = 1.44

method	result	size
risch	$-Bx+\frac{2A}{e^{x}-i}+\frac{2iB}{e^{x}-i}$	27
default	$-\frac{2i(iB+A)}{\tanh\left(\frac{x}{2}\right)-i}+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(I*B+A)/(tanh(1/2*x)-I)+B*ln(tanh(1/2*x)-1)-B*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.00

$$-B\left(x-\frac{2i}{e^{(-x)}+i}\right)+\frac{2A}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(x - 2*I/(e^(-x) + I)) + 2*A/(e^(-x) + I)
```

Fricas [A]
time = 0.42, size = 24, normalized size = 0.89

$$-\frac{Bxe^{x}-iBx-2A-2iB}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x - I*B*x - 2*A - 2*I*B)/(e^x - I)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.56

$$-Bx+\frac{2A+2iB}{e^{x}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x)
```

```
[Out] -B*x + (2*A + 2*I*B)/(exp(x) - I)
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.67

$$-Bx + \frac{2\,(A + i\,B)}{e^x - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*(A + I*B)/(e^x - I)
```

Mupad [B]
time = 0.11, size = 21, normalized size = 0.78

$$-B\,x + \frac{2\,A + B\,2\mathrm{i}}{\mathrm{e}^x - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i),x)
```

```
[Out] (2*A + B*2i)/(exp(x) - 1i) - B*x
```

3.120 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx$

Optimal. Leaf size=49

$$\frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}$$

```
[Out] 1/3*(I*A-B)*cosh(x)/(I-sinh(x))^2+1/3*(A-2*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2829, 2727}

$$\frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)}+\frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) + ((A - (2*I)*B)*Cosh[x])/(3*(I - S
inh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx&=\frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{1}{3}(-iA-2B)\int \frac{1}{i-\sinh(x)}\,dx\\&=\frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.65

$$\frac{\cosh(x)(2iA + B - (A - 2iB)\sinh(x))}{3(-i + \sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*((2*I)*A + B - (A - (2*I)*B)*Sinh[x]))/(3*(-I + Sinh[x])^2)
```

Maple [A]
time = 0.53, size = 52, normalized size = 1.06

method	result	size
risch	$-\frac{2(3A\,\mathrm{e}^{x}-3iB\,\mathrm{e}^{x}+3B\,\mathrm{e}^{2x}-iA-2B)}{3(\mathrm{e}^{x}-i)^{3}}$	36
default	$-\frac{2A}{\tanh(\frac{x}{2})-i}-\frac{2iA-2B}{(\tanh(\frac{x}{2})-i)^{2}}-\frac{2(-2iB-2A)}{3(\tanh(\frac{x}{2})-i)^{3}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/(tanh(1/2*x)-I)-(2*I*A-2*B)/(tanh(1/2*x)-I)^2-2/3*(-2*I*B-2*A)/(tanh(1
/2*x)-I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(31) = 62$.
time = 0.32, size = 141, normalized size = 2.88

$$-\frac{2}{3}A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{i}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}\right)-\frac{2}{3}B\left(-\frac{3i\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{2}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*A*(3*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) + I/(3*e^(-x) - 3
*I*e^(-2*x) - e^(-3*x) + I)) - 2/3*B*(-3*I*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x)
- e^(-3*x) + I) - 3*e^(-2*x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) + 2/(
3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I))
```

Fricas [A]
time = 0.43, size = 43, normalized size = 0.88

$$-\frac{2\left(3\,Be^{(2\,x)}+3\,(A-i\,B)e^{x}-i\,A-2\,B\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*(A - I*B)*e^x - I*A - 2*B)/(e^(3*x) - 3*I*e^(2*x) - 3
*e^x + I)
```

Sympy [A]
time = 0.12, size = 51, normalized size = 1.04

$$\frac{2iA - 6Be^{2x} + 4B + (-6A + 6iB)\,e^{x}}{3e^{3x} - 9ie^{2x} - 9e^{x} + 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**2,x)
```

```
[Out] (2*I*A - 6*B*exp(2*x) + 4*B + (-6*A + 6*I*B)*exp(x))/(3*exp(3*x) - 9*I*exp(
2*x) - 9*exp(x) + 3*I)
```

Giac [A]
time = 0.41, size = 32, normalized size = 0.65

$$-\frac{2\left(3\,Be^{(2\,x)} + 3\,Ae^{x} - 3i\,Be^{x} - i\,A - 2\,B\right)}{3\,(e^{x} - i)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x - 3*I*B*e^x - I*A - 2*B)/(e^x - I)^3
```

Mupad [B]
time = 0.60, size = 37, normalized size = 0.76

$$\frac{\frac{2\,A}{3} - \frac{B\,4\mathrm{i}}{3} + \mathrm{e}^{x}\,(2\,B + A\,2\mathrm{i}) + B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{(1 + \mathrm{e}^{x}\,1\mathrm{i})^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^2,x)
```

```
[Out] ((2*A)/3 - (B*4i)/3 + exp(x)*(A*2i + 2*B) + B*exp(2*x)*2i)/(exp(x)*1i + 1)^
3
```

3.121 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$

Optimal. Leaf size=76

$$\frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}-\frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}$$

```
[Out] 1/5*(I*A-B)*cosh(x)/(I-sinh(x))^3+1/15*(2*A-3*I*B)*cosh(x)/(I-sinh(x))^2-1/
15*(2*I*A+3*B)*cosh(x)/(I-sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2829, 2729, 2727}

$$-\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)}+\frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2}+\frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3) + ((2*A - (3*I)*B)*Cosh[x])/(15*(I
- Sinh[x])^2) - (((2*I)*A + 3*B)*Cosh[x])/(15*(I - Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3} + \frac{1}{5}(-2iA-3B)\int \frac{1}{(i-\sinh(x))^2}\,dx$$
$$= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3} + \frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2} + \frac{1}{15}(-2A+3iB)\int \frac{1}{i-\sinh(x)}\,dx$$
$$= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3} + \frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2} - \frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 1.21

$$-\frac{15iB\cosh\left(\frac{x}{2}\right)+5(2A-3iB)\cosh\left(\frac{3x}{2}\right)-20iA\sinh\left(\frac{x}{2}\right)-15B\sinh\left(\frac{x}{2}\right)+2iA\sinh\left(\frac{5x}{2}\right)+3B\sinh\left(\frac{5x}{2}\right)}{30\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] -1/30*((15*I)*B*Cosh[x/2] + 5*(2*A - (3*I)*B)*Cosh[(3*x)/2] - (20*I)*A*Sinh
[x/2] - 15*B*Sinh[x/2] + (2*I)*A*Sinh[(5*x)/2] + 3*B*Sinh[(5*x)/2])/(Cosh[x
/2] + I*Sinh[x/2])^5
```

Maple [A]
time = 0.57, size = 91, normalized size = 1.20

method	result	size
risch	$\frac{2B\,\mathrm{e}^{3x}-2B\,\mathrm{e}^{x}-2iB\,\mathrm{e}^{2x}-\frac{4A}{15}-\frac{4iA\,\mathrm{e}^{x}}{3}+\frac{2iB}{5}+\frac{8A\,\mathrm{e}^{2x}}{3}}{(\mathrm{e}^{x}-i)^5}$	51
default	$-\frac{2(-4iA+4B)}{5(\tanh(\frac{x}{2})-i)^5}-\frac{2iB+4A}{(\tanh(\frac{x}{2})-i)^2}-\frac{-8iB-8A}{2(\tanh(\frac{x}{2})-i)^4}-\frac{2(8iA-6B)}{3(\tanh(\frac{x}{2})-i)^3}+\frac{2iA}{\tanh(\frac{x}{2})-i}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-4*I*A+4*B)/(tanh(1/2*x)-I)^5-(4*A+2*I*B)/(tanh(1/2*x)-I)^2-1/2*(-8*A
-8*I*B)/(tanh(1/2*x)-I)^4-2/3*(8*I*A-6*B)/(tanh(1/2*x)-I)^3+2*I*A/(tanh(1/2
*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(50) = 100.
time = 0.32, size = 267, normalized size = 3.51

$$\frac{2}{5}B\left(\frac{5e^{(-x)}}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}-\frac{5ie^{(-2x)}}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}-\frac{5e^{(-3x)}}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}+\frac{i}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}\right)+\frac{4}{15}A\left(\frac{5ie^{(-x)}}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}+\frac{10e^{(-2x)}}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}-\frac{1}{5e^{(-x)}-10ie^{(-2x)}-10e^{(-3x)}+5ie^{(-4x)}+e^{(-5x)}+i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 2/5*B*(5*e^(-x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^
(-5*x) + I) - 5*I*e^(-2*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^
(-4*x) + e^(-5*x) + I) - 5*e^(-3*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x)
 + 5*I*e^(-4*x) + e^(-5*x) + I) + I/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x)
 + 5*I*e^(-4*x) + e^(-5*x) + I)) + 4/15*A*(5*I*e^(-x)/(5*e^(-x) - 10*I*e^(-
2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) + 10*e^(-2*x)/(5*e^(-x) -
 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) - 1/(5*e^(-x) -
 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I))
```

Fricas [**A**]
time = 0.47, size = 70, normalized size = 0.92

$$\frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A-3i\,B)e^{(2\,x)}-5\,(2i\,A+3\,B)e^{x}-2\,A+3i\,B\right)}{15\,(e^{(5\,x)}-5i\,e^{(4\,x)}-10\,e^{(3\,x)}+10i\,e^{(2\,x)}+5\,e^{x}-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(15*B*e^(3*x) + 5*(4*A - 3*I*B)*e^(2*x) - 5*(2*I*A + 3*B)*e^x - 2*A +
3*I*B)/(e^(5*x) - 5*I*e^(4*x) - 10*e^(3*x) + 10*I*e^(2*x) + 5*e^x - I)
```

Sympy [**A**]
time = 0.21, size = 82, normalized size = 1.08

$$\frac{-4A+30Be^{3x}+6iB+(40A-30iB)\,e^{2x}+(-20iA-30B)\,e^{x}}{15e^{5x}-75ie^{4x}-150e^{3x}+150ie^{2x}+75e^{x}-15i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**3,x)
```

```
[Out] (-4*A + 30*B*exp(3*x) + 6*I*B + (40*A - 30*I*B)*exp(2*x) + (-20*I*A - 30*B)
*exp(x))/(15*exp(5*x) - 75*I*exp(4*x) - 150*exp(3*x) + 150*I*exp(2*x) + 75*
exp(x) - 15*I)
```

Giac [**A**]
time = 0.39, size = 46, normalized size = 0.61

$$\frac{2\left(15\,Be^{(3\,x)}+20\,Ae^{(2\,x)}-15i\,Be^{(2\,x)}-10i\,Ae^{x}-15\,Be^{x}-2\,A+3i\,B\right)}{15\,(e^{x}-i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*I*B*e^(2*x) - 10*I*A*e^x - 15*B*e^x
- 2*A + 3*I*B)/(e^x - I)^5
```

Mupad [B]
time = 0.80, size = 52, normalized size = 0.68

$$\frac{2\,B\,{\mathrm{e}}^{2\,x}-\frac{2\,B}{5}+\frac{A\,{\mathrm{e}}^{2\,x}\,8{}\mathrm{i}}{3}+{\mathrm{e}}^x\,\left(\frac{4\,A}{3}-B\,2{}\mathrm{i}\right)-\frac{A\,4{}\mathrm{i}}{15}+B\,{\mathrm{e}}^{3\,x}\,2{}\mathrm{i}}{{\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i)^3,x)
```

```
[Out] ((A*exp(2*x)*8i)/3 - (2*B)/5 - (A*4i)/15 + exp(x)*((4*A)/3 - B*2i) + 2*B*ex
p(2*x) + B*exp(3*x)*2i)/(exp(x)*1i + 1)^5
```

3.122 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$

Optimal. Leaf size=101

$$\frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}$$

```
[Out] 1/7*(I*A-B)*cosh(x)/(I-sinh(x))^4+1/35*(3*A-4*I*B)*cosh(x)/(I-sinh(x))^3-2/
105*(3*I*A+4*B)*cosh(x)/(I-sinh(x))^2-2/105*(3*A-4*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2829, 2729, 2727}

$$-\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2}+\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(7*(I - Sinh[x])^4) + ((3*A - (4*I)*B)*Cosh[x])/(35*(I
 - Sinh[x])^3) - (2*((3*I)*A + 4*B)*Cosh[x])/(105*(I - Sinh[x])^2) - (2*(3*A
 - (4*I)*B)*Cosh[x])/(105*(I - Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{1}{7}(-3iA-4B)\int\frac{1}{(i-\sinh(x))^3}\,dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{1}{35}(2(3A-4iB))\int\frac{1}{(i-\sinh(x))^2}\,dx\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}+\frac{1}{105}(2(3iA+4\\ &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\,\mathrm{co}}{105(i-\sinh(}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 63, normalized size = 0.62

$$\frac{\cosh(x)\left(36iA+13B+(-39A+52iB)\sinh(x)+(-24iA-32B)\sinh^2(x)+(6A-8iB)\sinh^3(x)\right)}{105(-i+\sinh(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] (Cosh[x]*((36*I)*A + 13*B + (-39*A + (52*I)*B)*Sinh[x] + ((-24*I)*A - 32*B)
*Sinh[x]^2 + (6*A - (8*I)*B)*Sinh[x]^3))/(105*(-I + Sinh[x])^4)
```

Maple [**A**]
time = 0.61, size = 128, normalized size = 1.27

method	result
risch	$-\frac{4\left(4B-84B\,\mathrm{e}^{2x}-21A\,\mathrm{e}^{x}-70iB\,\mathrm{e}^{3x}+70B\,\mathrm{e}^{4x}-63iA\,\mathrm{e}^{2x}+28iB\,\mathrm{e}^{x}+3iA+105A\,\mathrm{e}^{3x}\right)}{105\left(\mathrm{e}^{x}-i\right)^{7}}$
default	$-\frac{2(10iB+18A)}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{2(-32iB-36A)}{5\left(\tanh\left(\frac{x}{2}\right)-i\right)^{5}}-\frac{-6iA+2B}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2(8iB+8A)}{7\left(\tanh\left(\frac{x}{2}\right)-i\right)^{7}}-\frac{-24iA+24B}{3\left(\tanh\left(\frac{x}{2}\right)-i\right)^{6}}-\frac{32iA-24B}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}+\frac{2A}{\tanh\left(\frac{x}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(I-sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(18*A+10*I*B)/(tanh(1/2*x)-I)^3-2/5*(-36*A-32*I*B)/(tanh(1/2*x)-I)^5-(
-6*I*A+2*B)/(tanh(1/2*x)-I)^2-2/7*(8*A+8*I*B)/(tanh(1/2*x)-I)^7-1/3*(-24*I*
A+24*B)/(tanh(1/2*x)-I)^6-1/2*(32*I*A-24*B)/(tanh(1/2*x)-I)^4+2*A/(tanh(1/2
*x)-I)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.
time = 0.30, size = 469, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) +
21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 21*I*e^(-2*x)/(7*e^(-x) - 21*I
*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-
7*x) + I) - 35*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-
4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + I/(7*e^(-x) - 21*I*e^(-
2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x)
+ I)) - 8/105*B*(14*I*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I
*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + 42*e^(-2*x)/(7*e^(
-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6
*x) - e^(-7*x) + I) - 35*I*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x)
 + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 35*e^(-4*x)
/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*
I*e^(-6*x) - e^(-7*x) + I) - 2/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35
*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I))
```

Fricas [A]
time = 0.40, size = 95, normalized size = 0.94

$$-\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A-2i\,B)e^{(3\,x)}+21\,(-3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A-4i\,B)e^{x}+3i\,A+4\,B\right)}{105\,\left(e^{(7\,x)}-7i\,e^{(6\,x)}-21\,e^{(5\,x)}+35i\,e^{(4\,x)}+35\,e^{(3\,x)}-21i\,e^{(2\,x)}-7\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -4/105*(70*B*e^(4*x) + 35*(3*A - 2*I*B)*e^(3*x) + 21*(-3*I*A - 4*B)*e^(2*x)
 - 7*(3*A - 4*I*B)*e^x + 3*I*A + 4*B)/(e^(7*x) - 7*I*e^(6*x) - 21*e^(5*x) +
 35*I*e^(4*x) + 35*e^(3*x) - 21*I*e^(2*x) - 7*e^x + I)
```

Sympy [A]
time = 0.37, size = 109, normalized size = 1.08

$$\frac{-12iA-280Be^{4x}-16B+(-420A+280iB)\,e^{3x}+(84A-112iB)\,e^{x}+(252iA+336B)\,e^{2x}}{105e^{7x}-735ie^{6x}-2205e^{5x}+3675ie^{4x}+3675e^{3x}-2205ie^{2x}-735e^{x}+105i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**4,x)
```

```
[Out] (-12*I*A - 280*B*exp(4*x) - 16*B + (-420*A + 280*I*B)*exp(3*x) + (84*A - 11
2*I*B)*exp(x) + (252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) - 735*I*exp(6*x)
- 2205*exp(5*x) + 3675*I*exp(4*x) + 3675*exp(3*x) - 2205*I*exp(2*x) - 735*e
xp(x) + 105*I)
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.59

$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70i\,Be^{(3\,x)}-63i\,Ae^{(2\,x)}-84\,Be^{(2\,x)}-21\,Ae^{x}+28i\,Be^{x}+3i\,A+4\,B\right)}{105\,(e^{x}-i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*I*B*e^(3*x) - 63*I*A*e^(2*x) - 84
*B*e^(2*x) - 21*A*e^x + 28*I*B*e^x + 3*I*A + 4*B)/(e^x - I)^7
```

Mupad [B]
time = 1.00, size = 68, normalized size = 0.67

$$\frac{\frac{12\,A\,e^{2\,x}}{5}+\frac{B\,16\mathrm{i}}{105}-\frac{4\,A}{35}+A\,e^{3\,x}\,4\mathrm{i}-e^{x}\left(\frac{16\,B}{15}+\frac{A\,4\mathrm{i}}{5}\right)-\frac{B\,e^{2\,x}\,16\mathrm{i}}{5}+\frac{8\,B\,e^{3\,x}}{3}+\frac{B\,e^{4\,x}\,8\mathrm{i}}{3}}{(1+e^{x}\,1\mathrm{i})^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^4,x)
```

```
[Out] ((B*16i)/105 - (4*A)/35 + (12*A*exp(2*x))/5 + A*exp(3*x)*4i - exp(x)*((A*4i
)/5 + (16*B)/15) - (B*exp(2*x)*16i)/5 + (8*B*exp(3*x))/3 + (B*exp(4*x)*8i)/
3)/(exp(x)*1i + 1)^7
```

3.123 $\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{2}\,(iA-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] (I*A-B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/
a^(1/2)+2*B*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2830, 2728, 212}

$$\frac{\sqrt{2}\,(-B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*(I*A - B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]]
)])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}} + (A+iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx$$
$$= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}} + (2(iA-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)$$
$$= \frac{\sqrt{2}\,(iA-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cosh(x)}{\sqrt{2}\,\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 1.29

$$\frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((1+i)\sqrt[4]{-1}\,(-iA+B)\text{ArcTan}\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+B\cosh\left(\frac{x}{2}\right)-iB\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
[Out] (2*(Cosh[x/2] + I*Sinh[x/2])*((1 + I)*(-1)^(1/4)*((-I)*A + B)*ArcTan[(I + T
anh[x/4])/Sqrt[2]] + B*Cosh[x/2] - I*B*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [F]
time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(49) = 98$.
time = 0.41, size = 188, normalized size = 2.85

$$\frac{\sqrt{2}\,a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(-\frac{2\left(\sqrt{2}\,a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}+2\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A-B)\right)}{-4i\,A+4\,B}\right)-\sqrt{2}\,a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(\frac{2\left(\sqrt{2}\,a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}-2\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A-B)\right)}{-4i\,A+4\,B}\right)-2\sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,Be^{x}-B)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(-2*(sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a) + 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(2*(sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a) - 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - 2*sqrt(1/2*I*a*e^(-x))*(I*B*e^x - B))/a

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sinh{(x)}}{\sqrt{ia\left(\sinh{(x)}-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(1/2),x)

[Out] Integral((A + B*sinh(x))/sqrt(I*a*(sinh(x) - I)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")

[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\,\mathrm{sinh}(x)}{\sqrt{a+a\,\mathrm{sinh}\left(x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2),x)

[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2), x)

3.124 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{(iA+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

```
[Out] 1/2*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(3/2)+1/4*(I*A+3*B)*arctanh(1/2*cosh(x)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2829, 2728, 212}

$$\frac{(3B+iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((I*A + 3*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(2
*Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(A-3iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{4a}$$
$$= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(iA+3B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{2a}$$
$$= \frac{(iA+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

Mathematica [A]
time = 0.18, size = 105, normalized size = 1.33

$$\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(i(A+iB)\cosh\left(\frac{x}{2}\right)+(A+iB)\sinh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}\,(A-3iB)\text{ArcTan}\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)(-i+\sinh(x))\right)}{2(a+ia\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
[Out] ((Cosh[x/2] + I*Sinh[x/2])*(I*(A + I*B)*Cosh[x/2] + (A + I*B)*Sinh[x/2] + (
1 + I)*(-1)^(1/4)*(A - (3*I)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(-I + Sinh[
x])))/(2*(a + I*a*Sinh[x])^(3/2))
```

Maple [F]
time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(54) = 108$.
time = 0.46, size = 264, normalized size = 3.34

$$\frac{\sqrt{\frac{1}{2}}\left(a^2e^{(2x)} - 2i\,a^2e^x - a^2\right)\sqrt{-\frac{A^2 - 6i\,AB - 9\,B^2}{a^3}}\log\left(\frac{\sqrt{\frac{1}{2}}a^2\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}} + \sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A+3\,B)}{i\,A+3\,B}\right) - \sqrt{\frac{1}{2}}\left(a^2e^{(2x)} - 2i\,a^2e^x - a^2\right)\sqrt{-\frac{A^2 - 6i\,AB - 9\,B^2}{a^3}}\log\left(-\frac{\sqrt{\frac{1}{2}}a^2\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}} - \sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(i\,A+3\,B)}{i\,A+3\,B}\right) - 2\left((i\,A - B)e^{(2x)} - (A + i\,B)e^x\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}}{2\left(a^2e^{(2x)} - 2i\,a^2e^x - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B
^2)/a^3)*log((sqrt(1/2)*a^2*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3) + sqrt(1/2*I
*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x
 - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3)*log(-(sqrt(1/2)*a^2*sqrt(-(A^2 -
 6*I*A*B - 9*B^2)/a^3) - sqrt(1/2*I*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - 2
*((I*A - B)*e^(2*x) - (A + I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^2*e^(2*x) - 2
*I*a^2*e^x - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh(x)}{(ia(\sinh(x) - i))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(3/2),x)
```

```
[Out] Integral((A + B*sinh(x))/(I*a*(sinh(x) - I))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\sinh(x)}{(a + a\sinh(x)\ 1\mathrm{i})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2), x)
```

3.125 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{(3iA+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}+\frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}$$

```
[Out] 1/4*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(5/2)+1/16*(3*I*A+5*B)*cosh(x)/a/(a+I*a
*sinh(x))^(3/2)+1/32*(3*I*A+5*B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a
*sinh(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2829, 2729, 2728, 212}

$$\frac{(5B+3iA)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}+\frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] (((3*I)*A + 5*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])]
)/(16*Sqrt[2]*a^(5/2)) + ((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) +
(((3*I)*A + 5*B)*Cosh[x])/(16*a*(a + I*a*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3A-5iB)\int \frac{1}{(a+ia\sinh(x))^{3/2}}\,dx}{8a}$$

$$= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3A-5iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}}{32a^2}$$

$$= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3iA+5B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x,\right.}{16a^2}$$

$$= \frac{(3iA+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}\,a^{5/2}} + \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5}{16a(a+i}$$

Mathematica [A]
time = 0.16, size = 184, normalized size = 1.67

$$\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(4i(A+iB)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)+(3iA+5B)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3+(1-i)\sqrt[4]{-1}\,(3A-5iB)\text{ArcTan}\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^4+8(A+iB)\sinh\left(\frac{x}{2}\right)+2(3iA+5B)\sinh\left(\frac{x}{2}\right)(-i+\sinh(x))\right)}{16(a+ia\sinh(x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
[Out] ((Cosh[x/2] + I*Sinh[x/2])*((4*I)*(A + I*B)*(Cosh[x/2] + I*Sinh[x/2]) + ((3
*I)*A + 5*B)*(Cosh[x/2] + I*Sinh[x/2])^3 + (1 - I)*(-1)^(1/4)*(3*A - (5*I)*
B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(Cosh[x/2] + I*Sinh[x/2])^4 + 8*(A + I*B
)*Sinh[x/2] + 2*((3*I)*A + 5*B)*Sinh[x/2]*(-I + Sinh[x])))/(16*(a + I*a*Sin
h[x])^(5/2))
```

Maple [F]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{A+B\sinh\left(x\right)}{\left(a+ia\sinh\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(77) = 154$.
time = 0.39, size = 347, normalized size = 3.15

$$\frac{\sqrt{\frac{1}{2}}\left(a^{3}e^{(4x)} - 4i\,a^{3}e^{(3x)} - 6\,a^{3}e^{(2x)} + 4i\,a^{3}e^{x} + a^{3}\right)\sqrt{-\frac{9\,A^{2} - 30i\,AB - 25\,B^{2}}{a^{5}}}\log\left(\frac{\sqrt{\frac{1}{2}}a^{3}\sqrt{-\frac{9\,A^{2} - 30i\,AB - 25\,B^{2}}{a^{5}}} + \sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(3i\,A + 5\,B)}{3i\,A + 5\,B}\right) - \sqrt{\frac{1}{2}}\left(a^{3}e^{(4x)} - 4i\,a^{3}e^{(3x)} - 6\,a^{3}e^{(2x)} + 4i\,a^{3}e^{x} + a^{3}\right)\sqrt{-\frac{9\,A^{2} - 30i\,AB - 25\,B^{2}}{a^{5}}}\log\left(-\frac{\sqrt{\frac{1}{2}}a^{3}\sqrt{-\frac{9\,A^{2} - 30i\,AB - 25\,B^{2}}{a^{5}}} - \sqrt{\frac{1}{2}i\,ae^{(-x)}}\,(3i\,A + 5\,B)}{3i\,A + 5\,B}\right) + 2\left((-3i\,A - 5\,B)e^{(4x)} - (11\,A + 3i\,B)e^{(3x)} + (-11i\,A + 3\,B)e^{(2x)} - (3\,A - 5i\,B)e^{x}\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}}{16\left(a^{3}e^{(4x)} - 4i\,a^{3}e^{(3x)} - 6\,a^{3}e^{(2x)} + 4i\,a^{3}e^{x} + a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e^
x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log((sqrt(1/2)*a^3*sqrt(-(9
*A^2 - 30*I*A*B - 25*B^2)/a^5) + sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/(3*I*A
 + 5*B)) - sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a
^3*e^x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log(-(sqrt(1/2)*a^3*sq
rt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5) - sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/
(3*I*A + 5*B)) + 2*((-3*I*A - 5*B)*e^(4*x) - (11*A + 3*I*B)*e^(3*x) + (-11*
I*A + 3*B)*e^(2*x) - (3*A - 5*I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^3*e^(4*x)
- 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e^x + a^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\sinh(x)}{(a + a\sinh(x)\ 1\mathrm{i})^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2), x)
```

3.126 $\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=259

$$\frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh($$

```
[Out] 2/35*(7*A*b+5*B*a)*cosh(x)*(a+b*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+b*sinh(x))^
(5/2)+2/105*(56*A*a*b+15*B*a^2-25*B*b^2)*cosh(x)*(a+b*sinh(x))^(1/2)+2/105*
I*(161*A*a^2*b-63*A*b^3+15*B*a^3-145*B*a*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/105*I*(a^2+b^2)*(5
6*A*a*b+15*B*a^2-25*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x
)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(
a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{2}{105}\cosh(x)\left(15a^2B+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}-\frac{2i(a^2+b^2)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{105b\sqrt{a+b\sinh(x)}}+\frac{2i\left(15a^3B+161a^2Ab-145ab^2B-63Ab^3\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{35}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/105 + (2*(
7*A*b + 5*a*B)*Cosh[x]*(a + b*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + b*Sinh
[x])^(5/2))/7 + (((2*I)/105)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2
*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt
[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/105)*(a^2 + b^2)*(56*a*A*b + 15*a^2*
B - 25*b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+\frac{2}{7}\int (a+b\sinh(x))^{3/2}\left(\frac{1}{2}(7aA-\right.$$

$$= \frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sin$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7A$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7A$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7A$$

$$= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7A$$

Mathematica [A]
time = 0.61, size = 241, normalized size = 0.93

$$\frac{\frac{2i\left(b(105a^3A-119aAb^2-135a^2bB+25b^3B)F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)+(161a^2Ab-63Ab^3+15a^3B-145ab^2B)\left((a-ib)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-aF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b}+\cosh(x)(a+b\sinh(x))(154aAb+90a^2B-65b^2B+15b^2B\cosh(2x)+6b(7Ab+15aB)\sinh(x))}{105\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (((2*I)*(b*(105*a^3*A - 119*a*A*b^2 - 135*a^2*b*B + 25*b^3*B)*EllipticF[(Pi
 - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + (161*a^2*A*b - 63*A*b^3 + 15*a^3*B -
 145*a*b^2*B)*((a - I*b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]
- a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x]
)/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x])*(154*a*A*b + 90*a^2*B - 65*b^2*B
+ 15*b^2*B*Cosh[2*x] + 6*b*(7*A*b + 15*a*B)*Sinh[x]))/(105*Sqrt[a + b*Sinh[
x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1892 vs. 2(279) = 558.
time = 1.19, size = 1893, normalized size = 7.31

method	result	size
default	Expression too large to display	1893

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(-10*B*b^5*sinh(x)^3+21*A*b^5*sinh(x)^2-25*B*b^5*sinh(x)+15*B*b^5*sin
h(x)^5+21*A*b^5*sinh(x)^4+77*A*a^2*b^3+45*B*a^3*b^2-25*B*a*b^4+130*B*(-(a+b
*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-
a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2)
)*a^3*b^2+145*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2
)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-
(I*b-a)/(I*b+a))^(1/2))*a*b^4-161*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh
(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x
))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b+42*A*(-(a+b*sinh(x))/(I*b
-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elli
pticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^3-98*A
*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*
b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a)
)^(1/2))*a^2*b^3-120*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a
))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(
1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2-120*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4+15*I*B*(-(a+b*si
nh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))
^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a
^4*b-10*I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*a^2*b^3+56*I*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(
x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x)
)/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2+56*I*A*(-(a+b*sinh(x))/(
I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*E
llipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4+45*
B*a^3*b^2*sinh(x)^2+35*B*a*b^4*sinh(x)^2+98*A*a*b^4*sinh(x)+90*B*a^2*b^3*si
nh(x)-25*I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
(I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b^5+60*B*a*b^4*sinh(x)^4+98*A*a*b^4*sinh(x)^3+90*B*a^2
*b^3*sinh(x)^3+77*A*a^2*b^3*sinh(x)^2+105*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*
((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+
b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b-15*B*(-(a+b*sinh(
x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1
/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^5-
63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(
```

```
x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*
b+a))^(1/2))*b^5+63*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a)
)^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1
/2),(-(I*b-a)/(I*b+a))^(1/2))*b^5)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 1139, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
[Out] -1/1260*(8*(sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a*b^3 - 7
5*B*b^4)*cosh(x)^3 + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*
A*a*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 1
15*B*a^2*b^2 + 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(30*B*a^
4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqrt(b)*
weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 24*(sqrt(2)*(15*B*a^3*b + 161*A*a
^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^3 + 3*sqrt(2)*(15*B*a^3*b + 161*A*
a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(15*B*a^3*b
 + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(15*
B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*sinh(x)^3)*sqrt(b)*weiers
trassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cos
h(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(15*B*b^4*cosh(x)^6 + 15*B*b^4*sinh(x)^6
+ 6*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^5 + 6*(15*B*b^4*cosh(x) + 15*B*a*b^3 + 7
*A*b^4)*sinh(x)^5 + 15*B*b^4 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*co
sh(x)^4 + (225*B*b^4*cosh(x)^2 + 180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4 +
30*(15*B*a*b^3 + 7*A*b^4)*cosh(x))*sinh(x)^4 - 8*(15*B*a^3*b + 161*A*a^2*b^
2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^3 + 4*(75*B*b^4*cosh(x)^3 - 30*B*a^3*b
- 322*A*a^2*b^2 + 290*B*a*b^3 + 126*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(
x)^2 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x))*sinh(x)^3 + (180*
B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x)^2 + (225*B*b^4*cosh(x)^4 + 180
*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4 + 60*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^3
```

```
+ 6*(180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x)^2 - 24*(15*B*a^3*b +
161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x))*sinh(x)^2 - 6*(15*B*a*b^3
+ 7*A*b^4)*cosh(x) + 2*(45*B*b^4*cosh(x)^5 - 45*B*a*b^3 - 21*A*b^4 + 15*(15
*B*a*b^3 + 7*A*b^4)*cosh(x)^4 + 2*(180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)
*cosh(x)^3 - 12*(15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(
x)^2 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x))*sinh(x))*sqrt(b*s
inh(x) + a))/(b^2*cosh(x)^3 + 3*b^2*cosh(x)^2*sinh(x) + 3*b^2*cosh(x)*sinh(
x)^2 + b^2*sinh(x)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(5/2)*(A+B*sinh(x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2), x)
```

3.127 $\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal. Leaf size=207

$$\frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2i(20aAb+3a^2B-9b^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\right.}{15b\sqrt{\frac{a+b\sinh(}{a-ib}}}$$

```
[Out] 2/5*B*cosh(x)*(a+b*sinh(x))^(3/2)+2/15*(5*A*b+3*B*a)*cosh(x)*(a+b*sinh(x))^
(1/2)+2/15*I*(20*A*a*b+3*B*a^2-9*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1
/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+
b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/15*I*(a^2+b^2)*(5*A*b+3*
B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi
+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*s
inh(x))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2i(a^2+b^2)(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{15b\sqrt{a+b\sinh(x)}}+\frac{2i(3a^2B+20aAb-9b^2B)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{15}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*B*Cosh[x]*(a + b*Si
nh[x])^(3/2))/5 + (((2*I)/15)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/15)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2}{5}\int \sqrt{a+b\sinh(x)}\left(\frac{1}{2}(5aA - 3\right.$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh$$

Mathematica [A]
time = 0.46, size = 196, normalized size = 0.95

$$\frac{2\left(\frac{i\left(b(15a^2A-5Ab^2-12abB)F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+(20aAb+3a^2B-9b^2B)\left((a-ib)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)-aF\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b}+\cosh(x)(a+b\sinh(x))(5Ab+6aB+3bB\sinh(x))\right)}{15\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*((I*(b*(15*a^2*A - 5*A*b^2 - 12*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*
I)*b)/(a - I*b)] + (20*a*A*b + 3*a^2*B - 9*b^2*B)*((a - I*b)*EllipticE[(Pi
- (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)
*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x])/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x
])*(5*A*b + 6*a*B + 3*b*B*Sinh[x])))/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(231) = 462.
time = 1.29, size = 1037, normalized size = 5.01

method	result	size
default	Expression too large to display	1037

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(B*b^2*(2/5/b*sinh(x)*(cosh(x)^2*(a+b*sinh(
x)))^(1/2)-8/15*a/b^2*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-4/15*a/b*(a/b-I)*((-b
*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*
b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*(-3/5+8/15*a^2/b^2)*(a/b-I)*((-b*sinh(
x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^
(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(
I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))
^(1/2),((a-I*b)/(I*b+a))^(1/2))))+(A*b^2+2*B*a*b)*(2/3/b*(cosh(x)^2*(a+b*si
nh(x)))^(1/2)-2/3*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*
b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*E
llipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))-4/3*a/b*(a
/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE((
(-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sin
h(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+2*(2*A*a*b+B*a^2)*(a/b-I)
*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*
b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-b*s
inh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-
a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))+2*a^2*A*(a/b-I)*((-b*sinh(x)-a)
/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)
/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),(
(a-I*b)/(I*b+a))^(1/2)))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 635, normalized size = 3.07

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)^2 +
 2*sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)*sinh(x) +
```

```
sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*sinh(x)^2)*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3
*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(3*B*a^2*b + 20*A*a*b^2
 - 9*B*b^3)*cosh(x)^2 + 2*sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x
)*sinh(x) + sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*sinh(x)^2)*sqrt(b)*w
eierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*
b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b^3*sinh(x)
^4 - 3*B*b^3 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*cosh(x) + 6*B
*a*b^2 + 5*A*b^3)*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x)^
2 + 2*(9*B*b^3*cosh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 + 18*B*b^3 + 3*(6*B*a*b^2
 + 5*A*b^3)*cosh(x))*sinh(x)^2 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x) + 2*(6*B*b
^3*cosh(x)^3 + 6*B*a*b^2 + 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^2 - 4*
(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(
b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sinh(x))(a + b\sinh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(3/2)*(A+B*sinh(x)),x)
[Out] Integral((A + B*sinh(x))*(a + b*sinh(x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\sinh(x))\,(a + b\sinh(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2), x)
```

3.128 $\int \sqrt{a+b\sinh(x)}\,(A+B\sinh(x))\,dx$

Optimal. Leaf size=164

$$\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)\,BF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\ldots}}{3b\sqrt{a+b\sinh(x\ldots}}$$

```
[Out] 2/3*B*cosh(x)*(a+b*sinh(x))^(1/2)+2/3*I*(3*A*b+B*a)*(sin(1/4*Pi+1/2*I*x)^2)
^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b)
)^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2
)*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*si
nh(x))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2iB(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\sqrt{a+b\sinh(x)}}+\frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)/3)*(3*A*b + a*B)*EllipticE[Pi
/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])
/(a - I*b)]) - (((2*I)/3)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*
a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\sinh(x)}\,(A+B\sinh(x))\,dx = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA-bB)+\frac{1}{2}(3Ab+aB)\,\mathrm{si}}{\sqrt{a+b\sinh(x)}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} - \frac{((a^2+b^2)\,B)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{\left((3Ab+aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{}}{3b\sqrt{\frac{a+b\sinh(x}{a-ib}}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [A]
time = 0.26, size = 151, normalized size = 0.92

$$\frac{2bB\cosh(x)(a+b\sinh(x))+2(ia+b)(3Ab+aB)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)\,BF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*b*B*Cosh[x]*(a + b*Sinh[x]) + 2*(I*a + b)*(3*A*b + a*B)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(
a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/(3*b*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(192) = 384.
time = 1.12, size = 897, normalized size = 5.47

method	result
default	$\dfrac{\dfrac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\dfrac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+s
inh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
/(I*b+a))^(1/2))*a^2*b+I*B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*(
(I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b
*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b+3*A*(-(a+b*sinh(x)
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3-3*
A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*a^2*b-3*A*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^
(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2
),(-(I*b-a)/(I*b+a))^(1/2))*b^3-B*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x
))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3-B*(-(a+b*sinh(x))/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((
-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2+B*b^3*sinh(x)
^3+B*a*b^2*sinh(x)^2+B*b^3*sinh(x)+B*a*b^2)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 324, normalized size = 1.98

$$-\frac{2\left(\sqrt{2}\,(2\,Ba^2-3\,Aab+3\,Bb^2)\cosh(x)+\sqrt{2}\,(2\,Ba^2-3\,Aab+3\,Bb^2)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+6\left(\sqrt{2}\,(Bab+3\,Ab^2)\cosh(x)+\sqrt{2}\,(Bab+3\,Ab^2)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)\right)-3\left(Bb^2\cosh(x)^2+Bb^2\sinh(x)^2+Bb^2-2\,(Bab+3\,Ab^2)\cosh(x)+2\,(Bb^2\cosh(x)-Bab-3\,Ab^2)\sinh(x)\right)\sqrt{b\sinh(x)+a}}{9\,(b^2\cosh(x)+b^2\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(sqrt(2)*(2*B*a^2 - 3*A*a*b + 3*B*b^2)*cosh(x) + sqrt(2)*(2*B*a^2 -
 3*A*a*b + 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2
)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b
) + 6*(sqrt(2)*(B*a*b + 3*A*b^2)*cosh(x) + sqrt(2)*(B*a*b + 3*A*b^2)*sinh(x
))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/
```

```
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2 + B*b^2
*sinh(x)^2 + B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x) - B*a*b
 - 3*A*b^2)*sinh(x))*sqrt(b*sinh(x) + a))/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B \sinh(x)) \sqrt{a + b \sinh(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))**(1/2)*(A+B*sinh(x)),x)
[Out] Integral((A + B*sinh(x))*sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sinh(x)) \sqrt{a + b \sinh(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2), x)
```

3.129 $\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=55

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] B*x/b-2*(A*b-B*a)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2814, 2739, 632, 212}

$$\frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b - (2*(A*b - a*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqr
t[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{Bx}{b}-\frac{(i(iAb-iaB))\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{(2i(iAb-iaB))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{(4i(iAb-iaB))\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 1.11

$$\frac{Bx+\frac{2(Ab-aB)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*x + (2*(A*b - a*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2])/b
```

Maple [A]
time = 0.36, size = 72, normalized size = 1.31

method	result
default	$-\frac{2(-Ab+Ba)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$\frac{Bx}{b}+\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)Ba}{\sqrt{a^2+b^2}\,b}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-A*b+B*a)/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)
^(1/2))-B/b*ln(tanh(1/2*x)-1)+B/b*ln(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.
time = 0.50, size = 124, normalized size = 2.25

$$-B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{x}{b}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^
(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(51) = 102$.
time = 0.45, size = 147, normalized size = 2.67

$$-\frac{(Ba-Ab)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(Ba^2+Bb^2)x}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a - A*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x)/(a^2*b + b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(46) = 92$.

time = 48.92, size = 422, normalized size = 7.67

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx\right) & \text{for } a=0 \wedge b=0 \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx}{b} & \text{for } a=0 \\ \frac{Ax+B\cosh(x)}{a} & \text{for } b=0 \\ -\frac{2Ab}{b^2+b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}+\frac{Bbx}{b^2+b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}+\frac{Bx\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}{b^2+b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}-\frac{2B\sqrt{-b^2}}{b^2+b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)} & \text{for } a=-\sqrt{-b^2} \\ -\frac{2Ab}{b^2-b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}+\frac{Bbx}{b^2-b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}-\frac{Bx\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}{b^2-b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)}+\frac{2B\sqrt{-b^2}}{b^2-b\sqrt{-b^2}\tanh\left(\frac{x}{2}\right)} & \text{for } a=\sqrt{-b^2} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}-\frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}}+\frac{Bx}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh
(x/2)) + B*x)/b, Eq(a, 0)), ((A*x + B*cosh(x))/a, Eq(b, 0)), (-2*A*b/(b**2
+ b*sqrt(-b**2)*tanh(x/2)) + B*b*x/(b**2 + b*sqrt(-b**2)*tanh(x/2)) + B*x*s
qrt(-b**2)*tanh(x/2)/(b**2 + b*sqrt(-b**2)*tanh(x/2)) - 2*B*sqrt(-b**2)/(b*
*2 + b*sqrt(-b**2)*tanh(x/2)), Eq(a, -sqrt(-b**2))), (-2*A*b/(b**2 - b*sqrt
(-b**2)*tanh(x/2)) + B*b*x/(b**2 - b*sqrt(-b**2)*tanh(x/2)) - B*x*sqrt(-b**
2)*tanh(x/2)/(b**2 - b*sqrt(-b**2)*tanh(x/2)) + 2*B*sqrt(-b**2)/(b**2 - b*s
qrt(-b**2)*tanh(x/2)), Eq(a, sqrt(-b**2))), (-A*log(tanh(x/2) - b/a - sqrt(
a**2 + b**2)/a)/sqrt(a**2 + b**2) + A*log(tanh(x/2) - b/a + sqrt(a**2 + b**
2)/a)/sqrt(a**2 + b**2) + B*a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b
*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqr
t(a**2 + b**2)) + B*x/b, True))
```

Giac [A]
time = 0.43, size = 75, normalized size = 1.36

$$\frac{Bx}{b}-\frac{(Ba-Ab)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Mupad [B]
time = 0.93, size = 269, normalized size = 4.89

$$\frac{B\,x}{b}-\frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{-a^2\,b^2-b^4}\,\left(\frac{2\,\left(A\,b\,\sqrt{-a^2\,b^2-b^4}-B\,a\,\sqrt{-a^2\,b^2-b^4}\right)}{b^4\,\sqrt{-a^2\,b^2-b^4}\,\sqrt{{\left(A\,b-B\,a\right)}^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\sqrt{-b^2\,\left(a^2+b^2\right)}\,\sqrt{-a^2\,b^2-b^4}\,\left(A\,b-B\,a\right)}\right)}{2}-\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{-b^2\,\left(a^2+b^2\right)}\,\left(A\,b-B\,a\right)}\right)\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{-a^2\,b^2-b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*x)/b - (2*atan((b^2*exp(x)*(- b^4 - a^2*b^2)^(1/2)*((2*(A*b*(- b^4 - a^2
*b^2)^(1/2) - B*a*(- b^4 - a^2*b^2)^(1/2)))/(b^4*(- b^4 - a^2*b^2)^(1/2)*((
A*b - B*a)^2)^(1/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(
-b^2*(a^2 + b^2))^(1/2)*(- b^4 - a^2*b^2)^(1/2)*(A*b - B*a))))/2 - (a*b*(A^
2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((-b^2*(a^2 + b^2))^(1/2)*(A*b - B*a)))
*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(- b^4 - a^2*b^2)^(1/2)
```

3.130 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

```
[Out] -2*(A*a+B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-(A*
b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2833, 12, 2739, 632, 212}

$$-\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{(a^2+b^2)\,(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}-\frac{\int \frac{-aA-bB}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{(aA+bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{(2(aA+bB))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}-\frac{(4(aA+bB))\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\right.}{a^2+b^2}\\
&= -\frac{2(aA+bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 1.11

$$
\frac{\frac{2(aA+bB)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{(-Ab+aB)\cosh(x)}{a+b\sinh(x)}}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*(a*A + b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + ((-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x]))/(a^2 + b^2)
```

Maple [A]
time = 0.46, size = 113, normalized size = 1.53

method	result
default	$-\frac{2\left(-\frac{b(Ab-Ba)\tanh\left(\frac{x}{2}\right)}{a\left(a^{2}+b^{2}\right)}-\frac{Ab-Ba}{a^{2}+b^{2}}\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{2(Aa+bB)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2(Ab-Ba)(a\,\mathrm{e}^{x}-b)}{b(a^{2}+b^{2})(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)Aa}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)bB}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\left(a^{2}+b\right.}{}\right.}{(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-b*(A*b-B*a)/a/(a^2+b^2)*tanh(1/2*x)-(A*b-B*a)/(a^2+b^2))/(a*tanh(1/2*x
)^2-2*b*tanh(1/2*x)-a)+2*(A*a+B*b)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(69) = 138.
time = 0.51, size = 229, normalized size = 3.09

$$A\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\left(ae^{(-x)}+b\right)}{a^{2}b+b^{3}+2\left(a^{3}+ab^{2}\right)e^{(-x)}-\left(a^{2}b+b^{3}\right)e^{(-2x)}}\right)+B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{2\left(a^{2}e^{(-x)}+ab\right)}{a^{2}b^{2}+b^{4}+2\left(a^{3}b+ab^{3}\right)e^{(-x)}-\left(a^{2}b^{2}+b^{4}\right)e^{(-2x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] A*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(a*e^(-x) + b)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x)
 - (a^2*b + b^3)*e^(-2*x))) + B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^
2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e^(-x) - (a^2*b^2 + b^4)*e^(-2*x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(69) = 138.
time = 0.37, size = 444, normalized size = 6.00

$$-\frac{2\,Ba^{3}b-2\,Aa^{2}b^{2}+2\,Bab^{3}-2\,Ab^{4}-\left(Aab^{2}+Bb^{3}-\left(Aab^{2}+Bb^{3}\right)\cosh\left(x\right)^{2}-\left(Aab^{2}+Bb^{3}\right)\sinh\left(x\right)^{2}-2\left(Aa^{2}b+Bab^{2}\right)\cosh\left(x\right)-2\left(Aa^{2}b+Bab^{2}+\left(Aab^{2}+Bb^{3}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}+b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)-2\sqrt{a^{2}+b^{2}}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-2\left(Ba^{4}-Aa^{3}b+Ba^{2}b^{2}-Aab^{3}\right)\cosh\left(x\right)-2\left(Ba^{4}-Aa^{3}b+Ba^{2}b^{2}-Aab^{3}\right)\sinh\left(x\right)}{a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}-\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)\cosh\left(x\right)^{2}-\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)\sinh\left(x\right)^{2}-2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\cosh\left(x\right)-2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}+\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)\cosh\left(x\right)\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4 - (A*a*b^2 + B*b^3 - (A*a*b
^2 + B*b^3)*cosh(x)^2 - (A*a*b^2 + B*b^3)*sinh(x)^2 - 2*(A*a^2*b + B*a*b^2)
```

```
*cosh(x) - 2*(A*a^2*b + B*a*b^2 + (A*a*b^2 + B*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*cosh(x) - 2*(B*a^4 - A*a
^3*b + B*a^2*b^2 - A*a*b^3)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2
 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 119, normalized size = 1.61

$$\frac{(Aa+Bb)\log\left(\left|\frac{2\,be^x+2\,a-2\,\sqrt{a^2+b^2}}{2\,be^x+2\,a+2\,\sqrt{a^2+b^2}}\right|\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(Ba^2e^x-Aabe^x-Bab+Ab^2)}{(a^2b+b^3)(be^{(2\,x)}+2\,ae^x-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (A*a + B*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a +
2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(B*a^2*e^x - A*a*b*e^x - B*a*b +
A*b^2)/((a^2*b + b^3)*(b*e^(2*x) + 2*a*e^x - b))
```

Mupad [**B**]
time = 0.88, size = 223, normalized size = 3.01

$$\frac{\ln\left(\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}}-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}}-\frac{\ln\left(-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}-\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}}-\frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b+b^3)}+\frac{2\,e^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^2,x)
```

```
[Out] (log((2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3/2)) - (2*exp(x)*(A*a
+ B*b))/(a^2*b + b^3))*(A*a + B*b))/(a^2 + b^2)^(3/2) - (log(- (2*exp(x)*(A
*a + B*b))/(a^2*b + b^3) - (2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3
/2)))*(A*a + B*b))/(a^2 + b^2)^(3/2) - ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b + b
^3)) + (2*exp(x)*(B*a^2*b^2 - A*a*b^3))/(b^2*(a^2*b + b^3)))/(2*a*exp(x) -
b + b*exp(2*x))
```

3.131 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{(2a^2A-Ab^2+3abB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2}-\frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2(a^2+b^2)^2(a+b\sinh(x))}$$

```
[Out] -(2*A*a^2-A*b^2+3*B*a*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(5/2)-1/2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^2-1/2*(3*A*a*b-B*a^2
+2*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))
```

Rubi [A]
time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2833, 12, 2739, 632, 212}

$$-\frac{(2a^2A+3abB-Ab^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2(a^2+b^2)^2(a+b\sinh(x))}-\frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - A*b^2 + 3*a*b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(5/2)) - ((A*b - a*B)*Cosh[x])/(2*(a^2 + b^2)*(a + b*Sinh[x])^2
) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx &= -\frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{\int \frac{-2(aA+bB)+(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}\,dx}{2\,(a^2+b^2)} \\
&= -\frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2\,(a^2+b^2)^2\,(a+b\sinh(x))} + \frac{\int \frac{2a^2A-Ab^2+3abB}{a+b\sinh(x)}\,d}{2\,(a^2+b^2)^2} \\
&= -\frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2\,(a^2+b^2)^2\,(a+b\sinh(x))} + \frac{(2a^2A-Ab^2+3}{2\,(a^2} \\
&= -\frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2\,(a^2+b^2)^2\,(a+b\sinh(x))} + \frac{(2a^2A-Ab^2+3}{} \\
&= -\frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2\,(a^2+b^2)^2\,(a+b\sinh(x))} - \frac{(2(2a^2A-Ab^2+}{} \\
&= -\frac{(2a^2A-Ab^2+3abB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2\,(a^2+b^2)\,(a+b\sinh(x))^2} - \frac{(3a}{2}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 131, normalized size = 1.02

$$\frac{\frac{2(2a^2A-Ab^2+3abB)\,\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{(a^2+b^2)(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^2} + \frac{(-3aAb+a^2B-2b^2B)\cosh(x)}{a+b\sinh(x)}}{2\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] ((2*(2*a^2*A - A*b^2 + 3*a*b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] + ((a^2 + b^2)*(-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x])^2
+ ((-3*a*A*b + a^2*B - 2*b^2*B)*Cosh[x])/(a + b*Sinh[x]))/(2*(a^2 + b^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(118) = 236$.
time = 0.52, size = 314, normalized size = 2.45

method	result
default	$-\frac{2\left(-\frac{b\left(5A\,a^2b+2A\,b^3-3a^3B\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(4A\,a^4b-7A\,a^2b^3-2A\,b^5-2B\,a^5+5B\,a^3b^2-2Ba\,b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b\left(11A\,a^2b+2A\,b^3-5a^3B+4Ba\right.}{2\left(a^4+2a^2b^2+b^4\right)a}\right.}{\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)^2}$
risch	$\frac{2A\,a^2b^2\mathrm{e}^{3x}-A\,b^4\mathrm{e}^{3x}+3Ba\,b^3\mathrm{e}^{3x}+6A\,a^3b\,\mathrm{e}^{2x}-3Aa\,b^3\mathrm{e}^{2x}-2B\,a^4\mathrm{e}^{2x}+5B\,a^2b^2\mathrm{e}^{2x}-2B\,b^4\mathrm{e}^{2x}-10A\,a^2b^2\mathrm{e}^{x}-A\,b^4\mathrm{e}^{x}+4B\,a^3b\,\mathrm{e}^{x}-5Ba\,b^3}{b(a^2+b^2)^2(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*b*(5*A*a^2*b+2*A*b^3-3*B*a^3)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*x)^3-
1/2*(4*A*a^4*b-7*A*a^2*b^3-2*A*b^5-2*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4+2*a^
2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/2*b*(11*A*a^2*b+2*A*b^3-5*B*a^3+4*B*a*b^2)/(
a^4+2*a^2*b^2+b^4)/a*tanh(1/2*x)+1/2*(4*A*a^2*b+A*b^3-2*B*a^3+B*a*b^2)/(a^4
+2*a^2*b^2+b^4))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)^2+(2*A*a^2-A*b^2+3*B*a
*b)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(119) = 238$.
time = 0.54, size = 537, normalized size = 4.20

$$\frac{1}{2}\left(\frac{3\,ab\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(3\,ab^3e^{(-3\,x)}+a^2b^2-2\,b^4+(4\,a^3b-5\,ab^3)e^{(-x)}+(2\,a^4-5\,a^2b^2+2\,b^4)e^{(-2\,x)}\right)}{a^4b^3+2\,a^2b^5+b^7+4\,(a^5b^2+2\,a^3b^4+ab^6)e^{(-x)}+2\,(2\,a^6b+3\,a^4b^3-b^7)e^{(-2\,x)}-4\,(a^5b^2+2\,a^3b^4+ab^6)e^{(-3\,x)}+(a^4b^3+2\,a^2b^5+b^7)e^{(-4\,x)}}\right)B+\frac{1}{2}\,A\left(\frac{(2\,a^2-b^2)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,ab^2+(10\,a^2b+b^3)e^{(-x)}+3\,(2\,a^3-ab^2)e^{(-2\,x)}-(2\,a^2b-b^3)e^{(-3\,x)}\right)}{a^4b^2+2\,a^2b^4+b^6+4\,(a^5b+2\,a^3b^3+ab^5)e^{(-x)}+2\,(2\,a^6+3\,a^4b^2-b^6)e^{(-2\,x)}-4\,(a^5b+2\,a^3b^3+ab^5)e^{(-3\,x)}+(a^4b^2+2\,a^2b^4+b^6)e^{(-4\,x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^3*e^(-3*x) + a^
2*b^2 - 2*b^4 + (4*a^3*b - 5*a*b^3)*e^(-x) + (2*a^4 - 5*a^2*b^2 + 2*b^4)*e^
(-2*x))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*e^(-x)
 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*e^(-2*x) - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)
*e^(-3*x) + (a^4*b^3 + 2*a^2*b^5 + b^7)*e^(-4*x)))*B + 1/2*A*((2*a^2 - b^2)
*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a
```

```
^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a*b^2 + (10*a^2*b + b^3)*e^(-
x) + 3*(2*a^3 - a*b^2)*e^(-2*x) - (2*a^2*b - b^3)*e^(-3*x))/(a^4*b^2 + 2*a^
2*b^4 + b^6 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-x) + 2*(2*a^6 + 3*a^4*b^2 -
 b^6)*e^(-2*x) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*x) + (a^4*b^2 + 2*a^2*
b^4 + b^6)*e^(-4*x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. $2(119) = 238$.
time = 0.45, size = 1614, normalized size = 12.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 - 2*B*a^2*b^4 - 6*A*a*b^5 - 4*B*b^6 - 2*(2*
A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*A
*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B*
a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B
*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 2*B*b^6 - 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5 +
(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*
b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^3 + 4*(2*A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x
))*sinh(x)^3 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ 3*(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 + 3*B*a^2*
b^3 - A*a*b^4)*cosh(x))*sinh(x)^2 - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)
*cosh(x) - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 - (2*A*a^2*b^3 + 3*B*a*b^
4 - A*b^5)*cosh(x)^3 - 3*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^2 -
(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x
))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*a^2*b^4 - 5
*B*a*b^5 - A*b^6)*cosh(x) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*
B*a*b^5 - A*b^6)*cosh(x)^2 - 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3
*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 + 3*a^
4*b^5 + 3*a^2*b^7 + b^9 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^4
 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sinh(x)^4 + 4*(a^7*b^2 + 3*a^5*b
^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^3 + 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*
b^8 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^8
*b + 5*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b + 5*a^6*
```

```
b^3 + 3*a^4*b^5 - a^2*b^7 - b^9 + 3*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)
*cosh(x)^2 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2
 - 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x) - 4*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8 - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^
3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^2 - (2*a^8*b + 5*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(119) = 238.
time = 0.44, size = 279, normalized size = 2.18

$$-\frac{\left(2 \, A a^{2} + 3 \, B a b - A b^{2}\right) \log\left(\frac{{\left| -2 \, b e^{x} - 2 \, a - 2 \, \sqrt{a^{2} + b^{2}} \right|}}{{\left| -2 \, b e^{x} - 2 \, a + 2 \, \sqrt{a^{2} + b^{2}} \right|}}\right)}{2 \, {\left(a^{4} + 2 \, a^{2} b^{2} + b^{4}\right)} \sqrt{a^{2} + b^{2}}} + \frac{2 \, A a^{2} b^{2} e^{\left(3 \, x\right)} + 3 \, B a b^{3} e^{\left(3 \, x\right)} - A b^{4} e^{\left(3 \, x\right)} - 2 \, B a^{4} e^{\left(2 \, x\right)} + 6 \, A a^{3} b e^{\left(2 \, x\right)} + 5 \, B a^{2} b^{2} e^{\left(2 \, x\right)} - 3 \, A a b^{3} e^{\left(2 \, x\right)} - 2 \, B b^{4} e^{\left(2 \, x\right)} + 4 \, B a^{3} b e^{x} - 10 \, A a^{2} b^{2} e^{x} - 5 \, B a b^{3} e^{x} - A b^{4} e^{x} - B a^{2} b^{2} + 3 \, A a b^{3} + 2 \, B b^{4}}{{\left(a^{4} b + 2 \, a^{2} b^{3} + b^{5}\right)} {\left(b e^{\left(2 \, x\right)} + 2 \, a e^{x} - b\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*A*a^2 + 3*B*a*b - A*b^2)*log(abs(-2*b*e^x - 2*a - 2*sqrt(a^2 + b^2)
)/abs(-2*b*e^x - 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + (2*A*a^2*b^2*e^(3*x) + 3*B*a*b^3*e^(3*x) - A*b^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 4*B*a^3*b*e^x - 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^x - A*b^4
*e^x - B*a^2*b^2 + 3*A*a*b^3 + 2*B*b^4)/((a^4*b + 2*a^2*b^3 + b^5)*(b*e^(2*
x) + 2*a*e^x - b)^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\mathrm{sinh}\left(x\right)}{{\left(a+b\,\mathrm{sinh}\left(x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^3,x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^3, x)
```

3.132 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$

Optimal. Leaf size=187

$$-\frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3}-\frac{(5aAb-2a^2B+3b^2}{6(a^2+b^2)^2(a+b}$$

```
[Out] -(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1
/2))/(a^2+b^2)^(7/2)-1/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^3-1/6*(5
*A*a*b-2*B*a^2+3*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^2-1/6*(11*A*a^2*b
-4*A*b^3-2*B*a^3+13*B*a*b^2)*cosh(x)/(a^2+b^2)^3/(a+b*sinh(x))
```

Rubi [A]
time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2833, 12, 2739, 632, 212}

$$-\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2}-\frac{\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}-\frac{\left(2a^3A+4a^2bB-3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{6\left(a^2+b^2\right)^3(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2 + b^2)^(7/2)) - ((A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a
+ b*Sinh[x])^3) - ((5*a*A*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/(6*(a^2 + b^2)^2*
(a + b*Sinh[x])^2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 13*a*b^2*B)*Cosh[x]
)/(6*(a^2 + b^2)^3*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\int \frac{-3(aA+bB)+2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} + \frac{\int \frac{2(3a^2A-2Ab^2+5ab\ldots}{(\ldots}}{(\ldots}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3\ldots}{6\left(a^2+b\ldots}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3\ldots}{6\left(a^2+b\ldots}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3\ldots}{6\left(a^2+b\ldots}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3} - \frac{\left(5aAb-2a^2B+3b^2B\right)\cosh(x)}{6\left(a^2+b^2\right)^2(a+b\sinh(x))^2} - \frac{\left(11a^2Ab-4Ab^3\ldots}{6\left(a^2+b\ldots}$$

$$= -\frac{\left(2a^3A-3aAb^2+4a^2bB-b^3B\right)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x\ldots}$$

Mathematica [A]
time = 0.33, size = 189, normalized size = 1.01

$$\frac{\frac{6\left(2a^3A-3aAb^2+4a^2bB-b^3B\right)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{2\left(a^2+b^2\right)^2(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^3}+\frac{\left(a^2+b^2\right)\left(-5aAb+2a^2B-3b^2B\right)\cosh(x)}{(a+b\sinh(x))^2}+\frac{\left(-11a^2Ab+4Ab^3+2a^3B-13ab^2B\right)\cosh(x)}{a+b\sinh(x)}}{6\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Cosh[x])/
(a + b*Sinh[x])^3 + ((a^2 + b^2)*(-5*a*A*b + 2*a^2*B - 3*b^2*B)*Cosh[x])/(a
 + b*Sinh[x])^2 + ((-11*a^2*A*b + 4*A*b^3 + 2*a^3*B - 13*a*b^2*B)*Cosh[x])/
(a + b*Sinh[x]))/(6*(a^2 + b^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(175) = 350$.
time = 0.58, size = 633, normalized size = 3.39

method	result
default	$-\frac{2\left(-\frac{b\left(9A\,a^4b+6A\,a^2b^3+2A\,b^5-4B\,a^5+B\,a^3b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\left(6A\,a^6b-27A\,a^4b^3-12A\,a^2b^5-4A\,b^7-2B\,a^7+14B\,a^5b^2-11B\,a^3b^4-2Ba\,b^6\right)(}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)a^2}$
risch	$\frac{-13Ba\,b^5-11A\,a^2b^4+2B\,a^3b^3+66B\,a^2b^4\mathrm{e}^x+4A\,b^6-8B\,a^6\mathrm{e}^{3x}-12A\,b^6\mathrm{e}^{2x}-3B\,b^6\mathrm{e}^{5x}+3B\,b^6\mathrm{e}^x-102B\,a^3b^3\mathrm{e}^{2x}+24Ba\,b^5\mathrm{e}^{2x}+60A\,a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*b*(9*A*a^4*b+6*A*a^2*b^3+2*A*b^5-4*B*a^5+B*a^3*b^2)/a/(a^6+3*a^4*b
^2+3*a^2*b^4+b^6)*tanh(1/2*x)^5-1/2*(6*A*a^6*b-27*A*a^4*b^3-12*A*a^2*b^5-4*
A*b^7-2*B*a^7+14*B*a^5*b^2-11*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/a^2*tanh(1/2*x)^4+1/3/a^3*b*(54*A*a^6*b-21*A*a^4*b^3-4*A*a^2*b^5-4*A*
b^7-18*B*a^7+42*B*a^5*b^2-17*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6)*tanh(1/2*x)^3+1/a^2*(6*A*a^6*b-20*A*a^4*b^3-3*A*a^2*b^5-2*A*b^7-2*B*a^
7+10*B*a^5*b^2-14*B*a^3*b^4-B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*x)^2-1/2/a*b*(27*A*a^4*b+4*A*a^2*b^3+2*A*b^5-8*B*a^5+19*B*a^3*b^2+2*B*a*b^
4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*x)-1/6*(18*A*a^4*b+5*A*a^2*b^3+2*
A*b^5-6*B*a^5+10*B*a^3*b^2+B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6))/(a*tanh(
1/2*x)^2-2*b*tanh(1/2*x)-a)^3+(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(177) = 354$.
time = 0.57, size = 982, normalized size = 5.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*a^2 - 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a
 + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2))
- 2*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-x) + 6*(17*a^4*b - 6*a
^2*b^3 + 2*b^5)*e^(-2*x) + 2*(22*a^5 - 41*a^3*b^2 + 12*a*b^4)*e^(-3*x) - 15
*(2*a^4*b - 3*a^2*b^3)*e^(-4*x) + 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*x))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^
8)*e^(-x) + 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*x) +
 4*(2*a^9 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^
8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*x) + 6*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*x) - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9
)*e^(-6*x)))*A + 1/6*B*(3*(4*a^2*b - b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sqrt(a^2 + b^2)) + 2*(2*a^3*b^3 - 13*a*b^5 + 3*(4*a^4*b^2 - 22*a^2*b^4 - b^
6)*e^(-x) + 6*(4*a^5*b - 17*a^3*b^3 + 4*a*b^5)*e^(-2*x) + 2*(4*a^6 - 32*a^4
*b^2 + 39*a^2*b^4)*e^(-3*x) + 15*(4*a^3*b^3 - a*b^5)*e^(-4*x) - 3*(4*a^2*b^
4 - b^6)*e^(-5*x))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10 + 6*(a^7*b^3 + 3
*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-x) + 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^
6 + a^2*b^8 - b^10)*e^(-2*x) + 4*(2*a^9*b + 3*a^7*b^3 - 3*a^5*b^5 - 7*a^3*b
^7 - 3*a*b^9)*e^(-3*x) - 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^6 + a^2*b^8 -
b^10)*e^(-4*x) + 6*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-5*x) - (a^
6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*e^(-6*x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3870 vs. $2(177) = 354$.
time = 0.40, size = 3870, normalized size = 20.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 - 22*B*a^3*b^5 - 14*A*a^2*b^6 - 26*B*a*b^7
 + 8*A*b^8 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 - B*b^8)*cosh(x)^5 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^
2*b^6 - 3*A*a*b^7 - B*b^8)*sinh(x)^5 + 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^
4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^4 + 30*(2*A*a^6*b^2 +
4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7 + (2*A*a^5*b^
3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x))*sin
h(x)^4 - 4*(4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^
4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x)^3 - 4*(4*B*a^8 - 22*A
*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^
2*b^6 - 12*A*a*b^7 - 15*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^
6 - 3*A*a*b^7 - B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^
4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*b
```

- 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
+ 4*B*a*b^7 - 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*
b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8 + 5*(
2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*co
sh(x)^3 + 15*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2
*b^6 - B*a*b^7)*cosh(x)^2 - (4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5
*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x))*sin
h(x)^2 + 3*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7 - (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^6 - (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3
*A*a*b^6 - B*b^7)*sinh(x)^6 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^5 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 +
(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 - 3*(8*
A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*
cosh(x)^4 - 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*
A*a*b^6 + B*b^7 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)
^2 + 10*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x
)^4 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^
5 + 3*B*a*b^6)*cosh(x)^3 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B
*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b
^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b
^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5
+ 3*A*a*b^6 + B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 1
4*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 +
16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7 - 5*(2*A*a^3
*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^4 - 20*(2*A*a^4*b^3 + 4*B*a
^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 - 6*(8*A*a^5*b^2 + 16*B*a^4*b^3 -
14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 - 4*(4*A*a^6*b +
8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(
x))*sinh(x)^2 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(
x) - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 + 4*B*a^3*b^4 -
3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^
3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b + 8*B*a^5
*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x)^2 - (
8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7
)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*
a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^
2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 - 18*B*a^4*
b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 - B*b^8)*cosh(x) - 6*(4*B*a^6
*b^2 - 20*A*a^5*b^3 - 18*B*a^4*b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^
7 - B*b^8 - 5*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*
b^7 - B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^
3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b - 28*B*a
^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*

```
b^7)*cosh(x)^2 - 4*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4
- 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8)*cosh(x))*sinh(x))/(a^8*
b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12 - (a^8*b^4 + 4*a^6*b^6 + 6*
a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^6 - (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 +
 4*a^2*b^10 + b^12)*sinh(x)^6 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*
b^9 + a*b^11)*cosh(x)^5 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
a*b^11 + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x))*sin
h(x)^5 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(
x)^4 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(177) = 354.
time = 0.43, size = 477, normalized size = 2.55

$$\frac{\left(2\,Aa^3+4\,Ba^2b-3\,Aab^2-Bb^3\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{6\,Aa^3b^3e^{(5\,x)}+12\,Ba^2b^4e^{(5\,x)}-9\,Aab^5e^{(5\,x)}-3\,Bb^6e^{(5\,x)}+30\,Aa^4b^2e^{(4\,x)}+60\,Ba^3b^3e^{(4\,x)}-45\,Aa^2b^4e^{(4\,x)}-15\,Bab^5e^{(4\,x)}-8\,Ba^6e^{(3\,x)}+44\,Aa^5be^{(3\,x)}+64\,Ba^4b^2e^{(3\,x)}-82\,Aa^3b^3e^{(3\,x)}-78\,Ba^2b^4e^{(3\,x)}+24\,Aab^5e^{(3\,x)}+24\,Ba^5be^{(2\,x)}-102\,Aa^4b^2e^{(2\,x)}-102\,Ba^3b^3e^{(2\,x)}+36\,Aa^2b^4e^{(2\,x)}+24\,Bab^5e^{(2\,x)}-12\,Ab^6e^{(2\,x)}-12\,Ba^4b^2e^x+60\,Aa^3b^3e^x+66\,Ba^2b^4e^x-15\,Aab^5e^x+3\,Bb^6e^x+2\,Ba^3b^3-11\,Aa^2b^4-13\,Bab^5+4\,Ab^6}{3\,\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\left(be^{(2\,x)}+2\,ae^x-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a^3 + 4*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(6*A*a^3*b^3*e^(5*x) + 12*B*a^2*b^4*
e^(5*x) - 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 30*A*a^4*b^2*e^(4*x) + 60*B
*a^3*b^3*e^(4*x) - 45*A*a^2*b^4*e^(4*x) - 15*B*a*b^5*e^(4*x) - 8*B*a^6*e^(3
*x) + 44*A*a^5*b*e^(3*x) + 64*B*a^4*b^2*e^(3*x) - 82*A*a^3*b^3*e^(3*x) - 78
*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) + 24*B*a^5*b*e^(2*x) - 102*A*a^4*b^
2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*e^(2*x) + 24*B*a*b^5*e^(2*
x) - 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a^3*b^3*e^x + 66*B*a^2*b^4*
e^x - 15*A*a*b^5*e^x + 3*B*b^6*e^x + 2*B*a^3*b^3 - 11*A*a^2*b^4 - 13*B*a*b^
5 + 4*A*b^6)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*e^(2*x) + 2*a*e^x -
b)^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sinh(x)}{{\left(a+b\,\sinh\left(x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^4,x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^4, x)
```

3.133 $\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$\frac{Bx}{b}+\frac{2(a^2-b^2)\,B\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}$$

```
[Out] B*x/b+2*(a^2-b^2)*B*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/b/(a^2+b^2
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2814, 2739, 632, 212}

$$\frac{2B(a^2-b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{Bx}{b}-\frac{\left(i\left(-iaB+\frac{ib^2B}{a}\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{\left(2i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{\left(4i\left(-iaB+\frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b}+\frac{2(a^2-b^2)\,B\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 1.10

$$\frac{B\left(ax-\frac{2(a^2-b^2)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*(a*x - (2*(a^2 - b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2]))/(a*b)
```

Maple [A]
time = 0.41, size = 80, normalized size = 1.33

method	result
default	$\frac{2B\left(\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b}-\frac{\left(a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}\right)}{a}$

risch	$\frac{Bx}{b}+\frac{Ba\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}-\frac{Bb\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{Ba\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}+\frac{Bb}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/a*(1/2*a/b*ln(tanh(1/2*x)+1)-(a^2-b^2)/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2*a/b*ln(tanh(1/2*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.
time = 0.52, size = 128, normalized size = 2.13

$$-B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{x}{b}\right)+\frac{Bb\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(56) = 112$.
time = 0.41, size = 154, normalized size = 2.57

$$-\frac{(Ba^2-Bb^2)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2(b^2\cosh(x)+ab)\sinh(x)-2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)-b}\right)-(Ba^3+Bab^2)x}{a^3b+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a^2 - B*b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*
b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)
*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) - b)) - (B*a^3 + B*a*b^2)*x)/(a^3*b + a*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(49) = 98$.

time = 44.71, size = 340, normalized size = 5.67

$$\begin{cases} \text{NaN} & \text{for}\ a=0 \wedge b=0 \\ \frac{B\cosh(x)}{a} & \text{for}\ b=0 \\ \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}} - \frac{4Bb}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}} - \frac{Bx\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right)-b\sqrt{-b^2}} & \text{for}\ a=-\sqrt{-b^2} \\ \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}} - \frac{4Bb}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}} + \frac{Bx\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right)+b\sqrt{-b^2}} & \text{for}\ a=\sqrt{-b^2} \\ \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} + \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*cosh(x)/a, Eq(b, 0)), (B*b*x*tanh(
x/2)/(b**2*tanh(x/2) - b*sqrt(-b**2)) - 4*B*b/(b**2*tanh(x/2) - b*sqrt(-b**
2)) - B*x*sqrt(-b**2)/(b**2*tanh(x/2) - b*sqrt(-b**2)), Eq(a, -sqrt(-b**2))
), (B*b*x*tanh(x/2)/(b**2*tanh(x/2) + b*sqrt(-b**2)) - 4*B*b/(b**2*tanh(x/2
) + b*sqrt(-b**2)) + B*x*sqrt(-b**2)/(b**2*tanh(x/2) + b*sqrt(-b**2)), Eq(a
, sqrt(-b**2))), (B*a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*sqrt(a*
*2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 +
 b**2)) + B*x/b - B*b*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a*sqrt(a*
*2 + b**2)) + B*b*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a*sqrt(a**2 +
 b**2)), True))
```

Giac [A]
time = 0.42, size = 82, normalized size = 1.37

$$\frac{Bx}{b} - \frac{(Ba^2 - Bb^2)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a^2 - B*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b)
```

Mupad [B]
time = 1.20, size = 331, normalized size = 5.52

$$\frac{2\,\mathrm{atan}\left(\left(\frac{a\,b^2\,e^x\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(\frac{2\,\left(B\,a^2\,\sqrt{-a^4\,b^2-a^2\,b^4}-B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\right)}{a^2\,b^4\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\sqrt{B^2\,{\left(a^2-b^2\right)}^2}}+\frac{2\,a^2\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right)}{2}-\frac{a^2\,b\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right)\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}\right)}{\sqrt{-a^4\,b^2-a^2\,b^4}}+\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sinh(x) + (B*b)/a)/(a + b*sinh(x)),x)

[Out] (2*atan((a*b^2*exp(x)*(- a^2*b^4 - a^4*b^2)^(1/2)*((2*(B*a^2*(- a^2*b^4 - a
^4*b^2)^(1/2) - B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)))/(a^2*b^4*(- a^2*b^4 - a
^4*b^2)^(1/2)*(B^2*(a^2 - b^2)^2)^(1/2)) + (2*a^2*(B^2*a^4 + B^2*b^4 - 2*B^
2*a^2*b^2)^(1/2))/(B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)*(a^2 - b^2)*(-a^2*b^2*
(a^2 + b^2))^(1/2))))/2 - (a^2*b*(B^2*a^4 + B^2*b^4 - 2*B^2*a^2*b^2)^(1/2))
/(B*(a^2 - b^2)*(-a^2*b^2*(a^2 + b^2))^(1/2)))*(B^2*a^4 + B^2*b^4 - 2*B^2*a
^2*b^2)^(1/2))/(- a^2*b^4 - a^4*b^2)^(1/2) + (B*x)/b
```

3.134 $\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b
```

Maple [A]
time = 0.26, size = 7, normalized size = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(6) = 12.
time = 0.51, size = 128, normalized size = 21.33

$$-B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}-\frac{x}{b}\right)+\frac{Ba\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Fricas [A]
time = 0.38, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [**A**]
time = 0.15, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] B*x/b
```

Giac [**A**]
time = 0.42, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b
```

Mupad [**B**]
time = 0.02, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*sinh(x) + (B*a)/b)/(a + b*sinh(x)),x)
```

```
[Out] (B*x)/b
```

3.135 $\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh(x)}{b+a\sinh(x)}$$

```
[Out] -cosh(x)/(b+a*sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2833, 8}

$$-\frac{\cosh(x)}{a\sinh(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b+a\sinh(x)}-\frac{\int 0\,dx}{a^2+b^2}\\ &= -\frac{\cosh(x)}{b+a\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 1.00

$$-\frac{\cosh(x)}{b+a\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 0.42, size = 36, normalized size = 3.00

method	result	size
risch	$-\frac{2(-e^{x} b+a)}{a(e^{2x} a+2 e^{x} b-a)}$	30
default	$-\frac{2\left(\frac{a \tanh\left(\frac{x}{2}\right)}{2b}+\frac{1}{2}\right)}{-\frac{b\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{2}+a \tanh\left(\frac{x}{2}\right)+\frac{b}{2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*sinh(x))/(b+a*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/2*a/b*tanh(1/2*x)+1/2)/(-1/2*b*tanh(1/2*x)^2+a*tanh(1/2*x)+1/2*b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(12) = 24$.
time = 0.53, size = 230, normalized size = 19.17

$$-b\left(\frac{a \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(b^2e^{(-x)}+ab\right)}{a^4+a^2b^2+2\left(a^3b+ab^3\right)e^{(-x)}-\left(a^4+a^2b^2\right)e^{(-2x)}}\right)+a\left(\frac{b \log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(be^{(-x)}+a\right)}{a^3+ab^2+2\left(a^2b+b^3\right)e^{(-x)}-\left(a^3+ab^2\right)e^{(-2x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -b*(a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) + a*b)/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^
3)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) + a*(b*log((a*e^(-x) - b - sqrt(a^2
+ b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) +
 a)/(a^3 + a*b^2 + 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.
time = 0.36, size = 58, normalized size = 4.83

$$\frac{2\left(b\cosh\left(x\right)+b\sinh\left(x\right)-a\right)}{a^2\cosh\left(x\right)^2+a^2\sinh\left(x\right)^2+2\,ab\cosh\left(x\right)-a^2+2\left(a^2\cosh\left(x\right)+ab\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*(b*cosh(x) + b*sinh(x) - a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x
) - a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.
time = 0.43, size = 30, normalized size = 2.50

$$\frac{2\,(be^{x} - a)}{(ae^{(2\,x)} + 2\,be^{x} - a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*(b*e^x - a)/((a*e^(2*x) + 2*b*e^x - a)*a)
```

Mupad [**B**]
time = 0.57, size = 49, normalized size = 4.08

$$\frac{\frac{2\,\mathrm{e}^{x}\,\left(a^3\,b+a\,b^3\right)}{a\,\left(a^3+a\,b^2\right)}-2}{2\,b\,\mathrm{e}^{x}-a+a\,\mathrm{e}^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*sinh(x))/(b + a*sinh(x))^2,x)
```

```
[Out] ((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) - 2)/(2*b*exp(x) - a + a*exp(
2*x))
```

3.136 $\int \frac{2-\sinh(x)}{2+\sinh(x)} \, dx$

Optimal. Leaf size=34

$$-x+\frac{4x}{\sqrt{5}}-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}$$

```
[Out] -x+4/5*x*5^(1/2)-8/5*arctanh(cosh(x)/(2+sinh(x)+5^(1/2)))*5^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2814, 2736}

$$\frac{4x}{\sqrt{5}}-x-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[5] - (8*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-\sinh(x)}{2+\sinh(x)} \, dx &= -x+4\int \frac{1}{2+\sinh(x)} \, dx\\ &= -x+\frac{4x}{\sqrt{5}}-\frac{8\tanh^{-1}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 28, normalized size = 0.82

$$-x-\frac{8\tanh^{-1}\left(\frac{1-2\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x - (8*ArcTanh[(1 - 2*Tanh[x/2])/Sqrt[5]])/Sqrt[5]
```

Maple [A]
time = 0.27, size = 37, normalized size = 1.09

method	result	size
risch	$-x+\frac{4\sqrt{5}\,\ln\left(e^{x}+2-\sqrt{5}\right)}{5}-\frac{4\sqrt{5}\,\ln\left(e^{x}+2+\sqrt{5}\right)}{5}$	33
default	$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{8\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-1\right)\sqrt{5}}{5}\right)}{5}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-sinh(x))/(2+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tanh(1/2*x)-1)-ln(tanh(1/2*x)+1)+8/5*5^(1/2)*arctanh(1/5*(2*tanh(1/2*x)-
1)*5^(1/2))
```

Maxima [A]
time = 0.50, size = 34, normalized size = 1.00

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-e^{(-x)}+2}{\sqrt{5}+e^{(-x)}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="maxima")
```

```
[Out] 4/5*sqrt(5)*log(-(sqrt(5) - e^(-x) + 2)/(sqrt(5) + e^(-x) - 2)) - x
```

Fricas [A]
time = 0.46, size = 42, normalized size = 1.24

$$\frac{4}{5}\sqrt{5}\log\left(-\frac{\left(2\sqrt{5}-5\right)\cosh\left(x\right)-2\left(\sqrt{5}-2\right)\sinh\left(x\right)+\sqrt{5}-2}{\sinh\left(x\right)+2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log(-((2*sqrt(5) - 5)*cosh(x) - 2*(sqrt(5) - 2)*sinh(x) + sqrt(
5) - 2)/(sinh(x) + 2)) - x
```

Sympy [A]
time = 0.96, size = 51, normalized size = 1.50

$$-x+\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{5}-\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] -x + 4*sqrt(5)*log(tanh(x/2) - 1/2 + sqrt(5)/2)/5 - 4*sqrt(5)*log(tanh(x/2)
 - sqrt(5)/2 - 1/2)/5
```

Giac [A]
time = 0.42, size = 33, normalized size = 0.97

$$\frac{4}{5}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2e^x+4\right|}{2\left(\sqrt{5}+e^x+2\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="giac")
```

```
[Out] 4/5*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*e^x + 4)/(sqrt(5) + e^x + 2)) - x
```

Mupad [B]
time = 0.59, size = 48, normalized size = 1.41

$$\frac{4\sqrt{5}\,\ln\left(-8\,\mathrm{e}^x-\frac{4\sqrt{5}\,\left(4\,\mathrm{e}^x-2\right)}{5}\right)}{5}-x-\frac{4\sqrt{5}\,\ln\left(\frac{4\sqrt{5}\,\left(4\,\mathrm{e}^x-2\right)}{5}-8\,\mathrm{e}^x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sinh(x) - 2)/(sinh(x) + 2),x)
```

```
[Out] (4*5^(1/2)*log(- 8*exp(x) - (4*5^(1/2)*(4*exp(x) - 2))/5))/5 - x - (4*5^(1/
2)*log((4*5^(1/2)*(4*exp(x) - 2))/5 - 8*exp(x)))/5
```

3.137 $\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=136

$$\frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2i(Ab-aB)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*P
i+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/
(a-I*b))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I
*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))
/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b
*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (I/
2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[
x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \frac{B\int \sqrt{a+b\sinh(x)}\,dx}{b} + \frac{(i(-iAb+iaB))\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}$$

$$= \frac{\left(B\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{\left(i(-iAb+iaB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right.}{b\sqrt{a+}}$$

$$= \frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2i(Ab-aB)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A]
time = 0.38, size = 109, normalized size = 0.80

$$\frac{2\left((ia+b)BE\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)+i(Ab-aB)F\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*B*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + I*(A*b
- a*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A]
time = 1.12, size = 266, normalized size = 1.96

method	result
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(-iB\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b+iB\,\mathrm{Elliptic}\right.}{b^2\cosh(x)}$
risch	$\frac{B\left(b\,e^{2x}+2a\,e^{x}-b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2a\,e^{x}-b\right)e^{-x}}}+\frac{\left(4A\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+}}{b}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*(-I*B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*b+I*B*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b+A*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*b-B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 183, normalized size = 1.35

$$-\frac{2\left(3\sqrt{2}Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(\frac{4\left(4a^2+3b^2\right)}{3b^2},-\frac{8\left(8a^3+9ab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^2+3b^2\right)}{3b^2},-\frac{8\left(8a^3+9ab^2\right)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)\right)+\sqrt{2}\left(2Ba-3Ab\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(4a^2+3b^2\right)}{3b^2},-\frac{8\left(8a^3+9ab^2\right)}{27b^3},\frac{3b\cosh(x)+3b\sinh(x)+2a}{3b}\right)+3\sqrt{b\sinh\left(x\right)+a}\,Bb\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8
*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + sqrt(2)*(2*
B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(b*sin
h(x) + a)*B*b)/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sinh\left(x\right)}{\sqrt{a + b\sinh\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral((A + B*sinh(x))/sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2),x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2), x)
```

3.138 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iBF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] -2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)/((a+b*sinh(x))/(a-I*b
))^(1/2)+2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(
cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2
)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$,
Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*(A*b -
a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*(a^
2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*B*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA-bB)-\frac{1}{2}(Ab-aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\sin}}{b\,(a^2+b^2)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(}{a-i}}}{b\,(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\,(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iI}{}$$

Mathematica [A]
time = 0.47, size = 159, normalized size = 0.90

$$\frac{2b(-Ab+aB)\cosh(x)+\frac{2i(Ab-aB)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+2i(a^2+b^2)\,BF\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\,(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*(-(A*b) + a*B)*Cosh[x] + ((2*I)*(A*b - a*B)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]))/Sqrt[(a + b*Sinh[x])/(a - I*b)] +
(2*I)*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[
(a + b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(210) = 420.
time = 1.30, size = 517, normalized size = 2.94

method	result

default	$\frac{\sqrt{\left(\cosh^2(x)\right)(a+b\sinh(x))}\left(\frac{2B\left(\frac{a}{b}-i\right)\sqrt{\frac{-b\sinh(x)-a}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-b\sinh(x}{ib-a}}\right.}{b\sqrt{\left(\cosh^2(x)\right)(a+b\sinh(x))}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(2*B/b*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(
a+b*sinh(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+
a))^(1/2))+(A*b-B*a)/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*(a+b*sinh(x)))^
(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*b/(a^2
+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*Ell
ipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(
((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(x)/(a+b*sin
h(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 633, normalized size = 3.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*B*b^3)*cosh(x)^2 + sqrt(2)*(2*B*a^2
*b + A*a*b^2 + 3*B*b^3)*sinh(x)^2 + 2*sqrt(2)*(2*B*a^3 + A*a^2*b + 3*B*a*b^
2)*cosh(x) + 2*(sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*B*b^3)*cosh(x) + sqrt(2)*(
2*B*a^3 + A*a^2*b + 3*B*a*b^2))*sinh(x) - sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*
B*b^3))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 +
 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a*b
^2 - A*b^3)*cosh(x)^2 + sqrt(2)*(B*a*b^2 - A*b^3)*sinh(x)^2 + 2*sqrt(2)*(B*
a^2*b - A*a*b^2)*cosh(x) + 2*(sqrt(2)*(B*a*b^2 - A*b^3)*cosh(x) + sqrt(2)*(
B*a^2*b - A*a*b^2))*sinh(x) - sqrt(2)*(B*a*b^2 - A*b^3))*sqrt(b)*weierstras
sZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInv
erse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*a*b^2 - A*b
^3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^2 + 2*(B*a*b
^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(a^2*b^3 + b^5 - (a^2*b^
3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^3*b^2 + a*b^4)*cosh(x
) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sinh(x)}{\left(a+b\sinh\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2),x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2), x)
```

3.139 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}+\frac{2i\left(4aAb-a^2B+3b^2B\right)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x}{a-ib}}}$$

```
[Out] -2/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-2/3*(4*A*a*b-B*a^2+3*B
*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^(1/2)+2/3*I*(4*A*a*b-B*a^2+3*B*b^2)
*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2
*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)^2/((a+b*si
nh(x))/(a-I*b))^(1/2)-2/3*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4
*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b
*sinh(x))/(a-I*b))^(1/2)/b/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}-\frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}+\frac{2i\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (2*(4*a*A*
b - a^2*B + 3*b^2*B)*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((2*
I)/3)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/(b*(a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) -
(((2*I)/3)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA+bB)+\frac{1}{2}(Ab-aB)\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\,(a^2+b^2)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3\,(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{1}{4}(3a^2A-A \cdots}{\cdots}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3\,(a^2+b^2)^2\sqrt{a+b\sinh(x)}} - \frac{(Ab-aB)\int \cdots}{3\cdots}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3\,(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{\left((4aAb-a^2 \cdots\right.}{\cdots}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3\,(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{2i(4aAb-a \cdots}{\cdots}$$

Mathematica [A]
time = 0.61, size = 236, normalized size = 0.94

$$\frac{2i\left(\left(b(3a^2A-Ab^2+4abB)\,F\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)+(4aAb-a^2B+3b^2B)\left((a-ib)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)-aF\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\right)\right)(a+b\sinh(x))\sqrt{\frac{a+b\sinh(x)}{a-ib}}+ib\cosh(x)\left(-\left((a^2+b^2)\,(-Ab+aB)\right)-(-4aAb+a^2B-3b^2B)\,(a+b\sinh(x))\right)\right)}{3b\,(a^2+b^2)^2\,(a+b\sinh(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (((2*I)/3)*((b*(3*a^2*A - A*b^2 + 4*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)] + (4*a*A*b - a^2*B + 3*b^2*B)*((a - I*b)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b
)/(a - I*b)]))*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*b)] + I*b*Cosh[x
]*(-((a^2 + b^2)*(-(A*b) + a*B)) - (-4*a*A*b + a^2*B - 3*b^2*B)*(a + b*Sinh
[x]))))/(b*(a^2 + b^2)^2*(a + b*Sinh[x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(275) = 550.
time = 1.44, size = 806, normalized size = 3.21

method	result

default	$\frac{\sqrt{\left(\cosh^2(x)\right)(a+b\sinh(x))}\left(\frac{(Ab-Ba)\left(-\frac{2\sqrt{\left(\cosh^2(x)\right)(a+b\sinh(x))}}{3b\left(a^2+b^2\right)\left(\sinh(x)+\frac{a}{b}\right)^2}-\frac{8b\left(\cosh^2(x)\right)a}{3\left(a^2+b^2\right)^2\sqrt{\left(\cosh^2(x)\right)(a}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*((A*b-B*a)/b*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(
a+b*sinh(x)))^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^
2*(a+b*sinh(x)))^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-b*s
inh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-
a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a)
)^(1/2),((a-I*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-b*sinh(x)-a
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/
2),((a-I*b)/(I*b+a))^(1/2))))+B/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*(a+b
*sinh(x)))^(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh
(x)))^(1/2)*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2
))+2*b/(a^2+b^2)*(a/b-I)*((-b*sinh(x)-a)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b
+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((
-a/b-I)*EllipticE(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I
*EllipticF(((-b*sinh(x)-a)/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(
x)/(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 2167, normalized size = 8.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^4 + s
qrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*sinh(x)^4 + 4*sqrt(2
)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*
(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x) + sqrt(2)*(2*B*a^4*
b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(4*B*a^5 +
2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^2 + 2
*(3*sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^2 + 6*s
qrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x) + sqrt(2)*
(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5))*s
inh(x)^2 - 4*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh
(x) + 4*(sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^3
+ 3*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x)^2 + s
qrt(2)*(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*
b^5)*cosh(x) - sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4))*s
inh(x) + sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/
3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4
- 3*B*b^5)*cosh(x)^4 + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*sinh(x)^4
+ 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*(B
*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x) + sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3
- 3*B*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3
+ 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + 2*(3*sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*
B*b^5)*cosh(x)^2 + 6*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x)
+ sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5))*si
nh(x)^2 - 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x) + 4*(sqrt
(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x)^3 + 3*sqrt(2)*(B*a^3*b^2 - 4*
A*a^2*b^3 - 3*B*a*b^4)*cosh(x)^2 + sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a
^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x) - sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 -
3*B*a*b^4))*sinh(x) + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5))*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b
*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*co
sh(x)^4 + (B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*sinh(x)^4 + (4*B*a^3*b^2 - 13*A
*a^2*b^3 - 8*B*a*b^4 - A*b^5)*cosh(x)^3 + (4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B
*a*b^4 - A*b^5 + 4*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x))*sinh(x)^3 + (
2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + (2
*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5 + 6*(B*a^2*b^3 -
 4*A*a*b^4 - 3*B*b^5)*cosh(x)^2 + 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B*a*b^4
 - A*b^5)*cosh(x))*sinh(x)^2 + (3*A*a^2*b^3 + 4*B*a*b^4 - A*b^5)*cosh(x) +
(3*A*a^2*b^3 + 4*B*a*b^4 - A*b^5 + 4*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh
(x)^3 + 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B*a*b^4 - A*b^5)*cosh(x)^2 + 2*(2
```

```
*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x))*sinh(x
))*sqrt(b*sinh(x) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8 + (a^4*b^4 + 2*a^2*b^6 +
 b^8)*cosh(x)^4 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(x)^4 + 4*(a^5*b^3 + 2*a^
3*b^5 + a*b^7)*cosh(x)^3 + 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7 + (a^4*b^4 + 2*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b^2 + 3*a^4*b^4 - b^8)*cosh(x)^2
 + 2*(2*a^6*b^2 + 3*a^4*b^4 - b^8 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^2
 + 6*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x)^2 - 4*(a^5*b^3 + 2*a^3*
b^5 + a*b^7)*cosh(x) - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7 - (a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^3 - 3*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 - (2*a^6*b^2
 + 3*a^4*b^4 - b^8)*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\sinh(x)}{\left(a + b\sinh\left(x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2),x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2), x)
```

3.140 $\int \left(a\sinh^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}$$

```
[Out] -4/15*a*coth(x)*(a*sinh(x)^2)^(3/2)+1/5*coth(x)*(a*sinh(x)^2)^(5/2)+8/15*a^
2*coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2718}

$$\frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Coth[x]*Sqrt[a*Sinh[x]^2])/15 - (4*a*Coth[x]*(a*Sinh[x]^2)^(3/2))/15
 + (Coth[x]*(a*Sinh[x]^2)^(5/2))/5
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx = \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2} - \frac{1}{5}(4a)\int \left(a\sinh^2(x)\right)^{3/2}\,dx$$
$$= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2} + \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2} + \frac{1}{15}(8a^2)\int \sqrt{a\sinh^2(x)}$$
$$= -\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2} + \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2} + \frac{1}{15}\Big(8a^2\operatorname{csch}(x)\sqrt{a\sinh^2}$$
$$= \frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)} - \frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2} + \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\cosh(x) - 25\cosh(3x) + 3\cosh(5x))\operatorname{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (a^2*(150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/
240
```

Maple [A]
time = 0.63, size = 32, normalized size = 0.60

method	result
default	$\frac{a^3\sinh(x)\cosh(x)\left(3\left(\sinh^4(x)\right)-4\left(\sinh^2(x)\right)+8\right)}{15\sqrt{a\left(\sinh^2(x)\right)}}$
risch	$\frac{a^2 e^{6x}\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}{160\,e^{2x}-160} - \frac{5a^2 e^{4x}\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}{96(e^{2x}-1)} + \frac{5a^2 e^{2x}\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}{16(e^{2x}-1)} + \frac{5\sqrt{a\left(e^{2x}-1\right)}}{16(e^{2x}-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*a^3*sinh(x)*cosh(x)*(3*sinh(x)^4-4*sinh(x)^2+8)/(a*sinh(x)^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 53, normalized size = 1.00

$$-\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)} + \frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)} - \frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)} + \frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)} - \frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)} - \frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="maxima")

[Out] -1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) + 5/96*a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) - 5/16*a^(5/2)*e^x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(41) = 82$.
time = 0.39, size = 511, normalized size = 9.64

$$\frac{\left(30\,a^{2}\cosh\left(x\right)e^{x}\sinh\left(x\right)^{9}+3\,a^{2}e^{x}\sinh\left(x\right)^{10}+5\,\left(27\,a^{2}\cosh\left(x\right)^{2}-5\,a^{2}\right)e^{x}\sinh\left(x\right)^{8}+40\,\left(9\,a^{2}\cosh\left(x\right)^{3}-5\,a^{2}\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{7}+10\,\left(63\,a^{2}\cosh\left(x\right)^{4}-70\,a^{2}\cosh\left(x\right)^{2}+15\,a^{2}\right)e^{x}\sinh\left(x\right)^{6}+4\,\left(189\,a^{2}\cosh\left(x\right)^{5}-350\,a^{2}\cosh\left(x\right)^{3}+225\,a^{2}\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{5}+10\,\left(63\,a^{2}\cosh\left(x\right)^{6}-175\,a^{2}\cosh\left(x\right)^{4}+225\,a^{2}\cosh\left(x\right)^{2}+15\,a^{2}\right)e^{x}\sinh\left(x\right)^{4}+40\,\left(9\,a^{2}\cosh\left(x\right)^{7}-35\,a^{2}\cosh\left(x\right)^{5}+75\,a^{2}\cosh\left(x\right)^{3}+15\,a^{2}\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{3}+5\,\left(27\,a^{2}\cosh\left(x\right)^{8}-140\,a^{2}\cosh\left(x\right)^{6}+450\,a^{2}\cosh\left(x\right)^{4}+180\,a^{2}\cosh\left(x\right)^{2}-5\,a^{2}\right)e^{x}\sinh\left(x\right)^{2}+10\,\left(3\,a^{2}\cosh\left(x\right)^{9}-20\,a^{2}\cosh\left(x\right)^{7}+90\,a^{2}\cosh\left(x\right)^{5}+60\,a^{2}\cosh\left(x\right)^{3}-5\,a^{2}\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)+\left(3\,a^{2}\cosh\left(x\right)^{10}-25\,a^{2}\cosh\left(x\right)^{8}+150\,a^{2}\cosh\left(x\right)^{6}+150\,a^{2}\cosh\left(x\right)^{4}-25\,a^{2}\cosh\left(x\right)^{2}+3\,a^{2}\right)e^{x}\right)\sqrt{a e^{\left(4\,x\right)}-2\,a e^{\left(2\,x\right)}+a}\,e^{\left(-x\right)}}{480\,\left(\cosh\left(x\right)^{5}e^{\left(2\,x\right)}+\left(e^{\left(2\,x\right)}-1\right)\sinh\left(x\right)^{5}-\cosh\left(x\right)^{5}+5\,\left(\cosh\left(x\right)e^{\left(2\,x\right)}-\cosh\left(x\right)\right)\sinh\left(x\right)^{4}+10\,\left(\cosh\left(x\right)^{2}e^{\left(2\,x\right)}-\cosh\left(x\right)^{2}\right)\sinh\left(x\right)^{3}+10\,\left(\cosh\left(x\right)^{3}e^{\left(2\,x\right)}-\cosh\left(x\right)^{3}\right)\sinh\left(x\right)^{2}+5\,\left(\cosh\left(x\right)^{4}e^{\left(2\,x\right)}-\cosh\left(x\right)^{4}\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="fricas")

[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*sinh(x)^7 + 10*(63*a^2*cosh(x)^4 - 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 + 4*(189*a^2*cosh(x)^5 - 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 + 10*(63*a^2*cosh(x)^6 - 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 - 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 - 140*a^2*cosh(x)^6 + 450*a^2*cosh(x)^4 + 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(x)^9 - 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 + 60*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 - 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6 + 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 + 3*a^2)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) - 1)*sinh(x)^5 - cosh(x)^5 + 5*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) - cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^4*e^(2*x) - cosh(x)^4)*sinh(x))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^{2}\left(x\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sinh(x)**2)**(5/2),x)

[Out] Integral((a*sinh(x)**2)**(5/2), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.
time = 0.41, size = 120, normalized size = 2.26

$$\frac{1}{480}\left(3\,a^{2}e^{\left(5\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)-25\,a^{2}e^{\left(3\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)+150\,a^{2}e^{x}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)+\left(150\,a^{2}e^{\left(4\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)-25\,a^{2}e^{\left(2\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)+3\,a^{2}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)\right)e^{\left(-5\,x\right)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*e^(5*x)*sgn(e^(3*x) - e^x) - 25*a^2*e^(3*x)*sgn(e^(3*x) - e^x)
 + 150*a^2*e^x*sgn(e^(3*x) - e^x) + (150*a^2*e^(4*x)*sgn(e^(3*x) - e^x) - 2
5*a^2*e^(2*x)*sgn(e^(3*x) - e^x) + 3*a^2*sgn(e^(3*x) - e^x))*e^(-5*x))*sqrt
(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a\sinh(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(5/2),x)
```

```
[Out] int((a*sinh(x)^2)^(5/2), x)
```

3.141 $\int \left(a\sinh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

```
[Out] 1/3*coth(x)*(a*sinh(x)^2)^(3/2)-2/3*a*coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2718}

$$\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}(2a)\int \sqrt{a\sinh^2(x)}\,dx$$
$$= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}\left(2a\text{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx$$
$$= -\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)} + \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.76

$$\frac{1}{12}a(-9\cosh(x)+\cosh(3x))\text{csch}(x)\sqrt{a\sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(3/2),x]
```

```
[Out] (a*(-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/12
```

Maple [A]
time = 0.59, size = 24, normalized size = 0.71

method	result
default	$\frac{a^2\sinh(x)\cosh(x)(\sinh^2(x)-2)}{3\sqrt{a\left(\sinh^2(x)\right)}}$
risch	$\frac{a\,e^{4x}\sqrt{a\left(e^{2x}-1\right)^2e^{-2x}}}{24\,e^{2x}-24} - \frac{3a\,e^{2x}\sqrt{a\left(e^{2x}-1\right)^2e^{-2x}}}{8(e^{2x}-1)} - \frac{3\sqrt{a\left(e^{2x}-1\right)^2e^{-2x}}\,a}{8(e^{2x}-1)} + \frac{a\,e^{-2x}\sqrt{a\left(e^{2x}-1\right)^2 \ldots}}{24\,e^{2x}-24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*sinh(x)*cosh(x)*(sinh(x)^2-2)/(a*sinh(x)^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 35, normalized size = 1.03

$$-\frac{1}{24}\,a^{\frac{3}{2}}e^{(3\,x)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{(-x)} - \frac{1}{24}\,a^{\frac{3}{2}}e^{(-3\,x)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

[Out] -1/24*a^(3/2)*e^(3*x) + 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(3/2)*e^x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(26) = 52$.
time = 0.43, size = 226, normalized size = 6.65

$$\frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5}+ae^{x}\sinh\left(x\right)^{6}+3\,\left(5\,a\cosh\left(x\right)^{2}-3\,a\right)e^{x}\sinh\left(x\right)^{4}+4\,\left(5\,a\cosh\left(x\right)^{3}-9\,a\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)^{3}+3\,\left(5\,a\cosh\left(x\right)^{4}-18\,a\cosh\left(x\right)^{2}-3\,a\right)e^{x}\sinh\left(x\right)^{2}+6\,\left(a\cosh\left(x\right)^{5}-6\,a\cosh\left(x\right)^{3}-3\,a\cosh\left(x\right)\right)e^{x}\sinh\left(x\right)+\left(a\cosh\left(x\right)^{6}-9\,a\cosh\left(x\right)^{4}-9\,a\cosh\left(x\right)^{2}+a\right)e^{x}\right)\sqrt{ae^{\left(4\,x\right)}-2\,ae^{\left(2\,x\right)}+a}\,e^{\left(-x\right)}}{24\,\left(\cosh\left(x\right)^{3}e^{\left(2\,x\right)}+\left(e^{\left(2\,x\right)}-1\right)\sinh\left(x\right)^{3}-\cosh\left(x\right)^{3}+3\,\left(\cosh\left(x\right)e^{\left(2\,x\right)}-\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+3\,\left(\cosh\left(x\right)^{2}e^{\left(2\,x\right)}-\cosh\left(x\right)^{2}\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 - 3*a)
*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cos
h(x)^4 - 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 - 6*a*cosh(x)
^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 - 9*a*cosh(x)^4 - 9*a*cosh(x)^
2 + a)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (
e^(2*x) - 1)*sinh(x)^3 - cosh(x)^3 + 3*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^
2 + 3*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.
time = 0.42, size = 70, normalized size = 2.06

$$-\frac{1}{24}\left(\left(9\,e^{\left(2\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)-\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)\right)e^{\left(-3\,x\right)}-e^{\left(3\,x\right)}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)+9\,e^{x}\mathrm{sgn}\left(e^{\left(3\,x\right)}-e^{x}\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) - e^(3*
x)*sgn(e^(3*x) - e^x) + 9*e^x*sgn(e^(3*x) - e^x))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a\,{\mathrm{sinh}\left(x\right)}^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(3/2),x)
```

```
[Out] int((a*sinh(x)^2)^(3/2), x)
```

3.142 $\int \sqrt{a \sinh^2(x)}\, dx$

Optimal. Leaf size=13

$$\coth(x) \sqrt{a \sinh^2(x)}$$

```
[Out] coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 2718}

$$\coth(x) \sqrt{a \sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \sinh^2(x)}\, dx &= \left(\operatorname{csch}(x) \sqrt{a \sinh^2(x)}\right) \int \sinh(x)\, dx \\ &= \coth(x) \sqrt{a \sinh^2(x)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\coth(x) \sqrt{a \sinh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Maple [A]
time = 0.64, size = 15, normalized size = 1.15

method	result	size
default	$\frac{a\sinh(x)\cosh(x)}{\sqrt{a\left(\sinh^{2}(x)\right)}}$	15
risch	$\frac{\sqrt{a\left(e^{2x}-1\right)^{2}e^{-2x}}\,e^{2x}}{2\,e^{2x}-2}+\frac{\sqrt{a\left(e^{2x}-1\right)^{2}e^{-2x}}}{2\,e^{2x}-2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*sinh(x)^2)^(1/2)*a*sinh(x)*cosh(x)
```

Maxima [A]
time = 0.51, size = 17, normalized size = 1.31

$$-\frac{1}{2}\sqrt{a}\,e^{(-x)}-\frac{1}{2}\sqrt{a}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*e^(-x) - 1/2*sqrt(a)*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(11) = 22$.
time = 0.38, size = 71, normalized size = 5.46

$$\frac{\left(2\cosh\left(x\right)e^{x}\sinh\left(x\right)+e^{x}\sinh\left(x\right)^{2}+\left(\cosh\left(x\right)^{2}+1\right)e^{x}\right)\sqrt{ae^{(4\,x)}-2\,ae^{(2\,x)}+a}\;e^{(-x)}}{2\left(\cosh\left(x\right)e^{(2\,x)}+\left(e^{(2\,x)}-1\right)\sinh\left(x\right)-\cosh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)*sqrt(a*e^
(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x) -
cosh(x))
```

Sympy [**A**]
time = 0.11, size = 15, normalized size = 1.15

$$\frac{\sqrt{a\sinh^2{(x)}}\cosh{(x)}}{\sinh{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**2)**(1/2),x)
```

```
[Out] sqrt(a*sinh(x)**2)*cosh(x)/sinh(x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(11) = 22.
time = 0.40, size = 34, normalized size = 2.62

$$\frac{1}{2}\left(e^{(-x)}\mathrm{sgn}\left(e^{(3x)}-e^{x}\right)+e^{x}\mathrm{sgn}\left(e^{(3x)}-e^{x}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x)*sgn(e^(3*x) - e^x) + e^x*sgn(e^(3*x) - e^x))*sqrt(a)
```

Mupad [**B**]
time = 0.46, size = 21, normalized size = 1.62

$$\sqrt{a}\,\coth(x)\,\sqrt{{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^2)^(1/2),x)
```

```
[Out] a^(1/2)*coth(x)*((exp(-x)/2 - exp(x)/2)^2)^(1/2)
```

3.143 $\int \frac{1}{\sqrt{a \sinh^2(x)}} \, dx$

Optimal. Leaf size=17

$$-\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

```
[Out] -arctanh(cosh(x))*sinh(x)/(a*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3855}

$$-\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sinh^2(x)}} \, dx &= \frac{\sinh(x)\int \text{csch}(x) \, dx}{\sqrt{a\sinh^2(x)}}\\ &= -\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.18

$$\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^2],x]
[Out] (Log[Tanh[x/2]]*Sinh[x])/Sqrt[a*Sinh[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.87, size = 49, normalized size = 2.88

method	result	size
default	$-\frac{\sinh(x)\sqrt{a\left(\cosh^2(x)\right)}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cosh^2(x)\right)}+2a}{\sinh(x)}\right)}{\sqrt{a}\,\cosh(x)\sqrt{a\left(\sinh^2(x)\right)}}$	49
risch	$-\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}+1\right)}{\sqrt{a\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}}}+\frac{\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\ln\left(\mathrm{e}^{x}-1\right)}{\sqrt{a\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -sinh(x)*(a*cosh(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/s
inh(x))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 24, normalized size = 1.41

$$\frac{\log\left(e^{(-x)}+1\right)}{\sqrt{a}}-\frac{\log\left(e^{(-x)}-1\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

time = 0.49, size = 110, normalized size = 6.47

$$\left[\frac{\sqrt{ae^{(4\,x)} - 2\,ae^{(2\,x)} + a}\,\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right)}{ae^{(2\,x)} - a}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{ae^{(4\,x)} - 2\,ae^{(2\,x)} + a}\,\sqrt{-a}}{a\cosh(x)e^{(2\,x)} - a\cosh(x) + \left(ae^{(2\,x)} - a\right)\sinh(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*log((cosh(x) + sinh(x) - 1)/(cosh(x) + s
inh(x) + 1))/(a*e^(2*x) - a), 2*sqrt(-a)*arctan(sqrt(a*e^(4*x) - 2*a*e^(2*x
) + a)*sqrt(-a)/(a*cosh(x)*e^(2*x) - a*cosh(x) + (a*e^(2*x) - a)*sinh(x)))/
a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**2), x)
```

Giac [A]
time = 0.41, size = 1, normalized size = 0.06

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 0
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\,{\sinh\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(1/2), x)
```

3.144 $\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}+\frac{\tanh^{-1}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

```
[Out] -1/2*coth(x)/a/(a*sinh(x)^2)^(1/2)+1/2*arctanh(cosh(x))*sinh(x)/a/(a*sinh(x
)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3283, 3286, 3855}

$$\frac{\sinh(x)\tanh^{-1}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -1/2*Coth[x]/(a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a
*Sinh[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{2a}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{2a\sqrt{a\sinh^2(x)}}$$
$$= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} + \frac{\tanh^{-1}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.05

$$-\frac{\left(\operatorname{csch}^2\left(\frac{x}{2}\right)+4\log\left(\tanh\left(\frac{x}{2}\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)\right)\sinh^3(x)}{8\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-3/2),x]
[Out] -1/8*((Csch[x/2]^2 + 4*Log[Tanh[x/2]] + Sech[x/2]^2)*Sinh[x]^3)/(a*Sinh[x]^
2)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.
time = 0.80, size = 71, normalized size = 1.69

method	result	size
default	$-\frac{\sqrt{a\left(\cosh^2(x)\right)}\left(-\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\cosh^2(x)\right)}+2a}{\sinh(x)}\right)a\left(\sinh^2(x)\right)+\sqrt{a}\sqrt{a\left(\cosh^2(x)\right)}\right)}{2a^{\frac{5}{2}}\sinh(x)\cosh(x)\sqrt{a\left(\sinh^2(x)\right)}}$	71
risch	$-\frac{1+e^{2x}}{a\left(e^{2x}-1\right)\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}+\frac{\left(e^{2x}-1\right)e^{-x}\ln\left(e^x+1\right)}{2a\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}-\frac{\left(e^{2x}-1\right)e^{-x}\ln\left(e^x-1\right)}{2a\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(5/2)/sinh(x)*(a*cosh(x)^2)^(1/2)*(-ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2
)+a)/sinh(x))*a*sinh(x)^2+a^(1/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2
)^(1/2)
```

Maxima [A]
time = 0.53, size = 62, normalized size = 1.48

$$-\frac{e^{(-x)} + e^{(-3\,x)}}{2\,a^{\frac{3}{2}}e^{(-2\,x)} - a^{\frac{3}{2}}e^{(-4\,x)} - a^{\frac{3}{2}}} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a^{\frac{3}{2}}} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + e^(-3*x))/(2*a^(3/2)*e^(-2*x) - a^(3/2)*e^(-4*x) - a^(3/2)) - 1/
2*log(e^(-x) + 1)/a^(3/2) + 1/2*log(e^(-x) - 1)/a^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(34) = 68$.
time = 0.40, size = 327, normalized size = 7.79

$$\frac{\left(6\,\cosh\left(x\right)e^{x}\sinh\left(x\right)^{2} + 2\,e^{x}\sinh\left(x\right)^{3} + 2\,\left(3\,\cosh\left(x\right)^{2} + 1\right)e^{x}\sinh\left(x\right) + 2\,\left(\cosh\left(x\right)^{3} + \cosh\left(x\right)\right)e^{x} - \left(4\,\cosh\left(x\right)e^{x}\sinh\left(x\right)^{3} + e^{x}\sinh\left(x\right)^{4} + 2\,\left(3\,\cosh\left(x\right)^{2} - 1\right)e^{x}\sinh\left(x\right)^{2} + 4\,\left(\cosh\left(x\right)^{3} - \cosh\left(x\right)\right)e^{x}\sinh\left(x\right) + \left(\cosh\left(x\right)^{4} - 2\,\cosh\left(x\right)^{2} + 1\right)e^{x}\right)\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)\right)\sqrt{ae^{(4\,x)} - 2\,ae^{(2\,x)} + a}\,e^{(-x)}}{2\,\left(a^{2}\cosh\left(x\right)^{4} - \left(a^{2}e^{(2\,x)} - a^{2}\right)\sinh\left(x\right)^{4} - 2\,a^{2}\cosh\left(x\right)^{2} - 4\,\left(a^{2}\cosh\left(x\right)e^{(2\,x)} - a^{2}\cosh\left(x\right)\right)\sinh\left(x\right)^{3} + 2\,\left(3\,a^{2}\cosh\left(x\right)^{2} - a^{2} - \left(3\,a^{2}\cosh\left(x\right)^{2} - a^{2}\right)e^{(2\,x)}\right)\sinh\left(x\right)^{2} + a^{2} - \left(a^{2}\cosh\left(x\right)^{4} - 2\,a^{2}\cosh\left(x\right)^{2} + a^{2}\right)e^{(2\,x)} + 4\,\left(a^{2}\cosh\left(x\right)^{3} - a^{2}\cosh\left(x\right) - \left(a^{2}\cosh\left(x\right)^{3} - a^{2}\cosh\left(x\right)\right)e^{(2\,x)}\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(x)*e^x*sinh(x)^2 + 2*e^x*sinh(x)^3 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x) + 2*(cosh(x)^3 + cosh(x))*e^x - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x
)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*sinh(
x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)*log((cosh(x) + sinh(x) + 1)/(cosh(x
) + sinh(x) - 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4
- (a^2*e^(2*x) - a^2)*sinh(x)^4 - 2*a^2*cosh(x)^2 - 4*(a^2*cosh(x)*e^(2*x)
- a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*cosh(x)^2 - a^2 - (3*a^2*cosh(x)^2 - a^
2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^4 - 2*a^2*cosh(x)^2 + a^2)*e^(2*
x) + 4*(a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x)
)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^{2}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-3/2), x)
```

Giac [A]
time = 0.41, size = 37, normalized size = 0.88

$$-\frac{e^{(-x)} + e^x}{\left(\left(e^{(-x)} + e^x\right)^2 - 4\right)a^{\frac{3}{2}}\operatorname{sgn}\left(e^{(3x)} - e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] -(e^(-x) + e^x)/(((e^(-x) + e^x)^2 - 4)*a^(3/2)*sgn(e^(3*x) - e^x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\sinh\left(x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(3/2),x)
[Out] int(1/(a*sinh(x)^2)^(3/2), x)
```

3.145 $\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\tanh^{-1}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}$$

```
[Out] -1/4*coth(x)/a/(a*sinh(x)^2)^(3/2)+3/8*coth(x)/a^2/(a*sinh(x)^2)^(1/2)-3/8*
arctanh(cosh(x))*sinh(x)/a^2/(a*sinh(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3283, 3286, 3855}

$$\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\sinh(x)\tanh^{-1}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -1/4*Coth[x]/(a*(a*Sinh[x]^2)^(3/2)) + (3*Coth[x])/(8*a^2*Sqrt[a*Sinh[x]^2]
) - (3*ArcTanh[Cosh[x]]*Sinh[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx &= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} - \frac{3\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx}{4a} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{8a^2} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} + \frac{(3\sinh(x))\int \operatorname{csch}(x)\,dx}{8a^2\sqrt{a\sinh^2(x)}} \\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}} + \frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}} - \frac{3\tanh^{-1}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 67, normalized size = 1.10

$$-\frac{\operatorname{csch}(x)\left(-6\operatorname{csch}^2\left(\frac{x}{2}\right)+\operatorname{csch}^4\left(\frac{x}{2}\right)-24\log\left(\tanh\left(\frac{x}{2}\right)\right)-6\operatorname{sech}^2\left(\frac{x}{2}\right)-\operatorname{sech}^4\left(\frac{x}{2}\right)\right)\sqrt{a\sinh^2(x)}}{64a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -1/64*(Csch[x]*(-6*Csch[x/2]^2 + Csch[x/2]^4 - 24*Log[Tanh[x/2]] - 6*Sech[x
/2]^2 - Sech[x/2]^4)*Sqrt[a*Sinh[x]^2])/a^3
```

Maple [A]
time = 0.80, size = 89, normalized size = 1.46

method	result
default	$-\frac{\sqrt{a\left(\cosh^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cosh^2(x)\right)}+2a}{\sinh(x)}\right)a\left(\sinh^4(x)\right)-3\sqrt{a\left(\cosh^2(x)\right)}\left(\sinh^2(x)\right)\sqrt{a}+2\sqrt{a}\right.}{8a^{\frac{7}{2}}\sinh(x)^3\cosh(x)\sqrt{a\left(\sinh^2(x)\right)}}$
risch	$\frac{3\,\mathrm{e}^{6x}-11\,\mathrm{e}^{4x}-11\,\mathrm{e}^{2x}+3}{4a^2\left(\mathrm{e}^{2x}-1\right)^3\sqrt{a\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}}}+\frac{3\left(\mathrm{e}^{2x}-1\right)\mathrm{e}^{-x}\ln\left(\mathrm{e}^{x}-1\right)}{8a^2\sqrt{a\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}}}-\frac{3\left(\mathrm{e}^{2x}-1\right)\mathrm{e}^{-x}\ln\left(\mathrm{e}^{x}+1\right)}{8a^2\sqrt{a\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(a*cosh(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/sinh(x))*a
*sinh(x)^4-3*(a*cosh(x)^2)^(1/2)*sinh(x)^2*a^(1/2)+2*a^(1/2)*(a*cosh(x)^2)^
(1/2))/a^(7/2)/sinh(x)^3/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Maxima [A]
time = 0.56, size = 96, normalized size = 1.57

$$\frac{3\,e^{(-x)}-11\,e^{(-3\,x)}-11\,e^{(-5\,x)}+3\,e^{(-7\,x)}}{4\left(4\,a^{\frac{5}{2}}e^{(-2\,x)}-6\,a^{\frac{5}{2}}e^{(-4\,x)}+4\,a^{\frac{5}{2}}e^{(-6\,x)}-a^{\frac{5}{2}}e^{(-8\,x)}-a^{\frac{5}{2}}\right)}+\frac{3\,\log\left(e^{(-x)}+1\right)}{8\,a^{\frac{5}{2}}}-\frac{3\,\log\left(e^{(-x)}-1\right)}{8\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*a^(5/2)*e^(-2*x)
 - 6*a^(5/2)*e^(-4*x) + 4*a^(5/2)*e^(-6*x) - a^(5/2)*e^(-8*x) - a^(5/2)) +
3/8*log(e^(-x) + 1)/a^(5/2) - 3/8*log(e^(-x) - 1)/a^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(49) = 98$.
time = 0.38, size = 875, normalized size = 14.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(42*cosh(x)*e^x*sinh(x)^6 + 6*e^x*sinh(x)^7 + 2*(63*cosh(x)^2 - 11)*e^
x*sinh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*e^x*sinh(x)^4 + 2*(105*cosh(x)
^4 - 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + 2*(63*cosh(x)^5 - 110*cosh(x)^3 -
33*cosh(x))*e^x*sinh(x)^2 + 2*(21*cosh(x)^6 - 55*cosh(x)^4 - 33*cosh(x)^2 +
 3)*e^x*sinh(x) + 2*(3*cosh(x)^7 - 11*cosh(x)^5 - 11*cosh(x)^3 + 3*cosh(x))
*e^x + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x
*sinh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 -
30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x)
)*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh
(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (
cosh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)*log((cosh(x)
+ sinh(x) - 1)/(cosh(x) + sinh(x) + 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*
e^(-x)/(a^3*cosh(x)^8 - 4*a^3*cosh(x)^6 - (a^3*e^(2*x) - a^3)*sinh(x)^8 - 8
*(a^3*cosh(x)*e^(2*x) - a^3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3
*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*co
sh(x)^3 - 3*a^3*cosh(x) - (7*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^5 - 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3 - (3
5*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*c
```

```
osh(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x) - (7*a^3*cosh(x)^5 - 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 - 15*
a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^6 - 15*a^3*cosh(x)^4
 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^2 - (a^3*cosh(x)^8 - 4*a^3*cosh(
x)^6 + 6*a^3*cosh(x)^4 - 4*a^3*cosh(x)^2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7
- 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x) - (a^3*cosh(x)^7 - 3*a^3*
cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^{2}\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**2)**(-5/2), x)
```

Giac [**A**]
time = 0.42, size = 52, normalized size = 0.85

$$\frac{3\left(e^{(-x)}+e^{x}\right)^{3}-20\,e^{(-x)}-20\,e^{x}}{4\left(\left(e^{(-x)}+e^{x}\right)^{2}-4\right)^{2}a^{\frac{5}{2}}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/(((e^(-x) + e^x)^2 - 4)^2*a^(
5/2)*sgn(e^(3*x) - e^x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a\,{\mathrm{sinh}\left(x\right)}^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(5/2), x)
```

3.146 $\int \left(a\sinh^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=135

$$-\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)}+\frac{26}{77}ia^2\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}+\frac{78}{385}a^2\cosh(x)\sinh(x$$

```
[Out] -26/77*a^2*coth(x)*(a*sinh(x)^3)^(1/2)+78/385*a^2*cosh(x)*sinh(x)*(a*sinh(x
)^3)^(1/2)-26/165*a^2*cosh(x)*sinh(x)^3*(a*sinh(x)^3)^(1/2)+2/15*a^2*cosh(x
)*sinh(x)^5*(a*sinh(x)^3)^(1/2)+26/77*I*a^2*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^
2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh
(x))^(1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2715, 2721, 2720}

$$-\frac{26}{165}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)}-\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)}+\frac{26}{77}ia^2\sqrt{i\sinh(x)}\,\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Coth[x]*Sqrt[a*Sinh[x]^3])/77 + ((26*I)/77)*a^2*Csch[x]^2*Elliptic
F[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3] + (78*a^2*Cosh[x]*Si
nh[x]*Sqrt[a*Sinh[x]^3])/385 - (26*a^2*Cosh[x]*Sinh[x]^3*Sqrt[a*Sinh[x]^3])
/165 + (2*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]^3])/15
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{15}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} - \frac{\left(13a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{11}{2}}(x)\,dx}{15\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{\left(39\right.}{}$$

$$= \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{26}{77}ia^2\text{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\,2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 0.50

$$\frac{a^2\text{csch}(x)\left(-15465\cosh(x)+3657\cosh(3x)-749\cosh(5x)+77\cosh(7x)-\frac{12480F\left(\frac{1}{4}(\pi-2ix)|2\right)}{\sqrt{i\sinh(x)}}\right)\sqrt{a\sinh^3(x)}}{36960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (a^2*Csch[x]*(-15465*Cosh[x] + 3657*Cosh[3*x] - 749*Cosh[5*x] + 77*Cosh[7*x
] - (12480*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]])*Sqrt[a*Sinh[x]^
3])/36960
```

Maple [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int \left(a\left(\sinh^{3}\left(x\right)\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(5/2),x)
```

```
[Out] int((a*sinh(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 823, normalized size = 6.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/73920*(24960*(sqrt(2)*a^2*cosh(x)^7 + 7*sqrt(2)*a^2*cosh(x)^6*sinh(x) + 2
1*sqrt(2)*a^2*cosh(x)^5*sinh(x)^2 + 35*sqrt(2)*a^2*cosh(x)^4*sinh(x)^3 + 35
*sqrt(2)*a^2*cosh(x)^3*sinh(x)^4 + 21*sqrt(2)*a^2*cosh(x)^2*sinh(x)^5 + 7*s
qrt(2)*a^2*cosh(x)*sinh(x)^6 + sqrt(2)*a^2*sinh(x)^7)*sqrt(a)*weierstrassPI
nverse(4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)^14 + 1078*a^2*cosh(x)*sin
h(x)^13 + 77*a^2*sinh(x)^14 - 749*a^2*cosh(x)^12 + 7*(1001*a^2*cosh(x)^2 -
107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*(1001*a^2*cosh(x)^3 - 321*a^
2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 - 49434*a^2*cosh(x)^2 + 3657*a
^2)*sinh(x)^10 - 15465*a^2*cosh(x)^8 + 2*(77077*a^2*cosh(x)^5 - 82390*a^2*c
osh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*(77077*a^2*cosh(x)^6 - 123585*a
^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 - 5155*a^2)*sinh(x)^8 - 15465*a^2*cosh(x
)^6 + 24*(11011*a^2*cosh(x)^7 - 24717*a^2*cosh(x)^5 + 18285*a^2*cosh(x)^3 -
 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a^2*cosh(x)^8 - 230692*a^2*cosh(x)^
```

```
6 + 255990*a^2*cosh(x)^4 - 144340*a^2*cosh(x)^2 - 5155*a^2)*sinh(x)^6 + 365
7*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^9 - 296604*a^2*cosh(x)^7 + 460782*a^
2*cosh(x)^5 - 433020*a^2*cosh(x)^3 - 46395*a^2*cosh(x))*sinh(x)^5 + (77077*
a^2*cosh(x)^10 - 370755*a^2*cosh(x)^8 + 767970*a^2*cosh(x)^6 - 1082550*a^2*
cosh(x)^4 - 231975*a^2*cosh(x)^2 + 3657*a^2)*sinh(x)^4 - 749*a^2*cosh(x)^2
+ 4*(7007*a^2*cosh(x)^11 - 41195*a^2*cosh(x)^9 + 109710*a^2*cosh(x)^7 - 216
510*a^2*cosh(x)^5 - 77325*a^2*cosh(x)^3 + 3657*a^2*cosh(x))*sinh(x)^3 + (70
07*a^2*cosh(x)^12 - 49434*a^2*cosh(x)^10 + 164565*a^2*cosh(x)^8 - 433020*a^
2*cosh(x)^6 - 231975*a^2*cosh(x)^4 + 21942*a^2*cosh(x)^2 - 749*a^2)*sinh(x)
^2 + 77*a^2 + 2*(539*a^2*cosh(x)^13 - 4494*a^2*cosh(x)^11 + 18285*a^2*cosh(
x)^9 - 61860*a^2*cosh(x)^7 - 46395*a^2*cosh(x)^5 + 7314*a^2*cosh(x)^3 - 749
*a^2*cosh(x))*sinh(x))*sqrt(a*sinh(x)))/(cosh(x)^7 + 7*cosh(x)^6*sinh(x) +
21*cosh(x)^5*sinh(x)^2 + 35*cosh(x)^4*sinh(x)^3 + 35*cosh(x)^3*sinh(x)^4 +
21*cosh(x)^2*sinh(x)^5 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^{3}(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(5/2),x)
[Out] Integral((a*sinh(x)**3)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\mathrm{sinh}\left(x\right)}^3\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(5/2),x)
[Out] int((a*sinh(x)^3)^(5/2), x)
```

3.147 $\int \left(a \sinh^3(x)\right)^{3/2} dx$

Optimal. Leaf size=83

$$-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}+\frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)}$$

```
[Out] -14/45*a*cosh(x)*(a*sinh(x)^3)^(1/2)+2/9*a*cosh(x)*sinh(x)^2*(a*sinh(x)^3)^
(1/2)+14/15*I*a*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*E
llipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*(a*sinh(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2715, 2721, 2719}

$$-\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)}+\frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)}+\frac{14ia\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cosh[x]*Sqrt[a*Sinh[x]^3])/45 + (((14*I)/15)*a*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[a*Sinh[x]^3])/Sqrt[I*Sinh[x]] + (2*a*Cosh[x]*Sinh[x]^2
*Sqrt[a*Sinh[x]^3])/9
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{9}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} - \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{5}{2}}(x)\,dx}{9\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\sqrt{a\sinh^3(x)}\right.}{15\,\mathrm{si}}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\mathrm{csch}(x)\sqrt{a\,\mathrm{s}}\right.}{15}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14ia\mathrm{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}} + \frac{2}{9}a\cosh(x)$$

Mathematica [**A**]
time = 0.05, size = 57, normalized size = 0.69

$$\frac{1}{180}a\mathrm{csch}(x)\sqrt{a\sinh^3(x)}\left(168\mathrm{csch}(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}-38\sinh(2x)+5\sinh(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(3/2),x]
[Out] (a*Csch[x]*Sqrt[a*Sinh[x]^3]*(168*Csch[x]*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]] - 38*Sinh[2*x] + 5*Sinh[4*x]))/180
```

Maple [**F**]
time = 1.06, size = 0, normalized size = 0.00

$$\int \left(a\left(\sinh^3(x)\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(3/2),x)
```

```
[Out] int((a*sinh(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 317, normalized size = 3.82

$$-\frac{336 \left(\sqrt{2} a \cosh\left(x\right)^{4} + 4 \sqrt{2} a \cosh\left(x\right)^{3} \sinh\left(x\right) + 6 \sqrt{2} a \cosh\left(x\right)^{2} \sinh\left(x\right)^{2} + 4 \sqrt{2} a \cosh\left(x\right) \sinh\left(x\right)^{3} + \sqrt{2} a \sinh\left(x\right)^{4}\right) \sqrt{a} \,\mathrm{weierstrassZeta}(4, 0, \mathrm{weierstrassPInverse}(4, 0, \cosh\left(x\right) + \sinh\left(x\right))) - \left(5 a \cosh\left(x\right)^{8} + 40 a \cosh\left(x\right) \sinh\left(x\right)^{7} + 5 a \sinh\left(x\right)^{8} - 38 a \cosh\left(x\right)^{6} + 2 \left(70 a \cosh\left(x\right)^{2} - 19 a\right) \sinh\left(x\right)^{6} + 4 \left(70 a \cosh\left(x\right)^{3} - 57 a \cosh\left(x\right)\right) \sinh\left(x\right)^{5} - 336 a \cosh\left(x\right)^{4} + 2 \left(175 a \cosh\left(x\right)^{4} - 285 a \cosh\left(x\right)^{2} - 168 a\right) \sinh\left(x\right)^{4} + 8 \left(35 a \cosh\left(x\right)^{5} - 95 a \cosh\left(x\right)^{3} - 168 a \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 38 a \cosh\left(x\right)^{2} + 2 \left(70 a \cosh\left(x\right)^{6} - 285 a \cosh\left(x\right)^{4} - 1008 a \cosh\left(x\right)^{2} + 19 a\right) \sinh\left(x\right)^{2} + 4 \left(10 a \cosh\left(x\right)^{7} - 57 a \cosh\left(x\right)^{5} - 336 a \cosh\left(x\right)^{3} + 19 a \cosh\left(x\right)\right) \sinh\left(x\right) - 5 a\right) \sqrt{a \sinh\left(x\right)}}{360 \left(\cosh\left(x\right)^{4} + 4 \cosh\left(x\right)^{3} \sinh\left(x\right) + 6 \cosh\left(x\right)^{2} \sinh\left(x\right)^{2} + 4 \cosh\left(x\right) \sinh\left(x\right)^{3} + \sinh\left(x\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/360*(336*(sqrt(2)*a*cosh(x)^4 + 4*sqrt(2)*a*cosh(x)^3*sinh(x) + 6*sqrt(2
)*a*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*a*cosh(x)*sinh(x)^3 + sqrt(2)*a*sinh(x)
^4)*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(
x))) - (5*a*cosh(x)^8 + 40*a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 - 38*a*cosh(
x)^6 + 2*(70*a*cosh(x)^2 - 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 - 57*a*cosh(
x))*sinh(x)^5 - 336*a*cosh(x)^4 + 2*(175*a*cosh(x)^4 - 285*a*cosh(x)^2 - 16
8*a)*sinh(x)^4 + 8*(35*a*cosh(x)^5 - 95*a*cosh(x)^3 - 168*a*cosh(x))*sinh(x
)^3 + 38*a*cosh(x)^2 + 2*(70*a*cosh(x)^6 - 285*a*cosh(x)^4 - 1008*a*cosh(x)
^2 + 19*a)*sinh(x)^2 + 4*(10*a*cosh(x)^7 - 57*a*cosh(x)^5 - 336*a*cosh(x)^3
 + 19*a*cosh(x))*sinh(x) - 5*a)*sqrt(a*sinh(x)))/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sinh^{3}{\left(x \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(3/2),x)
[Out] int((a*sinh(x)^3)^(3/2), x)
```

3.148 $\int \sqrt{a \sinh^3(x)}\, dx$

Optimal. Leaf size=62

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\operatorname{csch}^2(x)F\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}$$

```
[Out] 2/3*coth(x)*(a*sinh(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2715, 2721, 2720}

$$\frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\sqrt{i\sinh(x)}\operatorname{csch}^2(x)F\left(\frac{\pi}{4} - \frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (2*Coth[x]*Sqrt[a*Sinh[x]^3])/3 - ((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (I/2
)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
```

```
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sinh^3(x)}\,dx &= \frac{\sqrt{a\sinh^3(x)}\int \sinh^{\frac{3}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)} \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{\sqrt{a\sinh^3(x)}\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{3\sinh^{\frac{3}{2}}(x)} \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{1}{3}\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\operatorname{csch}^2(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.07, size = 60, normalized size = 0.97

$$\frac{2}{3}\sqrt{a\sinh^3(x)}\left(\coth(x)-\sqrt{2}\operatorname{csch}^2(x)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2x)+\sinh(2x)\right)\sqrt{-\sinh(x)(\cosh(x)+\sinh(x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^3],x]
[Out] (2*Sqrt[a*Sinh[x]^3]*(Coth[x] - Sqrt[2]*Csch[x]^2*Hypergeometric2F1[1/4, 1/
2, 5/4, Cosh[2*x] + Sinh[2*x]]*Sqrt[-(Sinh[x]*(Cosh[x] + Sinh[x]))]))/3
```

Maple [F]
time = 1.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^3(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 60, normalized size = 0.97

$$-\frac{2\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}(4,0,\cosh\left(x\right)+\sinh\left(x\right))-\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\sqrt{a\sinh\left(x\right)}}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
[Out] -1/3*(2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(a)*weierstrassPInverse(4,
0, cosh(x) + sinh(x)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqr
t(a*sinh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sinh^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*sinh(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a\,{\sinh\left(x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2), x)
```

3.149 $\int \frac{1}{\sqrt{a \sinh^3(x)}}\, dx$

Optimal. Leaf size=60

$$-\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\,2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\,\sqrt{a\sinh^3(x)}}$$

```
[Out] -2*cosh(x)*sinh(x)/(a*sinh(x)^3)^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/si
n(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/(I*sinh(
x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2716, 2721, 2719}

$$-\frac{2\sinh(x)\cosh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\,2\right)}{\sqrt{i\sinh(x)}\,\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^3] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2
]*Sinh[x]^2)/(Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx &= \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^{\frac{3}{2}}(x)\int \sqrt{\sinh(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^2(x)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.70

$$-\frac{2\left(\cosh(x)-E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)}\right)\sinh(x)}{\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/Sqr
t[a*Sinh[x]^3]
```

Maple [F]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\left(\sinh^3(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 97, normalized size = 1.62

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}-\sqrt{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(x\right)+\sinh\left(x\right)))+2\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\sqrt{a\sinh\left(x\right)}\right)}{a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sq
rt(2))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + si
nh(x))) + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a*sinh(x)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a \sinh(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
[Out] int(1/(a*sinh(x)^3)^(1/2), x)
```

3.150 $\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}+\frac{10iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\,\sinh(x)}{21a\sqrt{a\sinh^3(x)}}$$

```
[Out] 10/21*cosh(x)/a/(a*sinh(x)^3)^(1/2)-2/7*coth(x)*csch(x)/a/(a*sinh(x)^3)^(1/
2)+10/21*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(
1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(I*sinh(x))^(1/2)/a/(a*sinh(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3286, 2716, 2721, 2720}

$$\frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}+\frac{10i\sqrt{i\sinh(x)}\,\sinh(x)F\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (10*Cosh[x])/(21*a*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x])/(7*a*Sqrt[a*Sin
h[x]^3]) + (((10*I)/21)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sinh[x
])/(a*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} - \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{\left(5\sqrt{i\sinh(x)}\,\sinh(x)\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}$$

$$= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{10iF\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{i\sinh(x)}\,\sinh(x)}{21a\sqrt{a\sinh^3(x)}}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.61

$$\frac{2\left(5\cosh(x) - 3\coth(x)\text{csch}(x) + 5F\left(\frac{1}{4}(\pi-2ix)\middle|2\right)(i\sinh(x))^{3/2}\right)}{21a\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (2*(5*Cosh[x] - 3*Coth[x]*Csch[x] + 5*EllipticF[(Pi - (2*I)*x)/4, 2]*(I*Sin
h[x])^(3/2)))/(21*a*Sqrt[a*Sinh[x]^3])
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \left(\sinh^{3}\left(x \right)\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 639, normalized size = 7.34

$$\frac{2 \left(5 \left(\sqrt{2} \cosh\left(x\right)^{8} + 8 \sqrt{2} \cosh\left(x\right) \sinh\left(x\right)^{7} + \sqrt{2} \sinh\left(x\right)^{8} + 4 \left(7 \sqrt{2} \cosh\left(x\right)^{2} - \sqrt{2}\right) \sinh\left(x\right)^{6} - 4 \sqrt{2} \cosh\left(x\right)^{6} + 8 \left(7 \sqrt{2} \cosh\left(x\right)^{3} - 3 \sqrt{2} \cosh\left(x\right)\right) \sinh\left(x\right)^{5} + 2 \left(35 \sqrt{2} \cosh\left(x\right)^{4} - 30 \sqrt{2} \cosh\left(x\right)^{2} + 3 \sqrt{2}\right) \sinh\left(x\right)^{4} + 6 \sqrt{2} \cosh\left(x\right)^{4} + 8 \left(7 \sqrt{2} \cosh\left(x\right)^{5} - 10 \sqrt{2} \cosh\left(x\right)^{3} + 3 \sqrt{2} \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 4 \left(7 \sqrt{2} \cosh\left(x\right)^{6} - 15 \sqrt{2} \cosh\left(x\right)^{4} + 9 \sqrt{2} \cosh\left(x\right)^{2} - \sqrt{2}\right) \sinh\left(x\right)^{2} - 4 \sqrt{2} \cosh\left(x\right)^{2} + 8 \left(\sqrt{2} \cosh\left(x\right)^{7} - 3 \sqrt{2} \cosh\left(x\right)^{5} + 3 \sqrt{2} \cosh\left(x\right)^{3} - \sqrt{2} \cosh\left(x\right)\right) \sinh\left(x\right) + \sqrt{2}\right) \sqrt{a} {\rm weierstrassPInverse}\left(4, 0, \cosh\left(x\right) + \sinh\left(x\right)\right) + 2 \left(5 \cosh\left(x\right)^{7} + 35 \cosh\left(x\right) \sinh\left(x\right)^{6} + 5 \sinh\left(x\right)^{7} + \left(105 \cosh\left(x\right)^{2} - 17\right) \sinh\left(x\right)^{5} - 17 \cosh\left(x\right)^{5} + 5 \left(35 \cosh\left(x\right)^{3} - 17 \cosh\left(x\right)\right) \sinh\left(x\right)^{4} + \left(175 \cosh\left(x\right)^{4} - 170 \cosh\left(x\right)^{2} - 17\right) \sinh\left(x\right)^{3} - 17 \cosh\left(x\right)^{3} + \left(105 \cosh\left(x\right)^{5} - 170 \cosh\left(x\right)^{3} - 51 \cosh\left(x\right)\right) \sinh\left(x\right)^{2} + \left(35 \cosh\left(x\right)^{6} - 85 \cosh\left(x\right)^{4} - 51 \cosh\left(x\right)^{2} + 5\right) \sinh\left(x\right) + 5 \cosh\left(x\right)\right) \sqrt{a \sinh\left(x\right)}\right)}{21 \left(a^{2} \cosh\left(x\right)^{8} + 8 a^{2} \cosh\left(x\right) \sinh\left(x\right)^{7} + a^{2} \sinh\left(x\right)^{8} - 4 a^{2} \cosh\left(x\right)^{6} + 4 \left(7 a^{2} \cosh\left(x\right)^{2} - a^{2}\right) \sinh\left(x\right)^{6} + 6 a^{2} \cosh\left(x\right)^{4} + 8 \left(7 a^{2} \cosh\left(x\right)^{3} - 3 a^{2} \cosh\left(x\right)\right) \sinh\left(x\right)^{5} + 2 \left(35 a^{2} \cosh\left(x\right)^{4} - 30 a^{2} \cosh\left(x\right)^{2} + 3 a^{2}\right) \sinh\left(x\right)^{4} - 4 a^{2} \cosh\left(x\right)^{2} + 8 \left(7 a^{2} \cosh\left(x\right)^{5} - 10 a^{2} \cosh\left(x\right)^{3} + 3 a^{2} \cosh\left(x\right)\right) \sinh\left(x\right)^{3} + 4 \left(7 a^{2} \cosh\left(x\right)^{6} - 15 a^{2} \cosh\left(x\right)^{4} + 9 a^{2} \cosh\left(x\right)^{2} - a^{2}\right) \sinh\left(x\right)^{2} + a^{2} + 8 \left(a^{2} \cosh\left(x\right)^{7} - 3 a^{2} \cosh\left(x\right)^{5} + 3 a^{2} \cosh\left(x\right)^{3} - a^{2} \cosh\left(x\right)\right) \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^
8 + 4*(7*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^6 - 4*sqrt(2)*cosh(x)^6 + 8*(
7*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^
4 - 30*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^4 + 6*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 - 10*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 4*(7*sqrt(2)*cosh(x)^6 - 15*sqrt(2)*cosh(x)^4 + 9*sqrt(2)*cosh(x)^2 - sqrt
(2))*sinh(x)^2 - 4*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 3*sqrt(2)*cos
h(x)^5 + 3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sqrt(a)*
weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + 2*(5*cosh(x)^7 + 35*cosh(x)*
sinh(x)^6 + 5*sinh(x)^7 + (105*cosh(x)^2 - 17)*sinh(x)^5 - 17*cosh(x)^5 + 5
*(35*cosh(x)^3 - 17*cosh(x))*sinh(x)^4 + (175*cosh(x)^4 - 170*cosh(x)^2 - 1
7)*sinh(x)^3 - 17*cosh(x)^3 + (105*cosh(x)^5 - 170*cosh(x)^3 - 51*cosh(x))*
sinh(x)^2 + (35*cosh(x)^6 - 85*cosh(x)^4 - 51*cosh(x)^2 + 5)*sinh(x) + 5*co
sh(x))*sqrt(a*sinh(x)))/(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh
(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(
x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4
```

```
- 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^
5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a
^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 -
3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**3)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,{\mathrm{sinh}\left(x\right)}^3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(3/2), x)
```

3.151 $\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\,\sqrt{}}$$

```
[Out] -154/585*coth(x)/a^2/(a*sinh(x)^3)^(1/2)+22/117*coth(x)*csch(x)^2/a^2/(a*si
nh(x)^3)^(1/2)-2/13*coth(x)*csch(x)^4/a^2/(a*sinh(x)^3)^(1/2)+154/195*cosh(
x)*sinh(x)/a^2/(a*sinh(x)^3)^(1/2)-154/195*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/
sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/a^2/(I
*sinh(x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2716, 2721, 2719}

$$\frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}}-\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}}-\frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\,\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}}+\frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-154*Coth[x])/(585*a^2*Sqrt[a*Sinh[x]^3]) + (22*Coth[x]*Csch[x]^2)/(117*a^
2*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x]^4)/(13*a^2*Sqrt[a*Sinh[x]^3]) + (
154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Sinh[x]^3]) - (((154*I)/195)*EllipticE
[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/(a^2*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(11\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sinh^3(x)}}$$

$$= \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int}{195a^2\sqrt{a\sin}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh}{195a^2\sqrt{a\sinh^3(}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh}{195a^2\sqrt{a\sinh^3(}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh}{195a^2\sqrt{a\sinh^3(}}$$

Mathematica [A]
time = 0.19, size = 69, normalized size = 0.51

$$\frac{-2\coth(x)\left(77-55\text{csch}^2(x)+45\text{csch}^4(x)\right)+462iE\left(\frac{1}{4}(\pi-2ix)\middle|2\right)(i\sinh(x))^{3/2}+462\cosh(x)\sinh(x)}{585a^2\sqrt{a\sinh^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-2*Coth[x]*(77 - 55*Csch[x]^2 + 45*Csch[x]^4) + (462*I)*EllipticE[(Pi - (2
*I)*x)/4, 2]*(I*Sinh[x])^(3/2) + 462*Cosh[x]*Sinh[x])/(585*a^2*Sqrt[a*Sinh[
x]^3])
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \left(\sinh^{3}\left(x\right)\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 1676, normalized size = 12.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(231*(sqrt(2)*cosh(x)^14 + 14*sqrt(2)*cosh(x)*sinh(x)^13 + sqrt(2)*si
nh(x)^14 + 7*(13*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^12 - 7*sqrt(2)*cosh(x
)^12 + 28*(13*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^11 + 7*(143*sq
rt(2)*cosh(x)^4 - 66*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^10 + 21*sqrt(2)
*cosh(x)^10 + 14*(143*sqrt(2)*cosh(x)^5 - 110*sqrt(2)*cosh(x)^3 + 15*sqrt(2
)*cosh(x))*sinh(x)^9 + 7*(429*sqrt(2)*cosh(x)^6 - 495*sqrt(2)*cosh(x)^4 + 1
35*sqrt(2)*cosh(x)^2 - 5*sqrt(2))*sinh(x)^8 - 35*sqrt(2)*cosh(x)^8 + 8*(429
*sqrt(2)*cosh(x)^7 - 693*sqrt(2)*cosh(x)^5 + 315*sqrt(2)*cosh(x)^3 - 35*sqr
t(2)*cosh(x))*sinh(x)^7 + 7*(429*sqrt(2)*cosh(x)^8 - 924*sqrt(2)*cosh(x)^6
```

+ 630*sqrt(2)*cosh(x)^4 - 140*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh(x)^6 + 35
*sqrt(2)*cosh(x)^6 + 14*(143*sqrt(2)*cosh(x)^9 - 396*sqrt(2)*cosh(x)^7 + 37
8*sqrt(2)*cosh(x)^5 - 140*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x))*sinh(x)^5
 + 7*(143*sqrt(2)*cosh(x)^10 - 495*sqrt(2)*cosh(x)^8 + 630*sqrt(2)*cosh(x)^
6 - 350*sqrt(2)*cosh(x)^4 + 75*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^4 - 2
1*sqrt(2)*cosh(x)^4 + 28*(13*sqrt(2)*cosh(x)^11 - 55*sqrt(2)*cosh(x)^9 + 90
*sqrt(2)*cosh(x)^7 - 70*sqrt(2)*cosh(x)^5 + 25*sqrt(2)*cosh(x)^3 - 3*sqrt(2
)*cosh(x))*sinh(x)^3 + 7*(13*sqrt(2)*cosh(x)^12 - 66*sqrt(2)*cosh(x)^10 + 1
35*sqrt(2)*cosh(x)^8 - 140*sqrt(2)*cosh(x)^6 + 75*sqrt(2)*cosh(x)^4 - 18*sq
rt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 7*sqrt(2)*cosh(x)^2 + 14*(sqrt(2)*co
sh(x)^13 - 6*sqrt(2)*cosh(x)^11 + 15*sqrt(2)*cosh(x)^9 - 20*sqrt(2)*cosh(x)
^7 + 15*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x)
- sqrt(2))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x)
+ sinh(x))) + 2*(231*cosh(x)^14 + 3234*cosh(x)*sinh(x)^13 + 231*sinh(x)^14
+ 77*(273*cosh(x)^2 - 20)*sinh(x)^12 - 1540*cosh(x)^12 + 924*(91*cosh(x)^3
- 20*cosh(x))*sinh(x)^11 + 11*(21021*cosh(x)^4 - 9240*cosh(x)^2 + 397)*sinh
(x)^10 + 4367*cosh(x)^10 + 22*(21021*cosh(x)^5 - 15400*cosh(x)^3 + 1985*cos
h(x))*sinh(x)^9 + (693693*cosh(x)^6 - 762300*cosh(x)^4 + 196515*cosh(x)^2 -
 6808)*sinh(x)^8 - 6808*cosh(x)^8 + 8*(99099*cosh(x)^7 - 152460*cosh(x)^5 +
 65505*cosh(x)^3 - 6808*cosh(x))*sinh(x)^7 + (693693*cosh(x)^8 - 1422960*co
sh(x)^6 + 917070*cosh(x)^4 - 190624*cosh(x)^2 + 1277)*sinh(x)^6 + 1277*cosh
(x)^6 + 2*(231231*cosh(x)^9 - 609840*cosh(x)^7 + 550242*cosh(x)^5 - 190624*
cosh(x)^3 + 3831*cosh(x))*sinh(x)^5 + (231231*cosh(x)^10 - 762300*cosh(x)^8
 + 917070*cosh(x)^6 - 476560*cosh(x)^4 + 19155*cosh(x)^2 - 484)*sinh(x)^4 -
 484*cosh(x)^4 + 4*(21021*cosh(x)^11 - 84700*cosh(x)^9 + 131010*cosh(x)^7 -
 95312*cosh(x)^5 + 6385*cosh(x)^3 - 484*cosh(x))*sinh(x)^3 + (21021*cosh(x)
^12 - 101640*cosh(x)^10 + 196515*cosh(x)^8 - 190624*cosh(x)^6 + 19155*cosh(
x)^4 - 2904*cosh(x)^2 + 77)*sinh(x)^2 + 77*cosh(x)^2 + 2*(1617*cosh(x)^13 -
 9240*cosh(x)^11 + 21835*cosh(x)^9 - 27232*cosh(x)^7 + 3831*cosh(x)^5 - 968
*cosh(x)^3 + 77*cosh(x))*sinh(x))*sqrt(a*sinh(x)))/(a^3*cosh(x)^14 + 14*a^3
*cosh(x)*sinh(x)^13 + a^3*sinh(x)^14 - 7*a^3*cosh(x)^12 + 21*a^3*cosh(x)^10
 + 7*(13*a^3*cosh(x)^2 - a^3)*sinh(x)^12 + 28*(13*a^3*cosh(x)^3 - 3*a^3*cos
h(x))*sinh(x)^11 - 35*a^3*cosh(x)^8 + 7*(143*a^3*cosh(x)^4 - 66*a^3*cosh(x)
^2 + 3*a^3)*sinh(x)^10 + 14*(143*a^3*cosh(x)^5 - 110*a^3*cosh(x)^3 + 15*a^3
*cosh(x))*sinh(x)^9 + 35*a^3*cosh(x)^6 + 7*(429*a^3*cosh(x)^6 - 495*a^3*cos
h(x)^4 + 135*a^3*cosh(x)^2 - 5*a^3)*sinh(x)^8 + 8*(429*a^3*cosh(x)^7 - 693*
a^3*cosh(x)^5 + 315*a^3*cosh(x)^3 - 35*a^3*cosh(x))*sinh(x)^7 - 21*a^3*cosh
(x)^4 + 7*(429*a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 630*a^3*cosh(x)^4 - 140*
a^3*cosh(x)^2 + 5*a^3)*sinh(x)^6 + 14*(143*a^3*cosh(x)^9 - 396*a^3*cosh(x)^
7 + 378*a^3*cosh(x)^5 - 140*a^3*cosh(x)^3 + 15*a^3*cosh(x))*sinh(x)^5 + 7*a
^3*cosh(x)^2 + 7*(143*a^3*cosh(x)^10 - 495*a^3*cosh(x)^8 + 630*a^3*cosh(x)^
6 - 350*a^3*cosh(x)^4 + 75*a^3*cosh(x)^2 - 3*a^3)*sinh(x)^4 + 28*(13*a^3*co
sh(x)^11 - 55*a^3*cosh(x)^9 + 90*a^3*cosh(x)^7 - 70*a^3*cosh(x)^5 + 25*a^3*
cosh(x)^3 - 3*a^3*cosh(x))*sinh(x)^3 - a^3 + 7*(13*a^3*cosh(x)^12 - 66*a^3*
cosh(x)^10 + 135*a^3*cosh(x)^8 - 140*a^3*cosh(x)^6 + 75*a^3*cosh(x)^4 - 18*

```
a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 14*(a^3*cosh(x)^13 - 6*a^3*cosh(x)^11 + 15
*a^3*cosh(x)^9 - 20*a^3*cosh(x)^7 + 15*a^3*cosh(x)^5 - 6*a^3*cosh(x)^3 + a^
3*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**3)**(5/2),x)
[Out] Integral((a*sinh(x)**3)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,{\sinh\left(x\right)}^3\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
[Out] int(1/(a*sinh(x)^3)^(5/2), x)
```

3.152 $\int \left(a \sinh^4(x)\right)^{5/2} dx$

Optimal. Leaf size=132

$$\frac{63}{256}a^2 \coth(x)\sqrt{a \sinh^4(x)} - \frac{63}{256}a^2 x\text{csch}^2(x)\sqrt{a \sinh^4(x)} - \frac{21}{128}a^2 \cosh(x)\sinh(x)\sqrt{a \sinh^4(x)} + \frac{21}{160}a^2 \cosh$$

```
[Out] 63/256*a^2*coth(x)*(a*sinh(x)^4)^(1/2)-63/256*a^2*x*csch(x)^2*(a*sinh(x)^4)
^(1/2)-21/128*a^2*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+21/160*a^2*cosh(x)*si
nh(x)^3*(a*sinh(x)^4)^(1/2)-9/80*a^2*cosh(x)*sinh(x)^5*(a*sinh(x)^4)^(1/2)+
1/10*a^2*cosh(x)*sinh(x)^7*(a*sinh(x)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$-\frac{21}{128}a^2 \sinh(x)\cosh(x)\sqrt{a \sinh^4(x)} + \frac{1}{10}a^2 \sinh^7(x)\cosh(x)\sqrt{a \sinh^4(x)} - \frac{9}{80}a^2 \sinh^5(x)\cosh(x)\sqrt{a \sinh^4(x)} + \frac{21}{160}a^2 \sinh^3(x)\cosh(x)\sqrt{a \sinh^4(x)} + \frac{63}{256}a^2 \coth(x)\sqrt{a \sinh^4(x)} - \frac{63}{256}a^2 x\text{csch}^2(x)\sqrt{a \sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*Coth[x]*Sqrt[a*Sinh[x]^4])/256 - (63*a^2*x*Csch[x]^2*Sqrt[a*Sinh[x]
^4])/256 - (21*a^2*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/128 + (21*a^2*Cosh[x]
*Sinh[x]^3*Sqrt[a*Sinh[x]^4])/160 - (9*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]
^4])/80 + (a^2*Cosh[x]*Sinh[x]^7*Sqrt[a*Sinh[x]^4])/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \left(a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{10}\left(9a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^8(x)\,dx$$
$$= -\frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} + \frac{1}{80}\Big($$
$$= \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2$$
$$= -\frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)$$
$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{63}{256}a^2x\text{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh($$

Mathematica [A]
time = 0.12, size = 53, normalized size = 0.40

$$\frac{a\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}(-2520x+2100\sinh(2x)-600\sinh(4x)+150\sinh(6x)-25\sinh(8x)+2\sinh(10x))}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(5/2),x]
[Out] (a*Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x]
+ 150*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Maple [A]
time = 1.51, size = 171, normalized size = 1.30

method	result
default	$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\,a^{\frac{3}{2}}\left(8\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a}\,(\sinh^4(2x))-50\sqrt{a\,(\sinh}\right.}{}$
risch	$-\frac{63a^2e^{2x}\sqrt{a\left(e^{2x}-1\right)^4e^{-4x}}\,x}{256(e^{2x}-1)^2}+\frac{a^2e^{12x}\sqrt{a\left(e^{2x}-1\right)^4e^{-4x}}}{10240(e^{2x}-1)^2}-\frac{5a^2e^{10x}\sqrt{a\left(e^{2x}-1\right)^4e^{-4x}}}{4096(e^{2x}-1)^2}+\frac{15a^2e^{8x}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2560*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*a^(3/2)*(8*(a*
sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4-50*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*cos
h(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-325*cosh(2
*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-315*ln(
cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/(a*(-1+cosh(2*x))^2)^
(1/2)
```

Maxima [A]
time = 0.49, size = 100, normalized size = 0.76

$$-\frac{63}{256}\,a^{\frac{5}{2}}x-\frac{1}{20480}\,\left(25\,a^{\frac{5}{2}}e^{(-2\,x)}-150\,a^{\frac{5}{2}}e^{(-4\,x)}+600\,a^{\frac{5}{2}}e^{(-6\,x)}-2100\,a^{\frac{5}{2}}e^{(-8\,x)}+2100\,a^{\frac{5}{2}}e^{(-12\,x)}-600\,a^{\frac{5}{2}}e^{(-14\,x)}+150\,a^{\frac{5}{2}}e^{(-16\,x)}-25\,a^{\frac{5}{2}}e^{(-18\,x)}+2\,a^{\frac{5}{2}}e^{(-20\,x)}-2\,a^{\frac{5}{2}}\right)e^{(10\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -63/256*a^(5/2)*x - 1/20480*(25*a^(5/2)*e^(-2*x) - 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) - 2100*a^(5/2)*e^(-8*x) + 2100*a^(5/2)*e^(-12*x) - 600*
a^(5/2)*e^(-14*x) + 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) + 2*a^(5/2
)*e^(-20*x) - 2*a^(5/2))*e^(10*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(108) = 216$.
time = 0.44, size = 1597, normalized size = 12.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 - 15*a^
2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 - 255*a^2*cosh(x)^2 +
 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 - 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 - 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 - 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 - 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sin
h(x)^13 + 60*(4199*a^2*cosh(x)^8 - 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
- 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 -
 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 - 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 - 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 - 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 - 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 - 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 - 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 - 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 - 36465*a^2*cosh(x
```

```
)^10 + 64350*a^2*cosh(x)^8 - 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 - 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 -
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 - 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 - 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 - 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 - 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 - 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
 + 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 - 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 - 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 - 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 + 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
 + 30*(323*a^2*cosh(x)^16 - 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 - 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 - 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 - 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 - 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 - 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 + 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 - 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 - 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 - 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 + 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 - 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 - 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 -
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 + 360*a^2*cosh(x)^5 - 60*a^2*cosh
(x)^3 + 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 - 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 - 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 - 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 + 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 + 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) - 2*cosh(x)^10*e^(2
*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) - 2*cos
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) - 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) - 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) - 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) - 2*cosh(x)^7*e^(2*x) + cosh(x)^
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) - 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) - 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sinh^4(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(5/2),x)
[Out] Integral((a*sinh(x)**4)**(5/2), x)
```

Giac [A]
time = 0.42, size = 114, normalized size = 0.86

$$-\frac{1}{20480}\left(5040\,a^2x - 2\,a^2e^{(10\,x)} + 25\,a^2e^{(8\,x)} - 150\,a^2e^{(6\,x)} + 600\,a^2e^{(4\,x)} - 2100\,a^2e^{(2\,x)} - \left(5754\,a^2e^{(10\,x)} - 2100\,a^2e^{(8\,x)} + 600\,a^2e^{(6\,x)} - 150\,a^2e^{(4\,x)} + 25\,a^2e^{(2\,x)} - 2\,a^2\right)e^{(-10\,x)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*(5040*a^2*x - 2*a^2*e^(10*x) + 25*a^2*e^(8*x) - 150*a^2*e^(6*x) +
600*a^2*e^(4*x) - 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) - 2100*a^2*e^(8*x)
+ 600*a^2*e^(6*x) - 150*a^2*e^(4*x) + 25*a^2*e^(2*x) - 2*a^2)*e^(-10*x))*sq
rt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(5/2),x)
```

```
[Out] int((a*sinh(x)^4)^(5/2), x)
```

3.153 $\int \left(a \sinh^4(x)\right)^{3/2} dx$

Optimal. Leaf size=78

$$\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)}-\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)}+\frac{1}{6}a\cosh(x)\sinh$$

```
[Out] 5/16*a*coth(x)*(a*sinh(x)^4)^(1/2)-5/16*a*x*csch(x)^2*(a*sinh(x)^4)^(1/2)-5
/24*a*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+1/6*a*cosh(x)*sinh(x)^3*(a*sinh(x
)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$-\frac{5}{24}a\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{1}{6}a\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)}-\frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (5*a*Coth[x]*Sqrt[a*Sinh[x]^4])/16 - (5*a*x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/16
 - (5*a*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/24 + (a*Cosh[x]*Sinh[x]^3*Sqrt[a
*Sinh[x]^4])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \left(a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^6(x)\,dx$$
$$= \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{1}{6}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^4(x)\,dx$$
$$= -\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} + \frac{1}{8}\left(5a\text{csch}^2\right.$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)$$
$$= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sin}$$

Mathematica [A]
time = 0.07, size = 38, normalized size = 0.49

$$\frac{1}{192}\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}(-60x+45\sinh(2x)-9\sinh(4x)+\sinh(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6
*x]))/192
```

Maple [A]
time = 1.43, size = 125, normalized size = 1.60

method	result
default	$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\sqrt{a}\left(2\sqrt{a(\sinh^2(2x))}\sqrt{a}\,(\sinh^2(2x))-9\cosh(2x)\sqrt{\right.}{96\sinh(2x)\sqrt{a(-1+\cosh}}$
risch	$-\frac{5a\,e^{2x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,x}{16(e^{2x}-1)^2}+\frac{a\,e^{8x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{384(e^{2x}-1)^2}-\frac{3a\,e^{6x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{128(e^{2x}-1)^2}+\frac{15a\,e^{4x}\sqrt{a(e^{2}}}{128(e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*a^(1/2)*(2*(a*si
nh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2-9*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1
/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-15*ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2
)^(1/2))*a)/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 63, normalized size = 0.81

$$-\frac{5}{16}\,a^{\frac{3}{2}}x-\frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)}-45\,a^{\frac{3}{2}}e^{(-4\,x)}+45\,a^{\frac{3}{2}}e^{(-8\,x)}-9\,a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}e^{(-12\,x)}-a^{\frac{3}{2}}\right)e^{(6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^(3/2)*x - 1/384*(9*a^(3/2)*e^(-2*x) - 45*a^(3/2)*e^(-4*x) + 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) + a^(3/2)*e^(-12*x) - a^(3/2))*e^(6*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(62) = 124.
time = 0.41, size = 659, normalized size = 8.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 - 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 - 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 - 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 - 315*a*cosh(x)^4 + 210*a*cosh(x)^2 - 20*a*x)*e^(2*x)*sinh(x)
^6 + 36*(22*a*cosh(x)^7 - 63*a*cosh(x)^5 + 70*a*cosh(x)^3 - 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 - 42*a*cosh(x)^6 + 70*a*cosh(x)^4 -
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 - 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 - 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
+ 3*(22*a*cosh(x)^10 - 135*a*cosh(x)^8 + 420*a*cosh(x)^6 - 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 + 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 - 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 - 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 + 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 - 9*a*cosh(x)^10 + 45*a*cosh(x)^8 - 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 + 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) -
4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) -
2*cosh(x)^6*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
- 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) - 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sinh^{4}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(3/2),x)
[Out] Integral((a*sinh(x)**4)**(3/2), x)
```

Giac [A]
time = 0.42, size = 50, normalized size = 0.64

$$\frac{1}{384}\left(\left(110\,e^{(6\,x)}-45\,e^{(4\,x)}+9\,e^{(2\,x)}-1\right)e^{(-6\,x)}-120\,x+e^{(6\,x)}-9\,e^{(4\,x)}+45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sinh(x)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(3/2),x)
[Out] int((a*sinh(x)^4)^(3/2), x)
```

3.154 $\int \sqrt{a\sinh^4(x)}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\text{csch}^2(x)\sqrt{a\sinh^4(x)}$$

```
[Out] 1/2*coth(x)*(a*sinh(x)^4)^(1/2)-1/2*x*csch(x)^2*(a*sinh(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3286, 2715, 8}

$$\frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\text{csch}^2(x)\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] (Coth[x]*Sqrt[a*Sinh[x]^4])/2 - (x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a \sinh^4(x)}\, dx &= \left(\operatorname{csch}^2(x)\sqrt{a \sinh^4(x)}\right)\int \sinh^2(x)\, dx \\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}\left(\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\, dx \\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)} - \frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 24, normalized size = 0.67

$$\frac{1}{2}(\coth(x) - x\operatorname{csch}^2(x))\sqrt{a\sinh^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sinh[x]^4],x]
[Out] ((Coth[x] - x*Csch[x]^2)*Sqrt[a*Sinh[x]^4])/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(28) = 56.
time = 1.50, size = 84, normalized size = 2.33

method	result
default	$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(\cosh(2x)+1)}\left(\sqrt{a(\sinh^2(2x))}\sqrt{a}-\ln\left(\cosh(2x)\sqrt{a}+\sqrt{a(\sinh\right.\right.}{4\sqrt{a}\sinh(2x)\sqrt{a(-1+\cosh(2x))^2}}$
risch	$-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\, e^{2x} x}{2(e^{2x}-1)^2}+\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\, e^{4x}}{8(e^{2x}-1)^2}-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{8(e^{2x}-1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)*((a*sinh(2*x)^2)^
(1/2)*a^(1/2)-ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/a^(1/2)/sinh(2
*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [**A**]
time = 0.50, size = 27, normalized size = 0.75

$$-\frac{1}{8}\left(\sqrt{a}e^{(-4x)} - \sqrt{a}\right)e^{(2x)} - \frac{1}{2}\sqrt{a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) - 1/2*sqrt(a)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(28) = 56$.
time = 0.39, size = 180, normalized size = 5.00

$$\frac{\left(4\,\cosh\left(x\right)e^{\left(2\,x\right)}\sinh\left(x\right)^{3}+e^{\left(2\,x\right)}\sinh\left(x\right)^{4}+2\,\left(3\,\cosh\left(x\right)^{2}-2\,x\right)e^{\left(2\,x\right)}\sinh\left(x\right)^{2}+4\,\left(\cosh\left(x\right)^{3}-2\,x\cosh\left(x\right)\right)e^{\left(2\,x\right)}\sinh\left(x\right)+\left(\cosh\left(x\right)^{4}-4\,x\cosh\left(x\right)^{2}-1\right)e^{\left(2\,x\right)}\right)\sqrt{ae^{\left(8\,x\right)}-4\,ae^{\left(6\,x\right)}+6\,ae^{\left(4\,x\right)}-4\,ae^{\left(2\,x\right)}+a}\,e^{\left(-2\,x\right)}}{8\,\left(\cosh\left(x\right)^{2}e^{\left(4\,x\right)}-2\,\cosh\left(x\right)^{2}e^{\left(2\,x\right)}+\left(e^{\left(4\,x\right)}-2\,e^{\left(2\,x\right)}+1\right)\sinh\left(x\right)^{2}+\cosh\left(x\right)^{2}+2\,\left(\cosh\left(x\right)e^{\left(4\,x\right)}-2\,\cosh\left(x\right)e^{\left(2\,x\right)}+\cosh\left(x\right)\right)\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 - 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 - 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x
) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + (e
^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) - 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sinh^{4}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(x)**4), x)
```

Giac [A]
time = 0.40, size = 26, normalized size = 0.72

$$\frac{1}{8}\,\left(\left(2\,e^{\left(2\,x\right)}-1\right)e^{\left(-2\,x\right)}-4\,x+e^{\left(2\,x\right)}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\,{\sinh\left(x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sinh(x)^4)^(1/2),x)
[Out] int((a*sinh(x)^4)^(1/2), x)
```

3.155 $\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

```
[Out] -cosh(x)*sinh(x)/(a*sinh(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 3852, 8}

$$-\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sinh[x]^4],x]
```

```
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^2(x)\,dx}{\sqrt{a\sinh^4(x)}}$$
$$= -\frac{(i\sinh^2(x))\,\text{Subst}(\int 1\,dx, x, -i\coth(x))}{\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sinh[x]^4],x]
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.
time = 1.29, size = 50, normalized size = 3.12

method	result	size
risch	$-\frac{2\,e^{-2x}(e^{2x}-1)}{\sqrt{a\left(e^{2x}-1\right)^4 e^{-4x}}}$	29
default	$-\frac{\sqrt{a\left(-1+\cosh(2x)\right)\left(\cosh(2x)+1\right)}\,\sqrt{a\left(\sinh^2(2x)\right)}}{a\sinh(2x)\sqrt{a\left(-1+\cosh(2x)\right)^2}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/a*(a*sinh(2*x)^2)^(1/2)/sinh(2*x)/(
a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 18, normalized size = 1.12

$$\frac{2}{\sqrt{a}\,e^{(-2\,x)} - \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(a)*e^(-2*x) - sqrt(a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(14) = 28$.
time = 0.37, size = 122, normalized size = 7.62

$$-\frac{2\sqrt{ae^{(8x)} - 4ae^{(6x)} + 6ae^{(4x)} - 4ae^{(2x)} + a}}{a\cosh(x)^2 + (ae^{(4x)} - 2ae^{(2x)} + a)\sinh(x)^2 + (a\cosh(x)^2 - a)e^{(4x)} - 2(a\cosh(x)^2 - a)e^{(2x)} + 2(a\cosh(x)e^{(4x)} - 2a\cosh(x)e^{(2x)} + a\cosh(x))\sinh(x) - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 - a)*e^(4*x) -
2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**4), x)
```

Giac [A]
time = 0.45, size = 13, normalized size = 0.81

$$-\frac{2}{\sqrt{a}\,(e^{(2x)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(a)*(e^(2*x) - 1))
```

Mupad [B]
time = 0.47, size = 38, normalized size = 2.38

$$\frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^4}}{a\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(1/2),x)
[Out] (exp(-x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 - exp(x)/2)^3)
```

3.156 $\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}$$

```
[Out] 2/3*cosh(x)^2*coth(x)/a/(a*sinh(x)^4)^(1/2)-1/5*cosh(x)^2*coth(x)^3/a/(a*si
nh(x)^4)^(1/2)-cosh(x)*sinh(x)/a/(a*sinh(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*Coth[x])/(3*a*Sqrt[a*Sinh[x]^4]) - (Cosh[x]^2*Coth[x]^3)/(5*a*
Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a*Sqrt[a*Sinh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^6(x)\,dx}{a\sqrt{a\sinh^4(x)}}$$
$$= -\frac{(i\sinh^2(x))\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-i\coth(x)\right)}{a\sqrt{a\sinh^4(x)}}$$
$$= \frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}} - \frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.50

$$-\frac{\cosh(x)\left(8-4\operatorname{csch}^2(x)+3\operatorname{csch}^4(x)\right)\sinh^5(x)}{15\left(a\sinh^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] -1/15*(Cosh[x]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]^5)/(a*Sinh[x]^4)^(3/
2)
```

Maple [A]
time = 1.23, size = 74, normalized size = 1.09

method	result	size
risch	$-\frac{16\,\mathrm{e}^{-2x}\left(10\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+1\right)}{15a\left(\mathrm{e}^{2x}-1\right)^3\sqrt{a\left(\mathrm{e}^{2x}-1\right)^4\mathrm{e}^{-4x}}}$	48
default	$-\frac{4\left(2\left(\cosh^2(2x)\right)-6\cosh(2x)+7\right)\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a\left(-1+\cosh(2x)\right)\left(\cosh(2x)+1\right)}}{15a^2\left(-1+\cosh(2x)\right)^2\sinh(2x)\sqrt{a\left(-1+\cosh(2x)\right)^2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/15/a^2*(2*cosh(2*x)^2-6*cosh(2*x)+7)*(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2
*x))*(cosh(2*x)+1))^(1/2)/(-1+cosh(2*x))^2/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(
1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(58) = 116.

time = 0.52, size = 171, normalized size = 2.51

$$-\frac{16\,e^{(-2\,x)}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} + \frac{32\,e^{(-4\,x)}}{3\,\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} + \frac{16}{15\,\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 32/3*e^(-4*x)/(5*
a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^
(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) - 10*a^(3
/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
 - a^(3/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(58) = 116$.
time = 0.40, size = 1163, normalized size = 17.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 - 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 - 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) - 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 - 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) - 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 - a^2 + (9
*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(9*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^3
 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 - 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 - 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 - 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) - 2*(63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 - a^2 + (21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(4*x) - 2*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 -
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
```

```
(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) - 2*(9*a^2*cosh(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 - a^2 + (a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(
a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 - a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 - 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) - 2*(a^2*cosh(
x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-3/2), x)
```

Giac [A]
time = 0.41, size = 27, normalized size = 0.40

$$-\frac{16\left(10\,e^{(4\,x)}-5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) - 1)^5)
```

Mupad [B]
time = 0.53, size = 48, normalized size = 0.71

$$-\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}-1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(3/2),x)
```

```
[Out] -(64*exp(2*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x)
+ 1))/(15*a^2*(exp(2*x) - 1)^7)
```

3.157 $\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\sinh^4(x)}}$$

```
[Out] 4/3*cosh(x)^2*coth(x)/a^2/(a*sinh(x)^4)^(1/2)-6/5*cosh(x)^2*coth(x)^3/a^2/(
a*sinh(x)^4)^(1/2)+4/7*cosh(x)^2*coth(x)^5/a^2/(a*sinh(x)^4)^(1/2)-1/9*cosh
(x)^2*coth(x)^7/a^2/(a*sinh(x)^4)^(1/2)-cosh(x)*sinh(x)/a^2/(a*sinh(x)^4)^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$-\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}+\frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] (4*Cosh[x]^2*Coth[x])/(3*a^2*Sqrt[a*Sinh[x]^4]) - (6*Cosh[x]^2*Coth[x]^3)/(
5*a^2*Sqrt[a*Sinh[x]^4]) + (4*Cosh[x]^2*Coth[x]^5)/(7*a^2*Sqrt[a*Sinh[x]^4]
) - (Cosh[x]^2*Coth[x]^7)/(9*a^2*Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a^
2*Sqrt[a*Sinh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \frac{\sinh^2(x)\int \operatorname{csch}^{10}(x)\,dx}{a^2\sqrt{a\sinh^4(x)}}$$
$$= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\coth(x)\right)}{a^2\sqrt{a\sinh^4(x)}}$$
$$= \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 0.40

$$-\frac{\cosh(x)\left(128-64\operatorname{csch}^2(x)+48\operatorname{csch}^4(x)-40\operatorname{csch}^6(x)+35\operatorname{csch}^8(x)\right)\sinh(x)}{315a^2\sqrt{a\sinh^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] -1/315*(Cosh[x]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csch[x]^6 + 35*Csch
[x]^8)*Sinh[x])/(a^2*Sqrt[a*Sinh[x]^4])
```

Maple [A]
time = 1.19, size = 90, normalized size = 0.76

method	result
risch	$-\frac{256\,\mathrm{e}^{-2x}\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315a^2\left(\mathrm{e}^{2x}-1\right)^7\sqrt{a\left(\mathrm{e}^{2x}-1\right)^4\mathrm{e}^{-4x}}}$
default	$-\frac{16\left(8\left(\cosh^4(2x)\right)-40\left(\cosh^3(2x)\right)+84\left(\cosh^2(2x)\right)-100\cosh(2x)+83\right)\sqrt{a\left(\sinh^2(2x)\right)}\,\sqrt{a\left(-1+\cosh(2x)\right)\left(\cosh\right.}}{315a^3\left(-1+\cosh(2x)\right)^4\sinh(2x)\sqrt{a\left(-1+\cosh(2x)\right)^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/315/a^3*(8*cosh(2*x)^4-40*cosh(2*x)^3+84*cosh(2*x)^2-100*cosh(2*x)+83)*
(a*sinh(2*x)^2)^(1/2)*(a*(-1+cosh(2*x))*(cosh(2*x)+1))^(1/2)/(-1+cosh(2*x))
^4/sinh(2*x)/(a*(-1+cosh(2*x))^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(100) = 200.

time = 0.49, size = 467, normalized size = 3.96

$$-\frac{256\,e^{(-2x)}}{35\left(9a^{\frac{5}{2}}e^{(-2x)}-36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}-126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}-84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}-9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}-a^{\frac{5}{2}}\right)}+\frac{1024\,e^{(-4x)}}{35\left(9a^{\frac{5}{2}}e^{(-2x)}-36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}-126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}-84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}-9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}-a^{\frac{5}{2}}\right)}-\frac{1024\,e^{(-6x)}}{15\left(9a^{\frac{5}{2}}e^{(-2x)}-36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}-126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}-84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}-9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}-a^{\frac{5}{2}}\right)}+\frac{512\,e^{(-8x)}}{5\left(9a^{\frac{5}{2}}e^{(-2x)}-36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}-126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}-84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}-9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}-a^{\frac{5}{2}}\right)}+\frac{256}{315\left(9a^{\frac{5}{2}}e^{(-2x)}-36a^{\frac{5}{2}}e^{(-4x)}+84a^{\frac{5}{2}}e^{(-6x)}-126a^{\frac{5}{2}}e^{(-8x)}+126a^{\frac{5}{2}}e^{(-10x)}-84a^{\frac{5}{2}}e^{(-12x)}+36a^{\frac{5}{2}}e^{(-14x)}-9a^{\frac{5}{2}}e^{(-16x)}+a^{\frac{5}{2}}e^{(-18x)}-a^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(
-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x)
 + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)
) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(
5/2)) - 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(
5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e
^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) -
 a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a
^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)
*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
 - a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2
)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-
12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^
(5/2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3093 vs. $2(100) = 200$.
time = 0.48, size = 3093, normalized size = 26.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 - 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 - cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 - 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 - 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 -
 140*cosh(x)^4 + 24*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 - 2
8*cosh(x)^5 + 8*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 - 84*
cosh(x)^6 + 36*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e
^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 - 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) - 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) - 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
^14 + 9*(17*a^3*cosh(x)^2 - a^3 + (17*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 - 3*a^3*cos
```

h(x) + (17*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^3 - 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 - 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
4 - 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) - 2*(85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
 504*(17*a^3*cosh(x)^5 - 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
 - 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^5 - 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3 + (221*a^3*cosh(
x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(221*a^3*cos
h(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
 - 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 - 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) - 2*(2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 - 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) - 2*(1215
5*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 - 7*a^3 + (2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 +
 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(4*
x) - 2*(2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 - 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 - 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 - 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(221
*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^13 - 78*
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 -
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*

```
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3 +
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3)*e
^(4*x) - 2*(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 -
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 - a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 - 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 - 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sinh(x)**4)**(-5/2), x)
```

Giac [**A**]
time = 0.43, size = 39, normalized size = 0.33

$$-\frac{256\left(126\,e^{(8\,x)}-84\,e^{(6\,x)}+36\,e^{(4\,x)}-9\,e^{(2\,x)}+1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(a^(5/2)*(
e^(2*x) - 1)^9)
```

Mupad [**B**]
time = 0.54, size = 256, normalized size = 2.17

$$-\frac{2048\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{5\,a^3\left(e^{2x}-1\right)^5\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}-\frac{4096\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{3\,a^3\left(e^{2x}-1\right)^6\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}-\frac{12288\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{7\,a^3\left(e^{2x}-1\right)^7\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}-\frac{1024\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{a^3\left(e^{2x}-1\right)^8\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}-\frac{2048\,e^{4x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{9\,a^3\left(e^{2x}-1\right)^9\left(e^{2x}-2\,e^{4x}+e^{6x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sinh(x)^4)^(5/2),x)
```

```
[Out] - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) - 1)^
5*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (4096*exp(4*x)*(a*(exp(-x)/2 - exp(
x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

```
 - (12288*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) - 1
)^7*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*exp(4*x)*(a*(exp(-x)/2 - ex
p(x)/2)^4)^(1/2))/(a^3*(exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
 - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x) - 1)
^9*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.158 $\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=50

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{5}{16}i\cosh(x)\sinh(x)-\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)$$

```
[Out] -5/16*I*x+1/7*cosh(x)^7-5/16*I*cosh(x)*sinh(x)-5/24*I*cosh(x)^3*sinh(x)-1/6
*I*cosh(x)^5*sinh(x)
```

Rubi [A]

time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$-\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x)-\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*x + Cosh[x]^7/7 - ((5*I)/16)*Cosh[x]*Sinh[x] - ((5*I)/24)*Cosh[
x]^3*Sinh[x] - (I/6)*Cosh[x]^5*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^7(x)}{7} - i\int \cosh^6(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{6}i\int \cosh^4(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{8}i\int \cosh^2(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{16}i\int 1 \\
&= -\frac{5ix}{16} + \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x)
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. $2(50) = 100$.
time = 0.13, size = 219, normalized size = 4.38

$$
\frac{\cosh^9(x)\left(6i\left(35\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}+8\sqrt{1+i\sinh(x)}\right)+279\sqrt{1+i\sinh(x)}\sinh(x)-87i\sqrt{1+i\sinh(x)}\sinh^2(x)+326\sqrt{1+i\sinh(x)}\sinh^3(x)-38i\sqrt{1+i\sinh(x)}\sinh^4(x)+200\sqrt{1+i\sinh(x)}\sinh^5(x)-8i\sqrt{1+i\sinh(x)}\sinh^6(x)+48\sqrt{1+i\sinh(x)}\sinh^7(x)\right)}{336\sqrt{1+i\sinh(x)}(-i+\sinh(x))^4(i+\sinh(x))^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^9*((6*I)*(35*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x
]] + 8*Sqrt[1 + I*Sinh[x]]) + 279*Sqrt[1 + I*Sinh[x]]*Sinh[x] - (87*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^2 + 326*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3 - (38*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^4 + 200*Sqrt[1 + I*Sinh[x]]*Sinh[x]^5 - (8*I)*Sqrt[
1 + I*Sinh[x]]*Sinh[x]^6 + 48*Sqrt[1 + I*Sinh[x]]*Sinh[x]^7))/(336*Sqrt[1 +
 I*Sinh[x]]*(-I + Sinh[x])^4*(I + Sinh[x])^5)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(36) = 72$.
time = 0.60, size = 186, normalized size = 3.72

method	result
risch	$-\frac{5ix}{16}+\frac{e^{7x}}{896}-\frac{ie^{6x}}{384}+\frac{e^{5x}}{128}-\frac{3ie^{4x}}{128}+\frac{3\,e^{3x}}{128}-\frac{15ie^{2x}}{128}+\frac{5\,e^{x}}{128}+\frac{5\,e^{-x}}{128}+\frac{15ie^{-2x}}{128}+\frac{3\,e^{-3x}}{128}+\frac{3ie^{-4x}}{128}+\frac{e^{-5x}}{128}+\frac{ie^{-}}{38}$
default	$\frac{-\frac{5}{16}-\frac{11i}{16}}{\tanh(\frac{x}{2})-1}+\frac{-\frac{1}{2}-\frac{i}{6}}{(\tanh(\frac{x}{2})-1)^6}+\frac{5i\ln(\tanh(\frac{x}{2})-1)}{16}+\frac{-\frac{1}{2}+\frac{i}{6}}{(\tanh(\frac{x}{2})+1)^6}+\frac{-\frac{9}{8}-\frac{7i}{6}}{(\tanh(\frac{x}{2})-1)^3}+\frac{-\frac{5}{4}+i}{(\tanh(\frac{x}{2})+1)^4}+\frac{-\frac{5}{4}-i}{(\tanh(\frac{x}{2})-1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^8/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (-5/16-11/16*I)/(tanh(1/2*x)-1)-(1/2+1/6*I)/(tanh(1/2*x)-1)^6+5/16*I*ln(tan
h(1/2*x)-1)+(-1/2+1/6*I)/(tanh(1/2*x)+1)^6-(9/8+7/6*I)/(tanh(1/2*x)-1)^3+(-
```

```
5/4+I)/(tanh(1/2*x)+1)^4-(5/4+I)/(tanh(1/2*x)-1)^4+1/7/(tanh(1/2*x)+1)^7-5/
16*I*ln(tanh(1/2*x)+1)+(1-1/2*I)/(tanh(1/2*x)+1)^5-(1+1/2*I)/(tanh(1/2*x)-1
)^5+(-11/16+19/16*I)/(tanh(1/2*x)+1)^2-(11/16+19/16*I)/(tanh(1/2*x)-1)^2+(9
/8-7/6*I)/(tanh(1/2*x)+1)^3+(5/16-11/16*I)/(tanh(1/2*x)+1)-1/7/(tanh(1/2*x)
-1)^7
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.
time = 0.29, size = 90, normalized size = 1.80

$$-\frac{1}{2688}\left(7i e^{(-x)} - 21 e^{(-2x)} + 63i e^{(-3x)} - 63 e^{(-4x)} + 315i e^{(-5x)} - 105 e^{(-6x)} - 3\right)e^{(7x)} - \frac{5}{16}i x + \frac{5}{128} e^{(-x)} + \frac{15}{128} i e^{(-2x)} + \frac{3}{128} e^{(-3x)} + \frac{3}{128} i e^{(-4x)} + \frac{1}{128} e^{(-5x)} + \frac{1}{384} i e^{(-6x)} + \frac{1}{896} e^{(-7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/2688*(7*I*e^(-x) - 21*e^(-2*x) + 63*I*e^(-3*x) - 63*e^(-4*x) + 315*I*e^(
-5*x) - 105*e^(-6*x) - 3)*e^(7*x) - 5/16*I*x + 5/128*e^(-x) + 15/128*I*e^(-
2*x) + 3/128*e^(-3*x) + 3/128*I*e^(-4*x) + 1/128*e^(-5*x) + 1/384*I*e^(-6*x
) + 1/896*e^(-7*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(32) = 64$.
time = 0.36, size = 91, normalized size = 1.82

$$\frac{1}{2688}\left(-840i x e^{(7x)} + 3 e^{(14x)} - 7i e^{(13x)} + 21 e^{(12x)} - 63i e^{(11x)} + 63 e^{(10x)} - 315i e^{(9x)} + 105 e^{(8x)} + 105 e^{(6x)} + 315i e^{(5x)} + 63 e^{(4x)} + 63i e^{(3x)} + 21 e^{(2x)} + 7i e^{x} + 3\right)e^{(-7x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2688*(-840*I*x*e^(7*x) + 3*e^(14*x) - 7*I*e^(13*x) + 21*e^(12*x) - 63*I*e
^(11*x) + 63*e^(10*x) - 315*I*e^(9*x) + 105*e^(8*x) + 105*e^(6*x) + 315*I*e
^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x) + 7*I*e^x + 3)*e^(-7*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.
time = 0.14, size = 124, normalized size = 2.48

$$-\frac{5ix}{16} + \frac{e^{7x}}{896} - \frac{ie^{6x}}{384} + \frac{e^{5x}}{128} - \frac{3ie^{4x}}{128} + \frac{3e^{3x}}{128} - \frac{15ie^{2x}}{128} + \frac{5e^{x}}{128} + \frac{5e^{-x}}{128} + \frac{15ie^{-2x}}{128} + \frac{3e^{-3x}}{128} + \frac{3ie^{-4x}}{128} + \frac{e^{-5x}}{128} + \frac{ie^{-6x}}{384} + \frac{e^{-7x}}{896}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**8/(I+sinh(x)),x)
```

```
[Out] -5*I*x/16 + exp(7*x)/896 - I*exp(6*x)/384 + exp(5*x)/128 - 3*I*exp(4*x)/128
 + 3*exp(3*x)/128 - 15*I*exp(2*x)/128 + 5*exp(x)/128 + 5*exp(-x)/128 + 15*I
*exp(-2*x)/128 + 3*exp(-3*x)/128 + 3*I*exp(-4*x)/128 + exp(-5*x)/128 + I*ex
p(-6*x)/384 + exp(-7*x)/896
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(32) = 64$.
time = 0.43, size = 86, normalized size = 1.72

$$\frac{1}{2688}\left(105\,e^{(6\,x)} + 315i\,e^{(5\,x)} + 63\,e^{(4\,x)} + 63i\,e^{(3\,x)} + 21\,e^{(2\,x)} + 7i\,e^{x} + 3\right)e^{(-7\,x)} - \frac{5}{16}i\,x + \frac{1}{896}\,e^{(7\,x)} - \frac{1}{384}i\,e^{(6\,x)} + \frac{1}{128}\,e^{(5\,x)} - \frac{3}{128}i\,e^{(4\,x)} + \frac{3}{128}\,e^{(3\,x)} - \frac{15}{128}i\,e^{(2\,x)} + \frac{5}{128}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*(105*e^(6*x) + 315*I*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x
) + 7*I*e^x + 3)*e^(-7*x) - 5/16*I*x + 1/896*e^(7*x) - 1/384*I*e^(6*x) + 1/
128*e^(5*x) - 3/128*I*e^(4*x) + 3/128*e^(3*x) - 15/128*I*e^(2*x) + 5/128*e^
x
```

Mupad [**B**]
time = 0.78, size = 93, normalized size = 1.86

$$\frac{5\,\mathrm{e}^{-x}}{128} + \frac{3\,\mathrm{e}^{-3\,x}}{128} + \frac{3\,\mathrm{e}^{3\,x}}{128} + \frac{\mathrm{e}^{-5\,x}}{128} + \frac{\mathrm{e}^{5\,x}}{128} + \frac{\mathrm{e}^{-7\,x}}{896} + \frac{\mathrm{e}^{7\,x}}{896} + \frac{5\,\mathrm{e}^{x}}{128} - \frac{x\,5\mathrm{i}}{16} + \frac{\mathrm{e}^{-2\,x}\,15\mathrm{i}}{128} - \frac{\mathrm{e}^{2\,x}\,15\mathrm{i}}{128} + \frac{\mathrm{e}^{-4\,x}\,3\mathrm{i}}{128} - \frac{\mathrm{e}^{4\,x}\,3\mathrm{i}}{128} + \frac{\mathrm{e}^{-6\,x}\,1\mathrm{i}}{384} - \frac{\mathrm{e}^{6\,x}\,1\mathrm{i}}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^8/(sinh(x) + 1i),x)
```

```
[Out] (5*exp(-x))/128 - (x*5i)/16 + (exp(-2*x)*15i)/128 - (exp(2*x)*15i)/128 + (3
*exp(-3*x))/128 + (3*exp(3*x))/128 + (exp(-4*x)*3i)/128 - (exp(4*x)*3i)/128
 + exp(-5*x)/128 + exp(5*x)/128 + (exp(-6*x)*1i)/384 - (exp(6*x)*1i)/384 +
exp(-7*x)/896 + exp(7*x)/896 + (5*exp(x))/128
```

3.159 $\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=43

$$-(i-\sinh(x))^4-\frac{4}{5}i(i-\sinh(x))^5+\frac{1}{6}(i-\sinh(x))^6$$

```
[Out] -(I-sinh(x))^4-4/5*I*(I-sinh(x))^5+1/6*(I-sinh(x))^6
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\frac{1}{6}(-\sinh(x)+i)^6-\frac{4}{5}i(-\sinh(x)+i)^5-(-\sinh(x)+i)^4$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] -(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int (i-x)^3(i+x)^2\,dx,x,\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-4(i-x)^3-4i(i-x)^4+(i-x)^5\right)\,dx,x,\sinh(x)\right)\\
&= -(i-\sinh(x))^4-\frac{4}{5}i(i-\sinh(x))^5+\frac{1}{6}(i-\sinh(x))^6
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.98

$$\frac{1}{30}\sinh(x)\left(-30i+15\sinh(x)-20i\sinh^2(x)+15\sinh^3(x)-6i\sinh^4(x)+5\sinh^5(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-30*I + 15*Sinh[x] - (20*I)*Sinh[x]^2 + 15*Sinh[x]^3 - (6*I)*Sinh
[x]^4 + 5*Sinh[x]^5))/30
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(35) = 70.
time = 0.49, size = 142, normalized size = 3.30

method	result
risch	$\frac{e^{6x}}{384}-\frac{ie^{5x}}{160}+\frac{e^{4x}}{64}-\frac{5ie^{3x}}{96}+\frac{5\,e^{2x}}{128}-\frac{5ie^{x}}{16}+\frac{5ie^{-x}}{16}+\frac{5\,e^{-2x}}{128}+\frac{5ie^{-3x}}{96}+\frac{e^{-4x}}{64}+\frac{ie^{-5x}}{160}+\frac{e^{-6x}}{384}$
default	$\frac{\frac{11}{12}+\frac{11i}{12}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{\frac{11}{16}+\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{\frac{7}{8}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{\frac{1}{2}+\frac{i}{5}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}+\frac{\frac{5}{16}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^6}+\frac{\frac{11}{16}-\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (11/12+11/12*I)/(tanh(1/2*x)-1)^3+(11/16+7/8*I)/(tanh(1/2*x)-1)^2+(7/8+1/2*
I)/(tanh(1/2*x)-1)^4+(1/2+1/5*I)/(tanh(1/2*x)-1)^5+(5/16+I)/(tanh(1/2*x)-1)
+1/6/(tanh(1/2*x)-1)^6+(11/16-7/8*I)/(tanh(1/2*x)+1)^2+(7/8-1/2*I)/(tanh(1/
2*x)+1)^4+(-1/2+1/5*I)/(tanh(1/2*x)+1)^5+(-5/16+I)/(tanh(1/2*x)+1)+(-11/12+
11/12*I)/(tanh(1/2*x)+1)^3+1/6/(tanh(1/2*x)+1)^6
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(25) = 50.
time = 0.29, size = 75, normalized size = 1.74

$$-\frac{1}{1920}\left(12i\,e^{(-x)}-30\,e^{(-2x)}+100i\,e^{(-3x)}-75\,e^{(-4x)}+600i\,e^{(-5x)}-5\right)e^{(6x)}+\frac{5}{16}i\,e^{(-x)}+\frac{5}{128}\,e^{(-2x)}+\frac{5}{96}i\,e^{(-3x)}+\frac{1}{64}\,e^{(-4x)}+\frac{1}{160}i\,e^{(-5x)}+\frac{1}{384}\,e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*I*e^(-x) - 30*e^(-2*x) + 100*I*e^(-3*x) - 75*e^(-4*x) + 600*I*e
^(-5*x) - 5)*e^(6*x) + 5/16*I*e^(-x) + 5/128*e^(-2*x) + 5/96*I*e^(-3*x) + 1
/64*e^(-4*x) + 1/160*I*e^(-5*x) + 1/384*e^(-6*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(25) = 50.
time = 0.40, size = 72, normalized size = 1.67

$$\frac{1}{1920}\left(5\,e^{(12x)}-12i\,e^{(11x)}+30\,e^{(10x)}-100i\,e^{(9x)}+75\,e^{(8x)}-600i\,e^{(7x)}+600i\,e^{(5x)}+75\,e^{(4x)}+100i\,e^{(3x)}+30\,e^{(2x)}+12i\,e^{x}+5\right)e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*e^(12*x) - 12*I*e^(11*x) + 30*e^(10*x) - 100*I*e^(9*x) + 75*e^(8*
x) - 600*I*e^(7*x) + 600*I*e^(5*x) + 75*e^(4*x) + 100*I*e^(3*x) + 30*e^(2*x
) + 12*I*e^x + 5)*e^(-6*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(26) = 52$.
time = 0.12, size = 100, normalized size = 2.33

$$\frac{e^{6x}}{384} - \frac{ie^{5x}}{160} + \frac{e^{4x}}{64} - \frac{5ie^{3x}}{96} + \frac{5e^{2x}}{128} - \frac{5ie^{x}}{16} + \frac{5ie^{-x}}{16} + \frac{5e^{-2x}}{128} + \frac{5ie^{-3x}}{96} + \frac{e^{-4x}}{64} + \frac{ie^{-5x}}{160} + \frac{e^{-6x}}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(I+sinh(x)),x)
```

```
[Out] exp(6*x)/384 - I*exp(5*x)/160 + exp(4*x)/64 - 5*I*exp(3*x)/96 + 5*exp(2*x)/
128 - 5*I*exp(x)/16 + 5*I*exp(-x)/16 + 5*exp(-2*x)/128 + 5*I*exp(-3*x)/96 +
 exp(-4*x)/64 + I*exp(-5*x)/160 + exp(-6*x)/384
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.
time = 0.44, size = 71, normalized size = 1.65

$$-\frac{1}{1920}\left(-600i\,e^{(5\,x)} - 75\,e^{(4\,x)} - 100i\,e^{(3\,x)} - 30\,e^{(2\,x)} - 12i\,e^{x} - 5\right)e^{(-6\,x)} + \frac{1}{384}\,e^{(6\,x)} - \frac{1}{160}i\,e^{(5\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{5}{96}i\,e^{(3\,x)} + \frac{5}{128}\,e^{(2\,x)} - \frac{5}{16}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*(-600*I*e^(5*x) - 75*e^(4*x) - 100*I*e^(3*x) - 30*e^(2*x) - 12*I*e^
x - 5)*e^(-6*x) + 1/384*e^(6*x) - 1/160*I*e^(5*x) + 1/64*e^(4*x) - 5/96*I*e
^(3*x) + 5/128*e^(2*x) - 5/16*I*e^x
```

Mupad [B]
time = 0.67, size = 77, normalized size = 1.79

$$\frac{\mathrm{e}^{-x}\,5\mathrm{i}}{16} + \frac{5\,\mathrm{e}^{-2\,x}}{128} + \frac{5\,\mathrm{e}^{2\,x}}{128} + \frac{\mathrm{e}^{-3\,x}\,5\mathrm{i}}{96} - \frac{\mathrm{e}^{3\,x}\,5\mathrm{i}}{96} + \frac{\mathrm{e}^{-4\,x}}{64} + \frac{\mathrm{e}^{4\,x}}{64} + \frac{\mathrm{e}^{-5\,x}\,1\mathrm{i}}{160} - \frac{\mathrm{e}^{5\,x}\,1\mathrm{i}}{160} + \frac{\mathrm{e}^{-6\,x}}{384} + \frac{\mathrm{e}^{6\,x}}{384} - \frac{\mathrm{e}^{x}\,5\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*5i)/16 + (5*exp(-2*x))/128 + (5*exp(2*x))/128 + (exp(-3*x)*5i)/96
- (exp(3*x)*5i)/96 + exp(-4*x)/64 + exp(4*x)/64 + (exp(-5*x)*1i)/160 - (exp
(5*x)*1i)/160 + exp(-6*x)/384 + exp(6*x)/384 - (exp(x)*5i)/16
```

3.160 $\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{3}{8}i\cosh(x)\sinh(x)-\frac{1}{4}i\cosh^3(x)\sinh(x)$$

```
[Out] -3/8*I*x+1/5*cosh(x)^5-3/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2761, 2715, 8}

$$-\frac{3ix}{8}+\frac{\cosh^5(x)}{5}-\frac{1}{4}i\sinh(x)\cosh^3(x)-\frac{3}{8}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*x + Cosh[x]^5/5 - ((3*I)/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Si
nh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^5(x)}{5} - i\int \cosh^4(x)\,dx\\ &= \frac{\cosh^5(x)}{5} - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{4}i\int \cosh^2(x)\,dx\\ &= \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{8}i\int 1\,dx\\ &= -\frac{3ix}{8} + \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x)\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 131 vs. $2(38) = 76$.
time = 0.18, size = 131, normalized size = 3.45

$$-\frac{i\cosh^7(x)\left(8i + \frac{30i\,\mathrm{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}} + 33\sinh(x) - 9i\sinh^2(x) + 26\sinh^3(x) - 2i\sinh^4(x) + 8\sinh^5(x)\right)}{40\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-1/40*I)*Cosh[x]^7*(8*I + ((30*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqr
t[1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] + 33*Sinh[x] - (9*I)*Sinh[x]^2 + 26*S
inh[x]^3 - (2*I)*Sinh[x]^4 + 8*Sinh[x]^5))/((Cosh[x/2] - I*Sinh[x/2])^8*(Co
sh[x/2] + I*Sinh[x/2])^6)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(27) = 54$.
time = 0.57, size = 138, normalized size = 3.63

method	result
risch	$-\frac{3ix}{8} + \frac{e^{5x}}{160} - \frac{i e^{4x}}{64} + \frac{e^{3x}}{32} - \frac{i e^{2x}}{8} + \frac{e^{x}}{16} + \frac{e^{-x}}{16} + \frac{i e^{-2x}}{8} + \frac{e^{-3x}}{32} + \frac{i e^{-4x}}{64} + \frac{e^{-5x}}{160}$
default	$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8} + \frac{-\frac{1}{2}+\frac{i}{4}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} + \frac{\frac{3}{4}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} + \frac{\frac{3}{8}-\frac{5i}{8}}{\tanh\left(\frac{x}{2}\right)+1} + \frac{-\frac{5}{8}+\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{1}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^5} + \frac{3i\ln(\tanh}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*I*ln(tanh(1/2*x)+1)+(-1/2+1/4*I)/(tanh(1/2*x)+1)^4+(3/4-1/2*I)/(tanh(1
/2*x)+1)^3+(3/8-5/8*I)/(tanh(1/2*x)+1)+(-5/8+7/8*I)/(tanh(1/2*x)+1)^2+1/5/(
tanh(1/2*x)+1)^5+3/8*I*ln(tanh(1/2*x)-1)-(1/2+1/4*I)/(tanh(1/2*x)-1)^4-(3/8
```

```
+5/8*I)/(tanh(1/2*x)-1)-(5/8+7/8*I)/(tanh(1/2*x)-1)^2-(3/4+1/2*I)/(tanh(1/2
*x)-1)^3-1/5/(tanh(1/2*x)-1)^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.
time = 0.27, size = 66, normalized size = 1.74

$$-\frac{1}{320}\left(5i\,e^{(-x)}-10\,e^{(-2\,x)}+40i\,e^{(-3\,x)}-20\,e^{(-4\,x)}-2\right)e^{(5\,x)}-\frac{3}{8}i\,x+\frac{1}{16}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{32}\,e^{(-3\,x)}+\frac{1}{64}i\,e^{(-4\,x)}+\frac{1}{160}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="maxima")
[Out] -1/320*(5*I*e^(-x) - 10*e^(-2*x) + 40*I*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)
 - 3/8*I*x + 1/16*e^(-x) + 1/8*I*e^(-2*x) + 1/32*e^(-3*x) + 1/64*I*e^(-4*x)
 + 1/160*e^(-5*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(24) = 48$.
time = 0.37, size = 67, normalized size = 1.76

$$\frac{1}{320}\left(-120i\,xe^{(5\,x)}+2\,e^{(10\,x)}-5i\,e^{(9\,x)}+10\,e^{(8\,x)}-40i\,e^{(7\,x)}+20\,e^{(6\,x)}+20\,e^{(4\,x)}+40i\,e^{(3\,x)}+10\,e^{(2\,x)}+5i\,e^{x}+2\right)e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/320*(-120*I*x*e^(5*x) + 2*e^(10*x) - 5*I*e^(9*x) + 10*e^(8*x) - 40*I*e^(7
*x) + 20*e^(6*x) + 20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^
(-5*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 0.11, size = 82, normalized size = 2.16

$$-\frac{3ix}{8}+\frac{e^{5x}}{160}-\frac{ie^{4x}}{64}+\frac{e^{3x}}{32}-\frac{ie^{2x}}{8}+\frac{e^{x}}{16}+\frac{e^{-x}}{16}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{32}+\frac{ie^{-4x}}{64}+\frac{e^{-5x}}{160}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x)),x)
[Out] -3*I*x/8 + exp(5*x)/160 - I*exp(4*x)/64 + exp(3*x)/32 - I*exp(2*x)/8 + exp(
x)/16 + exp(-x)/16 + I*exp(-2*x)/8 + exp(-3*x)/32 + I*exp(-4*x)/64 + exp(-5
*x)/160
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.
time = 0.41, size = 62, normalized size = 1.63

$$\frac{1}{320}\left(20\,e^{(4\,x)}+40i\,e^{(3\,x)}+10\,e^{(2\,x)}+5i\,e^{x}+2\right)e^{(-5\,x)}-\frac{3}{8}i\,x+\frac{1}{160}\,e^{(5\,x)}-\frac{1}{64}i\,e^{(4\,x)}+\frac{1}{32}\,e^{(3\,x)}-\frac{1}{8}i\,e^{(2\,x)}+\frac{1}{16}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/320*(20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^(-5*x) - 3/8
*I*x + 1/160*e^(5*x) - 1/64*I*e^(4*x) + 1/32*e^(3*x) - 1/8*I*e^(2*x) + 1/16
*e^x
```

Mupad [B]
time = 0.62, size = 67, normalized size = 1.76

$$\frac{e^{-x}}{16}+\frac{e^{-3\,x}}{32}+\frac{e^{3\,x}}{32}+\frac{e^{-5\,x}}{160}+\frac{e^{5\,x}}{160}+\frac{e^{x}}{16}-\frac{x\,3\mathrm{i}}{8}+\frac{e^{-2\,x}\,1\mathrm{i}}{8}-\frac{e^{2\,x}\,1\mathrm{i}}{8}+\frac{e^{-4\,x}\,1\mathrm{i}}{64}-\frac{e^{4\,x}\,1\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/16 - (x*3i)/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/32 +
 exp(3*x)/32 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 + exp(-5*x)/160 + exp(5
*x)/160 + exp(x)/16
```

3.161 $\int \frac{\cosh^5(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=33

$$-i\sinh(x)+\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^4(x)}{4}$$

```
[Out] -I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$\frac{\sinh^4(x)}{4}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^2(x)}{2}-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(x)}{i+\sinh(x)} \, dx &= \text{Subst}\left(\int (i-x)^2(i+x) \, dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \left(-i+x-ix^2+x^3\right) \, dx, x, \sinh(x)\right) \\
&= -i\sinh(x)+\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^4(x)}{4}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 28, normalized size = 0.85

$$\frac{1}{12}\sinh(x)\left(-12i+6\sinh(x)-4i\sinh^2(x)+3\sinh^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-12*I + 6*Sinh[x] - (4*I)*Sinh[x]^2 + 3*Sinh[x]^3))/12
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(25) = 50.
time = 0.46, size = 94, normalized size = 2.85

method	result
risch	$\frac{e^{4x}}{64}-\frac{ie^{3x}}{24}+\frac{e^{2x}}{16}-\frac{3ie^{x}}{8}+\frac{3ie^{-x}}{8}+\frac{e^{-2x}}{16}+\frac{ie^{-3x}}{24}+\frac{e^{-4x}}{64}$
default	$\frac{\frac{5}{8}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{\frac{1}{2}+\frac{i}{3}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{\frac{5}{8}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{-\frac{1}{2}+\frac{i}{3}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{-\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)+1}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (5/8+1/2*I)/(tanh(1/2*x)-1)^2+(1/2+1/3*I)/(tanh(1/2*x)-1)^3+(3/8+I)/(tanh(1
/2*x)-1)+1/4/(tanh(1/2*x)-1)^4+(5/8-1/2*I)/(tanh(1/2*x)+1)^2+(-1/2+1/3*I)/(
tanh(1/2*x)+1)^3+(-3/8+I)/(tanh(1/2*x)+1)+1/4/(tanh(1/2*x)+1)^4
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.
time = 0.27, size = 51, normalized size = 1.55

$$-\frac{1}{192}\left(8i\,e^{(-x)}-12\,e^{(-2\,x)}+72i\,e^{(-3\,x)}-3\right)e^{(4\,x)}+\frac{3}{8}i\,e^{(-x)}+\frac{1}{16}\,e^{(-2\,x)}+\frac{1}{24}i\,e^{(-3\,x)}+\frac{1}{64}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*I*e^(-x) - 12*e^(-2*x) + 72*I*e^(-3*x) - 3)*e^(4*x) + 3/8*I*e^(-x
) + 1/16*e^(-2*x) + 1/24*I*e^(-3*x) + 1/64*e^(-4*x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.
time = 0.43, size = 48, normalized size = 1.45

$$\frac{1}{192}\left(3\,e^{(8\,x)}-8i\,e^{(7\,x)}+12\,e^{(6\,x)}-72i\,e^{(5\,x)}+72i\,e^{(3\,x)}+12\,e^{(2\,x)}+8i\,e^{x}+3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*e^(8*x) - 8*I*e^(7*x) + 12*e^(6*x) - 72*I*e^(5*x) + 72*I*e^(3*x) +
 12*e^(2*x) + 8*I*e^x + 3)*e^(-4*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.09, size = 63, normalized size = 1.91

$$\frac{e^{4x}}{64} - \frac{ie^{3x}}{24} + \frac{e^{2x}}{16} - \frac{3ie^{x}}{8} + \frac{3ie^{-x}}{8} + \frac{e^{-2x}}{16} + \frac{ie^{-3x}}{24} + \frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x)),x)
```

```
[Out] exp(4*x)/64 - I*exp(3*x)/24 + exp(2*x)/16 - 3*I*exp(x)/8 + 3*I*exp(-x)/8 +
exp(-2*x)/16 + I*exp(-3*x)/24 + exp(-4*x)/64
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.
time = 0.41, size = 47, normalized size = 1.42

$$-\frac{1}{192}\left(-72i\,e^{(3\,x)} - 12\,e^{(2\,x)} - 8i\,e^{x} - 3\right)e^{(-4\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)} - \frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/192*(-72*I*e^(3*x) - 12*e^(2*x) - 8*I*e^x - 3)*e^(-4*x) + 1/64*e^(4*x) -
 1/24*I*e^(3*x) + 1/16*e^(2*x) - 3/8*I*e^x
```

Mupad [B]
time = 0.56, size = 51, normalized size = 1.55

$$\frac{e^{-x}\,3i}{8} + \frac{e^{-2\,x}}{16} + \frac{e^{2\,x}}{16} + \frac{e^{-3\,x}\,1i}{24} - \frac{e^{3\,x}\,1i}{24} + \frac{e^{-4\,x}}{64} + \frac{e^{4\,x}}{64} - \frac{e^{x}\,3i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*3i)/8 + exp(-2*x)/16 + exp(2*x)/16 + (exp(-3*x)*1i)/24 - (exp(3*x)
*1i)/24 + exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*3i)/8
```

3.162 $\int \frac{\cosh^4(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=26

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)$$

```
[Out] -1/2*I*x+1/3*cosh(x)^3-1/2*I*cosh(x)*sinh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2761, 2715, 8}

$$-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-1/2*I)*x + Cosh[x]^3/3 - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx &= \frac{\cosh^3(x)}{3}-i\int \cosh^2(x)\,dx\\ &= \frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)-\frac{1}{2}i\int 1\,dx\\ &= -\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 93 vs. 2(26) = 52.
time = 0.11, size = 93, normalized size = 3.58

$$\frac{\cosh^5(x)\left(2i+\frac{6i\operatorname{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}+5\sinh(x)-i\sinh^2(x)+2\sinh^3(x)\right)}{6(-i+\sinh(x))^2(i+\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^5*(2*I + ((6*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sin
h[x]])/Sqrt[1 + I*Sinh[x]] + 5*Sinh[x] - I*Sinh[x]^2 + 2*Sinh[x]^3))/(6*(-I
 + Sinh[x])^2*(I + Sinh[x])^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(18) = 36.
time = 0.54, size = 90, normalized size = 3.46

method	result
risch	$-\frac{ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}+\frac{e^{x}}{8}+\frac{e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}$
default	$-\frac{i\ln(\tanh(\frac{x}{2})+1)}{2}+\frac{\frac{1}{2}-\frac{i}{2}}{\tanh(\frac{x}{2})+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{(\tanh(\frac{x}{2})+1)^2}+\frac{1}{3(\tanh(\frac{x}{2})+1)^3}+\frac{i\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{-\frac{1}{2}-\frac{i}{2}}{(\tanh(\frac{x}{2})-1)^2}+\frac{-\frac{1}{2}-\frac{i}{2}}{\tanh(\frac{x}{2})-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*ln(tanh(1/2*x)+1)+(1/2-1/2*I)/(tanh(1/2*x)+1)+(-1/2+1/2*I)/(tanh(1/2
*x)+1)^2+1/3/(tanh(1/2*x)+1)^3+1/2*I*ln(tanh(1/2*x)-1)-(1/2+1/2*I)/(tanh(1/
2*x)-1)^2-(1/2+1/2*I)/(tanh(1/2*x)-1)-1/3/(tanh(1/2*x)-1)^3
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(16) = 32.

time = 0.29, size = 42, normalized size = 1.62

$$-\frac{1}{24}\left(3i\,e^{(-x)}-3\,e^{(-2\,x)}-1\right)e^{(3\,x)}-\frac{1}{2}i\,x+\frac{1}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*I*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x) - 1/2*I*x + 1/8*e^(-x) + 1/8*I*
e^(-2*x) + 1/24*e^(-3*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(16) = 32$.
time = 0.39, size = 41, normalized size = 1.58

$$\frac{1}{24}\left(-12i\,xe^{(3\,x)}+e^{(6\,x)}-3i\,e^{(5\,x)}+3\,e^{(4\,x)}+3\,e^{(2\,x)}+3i\,e^{x}+1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-12*I*x*e^(3*x) + e^(6*x) - 3*I*e^(5*x) + 3*e^(4*x) + 3*e^(2*x) + 3*I
*e^x + 1)*e^(-3*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.08, size = 48, normalized size = 1.85

$$-\frac{ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}+\frac{e^{x}}{8}+\frac{e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x)),x)
```

```
[Out] -I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 + exp(x)/8 + exp(-x)/8 + I*exp(-2*x)/8
+ exp(-3*x)/24
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.
time = 0.40, size = 38, normalized size = 1.46

$$\frac{1}{24}\left(3\,e^{(2\,x)}+3i\,e^{x}+1\right)e^{(-3\,x)}-\frac{1}{2}i\,x+\frac{1}{24}\,e^{(3\,x)}-\frac{1}{8}i\,e^{(2\,x)}+\frac{1}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) + 3*I*e^x + 1)*e^(-3*x) - 1/2*I*x + 1/24*e^(3*x) - 1/8*I*e^
(2*x) + 1/8*e^x
```

Mupad [B]
time = 0.12, size = 41, normalized size = 1.58

$$\frac{e^{-x}}{8}+\frac{e^{-3\,x}}{24}+\frac{e^{3\,x}}{24}+\frac{e^{x}}{8}-\frac{x\,1\mathrm{i}}{2}+\frac{e^{-2\,x}\,1\mathrm{i}}{8}-\frac{e^{2\,x}\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/8 - (x*1i)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/24 +
exp(3*x)/24 + exp(x)/8
```

3.163 $\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=15

$$-i\sinh(x)+\frac{\sinh^2(x)}{2}$$

```
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2746}

$$\frac{\sinh^2(x)}{2}-i\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int (i-x)\,dx,x,\sinh(x)\right)\\ &= -i\sinh(x)+\frac{\sinh^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.80

$$\frac{1}{2}\sinh(x)(-2i+\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (Sinh[x]*(-2*I + Sinh[x]))/2
```

Maple [A]
time = 0.44, size = 13, normalized size = 0.87

method	result	size
derivativedivides	$-i \sinh\left(x\right)+\frac{\left(\sinh^{2}\left(x\right)\right)}{2}$	13
default	$-i \sinh\left(x\right)+\frac{\left(\sinh^{2}\left(x\right)\right)}{2}$	13
risch	$\frac{{\mathrm e}^{2 x}}{8}-\frac{i {\mathrm e}^{x}}{2}+\frac{i {\mathrm e}^{-x}}{2}+\frac{{\mathrm e}^{-2 x}}{8}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.
time = 0.28, size = 27, normalized size = 1.80

$$\frac{1}{8}\left(-4i\,e^{(-x)}+1\right)e^{(2\,x)}+\frac{1}{2}i\,e^{(-x)}+\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*e^(-x) + 1)*e^(2*x) + 1/2*I*e^(-x) + 1/8*e^(-2*x)
```

Fricas [A]
time = 0.38, size = 22, normalized size = 1.47

$$\frac{1}{8}\left(e^{(4\,x)}-4i\,e^{(3\,x)}+4i\,e^{x}+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(e^(4*x) - 4*I*e^(3*x) + 4*I*e^x + 1)*e^(-2*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.
time = 0.06, size = 27, normalized size = 1.80

$$\frac{e^{2x}}{8}-\frac{ie^{x}}{2}+\frac{ie^{-x}}{2}+\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x)),x)
```

```
[Out] exp(2*x)/8 - I*exp(x)/2 + I*exp(-x)/2 + exp(-2*x)/8
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.41, size = 23, normalized size = 1.53

$$-\frac{1}{8}\left(-4i\,e^{x}-1\right)e^{(-2\,x)}+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/8*(-4*I*e^x - 1)*e^(-2*x) + 1/8*e^(2*x) - 1/2*I*e^x
```

Mupad [B]
time = 0.47, size = 31, normalized size = 2.07

$$\frac{\mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{4\,x}+1\right)}{8}-\frac{\mathrm{e}^{-2\,x}\,\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)\,1{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (exp(-2*x)*(exp(4*x) + 1))/8 - (exp(-2*x)*(4*exp(3*x) - 4*exp(x))*1i)/8
```

3.164 $\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=8

$$-ix+\cosh(x)$$

```
[Out] -I*x+cosh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2761, 8}

$$\cosh(x)-ix$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx &= \cosh(x)-i\int 1\,dx\\ &= -ix+\cosh(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.
time = 0.04, size = 34, normalized size = 4.25

$$\cosh(x)+2\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}\,\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] Cosh[x] + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.
time = 0.54, size = 40, normalized size = 5.00

method	result	size
risch	$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}$	16
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1
)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.27, size = 14, normalized size = 1.75

$$-i\,x+\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.50, size = 17, normalized size = 2.12

$$\frac{1}{2}\left(-2i\,xe^{x}+e^{(2\,x)}+1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(-2*I*x*e^x + e^(2*x) + 1)*e^(-x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.
time = 0.04, size = 14, normalized size = 1.75

$$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x)),x)
```

```
[Out] -I*x + exp(x)/2 + exp(-x)/2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.42, size = 14, normalized size = 1.75

$$-i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

Mupad [B]
time = 0.46, size = 7, normalized size = 0.88

$$\cosh(x) - x\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] cosh(x) - x*1i
```

3.165 $\int \frac{\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=7

$$\log(i+\sinh(x))$$

```
[Out] ln(I+sinh(x))
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 31}

$$\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(I + Sinh[x]),x]
```

```
[Out] Log[I + Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{i+\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)\\ &= \log(i+\sinh(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$\log(i+\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x]),x]
```

```
[Out] Log[I + Sinh[x]]
```

Maple [A]
time = 0.31, size = 7, normalized size = 1.00

method	result	size
derivativedivides	$\ln(i+\sinh(x))$	7
default	$\ln(i+\sinh(x))$	7
risch	$-x+2\ln(e^{x}+i)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(I+sinh(x))
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.71

$$\log(\sinh(x)+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] log(sinh(x) + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.
time = 0.37, size = 11, normalized size = 1.57

$$-x+2\log(e^{x}+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -x + 2*log(e^x + I)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 1.14

$$-x+2\log(e^{x}+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x)
```

```
[Out] -x + 2*log(exp(x) + I)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.41, size = 11, normalized size = 1.57

$$-x + 2 \log(e^x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -x + 2*log(e^x + I)
```

Mupad [B]
time = 0.47, size = 10, normalized size = 1.43

$$\ln(\cosh(x)) - \operatorname{atan}(\sinh(x))\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x) + 1i),x)
```

```
[Out] log(cosh(x)) - atan(sinh(x))*1i
```

3.166 $\int \frac{\text{sech}(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=24

$$-\frac{1}{2}i\text{ArcTan}(\sinh(x)) - \frac{i}{2(i+\sinh(x))}$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*I/(I+sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2746, 46, 209}

$$-\frac{1}{2}i\text{ArcTan}(\sinh(x)) - \frac{i}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x]),x]
```

```
[Out] (-1/2*I)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^2}\,dx, x, \sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^2}+\frac{i}{2\,(1+x^2)}\right)dx, x, \sinh(x)\right)\\ &= -\frac{i}{2(i+\sinh(x))}-\frac{1}{2}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{2}i\tan^{-1}(\sinh(x))-\frac{i}{2(i+\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.75

$$-\frac{1}{2}i\left(\text{ArcTan}(\sinh(x))+\frac{1}{i+\sinh(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x]),x]
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I + Sinh[x])^(-1))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 0.48, size = 43, normalized size = 1.79

method	result	size
risch	$-\frac{i e^{x}}{(e^{x}+i)^{2}}-\frac{\ln(e^{x}-i)}{2}+\frac{\ln(e^{x}+i)}{2}$	30
default	$\frac{i}{\tanh(\frac{x}{2})+i}+\frac{1}{(\tanh(\frac{x}{2})+i)^{2}}+\frac{\ln(\tanh(\frac{x}{2})+i)}{2}-\frac{\ln(\tanh(\frac{x}{2})-i)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^2+1/2*ln(tanh(1/2*x)+I)-1/2*ln(tanh(1/2
*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.
time = 0.28, size = 41, normalized size = 1.71

$$\frac{2i\,e^{(-x)}}{-4i\,e^{(-x)}+2\,e^{(-2\,x)}-2}-\frac{1}{2}\log\left(e^{(-x)}+i\right)+\frac{1}{2}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(14) = 28$.
time = 0.40, size = 52, normalized size = 2.17

$$\frac{\left(e^{(2x)} + 2i\,e^{x} - 1\right)\log\left(e^{x} + i\right) - \left(e^{(2x)} + 2i\,e^{x} - 1\right)\log\left(e^{x} - i\right) - 2i\,e^{x}}{2\left(e^{(2x)} + 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) + 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) + 2*I*e^x - 1)*log(e^x
 - I) - 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{\sinh(x) + i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.
time = 0.42, size = 51, normalized size = 2.12

$$-\frac{e^{(-x)} - e^{x} - 6i}{4\left(e^{(-x)} - e^{x} - 2i\right)} + \frac{1}{4}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{4}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) - e^x - 6*I)/(e^(-x) - e^x - 2*I) + 1/4*log(-e^(-x) + e^x + 2*
I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.20, size = 46, normalized size = 1.92

$$\frac{\ln\left(-1 + \mathrm{e}^{x}\,1\mathrm{i}\right)}{2} - \frac{\ln\left(1 + \mathrm{e}^{x}\,1\mathrm{i}\right)}{2} - \frac{1}{\mathrm{e}^{2x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} - \frac{1\mathrm{i}}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 - 1/(exp(2*x) + exp(x)*2i - 1)
- 1i/(exp(x) + 1i)
```

3.167 $\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$-\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))}-\frac{2}{3}i\tanh(x)$$

```
[Out] -1/3*I*sech(x)/(I+sinh(x))-2/3*I*tanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2751, 3852, 8}

$$-\frac{2}{3}i\tanh(x)-\frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] ((-1/3*I)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\int \operatorname{sech}^2(x)\,dx\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))} + \frac{2}{3}\text{Subst}\Big(\int 1\,dx, x, -i\tanh(x)\Big)\\ &= -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\tanh(x)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 22, normalized size = 0.88

$$-\frac{1}{3}i\left(\frac{\operatorname{sech}(x)}{i+\sinh(x)} + 2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-1/3*I)*(Sech[x]/(I + Sinh[x]) + 2*Tanh[x])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.50, size = 49, normalized size = 1.96

method	result	size
risch	$-\frac{4(i+2\,\mathrm{e}^x)}{3(\mathrm{e}^x+i)^3(\mathrm{e}^x-i)}$	24
default	$-\frac{1}{(\tanh(\frac{x}{2})+i)^2}+\frac{2i}{3(\tanh(\frac{x}{2})+i)^3}-\frac{3i}{2(\tanh(\frac{x}{2})+i)}-\frac{i}{2(\tanh(\frac{x}{2})-i)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(tanh(1/2*x)+I)^2+2/3*I/(tanh(1/2*x)+I)^3-3/2*I/(tanh(1/2*x)+I)-1/2*I/(t
anh(1/2*x)-I)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.27, size = 53, normalized size = 2.12

$$-\frac{8\,e^{(-x)}}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3} + \frac{4i}{-6i\,e^{(-x)} - 6i\,e^{(-3\,x)} + 3\,e^{(-4\,x)} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -8*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 4*I/(-6*I*e^(-x)
- 6*I*e^(-3*x) + 3*e^(-4*x) - 3)
```

Fricas [A]
time = 0.36, size = 26, normalized size = 1.04

$$-\frac{4\left(2 e^{x}+i\right)}{3\left(e^{(4 x)}+2 i e^{(3 x)}+2 i e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(2*e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{\sinh(x)+i}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I), x)
```

Giac [A]
time = 0.41, size = 29, normalized size = 1.16

$$\frac{1}{2\left(e^{x}-i\right)}-\frac{3 e^{(2 x)}+12 i e^{x}-5}{6\left(e^{x}+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2/(e^x - I) - 1/6*(3*e^(2*x) + 12*I*e^x - 5)/(e^x + I)^3
```

Mupad [B]
time = 0.58, size = 63, normalized size = 2.52

$$-\frac{8 \mathrm{e}^{x}}{3\left(\mathrm{e}^{2 x}+1\right)^{3}}-\frac{8 \mathrm{e}^{x}\left(\mathrm{e}^{2 x}-1\right)}{3\left(\mathrm{e}^{2 x}+1\right)^{3}}+\frac{\mathrm{e}^{2 x}\, 16\mathrm{i}}{3\left(\mathrm{e}^{2 x}+1\right)^{3}}-\frac{\left(\mathrm{e}^{2 x}-1\right) 4\mathrm{i}}{3\left(\mathrm{e}^{2 x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)),x)
```

```
[Out] (exp(2*x)*16i)/(3*(exp(2*x) + 1)^3) - (8*exp(x))/(3*(exp(2*x) + 1)^3) - ((e
xp(2*x) - 1)*4i)/(3*(exp(2*x) + 1)^3) - (8*exp(x)*(exp(2*x) - 1))/(3*(exp(2
*x) + 1)^3)
```

3.168 $\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=52

$$-\frac{3}{8}i\text{ArcTan}(\sinh(x))+\frac{i}{8(i-\sinh(x))}+\frac{1}{8(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}$$

```
[Out] -3/8*I*arctan(sinh(x))+1/8*I/(I-sinh(x))+1/8/(I+sinh(x))^2-1/4*I/(I+sinh(x)
)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2746, 46, 209}

$$-\frac{3}{8}i\text{ArcTan}(\sinh(x))+\frac{i}{8(-\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)}+\frac{1}{8(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
(I/4)/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^3}\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{i}{8(-i+x)^2} - \frac{1}{4(i+x)^3} + \frac{i}{4(i+x)^2} - \frac{3i}{8\,(1+x^2)}\right)dx, x, \sinh(x)\right)\\ &= \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))} - \frac{3}{8}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{3}{8}i\tan^{-1}(\sinh(x)) + \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.17

$$-\frac{i\operatorname{sech}^2(x)\left(2+3i\text{ArcTan}(\sinh(x))+3(i+\text{ArcTan}(\sinh(x)))\sinh(x)+(3+3i\text{ArcTan}(\sinh(x)))\sinh^2(x)+3\text{ArcTan}(\sinh(x))\sinh^3(x)\right)}{8(i+\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-1/8*I)*Sech[x]^2*(2 + (3*I)*ArcTan[Sinh[x]] + 3*(I + ArcTan[Sinh[x]])*Si
nh[x] + (3 + (3*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 3*ArcTan[Sinh[x]]*Sinh[x]^3
))/(I + Sinh[x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(38) = 76.
time = 0.53, size = 91, normalized size = 1.75

method	result
risch	$-\frac{i e^{x}\left(6i e^{3x}+3\,e^{4x}-6i e^{x}+2\,e^{2x}+3\right)}{4\left(e^{x}+i\right)^{4}\left(e^{x}-i\right)^{2}}-\frac{3\ln\left(e^{x}-i\right)}{8}+\frac{3\ln\left(e^{x}+i\right)}{8}$
default	$\frac{i}{4\tanh\left(\frac{x}{2}\right)-4i}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{8}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/(tanh(1/2*x)-I)-1/4/(tanh(1/2*x)-I)^2-3/8*ln(tanh(1/2*x)-I)-1/2/(tanh
(1/2*x)+I)^4+I/(tanh(1/2*x)+I)-I/(tanh(1/2*x)+I)^3+3/2/(tanh(1/2*x)+I)^2+3/
8*ln(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.
time = 0.27, size = 92, normalized size = 1.77

$$\frac{8\left(3i\,e^{(-x)}-6\,e^{(-2x)}+2i\,e^{(-3x)}+6\,e^{(-4x)}+3i\,e^{(-5x)}\right)}{-64i\,e^{(-x)}-32\,e^{(-2x)}-128i\,e^{(-3x)}+32\,e^{(-4x)}-64i\,e^{(-5x)}+32\,e^{(-6x)}-32}-\frac{3}{8}\log\left(e^{(-x)}+i\right)+\frac{3}{8}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 8*(3*I*e^(-x) - 6*e^(-2*x) + 2*I*e^(-3*x) + 6*e^(-4*x) + 3*I*e^(-5*x))/(-64
*I*e^(-x) - 32*e^(-2*x) - 128*I*e^(-3*x) + 32*e^(-4*x) - 64*I*e^(-5*x) + 32
*e^(-6*x) - 32) - 3/8*log(e^(-x) + I) + 3/8*log(e^(-x) - I)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(30) = 60$.
time = 0.36, size = 143, normalized size = 2.75

$$\frac{3\left(e^{(6x)}+2i e^{(5x)}+e^{(4x)}+4i e^{(3x)}-e^{(2x)}+2i e^{x}-1\right)\log\left(e^{x}+i\right)-3\left(e^{(6x)}+2i e^{(5x)}+e^{(4x)}+4i e^{(3x)}-e^{(2x)}+2i e^{x}-1\right)\log\left(e^{x}-i\right)-6i e^{(5x)}+12 e^{(4x)}-4i e^{(3x)}-12 e^{(2x)}-6i e^{x}}{8\left(e^{(6x)}+2i e^{(5x)}+e^{(4x)}+4i e^{(3x)}-e^{(2x)}+2i e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*x) + 2*I*e^x -
 1)*log(e^x + I) - 3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)*log(e^x - I) - 6*I*e^(5*x) + 12*e^(4*x) - 4*I*e^(3*x) - 1
2*e^(2*x) - 6*I*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(30) = 60$.
time = 0.41, size = 92, normalized size = 1.77

$$\frac{3 e^{(-x)}-3 e^{x}+10i}{16\left(e^{(-x)}-e^{x}+2i\right)}-\frac{9\left(e^{(-x)}-e^{x}\right)^{2}-52i e^{(-x)}+52i e^{x}-84}{32\left(e^{(-x)}-e^{x}-2i\right)^{2}}+\frac{3}{16}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{3}{16}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*e^(-x) - 3*e^x + 10*I)/(e^(-x) - e^x + 2*I) - 1/32*(9*(e^(-x) - e^x
)^2 - 52*I*e^(-x) + 52*I*e^x - 84)/(e^(-x) - e^x - 2*I)^2 + 3/16*log(-e^(-x
) + e^x + 2*I) - 3/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.90, size = 115, normalized size = 2.21

$$\frac{3\,\ln\left(-\frac{3}{4}+\frac{\mathrm{e}^x\,3\mathrm{i}}{4}\right)}{8}-\frac{3\,\ln\left(\frac{3}{4}+\frac{\mathrm{e}^x\,3\mathrm{i}}{4}\right)}{8}-\frac{1}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}-\frac{1}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^x-\mathrm{i}\right)}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^x+1\mathrm{i}\right)}-\frac{1\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (3*log((exp(x)*3i)/4 - 3/4))/8 - (3*log((exp(x)*3i)/4 + 3/4))/8 - 1/(2*(exp
(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - 1/(4*(exp(x)*2i - exp(
2*x) + 1)) - 1i/(4*(exp(x) - 1i)) - 1i/(2*(exp(x) + 1i)) - 1i/(exp(2*x)*3i
+ exp(3*x) - 3*exp(x) - 1i)
```

3.169 $\int \frac{\text{sech}^4(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=37

$$-\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}-\frac{4}{5}i\tanh(x)+\frac{4}{15}i\tanh^3(x)$$

```
[Out] -1/5*I*sech(x)^3/(I+sinh(x))-4/5*I*tanh(x)+4/15*I*tanh(x)^3
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2751, 3852}

$$\frac{4}{15}i\tanh^3(x)-\frac{4}{5}i\tanh(x)-\frac{i\text{sech}^3(x)}{5(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((-1/5*I)*Sech[x]^3)/(I + Sinh[x]) - ((4*I)/5)*Tanh[x] + ((4*I)/15)*Tanh[x]
^3
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx &= -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\int \operatorname{sech}^4(x)\,dx \\
&= -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} + \frac{4}{5}\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(x)\right) \\
&= -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\tanh(x) + \frac{4}{15}i\tanh^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.95

$$
-\frac{1}{15}i\left(\frac{3\operatorname{sech}^3(x)}{i+\sinh(x)} + 12\operatorname{sech}^2(x)\tanh(x) + 8\tanh^3(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-1/15*I)*((3*Sech[x]^3)/(I + Sinh[x]) + 12*Sech[x]^2*Tanh[x] + 8*Tanh[x]^3
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(27) = 54.
time = 0.53, size = 93, normalized size = 2.51

method	result
risch	$-\frac{16\left(6\,\mathrm{e}^{3x}+2i\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+i\right)}{15\left(\mathrm{e}^{x}+i\right)^{5}\left(\mathrm{e}^{x}-i\right)^{3}}$
default	$\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}-\frac{5i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{5i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{11i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I/(tanh(1/2*x)-I)^3-5/8*I/(tanh(1/2*x)-I)+1/4/(tanh(1/2*x)-I)^2-2/5*I/(
tanh(1/2*x)+I)^5+5/3*I/(tanh(1/2*x)+I)^3-11/8*I/(tanh(1/2*x)+I)+1/(tanh(1/2
*x)+I)^4-3/2/(tanh(1/2*x)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(23) = 46.
time = 0.27, size = 205, normalized size = 5.54

$$
-\frac{32\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2x)}-90i\,e^{(-3x)}-90i\,e^{(-5x)}+30\,e^{(-6x)}-30i\,e^{(-7x)}+15\,e^{(-8x)}-15}+\frac{32i\,e^{(-2x)}}{-30i\,e^{(-x)}-30\,e^{(-2x)}-90i\,e^{(-3x)}-90i\,e^{(-5x)}+30\,e^{(-6x)}-30i\,e^{(-7x)}+15\,e^{(-8x)}-15}-\frac{96\,e^{(-3x)}}{-30i\,e^{(-x)}-30\,e^{(-2x)}-90i\,e^{(-3x)}-90i\,e^{(-5x)}+30\,e^{(-6x)}-30i\,e^{(-7x)}+15\,e^{(-8x)}-15}+\frac{16i}{-30i\,e^{(-x)}-30\,e^{(-2x)}-90i\,e^{(-3x)}-90i\,e^{(-5x)}+30\,e^{(-6x)}-30i\,e^{(-7x)}+15\,e^{(-8x)}-15}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -32*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 32*I*e^(-2*x)/(-30*I*e^(-x)
 - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x
) + 15*e^(-8*x) - 15) - 96*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 16
*I/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x
) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.
time = 0.42, size = 62, normalized size = 1.68

$$-\frac{16\left(6\,e^{(3\,x)}+2i\,e^{(2\,x)}+2\,e^{x}+i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -16/15*(6*e^(3*x) + 2*I*e^(2*x) + 2*e^x + I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^(
6*x) + 6*I*e^(5*x) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^{4}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(sinh(x) + I), x)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(23) = 46$.
time = 0.43, size = 53, normalized size = 1.43

$$\frac{9\,e^{(2\,x)}-24i\,e^{x}-11}{24\left(e^{x}-i\right)^{3}}-\frac{45\,e^{(4\,x)}+240i\,e^{(3\,x)}-490\,e^{(2\,x)}-320i\,e^{x}+73}{120\left(e^{x}+i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*I*e^x - 11)/(e^x - I)^3 - 1/120*(45*e^(4*x) + 240*I*e^
(3*x) - 490*e^(2*x) - 320*I*e^x + 73)/(e^x + I)^5
```

Mupad [B]
time = 1.01, size = 231, normalized size = 6.24

$$-\frac{1}{6\,\left({\mathrm{e}}^{2\,x}\,3{}\mathrm{i}-{\mathrm{e}}^{3\,x}+3\,{\mathrm{e}}^x-\mathrm{i}\right)}-\frac{\frac{3\,{\mathrm{e}}^x}{40}+\frac{1}{8}{}\mathrm{i}}{{\mathrm{e}}^{2\,x}-1+{\mathrm{e}}^x\,2{}\mathrm{i}}-\frac{\frac{3\,{\mathrm{e}}^{2\,x}}{40}-\frac{5}{24}+\frac{{\mathrm{e}}^x\,1{}\mathrm{i}}{4}}{{\mathrm{e}}^{2\,x}\,3{}\mathrm{i}+{\mathrm{e}}^{3\,x}-3\,{\mathrm{e}}^x-\mathrm{i}}+\frac{1{}\mathrm{i}}{4\,\left(1-{\mathrm{e}}^{2\,x}+{\mathrm{e}}^x\,2{}\mathrm{i}\right)}+\frac{3}{8\,\left({\mathrm{e}}^x-\mathrm{i}\right)}-\frac{3}{40\,\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}-\frac{\frac{{\mathrm{e}}^{2\,x}\,3{}\mathrm{i}}{8}+\frac{3\,{\mathrm{e}}^{3\,x}}{40}-\frac{5\,{\mathrm{e}}^x}{8}-\frac{1}{8}{}\mathrm{i}}{{\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1+{\mathrm{e}}^{3\,x}\,4{}\mathrm{i}-{\mathrm{e}}^x\,4{}\mathrm{i}}-\frac{\frac{3\,{\mathrm{e}}^{4\,x}}{40}-\frac{5\,{\mathrm{e}}^{2\,x}}{4}+\frac{3}{40}+\frac{{\mathrm{e}}^{3\,x}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^x\,1{}\mathrm{i}}{2}}{{\mathrm{e}}^{5\,x}-10\,{\mathrm{e}}^{3\,x}+{\mathrm{e}}^{4\,x}\,5{}\mathrm{i}-{\mathrm{e}}^{2\,x}\,10{}\mathrm{i}+5\,{\mathrm{e}}^x+1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((3*exp(x))/40 + 1i/8)/(exp(2*x) + exp(
x)*2i - 1) - ((3*exp(2*x))/40 + (exp(x)*1i)/4 - 5/24)/(exp(2*x)*3i + exp(3*
x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 3/(8
*(exp(x) - 1i)) - 3/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (3*exp(3*x))/40
 - (5*exp(x))/8 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i +
1) - ((exp(3*x)*1i)/2 - (5*exp(2*x))/4 + (3*exp(4*x))/40 - (exp(x)*1i)/2 +
3/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i)
```

3.170 $\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=80

$$-\frac{5}{16}i\text{ArcTan}(\sinh(x))-\frac{1}{32(i-\sinh(x))^2}+\frac{i}{8(i-\sinh(x))}+\frac{i}{24(i+\sinh(x))^3}+\frac{3}{32(i+\sinh(x))^2}-\frac{3i}{16(i+\sinh(x))}$$

```
[Out] -5/16*I*arctan(sinh(x))-1/32/(I-sinh(x))^2+1/8*I/(I-sinh(x))+1/24*I/(I+sinh
(x))^3+3/32/(I+sinh(x))^2-3/16*I/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2746, 46, 209}

$$-\frac{5}{16}i\text{ArcTan}(\sinh(x))+\frac{i}{8(-\sinh(x)+i)}-\frac{3i}{16(\sinh(x)+i)}-\frac{1}{32(-\sinh(x)+i)^2}+\frac{3}{32(\sinh(x)+i)^2}+\frac{i}{24(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
+ (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x]
)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(i-x)^3(i+x)^4}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{16(-i+x)^3} - \frac{i}{8(-i+x)^2} + \frac{i}{8(i+x)^4} + \frac{3}{16(i+x)^3} - \frac{3i}{16(i+x)^2} + \frac{}{16}\right.\right.\\
&= -\frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} - \frac{3i}{16(i+\sin}\\
&= -\frac{5}{16} i \tan^{-1}(\sinh(x)) - \frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{}{32(i+\text{s}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 1.18

$$
-\frac{i\operatorname{sech}^4(x)\,(8+15i\text{ArcTan}(\sinh(x))+5(5i+3\text{ArcTan}(\sinh(x)))\sinh(x)+5(5+6i\text{ArcTan}(\sinh(x)))\sinh^2(x)+15(i+2\text{ArcTan}(\sinh(x)))\sinh^3(x)+15(1+i\text{ArcTan}(\sinh(x)))\sinh^4(x)+15\text{ArcTan}(\sinh(x))\sinh^5(x))}{48(i+\sinh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-1/48*I)*Sech[x]^4*(8 + (15*I)*ArcTan[Sinh[x]] + 5*(5*I + 3*ArcTan[Sinh[x
]])*Sinh[x] + 5*(5 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 15*(I + 2*ArcTan[Si
nh[x]])*Sinh[x]^3 + 15*(1 + I*ArcTan[Sinh[x]])*Sinh[x]^4 + 15*ArcTan[Sinh[x
]]*Sinh[x]^5))/(I + Sinh[x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(59) = 118$.
time = 0.57, size = 137, normalized size = 1.71

method	result
risch	$-\frac{i e^{x}\left(30 i e^{7 x}+15 e^{8 x}+110 i e^{5 x}+40 e^{6 x}-110 i e^{3 x}+18 e^{4 x}-30 i e^{x}+40 e^{2 x}+15\right)}{24\left(e^{x}+i\right)^{6}\left(e^{x}-i\right)^{4}}-\frac{5 \ln \left(e^{x}-i\right)}{16}+\frac{5 \ln \left(e^{x}+i\right)}{16}$
default	$\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}-\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^{3}}+\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)^{4}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{16}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*I/(tanh(1/2*x)-I)-1/4*I/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)^4-1/2/(ta
nh(1/2*x)-I)^2-5/16*ln(tanh(1/2*x)-I)+I/(tanh(1/2*x)+I)^5+I/(tanh(1/2*x)+I)
-25/12*I/(tanh(1/2*x)+I)^3+1/3/(tanh(1/2*x)+I)^6-15/8/(tanh(1/2*x)+I)^4+15/
8/(tanh(1/2*x)+I)^2+5/16*ln(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.
time = 0.27, size = 140, normalized size = 1.75

$$\frac{32\left(15i\,e^{(-x)}-30\,e^{(-2x)}+40i\,e^{(-3x)}-110\,e^{(-4x)}+18i\,e^{(-5x)}+110\,e^{(-6x)}+40i\,e^{(-7x)}+30\,e^{(-8x)}+15i\,e^{(-9x)}\right)}{-1536i\,e^{(-x)}-2304\,e^{(-2x)}-6144i\,e^{(-3x)}-1536\,e^{(-4x)}-9216i\,e^{(-5x)}+1536\,e^{(-6x)}-6144i\,e^{(-7x)}+2304\,e^{(-8x)}-1536i\,e^{(-9x)}+768\,e^{(-10x)}-768}-\frac{5}{16}\log\left(e^{(-x)}+i\right)+\frac{5}{16}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 32*(15*I*e^(-x) - 30*e^(-2*x) + 40*I*e^(-3*x) - 110*e^(-4*x) + 18*I*e^(-5*x
) + 110*e^(-6*x) + 40*I*e^(-7*x) + 30*e^(-8*x) + 15*I*e^(-9*x))/(-1536*I*e^
(-x) - 2304*e^(-2*x) - 6144*I*e^(-3*x) - 1536*e^(-4*x) - 9216*I*e^(-5*x) +
1536*e^(-6*x) - 6144*I*e^(-7*x) + 2304*e^(-8*x) - 1536*I*e^(-9*x) + 768*e^(
-10*x) - 768) - 5/16*log(e^(-x) + I) + 5/16*log(e^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(46) = 92$.
time = 0.38, size = 245, normalized size = 3.06

$$\frac{15\left(e^{(10x)}+2i\,e^{(9x)}+3\,e^{(8x)}+8i\,e^{(7x)}+2\,e^{(6x)}+12i\,e^{(5x)}-2\,e^{(4x)}+8i\,e^{(3x)}-3\,e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-15\left(e^{(10x)}+2i\,e^{(9x)}+3\,e^{(8x)}+8i\,e^{(7x)}+2\,e^{(6x)}+12i\,e^{(5x)}-2\,e^{(4x)}+8i\,e^{(3x)}-3\,e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}-i\right)-30i\,e^{(9x)}+60\,e^{(8x)}-80i\,e^{(7x)}+220\,e^{(6x)}-36i\,e^{(5x)}-220\,e^{(4x)}-80i\,e^{(3x)}-60\,e^{(2x)}-30i\,e^{x}}{48\left(e^{(10x)}+2i\,e^{(9x)}+3\,e^{(8x)}+8i\,e^{(7x)}+2\,e^{(6x)}+12i\,e^{(5x)}-2\,e^{(4x)}+8i\,e^{(3x)}-3\,e^{(2x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12
*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x + I
) - 15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12*I
*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x - I)
- 30*I*e^(9*x) + 60*e^(8*x) - 80*I*e^(7*x) + 220*e^(6*x) - 36*I*e^(5*x) - 2
20*e^(4*x) - 80*I*e^(3*x) - 60*e^(2*x) - 30*I*e^x)/(e^(10*x) + 2*I*e^(9*x)
+ 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3
*x) - 3*e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{5}(x)}{\sinh(x)+i}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(sinh(x) + I), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.
time = 0.41, size = 118, normalized size = 1.48

$$\frac{15\left(e^{(-x)}-e^{x}\right)^{2}+76i\,e^{(-x)}-76i\,e^{x}-100}{64\left(e^{(-x)}-e^{x}+2i\right)^{2}}-\frac{55\left(e^{(-x)}-e^{x}\right)^{3}-402i\left(e^{(-x)}-e^{x}\right)^{2}-1020\,e^{(-x)}+1020\,e^{x}+936i}{192\left(e^{(-x)}-e^{x}-2i\right)^{3}}+\frac{5}{32}\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{5}{32}\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/64*(15*(e^(-x) - e^x)^2 + 76*I*e^(-x) - 76*I*e^x - 100)/(e^(-x) - e^x + 2
*I)^2 - 1/192*(55*(e^(-x) - e^x)^3 - 402*I*(e^(-x) - e^x)^2 - 1020*e^(-x) +
 1020*e^x + 936*I)/(e^(-x) - e^x - 2*I)^3 + 5/32*log(-e^(-x) + e^x + 2*I) -
 5/32*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 1.91, size = 249, normalized size = 3.11

$$\frac{5\,\ln\left(-\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16}-\frac{5\,\ln\left(\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16}-\frac{1i}{e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,5i-e^{2\,x}\,10i+5\,e^x+1i}+\frac{1i}{4\,\left(e^{2\,x}\,3i-e^{3\,x}+3\,e^x-i\right)}+\frac{1}{8\,\left(e^{4\,x}-6\,e^{2\,x}+1-e^{3\,x}\,4i+e^x\,4i\right)}+\frac{5}{8\,\left(e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i\right)}-\frac{1}{8\,\left(1-e^{2\,x}+e^x\,2i\right)}-\frac{1i}{4\,\left(e^x-i\right)}-\frac{3i}{8\,\left(e^x+1i\right)}-\frac{1}{3\,\left(15\,e^{2\,x}-15\,e^{4\,x}+e^{6\,x}-1-e^{3\,x}\,20i+e^{5\,x}\,6i+e^x\,6i\right)}-\frac{5i}{12\,\left(e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(sinh(x) + 1i)),x)
```

```
[Out] (5*log((exp(x)*5i)/8 - 5/8))/16 - (5*log((exp(x)*5i)/8 + 5/8))/16 - 1i/(exp
(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) + 1i/(4*(
exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 1/(8*(exp(4*x) - exp(3*x)*4i - 6
*exp(2*x) + exp(x)*4i + 1)) + 5/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - e
xp(x)*4i + 1)) - 1/(8*(exp(x)*2i - exp(2*x) + 1)) - 1i/(4*(exp(x) - 1i)) -
3i/(8*(exp(x) + 1i)) - 1/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp
(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 5i/(12*(exp(2*x)*3i + exp(3*x) - 3*
exp(x) - 1i))
```

3.171 $\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=40

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)-\frac{5}{8}\cosh(x)\sinh(x)+\frac{\cosh^5(x)}{4(i+\sinh(x))}$$

```
[Out] -5/8*x-5/12*I*cosh(x)^3-5/8*cosh(x)*sinh(x)+1/4*cosh(x)^5/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2758, 2761, 2715, 8}

$$-\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(\sinh(x)+i)}-\frac{5}{8}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-5*x)/8 - ((5*I)/12)*Cosh[x]^3 - (5*Cosh[x]*Sinh[x])/8 + Cosh[x]^5/(4*(I +
 Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx &= \frac{\cosh^5(x)}{4(i+\sinh(x))}-\frac{5}{4}i\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx\\ &= -\frac{5}{12}i\cosh^3(x)+\frac{\cosh^5(x)}{4(i+\sinh(x))}-\frac{5}{4}\int \cosh^2(x)\,dx\\ &= -\frac{5}{12}i\cosh^3(x)-\frac{5}{8}\cosh(x)\sinh(x)+\frac{\cosh^5(x)}{4(i+\sinh(x))}-\frac{5\int 1\,dx}{8}\\ &= -\frac{5x}{8}-\frac{5}{12}i\cosh^3(x)-\frac{5}{8}\cosh(x)\sinh(x)+\frac{\cosh^5(x)}{4(i+\sinh(x))}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 121 vs. 2(40) = 80.
time = 0.13, size = 121, normalized size = 3.02

$$-\frac{i\cosh^7(x)\left(16+\frac{30\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}-25i\sinh(x)+7\sinh^2(x)-10i\sinh^3(x)+6\sinh^4(x)\right)}{24\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/24*I)*Cosh[x]^7*(16 + (30*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 -
 I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] - (25*I)*Sinh[x] + 7*Sinh[x]^2 - (10*I)*Si
nh[x]^3 + 6*Sinh[x]^4))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh[x/2] + I*Sinh[x/
2])^6)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(30) = 60.
time = 0.56, size = 112, normalized size = 2.80

method	result
risch	$-\frac{5x}{8}+\frac{e^{4x}}{64}-\frac{ie^{3x}}{12}-\frac{e^{2x}}{8}-\frac{ie^{x}}{4}-\frac{ie^{-x}}{4}+\frac{e^{-2x}}{8}-\frac{ie^{-3x}}{12}-\frac{e^{-4x}}{64}$

default	$\frac{\frac{1}{2}+\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{-\frac{1}{8}+i}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{\frac{1}{2}-\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{\frac{1}{8}+i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+2/3*I)/(tanh(1/2*x)-1)^3+(-1/8+I)/(tanh(1/2*x)-1)^2+(-3/8+I)/(tanh(1/2
*x)-1)+1/4/(tanh(1/2*x)-1)^4+5/8*ln(tanh(1/2*x)-1)+(1/2-2/3*I)/(tanh(1/2*x)
+1)^3+(1/8+I)/(tanh(1/2*x)+1)^2-(3/8+I)/(tanh(1/2*x)+1)-1/4/(tanh(1/2*x)+1)
^4-5/8*ln(tanh(1/2*x)+1)
```

Maxima [A]
time = 0.32, size = 54, normalized size = 1.35

$$-\frac{1}{192}\left(16i\,e^{(-x)}+24\,e^{(-2\,x)}+48i\,e^{(-3\,x)}-3\right)e^{(4\,x)}-\frac{5}{8}\,x-\frac{1}{4}i\,e^{(-x)}+\frac{1}{8}\,e^{(-2\,x)}-\frac{1}{12}i\,e^{(-3\,x)}-\frac{1}{64}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(16*I*e^(-x) + 24*e^(-2*x) + 48*I*e^(-3*x) - 3)*e^(4*x) - 5/8*x - 1/
4*I*e^(-x) + 1/8*e^(-2*x) - 1/12*I*e^(-3*x) - 1/64*e^(-4*x)
```

Fricas [A]
time = 0.49, size = 55, normalized size = 1.38

$$-\frac{1}{192}\left(120\,xe^{(4\,x)}-3\,e^{(8\,x)}+16i\,e^{(7\,x)}+24\,e^{(6\,x)}+48i\,e^{(5\,x)}+48i\,e^{(3\,x)}-24\,e^{(2\,x)}+16i\,e^{x}+3\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/192*(120*x*e^(4*x) - 3*e^(8*x) + 16*I*e^(7*x) + 24*e^(6*x) + 48*I*e^(5*x
) + 48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x)
```

Sympy [A]
time = 0.10, size = 65, normalized size = 1.62

$$-\frac{5x}{8}+\frac{e^{4x}}{64}-\frac{ie^{3x}}{12}-\frac{e^{2x}}{8}-\frac{ie^{x}}{4}-\frac{ie^{-x}}{4}+\frac{e^{-2x}}{8}-\frac{ie^{-3x}}{12}-\frac{e^{-4x}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(I+sinh(x))**2,x)
```

```
[Out] -5*x/8 + exp(4*x)/64 - I*exp(3*x)/12 - exp(2*x)/8 - I*exp(x)/4 - I*exp(-x)/
4 + exp(-2*x)/8 - I*exp(-3*x)/12 - exp(-4*x)/64
```

Giac [A]
time = 0.42, size = 50, normalized size = 1.25

$$-\frac{1}{192}\left(48i\,e^{(3\,x)}-24\,e^{(2\,x)}+16i\,e^{x}+3\right)e^{(-4\,x)}-\frac{5}{8}\,x+\frac{1}{64}\,e^{(4\,x)}-\frac{1}{12}i\,e^{(3\,x)}-\frac{1}{8}\,e^{(2\,x)}-\frac{1}{4}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/192*(48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x) - 5/8*x + 1/64*e
^(4*x) - 1/12*I*e^(3*x) - 1/8*e^(2*x) - 1/4*I*e^x
```

Mupad [B]
time = 0.14, size = 54, normalized size = 1.35

$$\frac{\mathrm{e}^{-2\,x}}{8}-\frac{\mathrm{e}^{-x}\,1\mathrm{i}}{4}-\frac{5\,x}{8}-\frac{\mathrm{e}^{2\,x}}{8}-\frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{12}-\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{12}-\frac{\mathrm{e}^{-4\,x}}{64}+\frac{\mathrm{e}^{4\,x}}{64}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/4 - (5*x)/8 - exp(2*x)/8 - (exp(-3*x)*1i)/12 - (
exp(3*x)*1i)/12 - exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*1i)/4
```

3.172 $\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3}(i-\sinh(x))^3$$

```
[Out] -1/3*(I-sinh(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 32}

$$-\frac{1}{3}(-\sinh(x)+i)^3$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*(I - Sinh[x])^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx &= \text{Subst}\left(\int (i-x)^2\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{3}(i-\sinh(x))^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.79

$$-\frac{1}{2}i\cosh(2x)-\frac{5\sinh(x)}{4}+\frac{1}{12}\sinh(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] (-1/2*I)*Cosh[2*x] - (5*Sinh[x])/4 + Sinh[3*x]/12
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(11) = 22.
time = 0.56, size = 70, normalized size = 5.00

method	result	size
risch	$\frac{e^{3x}}{24}-\frac{i e^{2x}}{4}-\frac{5\,e^{x}}{8}+\frac{5\,e^{-x}}{8}-\frac{i e^{-2x}}{4}-\frac{e^{-3x}}{24}$	38
default	$\frac{\frac{1}{2}-i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1+i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1-i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{1}{2}-i}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-I)/(tanh(1/2*x)+1)^2+(1+I)/(tanh(1/2*x)+1)-1/3/(tanh(1/2*x)+1)^3+(1-I)
/(tanh(1/2*x)-1)-(1/2+I)/(tanh(1/2*x)-1)^2-1/3/(tanh(1/2*x)-1)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(8) = 16.
time = 0.28, size = 39, normalized size = 2.79

$$-\frac{1}{24}\left(6 i\,e^{(-x)}+15\,e^{(-2\,x)}-1\right)e^{(3\,x)}+\frac{5}{8}\,e^{(-x)}-\frac{1}{4}i\,e^{(-2\,x)}-\frac{1}{24}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(6*I*e^(-x) + 15*e^(-2*x) - 1)*e^(3*x) + 5/8*e^(-x) - 1/4*I*e^(-2*x)
- 1/24*e^(-3*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.
time = 0.37, size = 34, normalized size = 2.43

$$\frac{1}{24}\left(e^{(6\,x)}-6 i\,e^{(5\,x)}-15\,e^{(4\,x)}+15\,e^{(2\,x)}-6 i\,e^{x}-1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(e^(6*x) - 6*I*e^(5*x) - 15*e^(4*x) + 15*e^(2*x) - 6*I*e^x - 1)*e^(-3*
x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(8) = 16$.
time = 0.08, size = 44, normalized size = 3.14

$$\frac{e^{3x}}{24} - \frac{ie^{2x}}{4} - \frac{5e^{x}}{8} + \frac{5e^{-x}}{8} - \frac{ie^{-2x}}{4} - \frac{e^{-3x}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(I+sinh(x))**2,x)
```

```
[Out] exp(3*x)/24 - I*exp(2*x)/4 - 5*exp(x)/8 + 5*exp(-x)/8 - I*exp(-2*x)/4 - exp
(-3*x)/24
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(8) = 16$.
time = 0.42, size = 35, normalized size = 2.50

$$\frac{1}{24}\left(15\,e^{(2\,x)} - 6i\,e^{x} - 1\right)e^{(-3\,x)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{4}i\,e^{(2\,x)} - \frac{5}{8}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*e^(2*x) - 6*I*e^x - 1)*e^(-3*x) + 1/24*e^(3*x) - 1/4*I*e^(2*x) - 5
/8*e^x
```

Mupad [B]
time = 0.09, size = 37, normalized size = 2.64

$$\frac{5\,\mathrm{e}^{-x}}{8} - \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{5\,\mathrm{e}^{x}}{8} - \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{4} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] (5*exp(-x))/8 - (exp(-2*x)*1i)/4 - (exp(2*x)*1i)/4 - exp(-3*x)/24 + exp(3*x
)/24 - (5*exp(x))/8
```

3.173 $\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=30

$$-\frac{3x}{2}-\frac{3}{2}i\cosh(x)+\frac{\cosh^3(x)}{2(i+\sinh(x))}$$

```
[Out] -3/2*x-3/2*I*cosh(x)+1/2*cosh(x)^3/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2758, 2761, 8}

$$-\frac{3x}{2}-\frac{3}{2}i\cosh(x)+\frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*x)/2 - ((3*I)/2)*Cosh[x] + Cosh[x]^3/(2*(I + Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3}{2}i\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$$
$$= -\frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3\int 1\,dx}{2}$$
$$= -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.53

$$-3i\text{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}\,\text{sech}(x) + \frac{1}{2}\cosh(x)(-4i+\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x] + (Cosh[
x]*(-4*I + Sinh[x]))/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(22) = 44.
time = 0.68, size = 64, normalized size = 2.13

method	result	size
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$	29
default	$\frac{\frac{1}{2}-2i}{\tanh\left(\frac{x}{2}\right)+1}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{\frac{1}{2}+2i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-2*I)/(tanh(1/2*x)+1)-1/2/(tanh(1/2*x)+1)^2-3/2*ln(tanh(1/2*x)+1)+(1/2+
2*I)/(tanh(1/2*x)-1)+1/2/(tanh(1/2*x)-1)^2+3/2*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.00

$$-\frac{1}{8}\left(8i\,e^{(-x)}-1\right)e^{(2\,x)} - \frac{3}{2}\,x - i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(8*I*e^(-x) - 1)*e^(2*x) - 3/2*x - I*e^(-x) - 1/8*e^(-2*x)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 1.03

$$-\frac{1}{8}\left(12\,xe^{(2\,x)}-e^{(4\,x)}+8i\,e^{(3\,x)}+8i\,e^{x}+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*x*e^(2*x) - e^(4*x) + 8*I*e^(3*x) + 8*I*e^x + 1)*e^(-2*x)
```

Sympy [A]
time = 0.07, size = 29, normalized size = 0.97

$$-\frac{3x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.87

$$-\frac{1}{8}\,(8i\,e^{x}+1)e^{(-2\,x)}-\frac{3}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/8*(8*I*e^x + 1)*e^(-2*x) - 3/2*x + 1/8*e^(2*x) - I*e^x
```

Mupad [B]
time = 0.48, size = 28, normalized size = 0.93

$$\frac{\mathrm{e}^{2\,x}}{8}-\mathrm{e}^{-x}\,1\mathrm{i}-\frac{\mathrm{e}^{-2\,x}}{8}-\frac{3\,x}{2}-\mathrm{e}^{x}\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (3*x)/2 - exp(x)*1i
```

3.174 $\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=14

$$-2i\log(i+\sinh(x))+\sinh(x)$$

```
[Out] -2*I*ln(I+sinh(x))+sinh(x)
```

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$\sinh(x)-2i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(x)}{(i+\sinh(x))^2} \, dx &= -\text{Subst}\left(\int \frac{i-x}{i+x} \, dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-1+\frac{2i}{i+x}\right) \, dx, x, \sinh(x)\right)\\
&= -2i\log(i+\sinh(x))+\sinh(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-2i\log(i+\sinh(x))+\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(12) = 24.
time = 0.79, size = 53, normalized size = 3.79

method	result	size
risch	$2ix+\frac{e^{x}}{2}-\frac{e^{-x}}{2}-4i\ln\left(e^{x}+i\right)$	25
default	$-4i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+2i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*ln(tanh(1/2*x)+I)+2*I*ln(tanh(1/2*x)-1)-1/(tanh(1/2*x)-1)+2*I*ln(tanh(
1/2*x)+1)-1/(tanh(1/2*x)+1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.
time = 0.27, size = 23, normalized size = 1.64

$$-2i\,x-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}-4i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.
time = 0.41, size = 26, normalized size = 1.86

$$\frac{1}{2}\left(4i\,xe^{x}-8i\,e^{x}\log\left(e^{x}+i\right)+e^{(2\,x)}-1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*I*x*e^x - 8*I*e^x*log(e^x + I) + e^(2*x) - 1)*e^(-x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.08, size = 26, normalized size = 1.86

$$2ix + \frac{e^x}{2} - 4i \log \left(e^x + i\right) - \frac{e^{-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] 2*I*x + exp(x)/2 - 4*I*log(exp(x) + I) - exp(-x)/2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.43, size = 21, normalized size = 1.50

$$2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x - 4i\,\log\left(e^x + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^x + I)
```

Mupad [B]
time = 0.55, size = 24, normalized size = 1.71

$$\frac{\mathrm{e}^x}{2} - \frac{\mathrm{e}^{-x}}{2} + x\,2\mathrm{i} - \ln\left(\mathrm{e}^x + 1\mathrm{i}\right)\,4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] x*2i - exp(-x)/2 - log(exp(x) + 1i)*4i + exp(x)/2
```

3.175 $\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=14

$$x-\frac{2\cosh(x)}{i+\sinh(x)}$$

```
[Out] x-2*cosh(x)/(I+sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2759, 8}

$$x-\frac{2\cosh(x)}{\sinh(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x - (2*Cosh[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx &= -\frac{2\cosh(x)}{i+\sinh(x)}+\int 1\,dx\\ &= x-\frac{2\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.

time = 0.04, size = 69, normalized size = 4.93

$$\frac{2\cosh^3(x)\left(-1-\frac{\operatorname{ArcSin}\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}\right)}{(-i+\sinh(x))(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*Cosh[x]^3*(-1 - (ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x]]
)/Sqrt[1 + I*Sinh[x]]))/((-I + Sinh[x])*(I + Sinh[x])^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.82, size = 29, normalized size = 2.07

method	result	size
risch	$x+\frac{4i}{e^x+i}$	13
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tanh(1/2*x)-1)+ln(tanh(1/2*x)+1)-4/(tanh(1/2*x)+I)
```

Maxima [**A**]
time = 0.28, size = 12, normalized size = 0.86

$$x+\frac{4i}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] x + 4*I/(e^(-x) - I)
```

Fricas [**A**]
time = 0.38, size = 16, normalized size = 1.14

$$\frac{xe^x+i\,x+4i}{e^x+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.57

$$x + \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] x + 4*I/(exp(x) + I)
```

Giac [A]
time = 0.42, size = 10, normalized size = 0.71

$$x + \frac{4i}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] x + 4*I/(e^x + I)
```

Mupad [B]
time = 0.65, size = 12, normalized size = 0.86

$$x + \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] x + 4i/(exp(x) + 1i)
```

3.176 $\int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{i+\sinh(x)}$$

[Out] -1/(I+sinh(x))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 32}

$$-\frac{1}{\sinh(x)+i}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(I + Sinh[x])^2,x]

[Out] -(I + Sinh[x])^(-1)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx &= \text{Subst}\left(\int \frac{1}{(i+x)^2} \, dx, x, \sinh(x)\right) \\ &= -\frac{1}{i+\sinh(x)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -(I + Sinh[x])^(-1)
```

Maple [A]
time = 0.33, size = 10, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{1}{i+\sinh(x)}$	10
default	$-\frac{1}{i+\sinh(x)}$	10
risch	$-\frac{2\,e^{x}}{(e^{x}+i)^{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(I+sinh(x))
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{\sinh\left(x\right)+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(sinh(x) + I)
```

Fricas [A]
time = 0.43, size = 16, normalized size = 1.60

$$-\frac{2\,e^{x}}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*e^x/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.
time = 0.04, size = 19, normalized size = 1.90

$$-\frac{2e^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))**2,x)
```

```
[Out] -2*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Giac [A]
time = 0.41, size = 10, normalized size = 1.00

$$-\frac{2\,e^{x}}{\left(e^{x}+i\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*e^x/(e^x + I)^2
```

Mupad [B]
time = 0.58, size = 12, normalized size = 1.20

$$-\frac{1\mathrm{i}}{-1+\sinh\left(x\right)\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] -1i/(sinh(x)*1i - 1)
```

3.177 $\int \frac{\mathbf{sech}(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{4}\text{ArcTan}(\sinh(x))-\frac{i}{4(i+\sinh(x))^2}-\frac{1}{4(i+\sinh(x))}$$

```
[Out] -1/4*arctan(sinh(x))-1/4*I/(I+sinh(x))^2-1/4/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2746, 46, 209}

$$-\frac{1}{4}\text{ArcTan}(\sinh(x))-\frac{1}{4(\sinh(x)+i)}-\frac{i}{4(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^3} - \frac{1}{4(i+x)^2} + \frac{1}{4(1+x^2)}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) - \frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.76

$$\frac{1}{4}\left(-\text{ArcTan}(\sinh(x)) - \frac{2i+\sinh(x)}{(i+\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(I + Sinh[x])^2,x]
[Out] (-ArcTan[Sinh[x]] - (2*I + Sinh[x])/(I + Sinh[x])^2)/4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.
time = 0.67, size = 70, normalized size = 2.06

method	result	size
risch	$-\frac{-e^x+4ie^{2x}+e^{3x}}{2(e^x+i)^4} + \frac{i\ln(e^x-i)}{4} - \frac{i\ln(e^x+i)}{4}$	45
default	$\frac{i}{(\tanh(\frac{x}{2})+i)^4} - \frac{i\ln(\tanh(\frac{x}{2})+i)}{4} - \frac{5i}{2(\tanh(\frac{x}{2})+i)^2} - \frac{2}{(\tanh(\frac{x}{2})+i)^3} + \frac{3}{2(\tanh(\frac{x}{2})+i)} + \frac{i\ln(\tanh(\frac{x}{2})-i)}{4}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] I/(tanh(1/2*x)+I)^4-1/4*I*ln(tanh(1/2*x)+I)-5/2*I/(tanh(1/2*x)+I)^2-2/(tanh
(1/2*x)+I)^3+3/2/(tanh(1/2*x)+I)+1/4*I*ln(tanh(1/2*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.
time = 0.31, size = 70, normalized size = 2.06

$$-\frac{2\left(e^{(-x)} + 4i\,e^{(-2x)} - e^{(-3x)}\right)}{16i\,e^{(-x)} - 24\,e^{(-2x)} - 16i\,e^{(-3x)} + 4\,e^{(-4x)} + 4} - \frac{1}{4}i\,\log\left(i\,e^{(-x)}+1\right) + \frac{1}{4}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) + 4*I*e^(-2*x) - e^(-3*x))/(16*I*e^(-x) - 24*e^(-2*x) - 16*I*e^(
-3*x) + 4*e^(-4*x) + 4) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(22) = 44$.
time = 0.43, size = 103, normalized size = 3.03

$$\frac{\left(-i\,e^{(4\,x)}+4\,e^{(3\,x)}+6i\,e^{(2\,x)}-4\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(i\,e^{(4\,x)}-4\,e^{(3\,x)}-6i\,e^{(2\,x)}+4\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{(3\,x)}-8i\,e^{(2\,x)}+2\,e^{x}}{4\,\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) + (I*e
^(4*x) - 4*e^(3*x) - 6*I*e^(2*x) + 4*e^x + I)*log(e^x - I) - 2*e^(3*x) - 8*
I*e^(2*x) + 2*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{\left(\sinh\left(x\right)+i\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.
time = 0.41, size = 70, normalized size = 2.06

$$\frac{3i\left(e^{(-x)}-e^{x}\right)^{2}+20\,e^{(-x)}-20\,e^{x}-44i}{16\left(e^{(-x)}-e^{x}-2i\right)^{2}}-\frac{1}{8}i\,\log\left(i\,e^{(-x)}-i\,e^{x}+2\right)+\frac{1}{8}i\,\log\left(i\,e^{(-x)}-i\,e^{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(3*I*(e^(-x) - e^x)^2 + 20*e^(-x) - 20*e^x - 44*I)/(e^(-x) - e^x - 2*I
)^2 - 1/8*I*log(I*e^(-x) - I*e^x + 2) + 1/8*I*log(I*e^(-x) - I*e^x - 2)
```

Mupad [B]
time = 0.75, size = 86, normalized size = 2.53

$$-\frac{\mathrm{atan}\left(\mathrm{e}^{x}\right)}{2}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}-\frac{1}{2\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{2}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 1i/(2*(exp(2*x)
+ exp(x)*2i - 1)) - atan(exp(x))/2 - 1/(2*(exp(x) + 1i)) - 2/(exp(2*x)*3i +
 exp(3*x) - 3*exp(x) - 1i)
```

3.178 $\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=37

$$-\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2}-\frac{\operatorname{sech}(x)}{5(i+\sinh(x))}-\frac{2\tanh(x)}{5}$$

```
[Out] -1/5*I*sech(x)/(I+sinh(x))^2-1/5*sech(x)/(I+sinh(x))-2/5*tanh(x)
```

Rubi [A]

time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2751, 3852, 8}

$$-\frac{2\tanh(x)}{5}-\frac{\operatorname{sech}(x)}{5(\sinh(x)+i)}-\frac{i\operatorname{sech}(x)}{5(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/5*I)*Sech[x])/(I + Sinh[x])^2 - Sech[x]/(5*(I + Sinh[x])) - (2*Tanh[x]
)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx &= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{3}{5}i\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}\int \operatorname{sech}^2(x)\,dx \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right) \\
&= -\frac{i\operatorname{sech}(x)}{5(i+\sinh(x))^2} - \frac{\operatorname{sech}(x)}{5(i+\sinh(x))} - \frac{2\tanh(x)}{5}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 31, normalized size = 0.84

$$-\frac{\operatorname{sech}(x)(4i\cosh(2x) - 5\sinh(x) + \sinh(3x))}{10(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(I + Sinh[x])^2,x]
[Out] -1/10*(Sech[x]*((4*I)*Cosh[2*x] - 5*Sinh[x] + Sinh[3*x]))/(I + Sinh[x])^2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(28) = 56.
time = 0.60, size = 70, normalized size = 1.89

method	result	size
risch	$-\frac{4(4i e^x+5 e^{2x}-1)}{5(e^x+i)^5 (e^x-i)}$	30
default	$-\frac{1}{4(\tanh(\frac{x}{2})-i)} - \frac{2i}{(\tanh(\frac{x}{2})+i)^4} + \frac{5i}{2(\tanh(\frac{x}{2})+i)^2} - \frac{4}{5(\tanh(\frac{x}{2})+i)^5} + \frac{3}{(\tanh(\frac{x}{2})+i)^3} - \frac{7}{4(\tanh(\frac{x}{2})+i)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/4/(tanh(1/2*x)-I)-2*I/(tanh(1/2*x)+I)^4+5/2*I/(tanh(1/2*x)+I)^2-4/5/(tan
h(1/2*x)+I)^5+3/(tanh(1/2*x)+I)^3-7/4/(tanh(1/2*x)+I)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(25) = 50.
time = 0.31, size = 117, normalized size = 3.16

$$-\frac{16i\,e^{(-x)}}{20i\,e^{(-x)} - 25\,e^{(-2x)} - 25\,e^{(-4x)} - 20i\,e^{(-5x)} + 5\,e^{(-6x)} + 5} + \frac{20\,e^{(-2x)}}{20i\,e^{(-x)} - 25\,e^{(-2x)} - 25\,e^{(-4x)} - 20i\,e^{(-5x)} + 5\,e^{(-6x)} + 5} - \frac{4}{20i\,e^{(-x)} - 25\,e^{(-2x)} - 25\,e^{(-4x)} - 20i\,e^{(-5x)} + 5\,e^{(-6x)} + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -16*I*e^(-x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*e^(-5*x) + 5*e
^(-6*x) + 5) + 20*e^(-2*x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*
e^(-5*x) + 5*e^(-6*x) + 5) - 4/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 2
0*I*e^(-5*x) + 5*e^(-6*x) + 5)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.19

$$-\frac{4\left(5e^{(2x)} + 4i\,e^{x} - 1\right)}{5\left(e^{(6x)} + 4i\,e^{(5x)} - 5\,e^{(4x)} - 5\,e^{(2x)} - 4i\,e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -4/5*(5*e^(2*x) + 4*I*e^x - 1)/(e^(6*x) + 4*I*e^(5*x) - 5*e^(4*x) - 5*e^(2*
x) - 4*I*e^x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}(x)}{(\sinh(x) + i)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(sinh(x) + I)**2, x)
```

Giac [A]
time = 0.42, size = 41, normalized size = 1.11

$$-\frac{i}{4\left(e^{x} - i\right)} - \frac{-5i\,e^{(4x)} + 30\,e^{(3x)} + 80i\,e^{(2x)} - 50\,e^{x} - 11i}{20\left(e^{x} + i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*I/(e^x - I) - 1/20*(-5*I*e^(4*x) + 30*e^(3*x) + 80*I*e^(2*x) - 50*e^x
- 11*I)/(e^x + I)^5
```

Mupad [B]
time = 0.73, size = 109, normalized size = 2.95

$$-\frac{16\,\mathrm{e}^{x}\left(4\,\mathrm{e}^{3x} - 4\,\mathrm{e}^{x}\right)}{5\left(\mathrm{e}^{2x}+1\right)^{5}} - \frac{\left(4\,\mathrm{e}^{2x} - \frac{4}{5}\right)\left(\mathrm{e}^{4x} - 6\,\mathrm{e}^{2x} + 1\right)}{\left(\mathrm{e}^{2x}+1\right)^{5}} - \frac{\mathrm{e}^{x}\left(\mathrm{e}^{4x} - 6\,\mathrm{e}^{2x} + 1\right)\,16\mathrm{i}}{5\left(\mathrm{e}^{2x}+1\right)^{5}} + \frac{\left(4\,\mathrm{e}^{3x} - 4\,\mathrm{e}^{x}\right)\left(4\,\mathrm{e}^{2x} - \frac{4}{5}\right)\,1\mathrm{i}}{\left(\mathrm{e}^{2x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(4*exp(2*x) - 4/5)*1i)/(exp(2*x) + 1)^5 - (exp(x)*
(exp(4*x) - 6*exp(2*x) + 1)*16i)/(5*(exp(2*x) + 1)^5) - (16*exp(x)*(4*exp(3
*x) - 4*exp(x)))/(5*(exp(2*x) + 1)^5) - ((4*exp(2*x) - 4/5)*(exp(4*x) - 6*e
xp(2*x) + 1))/(exp(2*x) + 1)^5
```

3.179 $\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{4}\operatorname{ArcTan}(\sinh(x))+\frac{1}{16(i-\sinh(x))}+\frac{1}{12(i+\sinh(x))^3}-\frac{i}{8(i+\sinh(x))^2}-\frac{3}{16(i+\sinh(x))}$$

```
[Out] -1/4*arctan(sinh(x))+1/16/(I-sinh(x))+1/12/(I+sinh(x))^3-1/8*I/(I+sinh(x))^
2-3/16/(I+sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2746, 46, 209}

$$-\frac{1}{4}\operatorname{ArcTan}(\sinh(x))+\frac{1}{16(-\sinh(x)+i)}-\frac{3}{16(\sinh(x)+i)}-\frac{i}{8(\sinh(x)+i)^2}+\frac{1}{12(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/8
)/(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \operatorname{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= \operatorname{Subst}\left(\int \left(\frac{1}{16(-i+x)^2} - \frac{1}{4(i+x)^4} + \frac{i}{4(i+x)^3} + \frac{3}{16(i+x)^2} - \frac{1}{4(1+x^2)}\right) dx, x$$
$$= \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))} - \frac{1}{4}\operatorname{Subst}\Big($$
$$= -\frac{1}{4}\tan^{-1}(\sinh(x)) + \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{}{16(i}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 1.13

$$-\frac{\operatorname{sech}^2(x)\left(4i - 3\operatorname{ArcTan}(\sinh(x)) + (-1+6i\operatorname{ArcTan}(\sinh(x)))\sinh(x) + 6i\sinh^2(x) + (3+6i\operatorname{ArcTan}(\sinh(x)))\sinh^3(x) + 3\operatorname{ArcTan}(\sinh(x))\sinh^4(x)\right)}{12(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -1/12*(Sech[x]^2*(4*I - 3*ArcTan[Sinh[x]] + (-1 + (6*I)*ArcTan[Sinh[x]])*Si
nh[x] + (6*I)*Sinh[x]^2 + (3 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^3 + 3*ArcTan[
Sinh[x]]*Sinh[x]^4))/(I + Sinh[x])^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(45) = 90.
time = 0.75, size = 116, normalized size = 1.93

method	result
risch	$-\frac{8i e^{4x}+13\,e^{3x}+12i e^{2x}-13\,e^{5x}+12i e^{6x}+3\,e^{7x}-3\,e^{x}}{6(e^{x}-i)^2(e^{x}+i)^6}-\frac{i\ln(e^{x}+i)}{4}+\frac{i\ln(e^{x}-i)}{4}$
default	$\frac{i}{8(\tanh(\frac{x}{2})-i)^2}+\frac{i\ln(\tanh(\frac{x}{2})-i)}{4}+\frac{1}{8\tanh(\frac{x}{2})-8i}+\frac{7i}{2(\tanh(\frac{x}{2})+i)^4}-\frac{2i}{3(\tanh(\frac{x}{2})+i)^6}-\frac{i\ln(\tanh(\frac{x}{2})+i)}{4}-\frac{23}{8(\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/(tanh(1/2*x)-I)^2+1/4*I*ln(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)-I)+7/2*I/(
tanh(1/2*x)+I)^4-2/3*I/(tanh(1/2*x)+I)^6-1/4*I*ln(tanh(1/2*x)+I)-23/8*I/(ta
nh(1/2*x)+I)^2+2/(tanh(1/2*x)+I)^5-11/3/(tanh(1/2*x)+I)^3+11/8/(tanh(1/2*x)
+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.
time = 0.29, size = 120, normalized size = 2.00

$$-\frac{8\left(3\,e^{(-x)}+12i\,e^{(-2x)}-13\,e^{(-3x)}+8i\,e^{(-4x)}+13\,e^{(-5x)}+12i\,e^{(-6x)}-3\,e^{(-7x)}\right)}{192i\,e^{(-x)}-192\,e^{(-2x)}+192i\,e^{(-3x)}-480\,e^{(-4x)}-192i\,e^{(-5x)}-192\,e^{(-6x)}-192i\,e^{(-7x)}+48\,e^{(-8x)}+48}-\frac{1}{4}i\log\left(i\,e^{(-x)}+1\right)+\frac{1}{4}i\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(3*e^(-x) + 12*I*e^(-2*x) - 13*e^(-3*x) + 8*I*e^(-4*x) + 13*e^(-5*x) + 1
2*I*e^(-6*x) - 3*e^(-7*x))/(192*I*e^(-x) - 192*e^(-2*x) + 192*I*e^(-3*x) -
480*e^(-4*x) - 192*I*e^(-5*x) - 192*e^(-6*x) - 192*I*e^(-7*x) + 48*e^(-8*x)
 + 48) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(38) = 76$.
time = 0.46, size = 201, normalized size = 3.35

$$-\frac{3\left(i e^{(8x)} - 4e^{(7x)} - 4i e^{(6x)} - 4e^{(5x)} - 10i e^{(4x)} + 4e^{(3x)} - 4i e^{(2x)} + 4e^{x} + i\right)\log\left(e^{x} + i\right) + 3\left(-i e^{(8x)} + 4e^{(7x)} + 4i e^{(6x)} + 4e^{(5x)} + 10i e^{(4x)} - 4e^{(3x)} + 4i e^{(2x)} - 4e^{x} - i\right)\log\left(e^{x} - i\right) + 6e^{(7x)} + 24i e^{(6x)} - 26e^{(5x)} + 16i e^{(4x)} + 26e^{(3x)} + 24i e^{(2x)} - 6e^{x}}{12\left(e^{(8x)} + 4i e^{(7x)} - 4e^{(6x)} + 4i e^{(5x)} - 10e^{(4x)} - 4i e^{(3x)} - 4e^{(2x)} - 4i e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*(I*e^(8*x) - 4*e^(7*x) - 4*I*e^(6*x) - 4*e^(5*x) - 10*I*e^(4*x) +
4*e^(3*x) - 4*I*e^(2*x) + 4*e^x + I)*log(e^x + I) + 3*(-I*e^(8*x) + 4*e^(7*
x) + 4*I*e^(6*x) + 4*e^(5*x) + 10*I*e^(4*x) - 4*e^(3*x) + 4*I*e^(2*x) - 4*e
^x - I)*log(e^x - I) + 6*e^(7*x) + 24*I*e^(6*x) - 26*e^(5*x) + 16*I*e^(4*x)
 + 26*e^(3*x) + 24*I*e^(2*x) - 6*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) +
4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}(x)}{\left(\sinh(x) + i\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I)**2, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.
time = 0.41, size = 105, normalized size = 1.75

$$\frac{-i e^{(-x)} + i e^{x} + 3}{8\left(e^{(-x)} - e^{x} + 2i\right)} + \frac{11i\left(e^{(-x)} - e^{x}\right)^{3} + 84\left(e^{(-x)} - e^{x}\right)^{2} - 228i e^{(-x)} + 228i e^{x} - 240}{48\left(e^{(-x)} - e^{x} - 2i\right)^{3}} - \frac{1}{8}i\log\left(-e^{(-x)} + e^{x} + 2i\right) + \frac{1}{8}i\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(-I*e^(-x) + I*e^x + 3)/(e^(-x) - e^x + 2*I) + 1/48*(11*I*(e^(-x) - e^x
)^3 + 84*(e^(-x) - e^x)^2 - 228*I*e^(-x) + 228*I*e^x - 240)/(e^(-x) - e^x -
 2*I)^3 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 1.20, size = 198, normalized size = 3.30

$$-\frac{\mathrm{atan}\left(\mathrm{e}^x\right)}{2}-\frac{2}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^x+1\mathrm{i}}-\frac{1\mathrm{i}}{8\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{3\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{1\mathrm{i}}{8\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1}{8\,\left(\mathrm{e}^x-\mathrm{i}\right)}-\frac{3}{8\,\left(\mathrm{e}^x+1\mathrm{i}\right)}+\frac{2\mathrm{i}}{3\,\left(15\,\mathrm{e}^{2\,x}-15\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^x\,6\mathrm{i}\right)}-\frac{1}{3\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(8*(exp(x)*2i - exp(2*x) + 1)) - 2/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)
*10i + exp(5*x) + 5*exp(x) + 1i) - 1i/(8*(exp(2*x) + exp(x)*2i - 1)) - 3i/(
2*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - atan(exp(x))/2 -
 1/(8*(exp(x) - 1i)) - 3/(8*(exp(x) + 1i)) + 2i/(3*(15*exp(2*x) - exp(3*x)*
20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 1/(3*(exp(2*
x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.180 $\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=49

$$-\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\text{sech}^3(x)}{7(i+\sinh(x))}-\frac{4\tanh(x)}{7}+\frac{4\tanh^3(x)}{21}$$

```
[Out] -1/7*I*sech(x)^3/(I+sinh(x))^2-1/7*sech(x)^3/(I+sinh(x))-4/7*tanh(x)+4/21*t
anh(x)^3
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2751, 3852}

$$\frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\text{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\text{sech}^3(x)}{7(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/7*I)*Sech[x]^3)/(I + Sinh[x])^2 - Sech[x]^3/(7*(I + Sinh[x])) - (4*Tan
h[x])/7 + (4*Tanh[x]^3)/21
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx &= -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{5}{7}i\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx \\
&= -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\operatorname{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}\int \operatorname{sech}^4(x)\,dx \\
&= -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\operatorname{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh(x)\right) \\
&= -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\operatorname{sech}^3(x)}{7(i+\sinh(x))} - \frac{4\tanh(x)}{7} + \frac{4\tanh^3(x)}{21}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 47, normalized size = 0.96

$$
-\frac{\operatorname{sech}^3(x)(8i\cosh(2x)+4i\cosh(4x)-14\sinh(x)-3\sinh(3x)+\sinh(5x))}{42(i+\sinh(x))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -1/42*(Sech[x]^3*((8*I)*Cosh[2*x] + (4*I)*Cosh[4*x] - 14*Sinh[x] - 3*Sinh[3
*x] + Sinh[5*x]))/(I + Sinh[x])^2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(38) = 76.
time = 0.70, size = 116, normalized size = 2.37

method	result
risch	$-\frac{16\left(14\,e^{4x}+8i e^{3x}+3\,e^{2x}+4i e^{x}-1\right)}{21\left(e^{x}+i\right)^{7}\left(e^{x}-i\right)^{3}}$
default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{5i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{23i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}}-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{55}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{13}{8\left(\tanh\left(\frac{x}{2}\right)\right.}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^6-5*I/(tanh(1/2*x)+I)^4+23/8*I/(tanh(1/2*x)+I)^2+4/7/(t
anh(1/2*x)+I)^7-4/(tanh(1/2*x)+I)^5+55/12/(tanh(1/2*x)+I)^3-13/8/(tanh(1/2*
x)+I)-1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3-3/8/(tanh(1/2*x)-I)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(35) = 70.
time = 0.31, size = 317, normalized size = 6.47

$$
-\frac{64i e^{(-x)}}{84i e^{(-x)}-63 e^{(-2x)}+168i e^{(-3x)}-294 e^{(-4x)}-294 e^{(-6x)}-168i e^{(-7x)}-63 e^{(-8x)}-84i e^{(-9x)}+21 e^{(-10x)}+21}+\frac{48 e^{(-2x)}}{84i e^{(-x)}-63 e^{(-2x)}+168i e^{(-3x)}-294 e^{(-4x)}-294 e^{(-6x)}-168i e^{(-7x)}-63 e^{(-8x)}-84i e^{(-9x)}+21 e^{(-10x)}+21}-\frac{128i e^{(-3x)}}{84i e^{(-x)}-63 e^{(-2x)}+168i e^{(-3x)}-294 e^{(-4x)}-294 e^{(-6x)}-168i e^{(-7x)}-63 e^{(-8x)}-84i e^{(-9x)}+21 e^{(-10x)}+21}+\frac{224 e^{(-4x)}}{84i e^{(-x)}-63 e^{(-2x)}+168i e^{(-3x)}-294 e^{(-4x)}-294 e^{(-6x)}-168i e^{(-7x)}-63 e^{(-8x)}-84i e^{(-9x)}+21 e^{(-10x)}+21}-\frac{16}{84i e^{(-x)}-63 e^{(-2x)}+168i e^{(-3x)}-294 e^{(-4x)}-294 e^{(-6x)}-168i e^{(-7x)}-63 e^{(-8x)}-84i e^{(-9x)}+21 e^{(-10x)}+21}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -64*I*e^(-x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*x) - 2
94*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-10*x) +
 21) + 48*e^(-2*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*
x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-1
0*x) + 21) - 128*I*e^(-3*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 2
94*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) +
 21*e^(-10*x) + 21) + 224*e^(-4*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3
*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(
-9*x) + 21*e^(-10*x) + 21) - 16/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x)
 - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*
x) + 21*e^(-10*x) + 21)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.
time = 0.35, size = 80, normalized size = 1.63

$$-\frac{16\left(14\,e^{(4\,x)}+8i\,e^{(3\,x)}+3\,e^{(2\,x)}+4i\,e^{x}-1\right)}{21\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -16/21*(14*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x) + 4*I*e^x - 1)/(e^(10*x) + 4*I
*e^(9*x) - 3*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x)
- 3*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^4(x)}{(\sinh(x)+i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(sinh(x) + I)**2, x)
```

Giac [A]
time = 0.42, size = 65, normalized size = 1.33

$$-\frac{6i\,e^{(2\,x)}+15\,e^{x}-7i}{24\,(e^{x}-i)^{3}}-\frac{-42i\,e^{(6\,x)}+315\,e^{(5\,x)}+1015i\,e^{(4\,x)}-1750\,e^{(3\,x)}-1344i\,e^{(2\,x)}+511\,e^{x}+79i}{168\,(e^{x}+i)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(6*I*e^(2*x) + 15*e^x - 7*I)/(e^x - I)^3 - 1/168*(-42*I*e^(6*x) + 315
*e^(5*x) + 1015*I*e^(4*x) - 1750*e^(3*x) - 1344*I*e^(2*x) + 511*e^x + 79*I)
/(e^x + I)^7
```

Mupad [B]
time = 0.53, size = 139, normalized size = 2.84

$$\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(\frac{16\,{\mathrm{e}}^{2\,x}}{7}+\frac{32\,{\mathrm{e}}^{4\,x}}{3}-\frac{16}{21}\right)\,1{}\mathrm{i}}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{\left({\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1\right)\,\left(\frac{16\,{\mathrm{e}}^{2\,x}}{7}+\frac{32\,{\mathrm{e}}^{4\,x}}{3}-\frac{16}{21}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{\left(4\,{\mathrm{e}}^{3\,x}-4\,{\mathrm{e}}^{x}\right)\,\left(\frac{128\,{\mathrm{e}}^{3\,x}}{21}+\frac{64\,{\mathrm{e}}^{x}}{21}\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}-\frac{\left(\frac{128\,{\mathrm{e}}^{3\,x}}{21}+\frac{64\,{\mathrm{e}}^{x}}{21}\right)\,\left({\mathrm{e}}^{4\,x}-6\,{\mathrm{e}}^{2\,x}+1\right)\,1{}\mathrm{i}}{{\left({\mathrm{e}}^{2\,x}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*((16*exp(2*x))/7 + (32*exp(4*x))/3 - 16/21)*1i)/(e
xp(2*x) + 1)^7 - ((exp(4*x) - 6*exp(2*x) + 1)*((16*exp(2*x))/7 + (32*exp(4*
x))/3 - 16/21))/(exp(2*x) + 1)^7 - ((4*exp(3*x) - 4*exp(x))*((128*exp(3*x))
/21 + (64*exp(x))/21))/(exp(2*x) + 1)^7 - (((128*exp(3*x))/21 + (64*exp(x))
/21)*(exp(4*x) - 6*exp(2*x) + 1)*1i)/(exp(2*x) + 1)^7
```

3.181 $\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3} dx$

Optimal. Leaf size=28

$$i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2746, 45}

$$\frac{2i}{1+i\sinh(x)}+i\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx &= -\left(i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right)\\ &= -\left(i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, i\sinh(x)\right)\right)\\ &= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 1.61

$$\frac{2+2i\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))+\left(-2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+i\log(\cosh(x))\right)\sinh(x)}{-i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] (2 + (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] + (-2*ArcTan[Tanh[x/2]] + I*Log
[Cosh[x]])*Sinh[x])/(-I + Sinh[x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.
time = 0.92, size = 56, normalized size = 2.00

method	result	size
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-i)^{2}}+2i\ln\left(\mathrm{e}^{x}-i\right)$	26
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{4}{\tanh\left(\frac{x}{2}\right)-i}-i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -I*ln(tanh(1/2*x)+1)+2*I*ln(tanh(1/2*x)-I)-4*I/(tanh(1/2*x)-I)^2-4/(tanh(1/
2*x)-I)-I*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.18

$$i\,x-\frac{4\,e^{(-x)}}{2i\,e^{(-x)}+e^{(-2\,x)}-1}+2i\,\log\left(e^{(-x)}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.
time = 0.39, size = 50, normalized size = 1.79

$$\frac{-i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\left(-i\,e^{(2\,x)}-2\,e^{x}+i\right)\log\left(e^{x}-i\right)+i\,x}{e^{(2\,x)}-2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) + I
*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [A]
time = 0.09, size = 31, normalized size = 1.11

$$-ix + 2i\log\left(e^{x} - i\right) + \frac{4e^{x}}{e^{2x} - 2ie^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1+I*sinh(x))**3,x)
```

```
[Out] -I*x + 2*I*log(exp(x) - I) + 4*exp(x)/(exp(2*x) - 2*I*exp(x) - 1)
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.96

$$\frac{4\,e^{x}}{\left(e^{x} - i\right)^{2}} - i\,\log\left(i\,e^{x}\right) + 2i\,\log\left(-i\,e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x - I)^2 - I*log(I*e^x) + 2*I*log(-I*e^x - 1)
```

Mupad [B]
time = 0.19, size = 41, normalized size = 1.46

$$-x\,1\mathrm{i} + \ln\left(\mathrm{e}^{x} - \mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{4}{\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(sinh(x)*1i + 1)^3,x)
```

```
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

3.182 $\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=20

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

```
[Out] 1/3*I*cosh(x)^3/(1+I*sinh(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2750}

$$\frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

```
[Out] ((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 2.00

$$-\frac{i\left(-3\cosh\left(\frac{x}{2}\right)+\cosh\left(\frac{3x}{2}\right)\right)}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

```
[Out] ((-1/3*I)*(-3*Cosh[x/2] + Cosh[(3*x)/2]))/(Cosh[x/2] + I*Sinh[x/2])^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 0.89, size = 36, normalized size = 1.80

method	result	size
risch	$-\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}-i)^{3}}$	19
default	$\frac{2}{\tanh(\frac{x}{2})-i}+\frac{4i}{(\tanh(\frac{x}{2})-i)^{2}}-\frac{8}{3(\tanh(\frac{x}{2})-i)^{3}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(tanh(1/2*x)-I)+4*I/(tanh(1/2*x)-I)^2-8/3/(tanh(1/2*x)-I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.
time = 0.28, size = 53, normalized size = 2.65

$$\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}-\frac{2}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 6*e^(-2*x)/(-9*I*e^(-x) - 9*e^(-2*x) + 3*I*e^(-3*x) + 3) - 2/(-9*I*e^(-x) -
 9*e^(-2*x) + 3*I*e^(-3*x) + 3)
```

Fricas [A]
time = 0.35, size = 28, normalized size = 1.40

$$-\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(3*I*e^(2*x) - I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.06, size = 34, normalized size = 1.70

$$\frac{-6ie^{2x}+2i}{3e^{3x}-9ie^{2x}-9e^{x}+3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1+I*sinh(x))**3,x)
```

```
[Out] (-6*I*exp(2*x) + 2*I)/(3*exp(3*x) - 9*I*exp(2*x) - 9*exp(x) + 3*I)
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.80

$$-\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\left(e^{x}-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*I*e^(2*x) - I)/(e^x - I)^3
```

Mupad [B]
time = 0.66, size = 19, normalized size = 0.95

$$-\frac{2\,\mathrm{e}^{2\,x}-\frac{2}{3}}{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(sinh(x)*1i + 1)^3,x)
```

```
[Out] -(2*exp(2*x) - 2/3)/(exp(x)*1i + 1)^3
```

3.183 $\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx$

Optimal. Leaf size=16

$$\frac{i}{2(1+i\sinh(x))^2}$$

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 32}

$$\frac{i}{2(1+i\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

```
[Out] (I/2)/(1 + I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx &= -\left(i\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, i\sinh(x)\right)\right)\\
&= \frac{i}{2(1+i\sinh(x))^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 0.88

$$-\frac{i}{2(-i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 + I*Sinh[x])^3,x]
```

```
[Out] (-1/2*I)/(-I + Sinh[x])^2
```

Maple [A]
time = 0.33, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$\frac{i}{2(1+i\sinh(x))^2}$	13
default	$\frac{i}{2(1+i\sinh(x))^2}$	13
risch	$-\frac{2ie^{2x}}{(e^x-i)^4}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.62

$$\frac{i}{2\left(i\sinh\left(x\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*I/(I*sinh(x) + 1)^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(10) = 20$.
time = 0.36, size = 30, normalized size = 1.88

$$-\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2*I*e^(2*x)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.
time = 0.06, size = 37, normalized size = 2.31

$$-\frac{2ie^{2x}}{e^{4x}-4ie^{3x}-6e^{2x}+4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))**3,x)
```

```
[Out] -2*I*exp(2*x)/(exp(4*x) - 4*I*exp(3*x) - 6*exp(2*x) + 4*I*exp(x) + 1)
```

Giac [A]
time = 0.40, size = 12, normalized size = 0.75

$$-\frac{2i\,e^{(2\,x)}}{(e^x - i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2*I*e^(2*x)/(e^x - I)^4
```

Mupad [B]
time = 0.65, size = 16, normalized size = 1.00

$$-\frac{e^{2\,x}\,2i}{(1 + e^x\,1i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(sinh(x)*1i + 1)^3,x)
```

```
[Out] -(exp(2*x)*2i)/(exp(x)*1i + 1)^4
```

3.184 $\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=26

$$-i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}$$

```
[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2746, 45}

$$-\frac{2i}{1-i\sinh(x)}-i\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx &= i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -i\sinh(x)\right)\\ &= i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, -i\sinh(x)\right)\\ &= -i\log(i+\sinh(x))-\frac{2i}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 1.73

$$\frac{2-2i\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+\log(\cosh(x))-2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)\sinh(x)-i\log(\cosh(x))\sinh(x)}{i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
[Out] (2 - (2*I)*ArcTan[Tanh[x/2]] + Log[Cosh[x]] - 2*ArcTan[Tanh[x/2]]*Sinh[x] -
 I*Log[Cosh[x]]*Sinh[x])/(I + Sinh[x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(22) = 44.
time = 0.92, size = 56, normalized size = 2.15

method	result	size
risch	$ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^{2}}-2i\ln\left(\mathrm{e}^{x}+i\right)$	26
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{4i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] I*ln(tanh(1/2*x)+1)+I*ln(tanh(1/2*x)-1)+4*I/(tanh(1/2*x)+I)^2-2*I*ln(tanh(1
/2*x)+I)-4/(tanh(1/2*x)+I)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 1.27

$$-i\,x-\frac{4\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-2i\,\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="maxima")
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.
time = 0.38, size = 50, normalized size = 1.92

$$\frac{i\,xe^{(2\,x)}-2\,(x-2)e^{x}-2\,\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)-i\,x}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*x
)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A]
time = 0.08, size = 31, normalized size = 1.19

$$ix - 2i\log\left(e^{x} + i\right) + \frac{4e^{x}}{e^{2x} + 2ie^{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-I*sinh(x))**3,x)
```

```
[Out] I*x - 2*I*log(exp(x) + I) + 4*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Giac [A]
time = 0.40, size = 27, normalized size = 1.04

$$\frac{4\,e^{x}}{\left(e^{x} + i\right)^{2}} + i\,\log\left(-i\,e^{x}\right) - 2i\,\log\left(i\,e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 4*e^x/(e^x + I)^2 + I*log(-I*e^x) - 2*I*log(I*e^x - 1)
```

Mupad [B]
time = 0.16, size = 39, normalized size = 1.50

$$x\,1\mathrm{i} - \ln\left(\mathrm{e}^{x} + 1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{4}{\mathrm{e}^{x} + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^3/(sinh(x)*1i - 1)^3,x)
```

```
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

3.185 $\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx$

Optimal. Leaf size=20

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

```
[Out] -1/3*I*cosh(x)^3/(1-I*sinh(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2750\}$

$$-\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] ((-1/3*I)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 1.90

$$\frac{-3\cosh\left(\frac{x}{2}\right)+\cosh\left(\frac{3x}{2}\right)}{3\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-3*Cosh[x/2] + Cosh[(3*x)/2])/(3*(I*Cosh[x/2] + Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 0.66, size = 36, normalized size = 1.80

method	result	size
risch	$\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}+i)^{3}}$	19
default	$\frac{2}{\tanh(\frac{x}{2})+i}-\frac{4i}{(\tanh(\frac{x}{2})+i)^{2}}-\frac{8}{3(\tanh(\frac{x}{2})+i)^{3}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/(tanh(1/2*x)+I)-4*I/(tanh(1/2*x)+I)^2-8/3/(tanh(1/2*x)+I)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.
time = 0.27, size = 53, normalized size = 2.65

$$-\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3}+\frac{2}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="maxima")
[Out] -6*e^(-2*x)/(-9*I*e^(-x) + 9*e^(-2*x) + 3*I*e^(-3*x) - 3) + 2/(-9*I*e^(-x)
+ 9*e^(-2*x) + 3*I*e^(-3*x) - 3)
```

Fricas [A]
time = 0.36, size = 28, normalized size = 1.40

$$-\frac{2\left(-3i\,e^{(2\,x)}+i\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="fricas")
[Out] -2/3*(-3*I*e^(2*x) + I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A]
time = 0.07, size = 34, normalized size = 1.70

$$\frac{6ie^{2x}-2i}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-I*sinh(x))**3,x)
```

```
[Out] (6*I*exp(2*x) - 2*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.80

$$-\frac{2\left(-3i\,e^{(2\,x)}+i\right)}{3\left(e^{x}+i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2/3*(-3*I*e^(2*x) + I)/(e^x + I)^3
```

Mupad [B]
time = 0.59, size = 19, normalized size = 0.95

$$\frac{2\left(3\,\mathrm{e}^{2\,x}-1\right)}{3\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^2/(sinh(x)*1i - 1)^3,x)
```

```
[Out] (2*(3*exp(2*x) - 1))/(3*(exp(x)*1i - 1)^3)
```

3.186 $\int \frac{\cosh(x)}{(1-i\sinh(x))^3} dx$

Optimal. Leaf size=16

$$-\frac{i}{2(1-i\sinh(x))^2}$$

```
[Out] -1/2*I/(1-I*sinh(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 32}

$$-\frac{i}{2(1-i\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-1/2*I)/(1 - I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx &= i\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -i\sinh(x)\right)\\ &= -\frac{i}{2(1-i\sinh(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 0.88

$$\frac{i}{2(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(1 - I*Sinh[x])^3,x]
[Out] (I/2)/(I + Sinh[x])^2
```

Maple [A]
time = 0.36, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{i}{2(1-i\sinh(x))^2}$	13
default	$-\frac{i}{2(1-i\sinh(x))^2}$	13
risch	$\frac{2ie^{2x}}{(e^x+i)^4}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/2*I/(1-I*sinh(x))^2
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.62

$$-\frac{i}{2\left(-i\sinh\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="maxima")
[Out] -1/2*I/(-I*sinh(x) + 1)^2
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(10) = 20.
time = 0.37, size = 30, normalized size = 1.88

$$\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="fricas")
[Out] 2*I*e^(2*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.
time = 0.06, size = 36, normalized size = 2.25

$$\frac{2ie^{2x}}{e^{4x}+4ie^{3x}-6e^{2x}-4ie^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))**3,x)
```

```
[Out] 2*I*exp(2*x)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*exp(x) + 1)
```

Giac [A]
time = 0.42, size = 12, normalized size = 0.75

$$\frac{2i\,e^{(2\,x)}}{(e^x+i)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="giac")
```

```
[Out] 2*I*e^(2*x)/(e^x + I)^4
```

Mupad [B]
time = 0.20, size = 16, normalized size = 1.00

$$\frac{\mathrm{e}^{2\,x}\,2\mathrm{i}}{(-1+\mathrm{e}^x\,1\mathrm{i})^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)/(sinh(x)*1i - 1)^3,x)
```

```
[Out] (exp(2*x)*2i)/(exp(x)*1i - 1)^4
```

3.187 $\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=138

$$\frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7}-\frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6}+\frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5}-\frac{a(a^2+3b^2)\sinh^3(x}{3b^4}$$

```
[Out] (a^2+b^2)^3*ln(a+b*sinh(x))/b^7-a*(a^4+3*a^2*b^2+3*b^4)*sinh(x)/b^6+1/2*(a^
4+3*a^2*b^2+3*b^4)*sinh(x)^2/b^5-1/3*a*(a^2+3*b^2)*sinh(x)^3/b^4+1/4*(a^2+3
*b^2)*sinh(x)^4/b^3-1/5*a*sinh(x)^5/b^2+1/6*sinh(x)^6/b
```

Rubi [A]
time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7}-\frac{a(a^2+3b^2)\sinh^3(x)}{3b^4}+\frac{(a^2+3b^2)\sinh^4(x)}{4b^3}-\frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6}+\frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5}-\frac{a\sinh^5(x)}{5b^2}+\frac{\sinh^6(x)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^3*Log[a + b*Sinh[x]])/b^7 - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[
x])/b^6 + ((a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/(2*b^5) - (a*(a^2 + 3*b^2)*
Sinh[x]^3)/(3*b^4) + ((a^2 + 3*b^2)*Sinh[x]^4)/(4*b^3) - (a*Sinh[x]^5)/(5*b
^2) + Sinh[x]^6/(6*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^3}{a+x}\,dx, x, b\sinh(x)\right)}{b^7}$$

$$= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2(a^2+b^2)}{a^4}\right) - (a^4+3a^2b^2+3b^4)\,x + a(a^2+3b^2)\,x^2 - (a^2+3b^2)\,x\right.}{b^7}$$

$$= \frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7} - \frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6} + \frac{(a^4+3a^2b^2+3b^4)\sinh}{2b^5}$$

Mathematica [A]
time = 0.14, size = 137, normalized size = 0.99

$$\frac{15b^2(16a^4+40a^2b^2+29b^4)\cosh(2x)+30b^4(a^2+2b^2)\cosh(4x)+5b^6\cosh(6x)+960(a^2+b^2)^3\log(a+b\sinh(x))-120ab(8a^4+22a^2b^2+19b^4)\sinh(x)-20ab^3(4a^2+9b^2)\sinh(3x)-12ab^5\sinh(5x)}{960b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] (15*b^2*(16*a^4 + 40*a^2*b^2 + 29*b^4)*Cosh[2*x] + 30*b^4*(a^2 + 2*b^2)*Cos
h[4*x] + 5*b^6*Cosh[6*x] + 960*(a^2 + b^2)^3*Log[a + b*Sinh[x]] - 120*a*b*(
8*a^4 + 22*a^2*b^2 + 19*b^4)*Sinh[x] - 20*a*b^3*(4*a^2 + 9*b^2)*Sinh[3*x] -
 12*a*b^5*Sinh[5*x])/(960*b^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(128) = 256$.
time = 0.38, size = 516, normalized size = 3.74

method	result
risch	$-\frac{19a\,e^{x}}{16b^2}+\frac{e^{2x}a^4}{8b^5}+\frac{5\,e^{2x}a^2}{16b^3}+\frac{e^{4x}a^2}{64b^3}-\frac{3x\,a^2}{b^3}-\frac{x}{b}-\frac{3a\,e^{3x}}{32b^2}-\frac{a^5e^{x}}{2b^6}-\frac{a\,e^{5x}}{160b^2}+\frac{a\,e^{-5x}}{160b^2}-\frac{x\,a^6}{b^7}-\frac{3x\,a^4}{b^5}+\frac{a^5e^{-x}}{2b^6}$
default	$\frac{1}{6b\left(\tanh\left(\frac{x}{2}\right)-1\right)^6}-\frac{-5b-2a}{10b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{-2a^2-4ab-9b^2}{8b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{-4a^3-6a^2b-15a\,b^2-17b^3}{12b^4\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}+\frac{\left(-a^6-3a^4b^2-3a^2b^4-b^6\right)\ln(\tan}{b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/(tanh(1/2*x)-1)^6-1/10*(-5*b-2*a)/b^2/(tanh(1/2*x)-1)^5-1/8*(-2*a^2-4
*a*b-9*b^2)/b^3/(tanh(1/2*x)-1)^4-1/12*(-4*a^3-6*a^2*b-15*a*b^2-17*b^3)/b^4
/(tanh(1/2*x)-1)^3+(-a^6-3*a^4*b^2-3*a^2*b^4-b^6)/b^7*ln(tanh(1/2*x)-1)-1/1
6*(-8*a^4-8*a^3*b-26*a^2*b^2-22*a*b^3-29*b^4)/b^5/(tanh(1/2*x)-1)^2-1/16*(-
16*a^5-8*a^4*b-48*a^3*b^2-22*a^2*b^3-48*a*b^4-19*b^5)/b^6/(tanh(1/2*x)-1)+1
/6/b/(tanh(1/2*x)+1)^6-1/10*(5*b-2*a)/b^2/(tanh(1/2*x)+1)^5-1/8*(-2*a^2+4*a
*b-9*b^2)/b^3/(tanh(1/2*x)+1)^4-1/12*(-4*a^3+6*a^2*b-15*a*b^2+17*b^3)/b^4/(
```

```
tanh(1/2*x)+1)^3+(-a^6-3*a^4*b^2-3*a^2*b^4-b^6)/b^7*ln(tanh(1/2*x)+1)-1/16*
(-8*a^4+8*a^3*b-26*a^2*b^2+22*a*b^3-29*b^4)/b^5/(tanh(1/2*x)+1)^2-1/16*(-16
*a^5+8*a^4*b-48*a^3*b^2+22*a^2*b^3-48*a*b^4+19*b^5)/b^6/(tanh(1/2*x)+1)+2/b
^7*(1/2*a^6+3/2*a^4*b^2+3/2*a^2*b^4+1/2*b^6)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/
2*x)-a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(128) = 256$.
time = 0.28, size = 308, normalized size = 2.23

$$-\frac{\left(12\,ab^{4}e^{(-x)}-5\,b^{5}-30\,(a^{2}b^{3}+2\,b^{5})e^{(-2\,x)}+20\,(4\,a^{3}b^{2}+9\,ab^{4})e^{(-3\,x)}-15\,(16\,a^{4}b+40\,a^{2}b^{3}+29\,b^{5})e^{(-4\,x)}+120\,(8\,a^{5}+22\,a^{3}b^{2}+19\,ab^{4})e^{(-5\,x)}\right)e^{(6\,x)}}{1920\,b^{6}}+\frac{12\,ab^{4}e^{(-5\,x)}+5\,b^{5}e^{(-6\,x)}+120\,(8\,a^{5}+22\,a^{3}b^{2}+19\,ab^{4})e^{(-x)}+15\,(16\,a^{4}b+40\,a^{2}b^{3}+29\,b^{5})e^{(-2\,x)}+20\,(4\,a^{3}b^{2}+9\,ab^{4})e^{(-3\,x)}+30\,(a^{2}b^{3}+2\,b^{5})e^{(-4\,x)}}{1920\,b^{6}}+\frac{(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6})x}{b^{7}}+\frac{(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6})\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 + 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 + 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 + 1/1920*(12*a*b^
4*e^(-5*x) + 5*b^5*e^(-6*x) + 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-x) +
15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 + 9*a*b^4)*e^(
-3*x) + 30*(a^2*b^3 + 2*b^5)*e^(-4*x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*x/b^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*a*e^(-x) + b*e^(-2
*x) - b)/b^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2105 vs. $2(128) = 256$.
time = 0.48, size = 2105, normalized size = 15.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 + 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 + 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 - 9*a*b^5 + 15*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 + 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 + 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*x*cosh(x)^6 - 120*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b - 22*a^3*b^3 - 19*a*b
^5 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 + 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 + 2*b^6)*cosh(x)^
```

```
4 - 140*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 70*(8*a^5*b +
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 + 120*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 + 40
*a^5*b + 110*a^3*b^3 + 95*a*b^5 + 315*(a^2*b^4 + 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x) - 105*(8*a^5*b + 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 + 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 + 2*b^6)*cosh(x)^6 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 +
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 280*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 + 20*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 + 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 + 9*a*b^5)*
cosh(x)^6 + 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 210*(8*a^5*
b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 + 60*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)
*cosh(x)^2 + 3*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 + 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 + 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 + 2*b^6 - 960*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^4 - 84*(8*a^5*b + 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^2 + 2*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5*sinh(x) + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*sinh(x)^6)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 + 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 + 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 70*
(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 + a*b^5 + 50*(8*a^5*b + 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^3
+ 5*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**7/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 254, normalized size = 1.84

$$\frac{5\,b^5\left(e^{(-x)}-e^x\right)^6+12\,ab^4\left(e^{(-x)}-e^x\right)^5+30\,a^2b^3\left(e^{(-x)}-e^x\right)^4+90\,b^5\left(e^{(-x)}-e^x\right)^4+80\,a^3b^2\left(e^{(-x)}-e^x\right)^3+240\,ab^4\left(e^{(-x)}-e^x\right)^3+240\,a^4b\left(e^{(-x)}-e^x\right)^2+720\,a^2b^3\left(e^{(-x)}-e^x\right)^2+720\,b^5\left(e^{(-x)}-e^x\right)^2+960\,a^5\left(e^{(-x)}-e^x\right)+2880\,a^3b^2\left(e^{(-x)}-e^x\right)+2880\,ab^4\left(e^{(-x)}-e^x\right)}{1920\,b^6}+\frac{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) - e^x)^6 + 12*a*b^4*(e^(-x) - e^x)^5 + 30*a^2*b^3*(e^
(-x) - e^x)^4 + 90*b^5*(e^(-x) - e^x)^4 + 80*a^3*b^2*(e^(-x) - e^x)^3 + 240
*a*b^4*(e^(-x) - e^x)^3 + 240*a^4*b*(e^(-x) - e^x)^2 + 720*a^2*b^3*(e^(-x)
- e^x)^2 + 720*b^5*(e^(-x) - e^x)^2 + 960*a^5*(e^(-x) - e^x) + 2880*a^3*b^2
*(e^(-x) - e^x) + 2880*a*b^4*(e^(-x) - e^x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^7
```

Mupad [B]
time = 1.32, size = 287, normalized size = 2.08

$$\frac{\mathrm{e}^{-6\,x}}{384\,b}+\frac{\mathrm{e}^{6\,x}}{384\,b}+\frac{\mathrm{e}^{-x}\,\left(8\,a^5+22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{\mathrm{e}^{-3\,x}\,\left(4\,a^3+9\,a\,b^2\right)}{96\,b^4}-\frac{\mathrm{e}^{3\,x}\,\left(4\,a^3+9\,a\,b^2\right)}{96\,b^4}+\frac{\mathrm{e}^{-4\,x}\,\left(a^2+2\,b^2\right)}{64\,b^3}+\frac{\mathrm{e}^{4\,x}\,\left(a^2+2\,b^2\right)}{64\,b^3}+\frac{a\,\mathrm{e}^{-5\,x}}{160\,b^2}-\frac{a\,\mathrm{e}^{5\,x}}{160\,b^2}-\frac{x\,{\left(a^2+b^2\right)}^3}{b^7}+\frac{\mathrm{e}^{-2\,x}\,\left(16\,a^4+40\,a^2\,b^2+29\,b^4\right)}{128\,b^5}+\frac{\mathrm{e}^{2\,x}\,\left(16\,a^4+40\,a^2\,b^2+29\,b^4\right)}{128\,b^5}-\frac{\mathrm{e}^{x}\,\left(8\,a^5+22\,a^3\,b^2+19\,a\,b^4\right)}{16\,b^6}+\frac{\ln\left(2\,a\,\mathrm{e}^{x}-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^7/(a + b*sinh(x)),x)
```

```
[Out] exp(-6*x)/(384*b) + exp(6*x)/(384*b) + (exp(-x)*(19*a*b^4 + 8*a^5 + 22*a^3*
b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 + 4*a^3))/(96*b^4) - (exp(3*x)*(9*a*b^
2 + 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 + 2*b^2))/(64*b^3) + (exp(4*x)*(a^2
+ 2*b^2))/(64*b^3) + (a*exp(-5*x))/(160*b^2) - (a*exp(5*x))/(160*b^2) - (x*
(a^2 + b^2)^3)/b^7 + (exp(-2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) +
 (exp(2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*b^4 +
8*a^5 + 22*a^3*b^2))/(16*b^6) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2))/b^7
```

3.188 $\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=145

$$-\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6}-\frac{2(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh}{12b^3}$$

```
[Out] -1/8*a*(8*a^4+20*a^2*b^2+15*b^4)*x/b^6-2*(a^2+b^2)^(5/2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/b^6+1/5*cosh(x)^5/b+1/12*cosh(x)^3*(4*a^2+4*b^2-3*
a*b*sinh(x))/b^3+1/8*cosh(x)*(8*(a^2+b^2)^2-a*b*(4*a^2+7*b^2)*sinh(x))/b^5
```

Rubi [A]
time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2774, 2944, 2814, 2739, 632, 212}

$$-\frac{2(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5}+\frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3}-\frac{ax(8a^4+20a^2b^2+15b^4)}{8b^6}+\frac{\cosh^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] -1/8*(a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/b^6 - (2*(a^2 + b^2)^(5/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/b^6 + Cosh[x]^5/(5*b) + (Cosh[x]^3*(4*
(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^3) + (Cosh[x]*(8*(a^2 + b^2)^2 - a*b*(4
*a^2 + 7*b^2)*Sinh[x]))/(8*b^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \frac{\cosh^5(x)}{5b} + \frac{i\int \frac{\cosh^4(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3} - \frac{i\int \frac{\cosh^2(x)\left(ib(a^2+4b^2)-ia(4a^2+7b^2)\sinh(x\ldots\right.}{a+b\sinh(x)}}{4b^3}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7\ldots\right.}{8b^5}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh\ldots}{\ldots}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh\ldots}{\ldots}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3} + \frac{\cosh\ldots}{\ldots}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} - \frac{2(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.06, size = 463, normalized size = 3.19

$$\frac{\cosh(x)\left(8(15a^4+35a^2b^2+23b^4)-15ab(4a^2+9b^2)\sinh(x)+8b^2(5a^2+11b^2)\sinh^2(x)-30ab^3\sinh^3(x)+24b^4\sinh^4(x)-\frac{30(-1)^{3/4}\sqrt{b}\,(8a^4-4ia^3b+16a^2b^2-7iab^3+8b^4)\operatorname{ArcSin}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{b}}\right)}{\sqrt{a-ib}\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}-\frac{240(a^2+b^2)^2\tanh^{-1}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}+\frac{240(a-ib)^{5/2}(a+ib)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}\right)}{120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(8*(15*a^4 + 35*a^2*b^2 + 23*b^4) - 15*a*b*(4*a^2 + 9*b^2)*Sinh[x]
 + 8*b^2*(5*a^2 + 11*b^2)*Sinh[x]^2 - 30*a*b^3*Sinh[x]^3 + 24*b^4*Sinh[x]^4
 - (30*(-1)^(3/4)*Sqrt[b]*(8*a^4 - (4*I)*a^3*b + 16*a^2*b^2 - (7*I)*a*b^3 +
 8*b^4)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))])/Sqrt[b]])/(Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]) - (240*(a^2 + b^2)^2*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]])/(Sqrt[-((b*(-I + Sinh[x]))/(a +
 I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]) + (240*(a - I*b)^(5/2)*(a + I
*b)^(3/2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqr
t[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])])/(Sqrt[-((b*(-I + Sinh[x
]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(120*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(132) = 264$.
time = 0.42, size = 410, normalized size = 2.83

method	result
risch	$-\frac{a^5 x}{b^6}-\frac{5a^3x}{2b^4}-\frac{15ax}{8b^2}+\frac{e^{5x}}{160b}-\frac{a\,e^{4x}}{64b^2}+\frac{e^{3x}a^2}{24b^3}+\frac{7\,e^{3x}}{96b}-\frac{a^3e^{2x}}{8b^4}-\frac{a\,e^{2x}}{4b^2}+\frac{e^{x}a^4}{2b^5}+\frac{9\,e^{x}a^2}{8b^3}+\frac{11\,e^{x}}{16b}+\frac{e^{-x}a^4}{2b^5}+\frac{9\,e}{8}$
default	$-\frac{2\left(-a^6-3a^4b^2-3a^2b^4-b^6\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^6\sqrt{a^2+b^2}}+\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}-\frac{2b-a}{4b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{-4a^2+6ab-13b^2}{12b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^6*(-a^6-3*a^4*b^2-3*a^2*b^4-b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh
(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1/5/b/(tanh(1/2*x)+1)^5-1/4*(2*b-a)/b^2/(tanh
(1/2*x)+1)^4-1/12*(-4*a^2+6*a*b-13*b^2)/b^3/(tanh(1/2*x)+1)^3-1/8*(-4*a^3+4
*a^2*b-11*a*b^2+9*b^3)/b^4/(tanh(1/2*x)+1)^2-1/8*(-8*a^4+4*a^3*b-20*a^2*b^2
+9*a*b^3-15*b^4)/b^5/(tanh(1/2*x)+1)-1/8*a*(8*a^4+20*a^2*b^2+15*b^4)/b^6*ln
(tanh(1/2*x)+1)-1/5/b/(tanh(1/2*x)-1)^5-1/4*(2*b+a)/b^2/(tanh(1/2*x)-1)^4-1
/12*(4*a^2+6*a*b+13*b^2)/b^3/(tanh(1/2*x)-1)^3-1/8*(4*a^3+4*a^2*b+11*a*b^2+
9*b^3)/b^4/(tanh(1/2*x)-1)^2-1/8*(8*a^4+4*a^3*b+20*a^2*b^2+9*a*b^3+15*b^4)/
b^5/(tanh(1/2*x)-1)+1/8*a*(8*a^4+20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/2*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(133) = 266$.
time = 0.49, size = 283, normalized size = 1.95

$$-\frac{\left(15ab^3e^{(-x)}-6b^4-10\left(4a^2b^2+7b^4\right)e^{(-2x)}+120\left(a^3b+2ab^3\right)e^{(-3x)}-60\left(8a^4+18a^2b^2+11b^4\right)e^{(-4x)}\right)e^{(5x)}}{960\,b^5}+\frac{15ab^3e^{(-4x)}+6b^4e^{(-5x)}+60\left(8a^4+18a^2b^2+11b^4\right)e^{(-x)}+120\left(a^3b+2ab^3\right)e^{(-2x)}+10\left(4a^2b^2+7b^4\right)e^{(-3x)}}{960\,b^5}-\frac{\left(8a^5+20a^3b^2+15ab^4\right)x}{8\,b^6}+\frac{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a*b^3*e^(-x) - 6*b^4 - 10*(4*a^2*b^2 + 7*b^4)*e^(-2*x) + 120*(a^
3*b + 2*a*b^3)*e^(-3*x) - 60*(8*a^4 + 18*a^2*b^2 + 11*b^4)*e^(-4*x))*e^(5*x
)/b^5 + 1/960*(15*a*b^3*e^(-4*x) + 6*b^4*e^(-5*x) + 60*(8*a^4 + 18*a^2*b^2
+ 11*b^4)*e^(-x) + 120*(a^3*b + 2*a*b^3)*e^(-2*x) + 10*(4*a^2*b^2 + 7*b^4)*
e^(-3*x))/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. $2(133) = 266$.
time = 0.41, size = 1486, normalized size = 10.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5
*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 + 7*b^5)*cosh(x)^8 + 5*(54*b^5*
cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 + 14*b^5)*sinh(x)^8 - 120*(a^3*b^2
 + 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b
^2 - 12*a*b^4 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b + 54*a^2*b^3 + 3
3*b^5 + 14*(4*a^2*b^3 + 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 + 2*a*b^4)*cosh(x))*
sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 +
 280*(4*a^2*b^3 + 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6
0*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*c
osh(x))*sinh(x)^5 + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 + 48*a^4*b + 108*a^2*b^3 + 66*b
^5 + 70*(4*a^2*b^3 + 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 -
 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b + 18*a^2*b^3 +
11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 20*(36*b
^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 + 7*b^5)*cosh(x)^5 + 6*a^
3*b^2 + 12*a*b^4 - 210*(a^3*b^2 + 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 + 20*a^3*b
^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 +
 12*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 + 10*(4*a^2*b^3 + 7*
b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
+ 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 + 2*a*b^4)*cosh(x)^5 + 4*a^2*b^3 + 7*b^5
- 120*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b + 18*a^2*b^
3 + 11*b^5)*cosh(x)^4 + 36*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(
a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^5 + 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^5)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^
3 + 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 + 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^5 + 3*a*b^4 + 48*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
+ 2*a*b^4)*cosh(x)^2 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)
^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^
2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(133) = 266.
time = 0.42, size = 288, normalized size = 1.99

$$\frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} + 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} - 240\,ab^3e^{(2\,x)} + 480\,a^4e^x + 1080\,a^2b^2e^x + 660\,b^4e^x}{960\,b^5} - \frac{(8\,a^5 + 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6} + \frac{(15\,ab^4e^x + 6\,b^5 + 60\,(8\,a^4b + 18\,a^2b^3 + 11\,b^5)e^{(4\,x)} + 120\,(a^3b^2 + 2\,ab^4)e^{(3\,x)} + 10\,(4\,a^2b^3 + 7\,b^5)e^{(2\,x)})e^{(-5\,x)}}{960\,b^6} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{|2\,be^x + 2\,a - 2\,\sqrt{a^2 + b^2}|}{|2\,be^x + 2\,a + 2\,\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2}\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) + 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) - 240*a*b^3*e^(2*x) + 480*a^4*e^x + 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 + 2*a*b^4)*e^(3*x) + 10*(4*a^2*b^3 + 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + (a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Mupad [B]
time = 1.24, size = 302, normalized size = 2.08

$$\frac{e^{-5\,x}}{160\,b} + \frac{e^{5\,x}}{160\,b} - \frac{\ln\left(-\frac{2\,e^x\,(a^2+b^2)^3}{b^7} - \frac{2\,(b-a\,e^x)\,(a^2+b^2)^{5/2}}{b^7}\right)\,(a^2+b^2)^{5/2}}{b^6} + \frac{\ln\left(\frac{2\,(b-a\,e^x)\,(a^2+b^2)^{5/2}}{b^7} - \frac{2\,e^x\,(a^2+b^2)^3}{b^7}\right)\,(a^2+b^2)^{5/2}}{b^6} - \frac{x\,(8\,a^5 + 20\,a^3\,b^2 + 15\,a\,b^4)}{8\,b^6} + \frac{e^x\,(8\,a^4 + 18\,a^2\,b^2 + 11\,b^4)}{16\,b^5} + \frac{a\,e^{-4\,x}}{64\,b^2} - \frac{a\,e^{4\,x}}{64\,b^2} + \frac{e^{-x}\,(8\,a^4 + 18\,a^2\,b^2 + 11\,b^4)}{16\,b^5} + \frac{e^{-3\,x}\,(4\,a^2 + 7\,b^2)}{96\,b^3} + \frac{e^{3\,x}\,(4\,a^2 + 7\,b^2)}{96\,b^3} + \frac{e^{-2\,x}\,(a^3 + 2\,a\,b^2)}{8\,b^4} - \frac{e^{2\,x}\,(a^3 + 2\,a\,b^2)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^6/(a + b*sinh(x)),x)
```

```
[Out] exp(-5*x)/(160*b) + exp(5*x)/(160*b) - (log(- (2*exp(x)*(a^2 + b^2)^3)/b^7
- (2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7)*(a^2 + b^2)^(5/2))/b^6 + (log((
2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7 - (2*exp(x)*(a^2 + b^2)^3)/b^7)*(a^
2 + b^2)^(5/2))/b^6 - (x*(15*a*b^4 + 8*a^5 + 20*a^3*b^2))/(8*b^6) + (exp(x)
*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(64*b^2) - (a*exp(
4*x))/(64*b^2) + (exp(-x)*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (exp(-3
*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(
-2*x)*(2*a*b^2 + a^3))/(8*b^4) - (exp(2*x)*(2*a*b^2 + a^3))/(8*b^4)
```

3.189 $\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=81

$$\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

```
[Out] (a^2+b^2)^2*ln(a+b*sinh(x))/b^5-a*(a^2+2*b^2)*sinh(x)/b^4+1/2*(a^2+2*b^2)*s
inh(x)^2/b^3-1/3*a*sinh(x)^3/b^2+1/4*sinh(x)^4/b
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x)}{3b^2}+\frac{\sinh^4(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Sinh[x]])/b^5 - (a*(a^2 + 2*b^2)*Sinh[x])/b^4 + ((
a^2 + 2*b^2)*Sinh[x]^2)/(2*b^3) - (a*Sinh[x]^3)/(3*b^2) + Sinh[x]^4/(4*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-b^2-x^2\right)^2}{a+x}\,dx,x,b\sinh(x)\right)}{b^5}\\
&= \frac{\text{Subst}\left(\int \left(-a(a^2+2b^2)+(a^2+2b^2)\,x-ax^2+x^3+\frac{\left(a^2+b^2\right)^2}{a+x}\right)\,dx,x,b\sinh(x)\right)}{b^5}\\
&= \frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5}-\frac{a(a^2+2b^2)\sinh(x)}{b^4}+\frac{(a^2+2b^2)\sinh^2(x)}{2b^3}-\frac{a\sinh^3(x}{3b^2}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 1.01

$$\frac{12b^2(2a^2+3b^2)\cosh(2x)+3b^4\cosh(4x)+96(a^2+b^2)^2\log(a+b\sinh(x))-24ab(4a^2+7b^2)\sinh(x)-8ab^3\sinh(3x)}{96b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (12*b^2*(2*a^2 + 3*b^2)*Cosh[2*x] + 3*b^4*Cosh[4*x] + 96*(a^2 + b^2)^2*Log[
a + b*Sinh[x]] - 24*a*b*(4*a^2 + 7*b^2)*Sinh[x] - 8*a*b^3*Sinh[3*x])/(96*b^
5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(75) = 150.
time = 0.36, size = 299, normalized size = 3.69

method	result
risch	$-\frac{x\,a^4}{b^5}-\frac{2x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a\,e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}+\frac{3\,e^{2x}}{16b}-\frac{a^3e^{x}}{2b^4}-\frac{7a\,e^{x}}{8b^2}+\frac{a^3e^{-x}}{2b^4}+\frac{7a\,e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}+\frac{3\,e^{-2x}}{16b}+\frac{a\,e^{-}}{24}$
default	$\frac{1}{4b(\tanh(\frac{x}{2})-1)^4}-\frac{-3b-2a}{6b^2(\tanh(\frac{x}{2})-1)^3}-\frac{-4a^2-4ab-9b^2}{8b^3(\tanh(\frac{x}{2})-1)^2}+\frac{(-a^4-2a^2b^2-b^4)\ln(\tanh(\frac{x}{2})-1)}{b^5}-\frac{-8a^3-4a^2b-16a\,b^2-7b^3}{8b^4(\tanh(\frac{x}{2})-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/(tanh(1/2*x)-1)^4-1/6*(-3*b-2*a)/b^2/(tanh(1/2*x)-1)^3-1/8*(-4*a^2-4*
a*b-9*b^2)/b^3/(tanh(1/2*x)-1)^2+(-a^4-2*a^2*b^2-b^4)/b^5*ln(tanh(1/2*x)-1)
-1/8*(-8*a^3-4*a^2*b-16*a*b^2-7*b^3)/b^4/(tanh(1/2*x)-1)+1/4/b/(tanh(1/2*x)
+1)^4-1/6*(3*b-2*a)/b^2/(tanh(1/2*x)+1)^3-1/8*(-4*a^2+4*a*b-9*b^2)/b^3/(tan
h(1/2*x)+1)^2+(-a^4-2*a^2*b^2-b^4)/b^5*ln(tanh(1/2*x)+1)-1/8*(-8*a^3+4*a^2*
b-16*a*b^2+7*b^3)/b^4/(tanh(1/2*x)+1)+2/b^5*(1/2*a^4+a^2*b^2+1/2*b^4)*ln(a*
tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(75) = 150.
time = 0.27, size = 180, normalized size = 2.22

$$-\frac{(8\,ab^2e^{(-x)}-3\,b^3-12\,(2\,a^2b+3\,b^3)e^{(-2\,x)}+24\,(4\,a^3+7\,ab^2)e^{(-3\,x)})e^{(4\,x)}}{192\,b^4}+\frac{8\,ab^2e^{(-3\,x)}+3\,b^3e^{(-4\,x)}+24\,(4\,a^3+7\,ab^2)e^{(-x)}+12\,(2\,a^2b+3\,b^3)e^{(-2\,x)}}{192\,b^4}+\frac{(a^4+2\,a^2b^2+b^4)x}{b^5}+\frac{(a^4+2\,a^2b^2+b^4)\log(-2\,ae^{(-x)}+be^{(-2\,x)}-b)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b + 3*b^3)*e^(-2*x) + 24*(4*a^3
+ 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 + 1/192*(8*a*b^2*e^(-3*x) + 3*b^3*e^(-4*x)
 + 24*(4*a^3 + 7*a*b^2)*e^(-x) + 12*(2*a^2*b + 3*b^3)*e^(-2*x))/b^4 + (a^4
```

```
+ 2*a^2*b^2 + b^4)*x/b^5 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*a*e^(-x) + b*e^(-
2*x) - b)/b^5
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(75) = 150$.
time = 0.54, size = 865, normalized size = 10.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 + 9*b^4)*sinh(x)^6 - 192*(a^4 + 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b + 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b - 7*a*b^3 + 3*(2*a^2*b^2 + 3*b^4)*cosh(x
))*sinh(x)^5 + 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 - 96*(a^4 + 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 + 24*(4*a^3*b + 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 + 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 + 3*b^4)*cosh(x)^3 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b + 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 + 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 + 3*b^4 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b + 7*a*b^3)*cosh(x)^3 + 6*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
a^2*b^2 + 3*b^4)*cosh(x)^5 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b + 7*a*b^3)*cosh(x)^4 + a*b^3 + 9*(4*a^3*b + 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 139, normalized size = 1.72

$$\frac{3\,b^3\left(e^{(-x)}-e^x\right)^4+8\,ab^2\left(e^{(-x)}-e^x\right)^3+24\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2+96\,a^3\left(e^{(-x)}-e^x\right)+192\,ab^2\left(e^{(-x)}-e^x\right)}{192\,b^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) - e^x)^4 + 8*a*b^2*(e^(-x) - e^x)^3 + 24*a^2*b*(e^(-x)
 - e^x)^2 + 48*b^3*(e^(-x) - e^x)^2 + 96*a^3*(e^(-x) - e^x) + 192*a*b^2*(e^
(-x) - e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(-b*(e^(-x) - e^x) + 2*a)
)/b^5
```

Mupad [B]
time = 0.88, size = 169, normalized size = 2.09

$$\frac{e^{-4x}}{64\,b}+\frac{e^{4x}}{64\,b}+\frac{\ln\left(2\,a\,e^x-b+b\,e^{2x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5}+\frac{e^{-x}\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}+\frac{a\,e^{-3x}}{24\,b^2}-\frac{a\,e^{3x}}{24\,b^2}-\frac{x\,\left(a^2+b^2\right)^2}{b^5}+\frac{e^{-2x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}+\frac{e^{2x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}-\frac{e^x\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + b*sinh(x)),x)
```

```
[Out] exp(-4*x)/(64*b) + exp(4*x)/(64*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^4
 + b^4 + 2*a^2*b^2))/b^5 + (exp(-x)*(7*a*b^2 + 4*a^3))/(8*b^4) + (a*exp(-3*
x))/(24*b^2) - (a*exp(3*x))/(24*b^2) - (x*(a^2 + b^2)^2)/b^5 + (exp(-2*x)*(
2*a^2 + 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 + 3*b^2))/(16*b^3) - (exp(x)*(7
*a*b^2 + 4*a^3))/(8*b^4)
```

3.190 $\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=97

$$-\frac{a(2a^2+3b^2)\,x}{2b^4}-\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh^3(x)}{3b}+\frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3}$$

```
[Out] -1/2*a*(2*a^2+3*b^2)*x/b^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/b^4+1/3*cosh(x)^3/b+1/2*cosh(x)*(2*a^2+2*b^2-a*b*sinh(x))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2774, 2944, 2814, 2739, 632, 212}

$$-\frac{ax(2a^2+3b^2)}{2b^4}-\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}+\frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3}+\frac{\cosh^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + 3*b^2)*x)/b^4 - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x
/2])/Sqrt[a^2 + b^2]])/b^4 + Cosh[x]^3/(3*b) + (Cosh[x]*(2*(a^2 + b^2) - a*
b*Sinh[x]))/(2*b^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{\cosh^3(x)}{3b} + \frac{i\int \frac{\cosh^2(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} - \frac{i\int \frac{ib(a^2+2b^2)-ia(2a^2+3b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} + \frac{(a^2+b^2)^2\int \frac{1}{a+b\,\mathrm{si}}}{b^4}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} + \frac{\left(2(a^2+b^2)^2\right)\mathrm{Su}}{}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} - \frac{\left(4(a^2+b^2)^2\right)\mathrm{Su}}{}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} - \frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+\right.}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.50, size = 553, normalized size = 5.70

$$\frac{\cosh^3(x)\left(-12\sqrt{a-ib}\sqrt{a+ib}\,(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+12(a-ib)^2(a+ib)\tanh^{-1}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+\sqrt{a+ib}\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\left((3-3i)\sqrt{2}\sqrt{b}\,(2a^2-iab+2b^2)\,\mathrm{ArcSin}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{b}}\right)+2\sqrt{a-ib}\,(3a^2+4b^2)\sqrt{1+i\sinh(x)}\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}-3a\sqrt{a-ib}\,b\sqrt{1+i\sinh(x)}\sinh(x)\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}+2\sqrt{a-ib}\,b^2\sqrt{1+i\sinh(x)}\sinh^2(x)\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}\right)\right)}{6(a-ib)^{3/2}(a+ib)^{3/2}b\sqrt{1+i\sinh(x)}\left(-\frac{b(-i+\sinh(x))}{a+ib}\right)^{3/2}\left(-\frac{b(i+\sinh(x))}{a-ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]^3*(-12*Sqrt[a - I*b]*Sqrt[a + I*b]*(a^2 + b^2)*ArcTanh[Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I
*Sinh[x]] + 12*(a - I*b)^2*(a + I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]
)]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))
]*((3 - 3*I)*Sqrt[2]*Sqrt[b]*(2*a^2 - I*a*b + 2*b^2)*ArcSin[((1/2 + I/2)*Sq
rt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*Sqrt[a - I*b
]*(3*a^2 + 4*b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]
- 3*a*Sqrt[a - I*b]*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))] + 2*Sqrt[a - I*b]*b^2*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))])))/(6*(a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1 +
I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b)))^(3/2)*(-((b*(I + Sinh[x]))/(a
- I*b)))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(86) = 172$.
time = 0.40, size = 220, normalized size = 2.27

method	result
risch	$-\frac{a^3x}{b^4}-\frac{3ax}{2b^2}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^2}+\frac{e^x a^2}{2b^3}+\frac{5\,e^x}{8b}+\frac{e^{-x}a^2}{2b^3}+\frac{5\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^2}+\frac{e^{-3x}}{24b}+\frac{(a^2+b^2)^{\frac{3}{2}}\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right.}{b^4}$
default	$\frac{1}{3b(\tanh(\frac{x}{2})+1)^3}-\frac{-a+b}{2b^2(\tanh(\frac{x}{2})+1)^2}-\frac{-2a^2+ab-3b^2}{2b^3(\tanh(\frac{x}{2})+1)}-\frac{a(2a^2+3b^2)\ln(\tanh(\frac{x}{2})+1)}{2b^4}-\frac{1}{3b(\tanh(\frac{x}{2})-1)^3}-\frac{a+b}{2b^2(\tanh(\frac{x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/(tanh(1/2*x)+1)^3-1/2*(-a+b)/b^2/(tanh(1/2*x)+1)^2-1/2*(-2*a^2+a*b-3*
b^2)/b^3/(tanh(1/2*x)+1)-1/2*a*(2*a^2+3*b^2)/b^4*ln(tanh(1/2*x)+1)-1/3/b/(t
anh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+3*b^2)/b^3/(
tanh(1/2*x)-1)+1/2*a*(2*a^2+3*b^2)/b^4*ln(tanh(1/2*x)-1)-2/b^4*(-a^4-2*a^2*
b^2-b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.50, size = 170, normalized size = 1.75

$$-\frac{\left(3\,abe^{(-x)}-b^2-3\,(4\,a^2+5\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3}+\frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\,(4\,a^2+5\,b^2)e^{(-x)}}{24\,b^3}-\frac{(2\,a^3+3\,ab^2)x}{2\,b^4}+\frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 + 5*b^2)*e^(-2*x))*e^(3*x)/b^3 + 1/24*
(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 + 5*b^2)*e^(-x))/b^3 - 1/2*(2*a^3
 + 3*a*b^2)*x/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(87) = 174.
time = 0.43, size = 569, normalized size = 5.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
- a*b^2)*sinh(x)^5 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b + 5*b^3)
*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b + 5*b^3)*sinh(x
)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^3
 + 3*a*b^2)*x + 6*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x)^3 + b^3 + 3*(4*a^2*b +
```

```
 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 + 4*a^2*b + 5*b
^3 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b + 5*b^3)*cosh(x)^2)*sinh(x
)^2 + 24*((a^2 + b^2)*cosh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2*sinh(x) + 3*(a^2
+ b^2)*cosh(x)*sinh(x)^2 + (a^2 + b^2)*sinh(x)^3)*sqrt(a^2 + b^2)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 + 5*b^3)*cosh(x)^3 + a*b^2 + 2*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 168, normalized size = 1.73

$$\frac{b^2e^{(3x)}-3abe^{(2x)}+12a^2e^x+15b^2e^x}{24b^3}-\frac{(2a^3+3ab^2)x}{2b^4}+\frac{(3ab^2e^x+b^3+3(4a^2b+5b^3)e^{(2x)})e^{(-3x)}}{24b^4}+\frac{(a^4+2a^2b^2+b^4)\log\left(\frac{\left|2be^x+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^x+2a+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 + 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x + b^3 + 3*(4*a^2*b + 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2
 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Mupad [**B**]
time = 0.86, size = 200, normalized size = 2.06

$$\frac{e^{-3x}}{24b}+\frac{e^{3x}}{24b}-\frac{\ln\left(-\frac{2e^x(a^2+b^2)^2}{b^5}-\frac{2(b-ae^x)(a^2+b^2)^{3/2}}{b^5}\right)(a^2+b^2)^{3/2}}{b^4}+\frac{\ln\left(\frac{2(b-ae^x)(a^2+b^2)^{3/2}}{b^5}-\frac{2e^x(a^2+b^2)^2}{b^5}\right)(a^2+b^2)^{3/2}}{b^4}-\frac{x(2a^3+3ab^2)}{2b^4}+\frac{e^x(4a^2+5b^2)}{8b^3}+\frac{ae^{-2x}}{8b^2}-\frac{ae^{2x}}{8b^2}+\frac{e^{-x}(4a^2+5b^2)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (log(- (2*exp(x)*(a^2 + b^2)^2)/b^5 -
(2*(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5)*(a^2 + b^2)^(3/2))/b^4 + (log((2*
```

```
(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5 - (2*exp(x)*(a^2 + b^2)^2)/b^5)*(a^2
+ b^2)^(3/2))/b^4 - (x*(3*a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*(4*a^2 + 5*b^2)
)/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (exp(-x)*(4*a^2
+ 5*b^2))/(8*b^3)
```

3.191 $\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=38

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

```
[Out] (a^2+b^2)*ln(a+b*sinh(x))/b^3-a*sinh(x)/b^2+1/2*sinh(x)^2/b
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2 + Sinh[x]^2/(2*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx,x,b\sinh(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)\,dx,x,b\sinh(x)\right)}{b^3}\\ &= \frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.00

$$\frac{\cosh(2x)}{4b}+\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] Cosh[2*x]/(4*b) + ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(36) = 72.
time = 0.35, size = 146, normalized size = 3.84

method	result
risch	$-\frac{x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}+\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)a^2}{b^3}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$
default	$\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-a^2-b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^3}+\frac{2\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{b^3}+\frac{1}{2b(\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)-1)+(-a^2-b^2)/b^3*ln(
tanh(1/2*x)-1)+2/b^3*(1/2*a^2+1/2*b^2)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a
)+1/2/b/(tanh(1/2*x)+1)^2-1/2*(b-2*a)/b^2/(tanh(1/2*x)+1)+(-a^2-b^2)/b^3*ln
(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(36) = 72.
time = 0.28, size = 81, normalized size = 2.13

$$-\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2}+\frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2}+\frac{(a^2+b^2)x}{b^3}+\frac{(a^2+b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 + 1/8*(4*a*e^(-x) + b*e^(-2*x))/b^2 + (a^
2 + b^2)*x/b^3 + (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(36) = 72.
time = 0.48, size = 221, normalized size = 5.82

$$\frac{b^2\cosh(x)^4+b^2\sinh(x)^4-4\,ab\cosh(x)^3-8\,(a^2+b^2)x\cosh(x)^2+4\,(b^2\cosh(x)-ab)\sinh(x)^3+4\,ab\cosh(x)+2\left(3\,b^2\cosh(x)^2-6\,ab\cosh(x)-4\,(a^2+b^2)x\right)\sinh(x)^2+b^2+8\left((a^2+b^2)\cosh(x)^2+2\,(a^2+b^2)\cosh(x)\sinh(x)+(a^2+b^2)\sinh(x)^2\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+4\left(b^2\cosh(x)^3-3\,ab\cosh(x)^2-4\,(a^2+b^2)x\cosh(x)+ab\right)\sinh(x)}{8\left(b^3\cosh(x)^2+2\,b^3\cosh(x)\sinh(x)+b^3\sinh(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 + b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 + b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 + b^2)*cosh(
x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2)*log(2*(b*sinh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 + b^2)*x*cosh(x) + a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 61, normalized size = 1.61

$$\frac{b\left(e^{(-x)}-e^{x}\right)^{2}+4\,a\left(e^{(-x)}-e^{x}\right)}{8\,b^{2}}+\frac{\left(a^{2}+b^{2}\right)\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) - e^x)^2 + 4*a*(e^(-x) - e^x))/b^2 + (a^2 + b^2)*log(abs(-b*
(e^(-x) - e^x) + 2*a))/b^3
```

Mupad [**B**]
time = 0.62, size = 77, normalized size = 2.03

$$\frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(2\,a\,\mathrm{e}^{x}-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^{2}+b^{2}\right)}{b^{3}}-\frac{a\,\mathrm{e}^{x}}{2\,b^{2}}-\frac{x\,\left(a^{2}+b^{2}\right)}{b^{3}}+\frac{a\,\mathrm{e}^{-x}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^2 +
 b^2))/b^3 - (a*exp(x))/(2*b^2) - (x*(a^2 + b^2))/b^3 + (a*exp(-x))/(2*b^2)
```

3.192 $\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(
1/2)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2774, 2814, 2739, 632, 212}

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/b^2 + Cosh[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx &= \frac{\cosh(x)}{b}+\frac{i\int \frac{-ib+ia\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}+\frac{(2(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}+\frac{\cosh(x)}{b}-\frac{(4(a^2+b^2))\,\mathrm{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}+\frac{\cosh(x)}{b}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.58, size = 396, normalized size = 7.33

$$\frac{\cosh(x)\left(-2\sqrt{a-ib}\,\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+2(a-ib)\tanh^{-1}\left(\frac{\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\left(-2(-1)^{3/4}\sqrt{b}\,\mathrm{ArcSin}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{b}}\right)+\sqrt{a-ib}\,\sqrt{1+i\sinh(x)}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}\right)\right)}{\sqrt{a-ib}\,\sqrt{a+ib}\,b\sqrt{1+i\sinh(x)}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(
a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]] + 2*(
a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt
[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqr
t[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*Arc
```

```
Sin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b
]] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]
)))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh
[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(48) = 96$.
time = 0.37, size = 100, normalized size = 1.85

method	result	size
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}+\frac{e^{-x}}{2b}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{b^2}$	93
default	$\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(tanh(1/2*x)+1)-a/b^2*ln(tanh(1/2*x)+1)-2/b^2*(-a^2-b^2)/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/b/(tanh(1/2*x)-1)+a/
b^2*ln(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.50, size = 81, normalized size = 1.50

$$-\frac{ax}{b^2}+\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}+\frac{\sqrt{a^2+b^2}\,\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log((b*e^(-x) - a - s
qrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(50) = 100$.
time = 0.37, size = 171, normalized size = 3.17

$$-\frac{2\,ax\cosh\left(x\right)-b\cosh\left(x\right)^2-b\sinh\left(x\right)^2-2\,\sqrt{a^2+b^2}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,\left(b\cosh(x)+a\right)\sinh(x)-b}\right)+2\left(ax-b\cosh\left(x\right)\right)\sinh\left(x\right)-b}{2\left(b^2\cosh\left(x\right)+b^2\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 + b^2)*(cosh(x
) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b
^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(
x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) - b)) + 2*(a*x - b*cosh(x))*sinh(x) - b)/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.
time = 130.49, size = 377, normalized size = 6.98

$$\begin{cases} \tilde{\infty}\left(\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}}{a} & \text{for } b = 0 \\ \frac{\frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tanh\left(\frac{x}{2}\right)\right)}{\tanh^2\left(\frac{x}{2}\right)-1} - \frac{2}{\tanh^2\left(\frac{x}{2}\right)-1}}{b} & \text{for } a = 0 \\ -\frac{ax\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{ax}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{2b}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} + \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2\left(\frac{x}{2}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} - \frac{\sqrt{a^2+b^2}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b^2\tanh^2\left(\frac{x}{2}\right)-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x
/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((-x
*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), ((log(tan
h(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)**2 - 1)
 - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), (-a*x*tanh(x/2)**2/(b**2*tanh(x/2)**
2 - b**2) + a*x/(b**2*tanh(x/2)**2 - b**2) - 2*b/(b**2*tanh(x/2)**2 - b**2)
 - sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(x/2)**
2/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt
(a**2 + b**2)/a)/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/
2) - b/a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*tanh(x/2)**2 - b**2) - s
qrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)
**2 - b**2), True))
```

Giac [A]
time = 0.42, size = 83, normalized size = 1.54

$$-\frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log(abs(2*b*e^x + 2*a
 - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/b^2
```

Mupad [B]

time = 0.63, size = 87, normalized size = 1.61

$$\frac{\mathrm{e}^{-x}}{2\,b}+\frac{\mathrm{e}^{x}}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4}}{b^2\,\sqrt{a^2+b^2}}+\frac{\mathrm{e}^{x}\,\sqrt{-b^4}}{b\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4}}-\frac{a\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (2*atan((a*(-b^4)^(1/2))/(b^2*(a^2 + b^2)^(1
/2)) + (exp(x)*(-b^4)^(1/2))/(b*(a^2 + b^2)^(1/2)))*(a^2 + b^2)^(1/2))/(-b^
4)^(1/2) - (a*x)/b^2
```

3.193 $\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sinh(x))}{b}$$

[Out] ln(a+b*sinh(x))/b

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 31}

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Sinh[x]),x]

[Out] Log[a + b*Sinh[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x]),x]
[Out] Log[a + b*Sinh[x]]/b
```

Maple [A]
time = 0.21, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(x))}{b}$	12
default	$\frac{\ln(a+b\sinh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sinh(x))/b
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sinh\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="maxima")
[Out] log(b*sinh(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.
time = 0.39, size = 27, normalized size = 2.45

$$-\frac{x-\log\left(\frac{2\left(b\sinh\left(x\right)+a\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="fricas")
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A]
time = 0.17, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\sinh\left(x\right)\right)}{b} & \text{for}\: b\neq 0\\\frac{\sinh\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((log(a/b + sinh(x))/b, Ne(b, 0)), (sinh(x)/a, True))
```

Giac [A]
time = 0.42, size = 22, normalized size = 2.00

$$\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

Mupad [B]
time = 0.06, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\sinh(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*sinh(x)),x)
```

```
[Out] log(a + b*sinh(x))/b
```

3.194 $\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$\frac{a\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] a*arctan(sinh(x))/(a^2+b^2)-b*ln(cosh(x))/(a^2+b^2)+b*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2747, 720, 31, 649, 210, 266}

$$\frac{a\text{ArcTan}(\sinh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/(a^2 + b^2) - (b*Log[Cosh[x]])/(a^2 + b^2) + (b*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right)\right)\\ &= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{a^2+b^2}+\frac{b\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}\\ &= \frac{b\log(a+b\sinh(x))}{a^2+b^2}+\frac{b\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}-\frac{(ab)\text{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx,x,\right.}{a^2+b^2}\\ &= \frac{a\tan^{-1}(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(48) = 96.
time = 0.07, size = 99, normalized size = 2.06

$$-\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{2\sqrt{-b^2}\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a
 + b*Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/(Sqrt[-b^2]*
(a^2 + b^2))
```

Maple [A]
time = 0.44, size = 64, normalized size = 1.33

method	result	size
default	$\frac{-b\ln(\tanh^2(\frac{x}{2})+1)+2a\arctan(\tanh(\frac{x}{2}))}{a^2+b^2}+\frac{b\ln(a(\tanh^2(\frac{x}{2}))-2b\tanh(\frac{x}{2})-a)}{a^2+b^2}$	64
risch	$\frac{i\ln(e^x+i)a}{a^2+b^2}-\frac{\ln(e^x+i)b}{a^2+b^2}-\frac{i\ln(e^x-i)a}{a^2+b^2}-\frac{\ln(e^x-i)b}{a^2+b^2}+\frac{b\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^2+b^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)*(-1/2*b*ln(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))+b/(a^2+b^2)*
ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
```

Maxima [A]
time = 0.51, size = 66, normalized size = 1.38

$$-\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) + b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) - b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A]
time = 0.44, size = 57, normalized size = 1.19

$$\frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+b\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) + b*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) - b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.41, size = 89, normalized size = 1.85

$$\frac{b^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)}-\frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) - 1/2*b*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

Mupad [B]
time = 1.35, size = 93, normalized size = 1.94

$$\frac{b\,\ln\left(4\,b^3\,e^{2\,x}-a^2\,b-4\,b^3+2\,a^3\,e^x+8\,a\,b^2\,e^x+a^2\,b\,e^{2\,x}\right)}{a^2+b^2}-\frac{\ln\left(e^x+1\mathrm{i}\right)}{b+a\,1\mathrm{i}}-\frac{\ln\left(1+e^x\,1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*sinh(x))),x)
```

```
[Out] (b*log(4*b^3*exp(2*x) - a^2*b - 4*b^3 + 2*a^3*exp(x) + 8*a*b^2*exp(x) + a^2
*b*exp(2*x)))/(a^2 + b^2) - log(exp(x) + 1i)/(a*1i + b) - (log(exp(x)*1i +
1)*1i)/(a + b*1i)
```

3.195 $\int \frac{\mathrm{sech}^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{\mathrm{sech}(x)(b+a\sinh(x))}{a^2+b^2}$$

```
[Out] -2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+sech(x)*(
b+a*sinh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2775, 12, 2739, 632, 212}

$$\frac{\mathrm{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (Se
ch[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx &= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\int \frac{b^2}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}-\frac{(4b^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 67, normalized size = 1.14

$$\frac{\frac{2b^2\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+b\operatorname{sech}(x)+a\tanh(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] ((2*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + b*Se
ch[x] + a*Tanh[x])/(a^2 + b^2)
```

Maple [A]
time = 0.45, size = 71, normalized size = 1.20

method	result	size
default	$\frac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{x}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}$	71
risch	$-\frac{2\left(-\mathrm{e}^{x}b+a\right)}{\left(1+\mathrm{e}^{2x}\right)\left(a^{2}+b^{2}\right)}+\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{b^{2}\ln\left(\mathrm{e}^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}+a^{4}+2a^{2}b^{2}+b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/
(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)
```

Maxima [A]
time = 0.50, size = 89, normalized size = 1.51

$$\frac{b^{2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}+\frac{2\left(be^{(-x)}+a\right)}{a^{2}+b^{2}+\left(a^{2}+b^{2}\right)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) + 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(55) = 110$.
time = 0.37, size = 259, normalized size = 4.39

$$-\frac{2a^{3}+2ab^{2}-\left(b^{2}\cosh\left(x\right)^{2}+2b^{2}\cosh\left(x\right)\sinh\left(x\right)+b^{2}\sinh\left(x\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2ab\cosh(x)+2a^{2}+b^{2}+2\left(b^{2}\cosh(x)+ab\right)\sinh(x)-2\sqrt{a^{2}+b^{2}}\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2a\cosh(x)+2\left(b\cosh(x)+a\right)\sinh(x)-b}\right)-2\left(a^{2}b+b^{3}\right)\cosh\left(x\right)-2\left(a^{2}b+b^{3}\right)\sinh\left(x\right)}{a^{4}+2a^{2}b^{2}+b^{4}+\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{2}+2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)\sinh\left(x\right)+\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sinh\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
+ b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
```

```
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.41, size = 87, normalized size = 1.47

$$\frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(be^x-a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B]
time = 1.04, size = 321, normalized size = 5.44

$$-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,e^x}{a^2+b^2}}{e^{2\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\left(e^x\,\left(\frac{2}{\sqrt{b^4}\,{\left(a^2+b^2\right)}^2}+\frac{2\,a\,\left(a^3\,\sqrt{b^4}+a\,b^2\,\sqrt{b^4}\right)}{b^4\,\sqrt{-{\left(a^2+b^2\right)}^3}\,\left(a^2+b^2\right)\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}\right)-\frac{2\,a\,\left(b^3\,\sqrt{b^4}+a^2\,b\,\sqrt{b^4}\right)}{b^4\,\sqrt{-{\left(a^2+b^2\right)}^3}\,\left(a^2+b^2\right)\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}\right)\,\left(\frac{b^3\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}+\frac{a^2\,b\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}\right)\right)\,\sqrt{b^4}}{\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))),x)
```

```
[Out] - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan((
exp(x)*(2/((b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(a^3*(b^4)^(1/2) + a*b^2*(b^4)
^(1/2)))/(b^4*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 -
 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) + a^2*b*(b^4)^(1/2)))/(b^4*(-(a
^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))
)*((b^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^
6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(- a^6 - b^6 - 3*a^2*b^4
 - 3*a^4*b^2)^(1/2)
```

3.196 $\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=87

$$\frac{a(a^2+3b^2)\,\mathrm{ArcTan}(\sinh(x))}{2\,(a^2+b^2)^2}-\frac{b^3\log(\cosh(x))}{(a^2+b^2)^2}+\frac{b^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\,(a^2+b^2)}$$

```
[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(x))/(a^2+b^2)^2-b^3*ln(cosh(x))/(a^2+b^2)^2+b
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2747, 755, 815, 649, 209, 266}

$$\frac{a(a^2+3b^2)\,\mathrm{ArcTan}(\sinh(x))}{2\,(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(a\sinh(x)+b)}{2\,(a^2+b^2)}+\frac{b^3\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{b^3\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) - (b^3*Log[Cosh[x]])/(a
^2 + b^2)^2 + (b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(b + a*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx &= b^3\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^2}\,dx,x,b\sinh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b\text{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b\text{Subst}\left(\int \left(-\frac{2b^2}{\left(a^2+b^2\right)(a+x)}+\frac{-a^3-3ab^2+2b^2x}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)\,dx,x,b\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b\text{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}\\
&= \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}-\frac{b^3\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{\left(a^2+b^2\right)^2}+\\
&= \frac{a\left(a^2+3b^2\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2}-\frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}^2(x)(b+}{2\left(a^2+\right.}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 77, normalized size = 0.89

$$\frac{2a\left(a^2+3b^2\right)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+2b^3(-\log(\cosh(x))+\log(a+b\sinh(x)))+b\left(a^2+b^2\right)\operatorname{sech}^2(x)+a\left(a^2+b^2\right)\operatorname{sech}(x)\tanh(x)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[x/2]] + 2*b^3*(-Log[Cosh[x]] + Log[a + b*Sin
h[x]]) + b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 +
 b^2)^2)
```

Maple [A]
time = 0.57, size = 161, normalized size = 1.85

method	result
default	$\frac{b^3 \ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^4+2a^2b^2+b^4}+\frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-a^2b-b^3\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-b^3\ln\left(\tanh^2\left(\frac{x}{2}\right.\right.}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^x\left(e^{2x}a+2\,e^x b-a\right)}{\left(1+e^{2x}\right)^2\left(a^2+b^2\right)}+\frac{i\ln(e^x+i)a^3}{2a^4+4a^2b^2+2b^4}+\frac{3i\ln(e^x+i)a\,b^2}{2(a^4+2a^2b^2+b^4)}-\frac{\ln(e^x+i)b^3}{a^4+2a^2b^2+b^4}-\frac{i\ln(e^x-i)a^3}{2(a^4+2a^2b^2+b^4)}-\frac{3i\ln(e^x-i)a\,b^2}{2(a^4+2a^2b^2+b^4)}-\frac{\ln(e}{a^4+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b^3/(a^4+2*a^2*b^2+b^4)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+2/(a^4+2*a^2*
b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(-a^2*b-b^3)*tanh(1/2*x)^2+(1
/2*a^3+1/2*a*b^2)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2-1/2*b^3*ln(tanh(1/2*x)^2
+1)+1/2*(a^3+3*a*b^2)*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.56, size = 159, normalized size = 1.83

$$\frac{b^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^3+3\,ab^2\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{ae^{(-x)}+2\,be^{(-2\,x)}-ae^{(-3\,x)}}{a^2+b^2+2\,(a^2+b^2)e^{(-2\,x)}+(a^2+b^2)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(e^(
-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 + 3*a*b^2)*arctan(e^(-x))/(a^4 +
2*a^2*b^2 + b^4) + (a*e^(-x) + 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2*(a
^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(83) = 166.
time = 0.49, size = 652, normalized size = 7.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 + 2*(a^2*b + b^3)*cosh(x
)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 + 3*a*b
^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(
x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(
a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 +
 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(
x) + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(
x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*c
osh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (b^3*cosh(x)^
4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*
b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*l
og(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(x)^
2 - 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(83) = 166.
time = 0.41, size = 214, normalized size = 2.46

$$\frac{b^4\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2a\right|\right)}{a^4b + 2a^2b^3 + b^5} - \frac{b^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2a^2b^2 + b^4\right)} + \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)\left(a^3 + 3ab^2\right)}{4\left(a^4 + 2a^2b^2 + b^4\right)} + \frac{b^3\left(e^{(-x)} - e^x\right)^2 - 2a^3\left(e^{(-x)} - e^x\right) - 2ab^2\left(e^{(-x)} - e^x\right) + 4a^2b + 8b^3}{2\left(a^4 + 2a^2b^2 + b^4\right)\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*b^3*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^3 + 3*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(b^3*(e
^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 4*a^2*b +
8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [**B**]
time = 2.24, size = 291, normalized size = 3.34

$$\frac{\frac{2\left(a^2b+b^3\right)}{\left(a^2+b^2\right)^2} + \frac{e^x\left(a^3+ab^2\right)}{\left(a^2+b^2\right)^2}}{e^{2x}+1} - \frac{\frac{2b}{a^2+b^2} + \frac{2a\,e^x}{a^2+b^2}}{2\,e^{2x}+e^{4x}+1} - \frac{\ln\left(1+e^x\,1i\right)\,\left(a+b\,2i\right)}{2\left(-a^2\,1i+2\,a\,b+b^2\,1i\right)} + \frac{b^3\,\ln\left(16\,b^7\,e^{2x} - a^6\,b - 16\,b^7 - 9\,a^2\,b^5 - 6\,a^4\,b^3 + 2\,a^7\,e^x + 9\,a^2\,b^5\,e^{2x} + 6\,a^4\,b^3\,e^{2x} + 32\,a\,b^6\,e^x + a^6\,b\,e^{2x} + 18\,a^3\,b^4\,e^x + 12\,a^5\,b^2\,e^x\right)}{a^4+2\,a^2\,b^2+b^4} - \frac{\ln\left(e^x+1i\right)\,\left(2\,b+a\,1i\right)}{2\left(-a^2+a\,b\,2i+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] ((2*(a^2*b + b^3))/(a^2 + b^2)^2 + (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*b)/(a^2 + b^2) + (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) - (log(exp(x)*1i + 1)*(a + b*2i))/(2*(2*a*b - a^2*1i + b^2*1
i)) + (b^3*log(16*b^7*exp(2*x) - a^6*b - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 + 2
*a^7*exp(x) + 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6*exp(x) + a
^6*b*exp(2*x) + 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*
b^2) - (log(exp(x) + 1i)*(a*1i + 2*b))/(2*(a*b*2i - a^2 + b^2))
```

3.197 $\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\text{sech}^3(x)(b+a\sinh(x))}{3\left(a^2+b^2\right)}+\frac{\text{sech}(x)\left(3b^3+a\left(2a^2+5b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}$$

```
[Out] -2*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/3*sech(
x)^3*(b+a*sinh(x))/(a^2+b^2)+1/3*sech(x)*(3*b^3+a*(2*a^2+5*b^2)*sinh(x))/(a
^2+b^2)^2
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2775, 2945, 12, 2739, 632, 212}

$$\frac{\text{sech}^3(x)(a\sinh(x)+b)}{3\left(a^2+b^2\right)}-\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{\text{sech}(x)\left(a\left(2a^2+5b^2\right)\sinh(x)+3b^3\right)}{3\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (Se
ch[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + (Sech[x]*(3*b^3 + a*(2*a^2 + 5*b
^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} - \frac{\int \frac{\operatorname{sech}^2(x)(-2a^2-3b^2-2ab\sinh(x))}{a+b\sinh(x)}\,dx}{3(a^2+b^2)}$$
$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int \frac{3b^4}{a+b\sinh(x)}\,dx}{3(a^2+b^2)^2}$$
$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$
$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{(2b^4)\operatorname{Subst}\left(\int \frac{}{a+2bx}\right.}{(a^2}$$
$$= \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} - \frac{(4b^4)\operatorname{Subst}\left(\int \frac{}{4(a^2+}\right.}{}$$
$$= -\frac{2b^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{\operatorname{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\operatorname{sech}(x)\left(3b^3+a(2a^2+5b^2)\,\mathrm{s}\right.}{3(a^2+b^2)^2}$$

Mathematica [A]
time = 0.25, size = 102, normalized size = 1.02

$$\frac{\frac{6b^4\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3b^3\operatorname{sech}(x) + (a^2+b^2)\operatorname{sech}^3(x)(b+a\sinh(x)) + a(2a^2+5b^2)\tanh(x)}{3(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3*b^
3*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) + a*(2*a^2 + 5*b^2)*Tanh[
x])/(3*(a^2 + b^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(90) = 180$.
time = 0.50, size = 182, normalized size = 1.82

method	result
default	$-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tanh^4\left(\frac{x}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)-2b^3\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(-a^3-2a\,b^2\right)\tanh\left(\frac{x}{2}\right)-\frac{a^2}{3}\right.}{\left(a^4+2a^2b^2+b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3}$

risch	$-\frac{2(-3b^3 e^{5x}+3a b^2 e^{4x}-4a^2 b e^{3x}-10b^3 e^{3x}+6a^3 e^{2x}+12a b^2 e^{2x}-3b^3 e^{x}+2a^3+5a b^2)}{3(a^4+2a^2b^2+b^4)(1+e^{2x})^3}+\frac{b^4 \ln\left(e^x+\frac{(a^2+b^2)^{\frac{5}{2}} a-a^6-3a^4b^2-3a^2b^4-b^6}{b(a^2+b^2)^{\frac{5}{2}}}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+(-a^2*b-2*b^3)*tanh(1/2*x)^4+(-2/3*a^3-8/3*a*b^2)*tanh(1/2*x)^3-2*b^3*tanh(1/2*x)^2+(-a^3-2*a*b^2)*tanh(1/2*x)-1/3*a^2*b-4/3*b^3)/(tanh(1/2*x)^2+1)^3+2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(92) = 184$.
time = 0.54, size = 230, normalized size = 2.30

$$\frac{b^4 \log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(3\,b^3e^{(-x)}+3\,ab^2e^{(-4\,x)}+3\,b^3e^{(-5\,x)}+2\,a^3+5\,ab^2+6\,(a^3+2\,ab^2)e^{(-2\,x)}+2\,(2\,a^2b+5\,b^3)e^{(-3\,x)}\right)}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="maxima")

[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(-x) + 3*a*b^2*e^(-4*x) + 3*b^3*e^(-5*x) + 2*a^3 + 5*a*b^2 + 6*(a^3 + 2*a*b^2)*e^(-2*x) + 2*(2*a^2*b + 5*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-6*x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(92) = 184$.
time = 0.39, size = 1142, normalized size = 11.42

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="fricas")

[Out] 1/3*(6*(a^2*b^3 + b^5)*cosh(x)^5 + 6*(a^2*b^3 + b^5)*sinh(x)^5 - 4*a^5 - 14*a^3*b^2 - 10*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x)^3 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 + b^5)*cosh(x)^2 - 6*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x)^3 + 3*(a^3*

```
b^2 + a*b^4)*cosh(x)^2 - (2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 +
 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)
^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4
*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^
2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh
(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) + 6*(a^2*b^3 + b^5)*cosh(x) + 6*(a^2*b^3 + b^5 + 5*(a^2*b^
3 + b^5)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(2*a^4*b + 7*a^2*b^3
 + 5*b^5)*cosh(x)^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^6 +
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x)), x)
```

Giac [**A**]
time = 0.42, size = 180, normalized size = 1.80

$$\frac{b^4 \log\left(\frac{\left|2\, b e^x + 2\, a - 2\, \sqrt{a^2 + b^2}\right|}{\left|2\, b e^x + 2\, a + 2\, \sqrt{a^2 + b^2}\right|}\right)}{(a^4 + 2\, a^2 b^2 + b^4)\sqrt{a^2 + b^2}} + \frac{2\, \left(3\, b^3 e^{(5\, x)} - 3\, a b^2 e^{(4\, x)} + 4\, a^2 b e^{(3\, x)} + 10\, b^3 e^{(3\, x)} - 6\, a^3 e^{(2\, x)} - 12\, a b^2 e^{(2\, x)} + 3\, b^3 e^x - 2\, a^3 - 5\, a b^2\right)}{3\, (a^4 + 2\, a^2 b^2 + b^4)(e^{(2\, x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(5*x)
```

```
- 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) + 10*b^3*e^(3*x) - 6*a^3*e^(2*x) - 12*a
*b^2*e^(2*x) + 3*b^3*e^x - 2*a^3 - 5*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*
x) + 1)^3)
```

Mupad [B]
time = 1.61, size = 634, normalized size = 6.34

$$\frac{\frac{2\,b^3\,{\mathrm{e}}^x}{{\left(a^2+b^2\right)}^2}-\frac{2\,a\,b^2}{{\left(a^2+b^2\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{4\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}-\frac{8\,{\mathrm{e}}^x\,\left(a^2\,b+b^3\right)}{3\,{\left(a^2+b^2\right)}^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}+\frac{\frac{8\,a}{3\,\left(a^2+b^2\right)}-\frac{8\,b\,{\mathrm{e}}^x}{3\,\left(a^2+b^2\right)}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\left({\mathrm{e}}^x\,\left(\frac{2\,b^2}{\sqrt{b^8}\,{\left(a^2+b^2\right)}^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,\left(a^5\,\sqrt{b^8}+2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-{\left(a^2+b^2\right)}^5}\,\left(a^4+2\,a^2\,b^2+b^4\right)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right)-\frac{2\,a\,\left(b^5\,\sqrt{b^8}+2\,a^2\,b^3\,\sqrt{b^8}+a^4\,b\,\sqrt{b^8}\right)}{b^6\,\sqrt{-{\left(a^2+b^2\right)}^5}\,\left(a^4+2\,a^2\,b^2+b^4\right)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right)\,\left(\frac{b^5\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}{2}+\frac{a^4\,b\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}{2}+a^2\,b^3\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}\right)\right)\,\sqrt{b^8}}{\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^3*exp(x))/(a^2 + b^2)^2 - (2*a*b^2)/(a^2 + b^2)^2)/(exp(2*x) + 1) - (
(4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2
))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) - (8*b*exp(x))/(3*(
a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (2*atan((exp(x)*((2
*b^2)/((b^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(a^5*(b^8)
^(1/2) + 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^
(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*
a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*a*(b^5*(b^8)^(1/2) + 2*a^2*b^3*(b^8)^(1/2
) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)
*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*
((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/
2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8
*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b
^4 - 5*a^8*b^2)^(1/2)))*(b^8)^(1/2))/(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^
6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

3.198 $\int \frac{\text{sech}^5(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=135

$$\frac{a(3a^4+10a^2b^2+15b^4)\,\text{ArcTan}(\sinh(x))}{8\,(a^2+b^2)^3}-\frac{b^5\log(\cosh(x))}{(a^2+b^2)^3}+\frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{\text{sech}^4(x)(b+a\sinh(x))}{4\,(a^2+b^2)}+$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))/(a^2+b^2)^3-b^5*ln(cosh(x))
/(a^2+b^2)^3+b^5*ln(a+b*sinh(x))/(a^2+b^2)^3+1/4*sech(x)^4*(b+a*sinh(x))/(a
^2+b^2)+1/8*sech(x)^2*(4*b^3+a*(3*a^2+7*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2747, 755, 837, 815, 649, 209, 266}

$$\frac{\text{sech}^4(x)(a\sinh(x)+b)}{4\,(a^2+b^2)}+\frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3}-\frac{b^5\log(\cosh(x))}{(a^2+b^2)^3}+\frac{\text{sech}^2(x)\,(a(3a^2+7b^2)\sinh(x)+4b^3)}{8\,(a^2+b^2)^2}+\frac{a(3a^4+10a^2b^2+15b^4)\,\text{ArcTan}(\sinh(x))}{8\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*(a^2 + b^2)^3) - (b^5*
Log[Cosh[x]])/(a^2 + b^2)^3 + (b^5*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (Sec
h[x]^4*(b + a*Sinh[x]))/(4*(a^2 + b^2)) + (Sech[x]^2*(4*b^3 + a*(3*a^2 + 7*
b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = -\left(b^5\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\sinh(x)\right)\right)$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{b^3\text{Subst}\left(\int \frac{3a^2+4b^2+3ax}{(a+x)(-b^2-x^2)^2}\,dx, x, b\sinh(x)\right)}{4\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b\text{Subst}\left(\int \frac{-3a^4-7a^2}{(a}\right.}{}$$

$$= \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2} + \frac{b\text{Subst}\left(\int \left(\frac{8b^4}{(a^2+b^2)}\right.\right.}{}$$

$$= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x\right.}{8\left(a^2+b^2\right)^2}$$

$$= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x\right.}{8\left(a^2+b^2\right)^2}$$

$$= \frac{a(3a^4+10a^2b^2+15b^4)\tan^{-1}(\sinh(x))}{8\left(a^2+b^2\right)^3} - \frac{b^5\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{\text{se}}{}$$

Mathematica [A]
time = 0.17, size = 135, normalized size = 1.00

$$\frac{(6a^5+20a^3b^2+30ab^4)\operatorname{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+8b^5(-\log(\cosh(x))+\log(a+b\sinh(x)))+4b^3(a^2+b^2)\operatorname{sech}^2(x)+2b(a^2+b^2)^2\operatorname{sech}^4(x)+a(3a^4+10a^2b^2+7b^4)\operatorname{sech}(x)\tanh(x)+2a(a^2+b^2)^2\operatorname{sech}^3(x)\tanh(x)}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^5 + 20*a^3*b^2 + 30*a*b^4)*ArcTan[Tanh[x/2]] + 8*b^5*(-Log[Cosh[x]] +
 Log[a + b*Sinh[x]]) + 4*b^3*(a^2 + b^2)*Sech[x]^2 + 2*b*(a^2 + b^2)^2*Sech
[x]^4 + a*(3*a^4 + 10*a^2*b^2 + 7*b^4)*Sech[x]*Tanh[x] + 2*a*(a^2 + b^2)^2*
Sech[x]^3*Tanh[x])/(8*(a^2 + b^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(129) = 258$.
time = 0.70, size = 313, normalized size = 2.32

method	result
default	$\frac{2\left(\left(-\frac{5}{8}a^5-\frac{7}{4}a^3b^2-\frac{9}{8}a\,b^4\right)\left(\tanh^7\left(\frac{x}{2}\right)\right)+\left(-a^4b-3a^2b^3-2b^5\right)\left(\tanh^6\left(\frac{x}{2}\right)\right)+\left(\frac{3}{8}a^5+\frac{1}{4}a^3b^2-\frac{1}{8}a\,b^4\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-2a^2b^3-2b^5\right)\left(\tanh^4\left(\frac{x}{2}\right)\right)+\left(\right.\right.}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}$

risch	$\frac{(3a^3e^{6x}+7a\,b^2e^{6x}+8b^3e^{5x}+11a^3e^{4x}+15a\,b^2e^{4x}+16a^2b\,e^{3x}+32b^3e^{3x}-11a^3e^{2x}-15a\,b^2e^{2x}+8b^3e^x-3a^3-7a\,b^2)e^x}{4(a^4+2a^2b^2+b^4)(1+e^{2x})^4}+\frac{3i\ln(e^x+i}{8(a^6+3a^4b^2+3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*(((-5/8*a^5-7/4*a^3*b^2-9/8*a*b^4)*tanh(1/2
*x)^7+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^6+(3/8*a^5+1/4*a^3*b^2-1/8*a*b^4
)*tanh(1/2*x)^5+(-2*a^2*b^3-2*b^5)*tanh(1/2*x)^4+(-3/8*a^5-1/4*a^3*b^2+1/8*
a*b^4)*tanh(1/2*x)^3+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^2+(5/8*a^5+7/4*a^
3*b^2+9/8*a*b^4)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^4-1/2*b^5*ln(tanh(1/2*x)^2+
1)+1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tanh(1/2*x)))+b^5/(a^6+3*a^4*b^2+
3*a^2*b^4+b^6)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(129) = 258$.
time = 0.55, size = 345, normalized size = 2.56

$$\frac{b^5\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{b^5\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\arctan\left(e^{(-x)}\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+\left(3\,a^3+7\,ab^2\right)e^{(-x)}+\left(11\,a^3+15\,ab^2\right)e^{(-3\,x)}+16\left(a^2b+2\,b^3\right)e^{(-4\,x)}-\left(11\,a^3+15\,ab^2\right)e^{(-5\,x)}-\left(3\,a^3+7\,ab^2\right)e^{(-7\,x)}}{4\left(a^4+2\,a^2b^2+b^4+4\left(a^4+2\,a^2b^2+b^4\right)e^{(-2\,x)}+6\left(a^4+2\,a^2b^2+b^4\right)e^{(-4\,x)}+4\left(a^4+2\,a^2b^2+b^4\right)e^{(-6\,x)}+\left(a^4+2\,a^2b^2+b^4\right)e^{(-8\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 b^5*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^5 + 1
0*a^3*b^2 + 15*a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 + 7*a*b^2)*e^(-x) + (11*a^3 +
 15*a*b^2)*e^(-3*x) + 16*(a^2*b + 2*b^3)*e^(-4*x) - (11*a^3 + 15*a*b^2)*e^(
-5*x) - (3*a^3 + 7*a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(129) = 258$.
time = 0.52, size = 2707, normalized size = 20.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + (3*a^5 + 10*a^3*b^2 + 7*a*b
^4)*sinh(x)^7 + 8*(a^2*b^3 + b^5)*cosh(x)^6 + (8*a^2*b^3 + 8*b^5 + 7*(3*a^5
 + 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 26*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + (11*a^5 + 26*a^3*b^2 + 15*a*b^4 + 21*(3*a^5 + 10*a^3*b^2 +
```

7*a*b^4)*cosh(x)^2 + 48*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 + 16*(a^4*b + 3*
a^2*b^3 + 2*b^5)*cosh(x)^4 + (16*a^4*b + 48*a^2*b^3 + 32*b^5 + 35*(3*a^5 +
10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 + b^5)*cosh(x)^2 + 5*(11*a^5
 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 26*a^3*b^2 + 15*a*
b^4)*cosh(x)^3 - (11*a^5 + 26*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 +
 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 + b^5)*cosh(x)^3 - 10*(11*a^5 + 26*a^3*b
^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^
3 + 8*(a^2*b^3 + b^5)*cosh(x)^2 + (21*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x
)^5 + 8*a^2*b^3 + 8*b^5 + 120*(a^2*b^3 + b^5)*cosh(x)^4 + 10*(11*a^5 + 26*a
^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3
*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^
7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*
a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10
*a^3*b^2 + 15*a*b^4 + 6*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^
5 + 30*a^3*b^2 + 45*a*b^4 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 +
30*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*
a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^
3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
6 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^4 + 9*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^5 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (3*a^5 + 10*a^3*b^
2 + 7*a*b^4)*cosh(x) + 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sin
h(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cos
h(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b
^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*c
osh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6
 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 +
 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) - 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b
^5*sinh(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b
^5*cosh(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)
^5 + b^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7
*b^5*cosh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cos
h(x)^6 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(
x)^7 + 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + (7*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48
*(a^2*b^3 + b^5)*cosh(x)^5 - 3*a^5 - 10*a^3*b^2 - 7*a*b^4 + 5*(11*a^5 + 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^3 -
3*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 + b^5)*cosh(x))*

```
sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2
)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*
sinh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
)^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^7 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*sinh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(129) = 258.
time = 0.43, size = 369, normalized size = 2.73

$$\frac{b^6\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^6b+3a^4b^3+3a^2b^5+b^7}-\frac{b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3a^5+10a^3b^2+15ab^4\right)}{16\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{3b^5\left(e^{(-x)}-e^x\right)^4-3a^5\left(e^{(-x)}-e^x\right)^3-10a^3b^2\left(e^{(-x)}-e^x\right)^3-7ab^4\left(e^{(-x)}-e^x\right)^3+8a^2b^3\left(e^{(-x)}-e^x\right)^2+32b^5\left(e^{(-x)}-e^x\right)^2-20a^5\left(e^{(-x)}-e^x\right)-56a^3b^2\left(e^{(-x)}-e^x\right)-36ab^4\left(e^{(-x)}-e^x\right)+16a^4b+64a^2b^3+96b^5}{4\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(\left(e^{(-x)}-e^x\right)^2+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
 - 1/2*b^5*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b
^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(3*b^5*(e^(-x) - e^x)^4 - 3*a
^5*(e^(-x) - e^x)^3 - 10*a^3*b^2*(e^(-x) - e^x)^3 - 7*a*b^4*(e^(-x) - e^x)^
3 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 32*b^5*(e^(-x) - e^x)^2 - 20*a^5*(e^(-x) -
 e^x) - 56*a^3*b^2*(e^(-x) - e^x) - 36*a*b^4*(e^(-x) - e^x) + 16*a^4*b + 64
```

```
*a^2*b^3 + 96*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2 +
 4)^2)
```

Mupad [B]
time = 4.89, size = 548, normalized size = 4.06

$$\frac{\frac{2\,\left(2\,a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}-\frac{{\mathrm{e}}^x\,\left(3\,a\,b^2-a^3\right)}{2\,{\left(a^2+b^2\right)}^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{\frac{8\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}+\frac{6\,{\mathrm{e}}^x\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}+\frac{\frac{4\,b}{a^2+b^2}+\frac{4\,a\,{\mathrm{e}}^x}{a^2+b^2}}{4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1}+\frac{\frac{2\,\left(a^2\,b^3+b^5\right)}{{\left(a^2+b^2\right)}^3}+\frac{{\mathrm{e}}^x\,\left(3\,a^5+10\,a^3\,b^2+7\,a\,b^4\right)}{4\,{\left(a^2+b^2\right)}^3}}{{\mathrm{e}}^{2\,x}+1}+\frac{b^5\,\ln\left(256\,b^{11}\,{\mathrm{e}}^{2\,x}-9\,a^{10}\,b-256\,b^{11}-225\,a^2\,b^9-300\,a^4\,b^7-190\,a^6\,b^5-60\,a^8\,b^3+18\,a^{11}\,{\mathrm{e}}^x+225\,a^2\,b^9\,{\mathrm{e}}^{2\,x}+300\,a^4\,b^7\,{\mathrm{e}}^{2\,x}+190\,a^6\,b^5\,{\mathrm{e}}^{2\,x}+60\,a^8\,b^3\,{\mathrm{e}}^{2\,x}+512\,a\,b^{10}\,{\mathrm{e}}^x+9\,a^{10}\,b\,{\mathrm{e}}^{2\,x}+450\,a^3\,b^8\,{\mathrm{e}}^x+600\,a^5\,b^6\,{\mathrm{e}}^x+380\,a^7\,b^4\,{\mathrm{e}}^x+120\,a^9\,b^2\,{\mathrm{e}}^x\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,\left(-a^2\,3{}\mathrm{i}+9\,a\,b+b^2\,8{}\mathrm{i}\right)}{8\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}-\frac{\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,9{}\mathrm{i}+8\,b^2\right)}{8\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^5*(a + b*sinh(x))),x)
```

```
[Out] ((2*(2*a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(3*a*b^2 - a^3))/(2*(a^2 + b^2
)^2))/(2*exp(2*x) + exp(4*x) + 1) - ((8*(a^2*b + b^3))/(a^2 + b^2)^2 + (6*e
xp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1
) + ((4*b)/(a^2 + b^2) + (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((2*(b^5 + a^2*b^3))/(a^2 + b^2)^3 + (exp(x
)*(7*a*b^4 + 3*a^5 + 10*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (b^5*
log(256*b^11*exp(2*x) - 9*a^10*b - 256*b^11 - 225*a^2*b^9 - 300*a^4*b^7 - 1
90*a^6*b^5 - 60*a^8*b^3 + 18*a^11*exp(x) + 225*a^2*b^9*exp(2*x) + 300*a^4*b
^7*exp(2*x) + 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b^10*exp(x
) + 9*a^10*b*exp(2*x) + 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) + 380*a^7*b
^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (log
(exp(x)*1i + 1)*(9*a*b - a^2*3i + b^2*8i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b
^3*1i)) - (log(exp(x) + 1i)*(a*b*9i - 3*a^2 + 8*b^2))/(8*(a*b^2*3i - 3*a^2*
b - a^3*1i + b^3))
```

3.199 $\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\left(a^2+b^2\right)}+\frac{\operatorname{sech}^3(x)\left(5b^3+a\left(4a^2+9b^2\right)\sinh(x)\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(15b^5+\dots\right.}{}$$

```
[Out] -2*b^6*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+1/5*sech(
x)^5*(b+a*sinh(x))/(a^2+b^2)+1/15*sech(x)^3*(5*b^3+a*(4*a^2+9*b^2)*sinh(x))
/(a^2+b^2)^2+1/15*sech(x)*(15*b^5+a*(8*a^4+26*a^2*b^2+33*b^4)*sinh(x))/(a^2
+b^2)^3
```

Rubi [A]

time = 0.30, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2775, 2945, 12, 2739, 632, 212}

$$\frac{\operatorname{sech}^5(x)(a\sinh(x)+b)}{5\left(a^2+b^2\right)}-\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}^3(x)\left(a\left(4a^2+9b^2\right)\sinh(x)+5b^3\right)}{15\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(a\left(8a^4+26a^2b^2+33b^4\right)\sinh(x)+15b^5\right)}{15\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^6*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (Se
ch[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + (Sech[x]^3*(5*b^3 + a*(4*a^2 + 9
*b^2)*Sinh[x]))/(15*(a^2 + b^2)^2) + (Sech[x]*(15*b^5 + a*(8*a^4 + 26*a^2*b
^2 + 33*b^4)*Sinh[x]))/(15*(a^2 + b^2)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} - \frac{\int \frac{\operatorname{sech}^4(x)(-4a^2-5b^2-4ab\sinh(x))}{a+b\sinh(x)}\,dx}{5(a^2+b^2)}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\int \frac{\operatorname{sech}^2(x)(8a^4+18a^2b^2}{a-}}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8\right.}{15}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8\right.}{15}$$

$$= -\frac{2b^6\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\,\mathrm{s}\right.}{15(a^2+b^2)^2}$$

Mathematica [A]
time = 0.33, size = 146, normalized size = 1.00

$$\frac{\frac{30b^6\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 15b^5\operatorname{sech}(x) + 3(a^2+b^2)^2\operatorname{sech}^5(x)(b+a\sinh(x)) + (a^2+b^2)\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right) + a(8a^4+26a^2b^2+33b^4)\tanh(x)}{15(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] ((30*b^6*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 15*
b^5*Sech[x] + 3*(a^2 + b^2)^2*Sech[x]^5*(b + a*Sinh[x]) + (a^2 + b^2)*Sech[
x]^3*(5*b^3 + a*(4*a^2 + 9*b^2)*Sinh[x]) + a*(8*a^4 + 26*a^2*b^2 + 33*b^4)*
Tanh[x])/(15*(a^2 + b^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(134) = 268.
time = 0.55, size = 350, normalized size = 2.40

method	result

default	$\frac{2b^6 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(\left(-a^5-3a^3b^2-3a\,b^4\right)\left(\tanh^9\left(\frac{x}{2}\right)\right)+\left(-a^4b-3a^2b^3-3b^5\right)\left(\tanh^8\left(\frac{x}{2}\right)\right)+\left(-\frac{4}{3}a^5-\frac{16}{3}a^3b^2-8\right.\right.}{}$
risch	$-\frac{2\left(-15b^5e^{9x}+15a\,b^4e^{8x}-20a^2b^3e^{7x}-80b^5e^{7x}+30a^3b^2e^{6x}+90a\,b^4e^{6x}-48a^4b\,e^{5x}-136a^2b^3e^{5x}-178b^5e^{5x}+80a^5e^{4x}+230a^3b^2e^{4x}+\right.}{15(a^6+3a^4b^2+3a^2b^4+b^6)(1+e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^5-3*a^3*b^
2-3*a*b^4)*tanh(1/2*x)^9+(-a^4*b-3*a^2*b^3-3*b^5)*tanh(1/2*x)^8+(-4/3*a^5-1
6/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^7+(-2*a^2*b^3-6*b^5)*tanh(1/2*x)^6+(-58/15
*a^5-166/15*a^3*b^2-66/5*a*b^4)*tanh(1/2*x)^5+(-2*a^4*b-16/3*a^2*b^3-28/3*b
^5)*tanh(1/2*x)^4+(-4/3*a^5-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^3+(-2/3*a^2*b
^3-14/3*b^5)*tanh(1/2*x)^2+(-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)-1/5*a^4*b-1
1/15*a^2*b^3-23/15*b^5)/(tanh(1/2*x)^2+1)^5
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(136) = 272$.
time = 0.49, size = 438, normalized size = 3.00

$$\frac{b^6\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,b^5e^{(-x)}+15\,ab^4e^{(-8\,x)}+15\,b^5e^{(-9\,x)}+8\,a^5+26\,a^3b^2+33\,ab^4+10\,(4\,a^5+13\,a^3b^2+15\,ab^4)e^{(-2\,x)}+20\,(a^2b^3+4\,b^5)e^{(-3\,x)}+10\,(8\,a^5+23\,a^3b^2+24\,ab^4)e^{(-4\,x)}+2\,(24\,a^4b+68\,a^2b^3+89\,b^5)e^{(-5\,x)}+30\,(a^3b^2+3\,ab^4)e^{(-6\,x)}+20\,(a^2b^3+4\,b^5)e^{(-7\,x)}\right)}{15\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6+5\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-2\,x)}+10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-4\,x)}+10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-6\,x)}+5\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-8\,x)}+(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-10\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^6*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(15*b^5*e^(-x)
 + 15*a*b^4*e^(-8*x) + 15*b^5*e^(-9*x) + 8*a^5 + 26*a^3*b^2 + 33*a*b^4 + 10
*(4*a^5 + 13*a^3*b^2 + 15*a*b^4)*e^(-2*x) + 20*(a^2*b^3 + 4*b^5)*e^(-3*x) +
 10*(8*a^5 + 23*a^3*b^2 + 24*a*b^4)*e^(-4*x) + 2*(24*a^4*b + 68*a^2*b^3 + 8
9*b^5)*e^(-5*x) + 30*(a^3*b^2 + 3*a*b^4)*e^(-6*x) + 20*(a^2*b^3 + 4*b^5)*e^
(-7*x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*e^(-2*x) + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-4*x) + 10*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-6*x) + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*e^(-8*x) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-10*x))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3175 vs. $2(136) = 272$.
time = 0.41, size = 3175, normalized size = 21.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="fricas")

[Out] 1/15*(30*(a^2*b^5 + b^7)*cosh(x)^9 + 30*(a^2*b^5 + b^7)*sinh(x)^9 - 30*(a^3*b^4 + a*b^6)*cosh(x)^8 - 30*(a^3*b^4 + a*b^6 - 9*(a^2*b^5 + b^7)*cosh(x))*sinh(x)^8 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7 + 27*(a^2*b^5 + b^7)*cosh(x)^2 - 6*(a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^7 - 16*a^7 - 68*a^5*b^2 - 118*a^3*b^4 - 66*a*b^6 - 60*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 + 12*a^3*b^4 + 9*a*b^6 - 126*(a^2*b^5 + b^7)*cosh(x)^3 + 42*(a^3*b^4 + a*b^6)*cosh(x)^2 - 14*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^5 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7 + 945*(a^2*b^5 + b^7)*cosh(x)^4 - 420*(a^3*b^4 + a*b^6)*cosh(x)^3 + 210*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x))*sinh(x)^5 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^4 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6 - 189*(a^2*b^5 + b^7)*cosh(x)^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^4 - 70*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 + b^7)*cosh(x)^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^5 + 35*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^4 - 30*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^2 - 2*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x))*sinh(x)^3 - 20*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^4 + 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 + b^7)*cosh(x)^7 - 4*a^7 - 17*a^5*b^2 - 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^6 + 42*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^4 + 2*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^3 - 6*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^2 + 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9 + b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 + 10*b^6*cosh(x)^4 + 5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(x)^4 + 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5 + 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 30*(a^2*b^5 + b^7)*cosh(x) + 10*(27*(a^2*b^5 + b^7)*cosh(x)^8 - 24*(a^3*b^4 + a*b^6)*cosh(x)^7 + 3*a^2*b^5 + 3*b^7 + 28*(a^4*b^3 + 5*a^2*b^5 + 4*b^

```
7)*cosh(x)^6 - 36*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^4 - 8*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2
- 4*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^4 + 15*a*b^6)*cosh(x))*sinh(x))/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 + 4*a^6*b^2 +
 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*sinh(x)^10 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8)*cosh(x)^8 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 9
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 + a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 40*(3*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*cosh(x)^6 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21*(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 14*(a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 4*a^6
*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 70*(a^8 + 4*a^6*b^2 + 6*a^4
*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*cosh(x))*sinh(x)^5 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*cosh(x)^4 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21
*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 15*(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 7*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(
x)^3 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**6/(a + b*sinh(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(136) = 272.
time = 0.44, size = 323, normalized size = 2.21

$$\frac{b^6\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\,\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}+80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}-90\,ab^4e^{(6\,x)}+48\,a^4be^{(5\,x)}+136\,a^2b^3e^{(5\,x)}+178\,b^5e^{(5\,x)}-80\,a^5e^{(4\,x)}-230\,a^3b^2e^{(4\,x)}-240\,ab^4e^{(4\,x)}+20\,a^2b^3e^{(3\,x)}+80\,b^5e^{(3\,x)}-40\,a^5e^{(2\,x)}-130\,a^3b^2e^{(2\,x)}-150\,ab^4e^{(2\,x)}+15\,b^5e^x-8\,a^5-26\,a^3b^2-33\,ab^4\right)}{15\,\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(e^{(2\,x)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(1
5*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b^3*e^(7*x) + 80*b^5*e^(7*x) - 30
*a^3*b^2*e^(6*x) - 90*a*b^4*e^(6*x) + 48*a^4*b*e^(5*x) + 136*a^2*b^3*e^(5*x
) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) - 230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4
*x) + 20*a^2*b^3*e^(3*x) + 80*b^5*e^(3*x) - 40*a^5*e^(2*x) - 130*a^3*b^2*e^
(2*x) - 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*a^5 - 26*a^3*b^2 - 33*a*b^4)/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^5)
```

Mupad [B]
time = 2.20, size = 1010, normalized size = 6.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^6*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^5*exp(x))/(a^2 + b^2)^3 - (2*a*b^4)/(a^2 + b^2)^3)/(exp(2*x) + 1) - (
(8*(3*a*b^2 + 4*a^3))/(3*(a^2 + b^2)^2) - (8*exp(x)*(12*a^2*b + 7*b^3))/(15
*(a^2 + b^2)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a*b^4 + a^
3*b^2))/(a^2 + b^2)^3 - (8*exp(x)*(b^5 + a^2*b^3))/(3*(a^2 + b^2)^3))/(2*ex
p(2*x) + exp(4*x) + 1) - ((32*a)/(5*(a^2 + b^2)) - (32*b*exp(x))/(5*(a^2 +
b^2)))/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1
) + ((16*(a*b^2 + a^3))/(a^2 + b^2)^2 - (64*exp(x)*(a^2*b + b^3))/(5*(a^2 +
 b^2)^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) - (2*atan((
exp(x)*((2*b^4)/((b^12)^(1/2)*(a^2 + b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) + 3*a^5*b^2*(b^12)^
(1/2) + a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12)^(1/2) + 3
*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) + a^6*b*(b^12)^(1/2)))/(b^8*
(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(- a^14 - b^14 -
 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*
b^2)^(1/2)))*((b^7*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (3*a^2*b^5*(- a^14 - b^1
4 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^
12*b^2)^(1/2))/2 + (3*a^4*b^3*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 3
5*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (a^6*b*(- a^1
4 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4
 - 7*a^12*b^2)^(1/2))/2))*(b^12)^(1/2))/(- a^14 - b^14 - 7*a^2*b^12 - 21*a^
4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2)
```

3.200 $\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=94

$$\frac{3(2a^2+b^2)\,x}{2b^4}+\frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

```
[Out] 3/2*(2*a^2+b^2)*x/b^4-3/2*cosh(x)*(2*a-b*sinh(x))/b^3-cosh(x)^3/b/(a+b*sinh
(x))+6*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^4
```

Rubi [A]
time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2772, 2944, 2814, 2739, 632, 212}

$$\frac{3x(2a^2+b^2)}{2b^4}+\frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*x)/(2*b^4) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/b^4 - (3*Cosh[x]*(2*a - b*Sinh[x]))/(2*b^3) - Cosh[x]^3
/(b*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{3\int \frac{\cosh^2(x)\sinh(x)}{a+b\sinh(x)}\,dx}{b}$$

$$= -\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(3i)\int \frac{iab-i(2a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= \frac{3(2a^2+b^2)\,x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(3a(a^2+b^2))\int \frac{}{a+}}{b^4}$$

$$= \frac{3(2a^2+b^2)\,x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} - \frac{(6a(a^2+b^2))\,\mathrm{Subs}}{}$$

$$= \frac{3(2a^2+b^2)\,x}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))} + \frac{(12a(a^2+b^2))\,\mathrm{Sul}}{}$$

$$= \frac{3(2a^2+b^2)\,x}{2b^4} + \frac{6a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.26, size = 660, normalized size = 7.02

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]^3*(12*a*Sqrt[a - I*b]*(a + I*b)^(3/2)*ArcTanh[Sqrt[-((b*(I + Sinh[
x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]
*(a + b*Sinh[x]) - 12*a*(a^2 + b^2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + S
inh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])
]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[
x]))/(a + I*b))]*(6*(-1)^(3/4)*a*Sqrt[b]*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2
 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 6*(-
1)^(3/4)*b^(3/2)*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sq
rt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sinh[x] - 2*Sqrt[a - I*b]*(3*a
^3 + (3*I)*a^2*b + a*b^2 + I*b^3)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))] - 3*a*Sqrt[a - I*b]*(a + I*b)*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]
*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + Sqrt[a - I*b]*(a + I*b)*b^2*Sqrt[1
+ I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(2*(a - I*b)
^(3/2)*(a + I*b)^(5/2)*b*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b
)))^(3/2)*(-((b*(I + Sinh[x]))/(a - I*b)))^(3/2)*(a + b*Sinh[x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(84) = 168$.
time = 0.63, size = 205, normalized size = 2.18

method	result
risch	$\frac{3x\,a^2}{b^4}+\frac{3x}{2b^2}+\frac{\mathrm{e}^{2x}}{8b^2}-\frac{a\,\mathrm{e}^{x}}{b^3}-\frac{a\,\mathrm{e}^{-x}}{b^3}-\frac{\mathrm{e}^{-2x}}{8b^2}+\frac{2(a^2+b^2)(a\,\mathrm{e}^{x}-b)}{b^4(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}+\frac{3\sqrt{a^2+b^2}\,a\ln\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{b^4}-\frac{3\sqrt{a^2}}{}$
default	$\frac{1}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-b-4a}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-6a^2-3b^2)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^4}+\frac{\frac{2\left(\frac{(a^2+b^2)b^2\tanh\left(\frac{x}{2}\right)}{a}+b(a^2+b^2)\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-6a\sqrt{a^2+b^2}\ \mathrm{a}}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2/(tanh(1/2*x)-1)^2-1/2*(-b-4*a)/b^3/(tanh(1/2*x)-1)+1/2/b^4*(-6*a^2-
3*b^2)*ln(tanh(1/2*x)-1)+2/b^4*(((a^2+b^2)*b^2/a*tanh(1/2*x)+b*(a^2+b^2))/(
a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-3*a*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan
h(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/2/b^2/(tanh(1/2*x)+1)^2-1/2*(-b+4*a)/b^3/
(tanh(1/2*x)+1)+1/2/b^4*(6*a^2+3*b^2)*ln(tanh(1/2*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(85) = 170$.

time = 0.48, size = 176, normalized size = 1.87

$$-\frac{6\,ab^2e^{(-x)}-b^3+(32\,a^2b+17\,b^3)e^{(-2\,x)}+8\,(2\,a^3+ab^2)e^{(-3\,x)}}{8\,(b^5e^{(-2\,x)}+2\,ab^4e^{(-3\,x)}-b^5e^{(-4\,x)})}-\frac{3\,\sqrt{a^2+b^2}\,a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^4}-\frac{8\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^3}+\frac{3\,(2\,a^2+b^2)x}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(6*a*b^2*e^(-x) - b^3 + (32*a^2*b + 17*b^3)*e^(-2*x) + 8*(2*a^3 + a*b^
2)*e^(-3*x))/(b^5*e^(-2*x) + 2*a*b^4*e^(-3*x) - b^5*e^(-4*x)) - 3*sqrt(a^2
+ b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^
2)))/b^4 - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b^3 + 3/2*(2*a^2 + b^2)*x/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(85) = 170$.

time = 0.44, size = 833, normalized size = 8.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 6*a*b^2*cosh(x)^5 + 6*(b^3*cosh(x) - a
*b^2)*sinh(x)^5 - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^4 + (15*b
^3*cosh(x)^2 - 30*a*b^2*cosh(x) - 16*a^2*b - b^3 + 12*(2*a^2*b + b^3)*x)*si
```

```
nh(x)^4 + 6*a*b^2*cosh(x) + 8*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(
x)^3 + 4*(5*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 + 4*a^3 + 4*a*b^2 + 6*(2*a^3
 + a*b^2)*x - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x)^3 +
b^3 - (32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x)^2 + (15*b^3*cosh(x
)^4 - 60*a*b^2*cosh(x)^3 - 32*a^2*b - 17*b^3 - 6*(16*a^2*b + b^3 - 12*(2*a^
2*b + b^3)*x)*cosh(x)^2 - 12*(2*a^2*b + b^3)*x + 24*(2*a^3 + 2*a*b^2 + 3*(2
*a^3 + a*b^2)*x)*cosh(x))*sinh(x)^2 + 24*(a*b*cosh(x)^4 + a*b*sinh(x)^4 + 2
*a^2*cosh(x)^3 - a*b*cosh(x)^2 + 2*(2*a*b*cosh(x) + a^2)*sinh(x)^3 + (6*a*b
*cosh(x)^2 + 6*a^2*cosh(x) - a*b)*sinh(x)^2 + 2*(2*a*b*cosh(x)^3 + 3*a^2*co
sh(x)^2 - a*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*si
nh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*
a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(3*b^3*cosh(x)^5 - 15*a*b^2
*cosh(x)^4 - 2*(16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^3 + 3*a*b^2
+ 12*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x)^2 - (32*a^2*b + 17*b^3
 + 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 +
 2*a*b^4*cosh(x)^3 - b^5*cosh(x)^2 + 2*(2*b^5*cosh(x) + a*b^4)*sinh(x)^3 +
(6*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) - b^5)*sinh(x)^2 + 2*(2*b^5*cosh(x)^3 +
3*a*b^4*cosh(x)^2 - b^5*cosh(x))*sinh(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(85) = 170.
time = 0.41, size = 178, normalized size = 1.89

$$\frac{3\,(2\,a^2+b^2)x}{2\,b^4}-\frac{3\,(a^3+ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4}+\frac{\left(6\,ab^2e^x+b^3+8\,(2\,a^3+ab^2)e^{(3\,x)}-(32\,a^2b+17\,b^3)e^{(2\,x)}\right)e^{(-2\,x)}}{8\,(be^{(2\,x)}+2\,ae^x-b)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(2*a^2 + b^2)*x/b^4 - 3*(a^3 + a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1
/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(6*a*b^2*e^x + b^3 + 8*(2*a^3 + a*b^
2)*e^(3*x) - (32*a^2*b + 17*b^3)*e^(2*x))*e^(-2*x)/((b*e^(2*x) + 2*a*e^x -
b)*b^4)
```

Mupad [B]
time = 0.88, size = 256, normalized size = 2.72

$$\frac{e^{2x}}{8b^2}-\frac{e^{-2x}}{8b^2}-\frac{\frac{2\left(a^4b^2+2a^2b^4+b^6\right)}{b^4\left(a^2b+b^3\right)}-\frac{2e^x\left(a^5b^2+2a^3b^4+ab^6\right)}{b^5\left(a^2b+b^3\right)}}{2ae^x-b+be^{2x}}+\frac{x\left(6a^2+3b^2\right)}{2b^4}-\frac{ae^x}{b^3}-\frac{ae^{-x}}{b^3}-\frac{3a\ln\left(\frac{6ae^x\left(a^2+b^2\right)}{b^5}-\frac{6a\left(b-ae^x\right)\sqrt{a^2+b^2}}{b^5}\right)\sqrt{a^2+b^2}}{b^4}+\frac{3a\ln\left(\frac{6a\left(b-ae^x\right)\sqrt{a^2+b^2}}{b^5}+\frac{6ae^x\left(a^2+b^2\right)}{b^5}\right)\sqrt{a^2+b^2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*(b^6 + 2*a^2*b^4 + a^4*b^2))/(b^
4*(a^2*b + b^3)) - (2*exp(x)*(a*b^6 + 2*a^3*b^4 + a^5*b^2))/(b^5*(a^2*b + b
^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 + 3*b^2))/(2*b^4) - (a*exp(
x))/b^3 - (a*exp(-x))/b^3 - (3*a*log((6*a*exp(x)*(a^2 + b^2))/b^5 - (6*a*(b
 - a*exp(x))*(a^2 + b^2)^(1/2))/b^5)*(a^2 + b^2)^(1/2))/b^4 + (3*a*log((6*a
*(b - a*exp(x))*(a^2 + b^2)^(1/2))/b^5 + (6*a*exp(x)*(a^2 + b^2))/b^5)*(a^2
 + b^2)^(1/2))/b^4
```

3.201 $\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=40

$$-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$$

```
[Out] -2*a*ln(a+b*sinh(x))/b^3+sinh(x)/b^2+(-a^2-b^2)/b^3/(a+b*sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$-\frac{a^2+b^2}{b^3(a+b\sinh(x))}-\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Sinh[x]])/b^3 + Sinh[x]/b^2 - (a^2 + b^2)/(b^3*(a + b*Sinh[
x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^2}\,dx,x,b\sinh(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{-a^2-b^2}{(a+x)^2}+\frac{2a}{a+x}\right)\,dx,x,b\sinh(x)\right)}{b^3}\\ &= -\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 0.90

$$\frac{-2a\log(a+b\sinh(x))+b\sinh(x)-\frac{a^2+b^2}{a+b\sinh(x)}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x])^2,x]
[Out] (-2*a*Log[a + b*Sinh[x]] + b*Sinh[x] - (a^2 + b^2)/(a + b*Sinh[x]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(43) = 86$.
time = 0.59, size = 119, normalized size = 2.98

method	result
risch	$\frac{2ax}{b^3}+\frac{e^x}{2b^2}-\frac{e^{-x}}{2b^2}-\frac{2(a^2+b^2)e^x}{b^3(b\,e^{2x}+2a\,e^x-b)}-\frac{2a\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{b^3}$
default	$-\frac{2\left(\frac{b(a^2+b^2)\tanh\left(\frac{x}{2}\right)}{a\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}+a\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)\right)}{b^3}+\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^3}-\frac{1}{b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{2a\ln(\tan}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/b^3*(b*(a^2+b^2)/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+a*ln(
a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a))+2/b^3*a*ln(tanh(1/2*x)+1)-1/b^2/(tanh(1
/2*x)+1)+2/b^3*a*ln(tanh(1/2*x)-1)-1/b^2/(tanh(1/2*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(40) = 80$.
time = 0.28, size = 102, normalized size = 2.55

$$\frac{2\,abe^{(-x)}+b^2-(4\,a^2+5\,b^2)e^{(-2\,x)}}{2\,(b^4e^{(-x)}+2\,ab^3e^{(-2\,x)}-b^4e^{(-3\,x)})}-\frac{2\,ax}{b^3}-\frac{e^{(-x)}}{2\,b^2}-\frac{2\,a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] 1/2*(2*a*b*e^(-x) + b^2 - (4*a^2 + 5*b^2)*e^(-2*x))/(b^4*e^(-x) + 2*a*b^3*e
^(-2*x) - b^4*e^(-3*x)) - 2*a*x/b^3 - 1/2*e^(-x)/b^2 - 2*a*log(-2*a*e^(-x)
+ b*e^(-2*x) - b)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(40) = 80$.
time = 0.37, size = 370, normalized size = 9.25

$$\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+2\,(2\,abx+ab)\cosh(x)^3+2\,(2\,abx+2\,b^2\cosh(x)+ab)\sinh(x)^3+2\,(4\,a^2x-2\,a^2-3\,b^2)\cosh(x)^2+2\,(3\,b^2\cosh(x)^2+4\,a^2x-2\,a^2-3\,b^2+3\,(2\,abx+ab)\cosh(x))\sinh(x)^2+b^2-2\,(2\,abx+ab)\cosh(x)-4\,(ab\cosh(x)^3+ab\sinh(x)^3+2\,a^2\cosh(x)^2-ab\cosh(x)+(3\,ab\cosh(x)+2\,a^2)\sinh(x)^2+(3\,ab\cosh(x)^2+4\,a^2\cosh(x)-ab)\sinh(x))\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+2\,(2\,b^2\cosh(x)^3-2\,abx+3\,(2\,abx+ab)\cosh(x)^2-ab+2\,(4\,a^2x-2\,a^2-3\,b^2)\cosh(x))\sinh(x)}{2\,(b^4\cosh(x)^3+b^4\sinh(x)^3+2\,ab^3\cosh(x)^2-b^4\cosh(x)+(3\,b^4\cosh(x)+2\,ab^3)\sinh(x)^2+(3\,b^4\cosh(x)^2+4\,ab^3\cosh(x)-b^4)\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 2*(2*a*b*x + a*b)*cosh(x)^3 + 2*(2*a*b
*x + 2*b^2*cosh(x) + a*b)*sinh(x)^3 + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x)^2
 + 2*(3*b^2*cosh(x)^2 + 4*a^2*x - 2*a^2 - 3*b^2 + 3*(2*a*b*x + a*b)*cosh(x)
)*sinh(x)^2 + b^2 - 2*(2*a*b*x + a*b)*cosh(x) - 4*(a*b*cosh(x)^3 + a*b*sinh
(x)^3 + 2*a^2*cosh(x)^2 - a*b*cosh(x) + (3*a*b*cosh(x) + 2*a^2)*sinh(x)^2 +
 (3*a*b*cosh(x)^2 + 4*a^2*cosh(x) - a*b)*sinh(x))*log(2*(b*sinh(x) + a)/(co
sh(x) - sinh(x))) + 2*(2*b^2*cosh(x)^3 - 2*a*b*x + 3*(2*a*b*x + a*b)*cosh(x
)^2 - a*b + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x))*sinh(x))/(b^4*cosh(x)^3 +
b^4*sinh(x)^3 + 2*a*b^3*cosh(x)^2 - b^4*cosh(x) + (3*b^4*cosh(x) + 2*a*b^3)
*sinh(x)^2 + (3*b^4*cosh(x)^2 + 4*a*b^3*cosh(x) - b^4)*sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(39) = 78$.
time = 0.37, size = 133, normalized size = 3.32

$$\begin{cases} -\frac{2a^2\log\left(\frac{a}{b}+\sinh{(x)}\right)}{ab^3+b^4\sinh{(x)}} - \frac{2a^2}{ab^3+b^4\sinh{(x)}} - \frac{2ab\log\left(\frac{a}{b}+\sinh{(x)}\right)\sinh{(x)}}{ab^3+b^4\sinh{(x)}} + \frac{2b^2\sinh^2{(x)}}{ab^3+b^4\sinh{(x)}} - \frac{b^2\cosh^2{(x)}}{ab^3+b^4\sinh{(x)}} & \text{for } b \neq 0 \\ \frac{-\frac{2\sinh^3{(x)}}{3}+\sinh{(x)}\cosh^2{(x)}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-2*a**2*log(a/b + sinh(x))/(a*b**3 + b**4*sinh(x)) - 2*a**2/(a*b
**3 + b**4*sinh(x)) - 2*a*b*log(a/b + sinh(x))*sinh(x)/(a*b**3 + b**4*sinh(
x)) + 2*b**2*sinh(x)**2/(a*b**3 + b**4*sinh(x)) - b**2*cosh(x)**2/(a*b**3 +
 b**4*sinh(x)), Ne(b, 0)), ((-2*sinh(x)**3/3 + sinh(x)*cosh(x)**2)/a**2, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.
time = 0.40, size = 82, normalized size = 2.05

$$-\frac{e^{(-x)}-e^x}{2\,b^2} - \frac{2\,a\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^3} + \frac{2\left(ab\left(e^{(-x)}-e^x\right)-a^2+b^2\right)}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/b^2 - 2*a*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 + 2*(a*
b*(e^(-x) - e^x) - a^2 + b^2)/((b*(e^(-x) - e^x) - 2*a)*b^3)
```

Mupad [B]
time = 0.71, size = 60, normalized size = 1.50

$$\frac{\frac{\cosh(x)^2}{b} - \frac{2\sinh(x)^3}{a} + \frac{2\cosh(x)^2\sinh(x)}{a} + \frac{2\,a\sinh(x)}{b^2}}{a+b\sinh{(x)}} - \frac{2\,a\,\ln\left(a+b\sinh(x)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (cosh(x)^2/b - (2*sinh(x)^3)/a + (2*cosh(x)^2*sinh(x))/a + (2*a*sinh(x))/b^
2)/(a + b*sinh(x)) - (2*a*log(a + b*sinh(x)))/b^3
```

3.202 $\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=62

$$\frac{x}{b^2}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}$$

```
[Out] x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2772, 2814, 2739, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}+\frac{2a\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.35, size = 502, normalized size = 8.10

$$
\frac{\cosh(x)\left(2a\sqrt{a-ib}\,\sqrt{a+ib}\,\tanh^{-1}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}\,(a+b\sinh(x))-2a(a-ib)\tanh^{-1}\left(\frac{\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}\,(a+b\sinh(x))+\sqrt{a+ib}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\left(2\sqrt[4]{-1}\,a\sqrt{b}\,(ia+b)\text{ArcSin}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{b}}\right)+2\sqrt[4]{-1}\,b^{3/2}(ia+b)\text{ArcSin}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-ib}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{b}}\right)\sinh(x)-\sqrt{a-ib}\,(a^2+b^2)\,\sqrt{1+i\sinh(x)}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}\right)\right)}{(a-ib)^{3/2}(a+ib)^{3/2}b\sqrt{1+i\sinh(x)}\,\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}\,\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}\,(a+b\sinh(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(a +
 b*Sinh[x]) - 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))
/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1
```

```
 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a
+ I*b))]*(2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b
]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*(-1)^(1/4)*b^(3/2)*(I*
a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b)
)])/Sqrt[b]]*Sinh[x] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-
((b*(I + Sinh[x]))/(a - I*b))]))/((a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1
 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x])
)/(a - I*b))]*(a + b*Sinh[x]))
```

Maple [A]
time = 0.57, size = 101, normalized size = 1.63

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}+\frac{\frac{2\left(\frac{b^{2}\tanh\left(\frac{x}{2}\right)}{a}+b\right)}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}}{b^{2}}$	101
risch	$\frac{x}{b^{2}}+\frac{2a\,\mathrm{e}^{x}-2b}{b^{2}\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b\right)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}+a^{2}+b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*ln(tanh(1/2*x)+1)-1/b^2*ln(tanh(1/2*x)-1)+2/b^2*((b^2/a*tanh(1/2*x)+b
)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
nh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.47, size = 100, normalized size = 1.61

$$-\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^{2}e^{(-x)}-b^{3}e^{(-2\,x)}+b^{3}}-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x) -
 a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^
2) + x/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(58) = 116$.
time = 0.39, size = 362, normalized size = 5.84

$$-\frac{(a^{2}b+b^{3})x\cosh(x)^{2}+(a^{2}b+b^{3})x\sinh(x)^{2}-2\,a^{2}b-2\,b^{3}+\left(ab\cosh(x)^{2}+ab\sinh(x)^{2}+2\,a^{2}\cosh(x)-ab+2\,(ab\cosh(x)+a^{2})\sinh(x)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}+2\,ab\cosh(x)+2\,a^{2}+b^{2}+2\,(b^{2}\cosh(x)+ab)\sinh(x)+2\sqrt{a^{2}+b^{2}}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^{2}+b\sinh(x)^{2}+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(a^{2}b+b^{3})x+2\,(a^{3}+ab^{2}+(a^{3}+ab^{2})x)\cosh(x)+2\,(a^{3}+ab^{2}+(a^{2}b+b^{3})x\cosh(x)+(a^{3}+ab^{2})x)\sinh(x)}{a^{2}b^{3}+b^{5}-(a^{2}b^{3}+b^{5})\cosh(x)^{2}-(a^{2}b^{3}+b^{5})\sinh(x)^{2}-2\,(a^{3}b^{2}+ab^{4})\cosh(x)-2\,(a^{3}b^{2}+ab^{4}+(a^{2}b^{3}+b^{5})\cosh(x))\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 97, normalized size = 1.56

$$-\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{2}}+\frac{x}{b^{2}}+\frac{2\,(ae^{x}-b)}{(be^{(2\,x)}+2\,ae^{x}-b)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*
e^x - b)*b^2)
```

Mupad [B]
time = 0.67, size = 132, normalized size = 2.13

$$\frac{x}{b^{2}}-\frac{\frac{2}{b}-\frac{2\,a\,\mathrm{e}^{x}}{b^{2}}}{2\,a\,\mathrm{e}^{x}-b+b\,\mathrm{e}^{2\,x}}-\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{3}}-\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{3}\,\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\,\sqrt{a^{2}+b^{2}}}+\frac{a\,\ln\left(\frac{2\,a\,\mathrm{e}^{x}}{b^{3}}+\frac{2\,a\,(b-a\,\mathrm{e}^{x})}{b^{3}\,\sqrt{a^{2}+b^{2}}}\right)}{b^{2}\,\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2*
a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 +
b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^
2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))
```

3.203 $\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b(a+b\sinh(x))}$$

[Out] -1/b/(a+b*sinh(x))

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 32}

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + b*Sinh[x])^2,x]

[Out] -(1/(b*(a + b*Sinh[x])))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b}\\ &= -\frac{1}{b(a+b\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{b(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(1/(b*(a + b*Sinh[x])))
```

Maple [A]
time = 0.30, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{1}{b(a+b\sinh(x))}$	14
default	$-\frac{1}{b(a+b\sinh(x))}$	14
risch	$-\frac{2\,\mathrm{e}^{x}}{b(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(a+b*sinh(x))
```

Maxima [A]
time = 0.26, size = 13, normalized size = 1.00

$$-\frac{1}{(b\sinh(x)+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sinh(x) + a)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(13) = 26.
time = 0.42, size = 51, normalized size = 3.92

$$-\frac{2(\cosh(x)+\sinh(x))}{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)-b^2+2(b^2\cosh(x)+ab)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [A]
time = 0.26, size = 19, normalized size = 1.46

$$\begin{cases} -\frac{1}{ab+b^2\sinh(x)} & \text{for } b \neq 0 \\ \frac{\sinh(x)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-1/(a*b + b**2*sinh(x)), Ne(b, 0)), (sinh(x)/a**2, True))
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.69

$$\frac{2}{(b(e^{(-x)} - e^x) - 2a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/((b*(e^(-x) - e^x) - 2*a)*b)
```

Mupad [B]
time = 0.53, size = 14, normalized size = 1.08

$$\frac{\sinh(x)}{a\ (a + b\sinh\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] sinh(x)/(a*(a + b*sinh(x)))
```

3.204 $\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=79

$$\frac{(a^2-b^2)\operatorname{ArcTan}(\sinh(x))}{(a^2+b^2)^2}-\frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}+\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)\,(a+b\sinh(x))}$$

```
[Out] (a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^2-2*a*b*ln(cosh(x))/(a^2+b^2)^2+2*a*b*l
n(a+b*sinh(x))/(a^2+b^2)^2-b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2747, 724, 815, 649, 209, 266}

$$\frac{(a^2-b^2)\operatorname{ArcTan}(\sinh(x))}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^2 - (2*a*b*Log[Cosh[x]])/(a^2 + b
^2)^2 + (2*a*b*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a + b*Si
nh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
```

```
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right)\right)\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int \left(-\frac{2a}{\left(a^2+b^2\right)(a+x)}+\frac{-a^2+b^2+2ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)\,dx,x,b\sinh
\end{aligned}$$

$$\begin{aligned}
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int \frac{-a^2+b^2+2ax}{b^2+x^2}\,dx,x,b\sin\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{(2ab)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx,x,b\sinh\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{\left(a^2-b^2\right)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{2ab\log(\cosh(x))}{\left(a^2+b^2\right)^2}+\frac{2ab\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}-\frac{}{\left(a^2+b^2\right)(a}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 121, normalized size = 1.53

$$-\frac{b\left(\left(2a+\frac{-a^2+b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-4a\log(a+b\sinh(x))+\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)+\frac{2\left(a^2+b^2\right)}{a+b\sinh(x)}\right)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*(b*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a*
Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh
[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x])))/(a^2 + b^2)^2
```

Maple [A]
time = 0.64, size = 123, normalized size = 1.56

method	result
default	$\frac{2b\left(-\frac{b\left(a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{a\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}+a\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)\right)}{\left(a^2+b^2\right)^2}+\frac{-2ab\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+2\left(a^2-b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right.}{a^4+2a^2b^2+b^4}$
risch	$-\frac{2b\,\mathrm{e}^x}{\left(a^2+b^2\right)\left(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^x-b\right)}-\frac{i\ln\left(\mathrm{e}^x-i\right)a^2}{a^4+2a^2b^2+b^4}+\frac{i\ln\left(\mathrm{e}^x-i\right)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln\left(\mathrm{e}^x-i\right)ab}{a^4+2a^2b^2+b^4}+\frac{i\ln\left(\mathrm{e}^x+i\right)a^2}{a^4+2a^2b^2+b^4}-\frac{i\ln\left(\mathrm{e}^x+i\right)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln(\mathrm{e}^{}}{a^4+2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/(a^2+b^2)^2*(-b*(a^2+b^2)/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x
)-a)+a*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a))+2/(a^4+2*a^2*b^2+b^4)*(-a*b*l
n(tanh(1/2*x)^2+1)+(a^2-b^2)*arctan(tanh(1/2*x)))
```

Maxima [A]
time = 0.49, size = 149, normalized size = 1.89

$$\frac{2\,ab\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(a^2-b^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,be^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - 2*a*b*log
(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*arctan(e^(-x))/(a^4
+ 2*a^2*b^2 + b^4) - 2*b*e^(-x)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x) - (a^
2*b + b^3)*e^(-2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(79) = 158.
time = 0.39, size = 383, normalized size = 4.85

$$\frac{2\left(\left(a^2b-b^3-(a^2b-b^3)\cosh(x)^2-(a^2b-b^3)\sinh(x)^2-2\,(a^3-ab^2)\cosh(x)-2\,(a^3-ab^2+(a^2b-b^3)\cosh(x))\sinh(x)\right)\arctan\left(\cosh(x)+\sinh(x)\right)+(a^2b+b^3)\cosh(x)-\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\,(ab^2\cosh(x)+a^2b)\sinh(x)\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\,(ab^2\cosh(x)+a^2b)\sinh(x)\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+(a^2b+b^3)\sinh(x)\right)}{a^4b+2\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)\cosh(x)^2-(a^4b+2\,a^2b^3+b^5)\sinh(x)^2-2\,(a^5+2\,a^3b^2+ab^4)\cosh(x)-2\,(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\cosh(x))\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^
3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*arcta
```

```
n(cosh(x) + sinh(x)) + (a^2*b + b^3)*cosh(x) - (a*b^2*cosh(x)^2 + a*b^2*sin
h(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2
*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2
+ 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (a^2*b + b^3)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 -
 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b
+ 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(79) = 158.
time = 0.41, size = 186, normalized size = 2.35

$$\frac{2\,ab^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^2-b^2\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{2\left(ab^2\left(e^{(-x)}-e^x\right)-3\,a^2b-b^3\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^2 - b^2)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a*b^2*(e^(-x
) - e^x) - 3*a^2*b - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x) - 2*a)
)
```

Mupad [B]
time = 1.79, size = 186, normalized size = 2.35

$$\frac{2\,a\,b\,\ln\left(b^5\,e^{2\,x}-a^4\,b-b^5-14\,a^2\,b^3+2\,a^5\,e^x+14\,a^2\,b^3\,e^{2\,x}+2\,a\,b^4\,e^x+a^4\,b\,e^{2\,x}+28\,a^3\,b^2\,e^x\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b^2\,e^x}{\left(a^2\,b+b^3\right)\,\left(2\,a\,e^x-b+b\,e^{2\,x}\right)}-\frac{\ln\left(1+e^x\,1\mathrm{i}\right)}{-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}}-\frac{\ln\left(e^x+1\mathrm{i}\right)\,1\mathrm{i}}{-a^2+a\,b\,2\mathrm{i}+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] (2*a*b*log(b^5*exp(2*x) - a^4*b - b^5 - 14*a^2*b^3 + 2*a^5*exp(x) + 14*a^2*
b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) + 28*a^3*b^2*exp(x)))/(a^4 +
 b^4 + 2*a^2*b^2) - (log(exp(x) + 1i)*1i)/(a*b*2i - a^2 + b^2) - (2*b^2*exp
(x))/((a^2*b + b^3)*(2*a*exp(x) - b + b*exp(2*x))) - log(exp(x)*1i + 1)/(2*
a*b - a^2*1i + b^2*1i)
```

3.205 $\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=93

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\operatorname{sech}(x)\left(3ab+\left(a^2-2b^2\right)\sinh(x)\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] -6*a*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-b*sech(
x)/(a^2+b^2)/(a+b*sinh(x))+sech(x)*(3*a*b+(a^2-2*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2773, 2945, 12, 2739, 632, 212}

$$-\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}-\frac{b\operatorname{sech}(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\operatorname{sech}(x)\left(\left(a^2-2b^2\right)\sinh(x)+3ab\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]*(3*a*b + (a^2 - 2*b^2)*
Sinh[x]))/(a^2 + b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\operatorname{sech}^2(x)(-a+2b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2} \\
&= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{\int \frac{3ab^2}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2} \\
&= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(3ab^2)\int \frac{1}{a+b\sinh(x}}{(a^2+b^2)^2} \\
&= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(6ab^2)\operatorname{Subst}\left(\int \frac{\quad}{a}\right.}{(} \\
&= -\frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} - \frac{(12ab^2)\operatorname{Subst}\left(\int\right.}{} \\
&= -\frac{6ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\operatorname{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\operatorname{sech}(x)\left(3ab+(a^2-2b^2\right.}{(a^2+b^2)^2}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 1.01

$$\frac{\frac{6ab^2\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+2ab\text{sech}(x)-\frac{b^3\cosh(x)}{a+b\sinh(x)}+a^2\tanh(x)-b^2\tanh(x)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 2*
a*b*Sech[x] - (b^3*Cosh[x])/(a + b*Sinh[x]) + a^2*Tanh[x] - b^2*Tanh[x])/(a
^2 + b^2)^2
```

Maple [A]
time = 0.58, size = 138, normalized size = 1.48

method	result
default	$-\frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2}-\frac{2\left((-a^2+b^2)\tanh\left(\frac{x}{2}\right)-2ab\right)}{(a^4+2a^2b^2+b^4)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$
risch	$-\frac{2(-3a\,b^2e^{3x}-3a^2b\,e^{2x}+2a^3e^x-a\,b^2e^x-a^2b+2b^3)}{(a^2+b^2)^2(b\,e^{2x}+2a\,e^x-b)(1+e^{2x})}+\frac{3b^2a\ln\left(e^x+\frac{(a^2+b^2)^{\frac{5}{2}}a-a^6-3a^4b^2-3a^2b^4-b^6}{b(a^2+b^2)^{\frac{5}{2}}}\right)}{(a^2+b^2)^{\frac{5}{2}}}-\frac{3b^2a\ln\left(e^x+\frac{(a^2+b^2)}{}\right.}{(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2/(a^2+b^2)^2*((-b^2/a*tanh(1/2*x)-b)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)
-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(89) = 178.
time = 0.49, size = 215, normalized size = 2.31

$$\frac{3\,ab^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\,(3\,a^2be^{(-2\,x)}-3\,ab^2e^{(-3\,x)}+a^2b-2\,b^3+(2\,a^3-ab^2)e^{(-x)})}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*e^(-2*x) - 3*a*b
^2*e^(-3*x) + a^2*b - 2*b^3 + (2*a^3 - a*b^2)*e^(-x))/(a^4*b + 2*a^2*b^3 +
b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-
3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(89) = 178$.

time = 0.38, size = 802, normalized size = 8.62

$$-\frac{2a^4b-2a^2b^3-4b^5+6(a^3b^2+ab^4)\cosh(x)^3+6(a^3b^2+ab^4)\sinh(x)^3+6(a^4b+a^2b^3)\cosh(x)^2+6(a^4b+a^2b^3+3(a^3b^2+ab^4)\cosh(x))\sinh(x)^2+3\left(ab^3\cosh(x)^4+ab^3\sinh(x)^4+2a^2b^2\cosh(x)^3+2a^2b^2\cosh(x)-ab^3+2(2ab^3\cosh(x)+a^2b^2)\sinh(x)^3+6(ab^3\cosh(x)^2+a^2b^2\cosh(x))\sinh(x)^2+2(2ab^3\cosh(x)^3+3a^2b^2\cosh(x)^2+a^2b^2)\sinh(x)\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2ab\cosh(x)+2a^2+b^2+2(b^2\cosh(x)+ab)\sinh(x)-2\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2a\cosh(x)+2(b\cosh(x)+a)\sinh(x)-b}\right)-2(2a^5+a^3b^2-ab^4)\cosh(x)-2(2a^5+a^3b^2-ab^4-9(a^3b^2+ab^4)\cosh(x)^2-6(a^4b+a^2b^3)\cosh(x))\sinh(x)}{a^6b+3a^4b^3+3a^2b^5+b^7-(a^6b+3a^4b^3+3a^2b^5+b^7)\cosh(x)^4-(a^6b+3a^4b^3+3a^2b^5+b^7)\sinh(x)^4-2(a^7+3a^5b^2+3a^3b^4+ab^6)\cosh(x)^3-2(a^7+3a^5b^2+3a^3b^4+ab^6+2(a^6b+3a^4b^3+3a^2b^5+b^7)\cosh(x))\sinh(x)^3-6((a^6b+3a^4b^3+3a^2b^5+b^7)\cosh(x)^2+(a^7+3a^5b^2+3a^3b^4+ab^6)\cosh(x))\sinh(x)^2-2(a^7+3a^5b^2+3a^3b^4+ab^6)\cosh(x)-2\left(a^7+3a^5b^2+3a^3b^4+ab^6+2(a^6b+3a^4b^3+3a^2b^5+b^7)\cosh(x)^3+3(a^7+3a^5b^2+3a^3b^4+ab^6)\cosh(x)^2\right)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 6*(a^3*b^2 + a*b^4)*cosh(x)^3 + 6*(a^3*b^2
+ a*b^4)*sinh(x)^3 + 6*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b + a^2*b^3 + 3
*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^2 + 3*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^
4 + 2*a^2*b^2*cosh(x)^3 + 2*a^2*b^2*cosh(x) - a*b^3 + 2*(2*a*b^3*cosh(x) +
a^2*b^2)*sinh(x)^3 + 6*(a*b^3*cosh(x)^2 + a^2*b^2*cosh(x))*sinh(x)^2 + 2*(2
*a*b^3*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2 + a^2*b^2)*sinh(x))*sqrt(a^2 + b^2)*
log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*c
osh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*
(2*a^5 + a^3*b^2 - a*b^4)*cosh(x) - 2*(2*a^5 + a^3*b^2 - a*b^4 - 9*(a^3*b^2
 + a*b^4)*cosh(x)^2 - 6*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*
b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (
a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*((a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x
))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.43, size = 167, normalized size = 1.80

$$\frac{3\,ab^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,ab^2e^{(3\,x)}+3\,a^2be^{(2\,x)}-2\,a^3e^x+ab^2e^x+a^2b-2\,b^3\right)}{\left(a^4+2\,a^2b^2+b^4\right)\left(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3*a*b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^2*e^(3
*x) + 3*a^2*b*e^(2*x) - 2*a^3*e^x + a*b^2*e^x + a^2*b - 2*b^3)/((a^4 + 2*a^
2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b))
```

Mupad [B]
time = 0.97, size = 302, normalized size = 3.25

$$\frac{\frac{6\,a^4\,b^4\,\mathrm{e}^{2\,x}}{\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}-\frac{2\,\left(2\,a^2\,b^6-a^4\,b^4\right)}{\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}+\frac{6\,a^3\,b^5\,\mathrm{e}^{3\,x}}{\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}+\frac{2\,a\,\mathrm{e}^x\,\left(a^2\,b^6-2\,a^4\,b^4\right)}{b\,\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}}{2\,a\,\mathrm{e}^x-b+2\,a\,\mathrm{e}^{3\,x}+b\,\mathrm{e}^{4\,x}}-\frac{3\,a\,b^2\,\ln\left(-\frac{6\,a\,b\,\mathrm{e}^x}{{\left(a^2+b^2\right)}^2}-\frac{6\,a\,b\,\left(b-a\,\mathrm{e}^x\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{{\left(a^2+b^2\right)}^{5/2}}+\frac{3\,a\,b^2\,\ln\left(\frac{6\,a\,b\,\left(b-a\,\mathrm{e}^x\right)}{{\left(a^2+b^2\right)}^{5/2}}-\frac{6\,a\,b\,\mathrm{e}^x}{{\left(a^2+b^2\right)}^2}\right)}{{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((6*a^4*b^4*exp(2*x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*(2*a^2*b^6 - a
^4*b^4))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^3*b^5*exp(3*x))/((a*b^2 +
 a^3)*(a*b^5 + a^3*b^3)) + (2*a*exp(x)*(a^2*b^6 - 2*a^4*b^4))/(b*(a*b^2 + a
^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3*x) + b*exp(4*x)) - (3*a
*b^2*log(- (6*a*b*exp(x))/(a^2 + b^2)^2 - (6*a*b*(b - a*exp(x)))/(a^2 + b^2
)^(5/2)))/(a^2 + b^2)^(5/2) + (3*a*b^2*log((6*a*b*(b - a*exp(x)))/(a^2 + b^
2)^(5/2) - (6*a*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

3.206 $\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=136

$$\frac{(a^4+6a^2b^2-3b^4)\operatorname{ArcTan}(\sinh(x))}{2(a^2+b^2)^3}-\frac{4ab^3\log(\cosh(x))}{(a^2+b^2)^3}+\frac{4ab^3\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{b(a^2-3b^2)}{2(a^2+b^2)^2(a+b\sinh(x))}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(sinh(x))/(a^2+b^2)^3-4*a*b^3*ln(cosh(x))/(
a^2+b^2)^3+4*a*b^3*ln(a+b*sinh(x))/(a^2+b^2)^3+1/2*b*(a^2-3*b^2)/(a^2+b^2)^
2/(a+b*sinh(x))+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2747, 755, 815, 649, 209, 266}

$$\frac{b(a^2-3b^2)}{2(a^2+b^2)^2(a+b\sinh(x))}+\frac{\operatorname{sech}^2(x)(a\sinh(x)+b)}{2(a^2+b^2)(a+b\sinh(x))}+\frac{4ab^3\log(a+b\sinh(x))}{(a^2+b^2)^3}-\frac{4ab^3\log(\cosh(x))}{(a^2+b^2)^3}+\frac{(a^4+6a^2b^2-3b^4)\operatorname{ArcTan}(\sinh(x))}{2(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^3) - (4*a*b^3*Lo
g[Cosh[x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (b
*(a^2 - 3*b^2))/(2*(a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(b + a*Sinh[
x]))/(2*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx &= b^3\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\text{Subst}\left(\int \frac{a^2+3b^2+2ax}{(a+x)^2(-b^2-x^2)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\text{Subst}\left(\int \left(\frac{a^2-3b^2}{(a^2+b^2)(a+x)^2} - \frac{8ab^2}{(a^2+b^2)^2(a+x)} + \frac{-a^4-6a^2b^2+3b}{(a^2+b^2)^2(b^2}\right.\right.}{2\left(a^2+b^2\right)}\\
&= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b(a^2-3b^2)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))}\\
&= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b(a^2-3b^2)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))}\\
&= \frac{\left(a^4+6a^2b^2-3b^4\right)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^3} - \frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} -
\end{aligned}$$

Mathematica [A]
time = 1.58, size = 260, normalized size = 1.91

$$-\frac{-\frac{2\operatorname{sech}^2(x)(b+a\sinh(x))}{a+b\sinh(x)} + \frac{b\left(\frac{2a\left(a^2+b^2\right)\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+(-a^2+3b^2)\left(\left(2a+\frac{-a^2+b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-4a\log(a+b\sinh(x))+\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)+\frac{2\left(a^2+b^2\right)}{a+b\sinh(x)}\right)\right)}{(a^2+b^2)^2}}{4\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/4*((-2*Sech[x]^2*(b + a*Sinh[x]))/(a + b*Sinh[x]) + (b*((2*a*(a^2 + b^2)
*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b*Si
nh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (-a^2
+ 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a
*Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sin
h[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x]))))/(a^2 + b^2)^2)/(a^2 + b^2)
```

Maple [A]
time = 0.81, size = 212, normalized size = 1.56

method	result
default	$\frac{\frac{2\left(\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-2a^3b-2a\,b^3\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}-4a\,b^3\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+\left(a^4+6a^2b^2-3b^4\right)\arctan\left(\tanh\left(\frac{x}{2}\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}$
risch	$\frac{e^x\left(a^2b\,e^{4x}-3b^3e^{4x}+2a^3e^{3x}+2a\,b^2e^{3x}+6a^2b\,e^{2x}-2b^3e^{2x}-2a^3e^x-2a\,b^2e^x+a^2b-3b^3\right)}{\left(a^2+b^2\right)^2\left(1+e^{2x}\right)^2\left(b\,e^{2x}+2a\,e^x-b\right)}-\frac{i\ln\left(e^x-i\right)a^4}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{3i\ln\left(e^x-i\right)a}{a^6+3a^4b^2+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(((-1/2*a^4+1/2*b^4)*tanh(1/2*x)^3+(-2*a^3*
b-2*a*b^3)*tanh(1/2*x)^2+(1/2*a^4-1/2*b^4)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2
-2*a*b^3*ln(tanh(1/2*x)^2+1)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tanh(1/2*x)))
+2*b^3/(a^2+b^2)^3*(-b*(a^2+b^2)/a*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*b*tanh(1/
2*x)-a)+2*a*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(130) = 260.
time = 0.52, size = 375, normalized size = 2.76

$$\frac{4\,ab^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^2b-3\,b^3\right)e^{(-x)}+2\left(a^3+ab^2\right)e^{(-2\,x)}+2\left(3\,a^2b-b^3\right)e^{(-3\,x)}-2\left(a^3+ab^2\right)e^{(-4\,x)}+\left(a^2b-3\,b^3\right)e^{(-5\,x)}}{a^4b+2\,a^2b^3+b^5+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-x)}+\left(a^4b+2\,a^2b^3+b^5\right)e^{(-2\,x)}+4\left(a^5+2\,a^3b^2+ab^4\right)e^{(-3\,x)}-\left(a^4b+2\,a^2b^3+b^5\right)e^{(-4\,x)}+2\left(a^5+2\,a^3b^2+ab^4\right)e^{(-5\,x)}-\left(a^4b+2\,a^2b^3+b^5\right)e^{(-6\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - 4*a*b^3*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 +
 6*a^2*b^2 - 3*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((
a^2*b - 3*b^3)*e^(-x) + 2*(a^3 + a*b^2)*e^(-2*x) + 2*(3*a^2*b - b^3)*e^(-3*
x) - 2*(a^3 + a*b^2)*e^(-4*x) + (a^2*b - 3*b^3)*e^(-5*x))/(a^4*b + 2*a^2*b^
3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^
(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e
```

```
^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*
e^(-6*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(130) = 260$.
time = 0.44, size = 2615, normalized size = 19.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^5 + (a^4*b - 2*a^2*b^3 - 3*b^5)*sinh(
x)^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + (2*a^5 + 4*a^3*b^2 + 2*a*b^4
 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^4 + 2*(3*a^4*b + 2*a^2*b^
3 - b^5)*cosh(x)^3 + 2*(3*a^4*b + 2*a^2*b^3 - b^5 + 5*(a^4*b - 2*a^2*b^3 -
3*b^5)*cosh(x)^2 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 2*(a^5
+ 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 - 5*(a^4*b - 2*
a^2*b^3 - 3*b^5)*cosh(x)^3 - 6*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 3*(3*a
^4*b + 2*a^2*b^3 - b^5)*cosh(x))*sinh(x)^2 + ((a^4*b + 6*a^2*b^3 - 3*b^5)*c
osh(x)^6 + (a^4*b + 6*a^2*b^3 - 3*b^5)*sinh(x)^6 + 2*(a^5 + 6*a^3*b^2 - 3*a
*b^4)*cosh(x)^5 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 3*(a^4*b + 6*a^2*b^3 - 3*b
^5)*cosh(x))*sinh(x)^5 - a^4*b - 6*a^2*b^3 + 3*b^5 + (a^4*b + 6*a^2*b^3 - 3
*b^5)*cosh(x)^4 + (a^4*b + 6*a^2*b^3 - 3*b^5 + 15*(a^4*b + 6*a^2*b^3 - 3*b^
5)*cosh(x)^2 + 10*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^4 + 4*(a^5 +
 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^4*b +
 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^2 + (
a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - (a^4*b + 6*a^2*b^3 - 3*b^5)
*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5 - 15*(a^4*b + 6*a^2*b^3 - 3*b^5)*co
sh(x)^4 - 20*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 - 6*(a^4*b + 6*a^2*b^3 -
 3*b^5)*cosh(x)^2 - 12*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^2 + 2*(
a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x) + 2*(3*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(
x)^5 + a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^4
+ 2*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 6*(a^5 + 6*a^3*b^2 - 3*a*b^4)*c
osh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) + s
inh(x)) + (a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x) + 4*(a*b^4*cosh(x)^6 + a*b^4*
sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3 - a
*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh(x)^
5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4 + 4
*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*sinh(x
)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 + 12*a
^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*cosh(x
)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3)*si
nh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - 4*(a*b^4*cosh(x)^6 + a*
b^4*sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3
```

```
 - a*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh
(x)^5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4
 + 4*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*si
nh(x)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 +
12*a^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*co
sh(x)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3
)*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^4*b - 2*a^2*b^3 - 3*b^5
+ 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^4 + 8*(a^5 + 2*a^3*b^2 + a*b^4)*cos
h(x)^3 + 6*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x)^2 - 4*(a^5 + 2*a^3*b^2 + a*b
^4)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*s
inh(x)^6 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x
))*sinh(x)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3
*a^4*b^3 + 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x)^2 + 10*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*
a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7)*cosh(x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*cosh(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(a + b*sinh(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(130) = 260.
time = 0.45, size = 295, normalized size = 2.17

$$\frac{4\,ab^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{2\,ab^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^4+6\,a^2b^2-3\,b^4\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{a^2b\left(e^{(-x)}-e^x\right)^2-3\,b^3\left(e^{(-x)}-e^x\right)^2-2\,a^3\left(e^{(-x)}-e^x\right)-2\,ab^2\left(e^{(-x)}-e^x\right)+8\,a^2b-8\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\left(e^{(-x)}-e^x\right)^3-2\,a\left(e^{(-x)}-e^x\right)^2+4\,b\left(e^{(-x)}-e^x\right)-8\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 4*a*b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 2*a*b^3*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^4 + 6*a^2*b^2 - 3*b^4)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^2*b*(e^(-x) - e^x)^2 - 3*b^3*(e^(
-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 8*a^2*b - 8*
b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-x) - e^x)^2 +
4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B]
time = 5.07, size = 519, normalized size = 3.82

$$\frac{\frac{4\,\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}+\frac{\mathrm{e}^x\,\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{\mathrm{e}^{2\,x}+1}-\frac{\frac{4\,a\,b}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\mathrm{e}^x\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\ln\left(\mathrm{e}^x+1{}\mathrm{i}\right)\,\left(a-b\,3{}\mathrm{i}\right)}{2\,\left(-a^3\,1{}\mathrm{i}-3\,a^2\,b+a\,b^2\,3{}\mathrm{i}+b^3\right)}+\frac{\ln\left(1+\mathrm{e}^x\,1{}\mathrm{i}\right)\,\left(-3\,b+a\,1{}\mathrm{i}\right)}{2\,\left(-a^3-a^2\,b\,3{}\mathrm{i}+3\,a\,b^2+b^3\,1{}\mathrm{i}\right)}+\frac{4\,a\,b^3\,\ln\left(9\,b^9\,\mathrm{e}^{2\,x}-a^8\,b-9\,b^9-220\,a^2\,b^7-30\,a^4\,b^5-12\,a^6\,b^3+2\,a^9\,\mathrm{e}^x+220\,a^2\,b^7\,\mathrm{e}^{2\,x}+30\,a^4\,b^5\,\mathrm{e}^{2\,x}+12\,a^6\,b^3\,\mathrm{e}^{2\,x}+18\,a\,b^8\,\mathrm{e}^x+a^8\,b\,\mathrm{e}^{2\,x}+440\,a^3\,b^6\,\mathrm{e}^x+60\,a^5\,b^4\,\mathrm{e}^x+24\,a^7\,b^2\,\mathrm{e}^x\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\,\mathrm{e}^x\,\left(a^4\,b^6+2\,a^2\,b^8+b^{10}\right)}{b^2\,\left(a^2\,b+b^3\right)\,\left(a^2+b^2\right)\,\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)^2) + (exp(x)*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((4*a*b)/(a^4 + b^4 + 2*a^2*b^2) +
(2*exp(x)*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) + 1)
 + (log(exp(x) + 1i)*(a - b*3i))/(2*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) +
(log(exp(x)*1i + 1)*(a*1i - 3*b))/(2*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) +
 (4*a*b^3*log(9*b^9*exp(2*x) - a^8*b - 9*b^9 - 220*a^2*b^7 - 30*a^4*b^5 - 1
2*a^6*b^3 + 2*a^9*exp(x) + 220*a^2*b^7*exp(2*x) + 30*a^4*b^5*exp(2*x) + 12*
a^6*b^3*exp(2*x) + 18*a*b^8*exp(x) + a^8*b*exp(2*x) + 440*a^3*b^6*exp(x) +
60*a^5*b^4*exp(x) + 24*a^7*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
 - (2*exp(x)*(b^10 + 2*a^2*b^8 + a^4*b^6))/(b^2*(a^2*b + b^3)*(a^2 + b^2)*(
2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

3.207 $\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\operatorname{sech}^3(x)\left(5ab+\left(a^2-4b^2\right)\sinh(x)\right)}{3\left(a^2+b^2\right)^2}+\frac{\operatorname{sech}(x)\left(15ab\right.}{}$$

```
[Out] -10*a*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-b*sech
(x)^3/(a^2+b^2)/(a+b*sinh(x))+1/3*sech(x)^3*(5*a*b+(a^2-4*b^2)*sinh(x))/(a^
2+b^2)^2+1/3*sech(x)*(15*a*b^3+(2*a^4+9*a^2*b^2-8*b^4)*sinh(x))/(a^2+b^2)^3
```

Rubi [A]
time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2773, 2945, 12, 2739, 632, 212}

$$-\frac{b\operatorname{sech}^3(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}+\frac{\operatorname{sech}^3(x)\left(\left(a^2-4b^2\right)\sinh(x)+5ab\right)}{3\left(a^2+b^2\right)^2}-\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\operatorname{sech}(x)\left(\left(2a^4+9a^2b^2-8b^4\right)\sinh(x)+15ab^3\right)}{3\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-10*a*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) -
(b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]^3*(5*a*b + (a^2 - 4*
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2) + (Sech[x]*(15*a*b^3 + (2*a^4 + 9*a^2*b^2
- 8*b^4)*Sinh[x]))/(3*(a^2 + b^2)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\text{sech}^4(x)(-a+4b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int \frac{\text{sech}^2(x)(a(2a^2+}{a}}{3(}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+\right.}{}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+\right.}{}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+\right.}{}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+\right.}{}$$

$$= -\frac{10ab^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4\right.}{3(a^2+b^2)^2}$$

Mathematica [A]
time = 0.31, size = 137, normalized size = 0.95

$$\frac{\frac{30ab^4\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 12ab^3\text{sech}(x) - \frac{3b^5\cosh(x)}{a+b\sinh(x)} + (a^2+b^2)\,\text{sech}^3(x)\left(2ab+(a^2-b^2)\sinh(x)\right) + (2a^4+9a^2b^2-5b^4)\tanh(x)}{3(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((30*a*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 1
2*a*b^3*Sech[x] - (3*b^5*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2)*Sech[x]^3*(
2*a*b + (a^2 - b^2)*Sinh[x]) + (2*a^4 + 9*a^2*b^2 - 5*b^4)*Tanh[x])/(3*(a^2
 + b^2)^3)
```

Maple [A]
time = 0.72, size = 266, normalized size = 1.85

method	result

default	$-\frac{2b^4\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{5a\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tanh^5\left(\frac{x}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tanh^4\left(\right.\right.\right.}{}$
risch	$-\frac{2(-15a\,b^4\mathrm{e}^{7x}-15a^2b^3\mathrm{e}^{6x}+10a^3b^2\mathrm{e}^{5x}-35a\,b^4\mathrm{e}^{5x}-10a^4b\,\mathrm{e}^{4x}-55a^2b^3\mathrm{e}^{4x}+12a^5\mathrm{e}^{3x}+44a^3b^2\mathrm{e}^{3x}-13a\,b^4\mathrm{e}^{3x}-4a^4b\,\mathrm{e}^{2x}-33a^2b^3\mathrm{e}^{2x}+}{3(a^6+3a^4b^2+3a^2b^4+b^6)(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)(1+\mathrm{e}^{2x})^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tanh(1/2*x)-b)/(a*tanh(1/2*x)
^2-2*b*tanh(1/2*x)-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*
tanh(1/2*x)^5+(-2*a^3*b-6*a*b^3)*tanh(1/2*x)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4)
*tanh(1/2*x)^3-8*a*b^3*tanh(1/2*x)^2+(-a^4-3*a^2*b^2+2*b^4)*tanh(1/2*x)-2/3
*a^3*b-14/3*a*b^3)/(tanh(1/2*x)^2+1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(136) = 272$.
time = 0.51, size = 490, normalized size = 3.40

$$\frac{5\,ab^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\,(15\,a^2b^3e^{(-6\,x)}-15\,ab^4e^{(-7\,x)}+2\,a^4b+9\,a^2b^3-8\,b^5+(4\,a^5+18\,a^3b^2-ab^4)e^{(-x)}+(4\,a^4b+33\,a^2b^3-16\,b^5)e^{(-2\,x)}+(12\,a^5+44\,a^3b^2-13\,ab^4)e^{(-3\,x)}+5\,(2\,a^4b+11\,a^2b^3)e^{(-4\,x)}+5\,(2\,a^3b^2-7\,ab^4)e^{(-5\,x)})}{3\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x)}+6\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-3\,x)}+6\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-5\,x)}-2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-6\,x)}+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-7\,x)}-(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-8\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/3*(15*a^2*b^3
*e^(-6*x) - 15*a*b^4*e^(-7*x) + 2*a^4*b + 9*a^2*b^3 - 8*b^5 + (4*a^5 + 18*a
^3*b^2 - a*b^4)*e^(-x) + (4*a^4*b + 33*a^2*b^3 - 16*b^5)*e^(-2*x) + (12*a^5
 + 44*a^3*b^2 - 13*a*b^4)*e^(-3*x) + 5*(2*a^4*b + 11*a^2*b^3)*e^(-4*x) + 5*
(2*a^3*b^2 - 7*a*b^4)*e^(-5*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7)*e^(-2*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-6*x) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) -
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-8*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(136) = 272$.
time = 0.52, size = 3044, normalized size = 21.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] -1/3*(30*(a^3*b^4 + a*b^6)*cosh(x)^7 + 30*(a^3*b^4 + a*b^6)*sinh(x)^7 + 4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 30*(a^4*b^3 + a^2*b^5)*cosh(x)^6 + 30*(a^4*b^3 + a^2*b^5 + 7*(a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^6 - 10*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^5 - 10*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6 - 63*(a^3*b^4 + a*b^6)*cosh(x)^2 - 18*(a^4*b^3 + a^2*b^5)*cosh(x))*sinh(x)^5 + 10*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^4 + 10*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^3 + 45*(a^4*b^3 + a^2*b^5)*cosh(x)^2 - 5*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^4 - 2*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x)^3 - 2*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6 - 525*(a^3*b^4 + a*b^6)*cosh(x)^4 - 300*(a^4*b^3 + a^2*b^5)*cosh(x)^3 + 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^2 - 20*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x))*sinh(x)^3 + 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*cosh(x)^2 + 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7 + 315*(a^3*b^4 + a*b^6)*cosh(x)^5 + 225*(a^4*b^3 + a^2*b^5)*cosh(x)^4 - 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^3 + 30*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^2 - 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x))*sinh(x)^2 + 15*(a*b^5*cosh(x)^8 + a*b^5*sinh(x)^8 + 2*a^2*b^4*cosh(x)^7 + 2*a*b^5*cosh(x)^6 + 6*a^2*b^4*cosh(x)^5 + 6*a^2*b^4*cosh(x)^3 - 2*a*b^5*cosh(x)^2 + 2*(4*a*b^5*cosh(x) + a^2*b^4)*sinh(x)^7 + 2*a^2*b^4*cosh(x) + 2*(14*a*b^5*cosh(x)^2 + 7*a^2*b^4*cosh(x) + a*b^5)*sinh(x)^6 - a*b^5 + 2*(28*a*b^5*cosh(x)^3 + 21*a^2*b^4*cosh(x)^2 + 6*a*b^5*cosh(x) + 3*a^2*b^4)*sinh(x)^5 + 10*(7*a*b^5*cosh(x)^4 + 7*a^2*b^4*cosh(x)^3 + 3*a*b^5*cosh(x)^2 + 3*a^2*b^4*cosh(x))*sinh(x)^4 + 2*(28*a*b^5*cosh(x)^5 + 35*a^2*b^4*cosh(x)^4 + 20*a*b^5*cosh(x)^3 + 30*a^2*b^4*cosh(x)^2 + 3*a^2*b^4)*sinh(x)^3 + 2*(14*a*b^5*cosh(x)^6 + 21*a^2*b^4*cosh(x)^5 + 15*a*b^5*cosh(x)^4 + 30*a^2*b^4*cosh(x)^3 + 9*a^2*b^4*cosh(x) - a*b^5)*sinh(x)^2 + 2*(4*a*b^5*cosh(x)^7 + 7*a^2*b^4*cosh(x)^6 + 6*a*b^5*cosh(x)^5 + 15*a^2*b^4*cosh(x)^4 + 9*a^2*b^4*cosh(x)^2 - 2*a*b^5*cosh(x) + a^2*b^4)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(4*a^7 + 22*a^5*b^2 + 17*a^3*b^4 - a*b^6)*cosh(x) - 2*(4*a^7 + 22*a^5*b^2 + 17*a^3*b^4 - a*b^6 - 105*(a^3*b^4 + a*b^6)*cosh(x)^6 - 90*(a^4*b^3 + a^2*b^5)*cosh(x)^5 + 25*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^4 - 20*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^3 + 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x)^2 - 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*cosh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^8 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(

```
x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 2*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 + 14*(a^8*b + 4*a^6*b^3 + 6*
a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 +
 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 +
 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 + 6*(a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x)^5 - 10*(7*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 + 7*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b
^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 + 3*a
*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 35*
(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 20*(a^8*b + 4
*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 + 30*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^3 + 2*(a^8*b + 4*a^6*b^3 +
 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5
+ 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*co
sh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 -
15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 - 30*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 9*(a^9 + 4*a^7*b^2 +
 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 - 2*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(
x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(136) = 272.
time = 0.42, size = 287, normalized size = 1.99

$$\frac{5\,ab^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\,(ab^4e^x-b^5)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(be^{(2\,x)}+2\,ae^x-b)}+\frac{2\,\left(12\,ab^3e^{(5\,x)}-9\,a^2b^2e^{(4\,x)}+3\,b^4e^{(4\,x)}+8\,a^3be^{(3\,x)}+32\,ab^3e^{(3\,x)}-6\,a^4e^{(2\,x)}-18\,a^2b^2e^{(2\,x)}+12\,b^4e^{(2\,x)}+12\,ab^3e^x-2\,a^4-9\,a^2b^2+5\,b^4\right)}{3\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(e^{(2\,x)}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 5*a*b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(
a*b^4*e^x - b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x
- b)) + 2/3*(12*a*b^3*e^(5*x) - 9*a^2*b^2*e^(4*x) + 3*b^4*e^(4*x) + 8*a^3*b
*e^(3*x) + 32*a*b^3*e^(3*x) - 6*a^4*e^(2*x) - 18*a^2*b^2*e^(2*x) + 12*b^4*e
^(2*x) + 12*a*b^3*e^x - 2*a^4 - 9*a^2*b^2 + 5*b^4)/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.06, size = 476, normalized size = 3.31

$$\frac{\frac{8\,\left(a^2-b^2\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{16\,a\,b\,{\mathrm{e}}^x}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}-\frac{\frac{4\,\left(a^6+a^4\,b^2-a^2\,b^4-b^6\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}-\frac{16\,{\mathrm{e}}^x\,\left(a^5\,b+2\,a^3\,b^3+a\,b^5\right)}{3\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{\frac{2\,\left(3\,a^4\,b^2+2\,a^2\,b^4-b^6\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}-\frac{8\,{\mathrm{e}}^x\,\left(a^3\,b^3+a\,b^5\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{2\,\left(a^2\,b^9+b^{11}\right)}{b^3\,\left(a^2\,b+b^3\right)\,{\left(a^2+b^2\right)}^3}-\frac{2\,{\mathrm{e}}^x\,\left(a^3\,b^9+a\,b^{11}\right)}{b^4\,\left(a^2\,b+b^3\right)\,{\left(a^2+b^2\right)}^3}}{2\,a\,{\mathrm{e}}^x-b+b\,{\mathrm{e}}^{2\,x}}-\frac{5\,a\,b^4\,\ln\left(-\frac{10\,a\,b^3\,\left(b-a\,{\mathrm{e}}^x\right)}{{\left(a^2+b^2\right)}^{7/2}}-\frac{10\,a\,b^3\,{\mathrm{e}}^x}{{\left(a^2+b^2\right)}^3}\right)}{{\left(a^2+b^2\right)}^{7/2}}+\frac{5\,a\,b^4\,\ln\left(\frac{10\,a\,b^3\,\left(b-a\,{\mathrm{e}}^x\right)}{{\left(a^2+b^2\right)}^{7/2}}-\frac{10\,a\,b^3\,{\mathrm{e}}^x}{{\left(a^2+b^2\right)}^3}\right)}{{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(2*a^2*b^4 - b^6 + 3*a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (8*exp(x)*(
a*b^5 + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1) - ((2*(b^11 + a
^2*b^9))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a*b^11 + a^3*b^9))/
(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*exp(2*x)) - (5*a*b^4
*log(- (10*a*b^3*(b - a*exp(x)))/(a^2 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2
 + b^2)^3))/(a^2 + b^2)^(7/2) + (5*a*b^4*log((10*a*b^3*(b - a*exp(x)))/(a^2
 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2 + b^2)^3))/(a^2 + b^2)^(7/2)
```

3.208 $\int \frac{\tanh^4(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=31

$$-\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}-\frac{1}{5}i\tanh^5(x)$$

[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5-1/5*I*tanh(x)^5

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2686, 200}

$$-\frac{1}{5}i\tanh^5(x)-\frac{1}{5}\operatorname{sech}^5(x)+\frac{2\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(I + Sinh[x]),x]

[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^5(x)\,dx\\ &= -\operatorname{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)-\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)\\ &= -\frac{1}{5}i\tanh^5(x)-\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}-\frac{1}{5}i\tanh^5(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. $2(31) = 62$.
time = 0.10, size = 96, normalized size = 3.10

$$-\frac{200-534\cosh(x)+288\cosh(2x)-178\cosh(3x)+24\cosh(4x)+64i\sinh(x)+178i\sinh(2x)-192i\sinh(3x)+89i\sinh(4x)}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -1/960*(200 - 534*Cosh[x] + 288*Cosh[2*x] - 178*Cosh[3*x] + 24*Cosh[4*x] +
(64*I)*Sinh[x] + (178*I)*Sinh[2*x] - (192*I)*Sinh[3*x] + (89*I)*Sinh[4*x])/
((Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.
time = 0.70, size = 93, normalized size = 3.00

method	result
risch	$-\frac{2\left(25i\,e^{4x}+5\,e^{5x}+21i\,e^{2x}+13\,e^{3x}+15i\,e^{6x}+15\,e^{7x}-9\,e^{x}+3i\right)}{15\left(e^{x}+i\right)^{5}\left(e^{x}-i\right)^{3}}$
default	$\frac{i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{3i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{i}{6\left(\tanh\left(\frac{x}{2}\right)-i\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/(tanh(1/2*x)+I)^3-2/5*I/(tanh(1/2*x)+I)^5-3/8*I/(tanh(1/2*x)+I)+1/(ta
nh(1/2*x)+I)^4+1/2/(tanh(1/2*x)+I)^2+3/8*I/(tanh(1/2*x)-I)+1/6*I/(tanh(1/2*
x)-I)^3+1/4/(tanh(1/2*x)-I)^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(23) = 46$.
time = 0.29, size = 413, normalized size = 13.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 18*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 42*I*e^(-2*x)/(-30*I*e^(-x)
- 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
 + 15*e^(-8*x) - 15) - 26*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3
*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 50*
I*e^(-4*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 10*e^(-5*x)/(-30*I*e^(-x) -
 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
+ 15*e^(-8*x) - 15) + 30*I*e^(-6*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 30
*e^(-7*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 6*I/(-30*I*e^(-x) - 30*e^(-2
*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-
8*x) - 15)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.
time = 0.36, size = 86, normalized size = 2.77

$$-\frac{2\left(15\,e^{(7\,x)}+15i\,e^{(6\,x)}+5\,e^{(5\,x)}+25i\,e^{(4\,x)}+13\,e^{(3\,x)}+21i\,e^{(2\,x)}-9\,e^{x}+3i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*e^(7*x) + 15*I*e^(6*x) + 5*e^(5*x) + 25*I*e^(4*x) + 13*e^(3*x) +
21*I*e^(2*x) - 9*e^x + 3*I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^(6*x) + 6*I*e^(5*x
) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(26) = 52$.

time = 0.10, size = 107, normalized size = 3.45

$$\frac{-30e^{7x}-30ie^{6x}-10e^{5x}-50ie^{4x}-26e^{3x}-42ie^{2x}+18e^{x}-6i}{15e^{8x}+30ie^{7x}+30e^{6x}+90ie^{5x}+90ie^{3x}-30e^{2x}+30ie^{x}-15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x)),x)
```

```
[Out] (-30*exp(7*x) - 30*I*exp(6*x) - 10*exp(5*x) - 50*I*exp(4*x) - 26*exp(3*x) -
 42*I*exp(2*x) + 18*exp(x) - 6*I)/(15*exp(8*x) + 30*I*exp(7*x) + 30*exp(6*x
) + 90*I*exp(5*x) + 90*I*exp(3*x) - 30*exp(2*x) + 30*I*exp(x) - 15)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(23) = 46$.
time = 0.41, size = 53, normalized size = 1.71

$$-\frac{15\,e^{(2\,x)}-24i\,e^{x}-13}{24\,(e^{x}-i)^{3}}-\frac{165\,e^{(4\,x)}+480i\,e^{(3\,x)}-650\,e^{(2\,x)}-400i\,e^{x}+113}{120\,(e^{x}+i)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - 1/120*(165*e^(4*x) + 480*I
*e^(3*x) - 650*e^(2*x) - 400*I*e^x + 113)/(e^x + I)^5
```

Mupad [B]
time = 1.17, size = 231, normalized size = 7.45

$$-\frac{1}{6\,(e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i)}-\frac{\frac{11\,e^{x}}{40}+\frac{1}{8}i}{e^{2\,x}-1+e^{x}\,2i}-\frac{\frac{11\,e^{2\,x}}{40}-\frac{17}{120}+\frac{e^{x}\,1i}{4}}{e^{2\,x}\,3i+e^{3\,x}-3\,e^{x}-i}+\frac{1i}{4\,(1-e^{2\,x}+e^{x}\,2i)}-\frac{5}{8\,(e^{x}-i)}-\frac{11}{40\,(e^{x}+1i)}-\frac{\frac{e^{2\,x}\,3i}{8}+\frac{11\,e^{3\,x}}{40}-\frac{17\,e^{x}}{40}-\frac{1}{8}i}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^{x}\,4i}-\frac{\frac{11\,e^{4\,x}}{40}-\frac{17\,e^{2\,x}}{20}+\frac{11}{40}+\frac{e^{3\,x}\,1i}{2}-\frac{e^{x}\,1i}{2}}{e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,5i-e^{2\,x}\,10i+5\,e^{x}+1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((11*exp(x))/40 + 1i/8)/(exp(2*x) + exp
(x)*2i - 1) - ((11*exp(2*x))/40 + (exp(x)*1i)/4 - 17/120)/(exp(2*x)*3i + ex
p(3*x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5/(8*(exp(x) - 1i)) - 11/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (11*exp(3*
x))/40 - (17*exp(x))/40 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(
x)*4i + 1) - ((exp(3*x)*1i)/2 - (17*exp(2*x))/20 + (11*exp(4*x))/40 - (exp(
x)*1i)/2 + 11/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*
exp(x) + 1i)
```

3.209 $\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}\text{ArcTan}(\sinh(x))-\frac{3}{8}\text{sech}(x)\tanh(x)-\frac{1}{4}\text{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)$$

```
[Out] 3/8*arctan(sinh(x))-3/8*sech(x)*tanh(x)-1/4*sech(x)*tanh(x)^3-1/4*I*tanh(x)
^4
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{3}{8}\text{ArcTan}(\sinh(x))-\frac{1}{4}i\tanh^4(x)-\frac{1}{4}\tanh^3(x)\text{sech}(x)-\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 - (3*Sech[x]*Tanh[x])/8 - (Sech[x]*Tanh[x]^3)/4 - (I/
4)*Tanh[x]^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^3(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^4(x)\,dx\\
&= -\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-i\mathrm{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)+\frac{3}{4}\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= -\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\int \operatorname{sech}(x)\,dx\\
&= \frac{3}{8}\tan^{-1}(\sinh(x))-\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 1.17

$$\frac{1}{8}\left(3\mathrm{ArcTan}(\sinh(x))-\frac{2+i\sinh(x)+5\sinh^2(x)}{(-i+\sinh(x))(i+\sinh(x))^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]] - (2 + I*Sinh[x] + 5*Sinh[x]^2)/((-I + Sinh[x])*(I + Sin
h[x])^2))/8
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(27) = 54.
time = 0.72, size = 79, normalized size = 2.19

method	result
risch	$-\frac{2i e^{4x}-2e^{3x}-2i e^{2x}+5e^{5x}+5e^{x}}{4(e^{x}+i)^4(e^{x}-i)^2}-\frac{3i\ln(e^{x}-i)}{8}+\frac{3i\ln(e^{x}+i)}{8}$
default	$-\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{8}+\frac{i}{4\left(\tanh\left(\frac{x}{2}\right)-i\right)^2}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4i}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^4}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}+\frac{1}{2\tanh(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*I*ln(tanh(1/2*x)-I)+1/4*I/(tanh(1/2*x)-I)^2+1/4/(tanh(1/2*x)-I)-1/2*I/
(tanh(1/2*x)+I)^4+3/8*I*ln(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^3+1/2/(tanh(1/2
*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(26) = 52.
time = 0.26, size = 95, normalized size = 2.64

$$\frac{5\,e^{(-x)} + 2i\,e^{(-2\,x)} - 2\,e^{(-3\,x)} - 2i\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{-8i\,e^{(-x)} - 4\,e^{(-2\,x)} - 16i\,e^{(-3\,x)} + 4\,e^{(-4\,x)} - 8i\,e^{(-5\,x)} + 4\,e^{(-6\,x)} - 4} + \frac{3}{8}i\,\log\left(i\,e^{(-x)} + 1\right) - \frac{3}{8}i\,\log\left(i\,e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] (5*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I*e^(-4*x) + 5*e^(-5*x))/(-8*I*e^
(-x) - 4*e^(-2*x) - 16*I*e^(-3*x) + 4*e^(-4*x) - 8*I*e^(-5*x) + 4*e^(-6*x)
- 4) + 3/8*I*log(I*e^(-x) + 1) - 3/8*I*log(I*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(26) = 52.
time = 0.43, size = 151, normalized size = 4.19

$$-\frac{3\left(-i\,e^{(6\,x)} + 2\,e^{(5\,x)} - i\,e^{(4\,x)} + 4\,e^{(3\,x)} + i\,e^{(2\,x)} + 2\,e^{x} + i\right)\log\left(e^{x} + i\right) + 3\left(i\,e^{(6\,x)} - 2\,e^{(5\,x)} + i\,e^{(4\,x)} - 4\,e^{(3\,x)} - i\,e^{(2\,x)} - 2\,e^{x} - i\right)\log\left(e^{x} - i\right) + 10\,e^{(5\,x)} + 4i\,e^{(4\,x)} - 4\,e^{(3\,x)} - 4i\,e^{(2\,x)} + 10\,e^{x}}{8\left(e^{(6\,x)} + 2i\,e^{(5\,x)} + e^{(4\,x)} + 4i\,e^{(3\,x)} - e^{(2\,x)} + 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(-I*e^(6*x) + 2*e^(5*x) - I*e^(4*x) + 4*e^(3*x) + I*e^(2*x) + 2*e^x
 + I)*log(e^x + I) + 3*(I*e^(6*x) - 2*e^(5*x) + I*e^(4*x) - 4*e^(3*x) - I*e
^(2*x) - 2*e^x - I)*log(e^x - I) + 10*e^(5*x) + 4*I*e^(4*x) - 4*e^(3*x) - 4
*I*e^(2*x) + 10*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(36) = 72.
time = 0.13, size = 99, normalized size = 2.75

$$\frac{-5e^{5x} - 2ie^{4x} + 2e^{3x} + 2ie^{2x} - 5e^{x}}{4e^{6x} + 8ie^{5x} + 4e^{4x} + 16ie^{3x} - 4e^{2x} + 8ie^{x} - 4} + \operatorname{RootSum}\left(64z^{2} + 9, \left(i \mapsto i\log\left(\frac{8i}{3} + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x)),x)
```

```
[Out] (-5*exp(5*x) - 2*I*exp(4*x) + 2*exp(3*x) + 2*I*exp(2*x) - 5*exp(x))/(4*exp(
6*x) + 8*I*exp(5*x) + 4*exp(4*x) + 16*I*exp(3*x) - 4*exp(2*x) + 8*I*exp(x)
- 4) + RootSum(64*_z**2 + 9, Lambda(_i, _i*log(8*_i/3 + exp(x))))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(26) = 52$.
time = 0.43, size = 92, normalized size = 2.56

$$\frac{3i\,e^{(-x)} - 3i\,e^{x} - 2}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{9i\left(e^{(-x)} - e^{x}\right)^{2} + 4\,e^{(-x)} - 4\,e^{x} + 12i}{32\left(e^{(-x)} - e^{x} - 2i\right)^{2}} + \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{3}{16}i\,\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/32*(9*I*(e^(-x) -
e^x)^2 + 4*e^(-x) - 4*e^x + 12*I)/(e^(-x) - e^x - 2*I)^2 + 3/16*I*log(-e^(-
x) + e^x + 2*I) - 3/16*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.45, size = 113, normalized size = 3.14

$$\frac{3\,\mathrm{atan}\left(\mathrm{e}^{x}\right)}{4} + \frac{3\mathrm{i}}{2\,\left(\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}\right)} - \frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1 + \mathrm{e}^{3\,x}\,4\mathrm{i} - \mathrm{e}^{x}\,4\mathrm{i}\right)} + \frac{1\mathrm{i}}{4\,\left(1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,2\mathrm{i}\right)} - \frac{1}{4\,\left(\mathrm{e}^{x} - \mathrm{i}\right)} - \frac{1}{\mathrm{e}^{x} + 1\mathrm{i}} + \frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (3*atan(exp(x)))/4 + 3i/(2*(exp(2*x) + exp(x)*2i - 1)) - 1i/(2*(exp(3*x)*4i
 - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) + 1i/(4*(exp(x)*2i - exp(2*x) +
1)) - 1/(4*(exp(x) - 1i)) - 1/(exp(x) + 1i) + 1/(exp(2*x)*3i + exp(3*x) - 3
*exp(x) - 1i)
```

3.210 $\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$-\operatorname{sech}(x)+\frac{\operatorname{sech}^3(x)}{3}-\frac{1}{3} i \tanh^3(x)$$

```
[Out] -sech(x)+1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A]
time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2785, 2687, 30, 2686}

$$-\frac{1}{3} i \tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^3(x)\,dx\\ &= \operatorname{Subst}\left(\int x^2\,dx,x,i\tanh(x)\right)+\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{\operatorname{sech}^3(x)}{3}-\frac{1}{3}i\tanh^3(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 67 vs. $2(23) = 46$.
time = 0.04, size = 67, normalized size = 2.91

$$\frac{-3-\cosh(2x)+\cosh(x)(5-5i\sinh(x))+4i\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x]),x]
[Out] (-3 - Cosh[2*x] + Cosh[x]*(5 - (5*I)*Sinh[x]) + (4*I)*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.59, size = 47, normalized size = 2.04

method	result	size
risch	$-\frac{2(3i\mathrm{e}^{2x}+3\,\mathrm{e}^{3x}+i-\mathrm{e}^{x})}{3(\mathrm{e}^{x}-i)(\mathrm{e}^{x}+i)^3}$	37
default	$\frac{i}{2\tanh(\frac{x}{2})-2i}-\frac{2i}{3(\tanh(\frac{x}{2})+i)^3}-\frac{i}{2(\tanh(\frac{x}{2})+i)}+\frac{1}{(\tanh(\frac{x}{2})+i)^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I/(tanh(1/2*x)-I)-2/3*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh
(1/2*x)+I)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(17) = 34$.
time = 0.26, size = 109, normalized size = 4.74

$$\frac{2\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3x)}+3\,e^{(-4x)}-3}+\frac{6i\,e^{(-2x)}}{-6i\,e^{(-x)}-6i\,e^{(-3x)}+3\,e^{(-4x)}-3}-\frac{6\,e^{(-3x)}}{-6i\,e^{(-x)}-6i\,e^{(-3x)}+3\,e^{(-4x)}-3}+\frac{2i}{-6i\,e^{(-x)}-6i\,e^{(-3x)}+3\,e^{(-4x)}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 6*I*e^(-2*x)/(-6*I
*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) - 6*e^(-3*x)/(-6*I*e^(-x) - 6*I*e^
(-3*x) + 3*e^(-4*x) - 3) + 2*I/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.37, size = 38, normalized size = 1.65

$$-\frac{2\left(3\,e^{(3\,x)} + 3i\,e^{(2\,x)} - e^{x} + i\right)}{3\left(e^{(4\,x)} + 2i\,e^{(3\,x)} + 2i\,e^{x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*e^(3*x) + 3*I*e^(2*x) - e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x -
 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 0.06, size = 46, normalized size = 2.00

$$\frac{-6e^{3x} - 6ie^{2x} + 2e^{x} - 2i}{3e^{4x} + 6ie^{3x} + 6ie^{x} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x)),x)
```

```
[Out] (-6*exp(3*x) - 6*I*exp(2*x) + 2*exp(x) - 2*I)/(3*exp(4*x) + 6*I*exp(3*x) +
6*I*exp(x) - 3)
```

Giac [A]
time = 0.40, size = 29, normalized size = 1.26

$$-\frac{1}{2\left(e^{x} - i\right)} - \frac{9\,e^{(2\,x)} + 12i\,e^{x} - 7}{6\left(e^{x} + i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(e^x - I) - 1/6*(9*e^(2*x) + 12*I*e^x - 7)/(e^x + I)^3
```

Mupad [B]
time = 0.67, size = 80, normalized size = 3.48

$$-\frac{\frac{e^x}{2}+\frac{1}{6}\mathrm{i}}{e^{2x}-1+e^x\,2\mathrm{i}}-\frac{\frac{e^{2x}}{2}-\frac{1}{2}+\frac{e^x\,1\mathrm{i}}{3}}{e^{2x}\,3\mathrm{i}+e^{3x}-3\,e^x-\mathrm{i}}-\frac{1}{2\,\left(e^x-\mathrm{i}\right)}-\frac{1}{2\,\left(e^x+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] - (exp(x)/2 + 1i/6)/(exp(2*x) + exp(x)*2i - 1) - (exp(2*x)/2 + (exp(x)*1i)/
3 - 1/2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 1/(2*(exp(x) - 1i)) - 1
/(2*(exp(x) + 1i))
```

3.211 $\int \frac{\tanh(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=26

$$\frac{1}{2}\text{ArcTan}(\sinh(x)) + \frac{1}{2}i\text{sech}^2(x) - \frac{1}{2}\text{sech}(x)\tanh(x)$$

[Out] 1/2*arctan(sinh(x))+1/2*I*sech(x)^2-1/2*sech(x)*tanh(x)

Rubi [A]
time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2785, 2686, 30, 2691, 3855}

$$\frac{1}{2}\text{ArcTan}(\sinh(x)) + \frac{1}{2}i\text{sech}^2(x) - \frac{1}{2}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(I + Sinh[x]),x]

[Out] ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{sech}^2(x)\tanh(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^2(x)\,dx\\ &= -\frac{1}{2}\operatorname{sech}(x)\tanh(x)+i\mathrm{Subst}\left(\int x\,dx,x,\operatorname{sech}(x)\right)+\frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\tan^{-1}(\sinh(x))+\frac{1}{2}i\operatorname{sech}^2(x)-\frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.77

$$\frac{1}{2}\mathrm{ArcTan}(\sinh(x))-\frac{1}{2(i+\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x]),x]
[Out] ArcTan[Sinh[x]]/2 - 1/(2*(I + Sinh[x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.
time = 0.59, size = 45, normalized size = 1.73

method	result	size
risch	$-\frac{e^x}{(e^x+i)^2}-\frac{i\ln(e^x-i)}{2}+\frac{i\ln(e^x+i)}{2}$	31
default	$-\frac{i}{(\tanh(\frac{x}{2})+i)^2}+\frac{i\ln(\tanh(\frac{x}{2})+i)}{2}+\frac{1}{\tanh(\frac{x}{2})+i}-\frac{i\ln(\tanh(\frac{x}{2})-i)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I/(tanh(1/2*x)+I)^2+1/2*I*ln(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)-1/2*I*ln(tan
h(1/2*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.
time = 0.27, size = 42, normalized size = 1.62

$$\frac{e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}+\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) + 1/2*I*log(I*e^(-x) + 1) - 1/2*I*log(I
*e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(18) = 36$.
time = 0.43, size = 55, normalized size = 2.12

$$\frac{\left(i\,e^{(2\,x)}-2\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(-i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}-i\right)-2\,e^{x}}{2\left(e^{(2\,x)}+2i\,e^{x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*e^(2*x) - 2*e^x - I)*log(e^x + I) + (-I*e^(2*x) + 2*e^x + I)*log(e^
x - I) - 2*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A]
time = 0.08, size = 32, normalized size = 1.23

$$\operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)-\frac{e^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x)))) - exp(x)/(exp(2*x)
+ 2*I*exp(x) - 1)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(18) = 36$.
time = 0.41, size = 53, normalized size = 2.04

$$\frac{-i\,e^{(-x)}+i\,e^{x}+2}{4\left(e^{(-x)}-e^{x}-2i\right)}+\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(-I*e^(-x) + I*e^x + 2)/(e^(-x) - e^x - 2*I) + 1/4*I*log(-e^(-x) + e^x
+ 2*I) - 1/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.17, size = 29, normalized size = 1.12

$$\mathrm{atan}(\mathrm{e}^x) + \frac{1\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^x\,2\mathrm{i}} - \frac{1}{\mathrm{e}^x + 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(sinh(x) + 1i),x)
```

```
[Out] atan(exp(x)) + 1i/(exp(2*x) + exp(x)*2i - 1) - 1/(exp(x) + 1i)
```

3.212 $\int \frac{\coth(x)}{i+\sinh(x)} dx$

Optimal. Leaf size=19

$$-i\log(\sinh(x)) + i\log(i + \sinh(x))$$

```
[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2786, 36, 29, 31}

$$i\log(\sinh(x) + i) - i\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Sinh[x]),x]
```

```
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(i+x)}\,dx, x, \sinh(x)\right)$$
$$= -\left(i\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh(x)\right)\right) + i\text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)$$
$$= -i\log(\sinh(x)) + i\log(i+\sinh(x))$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-i\log(\sinh(x)) + i\log(i+\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x]),x]
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Maple [A]
time = 0.54, size = 17, normalized size = 0.89

method	result	size
derivativedivides	$-i\ln(\sinh(x)) + i\ln(i+\sinh(x))$	17
default	$-i\ln(\sinh(x)) + i\ln(i+\sinh(x))$	17
risch	$2i\ln(e^{x}+i) - i\ln(e^{2x}-1)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.
time = 0.27, size = 28, normalized size = 1.47

$$-i\,\log\left(e^{(-x)}+1\right) + 2i\,\log\left(e^{(-x)}-i\right) - i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*log(e^(-x) + 1) + 2*I*log(e^(-x) - I) - I*log(e^(-x) - 1)
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.89

$$-i \log \left(e^{(2x)} - 1\right) + 2i \log \left(e^{x} + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] -I*log(e^(2*x) - 1) + 2*I*log(e^x + I)
```

Sympy [A]
time = 0.09, size = 19, normalized size = 1.00

$$2i \log \left(e^{x} + i\right) - i \log \left(e^{2x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x)
[Out] 2*I*log(exp(x) + I) - I*log(exp(2*x) - 1)
```

Giac [A]
time = 0.41, size = 23, normalized size = 1.21

$$-i \log \left(e^{x} + 1\right) + 2i \log \left(e^{x} + i\right) - i \log \left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -I*log(e^x + 1) + 2*I*log(e^x + I) - I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.15, size = 24, normalized size = 1.26

$$\ln \left(-36\,\mathrm{e}^{x} - 36\mathrm{i}\right)\,2\mathrm{i} - \ln \left(3 - 3\,\mathrm{e}^{2\,x}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(sinh(x) + 1i),x)
[Out] log(- 36*exp(x) - 36i)*2i - log(3 - 3*exp(2*x))*1i
```

3.213 $\int \frac{\coth^2(x)}{i+\sinh(x)}\, dx$

Optimal. Leaf size=12

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

```
[Out] -arctanh(cosh(x))+I*coth(x)
```

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 3852, 8, 3855}

$$-\tanh^{-1}(\cosh(x)) + i\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + I*Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx &= -\left(i\int \operatorname{csch}^2(x)\,dx\right)+\int \operatorname{csch}(x)\,dx\\ &= -\tanh^{-1}(\cosh(x)) - \operatorname{Subst}(\int 1\,dx, x, -i\coth(x))\\ &= -\tanh^{-1}(\cosh(x)) + i\coth(x)\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 0.03, size = 32, normalized size = 2.67

$$\frac{1}{2}i\coth\left(\frac{x}{2}\right)+\log\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}i\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x]),x]
```

```
[Out] (I/2)*Coth[x/2] + Log[Tanh[x/2]] + (I/2)*Tanh[x/2]
```

Maple [**A**]
time = 0.44, size = 23, normalized size = 1.92

method	result	size
default	$\frac{i\tanh(\frac{x}{2})}{2}+\frac{i}{2\tanh(\frac{x}{2})}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	23
risch	$\frac{2i}{e^{2x}-1}+\ln\left(e^{x}-1\right)-\ln\left(e^{x}+1\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.28, size = 27, normalized size = 2.25

$$-\frac{2i}{e^{(-2x)}-1}-\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*I/(e^(-2*x) - 1) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.
time = 0.42, size = 37, normalized size = 3.08

$$-\frac{\left(e^{(2x)}-1\right)\log\left(e^{x}+1\right)-\left(e^{(2x)}-1\right)\log\left(e^{x}-1\right)-2i}{e^{(2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -((e^(2*x) - 1)*log(e^x + 1) - (e^(2*x) - 1)*log(e^x - 1) - 2*I)/(e^(2*x) -
 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.
time = 0.06, size = 22, normalized size = 1.83

$$\log\left(e^{x}-1\right)-\log\left(e^{x}+1\right)+\frac{2i}{e^{2x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x)),x)
```

```
[Out] log(exp(x) - 1) - log(exp(x) + 1) + 2*I/(exp(2*x) - 1)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.
time = 0.42, size = 24, normalized size = 2.00

$$\frac{2i}{e^{(2x)}-1}-\log\left(e^{x}+1\right)+\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*I/(e^(2*x) - 1) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B]
time = 0.18, size = 28, normalized size = 2.33

$$\ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)+\frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(sinh(x) + 1i),x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2i/(exp(2*x) - 1)
```

3.214 $\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=15

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)$$

```
[Out] -csch(x)+1/2*I*csch(x)^2
```

Rubi [A]
time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2785, 2686, 30, 8}

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = -\left(i\int \coth(x)\text{csch}^2(x)\,dx\right) + \int \coth(x)\text{csch}(x)\,dx$$
$$= -(i\text{Subst}(\int 1\,dx, x, -i\text{csch}(x))) - i\text{Subst}(\int x\,dx, x, -i\text{csch}(x))$$
$$= -\text{csch}(x) + \frac{1}{2}i\text{csch}^2(x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\text{csch}(x) + \frac{1}{2}i\text{csch}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.61, size = 34, normalized size = 2.27

method	result	size
risch	$-\frac{2\,e^{x}\left(-ie^{x}+e^{2x}-1\right)}{\left(e^{2x}-1\right)^{2}}$	24
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}+\frac{i}{8\tanh\left(\frac{x}{2}\right)^{2}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)+1/8*I*tanh(1/2*x)^2-1/2/tanh(1/2*x)+1/8*I/tanh(1/2*x)^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(11) = 22$.
time = 0.28, size = 67, normalized size = 4.47

$$\frac{2\,e^{(-x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2i\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} - \frac{2\,e^{(-3\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*I*e^(-2*x)/(2*e^(-2*x) - e^(-4*x)
- 1) - 2*e^(-3*x)/(2*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 0.41, size = 31, normalized size = 2.07

$$-\frac{2\left(e^{(3x)} - i\,e^{(2x)} - e^{x}\right)}{e^{(4x)} - 2\,e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2*(e^(3*x) - I*e^(2*x) - e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.
time = 0.06, size = 32, normalized size = 2.13

$$\frac{-2e^{3x} + 2ie^{2x} + 2e^{x}}{e^{4x} - 2e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x)),x)
```

```
[Out] (-2*exp(3*x) + 2*I*exp(2*x) + 2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 0.42, size = 23, normalized size = 1.53

$$\frac{2\left(e^{(-x)} - e^{x} + i\right)}{\left(e^{(-x)} - e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(-x) - e^x + I)/(e^(-x) - e^x)^2
```

Mupad [B]
time = 0.57, size = 25, normalized size = 1.67

$$\frac{2\,\mathrm{e}^{x}\,\left(1 - \mathrm{e}^{2\,x} + \mathrm{e}^{x}\,1\mathrm{i}\right)}{{\left(\mathrm{e}^{2\,x} - 1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (2*exp(x)*(exp(x)*1i - exp(2*x) + 1))/(exp(2*x) - 1)^2
```

3.215 $\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{3}i\coth^3(x)-\frac{1}{2}\coth(x)\text{csch}(x)$$

```
[Out] -1/2*arctanh(cosh(x))+1/3*I*coth(x)^3-1/2*coth(x)*csch(x)
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{1}{3}i\coth^3(x)-\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] -1/2*ArcTanh[Cosh[x]] + (I/3)*Coth[x]^3 - (Coth[x]*Csch[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^2(x)\operatorname{csch}^2(x)\,dx\right)+\int \coth^2(x)\operatorname{csch}(x)\,dx\\ &= -\frac{1}{2}\coth(x)\operatorname{csch}(x)+\frac{1}{2}\int \operatorname{csch}(x)\,dx-\operatorname{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cosh(x))+\frac{1}{3}i\coth^3(x)-\frac{1}{2}\coth(x)\operatorname{csch}(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 100 vs. $2(26) = 52$.
time = 0.03, size = 100, normalized size = 3.85

$$\frac{1}{6}i\coth\left(\frac{x}{2}\right)-\frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{24}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{1}{6}i\tanh\left(\frac{x}{2}\right)-\frac{1}{24}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x]),x]
[Out] (I/6)*Coth[x/2] - Csch[x/2]^2/8 + (I/24)*Coth[x/2]*Csch[x/2]^2 + Log[Tanh[x
/2]]/2 - Sech[x/2]^2/8 + (I/6)*Tanh[x/2] - (I/24)*Sech[x/2]^2*Tanh[x/2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(19) = 38$.
time = 0.75, size = 59, normalized size = 2.27

method	result	size
risch	$-\frac{-6i e^{4x}+3\,e^{5x}-2i-3\,e^{x}}{3\left(e^{2x}-1\right)^{3}}-\frac{\ln\left(e^{x}+1\right)}{2}+\frac{\ln\left(e^{x}-1\right)}{2}$	46
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\left(\tanh^3\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^2}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^3}+\frac{i}{8\tanh\left(\frac{x}{2}\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2+1/2*ln(tanh(1/2*x)
)-1/8/tanh(1/2*x)^2+1/24*I/tanh(1/2*x)^3+1/8*I/tanh(1/2*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(18) = 36$.
time = 0.28, size = 61, normalized size = 2.35

$$\frac{3\,e^{(-x)} - 6i\,e^{(-4\,x)} - 3\,e^{(-5\,x)} - 2i}{3\,(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1)} - \frac{1}{2}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{2}\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) - 6*I*e^(-4*x) - 3*e^(-5*x) - 2*I)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.
time = 0.37, size = 90, normalized size = 3.46

$$-\frac{3\,\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)\log\left(e^{x} + 1\right) - 3\,\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)\log\left(e^{x} - 1\right) + 6\,e^{(5\,x)} - 12i\,e^{(4\,x)} - 6\,e^{x} - 4i}{6\,\left(e^{(6\,x)} - 3\,e^{(4\,x)} + 3\,e^{(2\,x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x + 1) - 3*(e^(6*x) - 3
*e^(4*x) + 3*e^(2*x) - 1)*log(e^x - 1) + 6*e^(5*x) - 12*I*e^(4*x) - 6*e^x -
 4*I)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 0.09, size = 61, normalized size = 2.35

$$\frac{-3e^{5x} + 6ie^{4x} + 3e^{x} + 2i}{3e^{6x} - 9e^{4x} + 9e^{2x} - 3} + \frac{\log\left(e^{x} - 1\right)}{2} - \frac{\log\left(e^{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x)),x)
```

```
[Out] (-3*exp(5*x) + 6*I*exp(4*x) + 3*exp(x) + 2*I)/(3*exp(6*x) - 9*exp(4*x) + 9*
exp(2*x) - 3) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 0.40, size = 44, normalized size = 1.69

$$-\frac{3\,e^{(5\,x)} - 6i\,e^{(4\,x)} - 3\,e^{x} - 2i}{3\,\left(e^{(2\,x)} - 1\right)^{3}} - \frac{1}{2}\,\log\left(e^{x} + 1\right) + \frac{1}{2}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*e^(5*x) - 6*I*e^(4*x) - 3*e^x - 2*I)/(e^(2*x) - 1)^3 - 1/2*log(e^x
+ 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.31, size = 74, normalized size = 2.85

$$\frac{\ln\left(1-e^x\right)}{2}-\frac{\ln\left(e^x+1\right)}{2}-\frac{e^x}{e^{2\,x}-1}-\frac{2\,e^x}{{\left(e^{2\,x}-1\right)}^2}+\frac{2i}{e^{2\,x}-1}+\frac{4i}{{\left(e^{2\,x}-1\right)}^2}+\frac{8i}{3\,{\left(e^{2\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(exp(x) + 1)/2 - exp(x)/(exp(2*x) - 1) - (2*exp(x))/
(exp(2*x) - 1)^2 + 2i/(exp(2*x) - 1) + 4i/(exp(2*x) - 1)^2 + 8i/(3*(exp(2*x
) - 1)^3)
```

3.216 $\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{4}i\coth^4(x)-\operatorname{csch}(x)-\frac{\operatorname{csch}^3(x)}{3}$$

[Out] 1/4*I*coth(x)^4-csch(x)-1/3*csch(x)^3

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 2687, 30, 2686}

$$\frac{1}{4}i\coth^4(x)-\frac{\operatorname{csch}^3(x)}{3}-\operatorname{csch}(x)$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^5/(I + Sinh[x]),x]

[Out] (I/4)*Coth[x]^4 - Csch[x] - Csch[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^3(x)\text{csch}^2(x)\,dx\right)+\int \coth^3(x)\text{csch}(x)\,dx\\
&= i\text{Subst}\left(\int x^3\,dx,x,i\coth(x)\right)+i\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\text{csch}(x)\right)\\
&= \frac{1}{4}i\coth^4(x)-\text{csch}(x)-\frac{\text{csch}^3(x)}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.43

$$-\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}+\frac{1}{4}i\text{csch}^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 + (I/4)*Csch[x]^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(18) = 36.
time = 0.69, size = 68, normalized size = 2.96

method	result
risch	$-\frac{2\,e^{x}\left(-3i e^{5x}+3\,e^{6x}-5\,e^{4x}-3i e^{x}+5\,e^{2x}-3\right)}{3\left(e^{2x}-1\right)^{4}}$
default	$\frac{3\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{64}+\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}+\frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{16}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{16\tanh\left(\frac{x}{2}\right)^{2}}-\frac{3}{8\tanh\left(\frac{x}{2}\right)}+\frac{i}{64\tanh\left(\frac{x}{2}\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*tanh(1/2*x)+1/64*I*tanh(1/2*x)^4+1/24*tanh(1/2*x)^3+1/16*I*tanh(1/2*x)^
2-1/24/tanh(1/2*x)^3+1/16*I/tanh(1/2*x)^2-3/8/tanh(1/2*x)+1/64*I/tanh(1/2*x
)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(17) = 34.
time = 0.26, size = 205, normalized size = 8.91

$$\frac{2\,e^{(-x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{2i\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{10\,e^{(-3\,x)}}{3\,(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1)}+\frac{10\,e^{(-5\,x)}}{3\,(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1)}-\frac{2i\,e^{(-6\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{2\,e^{(-7\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-2*
x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 10/3*e^(-3*x)/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 10/3*e^(-5*x)/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-6*x)/(4*e^(-2*x) -
 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*e^(-7*x)/(4*e^(-2*x) - 6*e^(-4
*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.
time = 0.44, size = 63, normalized size = 2.74

$$-\frac{2\left(3\,e^{(7\,x)}-3i\,e^{(6\,x)}-5\,e^{(5\,x)}+5\,e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}\right)}{3\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*e^(7*x) - 3*I*e^(6*x) - 5*e^(5*x) + 5*e^(3*x) - 3*I*e^(2*x) - 3*e^x
)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(17) = 34$.
time = 0.09, size = 70, normalized size = 3.04

$$\frac{-6e^{7x}+6ie^{6x}+10e^{5x}-10e^{3x}+6ie^{2x}+6e^{x}}{3e^{8x}-12e^{6x}+18e^{4x}-12e^{2x}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x)),x)
```

```
[Out] (-6*exp(7*x) + 6*I*exp(6*x) + 10*exp(5*x) - 10*exp(3*x) + 6*I*exp(2*x) + 6*
exp(x))/(3*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(17) = 34$.
time = 0.41, size = 51, normalized size = 2.22

$$\frac{2\left(3\left(e^{(-x)}-e^{x}\right)^{3}+3i\left(e^{(-x)}-e^{x}\right)^{2}+4\,e^{(-x)}-4\,e^{x}+6i\right)}{3\left(e^{(-x)}-e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(e^(-x) - e^x)^3 + 3*I*(e^(-x) - e^x)^2 + 4*e^(-x) - 4*e^x + 6*I)/(e
^(-x) - e^x)^4
```

Mupad [B]
time = 0.61, size = 44, normalized size = 1.91

$$\frac{2\,{\mathrm{e}}^x\,\left(5\,{\mathrm{e}}^{4\,x}-5\,{\mathrm{e}}^{2\,x}-3\,{\mathrm{e}}^{6\,x}+3+{\mathrm{e}}^{5\,x}\,3{}\mathrm{i}+{\mathrm{e}}^x\,3{}\mathrm{i}\right)}{3\,{\left({\mathrm{e}}^{2\,x}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (2*exp(x)*(5*exp(4*x) - 5*exp(2*x) + exp(5*x)*3i - 3*exp(6*x) + exp(x)*3i +
 3))/(3*(exp(2*x) - 1)^4)
```

3.217 $\int \frac{\coth^6(x)}{i+\sinh(x)} \, dx$

Optimal. Leaf size=36

$$-\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\text{csch}(x)-\frac{1}{4}\coth^3(x)\text{csch}(x)$$

```
[Out] -3/8*arctanh(cosh(x))+1/5*I*coth(x)^5-3/8*coth(x)*csch(x)-1/4*coth(x)^3*csc
h(x)
```

Rubi [A]
time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{1}{5}i\coth^5(x)-\frac{3}{8}\tanh^{-1}(\cosh(x))-\frac{1}{4}\coth^3(x)\text{csch}(x)-\frac{3}{8}\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/8 + (I/5)*Coth[x]^5 - (3*Coth[x]*Csch[x])/8 - (Coth[x
]^3*Csch[x])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx &= -\left(i\int \coth^4(x)\operatorname{csch}^2(x)\,dx\right)+\int \coth^4(x)\operatorname{csch}(x)\,dx\\ &= -\frac{1}{4}\coth^3(x)\operatorname{csch}(x)+\frac{3}{4}\int \coth^2(x)\operatorname{csch}(x)\,dx+\operatorname{Subst}\left(\int x^4\,dx,x,i\coth(x)\right)\\ &= \frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\operatorname{csch}(x)-\frac{1}{4}\coth^3(x)\operatorname{csch}(x)+\frac{3}{8}\int \operatorname{csch}(x)\,dx\\ &= -\frac{3}{8}\tanh^{-1}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\operatorname{csch}(x)-\frac{1}{4}\coth^3(x)\operatorname{csch}(x)\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 164 vs. $2(36) = 72$.
time = 0.03, size = 164, normalized size = 4.56

$$\frac{1}{10}i\coth\left(\frac{x}{2}\right)-\frac{5}{32}\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{7}{160}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{1}{64}\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{1}{160}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{5}{32}\operatorname{sech}^2\left(\frac{x}{2}\right)+\frac{1}{64}\operatorname{sech}^4\left(\frac{x}{2}\right)+\frac{1}{10}i\tanh\left(\frac{x}{2}\right)-\frac{7}{160}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)+\frac{1}{160}i\operatorname{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x]),x]
[Out] (I/10)*Coth[x/2] - (5*Csch[x/2]^2)/32 + ((7*I)/160)*Coth[x/2]*Csch[x/2]^2 -
 Csch[x/2]^4/64 + (I/160)*Coth[x/2]*Csch[x/2]^4 + (3*Log[Tanh[x/2]])/8 - (5
*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + (I/10)*Tanh[x/2] - ((7*I)/160)*Sech[x/2
]^2*Tanh[x/2] + (I/160)*Sech[x/2]^4*Tanh[x/2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(27) = 54$.
time = 0.76, size = 93, normalized size = 2.58

method	result
risch	$-\frac{-40i\,e^{8x}+25\,e^{9x}-10\,e^{7x}-80i\,e^{4x}+10\,e^{3x}-8i-25\,e^{x}}{20\left(e^{2x}-1\right)^{5}}+\frac{3\ln\left(e^{x}-1\right)}{8}-\frac{3\ln\left(e^{x}+1\right)}{8}$
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{16}+\frac{i\left(\tanh^5\left(\frac{x}{2}\right)\right)}{160}+\frac{\left(\tanh^4\left(\frac{x}{2}\right)\right)}{64}+\frac{i\left(\tanh^3\left(\frac{x}{2}\right)\right)}{32}+\frac{\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{16\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^2}+\frac{i}{160\tanh\left(\frac{x}{2}\right)^5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*tanh(1/2*x)+1/160*I*tanh(1/2*x)^5+1/64*tanh(1/2*x)^4+1/32*I*tanh(1/2
*x)^3+1/8*tanh(1/2*x)^2+1/16*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2+1/160*I/tanh(1
/2*x)^5+3/8*ln(tanh(1/2*x))+1/32*I/tanh(1/2*x)^3-1/64/tanh(1/2*x)^4
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(26) = 52$.
time = 0.27, size = 91, normalized size = 2.53

$$\frac{25\,e^{(-x)}-10\,e^{(-3\,x)}-80i\,e^{(-4\,x)}+10\,e^{(-7\,x)}-40i\,e^{(-8\,x)}-25\,e^{(-9\,x)}-8i}{20\,\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}-\frac{3}{8}\,\log\left(e^{(-x)}+1\right)+\frac{3}{8}\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/20*(25*e^(-x) - 10*e^(-3*x) - 80*I*e^(-4*x) + 10*e^(-7*x) - 40*I*e^(-8*x)
 - 25*e^(-9*x) - 8*I)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
+ e^(-10*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(26) = 52$.
time = 0.36, size = 144, normalized size = 4.00

$$-\frac{15\,\left(e^{(10\,x)}-5\,e^{(8\,x)}+10\,e^{(6\,x)}-10\,e^{(4\,x)}+5\,e^{(2\,x)}-1\right)\log\left(e^{x}+1\right)-15\,\left(e^{(10\,x)}-5\,e^{(8\,x)}+10\,e^{(6\,x)}-10\,e^{(4\,x)}+5\,e^{(2\,x)}-1\right)\log\left(e^{x}-1\right)+50\,e^{(9\,x)}-80i\,e^{(8\,x)}-20\,e^{(7\,x)}-160i\,e^{(4\,x)}+20\,e^{(3\,x)}-50\,e^{x}-16i}{40\,\left(e^{(10\,x)}-5\,e^{(8\,x)}+10\,e^{(6\,x)}-10\,e^{(4\,x)}+5\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/40*(15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)*
log(e^x + 1) - 15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*
x) - 1)*log(e^x - 1) + 50*e^(9*x) - 80*I*e^(8*x) - 20*e^(7*x) - 160*I*e^(4*
x) + 20*e^(3*x) - 50*e^x - 16*I)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^
(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(36) = 72$.
time = 0.12, size = 100, normalized size = 2.78

$$\frac{3\log\left(e^{x}-1\right)}{8}-\frac{3\log\left(e^{x}+1\right)}{8}+\frac{-25e^{9x}+40ie^{8x}+10e^{7x}+80ie^{4x}-10e^{3x}+25e^{x}+8i}{20e^{10x}-100e^{8x}+200e^{6x}-200e^{4x}+100e^{2x}-20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x)),x)
```

```
[Out] 3*log(exp(x) - 1)/8 - 3*log(exp(x) + 1)/8 + (-25*exp(9*x) + 40*I*exp(8*x) +
 10*exp(7*x) + 80*I*exp(4*x) - 10*exp(3*x) + 25*exp(x) + 8*I)/(20*exp(10*x)
 - 100*exp(8*x) + 200*exp(6*x) - 200*exp(4*x) + 100*exp(2*x) - 20)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.
time = 0.42, size = 62, normalized size = 1.72

$$-\frac{25\,e^{(9\,x)} - 40i\,e^{(8\,x)} - 10\,e^{(7\,x)} - 80i\,e^{(4\,x)} + 10\,e^{(3\,x)} - 25\,e^{x} - 8i}{20\,\left(e^{(2\,x)} - 1\right)^{5}} - \frac{3}{8}\,\log\left(e^{x} + 1\right) + \frac{3}{8}\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/20*(25*e^(9*x) - 40*I*e^(8*x) - 10*e^(7*x) - 80*I*e^(4*x) + 10*e^(3*x) -
 25*e^x - 8*I)/(e^(2*x) - 1)^5 - 3/8*log(e^x + 1) + 3/8*log(abs(e^x - 1))
```

Mupad [B]
time = 0.97, size = 124, normalized size = 3.44

$$\frac{3\,\ln\left(\frac{3}{4}-\frac{3\,e^{x}}{4}\right)}{8}-\frac{3\,\ln\left(\frac{3\,e^{x}}{4}+\frac{3}{4}\right)}{8}-\frac{5\,e^{x}}{4\,\left(e^{2\,x}-1\right)}-\frac{9\,e^{x}}{2\,{\left(e^{2\,x}-1\right)}^2}-\frac{6\,e^{x}}{{\left(e^{2\,x}-1\right)}^3}-\frac{4\,e^{x}}{{\left(e^{2\,x}-1\right)}^4}+\frac{2i}{e^{2\,x}-1}+\frac{8i}{{\left(e^{2\,x}-1\right)}^2}+\frac{16i}{{\left(e^{2\,x}-1\right)}^3}+\frac{16i}{{\left(e^{2\,x}-1\right)}^4}+\frac{32i}{5\,{\left(e^{2\,x}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - (3*log((3*exp(x))/4 + 3/4))/8 - (5*exp(x))/
(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(exp(2*x)
 - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 2i/(exp(2*x) - 1) + 8i/(exp(2*x) -
1)^2 + 16i/(exp(2*x) - 1)^3 + 16i/(exp(2*x) - 1)^4 + 32i/(5*(exp(2*x) - 1)^
5)
```

3.218 $\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=47

$$\frac{2}{3}i\text{sech}^3(x)-\frac{4}{5}i\text{sech}^5(x)+\frac{2}{7}i\text{sech}^7(x)-\frac{\tanh^5(x)}{5}+\frac{2\tanh^7(x)}{7}$$

```
[Out] 2/3*I*sech(x)^3-4/5*I*sech(x)^5+2/7*I*sech(x)^7-1/5*tanh(x)^5+2/7*tanh(x)^7
```

Rubi [A]
time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2790, 2687, 14, 2686, 276, 30}

$$\frac{2\tanh^7(x)}{7}-\frac{\tanh^5(x)}{5}+\frac{2}{7}i\text{sech}^7(x)-\frac{4}{5}i\text{sech}^5(x)+\frac{2}{3}i\text{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^5
/5 + (2*Tanh[x]^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx &= \int \left(-\text{sech}^4(x)\tanh^4(x) - 2i\text{sech}^3(x)\tanh^5(x) + \text{sech}^2(x)\tanh^6(x)\right)\,dx\\
&= -\left(2i\int \text{sech}^3(x)\tanh^5(x)\,dx\right) - \int \text{sech}^4(x)\tanh^4(x)\,dx + \int \text{sech}^2(x)\tanh^6(x)\,dx\\
&= i\text{Subst}\left(\int x^6\,dx, x, i\tanh(x)\right) + i\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx, x, i\tanh(x)\right) + 2i\text{Subst}\\
&= \frac{\tanh^7(x)}{7} + i\text{Subst}\left(\int \left(x^4+x^6\right)\,dx, x, i\tanh(x)\right) + 2i\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,d\right.\\
&= \frac{2}{3}i\text{sech}^3(x) - \frac{4}{5}i\text{sech}^5(x) + \frac{2}{7}i\text{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2\tanh^7(x)}{7}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 112 vs. $2(47) = 94$.
time = 0.11, size = 112, normalized size = 2.38

$$-\frac{-672i + 1442i\cosh(x) - 1664i\cosh(2x) + 309i\cosh(3x) + 288i\cosh(4x) - 103i\cosh(5x) + 1232\sinh(x) + 824\sinh(2x) - 1896\sinh(3x) + 412\sinh(4x) + 72\sinh(5x)}{13440\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^7\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -1/13440*(-672*I + (1442*I)*Cosh[x] - (1664*I)*Cosh[2*x] + (309*I)*Cosh[3*x
] + (288*I)*Cosh[4*x] - (103*I)*Cosh[5*x] + 1232*Sinh[x] + 824*Sinh[2*x] -
1896*Sinh[3*x] + 412*Sinh[4*x] + 72*Sinh[5*x])/((Cosh[x/2] - I*Sinh[x/2])^7
*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(34) = 68$.
time = 0.82, size = 116, normalized size = 2.47

method	result
risch	$-\frac{2\left(-132\,e^{2x}-36i e^{x}+68i e^{3x}+14\,e^{4x}+9+84i e^{5x}-140\,e^{6x}+140i e^{7x}+105\,e^{8x}\right)}{105\left(e^{x}-i\right)^{3}\left(e^{x}+i\right)^{7}}$
default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{7\left(\tanh\left(\frac{x}{2}\right)+i\right)^{7}}-\frac{12}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{1}{12\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{8(\tanh(\frac{x}{2})+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^6-I/(tanh(1/2*x)+I)^4-1/8*I/(tanh(1/2*x)+I)^2+4/7/(tanh
(1/2*x)+I)^7-12/5/(tanh(1/2*x)+I)^5-1/12/(tanh(1/2*x)+I)^3-1/8/(tanh(1/2*x)
+I)-1/8*I/(tanh(1/2*x)-I)^2+1/12/(tanh(1/2*x)-I)^3+1/8/(tanh(1/2*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(31) = 62$.
time = 0.27, size = 573, normalized size = 12.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 72*I*e^(-x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) -
 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-1
0*x) + 105) - 264*e^(-2*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) -
1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9
*x) + 105*e^(-10*x) + 105) - 136*I*e^(-3*x)/(420*I*e^(-x) - 315*e^(-2*x) +
840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8
*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 28*e^(-4*x)/(420*I*e^(-x) - 3
15*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*
x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) - 168*I*e^(-5*x)/
(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*
x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
- 280*e^(-6*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x
) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^
(-10*x) + 105) - 280*I*e^(-7*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*
x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*
e^(-9*x) + 105*e^(-10*x) + 105) + 210*e^(-8*x)/(420*I*e^(-x) - 315*e^(-2*x)
 + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^
(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 18/(420*I*e^(-x) - 315*e^(
-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 3
15*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(31) = 62$.
time = 0.40, size = 104, normalized size = 2.21

$$-\frac{2\left(105\,e^{(8\,x)}+140i\,e^{(7\,x)}-140\,e^{(6\,x)}+84i\,e^{(5\,x)}+14\,e^{(4\,x)}+68i\,e^{(3\,x)}-132\,e^{(2\,x)}-36i\,e^{x}+9\right)}{105\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/105*(105*e^(8*x) + 140*I*e^(7*x) - 140*e^(6*x) + 84*I*e^(5*x) + 14*e^(4*
x) + 68*I*e^(3*x) - 132*e^(2*x) - 36*I*e^x + 9)/(e^(10*x) + 4*I*e^(9*x) - 3
*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x) - 3*e^(2*x)
- 4*I*e^x + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(44) = 88$.
time = 0.13, size = 128, normalized size = 2.72

$$\frac{-210e^{8x}-280ie^{7x}+280e^{6x}-168ie^{5x}-28e^{4x}-136ie^{3x}+264e^{2x}+72ie^{x}-18}{105e^{10x}+420ie^{9x}-315e^{8x}+840ie^{7x}-1470e^{6x}-1470e^{4x}-840ie^{3x}-315e^{2x}-420ie^{x}+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] (-210*exp(8*x) - 280*I*exp(7*x) + 280*exp(6*x) - 168*I*exp(5*x) - 28*exp(4*
x) - 136*I*exp(3*x) + 264*exp(2*x) + 72*I*exp(x) - 18)/(105*exp(10*x) + 420
*I*exp(9*x) - 315*exp(8*x) + 840*I*exp(7*x) - 1470*exp(6*x) - 1470*exp(4*x)
 - 840*I*exp(3*x) - 315*exp(2*x) - 420*I*exp(x) + 105)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.
time = 0.42, size = 65, normalized size = 1.38

$$-\frac{-6i\,e^{(2\,x)}-9\,e^{x}+5i}{24\left(e^{x}-i\right)^{3}}-\frac{210i\,e^{(6\,x)}-105\,e^{(5\,x)}+175i\,e^{(4\,x)}-910\,e^{(3\,x)}-756i\,e^{(2\,x)}+427\,e^{x}+31i}{840\left(e^{x}+i\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(-6*I*e^(2*x) - 9*e^x + 5*I)/(e^x - I)^3 - 1/840*(210*I*e^(6*x) - 105
*e^(5*x) + 175*I*e^(4*x) - 910*e^(3*x) - 756*I*e^(2*x) + 427*e^x + 31*I)/(e
^x + I)^7
```

Mupad [B]
time = 4.07, size = 395, normalized size = 8.40

$$-\frac{\frac{25\,e^{4\,x}}{168}-\frac{e^{2\,x}}{4}+\frac{5}{168}+\frac{e^{3\,x}\,5i}{42}+\frac{e^{5\,x}\,1i}{28}-\frac{e^{x}\,5i}{84}}{15\,e^{2\,x}-15\,e^{4\,x}+e^{6\,x}-1-e^{3\,x}\,20i+e^{5\,x}\,6i+e^{x}\,6i}+\frac{1i}{12\,\left(e^{2\,x}\,3i-e^{3\,x}+3\,e^{x}-i\right)}-\frac{\frac{5}{168}+\frac{e^{x}\,1i}{28}}{e^{2\,x}-1+e^{x}\,2i}-\frac{\frac{5\,e^{5\,x}}{28}-\frac{e^{3\,x}}{2}+\frac{e^{4\,x}\,5i}{28}-\frac{e^{2\,x}\,5i}{28}+\frac{e^{6\,x}\,1i}{28}+\frac{5\,e^{x}}{28}-\frac{1}{28}i}{e^{2\,x}\,21i+35\,e^{3\,x}-e^{4\,x}\,35i-21\,e^{5\,x}+e^{6\,x}\,7i+e^{7\,x}-7\,e^{x}-i}-\frac{\frac{e^{2\,x}\,1i}{28}+\frac{5\,e^{x}}{84}+\frac{1}{84}i}{e^{2\,x}\,3i+e^{3\,x}-3\,e^{x}-i}+\frac{1}{8\,\left(1-e^{2\,x}+e^{x}\,2i\right)}+\frac{1i}{4\,\left(e^{x}-i\right)}-\frac{1i}{28\,\left(e^{x}+1i\right)}-\frac{\frac{5\,e^{2\,x}}{56}-\frac{1}{40}+\frac{e^{3\,x}\,1i}{28}+\frac{e^{x}\,1i}{28}}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^{x}\,4i}-\frac{\frac{e^{2\,x}\,1i}{14}+\frac{5\,e^{3\,x}}{42}+\frac{e^{4\,x}\,1i}{28}-\frac{e^{x}}{10}-\frac{1}{84}i}{e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,5i-e^{2\,x}\,10i+5\,e^{x}+1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(sinh(x) + 1i)^2,x)

[Out] 1i/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) - ((exp(3*x)*5i)/42 - exp(2*x)/4 + (25*exp(4*x))/168 + (exp(5*x)*1i)/28 - (exp(x)*5i)/84 + 5/168)/(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1) - ((exp(x)*1i)/28 + 5/168)/(exp(2*x) + exp(x)*2i - 1) - ((exp(4*x)*5i)/28 - exp(3*x)/2 - (exp(2*x)*5i)/28 + (5*exp(5*x))/28 + (exp(6*x)*1i)/28 + (5*exp(x))/28 - 1i/28)/(exp(2*x)*21i + 35*exp(3*x) - exp(4*x)*35i - 21*exp(5*x) + exp(6*x)*7i + exp(7*x) - 7*exp(x) - 1i) - ((exp(2*x)*1i)/28 + (5*exp(x))/84 + 1i/84)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 1/(8*(exp(x)*2i - exp(2*x) + 1)) + 1i/(4*(exp(x) - 1i)) - 1i/(28*(exp(x) + 1i)) - ((5*exp(2*x))/56 + (exp(3*x)*1i)/28 + (exp(x)*1i)/28 - 1/40)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - ((exp(2*x)*1i)/14 + (5*exp(3*x))/42 + (exp(4*x)*1i)/28 - exp(x)/10 - 1i/84)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i)

3.219 $\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=66

$$-\frac{1}{8}i\text{ArcTan}(\sinh(x))-\frac{i}{16(i-\sinh(x))}+\frac{i}{12(i+\sinh(x))^3}-\frac{1}{4(i+\sinh(x))^2}-\frac{3i}{16(i+\sinh(x))}$$

```
[Out] -1/8*I*arctan(sinh(x))-1/16*I/(I-sinh(x))+1/12*I/(I+sinh(x))^3-1/4/(I+sinh(
x))^2-3/16*I/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2786, 90, 209}

$$-\frac{1}{8}i\text{ArcTan}(\sinh(x)) - \frac{i}{16(-\sinh(x)+i)} - \frac{3i}{16(\sinh(x)+i)} - \frac{1}{4(\sinh(x)+i)^2} + \frac{i}{12(\sinh(x)+i)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-1/8*I)*ArcTan[Sinh[x]] - (I/16)/(I - Sinh[x]) + (I/12)/(I + Sinh[x])^3 -
1/(4*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{x^3}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{i}{16(-i+x)^2} - \frac{i}{4(i+x)^4} + \frac{1}{2(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{8\,(1+x^2)}\right) dx, x$$
$$= -\frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))} - \frac{1}{8}i\text{Subst}$$
$$= -\frac{1}{8}i\tan^{-1}(\sinh(x)) - \frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{}{16(i+}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 0.79

$$\frac{1}{48}\left(-6i\text{ArcTan}(\sinh(x)) + \frac{2\left(2-7i\sinh(x)-6\sinh^2(x)-3i\sinh^3(x)\right)}{(-i+\sinh(x))(i+\sinh(x))^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((-6*I)*ArcTan[Sinh[x]] + (2*(2 - (7*I)*Sinh[x] - 6*Sinh[x]^2 - (3*I)*Sinh[
x]^3))/((-I + Sinh[x])*(I + Sinh[x])^3))/48
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(48) = 96.
time = 0.81, size = 114, normalized size = 1.73

method	result
risch	$-\frac{i e^{x}\left(-12i e^{5x}+3\,e^{6x}+40i e^{3x}+19\,e^{4x}-12i e^{x}-19\,e^{2x}-3\right)}{12\left(e^{x}+i\right)^{6}\left(e^{x}-i\right)^{2}}-\frac{\ln\left(e^{x}-i\right)}{8}+\frac{\ln\left(e^{x}+i\right)}{8}$
default	$-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)}+\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)-i\right)^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{8}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{2}{3(\tanh(\frac{x}{2})+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I/(tanh(1/2*x)-I)+1/8/(tanh(1/2*x)-I)^2-1/8*ln(tanh(1/2*x)-I)+2*I/(tan
h(1/2*x)+I)^5-2/3*I/(tanh(1/2*x)+I)^3-1/8*I/(tanh(1/2*x)+I)+2/3/(tanh(1/2*x
)+I)^6-2/(tanh(1/2*x)+I)^4-1/8/(tanh(1/2*x)+I)^2+1/8*ln(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(38) = 76.
time = 0.30, size = 115, normalized size = 1.74

$$\frac{-3i\,e^{(-x)} - 12\,e^{(-2\,x)} - 19i\,e^{(-3\,x)} + 40\,e^{(-4\,x)} + 19i\,e^{(-5\,x)} - 12\,e^{(-6\,x)} + 3i\,e^{(-7\,x)}}{48i\,e^{(-x)} - 48\,e^{(-2\,x)} + 48i\,e^{(-3\,x)} - 120\,e^{(-4\,x)} - 48i\,e^{(-5\,x)} - 48\,e^{(-6\,x)} - 48i\,e^{(-7\,x)} + 12\,e^{(-8\,x)} + 12} - \frac{1}{8}\log\left(e^{(-x)}+i\right) + \frac{1}{8}\log\left(e^{(-x)}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-3*I*e^(-x) - 12*e^(-2*x) - 19*I*e^(-3*x) + 40*e^(-4*x) + 19*I*e^(-5*x) -
12*e^(-6*x) + 3*I*e^(-7*x))/(48*I*e^(-x) - 48*e^(-2*x) + 48*I*e^(-3*x) - 12
0*e^(-4*x) - 48*I*e^(-5*x) - 48*e^(-6*x) - 48*I*e^(-7*x) + 12*e^(-8*x) + 12
) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(38) = 76$.
time = 0.37, size = 197, normalized size = 2.98

$$\frac{3\left(e^{(8x)} + 4i\,e^{(7x)} - 4\,e^{(6x)} + 4i\,e^{(5x)} - 10\,e^{(4x)} - 4i\,e^{(3x)} - 4\,e^{(2x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} + i\right) - 3\left(e^{(8x)} + 4i\,e^{(7x)} - 4\,e^{(6x)} + 4i\,e^{(5x)} - 10\,e^{(4x)} - 4i\,e^{(3x)} - 4\,e^{(2x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} - i\right) - 6i\,e^{(7x)} - 24\,e^{(6x)} - 38i\,e^{(5x)} + 80\,e^{(4x)} + 38i\,e^{(3x)} - 24\,e^{(2x)} + 6i\,e^{x}}{24\left(e^{(8x)} + 4i\,e^{(7x)} - 4\,e^{(6x)} + 4i\,e^{(5x)} - 10\,e^{(4x)} - 4i\,e^{(3x)} - 4\,e^{(2x)} - 4i\,e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I
*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - 3*(e^(8*x) + 4*I*e^(7*x)
 - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x
 + 1)*log(e^x - I) - 6*I*e^(7*x) - 24*e^(6*x) - 38*I*e^(5*x) + 80*e^(4*x) +
 38*I*e^(3*x) - 24*e^(2*x) + 6*I*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) +
4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.
time = 0.13, size = 129, normalized size = 1.95

$$\frac{-3ie^{7x} - 12e^{6x} - 19ie^{5x} + 40e^{4x} + 19ie^{3x} - 12e^{2x} + 3ie^{x}}{12e^{8x} + 48ie^{7x} - 48e^{6x} + 48ie^{5x} - 120e^{4x} - 48ie^{3x} - 48e^{2x} - 48ie^{x} + 12} - \frac{\log\left(e^{x} - i\right)}{8} + \frac{\log\left(e^{x} + i\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (-3*I*exp(7*x) - 12*exp(6*x) - 19*I*exp(5*x) + 40*exp(4*x) + 19*I*exp(3*x)
- 12*exp(2*x) + 3*I*exp(x))/(12*exp(8*x) + 48*I*exp(7*x) - 48*exp(6*x) + 48
*I*exp(5*x) - 120*exp(4*x) - 48*I*exp(3*x) - 48*exp(2*x) - 48*I*exp(x) + 12
) - log(exp(x) - I)/8 + log(exp(x) + I)/8
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(38) = 76$.
time = 0.42, size = 102, normalized size = 1.55

$$\frac{e^{(-x)} - e^{x}}{16\left(e^{(-x)} - e^{x} + 2i\right)} - \frac{11\left(e^{(-x)} - e^{x}\right)^{3} - 102i\left(e^{(-x)} - e^{x}\right)^{2} - 180\,e^{(-x)} + 180\,e^{x} + 104i}{96\left(e^{(-x)} - e^{x} - 2i\right)^{3}} + \frac{1}{16}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{16}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(e^(-x) - e^x)/(e^(-x) - e^x + 2*I) - 1/96*(11*(e^(-x) - e^x)^3 - 102*
I*(e^(-x) - e^x)^2 - 180*e^(-x) + 180*e^x + 104*I)/(e^(-x) - e^x - 2*I)^3 +
 1/16*log(-e^(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 1.50, size = 209, normalized size = 3.17

$$\frac{\ln\left(-\frac{1}{4}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}\right)}{8}-\frac{\ln\left(\frac{1}{4}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}\right)}{8}-\frac{2\mathrm{i}}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}-\frac{11}{8\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{3}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}+\frac{1}{8\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1\mathrm{i}}{8\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{3\mathrm{i}}{8\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{2}{3\,\left(15\,\mathrm{e}^{2\,x}-15\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^{x}\,6\mathrm{i}\right)}+\frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 2i/(exp(4*x)*5i -
 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) - 11/(8*(exp(2*x) +
 exp(x)*2i - 1)) + 3/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)
+ 1/(8*(exp(x)*2i - exp(2*x) + 1)) + 1i/(8*(exp(x) - 1i)) - 3i/(8*(exp(x) +
 1i)) - 2/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(
6*x) + exp(x)*6i - 1)) + 8i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.220 $\int \frac{\tanh^2(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=37

$$\frac{2}{3} i\text{sech}^3(x) - \frac{2}{5} i\text{sech}^5(x) - \frac{\tanh^3(x)}{3} + \frac{2\tanh^5(x)}{5}$$

```
[Out] 2/3*I*sech(x)^3-2/5*I*sech(x)^5-1/3*tanh(x)^3+2/5*tanh(x)^5
```

Rubi [A]
time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2790, 2687, 14, 2686, 30}

$$\frac{2\tanh^5(x)}{5} - \frac{\tanh^3(x)}{3} - \frac{2}{5} i\text{sech}^5(x) + \frac{2}{3} i\text{sech}^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx &= -\int \left(\operatorname{sech}^4(x)\tanh^2(x)+2i\operatorname{sech}^3(x)\tanh^3(x)-\operatorname{sech}^2(x)\tanh^4(x)\right)\,dx\\ &= -\left(2i\int \operatorname{sech}^3(x)\tanh^3(x)\,dx\right)-\int \operatorname{sech}^4(x)\tanh^2(x)\,dx+\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\\ &= -\left(i\text{Subst}\left(\int x^4\,dx,x,i\tanh(x)\right)\right)-i\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(x)\right)-2i\text{Su}\\ &= \frac{\tanh^5(x)}{5}-i\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(x)\right)-2i\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\text{s}\right.\\ &= \frac{2}{3}i\operatorname{sech}^3(x)-\frac{2}{5}i\operatorname{sech}^5(x)-\frac{\tanh^3(x)}{3}+\frac{2\tanh^5(x)}{5}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. $2(37) = 74$.
time = 0.07, size = 84, normalized size = 2.27

$$\frac{80i-55i\cosh(x)-16i\cosh(2x)+11i\cosh(3x)+140\sinh(x)-44\sinh(2x)-4\sinh(3x)}{240\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x])^2,x]
[Out] (80*I - (55*I)*Cosh[x] - (16*I)*Cosh[2*x] + (11*I)*Cosh[3*x] + 140*Sinh[x]
- 44*Sinh[2*x] - 4*Sinh[3*x])/(240*(Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] +
 I*Sinh[x/2]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(27) = 54$.
time = 0.77, size = 70, normalized size = 1.89

method	result	size
risch	$-\frac{2\left(-4i\,{\rm e}^{x}-20\,{\rm e}^{2x}+1+20i\,{\rm e}^{3x}+15\,{\rm e}^{4x}\right)}{15\left({\rm e}^{x}+i\right)^{5}\left({\rm e}^{x}-i\right)}$	43

default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{5}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4i}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(tanh(1/2*x)+I)^4-1/2*I/(tanh(1/2*x)+I)^2+4/5/(tanh(1/2*x)+I)^5-5/3/(ta
nh(1/2*x)+I)^3-1/4/(tanh(1/2*x)+I)+1/4/(tanh(1/2*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(25) = 50$.
time = 0.30, size = 197, normalized size = 5.32

$$\frac{8i\,e^{(-x)}}{60i\,e^{(-x)}-75\,e^{(-2x)}-75\,e^{(-4x)}-60i\,e^{(-5x)}+15\,e^{(-6x)}+15}-\frac{40\,e^{(-2x)}}{60i\,e^{(-x)}-75\,e^{(-2x)}-75\,e^{(-4x)}-60i\,e^{(-5x)}+15\,e^{(-6x)}+15}-\frac{40i\,e^{(-3x)}}{60i\,e^{(-x)}-75\,e^{(-2x)}-75\,e^{(-4x)}-60i\,e^{(-5x)}+15\,e^{(-6x)}+15}+\frac{30\,e^{(-4x)}}{60i\,e^{(-x)}-75\,e^{(-2x)}-75\,e^{(-4x)}-60i\,e^{(-5x)}+15\,e^{(-6x)}+15}+\frac{2}{60i\,e^{(-x)}-75\,e^{(-2x)}-75\,e^{(-4x)}-60i\,e^{(-5x)}+15\,e^{(-6x)}+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 8*I*e^(-x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^
(-6*x) + 15) - 40*e^(-2*x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*
e^(-5*x) + 15*e^(-6*x) + 15) - 40*I*e^(-3*x)/(60*I*e^(-x) - 75*e^(-2*x) - 7
5*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 30*e^(-4*x)/(60*I*e^(-x) -
 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 2/(60*I*e^
(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.
time = 0.38, size = 56, normalized size = 1.51

$$-\frac{2\left(15\,e^{(4\,x)}+20i\,e^{(3\,x)}-20\,e^{(2\,x)}-4i\,e^{x}+1\right)}{15\left(e^{(6\,x)}+4i\,e^{(5\,x)}-5\,e^{(4\,x)}-5\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(15*e^(4*x) + 20*I*e^(3*x) - 20*e^(2*x) - 4*I*e^x + 1)/(e^(6*x) + 4*I
*e^(5*x) - 5*e^(4*x) - 5*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A]
time = 0.09, size = 66, normalized size = 1.78

$$\frac{-30e^{4x}-40ie^{3x}+40e^{2x}+8ie^{x}-2}{15e^{6x}+60ie^{5x}-75e^{4x}-75e^{2x}-60ie^{x}+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-30*exp(4*x) - 40*I*exp(3*x) + 40*exp(2*x) + 8*I*exp(x) - 2)/(15*exp(6*x)
+ 60*I*exp(5*x) - 75*exp(4*x) - 75*exp(2*x) - 60*I*exp(x) + 15)
```

Giac [A]
time = 0.42, size = 41, normalized size = 1.11

$$\frac{i}{4\,(e^x - i)} - \frac{15i\,e^{(4\,x)} + 30\,e^{(3\,x)} + 40i\,e^{(2\,x)} - 50\,e^x - 7i}{60\,(e^x + i)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*I/(e^x - I) - 1/60*(15*I*e^(4*x) + 30*e^(3*x) + 40*I*e^(2*x) - 50*e^x -
 7*I)/(e^x + I)^5
```

Mupad [B]
time = 0.30, size = 139, normalized size = 3.76

$$\frac{(4\,\mathrm{e}^{3\,x} - 4\,\mathrm{e}^{x})\,\left(2\,\mathrm{e}^{4\,x} - \frac{8\,\mathrm{e}^{2\,x}}{3} + \frac{2}{15}\right)\,1\mathrm{i}}{(\mathrm{e}^{2\,x} + 1)^5} - \frac{(\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1)\,\left(2\,\mathrm{e}^{4\,x} - \frac{8\,\mathrm{e}^{2\,x}}{3} + \frac{2}{15}\right)}{(\mathrm{e}^{2\,x} + 1)^5} - \frac{(4\,\mathrm{e}^{3\,x} - 4\,\mathrm{e}^{x})\,\left(\frac{8\,\mathrm{e}^{3\,x}}{3} - \frac{8\,\mathrm{e}^{x}}{15}\right)}{(\mathrm{e}^{2\,x} + 1)^5} - \frac{\left(\frac{8\,\mathrm{e}^{3\,x}}{3} - \frac{8\,\mathrm{e}^{x}}{15}\right)\,(\mathrm{e}^{4\,x} - 6\,\mathrm{e}^{2\,x} + 1)\,1\mathrm{i}}{(\mathrm{e}^{2\,x} + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)*1i)/(exp(2*x)
 + 1)^5 - ((exp(4*x) - 6*exp(2*x) + 1)*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)
)/(exp(2*x) + 1)^5 - ((4*exp(3*x) - 4*exp(x))*((8*exp(3*x))/3 - (8*exp(x))/
15))/(exp(2*x) + 1)^5 - (((8*exp(3*x))/3 - (8*exp(x))/15)*(exp(4*x) - 6*exp
(2*x) + 1)*1i)/(exp(2*x) + 1)^5
```

3.221 $\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{4}i\text{ArcTan}(\sinh(x))-\frac{1}{4(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}$$

```
[Out] -1/4*I*arctan(sinh(x))-1/4/(I+sinh(x))^2-1/4*I/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2786, 78, 209}

$$-\frac{1}{4}i\text{ArcTan}(\sinh(x))-\frac{i}{4(\sinh(x)+i)}-\frac{1}{4(\sinh(x)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-1/4*I)*ArcTan[Sinh[x]] - 1/(4*(I + Sinh[x])^2) - (I/4)/(I + Sinh[x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{x}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{1}{2(i+x)^3} - \frac{i}{4(i+x)^2} + \frac{i}{4\,(1+x^2)}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))} - \frac{1}{4}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)$$
$$= -\frac{1}{4}i\tan^{-1}(\sinh(x)) - \frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.81

$$-\frac{i(\sinh(x) + \text{ArcTan}(\sinh(x))(i+\sinh(x))^2)}{4(i+\sinh(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(I + Sinh[x])^2,x]
[Out] ((-1/4*I)*(Sinh[x] + ArcTan[Sinh[x]]*(I + Sinh[x])^2))/(I + Sinh[x])^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(26) = 52.
time = 0.69, size = 66, normalized size = 1.83

method	result	size
risch	$-\frac{i e^{x}\left(e^{2x}-1\right)}{2\left(e^{x}+i\right)^{4}}+\frac{\ln\left(e^{x}+i\right)}{4}-\frac{\ln\left(e^{x}-i\right)}{4}$	36
default	$\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)}{4}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*I/(tanh(1/2*x)+I)^3-1/2*I/(tanh(1/2*x)+I)+1/(tanh(1/2*x)+I)^4-3/2/(tanh(1
/2*x)+I)^2+1/4*ln(tanh(1/2*x)+I)-1/4*ln(tanh(1/2*x)-I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 0.27, size = 61, normalized size = 1.69

$$\frac{-i\,e^{(-x)} + i\,e^{(-3\,x)}}{8i\,e^{(-x)} - 12\,e^{(-2\,x)} - 8i\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + 2} - \frac{1}{4}\log\left(e^{(-x)} + i\right) + \frac{1}{4}\log\left(e^{(-x)} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-I*e^(-x) + I*e^(-3*x))/(8*I*e^(-x) - 12*e^(-2*x) - 8*I*e^(-3*x) + 2*e^(-4
*x) + 2) - 1/4*log(e^(-x) + I) + 1/4*log(e^(-x) - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(22) = 44$.
time = 0.37, size = 94, normalized size = 2.61

$$\frac{\left(e^{(4x)} + 4i\,e^{(3x)} - 6\,e^{(2x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} + i\right) - \left(e^{(4x)} + 4i\,e^{(3x)} - 6\,e^{(2x)} - 4i\,e^{x} + 1\right)\log\left(e^{x} - i\right) - 2i\,e^{(3x)} + 2i\,e^{x}}{4\left(e^{(4x)} + 4i\,e^{(3x)} - 6\,e^{(2x)} - 4i\,e^{x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - (e^(4
*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 2*I*e^(3*x) + 2
*I*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A]
time = 0.10, size = 58, normalized size = 1.61

$$\frac{-ie^{3x} + ie^{x}}{2e^{4x} + 8ie^{3x} - 12e^{2x} - 8ie^{x} + 2} - \frac{\log\left(e^{x} - i\right)}{4} + \frac{\log\left(e^{x} + i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))**2,x)
```

```
[Out] (-I*exp(3*x) + I*exp(x))/(2*exp(4*x) + 8*I*exp(3*x) - 12*exp(2*x) - 8*I*exp
(x) + 2) - log(exp(x) - I)/4 + log(exp(x) + I)/4
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.
time = 0.44, size = 66, normalized size = 1.83

$$-\frac{3\left(e^{(-x)} - e^{x}\right)^{2} - 20i\,e^{(-x)} + 20i\,e^{x} - 12}{16\left(e^{(-x)} - e^{x} - 2i\right)^{2}} + \frac{1}{8}\log\left(-e^{(-x)} + e^{x} + 2i\right) - \frac{1}{8}\log\left(-e^{(-x)} + e^{x} - 2i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(3*(e^(-x) - e^x)^2 - 20*I*e^(-x) + 20*I*e^x - 12)/(e^(-x) - e^x - 2*
I)^2 + 1/8*log(-e^(-x) + e^x + 2*I) - 1/8*log(-e^(-x) + e^x - 2*I)
```

Mupad [B]
time = 0.87, size = 99, normalized size = 2.75

$$\frac{\ln\left(-\frac{1}{2} + \frac{e^{x}\,1i}{2}\right)}{4} - \frac{\ln\left(\frac{1}{2} + \frac{e^{x}\,1i}{2}\right)}{4} - \frac{3}{2\,\left(e^{2x} - 1 + e^{x}\,2i\right)} + \frac{1}{e^{4x} - 6\,e^{2x} + 1 + e^{3x}\,4i - e^{x}\,4i} - \frac{1i}{2\,\left(e^{x} + 1i\right)} + \frac{2i}{e^{2x}\,3i + e^{3x} - 3\,e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/2 - 1/2)/4 - log((exp(x)*1i)/2 + 1/2)/4 - 3/(2*(exp(2*x) +
exp(x)*2i - 1)) + 1/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) -
 1i/(2*(exp(x) + 1i)) + 2i/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.222 $\int \frac{\coth(x)}{(i+\sinh(x))^2} \, dx$

Optimal. Leaf size=25

$$-\log(\sinh(x)) + \log(i + \sinh(x)) - \frac{i}{i + \sinh(x)}$$

```
[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2786, 46}

$$-\frac{i}{\sinh(x) + i} - \log(\sinh(x)) + \log(\sinh(x) + i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{(i + \sinh(x))^2} \, dx &= \text{Subst}\left(\int \frac{1}{x(i + x)^2} \, dx, x, \sinh(x)\right) \\
&= \text{Subst}\left(\int \left(-\frac{1}{x} + \frac{i}{(i + x)^2} + \frac{1}{i + x}\right) dx, x, \sinh(x)\right) \\
&= -\log(\sinh(x)) + \log(i + \sinh(x)) - \frac{i}{i + \sinh(x)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\log(\sinh(x))+\log(i+\sinh(x))-\frac{i}{i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(I + Sinh[x])^2,x]
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Maple [A]
time = 0.58, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(i+\sinh\left(x\right)\right)-\frac{i}{i+\sinh(x)}$	23
default	$-\ln\left(\sinh\left(x\right)\right)+\ln\left(i+\sinh\left(x\right)\right)-\frac{i}{i+\sinh(x)}$	23
risch	$-\frac{2i\mathrm{e}^{x}}{(\mathrm{e}^{x}+i)^{2}}-\ln\left(\mathrm{e}^{2x}-1\right)+2\ln\left(\mathrm{e}^{x}+i\right)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.
time = 0.27, size = 48, normalized size = 1.92

$$\frac{2i\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-\log\left(e^{(-x)}+1\right)+2\,\log\left(e^{(-x)}-i\right)-\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="maxima")
[Out] 2*I*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - log(e^(-x) + 1) + 2*log(e^(-x) -
I) - log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(19) = 38.
time = 0.40, size = 54, normalized size = 2.16

$$-\frac{\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{(2\,x)}-1\right)-2\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)+2i\,e^{x}}{e^{(2\,x)}+2i\,e^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((e^(2*x) + 2*I*e^x - 1)*log(e^(2*x) - 1) - 2*(e^(2*x) + 2*I*e^x - 1)*log(
e^x + I) + 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.09, size = 36, normalized size = 1.44

$$2\log\left(e^{x}+i\right)-\log\left(e^{2x}-1\right)-\frac{2ie^{x}}{e^{2x}+2ie^{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))**2,x)
```

```
[Out] 2*log(exp(x) + I) - log(exp(2*x) - 1) - 2*I*exp(x)/(exp(2*x) + 2*I*exp(x) -
 1)
```

Giac [A]
time = 0.41, size = 33, normalized size = 1.32

$$-\frac{2i\,e^{x}}{\left(e^{x}+i\right)^{2}}-\log\left(e^{x}+1\right)+2\,\log\left(e^{x}+i\right)-\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*e^x/(e^x + I)^2 - log(e^x + 1) + 2*log(e^x + I) - log(abs(e^x - 1))
```

Mupad [B]
time = 0.30, size = 49, normalized size = 1.96

$$2\,\ln\left(36\,\mathrm{e}^{x}+36\mathrm{i}\right)-\ln\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-3\mathrm{i}\right)-\frac{2}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{2\mathrm{i}}{\mathrm{e}^{x}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] 2*log(36*exp(x) + 36i) - log(exp(2*x)*3i - 3i) - 2/(exp(2*x) + exp(x)*2i -
1) - 2i/(exp(x) + 1i)
```

3.223 $\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=26

$$2i\tanh^{-1}(\cosh(x))+\coth(x)+\frac{2i\coth(x)}{i-\operatorname{csch}(x)}$$

```
[Out] 2*I*arctanh(cosh(x))+3*coth(x)-2*I*coth(x)/(I+sinh(x))
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2788, 3855, 3852, 8, 3862}

$$\coth(x)+2i\tanh^{-1}(\cosh(x))+\frac{2i\coth(x)}{-\operatorname{csch}(x)+i}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx &= \int \left(2-2i\text{csch}(x)-\text{csch}^2(x)+\frac{2i}{-i+\text{csch}(x)}\right)dx\\ &= 2x-2i\int \text{csch}(x)\,dx+2i\int \frac{1}{-i+\text{csch}(x)}\,dx-\int \text{csch}^2(x)\,dx\\ &= 2x+2i\tanh^{-1}(\cosh(x))+\frac{2i\coth(x)}{i-\text{csch}(x)}+i\text{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)+2i\int i\,dx\\ &= 2i\tanh^{-1}(\cosh(x))+\coth(x)+\frac{2i\coth(x)}{i-\text{csch}(x)}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.11, size = 66, normalized size = 2.54

$$\frac{1}{2}\left(\coth\left(\frac{x}{2}\right)+4i\log\left(\cosh\left(\frac{x}{2}\right)\right)-4i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}+\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(I + Sinh[x])^2,x]
[Out] (Coth[x/2] + (4*I)*Log[Cosh[x/2]] - (4*I)*Log[Sinh[x/2]] + (8*Sinh[x/2])/(C
osh[x/2] - I*Sinh[x/2]) + Tanh[x/2])/2
```

Maple [A]
time = 0.75, size = 35, normalized size = 1.35

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}$	35
risch	$-\frac{2i\left(i\mathrm{e}^{x}+2\,\mathrm{e}^{2x}-3\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)}-2i\ln\left(\mathrm{e}^{x}-1\right)+2i\ln\left(\mathrm{e}^{x}+1\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)+4/(tanh(1/2*x)+I)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 0.26, size = 53, normalized size = 2.04

$$\frac{2\left(e^{(-x)} + 2i\,e^{(-2\,x)} - 3i\right)}{e^{(-x)} + i\,e^{(-2\,x)} - e^{(-3\,x)} - i} + 2i\,\log\left(e^{(-x)} + 1\right) - 2i\,\log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + 2*I*e^(-2*x) - 3*I)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) + 2*I*
log(e^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.
time = 0.46, size = 78, normalized size = 3.00

$$-\frac{2\left(\left(-i\,e^{(3\,x)} + e^{(2\,x)} + i\,e^{x} - 1\right)\log\left(e^{x} + 1\right) + \left(i\,e^{(3\,x)} - e^{(2\,x)} - i\,e^{x} + 1\right)\log\left(e^{x} - 1\right) + 2i\,e^{(2\,x)} - e^{x} - 3i\right)}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*((-I*e^(3*x) + e^(2*x) + I*e^x - 1)*log(e^x + 1) + (I*e^(3*x) - e^(2*x)
- I*e^x + 1)*log(e^x - 1) + 2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x) -
 e^x - I)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.10, size = 49, normalized size = 1.88

$$\frac{-4ie^{2x} + 2e^{x} + 6i}{e^{3x} + ie^{2x} - e^{x} - i} + 2\,\mathrm{RootSum}\left(z^{2} + 1, \left(i \mapsto i\log\left(-ii + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-4*I*exp(2*x) + 2*exp(x) + 6*I)/(exp(3*x) + I*exp(2*x) - exp(x) - I) + 2*R
ootSum(_z**2 + 1, Lambda(_i, _i*log(-_i*I + exp(x))))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.43, size = 48, normalized size = 1.85

$$-\frac{2\left(2i\,e^{(2\,x)} - e^{x} - 3i\right)}{e^{(3\,x)} + i\,e^{(2\,x)} - e^{x} - i} + 2i\,\log\left(e^{x} + 1\right) - 2i\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x) - e^x - I) + 2*I*log(e^x
+ 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.75, size = 60, normalized size = 2.31

$$-\ln\left(e^x\,4i-4i\right)\,2i+\ln\left(e^x\,4i+4i\right)\,2i+\frac{2\,e^x-e^{2\,x}\,4i+6i}{e^{2\,x}\,1i+e^{3\,x}-e^x-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + (2*exp(x) - exp(2*x)*4i +
 6i)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.224 $\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=29

$$2i\operatorname{csch}(x)+\frac{\operatorname{csch}^2(x)}{2}+2\log(\sinh(x))-2\log(i+\sinh(x))$$

```
[Out] 2*I*csch(x)+1/2*csch(x)^2+2*ln(sinh(x))-2*ln(I+sinh(x))
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2786, 78}

$$\frac{\operatorname{csch}^2(x)}{2}+2i\operatorname{csch}(x)+2\log(\sinh(x))-2\log(\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx &= -\operatorname{Subst}\left(\int \frac{i-x}{x^3(i+x)}\,dx,x,\sinh(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2i}{x^2}-\frac{2}{x}+\frac{2}{i+x}\right)\,dx,x,\sinh(x)\right)\\
&= 2i\operatorname{csch}(x)+\frac{\operatorname{csch}^2(x)}{2}+2\log(\sinh(x))-2\log(i+\sinh(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$2i\text{csch}(x) + \frac{\text{csch}^2(x)}{2} + 2\log(\sinh(x)) - 2\log(i + \sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Maple [A]
time = 0.97, size = 51, normalized size = 1.76

method	result	size
risch	$\frac{2i e^{x}\left(2\,e^{2x}-2-ie^{x}\right)}{\left(e^{2x}-1\right)^{2}}+2\ln\left(e^{2x}-1\right)-4\ln\left(e^{x}+i\right)$	45
default	$-i\tanh\left(\frac{x}{2}\right)+\frac{\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{8}-4\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\frac{i}{\tanh\left(\frac{x}{2}\right)}+\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I*tanh(1/2*x)+1/8*tanh(1/2*x)^2-4*ln(tanh(1/2*x)+I)+I/tanh(1/2*x)+1/8/tanh
(1/2*x)^2+2*ln(tanh(1/2*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(23) = 46.
time = 0.29, size = 63, normalized size = 2.17

$$-\frac{2\left(2i\,e^{(-x)}+e^{(-2\,x)}-2i\,e^{(-3\,x)}\right)}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}+2\,\log\left(e^{(-x)}+1\right)-4\,\log\left(e^{(-x)}-i\right)+2\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*I*e^(-x) + e^(-2*x) - 2*I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*l
og(e^(-x) + 1) - 4*log(e^(-x) - I) + 2*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(23) = 46.
time = 0.42, size = 70, normalized size = 2.41

$$\frac{2\left(\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{(2\,x)}-1\right)-2\left(e^{(4\,x)}-2\,e^{(2\,x)}+1\right)\log\left(e^{x}+i\right)+2i\,e^{(3\,x)}+e^{(2\,x)}-2i\,e^{x}\right)}{e^{(4\,x)}-2\,e^{(2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) - 2*(e^(4*x) - 2*e^(2*x) + 1)
*log(e^x + I) + 2*I*e^(3*x) + e^(2*x) - 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [A]
time = 0.12, size = 53, normalized size = 1.83

$$\frac{4ie^{3x} + 2e^{2x} - 4ie^{x}}{e^{4x} - 2e^{2x} + 1} - 4\log\left(e^{x} + i\right) + 2\log\left(e^{2x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (4*I*exp(3*x) + 2*exp(2*x) - 4*I*exp(x))/(exp(4*x) - 2*exp(2*x) + 1) - 4*lo
g(exp(x) + I) + 2*log(exp(2*x) - 1)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(23) = 46.
time = 0.41, size = 54, normalized size = 1.86

$$-\frac{2\left(-2i\,e^{(3\,x)} - e^{(2\,x)} + 2i\,e^{x}\right)}{(e^{x} + 1)^{2}(e^{x} - 1)^{2}} + 2\,\log\left(e^{x} + 1\right) - 4\,\log\left(e^{x} + i\right) + 2\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(-2*I*e^(3*x) - e^(2*x) + 2*I*e^x)/((e^x + 1)^2*(e^x - 1)^2) + 2*log(e^x
 + 1) - 4*log(e^x + I) + 2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.26, size = 56, normalized size = 1.93

$$\frac{2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} + 2\,\ln\left(-\mathrm{e}^{2\,x}\,6\mathrm{i} + 6\mathrm{i}\right) - 4\,\ln\left(144\,\mathrm{e}^{x} + 144\mathrm{i}\right) + \frac{2 + \mathrm{e}^{x}\,4\mathrm{i}}{\mathrm{e}^{2\,x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] 2*log(6i - exp(2*x)*6i) - 4*log(144*exp(x) + 144i) + 2/(exp(4*x) - 2*exp(2*
x) + 1) + (exp(x)*4i + 2)/(exp(2*x) - 1)
```

3.225 $\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=28

$$-i\tanh^{-1}(\cosh(x)) - 2\coth(x) + \frac{\coth^3(x)}{3} + i\coth(x)\operatorname{csch}(x)$$

```
[Out] -I*arctanh(cosh(x))-2*coth(x)+1/3*coth(x)^3+I*coth(x)*csch(x)
```

Rubi [A]
time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2787, 2836, 3852, 8, 3853, 3855}

$$\frac{\coth^3(x)}{3} - 2\coth(x) - i\tanh^{-1}(\cosh(x)) + i\coth(x)\operatorname{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx &= \int \operatorname{csch}^4(x)(i-\sinh(x))^2\,dx\\ &= \int \left(\operatorname{csch}^2(x)-2i\operatorname{csch}^3(x)-\operatorname{csch}^4(x)\right)\,dx\\ &= -\left(2i\int \operatorname{csch}^3(x)\,dx\right)+\int \operatorname{csch}^2(x)\,dx-\int \operatorname{csch}^4(x)\,dx\\ &= i\coth(x)\operatorname{csch}(x)+i\int \operatorname{csch}(x)\,dx-i\text{Subst}(\int 1\,dx,x,-i\coth(x))-i\text{Subst}\left(\int (1+\right.\\ &= -i\tanh^{-1}(\cosh(x))-2\coth(x)+\frac{\coth^3(x)}{3}+i\coth(x)\operatorname{csch}(x)\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 107 vs. 2(28) = 56.
time = 0.04, size = 107, normalized size = 3.82

$$-\frac{5}{6}\coth\left(\frac{x}{2}\right)+\frac{1}{4}i\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{24}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)-i\log\left(\cosh\left(\frac{x}{2}\right)\right)+i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{4}i\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{5}{6}\tanh\left(\frac{x}{2}\right)-\frac{1}{24}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*Coth[x/2])/6 + (I/4)*Csch[x/2]^2 + (Coth[x/2]*Csch[x/2]^2)/24 - I*Log[C
osh[x/2]] + I*Log[Sinh[x/2]] + (I/4)*Sech[x/2]^2 - (5*Tanh[x/2])/6 - (Sech[
x/2]^2*Tanh[x/2])/24
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.
time = 0.96, size = 58, normalized size = 2.07

method	result	size
risch	$\frac{2i(3ie^{4x}+3\,e^{5x}-12ie^{2x}+5i-3\,e^{x})}{3(e^{2x}-1)^{3}}+i\ln\left(e^{x}-1\right)-i\ln\left(e^{x}+1\right)$	56
default	$-\frac{7\tanh\left(\frac{x}{2}\right)}{8}+\frac{\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{24}-\frac{i\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{4}+i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^{2}}-\frac{7}{8\tanh\left(\frac{x}{2}\right)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7/8*tanh(1/2*x)+1/24*tanh(1/2*x)^3-1/4*I*tanh(1/2*x)^2+I*ln(tanh(1/2*x))+1
/24/tanh(1/2*x)^3+1/4*I/tanh(1/2*x)^2-7/8/tanh(1/2*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(22) = 44$.
time = 0.28, size = 67, normalized size = 2.39

$$-\frac{2\left(3i\,e^{(-x)}+12\,e^{(-2\,x)}-3\,e^{(-4\,x)}-3i\,e^{(-5\,x)}-5\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}-i\,\log\left(e^{(-x)}+1\right)+i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*I*e^(-x) + 12*e^(-2*x) - 3*e^(-4*x) - 3*I*e^(-5*x) - 5)/(3*e^(-2*x)
 - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(22) = 44$.
time = 0.40, size = 100, normalized size = 3.57

$$-\frac{3\left(i\,e^{(6\,x)}-3i\,e^{(4\,x)}+3i\,e^{(2\,x)}-i\right)\log\left(e^{x}+1\right)+3\left(-i\,e^{(6\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+i\right)\log\left(e^{x}-1\right)-6i\,e^{(5\,x)}+6\,e^{(4\,x)}-24\,e^{(2\,x)}+6i\,e^{x}+10}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^x + 1) + 3*(-I*e^
(6*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + I)*log(e^x - 1) - 6*I*e^(5*x) + 6*e^(4*
x) - 24*e^(2*x) + 6*I*e^x + 10)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.12, size = 66, normalized size = 2.36

$$\operatorname{RootSum}\left(z^{2}+1,\left(i\mapsto i\log\left(ii+e^{x}\right)\right)\right)+\frac{6ie^{5x}-6e^{4x}+24e^{2x}-6ie^{x}-10}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(I+sinh(x))**2,x)
```

```
[Out] RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*I + exp(x)))) + (6*I*exp(5*x) - 6*e
xp(4*x) + 24*exp(2*x) - 6*I*exp(x) - 10)/(3*exp(6*x) - 9*exp(4*x) + 9*exp(2
*x) - 3)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.
time = 0.40, size = 50, normalized size = 1.79

$$-\frac{2\left(-3 i e^{(5 x)}+3 e^{(4 x)}-12 e^{(2 x)}+3 i e^{x}+5\right)}{3\left(e^{(2 x)}-1\right)^{3}}-i \log \left(e^{x}+1\right)+i \log \left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-3*I*e^(5*x) + 3*e^(4*x) - 12*e^(2*x) + 3*I*e^x + 5)/(e^(2*x) - 1)^3
- I*log(e^x + 1) + I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.21, size = 111, normalized size = 3.96

$$-\ln\left(-e^x\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}+\ln\left(-e^x\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}-\frac{\frac{2\,e^{4\,x}}{3}-4\,e^{2\,x}+\frac{2}{3}-\frac{e^{3\,x}\,8\mathrm{i}}{3}+\frac{e^x\,8\mathrm{i}}{3}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1}+\frac{\frac{4}{3}+\frac{e^x\,4\mathrm{i}}{3}}{e^{4\,x}-2\,e^{2\,x}+1}+\frac{-\frac{4}{3}+e^x\,2\mathrm{i}}{e^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] log(2i - exp(x)*2i)*1i - log(- exp(x)*2i - 2i)*1i - ((2*exp(4*x))/3 - (exp(
3*x)*8i)/3 - 4*exp(2*x) + (exp(x)*8i)/3 + 2/3)/(3*exp(2*x) - 3*exp(4*x) + e
xp(6*x) - 1) + ((exp(x)*4i)/3 + 4/3)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*
2i - 4/3)/(exp(2*x) - 1)
```

3.226 $\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}\text{csch}^2(x)+\frac{2}{3}i\text{csch}^3(x)+\frac{\text{csch}^4(x)}{4}$$

```
[Out] -1/2*csch(x)^2+2/3*I*csch(x)^3+1/4*csch(x)^4
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2786, 45}

$$\frac{\text{csch}^4(x)}{4}+\frac{2}{3}i\text{csch}^3(x)-\frac{\text{csch}^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx &= \text{Subst}\left(\int \frac{(i-x)^2}{x^5}\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{x^5}-\frac{2i}{x^4}+\frac{1}{x^3}\right)\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{2}\text{csch}^2(x)+\frac{2}{3}i\text{csch}^3(x)+\frac{\text{csch}^4(x)}{4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-\frac{1}{2}\mathrm{csch}^2(x)+\frac{2}{3}i\mathrm{csch}^3(x)+\frac{\mathrm{csch}^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^5/(I + Sinh[x])^2,x]
[Out] -1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(20) = 40.
time = 0.96, size = 68, normalized size = 2.52

method	result	s
risch	$-\frac{2\,\mathrm{e}^{2x}\left(-8i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}+8i\mathrm{e}^{x}-12\,\mathrm{e}^{2x}+3\right)}{3\left(\mathrm{e}^{2x}-1\right)^{4}}$	4
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{4}+\frac{\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{64}-\frac{i\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{12}-\frac{3\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{16}-\frac{3}{16\tanh\left(\frac{x}{2}\right)^{2}}-\frac{i}{4\tanh\left(\frac{x}{2}\right)}+\frac{1}{64\tanh\left(\frac{x}{2}\right)^{4}}+\frac{i}{12\tanh\left(\frac{x}{2}\right)^{3}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*I*tanh(1/2*x)+1/64*tanh(1/2*x)^4-1/12*I*tanh(1/2*x)^3-3/16*tanh(1/2*x)^
2-3/16/tanh(1/2*x)^2-1/4*I/tanh(1/2*x)+1/64/tanh(1/2*x)^4+1/12*I/tanh(1/2*x
)^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(19) = 38.
time = 0.27, size = 171, normalized size = 6.33

$$\frac{2\,e^{(-2\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}-\frac{16i\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}-\frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}+\frac{16i\,e^{(-5\,x)}}{3\left(4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1\right)}+\frac{2\,e^{(-6\,x)}}{4\,e^{(-2\,x)}-6\,e^{(-4\,x)}+4\,e^{(-6\,x)}-e^{(-8\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
[Out] 2*e^(-2*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 16/3*I*e
^(-3*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 8*e^(-4*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 16/3*I*e^(-5*x)/(4*
e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 2*e^(-6*x)/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(19) = 38.
time = 0.38, size = 59, normalized size = 2.19

$$-\frac{2\left(3\,e^{(6\,x)}-8i\,e^{(5\,x)}-12\,e^{(4\,x)}+8i\,e^{(3\,x)}+3\,e^{(2\,x)}\right)}{3\left(e^{(8\,x)}-4\,e^{(6\,x)}+6\,e^{(4\,x)}-4\,e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*e^(6*x) - 8*I*e^(5*x) - 12*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x))/(e^(8
*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.
time = 0.10, size = 65, normalized size = 2.41

$$\frac{-6e^{6x} + 16ie^{5x} + 24e^{4x} - 16ie^{3x} - 6e^{2x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**5/(I+sinh(x))**2,x)
```

```
[Out] (-6*exp(6*x) + 16*I*exp(5*x) + 24*exp(4*x) - 16*I*exp(3*x) - 6*exp(2*x))/(3
*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Giac [A]
time = 0.42, size = 38, normalized size = 1.41

$$-\frac{2\left(3\left(e^{(-x)} - e^{x}\right)^{2} + 8i\,e^{(-x)} - 8i\,e^{x} - 6\right)}{3\left(e^{(-x)} - e^{x}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*(e^(-x) - e^x)^2 + 8*I*e^(-x) - 8*I*e^x - 6)/(e^(-x) - e^x)^4
```

Mupad [B]
time = 0.61, size = 40, normalized size = 1.48

$$-\frac{2\,e^{2\,x}\,\left(3\,e^{4\,x} - 12\,e^{2\,x} + 3 - e^{3\,x}\,8i + e^{x}\,8i\right)}{3\,\left(e^{2\,x} - 1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] -(2*exp(2*x)*(3*exp(4*x) - exp(3*x)*8i - 12*exp(2*x) + exp(x)*8i + 3))/(3*(
exp(2*x) - 1)^4)
```

3.227 $\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx$

Optimal. Leaf size=48

$$-\frac{1}{4}i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x)$$

```
[Out] -1/4*I*arctanh(cosh(x))-2/3*coth(x)^3+1/5*coth(x)^5+1/4*I*coth(x)*csch(x)+1
/2*I*coth(x)*csch(x)^3
```

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2788, 3852, 8, 3853, 3855}

$$\frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} - \frac{1}{4}i\tanh^{-1}(\cosh(x)) + \frac{1}{2}i\coth(x)\text{csch}^3(x) + \frac{1}{4}i\coth(x)\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-1/4*I)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*C
sch[x] + (I/2)*Coth[x]*Csch[x]^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx &= \int \left(\operatorname{csch}^2(x) - 2i\operatorname{csch}^3(x) - 2i\operatorname{csch}^5(x) - \operatorname{csch}^6(x)\right)\,dx \\
&= -\left(2i\int \operatorname{csch}^3(x)\,dx\right) - 2i\int \operatorname{csch}^5(x)\,dx + \int \operatorname{csch}^2(x)\,dx - \int \operatorname{csch}^6(x)\,dx \\
&= i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3(x) + i\int \operatorname{csch}(x)\,dx - i\text{Subst}(\int 1\,dx, x, -i\coth \\
&= -i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csch}^3 \\
&= -\frac{1}{4}i\tanh^{-1}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\operatorname{csch}(x) + \frac{1}{2}i\coth(x)\operatorname{csc}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. 2(48) = 96.
time = 0.04, size = 175, normalized size = 3.65

$$
-\frac{7}{30}\coth\left(\frac{x}{2}\right)+\frac{1}{16}i\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{19}{480}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{32}i\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{1}{160}\coth\left(\frac{x}{2}\right)\operatorname{csch}^4\left(\frac{x}{2}\right)-\frac{1}{4}i\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{4}i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{16}i\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{32}i\operatorname{sech}^4\left(\frac{x}{2}\right)-\frac{7}{30}\tanh\left(\frac{x}{2}\right)+\frac{19}{480}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)+\frac{1}{160}\operatorname{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-7*Coth[x/2])/30 + (I/16)*Csch[x/2]^2 - (19*Coth[x/2]*Csch[x/2]^2)/480 + (
I/32)*Csch[x/2]^4 + (Coth[x/2]*Csch[x/2]^4)/160 - (I/4)*Log[Cosh[x/2]] + (I
/4)*Log[Sinh[x/2]] + (I/16)*Sech[x/2]^2 - (I/32)*Sech[x/2]^4 - (7*Tanh[x/2]
)/30 + (19*Sech[x/2]^2*Tanh[x/2])/480 + (Sech[x/2]^4*Tanh[x/2])/160
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 1.03, size = 74, normalized size = 1.54

method	result

default	$-\frac{3\tanh\left(\frac{x}{2}\right)}{16}+\frac{\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{160}-\frac{i\left(\tanh^{4}\left(\frac{x}{2}\right)\right)}{32}-\frac{5\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{96}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4}-\frac{3}{16\tanh\left(\frac{x}{2}\right)}+\frac{i}{32\tanh\left(\frac{x}{2}\right)^{4}}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^{3}}$
risch	$\frac{i\left(60i\,e^{8x}+15\,e^{9x}-240i\,e^{6x}+90\,e^{7x}+40i\,e^{4x}-80i\,e^{2x}-90\,e^{3x}+28i-15\,e^{x}\right)}{30\left(e^{2x}-1\right)^{5}}+\frac{i\ln\left(e^{x}-1\right)}{4}-\frac{i\ln\left(e^{x}+1\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*tanh(1/2*x)+1/160*tanh(1/2*x)^5-1/32*I*tanh(1/2*x)^4-5/96*tanh(1/2*x)
^3+1/4*I*ln(tanh(1/2*x))-3/16/tanh(1/2*x)+1/32*I/tanh(1/2*x)^4-5/96/tanh(1/
2*x)^3+1/160/tanh(1/2*x)^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(32) = 64$.
time = 0.28, size = 103, normalized size = 2.15

$$\frac{-15i\,e^{(-x)}-80\,e^{(-2x)}-90i\,e^{(-3x)}+40\,e^{(-4x)}-240\,e^{(-6x)}+90i\,e^{(-7x)}+60\,e^{(-8x)}+15i\,e^{(-9x)}+28}{30\left(5\,e^{(-2x)}-10\,e^{(-4x)}+10\,e^{(-6x)}-5\,e^{(-8x)}+e^{(-10x)}-1\right)}-\frac{1}{4}i\,\log\left(e^{(-x)}+1\right)+\frac{1}{4}i\,\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(-15*I*e^(-x) - 80*e^(-2*x) - 90*I*e^(-3*x) + 40*e^(-4*x) - 240*e^(-6*
x) + 90*I*e^(-7*x) + 60*e^(-8*x) + 15*I*e^(-9*x) + 28)/(5*e^(-2*x) - 10*e^(
-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) - 1/4*I*log(e^(-x) + 1) +
 1/4*I*log(e^(-x) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(32) = 64$.
time = 0.43, size = 160, normalized size = 3.33

$$-\frac{15\left(i\,e^{(10x)}-5i\,e^{(8x)}+10i\,e^{(6x)}-10i\,e^{(4x)}+5i\,e^{(2x)}-i\right)\log\left(e^{x}+1\right)+15\left(-i\,e^{(10x)}+5i\,e^{(8x)}-10i\,e^{(6x)}+10i\,e^{(4x)}-5i\,e^{(2x)}+i\right)\log\left(e^{x}-1\right)-30i\,e^{(9x)}+120\,e^{(8x)}-180i\,e^{(7x)}-480\,e^{(6x)}+80\,e^{(4x)}+180i\,e^{(3x)}-160\,e^{(2x)}+30i\,e^{x}+56}{60\left(e^{(10x)}-5\,e^{(8x)}+10\,e^{(6x)}-10\,e^{(4x)}+5\,e^{(2x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*(I*e^(10*x) - 5*I*e^(8*x) + 10*I*e^(6*x) - 10*I*e^(4*x) + 5*I*e^(
2*x) - I)*log(e^x + 1) + 15*(-I*e^(10*x) + 5*I*e^(8*x) - 10*I*e^(6*x) + 10*
I*e^(4*x) - 5*I*e^(2*x) + I)*log(e^x - 1) - 30*I*e^(9*x) + 120*e^(8*x) - 18
0*I*e^(7*x) - 480*e^(6*x) + 80*e^(4*x) + 180*I*e^(3*x) - 160*e^(2*x) + 30*I
*e^x + 56)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.
time = 0.15, size = 114, normalized size = 2.38

$$\operatorname{RootSum}\left(16z^{2}+1,\left(i\mapsto i\log\left(4ii+e^{x}\right)\right)\right)+\frac{15ie^{9x}-60e^{8x}+90ie^{7x}+240e^{6x}-40e^{4x}-90ie^{3x}+80e^{2x}-15ie^{x}-28}{30e^{10x}-150e^{8x}+300e^{6x}-300e^{4x}+150e^{2x}-30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**6/(I+sinh(x))**2,x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i*I + exp(x)))) + (15*I*exp(9*x)
 - 60*exp(8*x) + 90*I*exp(7*x) + 240*exp(6*x) - 40*exp(4*x) - 90*I*exp(3*x)
 + 80*exp(2*x) - 15*I*exp(x) - 28)/(30*exp(10*x) - 150*exp(8*x) + 300*exp(6
*x) - 300*exp(4*x) + 150*exp(2*x) - 30)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.
time = 0.44, size = 74, normalized size = 1.54

$$-\frac{-15i\,e^{(9\,x)} + 60\,e^{(8\,x)} - 90i\,e^{(7\,x)} - 240\,e^{(6\,x)} + 40\,e^{(4\,x)} + 90i\,e^{(3\,x)} - 80\,e^{(2\,x)} + 15i\,e^{x} + 28}{30\,\left(e^{(2\,x)} - 1\right)^{5}} - \frac{1}{4}i\,\log\left(e^{x} + 1\right) + \frac{1}{4}i\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/30*(-15*I*e^(9*x) + 60*e^(8*x) - 90*I*e^(7*x) - 240*e^(6*x) + 40*e^(4*x)
 + 90*I*e^(3*x) - 80*e^(2*x) + 15*I*e^x + 28)/(e^(2*x) - 1)^5 - 1/4*I*log(e
^x + 1) + 1/4*I*log(abs(e^x - 1))
```

Mupad [B]
time = 0.77, size = 246, normalized size = 5.12

$$-\frac{80\,e^{4\,x} - 160\,e^{2\,x} - 480\,e^{6\,x} + 120\,e^{8\,x} + 56 - \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,15i + \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,15i + e^{3\,x}\,180i - e^{7\,x}\,180i - e^{9\,x}\,30i + e^{x}\,30i + \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,e^{2\,x}\,75i - \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,e^{2\,x}\,75i - \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,e^{4\,x}\,150i + \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,e^{4\,x}\,150i + \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,e^{6\,x}\,150i - \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,e^{6\,x}\,150i - \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,e^{8\,x}\,75i + \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,e^{8\,x}\,75i + \ln\left(-\frac{e^{x}\,1i}{2} - \frac{1}{2}i\right)\,e^{10\,x}\,15i - \ln\left(-\frac{e^{x}\,1i}{2} + \frac{1}{2}i\right)\,e^{10\,x}\,15i}{60\,\left(e^{2\,x} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] -(log(1i/2 - (exp(x)*1i)/2)*15i - log(- (exp(x)*1i)/2 - 1i/2)*15i - 160*exp
(2*x) + exp(3*x)*180i + 80*exp(4*x) - 480*exp(6*x) - exp(7*x)*180i + 120*ex
p(8*x) - exp(9*x)*30i + exp(x)*30i + log(- (exp(x)*1i)/2 - 1i/2)*exp(2*x)*7
5i - log(1i/2 - (exp(x)*1i)/2)*exp(2*x)*75i - log(- (exp(x)*1i)/2 - 1i/2)*e
xp(4*x)*150i + log(1i/2 - (exp(x)*1i)/2)*exp(4*x)*150i + log(- (exp(x)*1i)/
2 - 1i/2)*exp(6*x)*150i - log(1i/2 - (exp(x)*1i)/2)*exp(6*x)*150i - log(- (
exp(x)*1i)/2 - 1i/2)*exp(8*x)*75i + log(1i/2 - (exp(x)*1i)/2)*exp(8*x)*75i
+ log(- (exp(x)*1i)/2 - 1i/2)*exp(10*x)*15i - log(1i/2 - (exp(x)*1i)/2)*exp
(10*x)*15i + 56)/(60*(exp(2*x) - 1)^5)
```

3.228 $\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=124

$$-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}-\frac{a\tanh^3(x)}{3(a^2+b^2)}$$

```
[Out] -2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a^2*b*sec
h(x)/(a^2+b^2)^2-b*sech(x)/(a^2+b^2)+1/3*b*sech(x)^3/(a^2+b^2)-a^3*tanh(x)/
(a^2+b^2)^2-1/3*a*tanh(x)^3/(a^2+b^2)
```

Rubi [A]
time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {2806, 2687, 30, 2686, 3852, 8, 2739, 632, 212}

$$-\frac{a\tanh^3(x)}{3(a^2+b^2)}+\frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)}-\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (a^
2*b*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x])/(a^2 + b^2) + (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a^3*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x]^3)/(3*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a^2+b^2} + \frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx}{a^2+b^2} + \frac{b\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a^2+b^2}$$

$$= -\frac{a^3\int \operatorname{sech}^2(x)\,dx}{(a^2+b^2)^2} + \frac{a^4\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2} + \frac{(a^2b)\int \operatorname{sech}(x)\tanh(x)\,dx}{(a^2+b^2)^2} - \frac{(ia)\text{Subst}\left(\int x^2 \right.}{a^2}$$

$$= -\frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a\tanh^3(x)}{3(a^2+b^2)} - \frac{(ia^3)\,\text{Subst}(\int 1\,dx, x, -i\tanh(x))}{(a^2+b^2)^2} + \frac{(2a^4)\,\text{Su}}{}$$

$$= -\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a^3\tanh(x)}{(a^2+b^2)^2} - \frac{a\tanh^3(x)}{3(a^2+b^2)} - \frac{(4a^4)\,\text{Subst}\left(\int \frac{}{4(}\right.}{}$$

$$= -\frac{2a^4\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a^3\tanh(x)}{(a^2+b^2)^2} - \frac{a\,\mathrm{t}}{3\,(}$$

Mathematica [A]
time = 0.28, size = 108, normalized size = 0.87

$$\frac{\frac{6a^4\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 3b(2a^2+b^2)\operatorname{sech}(x) + (a^2+b^2)\operatorname{sech}^3(x)(b+a\sinh(x)) - a(4a^2+b^2)\tanh(x)}{3(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 3*b*
(2*a^2 + b^2)*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) - a*(4*a^2 +
b^2)*Tanh[x])/(3*(a^2 + b^2)^2)
```

Maple [A]
time = 0.53, size = 163, normalized size = 1.31

method	result
default	$\frac{-2a^3\left(\tanh^5\left(\frac{x}{2}\right)\right)-2a^2b\left(\tanh^4\left(\frac{x}{2}\right)\right)+2\left(-\frac{10}{3}a^3-\frac{4}{3}a\,b^2\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+2\left(-4a^2b-2b^3\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)-2a^3\tanh\left(\frac{x}{2}\right)-\frac{10a^2b}{3}-\frac{4b^3}{3}}{(a^2+b^2)^2\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^3} + \frac{32a}{(16a^4}$
risch	$\frac{-4a^2b\,e^{5x}-2b^3e^{5x}+4a^3e^{4x}+2a\,b^2e^{4x}-\frac{16a^2b\,e^{3x}}{3}-\frac{4b^3e^{3x}}{3}+4a^3e^{2x}-4a^2b\,e^{x}-2b^3e^{x}+\frac{8a^3}{3}+\frac{2a\,b^2}{3}}{(a^4+2a^2b^2+b^4)(1+e^{2x})^3} + \frac{a^4\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a^4b^2}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)^2*(-a^3*tanh(1/2*x)^5-a^2*b*tanh(1/2*x)^4+(-10/3*a^3-4/3*a*b^2)
*tanh(1/2*x)^3+(-4*a^2*b-2*b^3)*tanh(1/2*x)^2-a^3*tanh(1/2*x)-5/3*a^2*b-2/3
*b^3)/(tanh(1/2*x)^2+1)^3+32*a^4/(16*a^4+32*a^2*b^2+16*b^4)/(a^2+b^2)^(1/2)
*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(116) = 232$.
time = 0.49, size = 241, normalized size = 1.94

$$\frac{a^4 \log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^3e^{(-2\,x)}+4\,a^3+ab^2+3\,(2\,a^2b+b^3)e^{(-x)}+2\,(4\,a^2b+b^3)e^{(-3\,x)}+3\,(2\,a^3+ab^2)e^{(-4\,x)}+3\,(2\,a^2b+b^3)e^{(-5\,x)}\right)}{3\left(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^3*e^(-2*x) + 4*a^3 + a
*b^2 + 3*(2*a^2*b + b^3)*e^(-x) + 2*(4*a^2*b + b^3)*e^(-3*x) + 3*(2*a^3 + a
*b^2)*e^(-4*x) + 3*(2*a^2*b + b^3)*e^(-5*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^
4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 +
 2*a^2*b^2 + b^4)*e^(-6*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(116) = 232$.
time = 0.35, size = 1199, normalized size = 9.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b + 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 - 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 + 4*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^3 + 4*(4*a^4*b +
5*a^2*b^3 + b^5 + 15*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 6*(2*a^5 + 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + a^3*b^2)*cosh(x)^2 - 12*(a^5
 + a^3*b^2 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 + 3*(2*a^5 + 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh
(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
```

```
+ b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) - b)) + 6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b + 3*a^2*b^
3 + b^5 + 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 + 3*a^3*b^2 +
a*b^4)*cosh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2 - 4*(a^5 + a^3*b
^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
+ 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 +
3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh
(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x)),x)
[Out] Integral(tanh(x)**4/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.43, size = 197, normalized size = 1.59

$$\frac{a^4 \log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^2be^{(5\,x)} + 3\,b^3e^{(5\,x)} - 6\,a^3e^{(4\,x)} - 3\,ab^2e^{(4\,x)} + 8\,a^2be^{(3\,x)} + 2\,b^3e^{(3\,x)} - 6\,a^3e^{(2\,x)} + 6\,a^2be^x + 3\,b^3e^x - 4\,a^3 - ab^2\right)}{3\,(a^4 + 2\,a^2b^2 + b^4)(e^{(2\,x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^2*b*e^(5*x
) + 3*b^3*e^(5*x) - 6*a^3*e^(4*x) - 3*a*b^2*e^(4*x) + 8*a^2*b*e^(3*x) + 2*b
^3*e^(3*x) - 6*a^3*e^(2*x) + 6*a^2*b*e^x + 3*b^3*e^x - 4*a^3 - a*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.47, size = 654, normalized size = 5.27

$$\frac{\frac{2\,a\,(2\,a^2+b^2)}{(a^2+b^2)^2} - \frac{2\,b\,e^x\,(2\,a^2+b^2)}{(a^2+b^2)^2}}{e^{2\,x}+1} - \frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2} - \frac{8\,e^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\frac{8\,a}{3\,(a^2+b^2)} - \frac{8\,b\,e^x}{3\,(a^2+b^2)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,a^4}{b^2\,\sqrt{a^8}\,(a^2+b^2)^2\,(a^4+2\,a^2\,b^2+b^4)} + \frac{2\,\left(a^5\,\sqrt{a^8}+2\,a^3\,b^2\,\sqrt{a^8}+a\,b^4\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2+b^2)^5}\,(a^4+2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right) - \frac{2\,\left(b^5\,\sqrt{a^8}+2\,a^2\,b^3\,\sqrt{a^8}+a^4\,b\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2+b^2)^5}\,(a^4+2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right)\left(\frac{b^5\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}{2} + \frac{a^4\,b\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}{2} + a^2\,b^3\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}\right)\right)\,\sqrt{a^8}}{\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a + b*sinh(x)),x)

[Out] ((2*a*(2*a^2 + b^2))/(a^2 + b^2)^2 - (2*b*exp(x)*(2*a^2 + b^2))/(a^2 + b^2)^2)/(exp(2*x) + 1) - ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) - (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (2*atan((exp(x)*((2*a^4)/(b^2*(a^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)) + (2*(a^5*(a^8)^(1/2) + 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*(b^5*(a^8)^(1/2) + 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*(a^8)^(1/2))/(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)

3.229 $\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=88

$$\frac{b(3a^2+b^2)\,\mathrm{ArcTan}(\sinh(x))}{2\,(a^2+b^2)^2}+\frac{a^3\log(\cosh(x))}{(a^2+b^2)^2}-\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{\mathrm{sech}^2(x)(a-b\sinh(x))}{2\,(a^2+b^2)}$$

```
[Out] 1/2*b*(3*a^2+b^2)*arctan(sinh(x))/(a^2+b^2)^2+a^3*ln(cosh(x))/(a^2+b^2)^2-a
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(a-b*sinh(x))/(a^2+b^2)
```

Rubi [A]
time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2800, 1661, 815, 649, 209, 266}

$$\frac{b(3a^2+b^2)\,\mathrm{ArcTan}(\sinh(x))}{2\,(a^2+b^2)^2}+\frac{\mathrm{sech}^2(x)(a-b\sinh(x))}{2\,(a^2+b^2)}-\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a^3\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) + (a^3*Log[Cosh[x]])/(a
^2 + b^2)^2 - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(a - b*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$
$$= \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2\left(2a^2+b^2\right)x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$
$$= \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)} - \frac{b^2\left(3a^2b^2+b^4+2a^3x\right)}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)\,dx, x, b\sinh(x)\right)}{2b^2}$$
$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{\text{Subst}\left(\int \frac{3a^2b^2+b^4+2a^3x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$
$$= -\frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} + \frac{a^3\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^2}$$
$$= \frac{b(3a^2+b^2)\tan^{-1}(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{a^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a-}{2\left(a^2+\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.14, size = 153, normalized size = 1.74

$$-\frac{b\text{ArcTan}(\sinh(x))}{2\left(a^2+b^2\right)} + \frac{\left(a^3-i\left(2a^2b+b^3\right)\right)\log(i-\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{\left(a^3+i\left(2a^2b+b^3\right)\right)\log(i+\sinh(x))}{2\left(a^2+b^2\right)^2} - \frac{a^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{a\text{sech}^2(x)}{2\left(a^2+b^2\right)} - \frac{b\text{sech}(x)\tanh(x)}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*ArcTan[Sinh[x]])/(a^2 + b^2) + ((a^3 - I*(2*a^2*b + b^3))*Log[I - S
inh[x]])/(2*(a^2 + b^2)^2) + ((a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[x]])/(
2*(a^2 + b^2)^2) - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (a*Sech[x]^2)/(
2*(a^2 + b^2)) - (b*Sech[x]*Tanh[x])/(2*(a^2 + b^2))
```

Maple [A]
time = 0.58, size = 166, normalized size = 1.89

method	result
default	$\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-a^3-a\,b^2\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+a^3\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+\left(3a^2b+b^3\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}-$
risch	$\frac{e^x\left(-b\,e^{2x}+2a\,e^x+b\right)}{\left(1+e^{2x}\right)^2\left(a^2+b^2\right)}+\frac{3i\ln\left(e^x+i\right)a^2b}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{i\ln\left(e^x+i\right)b^3}{2a^4+4a^2b^2+2b^4}+\frac{\ln\left(e^x+i\right)a^3}{a^4+2a^2b^2+b^4}-\frac{3i\ln\left(e^x-i\right)a^2b}{2\left(a^4+2a^2b^2+b^4\right)}-\frac{i\ln\left(e^x-i\right)b^3}{2\left(a^4+2a^2b^2+b^4\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*x)^3+(-a^3-a*b^2)*tanh
(1/2*x)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+1/2*a^3*ln(
tanh(1/2*x)^2+1)+1/2*(3*a^2*b+b^3)*arctan(tanh(1/2*x)))-8*a^3/(8*a^4+16*a^2
*b^2+8*b^4)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
```

Maxima [A]
time = 0.50, size = 160, normalized size = 1.82

$$-\frac{a^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^3\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(3\,a^2b+b^3\right)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}-\frac{be^{(-x)}-2\,ae^{(-2\,x)}-be^{(-3\,x)}}{a^2+b^2+2\left(a^2+b^2\right)e^{(-2\,x)}+\left(a^2+b^2\right)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + a^3*log(e^
(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (3*a^2*b + b^3)*arctan(e^(-x))/(a^4 +
 2*a^2*b^2 + b^4) - (b*e^(-x) - 2*a*e^(-2*x) - b*e^(-3*x))/(a^2 + b^2 + 2*(
a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(85) = 170.
time = 0.45, size = 655, normalized size = 7.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(x)^3 + (a^2*b + b^3)*sinh(x)^3 - 2*(a^3 + a*b^2)*cosh(
x)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(x))*sinh(x)^2 - ((3*a^2*b +
b^3)*cosh(x)^4 + 4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (3*a^2*b + b^3)*sinh
(x)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(x)^2 + 2*(3*a^2*b + b^3 + 3*
(3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^2*b + b^3)*cosh(x)^3 + (3*a^
2*b + b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b + b^3)*cosh
(x) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh
(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*
cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^3*cosh(x)
^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3
*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*
log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(x)
^2 + 4*(a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh
(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^
4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(85) = 170.
time = 0.42, size = 211, normalized size = 2.40

$$-\frac{a^3 b\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4 b+2\,a^2 b^3+b^5}+\frac{a^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^4+2\,a^2 b^2+b^4\right)}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)\left(3\,a^2 b+b^3\right)}{4\left(a^4+2\,a^2 b^2+b^4\right)}-\frac{a^3\left(e^{(-x)}-e^x\right)^2-2\,a^2 b\left(e^{(-x)}-e^x\right)-2\,b^3\left(e^{(-x)}-e^x\right)-4\,ab^2}{2\left(a^4+2\,a^2 b^2+b^4\right)\left(\left(e^{(-x)}-e^x\right)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^
3*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*(3*a^2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^3
*(e^(-x) - e^x)^2 - 2*a^2*b*(e^(-x) - e^x) - 2*b^3*(e^(-x) - e^x) - 4*a*b^2
)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B]
time = 2.28, size = 291, normalized size = 3.31

$$\frac{\frac{2\,\left(a^3+a\,b^2\right)}{{\left(a^2+b^2\right)}^2}-\frac{{\mathrm{e}}^x\,\left(a^2\,b+b^3\right)}{{\left(a^2+b^2\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,{\mathrm{e}}^x}{a^2+b^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}+\frac{\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,\left(2\,a+b\,1{}\mathrm{i}\right)}{2\,\left(a^2+a\,b\,2{}\mathrm{i}-b^2\right)}-\frac{a^3\,\ln\left(b^7\,{\mathrm{e}}^{2\,x}-16\,a^6\,b-b^7-6\,a^2\,b^5-9\,a^4\,b^3+32\,a^7\,{\mathrm{e}}^x+6\,a^2\,b^5\,{\mathrm{e}}^{2\,x}+9\,a^4\,b^3\,{\mathrm{e}}^{2\,x}+2\,a\,b^6\,{\mathrm{e}}^x+16\,a^6\,b\,{\mathrm{e}}^{2\,x}+12\,a^3\,b^4\,{\mathrm{e}}^x+18\,a^5\,b^2\,{\mathrm{e}}^x\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)\,\left(b+a\,2{}\mathrm{i}\right)}{2\,\left(a^2\,1{}\mathrm{i}+2\,a\,b-b^2\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] ((2*(a*b^2 + a^3))/(a^2 + b^2)^2 - (exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) + (log(exp(x)*1i + 1)*(2*a + b*1i))/(2*(a*b*2i + a^2 - b^2))
 - (a^3*log(b^7*exp(2*x) - 16*a^6*b - b^7 - 6*a^2*b^5 - 9*a^4*b^3 + 32*a^7*
exp(x) + 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) + 16*a^6*
b*exp(2*x) + 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*b^2
) + (log(exp(x) + 1i)*(a*2i + b))/(2*(2*a*b + a^2*1i - b^2*1i))
```

3.230 $\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=69

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

```
[Out] -2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*sech(x)
/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2806, 3852, 8, 2686, 2739, 632, 212}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx &= -\frac{a\int \operatorname{sech}^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh(x)\,dx}{a^2+b^2}\\
&= -\frac{(ia)\text{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)}{a^2+b^2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\text{Subs}}{}\\
&= -\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}-\frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2a^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\operatorname{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 69, normalized size = 1.00

$$\frac{-b\operatorname{sech}(x)+a\left(\frac{2a\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\tanh(x)\right)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x]),x]
[Out] (-(b*Sech[x]) + a*((2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] - Tanh[x]))/(a^2 + b^2)
```

Maple [A]
time = 0.50, size = 84, normalized size = 1.22

method	result	size
default	$\frac{8a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}}+\frac{-2a\tanh\left(\frac{x}{2}\right)-2b}{(a^2+b^2)\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$	84
risch	$\frac{-2\,\mathrm{e}^x b+2a}{(1+\mathrm{e}^{2x})(a^2+b^2)}+\frac{a^2\ln\left(\mathrm{e}^x+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{a^2\ln\left(\mathrm{e}^x+\frac{a(a^2+b^2)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+
b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(tanh(1/2*x)^2+1)
```

Maxima [A]
time = 0.48, size = 89, normalized size = 1.29

$$\frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) - 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.
time = 0.35, size = 257, normalized size = 3.72

$$\frac{2\,a^3+2\,ab^2+\left(a^2\cosh\left(x\right)^2+2\,a^2\cosh\left(x\right)\sinh\left(x\right)+a^2\sinh\left(x\right)^2+a^2\right)\sqrt{a^2+b^2}\,\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-2\,\left(a^2b+b^3\right)\cosh\left(x\right)-2\,\left(a^2b+b^3\right)\sinh\left(x\right)}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\,\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)\sinh\left(x\right)+\left(a^4+2\,a^2b^2+b^4\right)\sinh\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 +
 a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^4
 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2
 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.41, size = 87, normalized size = 1.26

$$\frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,\left(be^x-a\right)}{\left(a^2+b^2\right)\left(e^{(2\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B]
time = 0.94, size = 330, normalized size = 4.78

$$\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2+b^2}}{\mathrm{e}^{2\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\frac{b^3\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}+\frac{a^2\,b\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}\right)\left(\mathrm{e}^x\left(\frac{2\,a^2}{b^2\,\sqrt{a^4}\,{\left(a^2+b^2\right)}^2}+\frac{2\,\left(a^3\,\sqrt{a^4}+a\,b^2\,\sqrt{a^4}\right)}{a\,b^2\,\sqrt{-{\left(a^2+b^2\right)}^3}\,\left(a^2+b^2\right)\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}\right)-\frac{2\,\left(b^3\,\sqrt{a^4}+a^2\,b\,\sqrt{a^4}\right)}{a\,b^2\,\sqrt{-{\left(a^2+b^2\right)}^3}\,\left(a^2+b^2\right)\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}\right)\right)\sqrt{a^4}}{\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan(((b
^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2))/2)*(exp(x)*((2*a^2)/(b^2*(a^4)^(1/2)*(a^2 + b^
2)^2) + (2*(a^3*(a^4)^(1/2) + a*b^2*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(
1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))) - (2*(b^3*(a
^4)^(1/2) + a^2*b*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(
- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))))*(a^4)^(1/2))/(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.231 $\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=48

$$\frac{b\text{ArcTan}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] b*arctan(sinh(x))/(a^2+b^2)+a*ln(cosh(x))/(a^2+b^2)-a*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2800, 815, 649, 209, 266}

$$\frac{b\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/(a^2 + b^2) + (a*Log[Cosh[x]])/(a^2 + b^2) - (a*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)}+\frac{-b^2-ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}-\frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\tan^{-1}(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.75

$$\frac{2b\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+a\log(\cosh(x))-a\log(a+b\sinh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x]),x]
[Out] (2*b*ArcTan[Tanh[x/2]] + a*Log[Cosh[x]] - a*Log[a + b*Sinh[x]])/(a^2 + b^2)
```

Maple [A]
time = 0.51, size = 73, normalized size = 1.52

method	result	size
default	$\frac{2a\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)+4b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^2+2b^2}-\frac{2a\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2a^2+2b^2}$	73
risch	$\frac{i\ln(e^x+i)b}{a^2+b^2}+\frac{\ln(e^x+i)a}{a^2+b^2}-\frac{i\ln(e^x-i)b}{a^2+b^2}+\frac{\ln(e^x-i)a}{a^2+b^2}-\frac{a\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^2+b^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*a^2+2*b^2)*(1/2*a*ln(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))-2*a/(2*a^
2+2*b^2)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)
```

Maxima [A]
time = 0.47, size = 66, normalized size = 1.38

$$-\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}+\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-x))/(a^2 + b^2) - a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Fricas [A]
time = 0.40, size = 57, normalized size = 1.19

$$\frac{2\,b\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-a\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+a\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(x) + sinh(x)) - a*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + a*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.42, size = 89, normalized size = 1.85

$$-\frac{ab\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a^2b+b^3}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\,(a^2+b^2)}+\frac{a\log\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)}{2\,(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

Mupad [B]
time = 1.40, size = 95, normalized size = 1.98

$$\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)}{a-b\,1\mathrm{i}}-\frac{a\,\ln\left(b^3\,\mathrm{e}^{2\,x}-4\,a^2\,b-b^3+8\,a^3\,\mathrm{e}^x+2\,a\,b^2\,\mathrm{e}^x+4\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2+b^2}+\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)\,1\mathrm{i}}{-b+a\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*sinh(x)),x)
```

```
[Out] (log(exp(x)*1i + 1)*1i)/(a*1i - b) + log(exp(x) + 1i)/(a - b*1i) - (a*log(b
^3*exp(2*x) - 4*a^2*b - b^3 + 8*a^3*exp(x) + 2*a*b^2*exp(x) + 4*a^2*b*exp(2
*x)))/(a^2 + b^2)
```

3.232 $\int \frac{\coth(x)}{a+b\sinh(x)} \, dx$

Optimal. Leaf size=20

$$\frac{\log(\sinh(x))}{a} - \frac{\log(a + b\sinh(x))}{a}$$

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2800, 36, 29, 31}

$$\frac{\log(\sinh(x))}{a} - \frac{\log(a + b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sinh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a}$$
$$= \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

Mathematica [**A**]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Maple [**A**]
time = 0.46, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\ln(\sinh(x))}{a} - \frac{\ln(a+b\sinh(x))}{a}$	21
default	$\frac{\ln(\sinh(x))}{a} - \frac{\ln(a+b\sinh(x))}{a}$	21
risch	$\frac{\ln(e^{2x}-1)}{a} - \frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{a}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.
time = 0.28, size = 46, normalized size = 2.30

$$-\frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a} + \frac{\log\left(e^{(-x)} + 1\right)}{a} + \frac{\log\left(e^{(-x)} - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-x) + b*e^(-2*x) - b)/a + log(e^(-x) + 1)/a + log(e^(-x) - 1)/
a
```

Fricas [A]
time = 0.38, size = 40, normalized size = 2.00

$$-\frac{\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="fricas")
[Out] -(log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - sin
h(x))))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth(x)}{a+b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x)
[Out] Integral(coth(x)/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.42, size = 39, normalized size = 1.95

$$-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="giac")
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a + log(abs(-e^(-x) + e^x))/a
```

Mupad [B]
time = 0.89, size = 195, normalized size = 9.75

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2}+b\,\mathrm{e}^x\,\sqrt{-a^2}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^2}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}}-\frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}+4\,a^2\,b^3\,\sqrt{-a^2}\right)\,\left(\frac{1}{8\,a\,b\,{\left(a^2+b^2\right)}^2}-\mathrm{e}^x\,\left(\frac{1}{16\,b^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^4\,b^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^3\,b\,{\left(a^2+b^2\right)}^2}\right)\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)),x)
[Out] (2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) - 2*a*exp(2*x)*(-a^2)^(1/2)
 - b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^(1
/2) + 4*a^2*b^3*(-a^2)^(1/2))*(1/(8*a*b*(a^2 + b^2)^2) - exp(x)*(1/(16*b^2*
(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2
)/(8*a^3*b*(a^2 + b^2)^2))))/(-a^2)^(1/2)
```

3.233 $\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/a^2
```

Rubi [A]
time = 0.17, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 212}

$$-\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/a^2 - Coth[x]/a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{a+b\sinh(x)}\,dx$$
$$= -\frac{\coth(x)}{a} + \frac{i\int \frac{\operatorname{csch}(x)(ib-ia\sinh(x))}{a+b\sinh(x)}\,dx}{a}$$
$$= -\frac{\coth(x)}{a} - \frac{b\int \operatorname{csch}(x)\,dx}{a^2} + \frac{(a^2+b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} + \frac{(2(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} - \frac{(4(a^2+b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\right.\right.}{a^2}$$
$$= \frac{b\tanh^{-1}(\cosh(x))}{a^2} - \frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2} - \frac{\coth(x)}{a}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 1.46

$$-\frac{\operatorname{csch}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\left(a\cosh(x)+\left(2\sqrt{-a^2-b^2}\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+b\log\left(\tanh\left(\frac{x}{2}\right)\right)\right)\sinh(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(Csch[x/2]*Sech[x/2]*(a*Cosh[x] + (2*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Ta
nh[x/2])/Sqrt[-a^2 - b^2]] + b*Log[Tanh[x/2]])*Sinh[x]))/a^2
```

Maple [A]
time = 0.49, size = 81, normalized size = 1.45

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a} - \frac{(-4a^2-4b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}} - \frac{1}{2a\tanh\left(\frac{x}{2}\right)} - \frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$
risch	$-\frac{2}{a(e^{2x}-1)} + \frac{b\ln(e^x+1)}{a^2} - \frac{b\ln(e^x-1)}{a^2} + \frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{a^2} - \frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2+b}}{b}\right.}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.47, size = 97, normalized size = 1.73

$$\frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{\sqrt{a^2+b^2}\,\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{a^2}+\frac{2}{ae^{(-2\,x)}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + sqrt(a^2 + b^2)*log((b*e^(-
x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/a^2 + 2/(a*e^(-
2*x) - a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(52) = 104$.
time = 0.43, size = 228, normalized size = 4.07

$$\frac{\sqrt{a^2+b^2}\,\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2+b^2}\,\left(b\cosh(x)+b\sinh(x)+a\right)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,\left(b\cosh(x)+a\right)\sinh(x)-b}\right)+\left(b\cosh\left(x\right)^2+2\,b\cosh\left(x\right)\sinh\left(x\right)+b\sinh\left(x\right)^2-b\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(b\cosh\left(x\right)^2+2\,b\cosh\left(x\right)\sinh\left(x\right)+b\sinh\left(x\right)^2-b\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-2\,a}{a^2\cosh\left(x\right)^2+2\,a^2\cosh\left(x\right)\sinh\left(x\right)+a^2\sinh\left(x\right)^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log((b^2*c
osh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a
*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) + 1) - (b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) - 1
) - 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.41, size = 95, normalized size = 1.70

$$\frac{b\log\left(e^{x}+1\right)}{a^{2}}-\frac{b\log\left(\left|e^{x}-1\right|\right)}{a^{2}}+\frac{\sqrt{a^{2}+b^{2}}\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{a^{2}}-\frac{2}{a\left(e^{(2\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1))/a^2 + sqrt(a^2 + b^2)*log(abs(2*b*
e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/a^2
- 2/(a*(e^(2*x) - 1))
```

Mupad [B]
time = 0.84, size = 304, normalized size = 5.43

$$\frac{2}{a-a\,\mathrm{e}^{2\,x}}-\frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,\mathrm{e}^x-32\,b^2\,\mathrm{e}^x\right)}{a^2}+\frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,\mathrm{e}^x+32\,b^2\,\mathrm{e}^x\right)}{a^2}+\frac{\ln\left(128\,a^4\,\mathrm{e}^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^x+128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2+b^2}+96\,a\,b^2\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2}-\frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x+64\,a^2\,b\,\sqrt{a^2+b^2}-96\,a\,b^2\,\mathrm{e}^x\,\sqrt{a^2+b^2}\right)\,\sqrt{a^2+b^2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*sinh(x)),x)
```

```
[Out] 2/(a - a*exp(2*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(x) - 32*b^2*exp(x)
))/a^2 + (b*log(32*a^2 + 32*b^2 + 32*a^2*exp(x) + 32*b^2*exp(x)))/a^2 + (lo
g(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*
exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) - 64*a^2*b*(
a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^
2 - (log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 - 64*a^3*b + 128*a^4*exp(x) +
32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) + 64*
a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1
/2))/a^2
```

3.234 $\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=52

$$\frac{b\text{csch}(x)}{a^2}-\frac{\text{csch}^2(x)}{2a}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}$$

```
[Out] b*csch(x)/a^2-1/2*csch(x)^2/a+(a^2+b^2)*ln(sinh(x))/a^3-(a^2+b^2)*ln(a+b*si
nh(x))/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2800, 908}

$$\frac{b\text{csch}(x)}{a^2}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}-\frac{\text{csch}^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[Sinh[x]])/a^3 - ((a^2
+ b^2)*Log[a + b*Sinh[x]])/a^3
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx &= -\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx, x, b\sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx, x, b\sinh(x)\right)\\ &= \frac{b\text{csch}(x)}{a^2}-\frac{\text{csch}^2(x)}{2a}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.87

$$\frac{2ab\text{csch}(x)-a^2\text{csch}^2(x)+2(a^2+b^2)\left(\log(\sinh(x))-\log(a+b\sinh(x))\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + b^2)*(Log[Sinh[x]] - Log[a + b*Si
nh[x]]))/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(50) = 100.
time = 0.45, size = 104, normalized size = 2.00

method	result
risch	$-\frac{2\,\mathrm{e}^{x}\left(-b\,\mathrm{e}^{2x}+a\,\mathrm{e}^{x}+b\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}a^{2}}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{a}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)b^{2}}{a^{3}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)b^{2}}{a^{3}}$
default	$-\frac{\frac{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)}{2}+2b\tanh\left(\frac{x}{2}\right)}{4a^{2}}+\frac{\left(-4a^{2}-4b^{2}\right)\ln\left(a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{4a^{3}}-\frac{1}{8a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^{3}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2*(1/2*a*tanh(1/2*x)^2+2*b*tanh(1/2*x))+1/4/a^3*(-4*a^2-4*b^2)*ln(a*
tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-1/8/a/tanh(1/2*x)^2+1/4/a^3*(4*a^2+4*b^2)*
ln(tanh(1/2*x))+1/2/a^2*b/tanh(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(50) = 100.
time = 0.26, size = 116, normalized size = 2.23

$$-\frac{2\left(be^{(-x)}-ae^{(-2x)}-be^{(-3x)}\right)}{2a^2e^{(-2x)}-a^2e^{(-4x)}-a^2}-\frac{(a^2+b^2)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^3}+\frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{a^3}+\frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) - b*e^(-3*x))/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^
2) - (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^3 + (a^2 + b^2)*log(e^
(-x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(50) = 100$.
time = 0.41, size = 427, normalized size = 8.21

$$\frac{2\,ab\cosh\left(x\right)^{3}+2\,ab\sinh\left(x\right)^{3}-2\,a^{2}\cosh\left(x\right)^{2}-2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^{2}\right)\sinh\left(x\right)^{2}-\left(\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{4}+4\left(a^{2}+b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)^{3}+\left(a^{2}+b^{2}\right)\sinh\left(x\right)^{4}-2\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{2}+2\left(3\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{2}-a^{2}-b^{2}\right)\sinh\left(x\right)^{2}+a^{2}+b^{2}+4\left(\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{3}-\left(a^{2}+b^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\left(b\sinh\left(x\right)+a\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+\left(\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{4}+4\left(a^{2}+b^{2}\right)\cosh\left(x\right)\sinh\left(x\right)^{3}+\left(a^{2}+b^{2}\right)\sinh\left(x\right)^{4}-2\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{2}+2\left(3\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{2}-a^{2}-b^{2}\right)\sinh\left(x\right)^{2}+a^{2}+b^{2}+4\left(\left(a^{2}+b^{2}\right)\cosh\left(x\right)^{3}-\left(a^{2}+b^{2}\right)\cosh\left(x\right)\right)\sinh\left(x\right)\right)\log\left(\frac{2\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)+2\left(3\,ab\cosh\left(x\right)^{2}-2\,a^{2}\cosh\left(x\right)-ab\right)\sinh\left(x\right)}{a^{3}\cosh\left(x\right)^{4}+4\,a^{3}\cosh\left(x\right)\sinh\left(x\right)^{3}+a^{3}\sinh\left(x\right)^{4}-2\,a^{3}\cosh\left(x\right)^{2}+a^{3}+2\left(3\,a^{3}\cosh\left(x\right)^{2}-a^{3}\right)\sinh\left(x\right)^{2}+4\left(a^{3}\cosh\left(x\right)^{3}-a^{3}\cosh\left(x\right)\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 - 2*a*b*cosh(x) + 2*(3
*a*b*cosh(x) - a^2)*sinh(x)^2 - ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh
(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2
 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(
x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(
x))) + ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^2 + b^
2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2 - a^2 -
 b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2)*cosh(x
))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^2
*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{3}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(50) = 100$.
time = 0.43, size = 125, normalized size = 2.40

$$\frac{\left(a^{2}+b^{2}\right)\log\left(\left|-e^{\left(-x\right)}+e^{x}\right|\right)}{a^{3}}-\frac{\left(a^{2}b+b^{3}\right)\log\left(\left|-b\left(e^{\left(-x\right)}-e^{x}\right)+2\,a\right|\right)}{a^{3}b}-\frac{3\,a^{2}\left(e^{\left(-x\right)}-e^{x}\right)^{2}+3\,b^{2}\left(e^{\left(-x\right)}-e^{x}\right)^{2}+4\,ab\left(e^{\left(-x\right)}-e^{x}\right)+4\,a^{2}}{2\,a^{3}\left(e^{\left(-x\right)}-e^{x}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(-e^(-x) + e^x))/a^3 - (a^2*b + b^3)*log(abs(-b*(e^(-x)
- e^x) + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) - e^x)^2 + 3*b^2*(e^(-x) - e^x)
^2 + 4*a*b*(e^(-x) - e^x) + 4*a^2)/(a^3*(e^(-x) - e^x)^2)
```

Mupad [B]
time = 1.24, size = 1163, normalized size = 22.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*sinh(x)),x)
```

```
[Out] ((2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/
2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*(a^2 + b^2)^2) + ((a^7 + a^5*b^2)*
(-a^6)^(1/2))/(2*a^6*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (a^6*b^2*exp(x)*(
-a^6)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*(a^2 + b^2)^2) - (4*(2*a^6*
b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/
2)*(a^2 + b^2)) + (2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b
^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)^(1/2)*
(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^
(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a^6)^(1/2)*(a^2 + b^2)^2)
))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2*exp(2*x)*(-a^6)^(1/2)*((4*(
a^2 + 2*b^2)*(a^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*(a^2 + b^2)^2) + (4*(a^2*(-a
^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2
*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^9*b^2*(-a^6)^(1/2)*(a^2
+ b^2)^2) + (2*(2*a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b
^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (4*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a
^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + b^4
 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2
*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2
*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*
(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a
^6)^(1/2)*(a^2 + b^2)^2))*(-a^6)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)))
- 2*atan((4*a^6*b*(-a^6)^(1/2)*(a^2 + b^2)^2 + 4*a^4*b^3*(-a^6)^(1/2)*(a^2
+ b^2)^2)*(1/(8*a^5*b*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - exp(x)*(1/(16*
a^4*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - (a^2 + 2*b^2)^2/(16*a^8*b^2*
((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)) + (a^2 + 2*b^2)/(8*a^7*b*((a^2 + b^2)
^2)^(1/2)*(a^2 + b^2)^3))))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-a^6)^(1/2) - 2
/(a*(exp(4*x) - 2*exp(2*x) + 1)) - (2/a - (2*b*exp(x))/a^2)/(exp(2*x) - 1)
```

3.235 $\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=108

$$\frac{b(3a^2+2b^2)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{(4a^2+3b^2)\coth(x)}{3a^3}+\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-$$

[Out] 1/2*b*(3*a^2+2*b^2)*arctanh(cosh(x))/a^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^4-1/3*(4*a^2+3*b^2)*coth(x)/a^3+1/2*b*coth(x)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a

Rubi [A]
time = 0.28, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2804, 3134, 3080, 3855, 2739, 632, 212}

$$\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{b(3a^2+2b^2)\tanh^{-1}(\cosh(x))}{2a^4}-\frac{(4a^2+3b^2)\coth(x)}{3a^3}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4/(a + b*Sinh[x]),x]

[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^4 - ((4*a^2 + 3*b^2)*Coth[x])/(3*a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{\int \frac{\operatorname{csch}^2(x)\left(2(4a^2+3b^2)-ab\sinh(x)+3(2a^2+b^2)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^2}$$

$$= -\frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{i\int \frac{\operatorname{csch}(x)(3ib(3a^2+2b^2)-}{a+b\sinh}}{6a^3}$$

$$= -\frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a} + \frac{(a^2+b^2)^2\int \frac{1}{a+b\sinh(x)}\,d}{a^4}$$

$$= \frac{b(3a^2+2b^2)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csc}}{3a}$$

$$= \frac{b(3a^2+2b^2)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csc}}{3a}$$

$$= \frac{b(3a^2+2b^2)\tanh^{-1}(\cosh(x))}{2a^4} - \frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{(4a^2+3b^2)\coth}{3a^3}$$

Mathematica [A]
time = 0.31, size = 176, normalized size = 1.63

$$\frac{48(-a^2-b^2)^{3/2}\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right) - 4a(4a^2+3b^2)\coth\left(\frac{x}{2}\right) + 3a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) - 12b(3a^2+2b^2)\log\left(\tanh\left(\frac{x}{2}\right)\right) + 3a^2b\operatorname{sech}^2\left(\frac{x}{2}\right) + 8a^3\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{1}{2}a^3\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x) - 4a(4a^2+3b^2)\tanh\left(\frac{x}{2}\right)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (48*(-a^2 - b^2)^(3/2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] - 4*a*(4*
a^2 + 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 - 12*b*(3*a^2 + 2*b^2)*Log[Tan
h[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2
]^4*Sinh[x])/2 - 4*a*(4*a^2 + 3*b^2)*Tanh[x/2])/(24*a^4)
```

Maple [A]
time = 0.58, size = 169, normalized size = 1.56

method	result
default	$-\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^2}{3}+ab\left(\tanh^2\left(\frac{x}{2}\right)\right)+5a^2\tanh\left(\frac{x}{2}\right)+4b^2\tanh\left(\frac{x}{2}\right)}{8a^3} - \frac{1}{24a\tanh\left(\frac{x}{2}\right)^3} - \frac{5a^2+4b^2}{8a^3\tanh\left(\frac{x}{2}\right)} + \frac{b}{8a^2\tanh\left(\frac{x}{2}\right)^2} - \frac{b(3a^2+2b^2)}{2a}$
risch	$-\frac{-3ab\,e^{5x}+12a^2e^{4x}+6b^2e^{4x}-12a^2e^{2x}-12b^2e^{2x}+3b\,e^{x}a+8a^2+6b^2}{3a^3(e^{2x}-1)^3} + \frac{(a^2+b^2)^{\frac{3}{2}}\ln\left(e^x-\frac{(a^2+b^2)^{\frac{3}{2}}-a^3-a\,b^2}{b(a^2+b^2)}\right)}{a^4} - \frac{(a^2+b^2)^{\frac{3}{2}}\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^3*(1/3*tanh(1/2*x)^3*a^2+a*b*tanh(1/2*x)^2+5*a^2*tanh(1/2*x)+4*b^2*t
anh(1/2*x))-1/24/a/tanh(1/2*x)^3-1/8*(5*a^2+4*b^2)/a^3/tanh(1/2*x)+1/8/a^2*
b/tanh(1/2*x)^2-1/2/a^4*b*(3*a^2+2*b^2)*ln(tanh(1/2*x))-1/8/a^4*(-16*a^4-32
*a^2*b^2-16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2
)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(96) = 192$.
time = 0.48, size = 212, normalized size = 1.96

$$-\frac{3\,abe^{(-x)} - 3\,abe^{(-5\,x)} - 8\,a^2 - 6\,b^2 + 12\,(a^2+b^2)e^{(-2\,x)} - 6\,(2\,a^2+b^2)e^{(-4\,x)}}{3\,(3\,a^3e^{(-2\,x)} - 3\,a^3e^{(-4\,x)} + a^3e^{(-6\,x)} - a^3)} + \frac{(3\,a^2b + 2\,b^3)\log\left(e^{(-x)}+1\right)}{2\,a^4} - \frac{(3\,a^2b + 2\,b^3)\log\left(e^{(-x)}-1\right)}{2\,a^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b*e^(-x) - 3*a*b*e^(-5*x) - 8*a^2 - 6*b^2 + 12*(a^2 + b^2)*e^(-2*
x) - 6*(2*a^2 + b^2)*e^(-4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6
*x) - a^3) + 1/2*(3*a^2*b + 2*b^3)*log(e^(-x) + 1)/a^4 - 1/2*(3*a^2*b + 2*b
^3)*log(e^(-x) - 1)/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(
a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(96) = 192$.
time = 0.45, size = 1303, normalized size = 12.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^2*b*cosh(x)^5 + 6*a^2*b*sinh(x)^5 - 12*(2*a^3 + a*b^2)*cosh(x)^4 +
 6*(5*a^2*b*cosh(x) - 4*a^3 - 2*a*b^2)*sinh(x)^4 - 6*a^2*b*cosh(x) + 12*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 + a*b^2)*cosh(x))*sinh(x)^3 - 16*a^3 - 12*a*b^2
+ 24*(a^3 + a*b^2)*cosh(x)^2 + 12*(5*a^2*b*cosh(x)^3 + 2*a^3 + 2*a*b^2 - 6*
(2*a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 + b^2)*cosh(x)^6 + 6*(a^2 +
b^2)*cosh(x)*sinh(x)^5 + (a^2 + b^2)*sinh(x)^6 - 3*(a^2 + b^2)*cosh(x)^4 +
3*(5*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 + b^2)*cosh(x
)^3 - 3*(a^2 + b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2 + 3*(5*(a^
2 + b^2)*cosh(x)^4 - 6*(a^2 + b^2)*cosh(x)^2 + a^2 + b^2)*sinh(x)^2 - a^2 -
 b^2 + 6*((a^2 + b^2)*cosh(x)^5 - 2*(a^2 + b^2)*cosh(x)^3 + (a^2 + b^2)*cos
```

```
h(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) - b)) + 3*((3*a^2*b + 2*b^3)*cosh(x)^6 + 6*(3*a^2*b +
2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh(x)^6 - 3*(3*a^2*b + 2*b^3
)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b + 2*b^3)*cosh(x))*sinh(x)^
3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b + 2*b^3
)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^2 +
6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b^3)*cosh(x)^3 + (3*a^2*b +
 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b + 2*b^3)
*cosh(x)^6 + 6*(3*a^2*b + 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh
(x)^6 - 3*(3*a^2*b + 2*b^3)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2
*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b
+ 2*b^3)*cosh(x))*sinh(x)^3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)
^2 + 3*(5*(3*a^2*b + 2*b^3)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^
3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b
^3)*cosh(x)^3 + (3*a^2*b + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 6*(5*a^2*b*cosh(x)^4 - 8*(2*a^3 + a*b^2)*cosh(x)^3 - a^2*b + 8*(a^3 +
 a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*si
nh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sin
h(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cos
h(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x
)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*sinh(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(96) = 192.
time = 0.41, size = 194, normalized size = 1.80

$$\frac{(3a^2b + 2b^3)\log(e^x + 1)}{2a^4} - \frac{(3a^2b + 2b^3)\log(|e^x - 1|)}{2a^4} + \frac{(a^4 + 2a^2b^2 + b^4)\log\left(\left|\frac{2be^x + 2a - 2\sqrt{a^2 + b^2}}{2be^x + 2a + 2\sqrt{a^2 + b^2}}\right|\right)}{\sqrt{a^2 + b^2}\, a^4} + \frac{3abe^{(5x)} - 12a^2e^{(4x)} - 6b^2e^{(4x)} + 12a^2e^{(2x)} + 12b^2e^{(2x)} - 3abe^x - 8a^2 - 6b^2}{3a^3(e^{(2x)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b + 2*b^3)*log(e^x + 1)/a^4 - 1/2*(3*a^2*b + 2*b^3)*log(abs(e^x
- 1))/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^
2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/3*(3*
a*b*e^(5*x) - 12*a^2*e^(4*x) - 6*b^2*e^(4*x) + 12*a^2*e^(2*x) + 12*b^2*e^(2
*x) - 3*a*b*e^x - 8*a^2 - 6*b^2)/(a^3*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.41, size = 778, normalized size = 7.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*sinh(x)),x)
```

```
[Out] (log(- (8*(18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9
*exp(x) - 14*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^
7*b^2*exp(x)))/(a^9*b^3) - (((a^2 + b^2)^3)^(1/2)*((8*(4*a^8 + 8*b^8 + 36*a
^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36*a^7*b*exp(x) - 52*a
^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) - (((16*(4*a^4*b + 4*b^5 + 8*
a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*exp(x)))/(a*b^5) + (32
*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(
x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4)*((a^2 + b^2)^3)^(1/2))/a^
4 - ((2*(2*a^2 + b^2))/a^3 - (b*exp(x))/a^2)/(exp(2*x) - 1) - (4/a - (2*b*e
xp(x))/a^2)/(exp(4*x) - 2*exp(2*x) + 1) - (log((((a^2 + b^2)^3)^(1/2)*((8*(
4*a^8 + 8*b^8 + 36*a^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36
*a^7*b*exp(x) - 52*a^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) + (((16*(
4*a^4*b + 4*b^5 + 8*a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*ex
p(x)))/(a*b^5) - (32*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp
(x) - 3*a^3*b^2*exp(x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4 - (8*(
18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9*exp(x) - 1
4*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^7*b^2*exp(x
)))/(a^9*b^3))*((a^2 + b^2)^3)^(1/2))/a^4 - 8/(3*a*(3*exp(2*x) - 3*exp(4*x)
 + exp(6*x) - 1)) - (log(exp(x) - 1)*(3*a^2*b + 2*b^3))/(2*a^4) + (log(exp(
x) + 1)*(3*a^2*b + 2*b^3))/(2*a^4)
```

3.236 $\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=224

$$-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3}+\frac{2ab\operatorname{sech}^3(x)}{3\,(a^2+b^2)^2}-\frac{a^4b\cosh(x)}{(a^2+b^2)^3\,(a+b\sinh(}$$

```
[Out] -2*a^5*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+8*a^3*b^2
*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-4*a^3*b*sech(x)
/(a^2+b^2)^3+2/3*a*b*sech(x)^3/(a^2+b^2)^2-a^4*b*cosh(x)/(a^2+b^2)^3/(a+b*s
inh(x))+(a^2-b^2)*tanh(x)/(a^2+b^2)^2-(2*a^4-3*a^2*b^2-b^4)*tanh(x)/(a^2+b^
2)^3-1/3*(a^2-b^2)*tanh(x)^3/(a^2+b^2)^2
```

Rubi [A]
time = 0.33, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {2810, 2743, 12, 2739, 632, 212, 2748, 3852, 8}

$$-\frac{(a^2-b^2)\tanh^3(x)}{3\,(a^2+b^2)^2}+\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}+\frac{2ab\operatorname{sech}^3(x)}{3\,(a^2+b^2)^2}-\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{a^4b\cosh(x)}{(a^2+b^2)^3\,(a+b\sinh(x))}-\frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3}+\frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8*
a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (4*
a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^4*b
*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2 + b^
2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2)*Tan
h[x]^3)/(3*(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \int \left(\frac{a^4}{(a^2+b^2)^2 (a+b\sinh(x))^2} - \frac{4a^3b^2}{(a^2+b^2)^3 (a+b\sinh(x))} + \frac{\operatorname{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right.\right.}{(a^2+b\right.}$$

$$= \frac{\int \operatorname{sech}^2(x)\left(-2a^4\left(1-\frac{3a^2b^2+b^4}{2a^4}\right)+4a^3b\sinh(x)\right)dx}{(a^2+b^2)^3} - \frac{(4a^3b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} + \frac{\int \operatorname{s}}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{a^4\int \frac{a}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} - \frac{(8a}{}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{a^5\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} + \frac{(16}{}$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x}$$

$$= \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x}$$

$$= -\frac{2a^5\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{8a^3b^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}}{3(a^2+}$$

Mathematica [A]
time = 0.30, size = 144, normalized size = 0.64

$$\frac{\frac{6a^3(a^2-4b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 12a^3b\operatorname{sech}(x) - \frac{3a^4b\cosh(x)}{a+b\sinh(x)} + (a^2+b^2)\operatorname{sech}^3(x)(2ab+(a^2-b^2)\sinh(x)) + (-4a^4+9a^2b^2+b^4)\tanh(x)}{3(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^3*(a^2 - 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2] - 12*a^3*b*Sech[x] - (3*a^4*b*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2
)*Sech[x]^3*(2*a*b + (a^2 - b^2)*Sinh[x]) + (-4*a^4 + 9*a^2*b^2 + b^4)*Tanh
[x])/(3*(a^2 + b^2)^3)
```

Maple [A]
time = 0.68, size = 262, normalized size = 1.17

method	result

default	$\frac{2(-a^4+3a^2b^2)(\tanh^5(\frac{x}{2}))+2(-2a^3b+2a\,b^3)(\tanh^4(\frac{x}{2}))+2(-\frac{10}{3}a^4+6a^2b^2+\frac{4}{3}b^4)(\tanh^3(\frac{x}{2}))-16a^3b(\tanh^2(\frac{x}{2}))+2(-a^4+3a^2b^2)}{(a^2+b^2)(a^4+2a^2b^2+b^4)(\tanh^2(\frac{x}{2})+1)^3}$
risch	$\frac{2a^5e^{7x}-8a^3b^2e^{7x}-14a^4b\,e^{6x}-6a^2b^3e^{6x}-2b^5e^{6x}+14a^5e^{5x}-\frac{44a^3b^2e^{5x}}{3}+\frac{4a\,b^4e^{5x}}{3}-\frac{82a^4b\,e^{4x}}{3}+\frac{14a^2b^3e^{4x}}{3}+2b^5e^{4x}+14a^5e^{3x}-\frac{64a^3b^2e}{3}}{(b\,e^{2x}+2a\,e^x-b)(a^4+2a^2b^2+b^4)(a^2+b^2)(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4+3*a^2*b^2)*tanh(1/2*x)^5+(-2*a^3*b+2
*a*b^3)*tanh(1/2*x)^4+(-10/3*a^4+6*a^2*b^2+4/3*b^4)*tanh(1/2*x)^3-8*a^3*b*t
anh(1/2*x)^2+(-a^4+3*a^2*b^2)*tanh(1/2*x)-10/3*a^3*b+2/3*a*b^3)/(tanh(1/2*x
)^2+1)^3-2*a^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tanh(1/2*x)-a*b)/(a*tan
h(1/2*x)^2-2*b*tanh(1/2*x)-a)-(a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(212) = 424$.
time = 0.51, size = 523, normalized size = 2.33

$$\frac{(a^2-4\,b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\,(7\,a^4b-9\,a^2b^3-b^5+(11\,a^5-6\,a^3b^2-2\,ab^4)e^{(-x)}+(35\,a^4b-9\,a^2b^3+b^5)e^{(-2\,x)}+(21\,a^5-32\,a^3b^2-8\,ab^4)e^{(-3\,x)}+(41\,a^4b-7\,a^2b^3-3\,b^5)e^{(-4\,x)}+(21\,a^5-22\,a^3b^2+2\,ab^4)e^{(-5\,x)}+3\,(7\,a^4b+3\,a^2b^3+b^5)e^{(-6\,x)}+3\,(a^5-4\,a^3b^2)e^{(-7\,x)})}{3\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x)}+6\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-3\,x)}+6\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-5\,x)}-2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-6\,x)}+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-7\,x)}-(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-8\,x)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (a^2 - 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2/3*(
7*a^4*b - 9*a^2*b^3 - b^5 + (11*a^5 - 6*a^3*b^2 - 2*a*b^4)*e^(-x) + (35*a^4
*b - 9*a^2*b^3 + b^5)*e^(-2*x) + (21*a^5 - 32*a^3*b^2 - 8*a*b^4)*e^(-3*x) +
 (41*a^4*b - 7*a^2*b^3 - 3*b^5)*e^(-4*x) + (21*a^5 - 22*a^3*b^2 + 2*a*b^4)*
e^(-5*x) + 3*(7*a^4*b + 3*a^2*b^3 + b^5)*e^(-6*x) + 3*(a^5 - 4*a^3*b^2)*e^(
-7*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-2*x) + 6*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-6*x) + 2
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) - (a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-8*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3534 vs. $2(212) = 424$.
time = 0.39, size = 3534, normalized size = 15.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^7 + 6*(a^7 - 3*a^5*b^2 - 4*a^
3*b^4)*sinh(x)^7 - 14*a^6*b + 4*a^4*b^3 + 20*a^2*b^5 + 2*b^7 - 6*(7*a^6*b +
 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^6 - 6*(7*a^6*b + 10*a^4*b^3 + 4*a^2*
b^5 + b^7 - 7*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x))*sinh(x)^6 + 2*(21*a^7
- a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^5 + 2*(21*a^7 - a^5*b^2 - 20*a^3*
b^4 + 2*a*b^6 + 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^2 - 18*(7*a^6*b +
10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x))*sinh(x)^5 - 2*(41*a^6*b + 34*a^4*b^3
 - 10*a^2*b^5 - 3*b^7)*cosh(x)^4 - 2*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 -
3*b^7 - 105*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^3 + 45*(7*a^6*b + 10*a^4*
b^3 + 4*a^2*b^5 + b^7)*cosh(x)^2 - 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b
^6)*cosh(x))*sinh(x)^4 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cos
h(x)^3 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6 + 105*(a^7 - 3*a^5*b
^2 - 4*a^3*b^4)*cosh(x)^4 - 60*(7*a^6*b + 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
h(x)^3 + 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^2 - 4*(41*a^6
*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 - 2*(35*a^6*b + 26
*a^4*b^3 - 8*a^2*b^5 + b^7)*cosh(x)^2 - 2*(35*a^6*b + 26*a^4*b^3 - 8*a^2*b^
5 + b^7 - 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^5 + 45*(7*a^6*b + 10*a^4
*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^4 - 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a
*b^6)*cosh(x)^3 + 6*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^2
 - 3*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x))*sinh(x)^2 - 3*((a
^5*b - 4*a^3*b^3)*cosh(x)^8 + (a^5*b - 4*a^3*b^3)*sinh(x)^8 + 2*(a^6 - 4*a^
4*b^2)*cosh(x)^7 + 2*(a^6 - 4*a^4*b^2 + 4*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh
(x)^7 + 2*(a^5*b - 4*a^3*b^3)*cosh(x)^6 + 2*(a^5*b - 4*a^3*b^3 + 14*(a^5*b
- 4*a^3*b^3)*cosh(x)^2 + 7*(a^6 - 4*a^4*b^2)*cosh(x))*sinh(x)^6 - a^5*b + 4
*a^3*b^3 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 2*(3*a^6 - 12*a^4*b^2 + 28*(a^5*
b - 4*a^3*b^3)*cosh(x)^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^2 + 6*(a^5*b - 4*a^
3*b^3)*cosh(x))*sinh(x)^5 + 10*(7*(a^5*b - 4*a^3*b^3)*cosh(x)^4 + 7*(a^6 -
4*a^4*b^2)*cosh(x)^3 + 3*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 3*(a^6 - 4*a^4*b^2
)*cosh(x))*sinh(x)^4 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 - 12*a^4*b^
2 + 28*(a^5*b - 4*a^3*b^3)*cosh(x)^5 + 35*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 20*
(a^5*b - 4*a^3*b^3)*cosh(x)^3 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^2)*sinh(x)^3 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 2*(14*(a^5*b - 4*a^3*b^3)*cosh(x)^6 - a^
5*b + 4*a^3*b^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 15*(a^5*b - 4*a^3*b^3)*c
osh(x)^4 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 9*(a^6 - 4*a^4*b^2)*cosh(x))*si
nh(x)^2 + 2*(a^6 - 4*a^4*b^2)*cosh(x) + 2*(4*(a^5*b - 4*a^3*b^3)*cosh(x)^7
+ 7*(a^6 - 4*a^4*b^2)*cosh(x)^6 + a^6 - 4*a^4*b^2 + 6*(a^5*b - 4*a^3*b^3)*c
osh(x)^5 + 15*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 9*(a^6 - 4*a^4*b^2)*cosh(x)^2 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(11*a^7 + 5*a^5*b^
```

```
2 - 8*a^3*b^4 - 2*a*b^6)*cosh(x) + 2*(11*a^7 + 5*a^5*b^2 - 8*a^3*b^4 - 2*a*
b^6 + 21*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^6 - 18*(7*a^6*b + 10*a^4*b^3
 + 4*a^2*b^5 + b^7)*cosh(x)^5 + 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)
*cosh(x)^4 - 4*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^3 + 3*(
21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x)^2 - 2*(35*a^6*b + 26*a^
4*b^3 - 8*a^2*b^5 + b^7)*cosh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
)^8 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9
+ 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*cosh(x))*sinh(x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b
^7 + b^9)*cosh(x)^6 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 +
14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4
*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a
^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 +
 18*a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*
a^2*b^7 + b^9)*cosh(x)^3 + 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*cosh(x)^2 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
))*sinh(x)^5 - 10*(7*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh
(x)^4 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(
a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b
^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*
a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*cosh(x)^5 + 35*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
*cosh(x)^4 + 20*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b\sinh(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.44, size = 292, normalized size = 1.30

$$\frac{(a^5 - 4a^3b^2)\log\left(\frac{\left|2be^x + 2a - 2\sqrt{a^2+b^2}\right|}{\left|2be^x + 2a + 2\sqrt{a^2+b^2}\right|}\right)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)\sqrt{a^2+b^2}} + \frac{2(a^5e^x - a^4b)}{(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)(be^{(2x)} + 2ae^x - b)} - \frac{2(12a^3be^{(5x)} - 6a^4e^{(4x)} + 9a^2b^2e^{(4x)} + 3b^4e^{(4x)} + 16a^3be^{(3x)} - 8ab^3e^{(3x)} - 6a^4e^{(2x)} + 18a^2b^2e^{(2x)} + 12a^3be^x - 4a^4 + 9a^2b^2 + b^4)}{3(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)(e^{(2x)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^5 - 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b
^2)) + 2*(a^5*e^x - a^4*b)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x)
+ 2*a*e^x - b)) - 2/3*(12*a^3*b*e^(5*x) - 6*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x)
 + 3*b^4*e^(4*x) + 16*a^3*b*e^(3*x) - 8*a*b^3*e^(3*x) - 6*a^4*e^(2*x) + 18*
a^2*b^2*e^(2*x) + 12*a^3*b*e^x - 4*a^4 + 9*a^2*b^2 + b^4)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B]
time = 1.30, size = 543, normalized size = 2.42

$$\frac{\frac{8\,\left(a^2-b^2\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{16\,a\,b\,{\mathrm{e}}^x}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1}-\frac{\frac{4\,\left(a^6+a^4\,b^2-a^2\,b^4-b^6\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}-\frac{16\,{\mathrm{e}}^x\,\left(a^5\,b+2\,a^3\,b^3+a\,b^5\right)}{3\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{\frac{2\,\left(a^6\,b^5+a^4\,b^7\right)}{b^3\,\left(a^2\,b+b^3\right)\,{\left(a^2+b^2\right)}^3}-\frac{2\,{\mathrm{e}}^x\,\left(a^7\,b^5+a^5\,b^7\right)}{b^4\,\left(a^2\,b+b^3\right)\,{\left(a^2+b^2\right)}^3}}{2\,a\,{\mathrm{e}}^x-b+b\,{\mathrm{e}}^{2\,x}}-\frac{\frac{2\,\left(-2\,a^6+a^4\,b^2+4\,a^2\,b^4+b^6\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}+\frac{8\,{\mathrm{e}}^x\,\left(a^5\,b+a^3\,b^3\right)}{{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\ln\left(-\frac{2\,{\mathrm{e}}^x\,\left(a^5-4\,a^3\,b^2\right)}{b\,{\left(a^2+b^2\right)}^3}-\frac{2\,\left(a^5-4\,a^3\,b^2\right)\,\left(b-a\,{\mathrm{e}}^x\right)}{b\,{\left(a^2+b^2\right)}^{7/2}}\right)\,\left(a^5-4\,a^3\,b^2\right)}{{\left(a^2+b^2\right)}^{7/2}}+\frac{\ln\left(\frac{2\,\left(a^5-4\,a^3\,b^2\right)\,\left(b-a\,{\mathrm{e}}^x\right)}{b\,{\left(a^2+b^2\right)}^{7/2}}-\frac{2\,{\mathrm{e}}^x\,\left(a^5-4\,a^3\,b^2\right)}{b\,{\left(a^2+b^2\right)}^3}\right)\,\left(a^5-4\,a^3\,b^2\right)}{{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(a^4*b^7 + a^6*b^5))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a^
5*b^7 + a^7*b^5))/(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*ex
p(2*x)) - ((2*(b^6 - 2*a^6 + 4*a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^
2 + (8*exp(x)*(a^5*b + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1)
- (log(- (2*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3) - (2*(a^5 - 4*a^3*b
^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)))*(a^5 - 4*a^3*b^2))/(a^2 + b^2)^(
7/2) + (log((2*(a^5 - 4*a^3*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)) - (2
*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3))*(a^5 - 4*a^3*b^2))/(a^2 + b^2
)^(7/2)
```

3.237 $\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=135

$$\frac{ab(3a^2-b^2)\operatorname{ArcTan}(\sinh(x))}{(a^2+b^2)^3}+\frac{a^2(a^2-3b^2)\log(\cosh(x))}{(a^2+b^2)^3}-\frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a^3}{(a^2+b^2)^2(a+$$

```
[Out] a*b*(3*a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^3+a^2*(a^2-3*b^2)*ln(cosh(x))/(a^
2+b^2)^3-a^2*(a^2-3*b^2)*ln(a+b*sinh(x))/(a^2+b^2)^3+a^3/(a^2+b^2)^2/(a+b*s
inh(x))+1/2*sech(x)^2*(a^2-b^2-2*a*b*sinh(x))/(a^2+b^2)^2
```

Rubi [A]
time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2800, 1661, 1643, 649, 209, 266}

$$\frac{ab(3a^2-b^2)\operatorname{ArcTan}(\sinh(x))}{(a^2+b^2)^3}-\frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3}+\frac{a^2(a^2-3b^2)\log(\cosh(x))}{(a^2+b^2)^3}+\frac{\operatorname{sech}^2(x)\left(a^2-2ab\sinh(x)-b^2\right)}{2\left(a^2+b^2\right)^2}+\frac{a^3}{(a^2+b^2)^2(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*b*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 + (a^2*(a^2 - 3*b^2)*Log[
Cosh[x]])/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2
)^3 + a^3/((a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(a^2 - b^2 - 2*a*b*S
inh[x]))/(2*(a^2 + b^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2+b^2\right)^2}+\frac{2a^2b^2x}{a^2+b^2}-\frac{2ab^4x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2(-b^2-x^2)}\,dx, x, b\sinh(x)\right.}{2b^2}$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{(a^2+b^2)^2(a+x)^2} + \frac{2a^2b^2\left(a^2-3b^2\right)}{(a^2+b^2)^3(a+x)} + \frac{2ab^2\left(-b^2\right.}{(}\right.\right.}{2b^2}$$

$$= -\frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-\right.}{2\left(a^2+b^2\right.}$$

$$= -\frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-\right.}{2\left(a^2+b^2\right.}$$

$$= \frac{ab(3a^2-b^2)\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^2(a^2-3b^2)\log(\cosh(x))}{\left(a^2+b^2\right)^3} - \frac{a^2(a^2-3b^2)\log(a+b\,\text{si}}{\left(a^2+b^2\right)^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.52, size = 150, normalized size = 1.11

$$\frac{-2ab(a^2+b^2)\,\text{ArcTan}(\sinh(x)) + a^2(a-ib)(a-3ib)\log(i-\sinh(x)) + a^2(a+ib)(a+3ib)\log(i+\sinh(x)) - 2a^2(a^2-3b^2)\log(a+b\sinh(x)) + (a^4-b^4)\,\text{sech}^2(x) + \frac{2a^3(a^2+b^2)}{a+b\sinh(x)} - 2ab(a^2+b^2)\,\text{sech}(x)\tanh(x)}{2\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^2*(a - I*b)*(a - (3*I)*b)*Log[I - S
inh[x]] + a^2*(a + I*b)*(a + (3*I)*b)*Log[I + Sinh[x]] - 2*a^2*(a^2 - 3*b^2
)*Log[a + b*Sinh[x]] + (a^4 - b^4)*Sech[x]^2 + (2*a^3*(a^2 + b^2))/(a + b*S
inh[x]) - 2*a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^2)^3)
```

Maple [A]
time = 0.75, size = 221, normalized size = 1.64

method	result
default	$\frac{\frac{2\left(\left(a^3b+a\,b^3\right)\left(\tanh^3\left(\frac{x}{2}\right)\right)+\left(-a^4+b^4\right)\left(\tanh^2\left(\frac{x}{2}\right)\right)+\left(-a^3b-a\,b^3\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2}+2a\left(\frac{\left(a^3-3a\,b^2\right)\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{2}+\left(3a^2b-b^3\right)\arctan(\tanh(\right.}{\left(a^2+b^2\right)^3}$
risch	$\frac{2\left(a^3e^{4x}-a\,b^2e^{4x}-a^2b\,e^{3x}-b^3e^{3x}+4a^3e^{2x}+a^2b\,e^x+b^3e^x+a^3-a\,b^2\right)e^x}{\left(b\,e^{2x}+2a\,e^x-b\right)\left(a^4+2a^2b^2+b^4\right)\left(1+e^{2x}\right)^2}-\frac{a^4\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{3a^2\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)^3*(((a^3*b+a*b^3)*tanh(1/2*x)^3+(-a^4+b^4)*tanh(1/2*x)^2+(-a^3*
b-a*b^3)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^2+a*(1/2*(a^3-3*a*b^2)*ln(tanh(1/2*
x)^2+1)+(3*a^2*b-b^3)*arctan(tanh(1/2*x))))-2*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+
b^2)*((-a^2*b-b^3)*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+1/2*(a^2
-3*b^2)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(133) = 266.
time = 0.50, size = 375, normalized size = 2.78

$$-\frac{2\,(3\,a^3b-ab^3)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(a^4-3\,a^2b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4-3\,a^2b^2)\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(4\,a^3e^{(-3\,x)}+(a^3-ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}+(a^2b+b^3)e^{(-4\,x)}+(a^3-ab^2)e^{(-5\,x)}\right)}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+(a^4b+2\,a^2b^3+b^5)e^{(-2\,x)}+4\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-5\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-6\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a^3*b - a*b^3)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
a^4 - 3*a^2*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6) + (a^4 - 3*a^2*b^2)*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + 2*(4*a^3*e^(-3*x) + (a^3 - a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2
*x) + (a^2*b + b^3)*e^(-4*x) + (a^3 - a*b^2)*e^(-5*x))/(a^4*b + 2*a^2*b^3 +
 b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^(-2
*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-
4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(
-6*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2850 vs. $2(133) = 266$.

time = 0.46, size = 2850, normalized size = 21.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^5 - a*b^4)*cosh(x)^5 + 2*(a^5 - a*b^4)*sinh(x)^5 - 2*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5 - 5*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^4 + 8*(a^5 + a^3*b^2)*cosh(x)^3 + 4*(2*a^5 + 2*a^3*b^2 + 5*(a^5 -
 a*b^4)*cosh(x)^2 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 2*(a^4
*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5 + 10*(a^5 - a*
b^4)*cosh(x)^3 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 12*(a^5 + a^3*b^2)
*cosh(x))*sinh(x)^2 + 2*((3*a^3*b^2 - a*b^4)*cosh(x)^6 + (3*a^3*b^2 - a*b^4
)*sinh(x)^6 + 2*(3*a^4*b - a^2*b^3)*cosh(x)^5 + 2*(3*a^4*b - a^2*b^3 + 3*(3
*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 - 3*a^3*b^2 + a*b^4 + (3*a^3*b^2 - a*b
^4)*cosh(x)^4 + (3*a^3*b^2 - a*b^4 + 15*(3*a^3*b^2 - a*b^4)*cosh(x)^2 + 10*
(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^4*b - a^2*b^3)*cosh(x)^3 +
4*(3*a^4*b - a^2*b^3 + 5*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 5*(3*a^4*b - a^2*b
^3)*cosh(x)^2 + (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - (3*a^3*b^2 - a*b^4
)*cosh(x)^2 - (3*a^3*b^2 - a*b^4 - 15*(3*a^3*b^2 - a*b^4)*cosh(x)^4 - 20*(3
*a^4*b - a^2*b^3)*cosh(x)^3 - 6*(3*a^3*b^2 - a*b^4)*cosh(x)^2 - 12*(3*a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^4*b - a^2*b^3)*cosh(x) + 2*(3*(3*a^
3*b^2 - a*b^4)*cosh(x)^5 + 3*a^4*b - a^2*b^3 + 5*(3*a^4*b - a^2*b^3)*cosh(x
)^4 + 2*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 6*(3*a^4*b - a^2*b^3)*cosh(x)^2 - (
3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^5 - a
*b^4)*cosh(x) - ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x
)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2
*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^
4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3
*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b
^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4
*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b
 - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cosh(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh
(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 - 12*(a^5 - 3*a^3*b^2)*cosh(x))*sin
h(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 +
 a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*co
sh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 - (a^4*b - 3*a^2*b^3)*cosh(x))*sinh
(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^4*b - 3*a^2*b^3)*cosh
(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(
a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*
b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^
2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3
```

```
*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*
(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4
*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cos
h(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 -
 12*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*
(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 + a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*co
sh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 -
 (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 2*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*cosh(x)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*c
osh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(
x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^6
- 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a^5*b^2 +
3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x
)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + 10
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*a^5*b^2 +
 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^6*b + 3*a
^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) -
2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh
(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(133) = 266.
time = 0.44, size = 307, normalized size = 2.27

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)(3a^3b - ab^3)}{2\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)} + \frac{(a^4 - 3a^2b^2)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^6 + 3a^4b^2 + 3a^2b^4 + b^6\right)} - \frac{(a^4b - 3a^2b^3)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2a\right|\right)}{a^6b + 3a^4b^3 + 3a^2b^5 + b^7} - \frac{2\left(a^3\left(e^{(-x)} - e^x\right)^2 - ab^2\left(e^{(-x)} - e^x\right)^2 + a^2b\left(e^{(-x)} - e^x\right) + b^3\left(e^{(-x)} - e^x\right) + 6a^3 - 2ab^2\right)}{(a^4 + 2a^2b^2 + b^4)\left(b\left(e^{(-x)} - e^x\right)^3 - 2a\left(e^{(-x)} - e^x\right)^2 + 4b\left(e^{(-x)} - e^x\right) - 8a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^3*b - a*b^3)/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b^2)*log((e^(-x) - e^x)^2 + 4)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2*b^3)*log(abs(-b*(e^(-x
) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 2*(a^3*(e^(-x) - e
^x)^2 - a*b^2*(e^(-x) - e^x)^2 + a^2*b*(e^(-x) - e^x) + b^3*(e^(-x) - e^x)
+ 6*a^3 - 2*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-
x) - e^x)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B]
time = 3.89, size = 501, normalized size = 3.71

$$\frac{\frac{2\,\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}-\frac{2\,{\mathrm{e}}^x\,\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\,{\left(a^4+2\,a^2\,b^2+b^4\right)}^2}}{{\mathrm{e}}^{2\,x}+1}-\frac{\frac{2\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a\,b\,{\mathrm{e}}^x}{a^4+2\,a^2\,b^2+b^4}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{a\,\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}{-a^3+a^2\,b\,3{}\mathrm{i}+3\,a\,b^2-b^3\,1{}\mathrm{i}}-\frac{\ln\left(15\,a^6\,b^3-a^2\,b^7-30\,a^4\,b^5-4\,a^8\,b+8\,a^9\,{\mathrm{e}}^x+a^2\,b^7\,{\mathrm{e}}^{2\,x}+30\,a^4\,b^5\,{\mathrm{e}}^{2\,x}-15\,a^6\,b^3\,{\mathrm{e}}^{2\,x}+4\,a^8\,b\,{\mathrm{e}}^{2\,x}+2\,a^3\,b^6\,{\mathrm{e}}^x+60\,a^5\,b^4\,{\mathrm{e}}^x-30\,a^7\,b^2\,{\mathrm{e}}^x\right)\,\left(a^4-3\,a^2\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,{\mathrm{e}}^x\,\left(a^7\,b^2+2\,a^5\,b^4+a^3\,b^6\right)}{b\,\left(a^2\,b+b^3\right)\,\left(a^2+b^2\right)\,\left(2\,a\,{\mathrm{e}}^x-b+b\,{\mathrm{e}}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\,\ln\left(1+{\mathrm{e}}^x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{-a^3\,1{}\mathrm{i}+3\,a^2\,b+a\,b^2\,3{}\mathrm{i}-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)^2) - (2*exp(x)*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^
4 + b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((2*(a^2 - b^2))/(a^4 + b^4 + 2*a
^2*b^2) - (4*a*b*exp(x))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) +
1) - (a*log(exp(x) + 1i))/(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i) - (log(15*a^6
*b^3 - a^2*b^7 - 30*a^4*b^5 - 4*a^8*b + 8*a^9*exp(x) + a^2*b^7*exp(2*x) + 3
0*a^4*b^5*exp(2*x) - 15*a^6*b^3*exp(2*x) + 4*a^8*b*exp(2*x) + 2*a^3*b^6*exp
(x) + 60*a^5*b^4*exp(x) - 30*a^7*b^2*exp(x))*(a^4 - 3*a^2*b^2))/(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2) - (a*log(exp(x)*1i + 1)*1i)/(a*b^2*3i + 3*a^2*b -
a^3*1i - b^3) + (2*exp(x)*(a^3*b^6 + 2*a^5*b^4 + a^7*b^2))/(b*(a^2*b + b^3)
*(a^2 + b^2)*(2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

3.238 $\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=144

$$-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{2ab\text{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{(a^2-b^2)}{(a^2+}$$

```
[Out] -2*a^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+4*a*b^2*a
rctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-2*a*b*sech(x)/(a^
2+b^2)^2-a^2*b*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))-(a^2-b^2)*tanh(x)/(a^2+b^2
)^2
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {2810, 2743, 12, 2739, 632, 212, 2748, 3852, 8}

$$\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}-\frac{2ab\text{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4*
a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*a*
b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]))
- ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = -\int \left(-\frac{a^2}{(a^2+b^2)(a+b\sinh(x))^2}+\frac{2ab^2}{(a^2+b^2)^2(a+b\sinh(x))}+\frac{\operatorname{sech}^2(x)\left(a^2\left(1-\right.\right.}{(a}\right.$$

$$=-\frac{\int \operatorname{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)dx}{(a^2+b^2)^2}-\frac{(2ab^2)\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}+\frac{a^2\int \frac{}{(a+b\,\mathrm{si}}}{a^2+}$$

$$=-\frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}+\frac{a^2\int \frac{a}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}-\frac{(4ab^2)\,\mathrm{Subst}\left(\int \frac{}{a+}\right.}{(a}$$

$$=-\frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}+\frac{a^3\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}+\frac{(8ab^2)\,\mathrm{Subst}\left(\int \frac{}{4(}\right.}{}$$

$$=\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{(a^2-b^2)\,\mathrm{t}}{(a^2+}$$

$$=\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))}-\frac{(a^2-b^2)\,\mathrm{t}}{(a^2+}$$

$$=-\frac{2a^3\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{4ab^2\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{a}{(a^2+b^2}$$

Mathematica [A]
time = 0.21, size = 100, normalized size = 0.69

$$\frac{\frac{2a(a^2-2b^2)\mathrm{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-2ab\operatorname{sech}(x)-\frac{a^2b\cosh(x)}{a+b\sinh(x)}+(-a^2+b^2)\tanh(x)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
 b^2] - 2*a*b*Sech[x] - (a^2*b*Cosh[x])/(a + b*Sinh[x]) + (-a^2 + b^2)*Tanh
[x])/(a^2 + b^2)^2
```

Maple [A]
time = 0.61, size = 142, normalized size = 0.99

method	result

default	$-\frac{2a\left(\frac{-b^{2}\tanh\left(\frac{x}{2}\right)-ab}{a\left(\tanh^{2}\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^{2}-2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{2\left(-a^{2}+b^{2}\right)\tanh\left(\frac{x}{2}\right)-4ab}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)}$
risch	$\frac{2a^{3}e^{3x}-4a\,b^{2}e^{3x}-8a^{2}b\,e^{2x}-2b^{3}e^{2x}+6a^{3}e^{x}-4a^{2}b+2b^{3}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(b\,e^{2x}+2a\,e^{x}-b\right)\left(1+e^{2x}\right)}+\frac{a^{3}\ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}a-a^{6}-3a^{4}b^{2}-3a^{2}b^{4}-b^{6}}{b\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}-\frac{2b^{2}a\ln\left(e^{x}+\frac{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}{}\right.}{\left(a^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/(a^2+b^2)^2*((-b^2*tanh(1/2*x)-a*b)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a
)-(a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(
1/2)))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(tanh(1/2*x)^2+
1)
```

Maxima [A]
time = 0.47, size = 223, normalized size = 1.55

$$\frac{\left(a^{2}-2\,b^{2}\right)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^{2}+b^{2}}}{be^{(-x)}-a+\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(3\,a^{3}e^{(-x)}+2\,a^{2}b-b^{3}+\left(4\,a^{2}b+b^{3}\right)e^{(-2\,x)}+\left(a^{3}-2\,ab^{2}\right)e^{(-3\,x)}\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}+2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)e^{(-x)}+2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)e^{(-3\,x)}-\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)e^{(-4\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (a^2 - 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*e^(-x) + 2
*a^2*b - b^3 + (4*a^2*b + b^3)*e^(-2*x) + (a^3 - 2*a*b^2)*e^(-3*x))/(a^4*b
+ 2*a^2*b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(136) = 272.
time = 0.42, size = 900, normalized size = 6.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^4*b + 2*a^2*b^3 - 2*b^5 - 2*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 2*(a
^5 - a^3*b^2 - 2*a*b^4)*sinh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2
 + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x))*sinh
```

```
(x)^2 + ((a^3*b - 2*a*b^3)*cosh(x)^4 + (a^3*b - 2*a*b^3)*sinh(x)^4 - a^3*b
+ 2*a*b^3 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 2*(a^4 - 2*a^2*b^2 + 2*(a^3*b -
 2*a*b^3)*cosh(x))*sinh(x)^3 + 6*((a^3*b - 2*a*b^3)*cosh(x)^2 + (a^4 - 2*a^
2*b^2)*cosh(x))*sinh(x)^2 + 2*(a^4 - 2*a^2*b^2)*cosh(x) + 2*(a^4 - 2*a^2*b^
2 + 2*(a^3*b - 2*a*b^3)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x))
*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) - 6*(a^5 + a^3*b^2)*cosh(x) - 2*(3*a^5 + 3*a^3*b^2 + 3*(a^5 -
a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(
x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*(
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
2)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.42, size = 181, normalized size = 1.26

$$\frac{(a^3-2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\,\left(a^3e^{(3\,x)}-2\,ab^2e^{(3\,x)}-4\,a^2be^{(2\,x)}-b^3e^{(2\,x)}+3\,a^3e^x-2\,a^2b+b^3\right)}{(a^4+2\,a^2b^2+b^4)(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^3 - 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*
a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*
e^(3*x) - 2*a*b^2*e^(3*x) - 4*a^2*b*e^(2*x) - b^3*e^(2*x) + 3*a^3*e^x - 2*a
^2*b + b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b
))
```

Mupad [B]
time = 1.15, size = 377, normalized size = 2.62

$$\frac{\frac{2\,\left(a^2\,b^9-2\,a^4\,b^7\right)}{b^3\,\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}-\frac{2\,{\mathrm{e}}^{2\,x}\,\left(4\,a^4\,b^7+a^2\,b^9\right)}{b^3\,\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}+\frac{6\,a^5\,b^3\,{\mathrm{e}}^x}{\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}-\frac{2\,a\,{\mathrm{e}}^{3\,x}\,\left(2\,a^2\,b^9-a^4\,b^7\right)}{b^4\,\left(a^3+a\,b^2\right)\,\left(a^3\,b^3+a\,b^5\right)}}{2\,a\,{\mathrm{e}}^x-b+2\,a\,{\mathrm{e}}^{3\,x}+b\,{\mathrm{e}}^{4\,x}}-\frac{a\,\ln\left(\frac{2\,{\mathrm{e}}^x\,\left(2\,a\,b^2-a^3\right)}{b\,{\left(a^2+b^2\right)}^2}-\frac{2\,a\,\left(a^2-2\,b^2\right)\,\left(b-a\,{\mathrm{e}}^x\right)}{b\,{\left(a^2+b^2\right)}^{5/2}}\right)\,\left(a^2-2\,b^2\right)}{{\left(a^2+b^2\right)}^{5/2}}+\frac{a\,\ln\left(\frac{2\,{\mathrm{e}}^x\,\left(2\,a\,b^2-a^3\right)}{b\,{\left(a^2+b^2\right)}^2}+\frac{2\,a\,\left(a^2-2\,b^2\right)\,\left(b-a\,{\mathrm{e}}^x\right)}{b\,{\left(a^2+b^2\right)}^{5/2}}\right)\,\left(a^2-2\,b^2\right)}{{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^2*b^9 - 2*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*exp(2
*x)*(a^2*b^9 + 4*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^5*b
^3*exp(x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*a*exp(3*x)*(2*a^2*b^9 - a
^4*b^7))/(b^4*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3
*x) + b*exp(4*x)) - (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) - (
2*a*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^
2 + b^2)^(5/2) + (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) + (2*a
*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^2 +
 b^2)^(5/2)
```

3.239 $\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2ab\text{ArcTan}(\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}-\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 2*a*b*arctan(sinh(x))/(a^2+b^2)^2+(a^2-b^2)*ln(cosh(x))/(a^2+b^2)^2-(a^2-b^
2)*ln(a+b*sinh(x))/(a^2+b^2)^2+a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2800, 815, 649, 209, 266}

$$\frac{2ab\text{ArcTan}(\sinh(x))}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)(a+b\sinh(x))}-\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cosh[x]])/(a^2 + b
^2)^2 - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + a/((a^2 + b^2)*(a
+ b*Sinh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx &= -\text{Subst}\left(\int \frac{x}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)^2}+\frac{a^2-b^2}{\left(a^2+b^2\right)^2(a+x)}+\frac{-2ab^2-\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)\,dx, x\\
&= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\text{Subst}\left(\int \frac{-2ab^2-\left(a^2-b^2\right)x}{b^2+x^2}\right.}{\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(2ab^2\right)\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx\right.}{\left(a^2+b^2\right)^2}\\
&= \frac{2ab\tan^{-1}(\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^2}-\frac{\left(a^2-b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2}+\frac{}{\left(a^2+\right.}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 146, normalized size = 1.72

$$\frac{a\left((a-ib)^2\log(i-\sinh(x))+(a+ib)^2\log(i+\sinh(x))+2\left(a^2+b^2+\left(-a^2+b^2\right)\log(a+b\sinh(x))\right)\right)+b\left((a-ib)^2\log(i-\sinh(x))+(a+ib)^2\log(i+\sinh(x))+2\left(-a^2+b^2\right)\log(a+b\sinh(x))\right)\sinh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*((a - I*b)^2*Log[I - Sinh[x]] + (a + I*b)^2*Log[I + Sinh[x]] + 2*(a^2 +
b^2 + (-a^2 + b^2)*Log[a + b*Sinh[x]])) + b*((a - I*b)^2*Log[I - Sinh[x]] +
 (a + I*b)^2*Log[I + Sinh[x]] + 2*(-a^2 + b^2)*Log[a + b*Sinh[x]])*Sinh[x])
/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Maple [A]
time = 0.66, size = 136, normalized size = 1.60

method	result

default	$\frac{2(a^2-b^2)\ln(\tanh^2(\frac{x}{2})+1)+8ab\arctan(\tanh(\frac{x}{2}))}{2a^4+4a^2b^2+2b^4}-\frac{2\left(\frac{(-a^2b-b^3)\tanh(\frac{x}{2})}{a(\tanh^2(\frac{x}{2}))-2b\tanh(\frac{x}{2})-a}+\frac{(a^2-b^2)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{(a^2+b^2)^2}$
risch	$\frac{2a\,e^x}{(a^2+b^2)(b\,e^{2x}+2a\,e^x-b)}-\frac{\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)a^2}{a^4+2a^2b^2+b^4}+\frac{\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)b^2}{a^4+2a^2b^2+b^4}+\frac{2i\ln(e^x+i)ab}{a^4+2a^2b^2+b^4}+\frac{\ln(e^x+i)a^2}{a^4+2a^2b^2+b^4}-\frac{\ln(e^x+i)b^2}{a^4+2a^2b^2+b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*a^4+4*a^2*b^2+2*b^4)*(1/2*(a^2-b^2)*ln(tanh(1/2*x)^2+1)+2*a*b*arctan(t
anh(1/2*x)))-2/(a^2+b^2)^2*((-a^2*b-b^3)*tanh(1/2*x)/(a*tanh(1/2*x)^2-2*b*t
anh(1/2*x)-a)+1/2*(a^2-b^2)*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a))
```

Maxima [A]
time = 0.48, size = 155, normalized size = 1.82

$$-\frac{4\,ab\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,ae^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}}-\frac{(a^2-b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -4*a*b*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + 2*a*e^(-x)/(a^2*b + b^3 + 2
*(a^3 + a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2*x)) - (a^2 - b^2)*log(-2*a*e^(-
x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(e^(-2*x) + 1
)/(a^4 + 2*a^2*b^2 + b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(85) = 170.
time = 0.46, size = 423, normalized size = 4.98

$$-\frac{4\left(ab^2\cosh(x)^2+ab^2\sinh(x)^2+2\,a^2b\cosh(x)-ab^2+2\,(ab^2\cosh(x)+a^2b)\sinh(x)\right)\arctan(\cosh(x)+\sinh(x))+2\,(a^3+ab^2)\cosh(x)+\left(a^2b-b^3-(a^2b-b^3)\cosh(x)^2-(a^2b-b^3)\sinh(x)^2-2\,(a^3-ab^2)\cosh(x)-2\,(a^3-ab^2+(a^2b-b^3)\cosh(x))\sinh(x)\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\left(a^2b-b^3-(a^2b-b^3)\cosh(x)^2-(a^2b-b^3)\sinh(x)^2-2\,(a^3-ab^2)\cosh(x)-2\,(a^3-ab^2+(a^2b-b^3)\cosh(x))\sinh(x)\right)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)+2\,(a^3+ab^2)\sinh(x)}{a^4b+2\,a^2b^3+b^5-(a^4b+2\,a^2b^3+b^5)\cosh(x)^2-(a^4b+2\,a^2b^3+b^5)\sinh(x)^2-2\,(a^5+2\,a^3b^2+ab^4)\cosh(x)-2\,(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\cosh(x))\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*(a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2
*cosh(x) + a^2*b)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^3 + a*b^2)*cosh
(x) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*
(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*lo
g(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^2*b - b^3 - (a^2*b - b^3)*cos
h(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2
 + (a^2*b - b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(
a^3 + a*b^2)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*
cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^
```

```
4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)
)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(a + b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.
time = 0.41, size = 199, normalized size = 2.34

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)ab}{a^4 + 2a^2b^2 + b^4} + \frac{(a^2 - b^2)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\left(a^4 + 2a^2b^2 + b^4\right)} - \frac{(a^2b - b^3)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2a\right|\right)}{a^4b + 2a^2b^3 + b^5} + \frac{a^2b\left(e^{(-x)} - e^x\right) - b^3\left(e^{(-x)} - e^x\right) - 4a^3}{\left(a^4 + 2a^2b^2 + b^4\right)\left(b\left(e^{(-x)} - e^x\right) - 2a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a*b/(a^4 + 2*a^2*b^2 + b^4) + 1/2
*(a^2 - b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b - b
^3)*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + (a^2*b*(e
^(-x) - e^x) - b^3*(e^(-x) - e^x) - 4*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(
-x) - e^x) - 2*a))
```

Mupad [B]
time = 1.75, size = 190, normalized size = 2.24

$$\frac{\ln\left(1 + e^x\,1i\right)}{a^2 + a\,b\,2i - b^2} - \frac{\ln\left(b^5\,e^{2x} - a^4\,b - b^5 + a^2\,b^3 + 2\,a^5\,e^x - a^2\,b^3\,e^{2x} + 2\,a\,b^4\,e^x + a^4\,b\,e^{2x} - 2\,a^3\,b^2\,e^x\right)\left(a^2 - b^2\right)}{a^4 + 2\,a^2\,b^2 + b^4} + \frac{2\,a\,b\,e^x}{\left(a^2\,b + b^3\right)\left(2\,a\,e^x - b + b\,e^{2x}\right)} + \frac{\ln\left(e^x + 1i\right)\,1i}{a^2\,1i + 2\,a\,b - b^2\,1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] log(exp(x)*1i + 1)/(a*b*2i + a^2 - b^2) + (log(exp(x) + 1i)*1i)/(2*a*b + a^
2*1i - b^2*1i) - (log(b^5*exp(2*x) - a^4*b - b^5 + a^2*b^3 + 2*a^5*exp(x) -
 a^2*b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) - 2*a^3*b^2*exp(x))*(a^
2 - b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b*exp(x))/((a^2*b + b^3)*(2*a*exp(
x) - b + b*exp(2*x)))
```

3.240 $\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sinh(x))}{a^2}-\frac{\log(a+b\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2800, 46}

$$-\frac{\log(a+b\sinh(x))}{a^2}+\frac{\log(\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx, x, b\sinh(x)\right)\\ &= \frac{\log(\sinh(x))}{a^2}-\frac{\log(a+b\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 0.84

$$\frac{\log(\sinh(x)) - \log(a + b\sinh(x)) + \frac{a}{a+b\sinh(x)}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (Log[Sinh[x]] - Log[a + b*Sinh[x]] + a/(a + b*Sinh[x]))/a^2
```

Maple [A]
time = 0.43, size = 33, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\ln(\sinh(x))}{a^2} - \frac{\ln(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$	33
default	$\frac{\ln(\sinh(x))}{a^2} - \frac{\ln(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$	33
risch	$\frac{2\,\mathrm{e}^{x}}{a(b\,\mathrm{e}^{2x}+2a\,\mathrm{e}^{x}-b)} + \frac{\ln(\mathrm{e}^{2x}-1)}{a^2} - \frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.
time = 0.28, size = 75, normalized size = 2.34

$$\frac{2\,e^{(-x)}}{2\,a^2e^{(-x)} - abe^{(-2\,x)} + ab} - \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^2} + \frac{\log\left(e^{(-x)} + 1\right)}{a^2} + \frac{\log\left(e^{(-x)} - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*a^2*e^(-x) - a*b*e^(-2*x) + a*b) - log(-2*a*e^(-x) + b*e^(-2*x)
 - b)/a^2 + log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(32) = 64.
time = 0.50, size = 158, normalized size = 4.94

$$\frac{2\,a\cosh\left(x\right) - \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\left(b\cosh\left(x\right) + a\right)\sinh\left(x\right) - b\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) + \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\left(b\cosh\left(x\right) + a\right)\sinh\left(x\right) - b\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) + 2\,a\sinh\left(x\right)}{a^2b\cosh\left(x\right)^2 + a^2b\sinh\left(x\right)^2 + 2\,a^3\cosh\left(x\right) - a^2b + 2\left(a^2b\cosh\left(x\right) + a^3\right)\sinh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*cosh(x) - (b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 2*a*sinh(x))/(a^2*b*cosh(x)^2 + a^2*b*sinh(x)^2 + 2*a^3
*cosh(x) - a^2*b + 2*(a^2*b*cosh(x) + a^3)*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth (x)}{(a + b \sinh (x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.
time = 0.42, size = 75, normalized size = 2.34

$$-\frac{\log \left(\left|-b\left(e^{(-x)} - e^{x}\right) + 2\,a\right|\right)}{a^{2}} + \frac{\log \left(\left|-e^{(-x)} + e^{x}\right|\right)}{a^{2}} + \frac{b\left(e^{(-x)} - e^{x}\right) - 4\,a}{\left(b\left(e^{(-x)} - e^{x}\right) - 2\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a^2 + log(abs(-e^(-x) + e^x))/a^2 + (b*(
e^(-x) - e^x) - 4*a)/((b*(e^(-x) - e^x) - 2*a)*a^2)
```

Mupad [B]
time = 0.97, size = 240, normalized size = 7.50

$$\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^4}+b\,\mathrm{e}^x\,\sqrt{-a^4}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^4}}{a^3}\right)-2\,\mathrm{atan}\left(\left(4\,a^5\,b\,\sqrt{-a^4}+4\,a^3\,b^3\,\sqrt{-a^4}\right)\,\left(\frac{1}{8\,a^3\,b\,{\left(a^2+b^2\right)}^2}-\mathrm{e}^x\,\left(\frac{1}{16\,a^2\,b^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^6\,b^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^5\,b\,{\left(a^2+b^2\right)}^2}\right)\right)}{\sqrt{-a^4}}+\frac{2\,b^3\,\mathrm{e}^x\,\left(a^2+b^2\right)}{a\,\left(a^2\,b^3+b^5\right)\,\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x))^2,x)
```

```
[Out] (2*atan((a*(-a^4)^(1/2) + b*exp(x)*(-a^4)^(1/2) - 2*a*exp(2*x)*(-a^4)^(1/2)
 - b*exp(3*x)*(-a^4)^(1/2))/a^3) - 2*atan((4*a^5*b*(-a^4)^(1/2) + 4*a^3*b^3
*(-a^4)^(1/2))*(1/(8*a^3*b*(a^2 + b^2)^2) - exp(x)*(1/(16*a^2*b^2*(a^2 + b^
2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^5*
b*(a^2 + b^2)^2))))/(-a^4)^(1/2) + (2*b^3*exp(x)*(a^2 + b^2))/(a*(b^5 + a^2
*b^3)*(2*a*exp(x) - b + b*exp(2*x)))
```

3.241 $\int \frac{\coth^2(x)}{(a+b\sinh(x))^2} dx$

Optimal. Leaf size=80

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}-\frac{2\coth(x)}{a^2}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*coth(x)/a^2+coth(x)/a/(a+b*sinh(x))-2*(a^2+2*b^2
)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 212}

$$\frac{2b\tanh^{-1}(\cosh(x))}{a^3}-\frac{2\coth(x)}{a^2}-\frac{2(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{\coth(x)}{a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) - (2*Coth[x])/a^2 + Coth[x]/(a*(a + b*
Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^2(x)\left(1+\sinh^2(x)\right)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}^2(x)(2(a^2+b^2)+(a^2+b^2)\sinh^2(x))}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{i\int \frac{\operatorname{csch}(x)(2ib(a^2+b^2)-ia(a^2+b^2)\sinh(x))}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{(2b)\int \operatorname{csch}(x)\,dx}{a^3} + \frac{(a^2+2b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{(2(a^2+2b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-}\right.}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{(4(a^2+2b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2}\right.}{a}$$

$$= \frac{2b\tanh^{-1}(\cosh(x))}{a^3} - \frac{2(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh}$$

Mathematica [A]
time = 0.38, size = 102, normalized size = 1.28

$$-\frac{-\frac{4(a^2+2b^2)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a\coth\left(\frac{x}{2}\right) + 4b\log\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{2ab\cosh(x)}{a+b\sinh(x)} + a\tanh\left(\frac{x}{2}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((-4*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] + a*Coth[x/2] + 4*b*Log[Tanh[x/2]] + (2*a*b*Cosh[x])/(a + b*Sinh[
x]) + a*Tanh[x/2])/a^3
```

Maple [A]
time = 0.65, size = 118, normalized size = 1.48

method	result

default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2}-\frac{2\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{2a^2\tanh\left(\frac{x}{2}\right)}-\frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}$
risch	$\frac{2a\,e^{3x}-4b\,e^{2x}-6a\,e^{x}+4b}{a^2(e^{2x}-1)(b\,e^{2x}+2a\,e^{x}-b)}+\frac{2b\ln(e^{x}+1)}{a^3}-\frac{2b\ln(e^{x}-1)}{a^3}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}+\frac{2\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-}{\sqrt{a^2+b^2}\,b}\right.}{\sqrt{a^2+b^2}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-2/a^3*((-b^2*tanh(1/2*x)-a*b)/(a*tanh(1/2*x)^2-2*b*tan
h(1/2*x)-a)-(a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2)))-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(76) = 152$.
time = 0.49, size = 165, normalized size = 2.06

$$-\frac{2\left(3\,ae^{(-x)}-2\,be^{(-2\,x)}-ae^{(-3\,x)}+2\,b\right)}{2\,a^3e^{(-x)}-2\,a^2be^{(-2\,x)}-2\,a^3e^{(-3\,x)}+a^2be^{(-4\,x)}+a^2b}+\frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3}-\frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}+\frac{\left(a^2+2\,b^2\right)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x) + 2*b)/(2*a^3*e^(-x) - 2*a^2*b*e
^(-2*x) - 2*a^3*e^(-3*x) + a^2*b*e^(-4*x) + a^2*b) + 2*b*log(e^(-x) + 1)/a^
3 - 2*b*log(e^(-x) - 1)/a^3 + (a^2 + 2*b^2)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. $2(76) = 152$.
time = 0.48, size = 1257, normalized size = 15.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*sinh(x
)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^2*b^2
)*cosh(x))*sinh(x)^2 + ((a^2*b + 2*b^3)*cosh(x)^4 + (a^2*b + 2*b^3)*sinh(x)
^4 + 2*(a^3 + 2*a*b^2)*cosh(x)^3 + 2*(a^3 + 2*a*b^2 + 2*(a^2*b + 2*b^3)*cos
```

```
h(x))*sinh(x)^3 + a^2*b + 2*b^3 - 2*(a^2*b + 2*b^3)*cosh(x)^2 - 2*(a^2*b +
2*b^3 - 3*(a^2*b + 2*b^3)*cosh(x)^2 - 3*(a^3 + 2*a*b^2)*cosh(x))*sinh(x)^2
- 2*(a^3 + 2*a*b^2)*cosh(x) + 2*(2*(a^2*b + 2*b^3)*cosh(x)^3 - a^3 - 2*a*b^
2 + 3*(a^3 + 2*a*b^2)*cosh(x)^2 - 2*(a^2*b + 2*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 6*(a^4 + a^2*b^2)*cosh(x) + 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^
2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^3*b
 + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh(x)
^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*cos
h(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*b^2
 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh(x)
)*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2
*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^
3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh
(x)^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*
cosh(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*
b^2 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(3*a^4 + 3*a^2*b^2 - 3*(a^4 +
a^2*b^2)*cosh(x)^2 + 4*(a^3*b + a*b^3)*cosh(x))*sinh(x))/(a^5*b + a^3*b^3 +
 (a^5*b + a^3*b^3)*cosh(x)^4 + (a^5*b + a^3*b^3)*sinh(x)^4 + 2*(a^6 + a^4*b
^2)*cosh(x)^3 + 2*(a^6 + a^4*b^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x)^3 -
 2*(a^5*b + a^3*b^3)*cosh(x)^2 - 2*(a^5*b + a^3*b^3 - 3*(a^5*b + a^3*b^3)*c
osh(x)^2 - 3*(a^6 + a^4*b^2)*cosh(x))*sinh(x)^2 - 2*(a^6 + a^4*b^2)*cosh(x)
 - 2*(a^6 + a^4*b^2 - 2*(a^5*b + a^3*b^3)*cosh(x)^3 - 3*(a^6 + a^4*b^2)*cos
h(x)^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.43, size = 148, normalized size = 1.85

$$\frac{2\,b\log\left(e^{x}+1\right)}{a^{3}}-\frac{2\,b\log\left(|e^{x}-1|\right)}{a^{3}}+\frac{\left(a^{2}+2\,b^{2}\right)\log\left(\frac{|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}|}{|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}|}\right)}{\sqrt{a^{2}+b^{2}}\,a^{3}}+\frac{2\,\left(ae^{(3\,x)}-2\,be^{(2\,x)}-3\,ae^{x}+2\,b\right)}{\left(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^{x}+b\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3 + (a^2 + 2*b^2)*log(abs(2*
b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*a^3) + 2*(a*e^(3*x) - 2*b*e^(2*x) - 3*a*e^x + 2*b)/((b*e^(4*
x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b)*a^2)
```

Mupad [B]
time = 1.78, size = 897, normalized size = 11.21

$$\frac{\frac{4\left(25a^8b^8+65a^6b^{10}+56a^4b^{12}+16a^2b^{14}\right)}{a^4b^4\left(25a^6b^3+65a^4b^5+56a^2b^7+16b^9\right)}-\frac{6e^x\left(25a^9b^8+65a^7b^{10}+56a^5b^{12}+16a^3b^{14}\right)}{a^4b^5\left(25a^6b^3+65a^4b^5+56a^2b^7+16b^9\right)}-\frac{4e^{2x}\left(25a^8b^8+65a^6b^{10}+56a^4b^{12}+16a^2b^{14}\right)}{a^4b^4\left(25a^6b^3+65a^4b^5+56a^2b^7+16b^9\right)}+\frac{2e^{3x}\left(25a^9b^8+65a^7b^{10}+56a^5b^{12}+16a^3b^{14}\right)}{a^4b^5\left(25a^6b^3+65a^4b^5+56a^2b^7+16b^9\right)}}{b-2ae^x+2ae^{3x}-2be^{2x}+be^{4x}}-\frac{2b\ln\left(64e^x-64\right)}{a^3}+\frac{2b\ln\left(64e^x+64\right)}{a^3}-\frac{\ln\left(\frac{\left(a^2+2b^2\right)\left(\frac{32\left(a^4+8b^4+12a^2b^2-12ab^3e^x-16a^3be^x\right)}{a^4b^4}+\frac{\left(a^2+2b^2\right)\left(\frac{32\left(2a^2b+4b^3-4a^3e^x-7ab^2e^x\right)}{b^5}-\frac{32\left(a^2+2b^2\right)\sqrt{a^2+b^2}\left(3a^4b+2a^2b^3-4a^5e^x-3a^3b^2e^x\right)}{b^5\left(a^5+a^3b^2\right)}\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}-\frac{64\left(a^2+2b^2\right)\left(4b-7ae^x\right)}{a^6b^3}\right)\left(a^2+2b^2\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}+\frac{\ln\left(-\frac{\left(a^2+2b^2\right)\left(\frac{32\left(a^4+8b^4+12a^2b^2-12ab^3e^x-16a^3be^x\right)}{a^4b^4}-\frac{\left(a^2+2b^2\right)\left(\frac{32\left(2a^2b+4b^3-4a^3e^x-7ab^2e^x\right)}{b^5}+\frac{32\left(a^2+2b^2\right)\sqrt{a^2+b^2}\left(3a^4b+2a^2b^3-4a^5e^x-3a^3b^2e^x\right)}{b^5\left(a^5+a^3b^2\right)}\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}-\frac{64\left(a^2+2b^2\right)\left(4b-7ae^x\right)}{a^6b^3}\right)\left(a^2+2b^2\right)\sqrt{a^2+b^2}}{a^5+a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((4*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 + 25*a^8*b^8))/(a^4*b^4*(16*b^
9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (6*exp(x)*(16*a^3*b^14 + 56*a^
5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (4*exp(2*x)*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 +
 25*a^8*b^8))/(a^4*b^4*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (
2*exp(3*x)*(16*a^3*b^14 + 56*a^5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5
*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*ex
p(3*x) - 2*b*exp(2*x) + b*exp(4*x)) - (2*b*log(64*exp(x) - 64))/a^3 + (2*b*
log(64*exp(x) + 64))/a^3 - (log(((a^2 + 2*b^2)*((32*(a^4 + 8*b^4 + 12*a^2*b
^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) + ((a^2 + 2*b^2)*((32*(2
*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 - (32*(a^2 + 2*b^2)*(a
^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b
^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))*(a^2 + b^2)^(1/2))
/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))/(a^6*b^3))*(a^2 +
2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (log(- ((a^2 + 2*b^2)*((32*(a^4
 + 8*b^4 + 12*a^2*b^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) - ((a
^2 + 2*b^2)*((32*(2*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 + (
32*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*
a^3*b^2*exp(x)))/(b^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))
*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))
/(a^6*b^3))*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

3.242 $\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=76

$$\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{(a^2+3b^2)\log(\sinh(x))}{a^4}-\frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4}+\frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

```
[Out] 2*b*csch(x)/a^3-1/2*csch(x)^2/a^2+(a^2+3*b^2)*ln(sinh(x))/a^4-(a^2+3*b^2)*l
n(a+b*sinh(x))/a^4+(a^2+b^2)/a^3/(a+b*sinh(x))
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2800, 908}

$$\frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{(a^2+3b^2)\log(\sinh(x))}{a^4}-\frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4}+\frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[Sinh[x]])/a^4 -
((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = -\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sinh(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{b^2}{a^2x^3}+\frac{2b^2}{a^3x^2}+\frac{-a^2-3b^2}{a^4x}+\frac{a^2+b^2}{a^3(a+x)^2}+\frac{a^2+3b^2}{a^4(a+x)}\right)dx, x, b\sin\right.$$
$$= \frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{(a^2+3b^2)\log(\sinh(x))}{a^4}-\frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4}+$$

Mathematica [A]
time = 0.17, size = 73, normalized size = 0.96

$$\frac{4ab\text{csch}(x)-a^2\text{csch}^2(x)+2(a^2+3b^2)\log(\sinh(x))-2(a^2+3b^2)\log(a+b\sinh(x))+\frac{2a(a^2+b^2)}{a+b\sinh(x)}}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (4*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + 3*b^2)*Log[Sinh[x]] - 2*(a^2 + 3*
b^2)*Log[a + b*Sinh[x]] + (2*a*(a^2 + b^2))/(a + b*Sinh[x]))/(2*a^4)
```

Maple [A]
time = 0.56, size = 142, normalized size = 1.87

method	result
default	$-\frac{\frac{a\left(\tanh^2\left(\frac{x}{2}\right)\right)}{2}+4b\tanh\left(\frac{x}{2}\right)}{4a^3}-\frac{2\left(\frac{(-a^2b-b^3)\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{(a^2+3b^2)\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{a^4}-\frac{1}{8a^2\tanh\left(\frac{x}{2}\right)^2}+$
risch	$\frac{2\,e^x\left(a^2e^{4x}+3b^2e^{4x}+3ab\,e^{3x}-4a^2e^{2x}-6b^2e^{2x}-3b\,e^x a+a^2+3b^2\right)}{a^3\left(e^{2x}-1\right)^2\left(b\,e^{2x}+2a\,e^x-b\right)}+\frac{\ln(e^{2x}-1)}{a^2}+\frac{3\ln(e^{2x}-1)b^2}{a^4}-\frac{\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^2}-\frac{3\ln(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^3*(1/2*a*tanh(1/2*x)^2+4*b*tanh(1/2*x))-2/a^4*((-a^2*b-b^3)*tanh(1/2
*x)/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+1/2*(a^2+3*b^2)*ln(a*tanh(1/2*x)^2-
2*b*tanh(1/2*x)-a))-1/8/a^2/tanh(1/2*x)^2+1/4/a^4*(4*a^2+12*b^2)*ln(tanh(1/
2*x))+1/a^3*b/tanh(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(74) = 148.
time = 0.29, size = 202, normalized size = 2.66

$$\frac{2\left(3abe^{(-2x)}-3abe^{(-4x)}+(a^2+3b^2)e^{(-x)}-2(2a^2+3b^2)e^{(-3x)}+(a^2+3b^2)e^{(-5x)}\right)}{2a^4e^{(-x)}-3a^3be^{(-2x)}-4a^4e^{(-3x)}+3a^3be^{(-4x)}+2a^4e^{(-5x)}-a^3be^{(-6x)}+a^3b}-\frac{(a^2+3b^2)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^4}+\frac{(a^2+3b^2)\log\left(e^{(-x)}+1\right)}{a^4}+\frac{(a^2+3b^2)\log\left(e^{(-x)}-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) + (a^2 + 3*b^2)*e^(-x) - 2*(2*a^2 + 3*b^
2)*e^(-3*x) + (a^2 + 3*b^2)*e^(-5*x))/(2*a^4*e^(-x) - 3*a^3*b*e^(-2*x) - 4*
a^4*e^(-3*x) + 3*a^3*b*e^(-4*x) + 2*a^4*e^(-5*x) - a^3*b*e^(-6*x) + a^3*b)
- (a^2 + 3*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^4 + (a^2 + 3*b^2)*log(e
^(-x) + 1)/a^4 + (a^2 + 3*b^2)*log(e^(-x) - 1)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(74) = 148$.
time = 0.45, size = 1463, normalized size = 19.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (6*a^2*b*cosh(x)^4 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2)*sinh(x
)^5 - 6*a^2*b*cosh(x)^2 + 2*(3*a^2*b + 5*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4
 - 4*(2*a^3 + 3*a*b^2)*cosh(x)^3 + 4*(6*a^2*b*cosh(x) - 2*a^3 - 3*a*b^2 + 5
*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^3 + 2*(18*a^2*b*cosh(x)^2 + 10*(a^3 + 3
*a*b^2)*cosh(x)^3 - 3*a^2*b - 6*(2*a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a
^3 + 3*a*b^2)*cosh(x) - ((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x
)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*co
sh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2
*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*
a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*cosh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3
+ 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^3 - a^2*b - 3*b^3
 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2*b + 3*b^3)*cosh(x)^4 + 20*(a^3 +
3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a
^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b
+ 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x
)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*cosh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(
x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^2*b + 3*b^3)*
cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^
3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh
(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2
)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*c
osh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3 + 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*
cosh(x))*sinh(x)^3 - a^2*b - 3*b^3 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2
*b + 3*b^3)*cosh(x)^4 + 20*(a^3 + 3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18
*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3
 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b + 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*co
sh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*c
```

```
osh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sin
h(x))) + 2*(12*a^2*b*cosh(x)^3 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*a^2*b*cosh
(x) + a^3 + 3*a*b^2 - 6*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x))/(a^4*b*cosh(x
)^6 + a^4*b*sinh(x)^6 + 2*a^5*cosh(x)^5 - 3*a^4*b*cosh(x)^4 - 4*a^5*cosh(x)
^3 + 3*a^4*b*cosh(x)^2 + 2*a^5*cosh(x) + 2*(3*a^4*b*cosh(x) + a^5)*sinh(x)^
5 - a^4*b + (15*a^4*b*cosh(x)^2 + 10*a^5*cosh(x) - 3*a^4*b)*sinh(x)^4 + 4*(
5*a^4*b*cosh(x)^3 + 5*a^5*cosh(x)^2 - 3*a^4*b*cosh(x) - a^5)*sinh(x)^3 + (1
5*a^4*b*cosh(x)^4 + 20*a^5*cosh(x)^3 - 18*a^4*b*cosh(x)^2 - 12*a^5*cosh(x)
+ 3*a^4*b)*sinh(x)^2 + 2*(3*a^4*b*cosh(x)^5 + 5*a^5*cosh(x)^4 - 6*a^4*b*cos
h(x)^3 - 6*a^5*cosh(x)^2 + 3*a^4*b*cosh(x) + a^5)*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(74) = 148.
time = 0.45, size = 190, normalized size = 2.50

$$\frac{(a^2+3b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^4} - \frac{(a^2b+3b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^4b} + \frac{a^2b\left(e^{(-x)}-e^x\right)+3b^3\left(e^{(-x)}-e^x\right)-4a^3-8ab^2}{\left(b\left(e^{(-x)}-e^x\right)-2a\right)a^4} - \frac{3a^2\left(e^{(-x)}-e^x\right)^2+9b^2\left(e^{(-x)}-e^x\right)^2+8ab\left(e^{(-x)}-e^x\right)+4a^2}{2a^4\left(e^{(-x)}-e^x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + 3*b^2)*log(abs(-e^(-x) + e^x))/a^4 - (a^2*b + 3*b^3)*log(abs(-b*(e^(
-x) - e^x) + 2*a))/(a^4*b) + (a^2*b*(e^(-x) - e^x) + 3*b^3*(e^(-x) - e^x) -
 4*a^3 - 8*a*b^2)/((b*(e^(-x) - e^x) - 2*a)*a^4) - 1/2*(3*a^2*(e^(-x) - e^x
)^2 + 9*b^2*(e^(-x) - e^x)^2 + 8*a*b*(e^(-x) - e^x) + 4*a^2)/(a^4*(e^(-x) -
 e^x)^2)
```

Mupad [**B**]
time = 1.63, size = 1375, normalized size = 18.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (2*exp(x)*(a*b^7 + 2*a^3*b^5 + a^5*b^3))/(a^4*(b^5 + a^2*b^3)*(2*a*exp(x) -
 b + b*exp(2*x))) - 2/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) - ((2*atan((4*a^9*b
```

```
*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2) + 12*a^5*b^5*((a^2 + 3*b^2)^2)^(1/2)*
(-a^8)^(1/2) + 16*a^7*b^3*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2))*(exp(x)*((a
^2 + 2*b^2)^2/(16*a^10*b^2*(a^4 + 3*b^4 + 4*a^2*b^2)^2) - 1/(16*a^6*b^2*(a^
2 + 3*b^2)^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^9*b*(a^4 + 3*b^4 + 4*a^2*
b^2)^2) + 1/(8*a^7*b*(a^2 + 3*b^2)^2*(a^2 + b^2)^2))) - 2*atan((a^2*(-a^8)^
(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6
*a^2*b^2)^(1/2))/(2*a^4*(a^4 + 3*b^4 + 4*a^2*b^2)) + ((a^8 + 3*a^6*b^2)*(-a
^8)^(1/2))/(2*a^8*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (a^8*b^2*exp(2*x)*
(-a^8)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2))/(a^12*b^2*(a^4 +
3*b^4 + 4*a^2*b^2)) + (4*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2)
+ 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*
b^2)^(1/2))/(a^12*b^2*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)) + (2*(2*a^7*b
 + 6*a^5*b^3)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^
(1/2)*(a^2 + b^2)) + (4*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2)) + (a^8*b^2*exp(3*x)*((2*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)
*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^
4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-
a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)))*(-a^8)^(1/2))/(8*(a^4 + 9*b^4 + 6*a^
2*b^2)^(1/2)) - (a^8*b^2*exp(x)*(-a^8)^(1/2)*((8*(a^4 + 9*b^4 + 6*a^2*b^2))
/(a^11*b*(a^4 + 3*b^4 + 4*a^2*b^2)) - (4*(2*a^7*b + 6*a^5*b^3)*(a^4 + 9*b^4
 + 6*a^2*b^2)^(1/2))/(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a
^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2
)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*
a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 +
 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)
)))/(8*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
(-a^8)^(1/2) - (2/a^2 - (4*b*exp(x))/a^3)/(exp(2*x) - 1)
```

3.243 $\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal. Leaf size=159

$$\frac{b(3a^2+4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{2\sqrt{a^2+b^2}\,(a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}-\frac{(7a^2+12b^2)\coth(x)}{3a^4}+\frac{(a^2+2\ldots}{}$$

```
[Out] b*(3*a^2+4*b^2)*arctanh(cosh(x))/a^5-1/3*(7*a^2+12*b^2)*coth(x)/a^4+(a^2+2*
b^2)*coth(x)*csch(x)/a^3/b-1/3*(3+4*b^2/a^2)*coth(x)*csch(x)/b/(a+b*sinh(x)
)-1/3*coth(x)*csch(x)^2/a/(a+b*sinh(x))-2*(a^2+4*b^2)*arctanh((b-a*tanh(1/2
*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^5
```

Rubi [A]
time = 0.48, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2803, 3134, 3080, 3855, 2739, 632, 212}

$$-\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))}-\frac{2\sqrt{a^2+b^2}\,(a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5}+\frac{b(3a^2+4b^2)\tanh^{-1}(\cosh(x))}{a^5}-\frac{(7a^2+12b^2)\coth(x)}{3a^4}+\frac{(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3b}-\frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a^5 - (2*Sqrt[a^2 + b^2]*(a^2 + 4*b^2)
*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - ((7*a^2 + 12*b^2)*Coth[x
])/(3*a^4) + ((a^2 + 2*b^2)*Coth[x]*Csch[x])/(a^3*b) - ((3 + (4*b^2)/a^2)*C
oth[x]*Csch[x])/(3*b*(a + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sin
h[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{\int \frac{\operatorname{csch}^3(x)(6(a^2+2b^2)-ab\sinh(x)+(3a}{a+b\sinh(x)}}{3a^2b}$$

$$= \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{i\int}{}$$

$$= -\frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} -$$

$$= -\frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} -$$

$$= \frac{b(3a^2+4b^2)\tanh^{-1}(\cosh(x))}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b}$$

$$= \frac{b(3a^2+4b^2)\tanh^{-1}(\cosh(x))}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b}$$

$$= \frac{b(3a^2+4b^2)\tanh^{-1}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}\,(a^2+4b^2)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{(7a}{}$$

Mathematica [A]
time = 0.64, size = 214, normalized size = 1.35

$$\frac{\frac{48(a^4+5a^2b^2+4b^4)\operatorname{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4a(4a^2+9b^2)\coth\left(\frac{x}{2}\right) + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) - 24b(3a^2+4b^2)\log\left(\tanh\left(\frac{x}{2}\right)\right) + 6a^2b\operatorname{sech}^2\left(\frac{x}{2}\right) + 8a^3\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{1}{2}a^3\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x) - \frac{24ab(a^2+b^2)\cosh(x)}{a+b\sinh(x)} - 4a(4a^2+9b^2)\tanh\left(\frac{x}{2}\right)}{24a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((48*(a^4 + 5*a^2*b^2 + 4*b^4)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] - 4*a*(4*a^2 + 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 - 24
*b*(3*a^2 + 4*b^2)*Log[Tanh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*S
inh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b*(a^2 + b^2)*Cosh[x])/(a
+ b*Sinh[x]) - 4*a*(4*a^2 + 9*b^2)*Tanh[x/2])/(24*a^5)
```

Maple [A]
time = 0.69, size = 221, normalized size = 1.39

method	result

default	$-\frac{\frac{\left(\tanh^3\left(\frac{x}{2}\right)\right)a^2}{3}+2ab\left(\tanh^2\left(\frac{x}{2}\right)\right)+5a^2\tanh\left(\frac{x}{2}\right)+12b^2\tanh\left(\frac{x}{2}\right)}{8a^4}-\frac{1}{24a^2\tanh\left(\frac{x}{2}\right)^3}-\frac{5a^2+12b^2}{8a^4\tanh\left(\frac{x}{2}\right)}+\frac{b}{4a^3\tanh\left(\frac{x}{2}\right)^2}-\frac{b(3a^2+4b}{}$
risch	$\frac{2a^3e^{7x}+4a\,b^2e^{7x}-2a^2b\,e^{6x}-8b^3e^{6x}-14a^3e^{5x}-20a\,b^2e^{5x}+14a^2b\,e^{4x}+24b^3e^{4x}+14a^3e^{3x}+28a\,b^2e^{3x}-\frac{50a^2b\,e^{2x}}{3}-24b^3e^{2x}-\frac{22a^3e^x}{3}-12}{a^4(e^{2x}-1)^3(b\,e^{2x}+2a\,e^x-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^4*(1/3*tanh(1/2*x)^3*a^2+2*a*b*tanh(1/2*x)^2+5*a^2*tanh(1/2*x)+12*b^
2*tanh(1/2*x))-1/24/a^2/tanh(1/2*x)^3-1/8*(5*a^2+12*b^2)/a^4/tanh(1/2*x)+1/
4/a^3*b/tanh(1/2*x)^2-1/a^5*b*(3*a^2+4*b^2)*ln(tanh(1/2*x))-2/a^5*((-b^2*(a
^2+b^2)*tanh(1/2*x)-b*a*(a^2+b^2))/(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(a^4
+5*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^
2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(149) = 298$.
time = 0.49, size = 339, normalized size = 2.13

$$-\frac{2\left(7a^2b+12b^3+(11a^3+18ab^2)e^{(-x)}-(25a^2b+36b^3)e^{(-2x)}-21(a^3+2ab^2)e^{(-3x)}+3(7a^2b+12b^3)e^{(-4x)}+3(7a^3+10ab^2)e^{(-5x)}-3(a^2b+4b^3)e^{(-6x)}-3(a^3+2ab^2)e^{(-7x)}\right)}{3\left(2a^5e^{(-x)}-4a^4be^{(-2x)}-6a^5e^{(-3x)}+6a^4be^{(-4x)}+6a^5e^{(-5x)}-4a^4be^{(-6x)}-2a^5e^{(-7x)}+a^4be^{(-8x)}+a^4b\right)}+\frac{(3a^2b+4b^3)\log\left(e^{(-x)}+1\right)}{a^5}-\frac{(3a^2b+4b^3)\log\left(e^{(-x)}-1\right)}{a^5}+\frac{(a^4+5a^2b^2+4b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(7*a^2*b + 12*b^3 + (11*a^3 + 18*a*b^2)*e^(-x) - (25*a^2*b + 36*b^3)*e
^(-2*x) - 21*(a^3 + 2*a*b^2)*e^(-3*x) + 3*(7*a^2*b + 12*b^3)*e^(-4*x) + 3*(
7*a^3 + 10*a*b^2)*e^(-5*x) - 3*(a^2*b + 4*b^3)*e^(-6*x) - 3*(a^3 + 2*a*b^2)
*e^(-7*x))/(2*a^5*e^(-x) - 4*a^4*b*e^(-2*x) - 6*a^5*e^(-3*x) + 6*a^4*b*e^(-
4*x) + 6*a^5*e^(-5*x) - 4*a^4*b*e^(-6*x) - 2*a^5*e^(-7*x) + a^4*b*e^(-8*x)
+ a^4*b) + (3*a^2*b + 4*b^3)*log(e^(-x) + 1)/a^5 - (3*a^2*b + 4*b^3)*log(e^
(-x) - 1)/a^5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. $2(149) = 298$.
time = 0.53, size = 3648, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] 1/3*(6*(a^4 + 2*a^2*b^2)*cosh(x)^7 + 6*(a^4 + 2*a^2*b^2)*sinh(x)^7 - 6*(a^3
*b + 4*a*b^3)*cosh(x)^6 - 6*(a^3*b + 4*a*b^3 - 7*(a^4 + 2*a^2*b^2)*cosh(x))
*sinh(x)^6 - 6*(7*a^4 + 10*a^2*b^2)*cosh(x)^5 - 6*(7*a^4 + 10*a^2*b^2 - 21*
(a^4 + 2*a^2*b^2)*cosh(x)^2 + 6*(a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^5 + 6*(7
*a^3*b + 12*a*b^3)*cosh(x)^4 + 6*(7*a^3*b + 12*a*b^3 + 35*(a^4 + 2*a^2*b^2)
*cosh(x)^3 - 15*(a^3*b + 4*a*b^3)*cosh(x)^2 - 5*(7*a^4 + 10*a^2*b^2)*cosh(x
))*sinh(x)^4 + 14*a^3*b + 24*a*b^3 + 42*(a^4 + 2*a^2*b^2)*cosh(x)^3 + 6*(35
*(a^4 + 2*a^2*b^2)*cosh(x)^4 + 7*a^4 + 14*a^2*b^2 - 20*(a^3*b + 4*a*b^3)*co
sh(x)^3 - 10*(7*a^4 + 10*a^2*b^2)*cosh(x)^2 + 4*(7*a^3*b + 12*a*b^3)*cosh(x
))*sinh(x)^3 - 2*(25*a^3*b + 36*a*b^3)*cosh(x)^2 + 2*(63*(a^4 + 2*a^2*b^2)*
cosh(x)^5 - 45*(a^3*b + 4*a*b^3)*cosh(x)^4 - 25*a^3*b - 36*a*b^3 - 30*(7*a^
4 + 10*a^2*b^2)*cosh(x)^3 + 18*(7*a^3*b + 12*a*b^3)*cosh(x)^2 + 63*(a^4 + 2
*a^2*b^2)*cosh(x))*sinh(x)^2 + 3*((a^2*b + 4*b^3)*cosh(x)^8 + (a^2*b + 4*b^
3)*sinh(x)^8 + 2*(a^3 + 4*a*b^2)*cosh(x)^7 + 2*(a^3 + 4*a*b^2 + 4*(a^2*b +
4*b^3)*cosh(x))*sinh(x)^7 - 4*(a^2*b + 4*b^3)*cosh(x)^6 - 2*(2*a^2*b + 8*b^
3 - 14*(a^2*b + 4*b^3)*cosh(x)^2 - 7*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^6 - 6
*(a^3 + 4*a*b^2)*cosh(x)^5 + 2*(28*(a^2*b + 4*b^3)*cosh(x)^3 - 3*a^3 - 12*a
*b^2 + 21*(a^3 + 4*a*b^2)*cosh(x)^2 - 12*(a^2*b + 4*b^3)*cosh(x))*sinh(x)^5
 + 6*(a^2*b + 4*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 4*b^3)*cosh(x)^4 + 35*(a^3
+ 4*a*b^2)*cosh(x)^3 + 3*a^2*b + 12*b^3 - 30*(a^2*b + 4*b^3)*cosh(x)^2 - 15
*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^4 + 6*(a^3 + 4*a*b^2)*cosh(x)^3 + 2*(28*(
a^2*b + 4*b^3)*cosh(x)^5 + 35*(a^3 + 4*a*b^2)*cosh(x)^4 - 40*(a^2*b + 4*b^3
)*cosh(x)^3 + 3*a^3 + 12*a*b^2 - 30*(a^3 + 4*a*b^2)*cosh(x)^2 + 12*(a^2*b +
 4*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*b^3 - 4*(a^2*b + 4*b^3)*cosh(x)^2 +
2*(14*(a^2*b + 4*b^3)*cosh(x)^6 + 21*(a^3 + 4*a*b^2)*cosh(x)^5 - 30*(a^2*b
+ 4*b^3)*cosh(x)^4 - 30*(a^3 + 4*a*b^2)*cosh(x)^3 - 2*a^2*b - 8*b^3 + 18*(a
^2*b + 4*b^3)*cosh(x)^2 + 9*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^2 - 2*(a^3 + 4
*a*b^2)*cosh(x) + 2*(4*(a^2*b + 4*b^3)*cosh(x)^7 + 7*(a^3 + 4*a*b^2)*cosh(x
)^6 - 12*(a^2*b + 4*b^3)*cosh(x)^5 - 15*(a^3 + 4*a*b^2)*cosh(x)^4 + 12*(a^2
*b + 4*b^3)*cosh(x)^3 - a^3 - 4*a*b^2 + 9*(a^3 + 4*a*b^2)*cosh(x)^2 - 4*(a^
2*b + 4*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(11*a^4 + 18*a^2*b^2)*cosh(x
) + 3*((3*a^2*b^2 + 4*b^4)*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3
*a^3*b + 4*a*b^3)*cosh(x)^7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^7 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4
 - 14*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x
)^6 - 6*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b
^2 + 4*b^4)*cosh(x)^3 - 21*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 +
4*b^4)*cosh(x))*sinh(x)^5 + 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*
b^2 + 4*b^4)*cosh(x)^4 + 9*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x
)^3 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*si
```

```
nh(x)^4 + 3*a^2*b^2 + 4*b^4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^
2*b^2 + 4*b^4)*cosh(x)^5 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*
a*b^3 - 40*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2
 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x
)^2 + 2*(14*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^
5 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*
a*b^3)*cosh(x)^3 + 18*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)
*cosh(x))*sinh(x)^2 - 2*(3*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b
^4)*cosh(x)^7 + 7*(3*a^3*b + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*co
sh(x)^5 - 15*(3*a^3*b + 4*a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*
b^2 + 4*b^4)*cosh(x)^3 + 9*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4
*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b^2 + 4*b^4)
*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3*a^3*b + 4*a*b^3)*cosh(x)^
7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^7 - 4*(3*
a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4 - 14*(3*a^2*b^2 + 4*b^4)*
cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^6 - 6*(3*a^3*b + 4*a*b^3
)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 21
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^5
+ 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 9
*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^3 - 30*(3*a^2*b^2 + 4*b^
4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + 3*a^2*b^2 + 4*b^
4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 +
 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*a*b^3 - 40*(3*a^2*b^2 + 4*
b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**4/(a+b*sinh(x))**2,x)
[Out] Integral(coth(x)**4/(a + b*sinh(x))**2, x)
```

Giac [A]
time = 0.43, size = 242, normalized size = 1.52

$$\frac{(3a^2b+4b^3)\log(e^x+1)}{a^5}-\frac{(3a^2b+4b^3)\log(|e^x-1|)}{a^5}+\frac{(a^4+5a^2b^2+4b^4)\log\left(\frac{|2be^x+2a-2\sqrt{a^2+b^2}|}{|2be^x+2a+2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}\,a^5}+\frac{2(a^3e^x+ab^2e^x-a^2b-b^3)}{(be^{(2x)}+2ae^x-b)a^4}+\frac{2(3abe^{(5x)}-6a^2e^{(4x)}-9b^2e^{(4x)}+6a^2e^{(2x)}+18b^2e^{(2x)}-3abe^x-4a^2-9b^2)}{3a^4(e^{(2x)}-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
[Out] (3*a^2*b + 4*b^3)*log(e^x + 1)/a^5 - (3*a^2*b + 4*b^3)*log(abs(e^x - 1))/a^
5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/ab
```

```
s(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5) + 2*(a^3*e^x +
a*b^2*e^x - a^2*b - b^3)/((b*e^(2*x) + 2*a*e^x - b)*a^4) + 2/3*(3*a*b*e^(5*
x) - 6*a^2*e^(4*x) - 9*b^2*e^(4*x) + 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b
*e^x - 4*a^2 - 9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B]
time = 1.53, size = 1450, normalized size = 9.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] (3*b*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 128*b^4*exp(x) +
224*a^2*b^2*exp(x)))/a^3 - 4/(a^2*exp(2*x) - a^2) - (6*b^2)/(a^4*exp(2*x) -
 a^4) - 8/(3*(3*a^2*exp(2*x) - 3*a^2*exp(4*x) + a^2*exp(6*x) - a^2)) - (4*a
^3*b^7)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^
6*exp(x) + 2*a^8*b^4*exp(x)) - (2*a^5*b^5)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^
5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*exp(x)) - (3*b*log(
96*a^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b
^2*exp(x)))/a^3 - 4/(a^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) - (4*b^3*log(96*a
^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b^2*e
xp(x)))/a^5 + (4*b^3*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 1
28*b^4*exp(x) + 224*a^2*b^2*exp(x)))/a^5 + (log(128*a^6*exp(x) - 256*a*b^5
- 64*a^5*b - 320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288
*a^2*b^3*(a^2 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4
*exp(x) + 672*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x
)*(a^2 + b^2)^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/a^3 - (log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b
^3 + 128*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*
a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64
*a^4*b*(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2
*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^3 - (2*a*b^9)/(a^5*b^7*exp(
2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*
exp(x)) + (4*b*exp(x))/(a^3*exp(4*x) - 2*a^3*exp(2*x) + a^3) + (2*b*exp(x))
/(a^3*exp(2*x) - a^3) + (4*b^2*log(128*a^6*exp(x) - 256*a*b^5 - 64*a^5*b -
320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288*a^2*b^3*(a^2
 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672
*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x)*(a^2 + b^2)
^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 - (4*
b^2*log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b^3 + 12
8*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*a^5*exp
(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64*a^4*b*
(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2*exp(x)
*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 + (2*a^2*b^9*exp(x))/(a^5*b^8*ex
```

```
p(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^
5*exp(x)) + (4*a^4*b^7*exp(x))/(a^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*
b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^5*exp(x)) + (2*a^6*b^5*exp(x))/(a
^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) +
 2*a^8*b^5*exp(x))
```

3.244 $\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$

Optimal. Leaf size=37

$$-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh(x)}$$

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2800, 52, 65, 213}

$$2\sqrt{a+b\sinh(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]],x]
```

```
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)} + a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\ &= 2\sqrt{a+b\sinh(x)} + (2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\ &= -2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.00

$$-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]],x]
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Maple [A]
time = 0.71, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh\left(x\right)}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{a+b\sinh\left(x\right)}$	30
default	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh\left(x\right)}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{a+b\sinh\left(x\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 0.58, size = 356, normalized size = 9.62

$$\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 16\,ab\cosh(x)^3 + 4\,(b^2\cosh(x) + 4\,ab)\sinh(x)^3 - 16\,ab\cosh(x) + 2\,(16\,a^2 - b^2)\cosh(x)^2 + 2\,(3\,b^2\cosh(x)^2 + 24\,ab\cosh(x) + 16\,a^2 - b^2)\sinh(x)^2 - 8\,(b\cosh(x)^3 + b\sinh(x)^3 + 4\,a\cosh(x)^2 + (3\,b\cosh(x) + 4\,a)\sinh(x)^2 - b\cosh(x) + (3\,b\cosh(x)^2 + 8\,a\cosh(x) - b)\sinh(x))\sqrt{b\sinh(x) + a}\sqrt{a} + b^2 + 4\,(b^2\cosh(x)^3 + 12\,ab\cosh(x)^2 - 4\,ab + (16\,a^2 - b^2)\cosh(x))\sinh(x)}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\,(3\cosh(x)^2 - 1)\sinh(x)^2 - 2\cosh(x)^2 + 4\,(\cosh(x)^3 - \cosh(x))\sinh(x) + 1}\right) + 2\sqrt{b\sinh(x) + a},\ \sqrt{-a}\arctan\left(\frac{4\sqrt{b\sinh(x) + a}\sqrt{-a}(\cosh(x) + \sinh(x))}{b\cosh(x)^2 + b\sinh(x)^2 + 4\,a\cosh(x) + 2\,(b\cosh(x) + 2\,a)\sinh(x) - b}\right) + 2\sqrt{b\sinh(x) + a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*sinh(x) + a), sqrt(-a)*arctan(4*sqrt(b*sinh(x) + a)*sqrt(-a)*(cosh(x
) + sinh(x))/(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)
*sinh(x) - b)) + 2*sqrt(b*sinh(x) + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh(x)}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x))*coth(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \coth(x)\ \sqrt{a+b\sinh\left(x\right)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x))^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*sinh(x))^(1/2), x)
```

3.245 $\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2800, 65, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.80, size = 19, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh\left(x\right)}}{\sqrt{a}}\right)}{\sqrt{a}}$	19
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sinh\left(x\right)}}{\sqrt{a}}\right)}{\sqrt{a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(18) = 36$.
time = 0.46, size = 370, normalized size = 15.42

$$\left[\frac{\log\left(\frac{b^{2} \cosh\left(x\right)^{4} + b^{2} \sinh\left(x\right)^{4} + 16 \, a b \cosh\left(x\right)^{3} + 4 \, {\left(b^{2} \cosh\left(x\right) + 4 \, a b\right)} \sinh\left(x\right)^{3} - 16 \, a b \cosh\left(x\right) + 2 \, {\left(16 \, a^{2} - b^{2}\right)} \cosh\left(x\right)^{2} + 2 \, {\left(3 \, b^{2} \cosh\left(x\right)^{2} + 24 \, a b \cosh\left(x\right) + 16 \, a^{2} - b^{2}\right)} \sinh\left(x\right)^{2} - 8 \, {\left(b \cosh\left(x\right)^{3} + b \sinh\left(x\right)^{3} + 4 \, a \cosh\left(x\right)^{2} + {\left(3 \, b \cosh\left(x\right) + 4 \, a\right)} \sinh\left(x\right)^{2} - b \cosh\left(x\right) + {\left(3 \, b \cosh\left(x\right)^{2} + 8 \, a \cosh\left(x\right) - b\right)} \sinh\left(x\right)\right)} \sqrt{b \sinh\left(x\right) + a} \sqrt{a} + b^{2} + 4 \, {\left(b^{2} \cosh\left(x\right)^{3} + 12 \, a b \cosh\left(x\right)^{2} - 4 \, a b + {\left(16 \, a^{2} - b^{2}\right)} \cosh\left(x\right)\right)} \sinh\left(x\right)}{\cosh\left(x\right)^{4} + 4 \, \cosh\left(x\right) \sinh\left(x\right)^{3} + \sinh\left(x\right)^{4} + 2 \, {\left(3 \, \cosh\left(x\right)^{2} - 1\right)} \sinh\left(x\right)^{2} - 2 \, \cosh\left(x\right)^{2} + 4 \, {\left(\cosh\left(x\right)^{3} - \cosh\left(x\right)\right)} \sinh\left(x\right) + 1}\right)}{2 \, \sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{{\left(b \cosh\left(x\right)^{2} + b \sinh\left(x\right)^{2} + 4 \, a \cosh\left(x\right) + 2 \, {\left(b \cosh\left(x\right) + 2 \, a\right)} \sinh\left(x\right) - b\right)} \sqrt{b \sinh\left(x\right) + a} \sqrt{-a}}{2 \, {\left(a b \cosh\left(x\right)^{2} + a b \sinh\left(x\right)^{2} + 2 \, a^{2} \cosh\left(x\right) - a b + 2 \, {\left(a b \cosh\left(x\right) + a^{2}\right)} \sinh\left(x\right)\right)}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*si
nh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(x\right)}{\sqrt{a + b \sinh\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x))^(1/2),x)
[Out] int(coth(x)/(a + b*sinh(x))^(1/2), x)
```

3.246 $\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

```
[Out] B*ln(a+b*sinh(x))/b-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4486, 2739, 632, 212, 2747, 31}

$$\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}\\
&= \frac{B\log(a+b\sinh(x))}{b}-(4A)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.16

$$\frac{2A\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.
time = 0.46, size = 92, normalized size = 1.80

method	result
default	$-\frac{B\ln(\tanh(\frac{x}{2})+1)}{b}+\frac{B\ln(a(\tanh^2(\frac{x}{2}))-2b\tanh(\frac{x}{2})-a)+\frac{2Ab\,\mathrm{arctanh}\left(\frac{2a\tanh(\frac{x}{2})-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}-\frac{B\ln(\tanh(\frac{x}{2})-1)}{b}$
risch	$\frac{Bx}{b}-\frac{2xB\,a^2b}{a^2b^2+b^4}-\frac{2xB\,b^3}{a^2b^2+b^4}+\frac{\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B\,a^2}{(a^2+b^2)b}+\frac{b\ln\left(\mathrm{e}^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B}{a^2+b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b*ln(tanh(1/2*x)+1)+2/b*(1/2*B*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+A*b
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-B/b*ln
(tanh(1/2*x)-1)
```

Maxima [A]
time = 0.49, size = 68, normalized size = 1.33

$$\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log\left(b\sinh\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sq
rt(a^2 + b^2) + B*log(b*sinh(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(47) = 94$.
time = 0.42, size = 170, normalized size = 3.33

$$\frac{\sqrt{a^2+b^2}\,Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(Ba^2+Bb^2)x+(Ba^2+Bb^2)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{a^2b+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
```

```
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x + (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(44) = 88$.

time = 52.42, size = 745, normalized size = 14.61

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) & \text{for}\ a = 0 \wedge b = 0 \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx - 2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right) + B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} & \text{for}\ a = 0 \\ \frac{Ax + B\sinh(x)}{a} & \text{for}\ b = 0 \\ \frac{2A\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} + \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} + \frac{2Bb\log\left(\frac{b}{\sqrt{-b^2}} + \tanh\left(\frac{x}{2}\right)\right)\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} - \frac{2Bb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} - \frac{Bx\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} - \frac{2B\sqrt{-b^2}\log\left(\frac{b}{\sqrt{-b^2}} + \tanh\left(\frac{x}{2}\right)\right)}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} + \frac{2B\sqrt{-b^2}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2\tanh\left(\frac{x}{2}\right) - b\sqrt{-b^2}} & \text{for}\ a = -\sqrt{-b^2} \\ -\frac{2A\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} + \frac{Bbx\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} + \frac{2Bb\log\left(-\frac{b}{\sqrt{-b^2}} + \tanh\left(\frac{x}{2}\right)\right)\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} - \frac{2Bb\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} + \frac{Bx\sqrt{-b^2}}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} + \frac{2B\sqrt{-b^2}\log\left(-\frac{b}{\sqrt{-b^2}} + \tanh\left(\frac{x}{2}\right)\right)}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} - \frac{2B\sqrt{-b^2}\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2\tanh\left(\frac{x}{2}\right) + b\sqrt{-b^2}} & \text{for}\ a = \sqrt{-b^2} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{A\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} + \frac{B\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)}{b} + \frac{B\log\left(\tanh\left(\frac{x}{2}\right) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/b, Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (
2*A*sqrt(-b**2)/(b**2*tanh(x/2) - b*sqrt(-b**2)) + B*b*x*tanh(x/2)/(b**2*ta
nh(x/2) - b*sqrt(-b**2)) + 2*B*b*log(b/sqrt(-b**2) + tanh(x/2))*tanh(x/2)/(
b**2*tanh(x/2) - b*sqrt(-b**2)) - 2*B*b*log(tanh(x/2) + 1)*tanh(x/2)/(b**2*
tanh(x/2) - b*sqrt(-b**2)) - B*x*sqrt(-b**2)/(b**2*tanh(x/2) - b*sqrt(-b**2
)) - 2*B*sqrt(-b**2)*log(b/sqrt(-b**2) + tanh(x/2))/(b**2*tanh(x/2) - b*sqr
t(-b**2)) + 2*B*sqrt(-b**2)*log(tanh(x/2) + 1)/(b**2*tanh(x/2) - b*sqrt(-b*
*2)), Eq(a, -sqrt(-b**2))), (-2*A*sqrt(-b**2)/(b**2*tanh(x/2) + b*sqrt(-b**
2)) + B*b*x*tanh(x/2)/(b**2*tanh(x/2) + b*sqrt(-b**2)) + 2*B*b*log(-b/sqrt(
-b**2) + tanh(x/2))*tanh(x/2)/(b**2*tanh(x/2) + b*sqrt(-b**2)) - 2*B*b*log(
tanh(x/2) + 1)*tanh(x/2)/(b**2*tanh(x/2) + b*sqrt(-b**2)) + B*x*sqrt(-b**2)
/(b**2*tanh(x/2) + b*sqrt(-b**2)) + 2*B*sqrt(-b**2)*log(-b/sqrt(-b**2) + ta
nh(x/2))/(b**2*tanh(x/2) + b*sqrt(-b**2)) - 2*B*sqrt(-b**2)*log(tanh(x/2) +
 1)/(b**2*tanh(x/2) + b*sqrt(-b**2)), Eq(a, sqrt(-b**2))), (-A*log(tanh(x/2
) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + A*log(tanh(x/2) - b/a +
sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) + B*x/b - 2*B*log(tanh(x/2) + 1)/b +
 B*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/b + B*log(tanh(x/2) - b/a + s
qrt(a**2 + b**2)/a)/b, True))
```

Giac [A]

time = 0.43, size = 87, normalized size = 1.71

$$\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} - \frac{Bx}{b} + \frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) - B*x/b + B*log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

Mupad [B]
time = 2.64, size = 198, normalized size = 3.88

$$\frac{B\,b^3\,\ln\left(8\,A^2\,a\,\mathrm{e}^x-4\,A^2\,b+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2+b^4}-\frac{B\,x}{b}-\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^x\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}}+\frac{B\,a^2\,b\,\ln\left(8\,A^2\,a\,\mathrm{e}^x-4\,A^2\,b+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*b^3*log(8*A^2*a*exp(x) - 4*A^2*b + 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2) -
(B*x)/b - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(
A^2)^(1/2)) + (A^2*a*b*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(A^2)^(1/2))
)*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) + (B*a^2*b*log(8*A^2*a*exp(x) - 4*A^2*b
+ 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2)
```

3.247 $\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx$

Optimal. Leaf size=25

$$B\log(i+\sinh(x))-\frac{A\cosh(x)}{1-i\sinh(x)}$$

[Out] B*ln(I+sinh(x))-A*cosh(x)/(1-I*sinh(x))

Rubi [A]
time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4486, 2727, 2746, 31}

$$B\log(\sinh(x)+i)-\frac{A\cosh(x)}{1-i\sinh(x)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cosh[x])/(I + Sinh[x]),x]

[Out] B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx &= \int \left(\frac{iA}{-1+i\sinh(x)}+\frac{iB\cosh(x)}{-1+i\sinh(x)}\right)dx\\ &= (iA)\int \frac{1}{-1+i\sinh(x)}\,dx+(iB)\int \frac{\cosh(x)}{-1+i\sinh(x)}\,dx\\ &= -\frac{A\cosh(x)}{1-i\sinh(x)}+B\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,i\sinh(x)\right)\\ &= B\log(i+\sinh(x))-\frac{A\cosh(x)}{1-i\sinh(x)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 1.92

$$-2iB\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))-\frac{2iA\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I + Sinh[x]),x]
[Out] (-2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]] - ((2*I)*A*Sinh[x/2])/(Cosh[x/2
] - I*Sinh[x/2])
```

Maple [A]
time = 0.59, size = 46, normalized size = 1.84

method	result	size
risch	$-Bx-\frac{2A}{e^{x}+i}+2B\ln\left(e^{x}+i\right)$	25
default	$-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+2B\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -B*ln(tanh(1/2*x)-1)+2*B*ln(tanh(1/2*x)+I)-2*I*A/(tanh(1/2*x)+I)-B*ln(tanh(
1/2*x)+1)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.76

$$B\log\left(\sinh\left(x\right)+i\right)-\frac{2\,A}{e^{(-x)}-i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] B*log(sinh(x) + I) - 2*A/(e^(-x) - I)
```

Fricas [**A**]
time = 0.41, size = 36, normalized size = 1.44

$$-\frac{Bxe^x + i\,Bx - 2\,(Be^x + i\,B)\log{(e^x + i)} + 2\,A}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -(B*x*e^x + I*B*x - 2*(B*e^x + I*B)*log(e^x + I) + 2*A)/(e^x + I)
```

Sympy [**A**]
time = 0.08, size = 20, normalized size = 0.80

$$-\frac{2A}{e^x + i} - Bx + 2B\log{(e^x + i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x)
```

```
[Out] -2*A/(exp(x) + I) - B*x + 2*B*log(exp(x) + I)
```

Giac [**A**]
time = 0.41, size = 22, normalized size = 0.88

$$-Bx + 2\,B\log{(e^x + i)} - \frac{2\,A}{e^x + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -B*x + 2*B*log(e^x + I) - 2*A/(e^x + I)
```

Mupad [**B**]
time = 0.14, size = 24, normalized size = 0.96

$$-B\,x - \frac{2\,A}{\mathrm{e}^x + 1\mathrm{i}} + 2\,B\,\ln{(\mathrm{e}^x + 1\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(x))/(sinh(x) + 1i),x)
```

```
[Out] 2*B*log(exp(x) + 1i) - (2*A)/(exp(x) + 1i) - B*x
```

3.248 $\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx$

Optimal. Leaf size=27

$$-B\log(i-\sinh(x))+\frac{A\cosh(x)}{1+i\sinh(x)}$$

```
[Out] -B*ln(I-sinh(x))+A*cosh(x)/(1+I*sinh(x))
```

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4486, 2727, 2746, 31}

$$\frac{A\cosh(x)}{1+i\sinh(x)}-B\log(-\sinh(x)+i)$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx &= \int \left(-\frac{iA}{1+i\sinh(x)}-\frac{iB\cosh(x)}{1+i\sinh(x)}\right)dx\\ &= -\left((iA)\int \frac{1}{1+i\sinh(x)}\,dx\right)-(iB)\int \frac{\cosh(x)}{1+i\sinh(x)}\,dx\\ &= \frac{A\cosh(x)}{1+i\sinh(x)}-B\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,i\sinh(x)\right)\\ &= -B\log(i-\sinh(x))+\frac{A\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 81 vs. 2(27) = 54.
time = 0.07, size = 81, normalized size = 3.00

$$-\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(B\cosh\left(\frac{x}{2}\right)\left(2\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))\right)+\left(2A+2iB\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))\right)\sinh\left(\frac{x}{2}\right)\right)}{-i+\sinh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[x])/(I - Sinh[x]),x]
[Out] -(((Cosh[x/2] + I*Sinh[x/2])*(B*Cosh[x/2]*(2*ArcTan[Tanh[x/2]] - I*Log[Cosh
[x]]) + (2*A + (2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]])*Sinh[x/2]))/(-I
+ Sinh[x]))
```

Maple [**A**]
time = 0.59, size = 44, normalized size = 1.63

method	result	size
risch	$Bx+\frac{2A}{e^{x}-i}-2B\ln\left(e^{x}-i\right)$	24
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)-i}-2B\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
[Out] B*ln(tanh(1/2*x)-1)+B*ln(tanh(1/2*x)+1)-2*I*A/(tanh(1/2*x)-I)-2*B*ln(tanh(1
/2*x)-I)
```

Maxima [**A**]
time = 0.26, size = 20, normalized size = 0.74

$$-B\log\left(\sinh\left(x\right)-i\right)+\frac{2\,A}{e^{(-x)}+i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*log(sinh(x) - I) + 2*A/(e^(-x) + I)
```

Fricas [A]
time = 0.46, size = 35, normalized size = 1.30

$$\frac{Bxe^{x} - i\,Bx - 2\left(Be^{x} - i\,B\right)\log\left(e^{x} - i\right) + 2\,A}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="fricas")
```

```
[Out] (B*x*e^x - I*B*x - 2*(B*e^x - I*B)*log(e^x - I) + 2*A)/(e^x - I)
```

Sympy [A]
time = 0.08, size = 20, normalized size = 0.74

$$\frac{2A}{e^{x} - i} + Bx - 2B\log\left(e^{x} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x)
```

```
[Out] 2*A/(exp(x) - I) + B*x - 2*B*log(exp(x) - I)
```

Giac [A]
time = 0.40, size = 21, normalized size = 0.78

$$Bx - 2\,B\log\left(e^{x} - i\right) + \frac{2\,A}{e^{x} - i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="giac")
```

```
[Out] B*x - 2*B*log(e^x - I) + 2*A/(e^x - I)
```

Mupad [B]
time = 0.12, size = 23, normalized size = 0.85

$$B\,x + \frac{2\,A}{\mathrm{e}^{x} - \mathrm{i}} - 2\,B\,\ln\left(\mathrm{e}^{x} - \mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A + B*cosh(x))/(sinh(x) - 1i),x)
```

```
[Out] B*x + (2*A)/(exp(x) - 1i) - 2*B*log(exp(x) - 1i)
```

3.249 $\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$\frac{bB\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{aB\log(\cosh(x))}{a^2+b^2}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] b*B*arctan(sinh(x))/(a^2+b^2)+a*B*ln(cosh(x))/(a^2+b^2)-a*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {4486, 2739, 632, 212, 2800, 815, 649, 209, 266}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}+\frac{aB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\tanh(x)}{a+b\sinh(x)}\right)dx$$

$$= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$$

$$= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - B\text{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)}\,dx$$

$$= -\left((4A)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) - B\text{Subst}\left(\int \left(\frac{}{(a^2+}\right.\right.$$

$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2} - \frac{B\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh\right.}{a^2+b^2}$$

$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2} + \frac{(aB)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sin\right.}{a^2+b^2}$$

$$= \frac{bB\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{aB\log(\cosh(x))}{a^2+b^2} - \frac{aB\log(a+b\sin}{a^2+b^2}$$

Mathematica [A]

time = 0.28, size = 132, normalized size = 1.48

$$-\frac{\cosh(x)\left(2b\sqrt{-a^2-b^2}\,B\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)+2A(a^2+b^2)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\sqrt{-a^2-b^2}\,B(\log(\cosh(x))-\log(a+b\sinh(x)))\right)(A+B\tanh(x))}{(-a^2-b^2)^{3/2}(A\cosh(x)+B\sinh(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((Cosh[x]*(2*b*Sqrt[-a^2 - b^2]*B*ArcTan[Tanh[x/2]] + 2*A*(a^2 + b^2)*ArcT
an[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Sqrt[-a^2 - b^2]*B*(Log[Cosh[x]]
 - Log[a + b*Sinh[x]]))*(A + B*Tanh[x]))/((-a^2 - b^2)^(3/2)*(A*Cosh[x] + B
*Sinh[x])))
```

Maple [A]

time = 0.72, size = 117, normalized size = 1.31

method	result
default	$\frac{2B\left(\frac{a\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{2}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2}+\frac{-Ba\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-Ab^2\right)\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}$

risch	$-\frac{2xBa}{a^2+b^2}-\frac{2x\,a^3B}{-a^4-2a^2b^2-b^4}-\frac{2xBa\,b^2}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(e^x+i)b}{a^2+b^2}+\frac{B\ln(e^x+i)a}{a^2+b^2}-\frac{iB\ln(e^x-i)b}{a^2+b^2}+\frac{B\ln(e^x-i)a}{a^2+b^2}-\frac{\ln\left(e^x+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*tanh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/(a^2+b^2)*(1/2*a*ln(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))+2/(a^2+b^2)
*(-1/2*B*a*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(-A*a^2-A*b^2)/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.49, size = 125, normalized size = 1.40

$$-B\left(\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(2*b*arctan(e^(-x))/(a^2 + b^2) + a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) - a*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.
time = 1.35, size = 172, normalized size = 1.93

$$\frac{2\,Bb\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-Ba\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+Ba\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*b*arctan(cosh(x) + sinh(x)) - B*a*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) + B*a*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\tanh\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x)
[Out] Integral((A + B*tanh(x))/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.42, size = 123, normalized size = 1.38

$$\frac{2\,Bb\arctan\left(e^{x}\right)}{a^{2}+b^{2}}+\frac{Ba\log\left(e^{(2\,x)}+1\right)}{a^{2}+b^{2}}-\frac{Ba\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^{2}+b^{2}}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="giac")
[Out] 2*B*b*arctan(e^x)/(a^2 + b^2) + B*a*log(e^(2*x) + 1)/(a^2 + b^2) - B*a*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B]
time = 8.67, size = 914, normalized size = 10.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*tanh(x))/(a + b*sinh(x)),x)
[Out] (B*log(exp(x) + 1i))/(a - b*1i) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*ex
p(x) + A^2*a*b + B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/
b^5 - (((32*(A^2*b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x)
 - 5*B^2*a*b^2*exp(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x)
 + 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2)*
(a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^
3*b^3*(128*A - 256*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*ex
p(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) + 96*A*a^2*b*((a^2 + b^2)^3)^(
1/2) - 128*A*a^3*exp(x)*((a^2 + b^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*((a^2 + b
^2)^3)^(1/2)))/(b^5*(a^2 + b^2)^3))*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*
b^2))/(a^2 + b^2)^2)*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2))/(a^4 + b^
4 + 2*a^2*b^2) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*exp(x) + A^2*a*b +
B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5 - (((32*(A^2*
b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x) - 5*B^2*a*b^2*ex
p(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x) + 2*A*B*a*b^2*ex
p(x)))/b^5 - ((A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2)*(a*b^5*(64*A - 12
8*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^3*b^3*(128*A - 25
6*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*exp(x)*(A - 3*B) +
```

```
96*a^4*b^2*exp(x)*(A - 3*B) - 96*A*a^2*b*((a^2 + b^2)^3)^(1/2) + 128*A*a^3*
exp(x)*((a^2 + b^2)^3)^(1/2) + 32*A*a*b^2*exp(x)*((a^2 + b^2)^3)^(1/2)))/(b
^5*(a^2 + b^2)^3))*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^2 + b^2)
^2)*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
(B*log(exp(x) - 1i)*1i)/(a*1i - b)
```

3.250 $\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(\sinh(x))}{a}-\frac{B\log(a+b\sinh(x))}{a}$$

```
[Out] B*ln(sinh(x))/a-B*ln(a+b*sinh(x))/a-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)
^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {4486, 2739, 632, 212, 2800, 36, 29, 31}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\coth(x)}{a+b\sinh(x)}\right)dx \\
&= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\coth(x)}{a+b\sinh(x)}\,dx \\
&= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + B\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh \right. \\
&= -\left((4A)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{B\text{Subst}\left(\int \frac{1}{x}\,dx, x \right.}{a} \\
&= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log(\sinh(x))}{a} - \frac{B\log(a+b\sinh(x))}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.13, size = 65, normalized size = 1.08

$$\frac{2A\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{B(\log(\sinh(x))-\log(a+b\sinh(x)))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Coth[x])/(a + b*Sinh[x]),x]
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*(Log
[Sinh[x]] - Log[a + b*Sinh[x]]))/a
```

Maple [A]
time = 0.63, size = 76, normalized size = 1.27

method	result
default	$\frac{-B\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)+\frac{2Aa\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
risch	$-\frac{2xB}{a}-\frac{2x\,a^3B}{-a^4-a^2b^2}-\frac{2xBa\,b^2}{-a^4-a^2b^2}+\frac{B\ln\left(\mathrm{e}^{2x}-1\right)}{a}-\frac{a\ln\left(\mathrm{e}^x+\frac{a^2A-\sqrt{A^2a^4+A^2a^2b^2}}{Aab}\right)B}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^x+\frac{a^2A-\sqrt{A^2a^4+}}{Aab}\right.}{(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*coth(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-B*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)+2*A*a/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+B/a*ln(tanh(1/2*x))
```

Maxima [A]
time = 0.49, size = 106, normalized size = 1.77

$$-B\left(\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}-\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -B*(log(-2*a*e^(-x) + b*e^(-2*x) - b)/a - log(e^(-x) + 1)/a - log(e^(-x) -
1)/a) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b
^2)))/sqrt(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(56) = 112.
time = 0.43, size = 183, normalized size = 3.05

$$\frac{\sqrt{a^2+b^2}\,Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(Ba^2+Bb^2)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+(Ba^2+Bb^2)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + (B*a^2 + B*b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\coth(x)}{a + b\sinh(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.43, size = 102, normalized size = 1.70

$$\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}} + \frac{B\log\left(e^{x}+1\right)}{a} - \frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a} + \frac{B\log\left(\left|e^{x}-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) + B*log(e^x + 1)/a - B*log(abs(b*e^(2*x) + 2*a*e^
x - b))/a + B*log(abs(e^x - 1))/a
```

Mupad [B]
time = 11.21, size = 164, normalized size = 2.73

$$\frac{B\,\ln\left(16\,B^{2}\,a^{2}+16\,B^{2}\,b^{2}-16\,B^{2}\,a^{2}\,\mathrm{e}^{2\,x}-16\,B^{2}\,b^{2}\,\mathrm{e}^{2\,x}\right)}{a} - \frac{2\,\mathrm{atan}\left(\frac{A^{2}\,b^{2}\,\mathrm{e}^{x}\,\sqrt{-a^{2}-b^{2}}+A^{2}\,a\,b\,\sqrt{-a^{2}-b^{2}}}{A\,b\,\sqrt{A^{2}}\,\left(a^{2}+b^{2}\right)}\right)\,\sqrt{A^{2}}}{\sqrt{-a^{2}-b^{2}}} - \frac{B\,\ln\left(32\,B^{2}\,a\,\mathrm{e}^{x}-16\,B^{2}\,b+16\,B^{2}\,b\,\mathrm{e}^{2\,x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*coth(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(16*B^2*a^2 + 16*B^2*b^2 - 16*B^2*a^2*exp(2*x) - 16*B^2*b^2*exp(2*x))
)/a - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2) + A^2*a*b*(- a^2 - b^2)^(
1/2))/(A*b*(A^2)^(1/2)*(a^2 + b^2)))*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) - (B*
log(32*B^2*a*exp(x) - 16*B^2*b + 16*B^2*b*exp(2*x)))/a
```

3.251 $\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=89

$$\frac{aB\text{ArcTan}(\sinh(x))}{a^2+b^2}-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{bB\log(\cosh(x))}{a^2+b^2}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] a*B*arctan(sinh(x))/(a^2+b^2)-b*B*ln(cosh(x))/(a^2+b^2)+b*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {4311, 4486, 2739, 632, 212, 2747, 720, 31, 649, 210, 266}

$$-\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{aB\text{ArcTan}(\sinh(x))}{a^2+b^2}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}-\frac{bB\log(\cosh(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4311

```
Int[(u_)*((A_) + (B_.)*sec[(a_.) + (b_.)*(x_)]), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Cos[a + b*x])/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \int \frac{(B+A\cosh(x))\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\operatorname{sech}(x)}{a+b\sinh(x)}\right)\,dx$$
$$= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) - (bB)\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\right.$$
$$= -\left((4A)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{(bB)\text{Subst}\left(\int \frac{1}{a+x}\,dx\right.}{a^2+b}$$
$$= -\frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2} + \frac{(bB)\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sin\right.}{a^2+b^2}$$
$$= \frac{aB\tan^{-1}(\sinh(x))}{a^2+b^2} - \frac{2A\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{bB\log(\cosh(x))}{a^2+b^2} + \frac{bB\log(a+b\sin}{a^2+b^2}$$

Mathematica [A]
time = 0.19, size = 93, normalized size = 1.04

$$\frac{2aB\text{ArcTan}\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} + \frac{2A\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{bB(\log(\cosh(x))-\log(a+b\sinh(x)))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sech[x])/(a + b*Sinh[x]),x]
[Out] (2*a*B*ArcTan[Tanh[x/2]])/(a^2 + b^2) + (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[
-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (b*B*(Log[Cosh[x]] - Log[a + b*Sinh[x]]))/
(a^2 + b^2)
```

Maple [A]
time = 0.64, size = 117, normalized size = 1.31

method	result
default	$\frac{2B\left(-\frac{b\ln\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}{2}+a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2} + \frac{bB\ln\left(a\left(\tanh^2\left(\frac{x}{2}\right)\right)-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-A\,b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}$

risch	$\frac{2xBb}{a^2+b^2}+\frac{2xB\,a^2b}{-a^4-2a^2b^2-b^4}+\frac{2xB\,b^3}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x+i)b}{a^2+b^2}-\frac{iB\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+\frac{Aa}{\ }\right.}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sech(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/(a^2+b^2)*(-1/2*b*ln(tanh(1/2*x)^2+1)+a*arctan(tanh(1/2*x)))+2/(a^2+b^2
)*(1/2*b*B*ln(a*tanh(1/2*x)^2-2*b*tanh(1/2*x)-a)-(-A*a^2-A*b^2)/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 125, normalized size = 1.40

$$-B\left(\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.
time = 2.11, size = 172, normalized size = 1.93

$$\frac{2\,Ba\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+Bb\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-Bb\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}\,A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*a*arctan(cosh(x) + sinh(x)) + B*b*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) - B*b*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\,\mathrm{sech}\,(x)}{a+b\sinh\,(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.44, size = 123, normalized size = 1.38

$$\frac{2\,Ba\arctan\left(e^{x}\right)}{a^{2}+b^{2}}-\frac{Bb\log\left(e^{(2\,x)}+1\right)}{a^{2}+b^{2}}+\frac{Bb\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^{2}+b^{2}}+\frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(e^x)/(a^2 + b^2) - B*b*log(e^(2*x) + 1)/(a^2 + b^2) + B*b*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B]
time = 10.64, size = 864, normalized size = 9.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (log(((A*((a^2 + b^2)^3)^(1/2) + B*b^3 + B*a^2*b)*(b^3*(32*A^2 + 64*B^2 - 9
6*A*B*exp(x)) - 128*A^2*exp(x)*((a^2 + b^2)^3)^(1/2) - a*b^2*(96*A^2*exp(x)
 + 128*B^2*exp(x) - 192*A*B) - 128*a^3*exp(x)*(A^2 + B^2) + a^2*b*(64*A^2 +
 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(
1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^2*
b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^3*b^3*(4*A - 19*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*exp(x)))/((a^2 + b^2)^
3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^2)^3)^(1/2)))/(b^5*(a
^2 + b^2)^2) - (32*B*(2*B^2*b^2 - A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(
x) + A^2*a*b*exp(x) - 4*B^2*a*b*exp(x) - 2*A*B*a*b))/b^5)*(A*((a^2 + b^2)^3
)^(1/2) + B*b^3 + B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2) - (B*log(exp(x) + 1i))/
(a*1i + b) - (B*log(exp(x) - 1i)*1i)/(a + b*1i) + (log(- (32*B*(2*B^2*b^2 -
 A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(x) + A^2*a*b*exp(x) - 4*B^2*a*b*e
xp(x) - 2*A*B*a*b))/b^5 - ((B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2*b)*(a*b
^2*(96*A^2*exp(x) + 128*B^2*exp(x) - 192*A*B) - 128*A^2*exp(x)*((a^2 + b^2)
^3)^(1/2) - b^3*(32*A^2 + 64*B^2 - 96*A*B*exp(x)) + 128*a^3*exp(x)*(A^2 + B
^2) - a^2*b*(64*A^2 + 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x
)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^
```

```
3)^(1/2) + (32*A*a^2*b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*
a^3*b^3*(4*A - 19*B*exp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^
2)^3)^(1/2)))/(b^5*(a^2 + b^2)^2))*(B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2
*b))/(a^4 + b^4 + 2*a^2*b^2)
```

3.252 $\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal. Leaf size=58

$$-\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] -B*arctanh(cosh(x))/a-2*(A*a-B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/a/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2907, 3080, 3855, 2739, 632, 212}

$$-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\tanh^{-1}(\cosh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((B*ArcTanh[Cosh[x]])/a) - (2*(a*A - b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a
^2 + b^2]])/(a*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx &= -\left(i\int \frac{\operatorname{csch}(x)(iB+iA\sinh(x))}{a+b\sinh(x)}\,dx\right)\\
&= \frac{B\int \operatorname{csch}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}+\frac{(2(aA-bB))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{(4(aA-bB))\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\tanh^{-1}(\cosh(x))}{a}-\frac{2(aA-bB)\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 1.16

$$
\frac{\frac{2(aA-bB)\text{ArcTan}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] ((2*(a*A - b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + B*Log[Tanh[x/2]])/a
```

Maple [A]
time = 0.56, size = 58, normalized size = 1.00

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{(-2Aa+2Bb)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$
risch	$\frac{B\ln(\mathrm{e}^x-1)}{a}-\frac{B\ln(\mathrm{e}^x+1)}{a}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)Bb}{\sqrt{a^2+b^2}\,a}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}}{\sqrt{a^2+}}\right.}{\sqrt{a^2+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*csch(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B/a*ln(tanh(1/2*x))-(-2*A*a+2*B*b)/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(
1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.
time = 0.49, size = 141, normalized size = 2.43

$$-B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*a) + log(e^(-x) + 1)/a - log(e^(-x) - 1)/a) + A*log((b*e
^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b
^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.
time = 0.69, size = 172, normalized size = 2.97

$$-\frac{(Aa-Bb)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}\,(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+(Ba^2+Bb^2)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(Ba^2+Bb^2)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((A*a - B*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) + (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) + 1) - (B
*a^2 + B*b^2)*log(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\operatorname{csch}(x)}{a + b\sinh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*sinh(x)), x)
```

Giac [A]
time = 0.42, size = 90, normalized size = 1.55

$$-\frac{B\log\left(e^{x}+1\right)}{a}+\frac{B\log\left(\left|e^{x}-1\right|\right)}{a}+\frac{(Aa-Bb)\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*log(e^x + 1)/a + B*log(abs(e^x - 1))/a + (A*a - B*b)*log(abs(2*b*e^x + 2
*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*a)
```

Mupad [B]
time = 2.18, size = 539, normalized size = 9.29

$$\frac{B\,\ln\left(e^{x}-1\right)}{a}-\frac{B\,\ln\left(e^{x}+1\right)}{a}-\frac{\ln\left(\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,e^{x}\,B^2\,a^3+2\,B^2\,a^2\,b-3\,e^{x}\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2+4\,A\,a^2\,e^{x}+A\,b^2\,e^{x}-2\,A\,a\,b-3\,B\,a\,b\,e^{x}\right)}{b^5}+\frac{32\,a\,(A\,a-B\,b)\left(-4\,e^{x}\,a^3+3\,a^2\,b-3\,e^{x}\,a\,b^2+2\,b^3\right)}{b^5\,\sqrt{a^2+b^2}}\right)}{a\,\sqrt{a^2+b^2}}\right)}{a\,\sqrt{a^2+b^2}}+\frac{32\,B\,(A\,a-B\,b)\,(A\,b\,e^{x}-2\,B\,b+4\,B\,a\,e^{x})}{b^5}\right)(A\,a-B\,b)\,\sqrt{a^2+b^2}}{a^3+a\,b^2}+\frac{\ln\left(\frac{32\,B\,(A\,a-B\,b)\,(A\,b\,e^{x}-2\,B\,b+4\,B\,a\,e^{x})}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,e^{x}\,B^2\,a^3+2\,B^2\,a^2\,b-3\,e^{x}\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2+4\,A\,a^2\,e^{x}+A\,b^2\,e^{x}-2\,A\,a\,b-3\,B\,a\,b\,e^{x}\right)}{b^5}-\frac{32\,a\,(A\,a-B\,b)\left(-4\,e^{x}\,a^3+3\,a^2\,b-3\,e^{x}\,a\,b^2+2\,b^3\right)}{b^5\,\sqrt{a^2+b^2}}\right)}{a\,\sqrt{a^2+b^2}}\right)}{a\,\sqrt{a^2+b^2}}\right)(A\,a-B\,b)\,\sqrt{a^2+b^2}}{a^3+a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(exp(x) - 1))/a - (B*log(exp(x) + 1))/a - (log(((A*a - B*b)*((32*(2*B
^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp(x) -
2*A*B*a*b^2))/b^5 - ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) + A*b^2
*exp(x) - 2*A*a*b - 3*B*a*b*exp(x))))/b^5 + (32*a*(A*a - B*b)*(3*a^2*b + 2*b
```

```
^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^2 + b^
2)^(1/2))))/(a*(a^2 + b^2)^(1/2)) + (32*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b +
 4*B*a*exp(x)))/b^5)*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*b^2 + a^3) + (log((3
2*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b + 4*B*a*exp(x)))/b^5 - ((A*a - B*b)*((3
2*(2*B^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp
(x) - 2*A*B*a*b^2))/b^5 + ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) +
 A*b^2*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 - (32*a*(A*a - B*b)*(3*a^2*b
 + 2*b^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^
2 + b^2)^(1/2))))/(a*(a^2 + b^2)^(1/2)))*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*
b^2 + a^3)
```

3.253 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$

Optimal. Leaf size=81

$$\frac{Cx}{c}-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}\,e}+\frac{B\log(a+c\sinh(d+ex))}{ce}$$

```
[Out] C*x/c+B*ln(a+c*sinh(e*x+d))/c/e-2*(A*c-C*a)*arctanh((c-a*tanh(1/2*e*x+1/2*d
))/(a^2+c^2)^(1/2))/c/e/(a^2+c^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.226$, Rules used = {4461, 2814, 2739, 632, 210, 2747, 31}

$$-\frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*x)/c - (2*(A*c - a*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]]
)/(c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = B\int \frac{\cosh(d+ex)}{a+c\sinh(d+ex)}\,dx + \int \frac{A+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$$

$$= \frac{Cx}{c} - \frac{(i(iAc-iaC))\int \frac{1}{a+c\sinh(d+ex)}\,dx}{c} + \frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx, x\right.}{ce}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sinh(d+ex))}{ce} - \frac{(2i(Ac-aC))\text{Subst}\left(\int \frac{}{a-}\right.}{}$$

$$= \frac{Cx}{c} + \frac{B\log(a+c\sinh(d+ex))}{ce} + \frac{(4i(Ac-aC))\text{Subst}\left(\int \frac{}{-4}\right.}{}$$

$$= \frac{Cx}{c} - \frac{2(Ac-aC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}\,e} + \frac{B\log(a+c\sinh}{ce}$$

Mathematica [A]

time = 0.22, size = 85, normalized size = 1.05

$$\frac{C(d+ex)+\frac{2(Ac-aC)\text{ArcTan}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}+B\log(a+c\sinh(d+ex))}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 -
c^2]])/Sqrt[-a^2 - c^2] + B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Maple [A]
time = 3.59, size = 136, normalized size = 1.68

method	result
derivativedivides	$\frac{\frac{(-B+C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}+\frac{B\ln\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}}}{c}+\frac{(\cdot}{}}{e}$
default	$\frac{\frac{(-B+C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}+\frac{B\ln\left(a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}}}{c}+\frac{(\cdot}{}}{e}$
risch	$\frac{xB}{c}+\frac{Cx}{c}-\frac{2B\,a^2ce^2x}{a^2c^2e^2+c^4e^2}-\frac{2B\,c^3e^2x}{a^2c^2e^2+c^4e^2}-\frac{2B\,a^2cde}{a^2c^2e^2+c^4e^2}-\frac{2B\,c^3de}{a^2c^2e^2+c^4e^2}+\frac{\ln\left(e^{ex+d}+\frac{Aac-a^2C-\sqrt{A^2a^2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/e*((-B+C)/c*ln(tanh(1/2*e*x+1/2*d)+1)+2/c*(1/2*B*ln(a*tanh(1/2*e*x+1/2*d)
^2-2*c*tanh(1/2*e*x+1/2*d)-a)-(-A*c+C*a)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*t
anh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2)))+(-B-C)/c*ln(tanh(1/2*e*x+1/2*d)-1
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(78) = 156$.
time = 0.48, size = 178, normalized size = 2.20

$$\frac{Ae^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}}-\left(\frac{ae^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}\,c}-\frac{(xe+d)e^{(-1)}}{c}\right)C+\frac{Be^{(-1)}\log\left(c\sinh\left(xe+d\right)+a\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] A*e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(-x*e - d) - a + s
qrt(a^2 + c^2)))/sqrt(a^2 + c^2) - (a*e^(-1)*log((c*e^(-x*e - d) - a - sqrt
(a^2 + c^2))/(c*e^(-x*e - d) - a + sqrt(a^2 + c^2)))/(sqrt(a^2 + c^2)*c) -
(x*e + d)*e^(-1)/c)*C + B*e^(-1)*log(c*sinh(x*e + d) + a)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(78) = 156$.
time = 0.50, size = 361, normalized size = 4.46

$$-\frac{((B-C)a^2+(B-C)c^2)x\cosh(1)+((B-C)a^2+(B-C)c^2)x\sinh(1)+(Ca-Ac)\sqrt{a^2+c^2}\log\left(\frac{c^2\cosh(x\cosh(1)+x\sinh(1)+d)^2+c^2\sinh(x\cosh(1)+x\sinh(1)+d)^2+2ac\cosh(x\cosh(1)+x\sinh(1)+d)+2a^2+c^2+2(c^2\cosh(x\cosh(1)+x\sinh(1)+d)+ac)\sinh(x\cosh(1)+x\sinh(1)+d)-2\sqrt{a^2+c^2}(c\cosh(x\cosh(1)+x\sinh(1)+d)+c\sinh(x\cosh(1)+x\sinh(1)+d)+a)}{c\cosh(x\cosh(1)+x\sinh(1)+d)^2+c\sinh(x\cosh(1)+x\sinh(1)+d)^2+2a\cosh(x\cosh(1)+x\sinh(1)+d)+2(c\cosh(x\cosh(1)+x\sinh(1)+d)+a)\sinh(x\cosh(1)+x\sinh(1)+d)-c}\right)-(Ba^2+Bc^2)\log\left(\frac{2(c\sinh(x\cosh(1)+x\sinh(1)+d)+a)}{\cosh(x\cosh(1)+x\sinh(1)+d)-\sinh(x\cosh(1)+x\sinh(1)+d)}\right)}{(a^2c+c^3)\cosh(1)+(a^2c+c^3)\sinh(1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="f
ricas")
```

```
[Out] -((((B - C)*a^2 + (B - C)*c^2)*x*cosh(1) + ((B - C)*a^2 + (B - C)*c^2)*x*sin
h(1) + (C*a - A*c)*sqrt(a^2 + c^2)*log((c^2*cosh(x*cosh(1) + x*sinh(1) + d)
^2 + c^2*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*c*cosh(x*cosh(1) + x*sinh(
1) + d) + 2*a^2 + c^2 + 2*(c^2*cosh(x*cosh(1) + x*sinh(1) + d) + a*c)*sinh(
x*cosh(1) + x*sinh(1) + d) - 2*sqrt(a^2 + c^2)*(c*cosh(x*cosh(1) + x*sinh(1
) + d) + c*sinh(x*cosh(1) + x*sinh(1) + d) + a))/(c*cosh(x*cosh(1) + x*sinh
(1) + d)^2 + c*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*cosh(x*cosh(1) + x*s
inh(1) + d) + 2*(c*cosh(x*cosh(1) + x*sinh(1) + d) + a)*sinh(x*cosh(1) + x*
sinh(1) + d) - c)) - (B*a^2 + B*c^2)*log(2*(c*sinh(x*cosh(1) + x*sinh(1) +
d) + a)/(cosh(x*cosh(1) + x*sinh(1) + d) - sinh(x*cosh(1) + x*sinh(1) + d))
))/((a^2*c + c^3)*cosh(1) + (a^2*c + c^3)*sinh(1))
```

Sympy [C] Result contains complex when optimal does not.
time = 21.37, size = 1318, normalized size = 16.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/sinh(d), Eq(a, 0) & Eq(c, 0) &
 Eq(e, 0)), (2*I*A/(c*e*tanh(d/2 + e*x/2) - I*c*e) + B*e*x*tanh(d/2 + e*x/2
)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - I*B*e*x/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 - 2*B*log(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
- I*c*e) + 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 + 2*B*log(tanh(d/2 + e*x/2) - I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2)
- I*c*e) - 2*I*B*log(tanh(d/2 + e*x/2) - I)/(c*e*tanh(d/2 + e*x/2) - I*c*e)
 + C*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - I*C*e*x/(c*e*t
```

```
anh(d/2 + e*x/2) - I*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) - I*c*e), Eq(a, -I*c
)), (-2*I*A/(c*e*tanh(d/2 + e*x/2) + I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*
tanh(d/2 + e*x/2) + I*c*e) + I*B*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*B*
log(tanh(d/2 + e*x/2) + 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e
) - 2*I*B*log(tanh(d/2 + e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*B*
log(tanh(d/2 + e*x/2) + I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e
) + 2*I*B*log(tanh(d/2 + e*x/2) + I)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + C*e*
x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + I*C*e*x/(c*e*tanh(d/2
 + e*x/2) + I*c*e) - 2*C/(c*e*tanh(d/2 + e*x/2) + I*c*e), Eq(a, I*c)), ((A*
x + B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cosh(d)
+ C*sinh(d))/(a + c*sinh(d)), Eq(e, 0)), ((A*log(tanh(d/2 + e*x/2))/e + B*x
 - 2*B*log(tanh(d/2 + e*x/2) + 1)/e + B*log(tanh(d/2 + e*x/2))/e + C*x)/c,
Eq(a, 0)), (-A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2
+ c**2)/a)/(a**2*c*e + c**3*e) + A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2
) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B*a**2*e*x/(a**2*c*e +
 c**3*e) - 2*B*a**2*log(tanh(d/2 + e*x/2) + 1)/(a**2*c*e + c**3*e) + B*a**2
*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B
*a**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e
) + B*c**2*e*x/(a**2*c*e + c**3*e) - 2*B*c**2*log(tanh(d/2 + e*x/2) + 1)/(a
**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/
a)/(a**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c
**2)/a)/(a**2*c*e + c**3*e) + C*a**2*e*x/(a**2*c*e + c**3*e) + C*a*sqrt(a**
2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c*
*3*e) - C*a*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**
2)/a)/(a**2*c*e + c**3*e) + C*c**2*e*x/(a**2*c*e + c**3*e), True))
```

Giac [A]
time = 0.43, size = 127, normalized size = 1.57

$$-\frac{\frac{(ex+d)(B-C)}{c}-\frac{B\log\left(\left|ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right|\right)}{c}+\frac{(Ca-Ac)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\sqrt{a^2+c^2}\,c}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] -((e*x + d)*(B - C)/c - B*log(abs(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c))
/c + (C*a - A*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c
*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(sqrt(a^2 + c^2)*c))/e
```

Mupad [B]
time = 1.85, size = 656, normalized size = 8.10

$$\frac{C\,x}{c}-\frac{B\,x}{c}-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c+A\,e\,a^2\,c^2-C\,e\,a\,c^3+A\,e\,c^4}-\frac{a^2\,c^2\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c^4+A\,e\,a^2\,c^5-C\,e\,a\,c^6+A\,e\,c^7}+\frac{A\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}{c\,e\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}-\frac{C\,a\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}{c^2\,e\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}\right)\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}+\frac{B\,c^3\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}+4\,A^2\,c^3\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}+8\,A^2\,a\,c^2\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}+4\,C^2\,a^2\,c\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}-16\,A\,C\,a^2\,c\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}-8\,A\,C\,a\,c^2\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}\right)}{a^2\,c^2\,e^2+c^4\,e^2}+\frac{B\,a^2\,c\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}+4\,A^2\,c^3\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}+8\,A^2\,a\,c^2\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}+4\,C^2\,a^2\,c\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}-16\,A\,C\,a^2\,c\,{\mathrm{e}}^{e\,x}\,{\mathrm{e}}^{d}-8\,A\,C\,a\,c^2\,{\mathrm{e}}^{2\,d}\,{\mathrm{e}}^{2\,e\,x}\right)}{a^2\,c^2\,e^2+c^4\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x)),x)
```

```
[Out] (C*x)/c - (B*x)/c - (2*atan((a*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C
^2*a^2 - 2*A*C*a*c)^(1/2))/(A*c^4*e - C*a*c^3*e - C*a^3*c*e + A*a^2*c^2*e)
- (a^2*c^2*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C^2*a
^2 - 2*A*C*a*c)^(1/2))/(A*c^7*e - C*a*c^6*e + A*a^2*c^5*e - C*a^3*c^4*e) +
(A*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2))/(c*e*(A^2*c^2 + C^2*a^2
 - 2*A*C*a*c)^(1/2)) - (C*a*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)
)/(c^2*e*(A^2*c^2 + C^2*a^2 - 2*A*C*a*c)^(1/2)))*(A^2*c^2 + C^2*a^2 - 2*A*C
*a*c)^(1/2))/(- c^4*e^2 - a^2*c^2*e^2)^(1/2) + (B*c^3*e*log(8*A*C*a*c^2 - 4
*C^2*a^2*c - 4*A^2*c^3 + 8*C^2*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp
(2*e*x) + 8*A^2*a*c^2*exp(e*x)*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 1
6*A*C*a^2*c*exp(e*x)*exp(d) - 8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 +
a^2*c^2*e^2) + (B*a^2*c*e*log(8*A*C*a*c^2 - 4*C^2*a^2*c - 4*A^2*c^3 + 8*C^2
*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*c^2*exp(e*x)
*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 16*A*C*a^2*c*exp(e*x)*exp(d) -
8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 + a^2*c^2*e^2)
```

3.254 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$

Optimal. Leaf size=113

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{3/2}\,e}-\frac{B}{ce(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))}$$

```
[Out] -2*(A*a+C*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))/(a^2+c^2)^(
3/2)/e-B/c/e/(a+c*sinh(e*x+d))-(A*c-C*a)*cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(
e*x+d))
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$,
Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$-\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\,(a^2+c^2)^{3/2}}-\frac{(Ac-aC)\cosh(d+ex)}{e\,(a^2+c^2)\,(a+c\sinh(d+ex))}-\frac{B}{ce(a+c\sinh(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]])/((a^2 +
 c^2)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x]
)/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} - \frac{\int \frac{-aA-cC}{a+c\sinh(d+ex)}\,dx}{a^2+c^2} + \frac{B\mathrm{Su}}{}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} +$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} -$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} +$$

$$= -\frac{2(aA+cC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{3/2}\,e} - \frac{B}{ce(a+c\sinh(d+}$$

Mathematica [A]
time = 0.38, size = 113, normalized size = 1.00

$$\frac{\frac{2(aA+cC)\mathrm{ArcTan}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{B(a^2+c^2)+c(Ac-aC)\cosh(d+ex)}{c(a+c\sinh(d+ex))}}{(a^2+c^2)\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A + c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 - c^2]])/Sqrt[-a
^2 - c^2] - (B*(a^2 + c^2) + c*(A*c - a*C)*Cosh[d + e*x])/(c*(a + c*Sinh[d
+ e*x])))/((a^2 + c^2)*e)
```

Maple [A]
time = 6.00, size = 151, normalized size = 1.34

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(A\,c^2-B\,a^2-B\,c^2-Cac\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^2+c^2\right)}-\frac{Ac-Ca}{a^2+c^2}\right)}{a\left(\tanh^2\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+Cc)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}}{e}$

default	$\frac{-\frac{2\left(-\frac{\left(A\,c^{2}-B\,a^{2}-B\,c^{2}-Cac\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^{2}+c^{2}\right)}-\frac{Ac-Ca}{a^{2}+c^{2}}\right)}{a\left(\tanh^{2}\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+Cc)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^{2}+c^{2}}}\right)}{\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}}{e}$
risch	$\frac{2Aac\,\mathrm{e}^{ex+d}-2B\,a^{2}\mathrm{e}^{ex+d}-2B\,c^{2}\mathrm{e}^{ex+d}-2C\,a^{2}\mathrm{e}^{ex+d}-2A\,c^{2}+2Cac}{ce(a^{2}+c^{2})\left(c\,\mathrm{e}^{2ex+2d}+2a\,\mathrm{e}^{ex+d}-c\right)}+\frac{\ln\left(\mathrm{e}^{ex+d}+\frac{\left(a^{2}+c^{2}\right)^{\frac{3}{2}}a-a^{4}-2a^{2}c^{2}-c^{4}}{c\left(a^{2}+c^{2}\right)^{\frac{3}{2}}}\right)Aa}{(a^{2}+c^{2})^{\frac{3}{2}}e}+-$ ln

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-(A*c^2-B*a^2-B*c^2-C*a*c)/a/(a^2+c^2)*tanh(1/2*e*x+1/2*d)-(A*c-C*
a)/(a^2+c^2))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2*(A*a+C*
c)/(a^2+c^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2
)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(110) = 220$.
time = 0.51, size = 346, normalized size = 3.06

$$\left(\frac{ae^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}}-\frac{2\left(ae^{(-xe-d)}+c\right)e^{(-1)}}{a^2c+c^3+2\left(a^3+ac^2\right)e^{(-xe-d)}-\left(a^2c+c^3\right)e^{(-2xe-2d)}}\right)A+\left(\frac{ce^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}}+\frac{2\left(a^2e^{(-xe-d)}+ac\right)e^{(-1)}}{a^2c^2+c^4+2\left(a^3c+ac^3\right)e^{(-xe-d)}-\left(a^2c^2+c^4\right)e^{(-2xe-2d)}}\right)C-\frac{2\,Be^{(-xe-d-1)}}{2\,ace^{(-xe-d)}-c^2e^{(-2xe-2d)}+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] (a*e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(-x*e - d) - a +
sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) - 2*(a*e^(-x*e - d) + c)*e^(-1)/(a^2*c
+ c^3 + 2*(a^3 + a*c^2)*e^(-x*e - d) - (a^2*c + c^3)*e^(-2*x*e - 2*d)))*A +
 (c*e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(-x*e - d) - a +
 sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) + 2*(a^2*e^(-x*e - d) + a*c)*e^(-1)/(a
^2*c^2 + c^4 + 2*(a^3*c + a*c^3)*e^(-x*e - d) - (a^2*c^2 + c^4)*e^(-2*x*e -
 2*d)))*C - 2*B*e^(-x*e - d - 1)/(2*a*c*e^(-x*e - d) - c^2*e^(-2*x*e - 2*d)
 + c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(110) = 220$.
time = 0.43, size = 837, normalized size = 7.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] (2*C*a^3*c - 2*A*a^2*c^2 + 2*C*a*c^3 - 2*A*c^4 - (A*a*c^2 + C*c^3 - (A*a*c^
2 + C*c^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (A*a*c^2 + C*c^3)*sinh(x*cos
h(1) + x*sinh(1) + d)^2 - 2*(A*a^2*c + C*a*c^2)*cosh(x*cosh(1) + x*sinh(1)
+ d) - 2*(A*a^2*c + C*a*c^2 + (A*a*c^2 + C*c^3)*cosh(x*cosh(1) + x*sinh(1)
+ d))*sinh(x*cosh(1) + x*sinh(1) + d))*sqrt(a^2 + c^2)*log((c^2*cosh(x*cosh
(1) + x*sinh(1) + d)^2 + c^2*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*c*cosh
(x*cosh(1) + x*sinh(1) + d) + 2*a^2 + c^2 + 2*(c^2*cosh(x*cosh(1) + x*sinh(
1) + d) + a*c)*sinh(x*cosh(1) + x*sinh(1) + d) - 2*sqrt(a^2 + c^2)*(c*cosh(
x*cosh(1) + x*sinh(1) + d) + c*sinh(x*cosh(1) + x*sinh(1) + d) + a))/(c*cos
h(x*cosh(1) + x*sinh(1) + d)^2 + c*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*
cosh(x*cosh(1) + x*sinh(1) + d) + 2*(c*cosh(x*cosh(1) + x*sinh(1) + d) + a)
*sinh(x*cosh(1) + x*sinh(1) + d) - c)) - 2*((B + C)*a^4 - A*a^3*c + (2*B +
C)*a^2*c^2 - A*a*c^3 + B*c^4)*cosh(x*cosh(1) + x*sinh(1) + d) - 2*((B + C)*
a^4 - A*a^3*c + (2*B + C)*a^2*c^2 - A*a*c^3 + B*c^4)*sinh(x*cosh(1) + x*sin
h(1) + d))/(((a^4*c^2 + 2*a^2*c^4 + c^6)*cosh(1) + (a^4*c^2 + 2*a^2*c^4 + c
^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 + ((a^4*c^2 + 2*a^2*c^4 + c^
6)*cosh(1) + (a^4*c^2 + 2*a^2*c^4 + c^6)*sinh(1))*sinh(x*cosh(1) + x*sinh(1
) + d)^2 - (a^4*c^2 + 2*a^2*c^4 + c^6)*cosh(1) + 2*((a^5*c + 2*a^3*c^3 + a*
c^5)*cosh(1) + (a^5*c + 2*a^3*c^3 + a*c^5)*sinh(1))*cosh(x*cosh(1) + x*sinh
(1) + d) - (a^4*c^2 + 2*a^2*c^4 + c^6)*sinh(1) + 2*((a^5*c + 2*a^3*c^3 + a*
c^5)*cosh(1) + ((a^4*c^2 + 2*a^2*c^4 + c^6)*cosh(1) + (a^4*c^2 + 2*a^2*c^4
+ c^6)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^5*c + 2*a^3*c^3 + a*c^
5)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 170, normalized size = 1.50

$$\frac{\frac{(Aa+Cc)\log\left(\left|\frac{2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}}{2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}}\right|\right)}{(a^2+c^2)^{\frac{3}{2}}}-\frac{2\,\left(Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aace^{(ex+d)}+Bc^2e^{(ex+d)}-Cac+Ac^2\right)}{(a^2c+c^3)\left(ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] ((A*a + C*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(
e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) - 2*(B*a^2*e^(e*x +
d) + C*a^2*e^(e*x + d) - A*a*c*e^(e*x + d) + B*c^2*e^(e*x + d) - C*a*c + A*
c^2)/((a^2*c + c^3)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)))/e
```

Mupad [B]
time = 1.20, size = 279, normalized size = 2.47

$$\frac{\ln\left(\frac{2\,(A\,a+C\,c)\,(c-a\,e^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}-\frac{2\,e^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}\right)\,(A\,a+C\,c)}{e\,{(a^2+c^2)}^{3/2}}-\frac{\ln\left(-\frac{2\,e^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}-\frac{2\,(A\,a+C\,c)\,(c-a\,e^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}\right)\,(A\,a+C\,c)}{e\,{(a^2+c^2)}^{3/2}}-\frac{\frac{2\,(A\,c^3-C\,a\,c^2)}{c\,e\,(a^2\,c+c^3)}+\frac{2\,e^{d+e\,x}\,(B\,c^4+B\,a^2\,c^2+C\,a^2\,c^2-A\,a\,c^3)}{c^2\,e\,(a^2\,c+c^3)}}{2\,a\,e^{d+e\,x}-c+c\,e^{2\,d+2\,e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^2,x)
```

```
[Out] (log((2*(A*a + C*c)*(c - a*exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)) - (2*exp(d
+ e*x)*(A*a + C*c))/(c*(a^2 + c^2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) - (
log(- (2*exp(d + e*x)*(A*a + C*c))/(c*(a^2 + c^2)) - (2*(A*a + C*c)*(c - a*
exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) -
((2*(A*c^3 - C*a*c^2))/(c*e*(a^2*c + c^3)) + (2*exp(d + e*x)*(B*c^4 + B*a^2
*c^2 + C*a^2*c^2 - A*a*c^3))/(c^2*e*(a^2*c + c^3)))/(2*a*exp(d + e*x) - c +
 c*exp(2*d + 2*e*x))
```

3.255 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{(2a^2A-Ac^2+3acC)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{5/2}e}-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex}$$

```
[Out] -(2*A*a^2-A*c^2+3*C*a*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))
/(a^2+c^2)^(5/2)/e-1/2*B/c/e/(a+c*sinh(e*x+d))^2-1/2*(A*c-C*a)*cosh(e*x+d)/
(a^2+c^2)/e/(a+c*sinh(e*x+d))^2-1/2*(3*A*a*c-C*a^2+2*C*c^2)*cosh(e*x+d)/(a^
2+c^2)^2/e/(a+c*sinh(e*x+d))
```

Rubi [A]
time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$-\frac{(2a^2A+3acC-Ac^2)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{5/2}}-\frac{\left(a^2(-C)+3aAc+2c^2C\right)\cosh(d+ex)}{2e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))}-\frac{(Ac-aC)\cosh(d+ex)}{2e\left(a^2+c^2\right)(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x]
```

```
[Out] -(((2*a^2*A - A*c^2 + 3*a*c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2]])/Sqrt[a^2 +
 c^2]])/((a^2 + c^2)^(5/2)*e)) - B/(2*c*e*(a + c*Sinh[d + e*x])^2) - ((A*c
- a*C)*Cosh[d + e*x])/(2*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) - ((3*a*A*c
 - a^2*C + 2*c^2*C)*Cosh[d + e*x])/(2*(a^2 + c^2)^2*e*(a + c*Sinh[d + e*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} - \frac{\int \frac{-2(aA+cC)+(Ac-aC)\sin\cdots}{(a+c\sinh(d+ex))^2}\cdots}{2\left(a^2+c^2\right)}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex\cdots}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex\cdots}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex\cdots}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex\cdots}$$

$$= -\frac{\left(2a^2A-Ac^2+3acC\right)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{5/2}e} - \frac{\cdots}{2ce(a\cdots}$$

Mathematica [**A**]
time = 0.48, size = 170, normalized size = 0.94

$$\frac{\frac{2\left(2a^2A-Ac^2+3acC\right)\text{ArcTan}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{\left(a^2+c^2\right)\left(B\left(a^2+c^2\right)+c(Ac-aC)\cosh(d+ex)\right)}{c(a+c\sinh(d+ex))^2} + \frac{\left(-3aAc+a^2C-2c^2C\right)\cosh(d+ex)}{a+c\sinh(d+ex)}}{2\left(a^2+c^2\right)^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x
]
```

```
[Out] ((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2
- c^2]])/Sqrt[-a^2 - c^2] - ((a^2 + c^2)*(B*(a^2 + c^2) + c*(A*c - a*C)*Cos
h[d + e*x]))/(c*(a + c*Sinh[d + e*x])^2) + ((-3*a*A*c + a^2*C - 2*c^2*C)*Co
sh[d + e*x])/(a + c*Sinh[d + e*x]))/(2*(a^2 + c^2)^2*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(169) = 338$.
time = 6.01, size = 416, normalized size = 2.31

method	result

derivativedivides	$-\frac{2\left(-\frac{\left(5A\,a^{2}c^{2}+2A\,c^{4}-2B\,a^{4}-4B\,a^{2}c^{2}-2B\,c^{4}-3C\,a^{3}c\right)\left(\tanh^{3}\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2a\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}-\frac{\left(4A\,a^{4}c-7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c+4B\,a^{2}c^{3}+2B\,c^{5}-2C}{2\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}\right.}{\left(a\left(\tanh^{2}\left(\frac{e}{}\right.\right.\right.}$
default	$-\frac{2\left(-\frac{\left(5A\,a^{2}c^{2}+2A\,c^{4}-2B\,a^{4}-4B\,a^{2}c^{2}-2B\,c^{4}-3C\,a^{3}c\right)\left(\tanh^{3}\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2a\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}-\frac{\left(4A\,a^{4}c-7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c+4B\,a^{2}c^{3}+2B\,c^{5}-2C}{2\left(a^{4}+2a^{2}c^{2}+c^{4}\right)}\right.}{\left(a\left(\tanh^{2}\left(\frac{e}{}\right.\right.\right.}$
risch	$\frac{2A\,a^{2}c^{2}\mathrm{e}^{3ex+3d}-A\,c^{4}\mathrm{e}^{3ex+3d}+3Ca\,c^{3}\mathrm{e}^{3ex+3d}+6A\,a^{3}c\,\mathrm{e}^{2ex+2d}-3Aa\,c^{3}\mathrm{e}^{2ex+2d}-2B\,a^{4}\mathrm{e}^{2ex+2d}-4B\,a^{2}c^{2}\mathrm{e}^{2ex+2d}-2B\,c^{4}}{ce(a^{2}+c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*(5*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^2*c^2-2*B*c^4-3*C*a^3*c)/a
/(a^4+2*a^2*c^2+c^4)*tanh(1/2*e*x+1/2*d)^3-1/2*(4*A*a^4*c-7*A*a^2*c^3-2*A*c
^5+2*B*a^4*c+4*B*a^2*c^3+2*B*c^5-2*C*a^5+5*C*a^3*c^2-2*C*a*c^4)/(a^4+2*a^2*
c^2+c^4)/a^2*tanh(1/2*e*x+1/2*d)^2+1/2*(11*A*a^2*c^2+2*A*c^4-2*B*a^4-4*B*a^
2*c^2-2*B*c^4-5*C*a^3*c+4*C*a*c^3)/(a^4+2*a^2*c^2+c^4)/a*tanh(1/2*e*x+1/2*d
)+1/2*(4*A*a^2*c+A*c^3-2*C*a^3+C*a*c^2)/(a^4+2*a^2*c^2+c^4))/(a*tanh(1/2*e*
x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)^2+(2*A*a^2-A*c^2+3*C*a*c)/(a^4+2*a^2*
c^2+c^4)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2
)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(172) = 344$.
time = 0.52, size = 743, normalized size = 4.13

$$\frac{1}{2}\left(\frac{(2a^2-c^2)e^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^4+2a^2c^2+c^4)\sqrt{a^2+c^2}}-\frac{2\left(3ac^2+(10a^2c+c^3)e^{(-xe-d)}+3(2a^3-ac^2)e^{(-2xe-2d)}-(2a^2c-c^3)e^{(-3xe-3d)}\right)e^{(-1)}}{a^4c^2+2a^2c^4+c^6+4(a^5c+2a^3c^3+ac^5)e^{(-xe-d)}+2(2a^6+3a^4c^2-c^6)e^{(-2xe-2d)}-4(a^5c+2a^3c^3+ac^5)e^{(-3xe-3d)}+(a^4c^2+2a^2c^4+c^6)e^{(-4xe-4d)}}\right)A+\frac{1}{2}\left(\frac{3ace^{(-1)}\log\left(\frac{ce^{(-xe-d)}-a-\sqrt{a^2+c^2}}{ce^{(-xe-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^4+2a^2c^2+c^4)\sqrt{a^2+c^2}}+\frac{2\left(3ac^3e^{(-3xe-3d)}+a^2c^2-2c^4+(4a^3c-5ac^3)e^{(-xe-d)}+(2a^4-5a^2c^2+2c^4)e^{(-2xe-2d)}\right)e^{(-1)}}{a^4c^3+2a^2c^5+c^7+4(a^5c^2+2a^3c^4+ac^6)e^{(-xe-d)}+2(2a^6c+3a^4c^3-c^7)e^{(-2xe-2d)}-4(a^5c^2+2a^3c^4+ac^6)e^{(-3xe-3d)}+(a^4c^3+2a^2c^5+c^7)e^{(-4xe-4d)}}\right)C-\frac{2Be^{(-2xe-2d-1)}}{4ac^2e^{(-xe-d)}-4ac^2e^{(-3xe-3d)}+c^3e^{(-4xe-4d)}+c^3+2(2a^2c-c^3)e^{(-2xe-2d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"maxima")
```

```
[Out] 1/2*((2*a^2 - c^2)*e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(
-x*e - d) - a + sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2))
 - 2*(3*a*c^2 + (10*a^2*c + c^3)*e^(-x*e - d) + 3*(2*a^3 - a*c^2)*e^(-2*x*e
 - 2*d) - (2*a^2*c - c^3)*e^(-3*x*e - 3*d))*e^(-1)/(a^4*c^2 + 2*a^2*c^4 + c
^6 + 4*(a^5*c + 2*a^3*c^3 + a*c^5)*e^(-x*e - d) + 2*(2*a^6 + 3*a^4*c^2 - c^
6)*e^(-2*x*e - 2*d) - 4*(a^5*c + 2*a^3*c^3 + a*c^5)*e^(-3*x*e - 3*d) + (a^4
*c^2 + 2*a^2*c^4 + c^6)*e^(-4*x*e - 4*d)))*A + 1/2*(3*a*c*e^(-1)*log((c*e^(
-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(-x*e - d) - a + sqrt(a^2 + c^2)))/((
```

```
a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)) + 2*(3*a*c^3*e^(-3*x*e - 3*d) + a^2
*c^2 - 2*c^4 + (4*a^3*c - 5*a*c^3)*e^(-x*e - d) + (2*a^4 - 5*a^2*c^2 + 2*c^
4)*e^(-2*x*e - 2*d))*e^(-1)/(a^4*c^3 + 2*a^2*c^5 + c^7 + 4*(a^5*c^2 + 2*a^3
*c^4 + a*c^6)*e^(-x*e - d) + 2*(2*a^6*c + 3*a^4*c^3 - c^7)*e^(-2*x*e - 2*d)
 - 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-3*x*e - 3*d) + (a^4*c^3 + 2*a^2*c^5
+ c^7)*e^(-4*x*e - 4*d)))*C - 2*B*e^(-2*x*e - 2*d - 1)/(4*a*c^2*e^(-x*e - d
) - 4*a*c^2*e^(-3*x*e - 3*d) + c^3*e^(-4*x*e - 4*d) + c^3 + 2*(2*a^2*c - c^
3)*e^(-2*x*e - 2*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2652 vs. $2(172) = 344$.
time = 0.42, size = 2652, normalized size = 14.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*C*a^4*c^2 - 6*A*a^3*c^3 - 2*C*a^2*c^4 - 6*A*a*c^5 - 4*C*c^6 - 2*(2*
A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(x*cosh(1) + x
*sinh(1) + d)^3 - 2*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*
c^6)*sinh(x*cosh(1) + x*sinh(1) + d)^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3*(
2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B + C
)*c^6)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3
*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B +
 C)*c^6 - 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cos
h(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 + (2*A*a^2*
c^3 + 3*C*a*c^4 - A*c^5 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(x*cosh(1)
+ x*sinh(1) + d)^4 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*sinh(x*cosh(1) + x*s
inh(1) + d)^4 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(x*cosh(1) + x*
sinh(1) + d)^3 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 + (2*A*a^2*c^3 + 3*
C*a*c^4 - A*c^5)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1
) + d)^3 + 2*(4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*co
sh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3
- 3*C*a*c^4 + A*c^5 + 3*(2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(x*cosh(1) +
x*sinh(1) + d)^2 + 6*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(x*cosh(1) +
 x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - 4*(2*A*a^3*c^2 + 3*C*a
^2*c^3 - A*a*c^4)*cosh(x*cosh(1) + x*sinh(1) + d) - 4*(2*A*a^3*c^2 + 3*C*a^
2*c^3 - A*a*c^4 - (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(x*cosh(1) + x*sinh
(1) + d)^3 - 3*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(x*cosh(1) + x*sin
h(1) + d)^2 - (4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*c
osh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d))*sqrt(a^2 +
 c^2)*log((c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + c^2*sinh(x*cosh(1) + x*s
inh(1) + d)^2 + 2*a*c*cosh(x*cosh(1) + x*sinh(1) + d) + 2*a^2 + c^2 + 2*(c^
```

```
2*cosh(x*cosh(1) + x*sinh(1) + d) + a*c)*sinh(x*cosh(1) + x*sinh(1) + d) +
2*sqrt(a^2 + c^2)*(c*cosh(x*cosh(1) + x*sinh(1) + d) + c*sinh(x*cosh(1) + x
*sinh(1) + d) + a))/(c*cosh(x*cosh(1) + x*sinh(1) + d)^2 + c*sinh(x*cosh(1)
 + x*sinh(1) + d)^2 + 2*a*cosh(x*cosh(1) + x*sinh(1) + d) + 2*(c*cosh(x*cos
h(1) + x*sinh(1) + d) + a)*sinh(x*cosh(1) + x*sinh(1) + d) - c)) - 2*(4*C*a
^5*c - 10*A*a^4*c^2 - C*a^3*c^3 - 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6)*cosh(x*
cosh(1) + x*sinh(1) + d) - 2*(4*C*a^5*c - 10*A*a^4*c^2 - C*a^3*c^3 - 11*A*a
^2*c^4 - 5*C*a*c^5 - A*c^6 + 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C
*a*c^5 - A*c^6)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 2*(2*(B + C)*a^6 - 6*A*
a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A*a*c^5
 + 2*(B + C)*c^6)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(
1) + d))/(((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*cosh(1) + (a^6*c^3 + 3*a
^4*c^5 + 3*a^2*c^7 + c^9)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 + ((a^
6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*cosh(1) + (a^6*c^3 + 3*a^4*c^5 + 3*a^2
*c^7 + c^9)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + 4*((a^7*c^2 + 3*a^
5*c^4 + 3*a^3*c^6 + a*c^8)*cosh(1) + (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c
^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 4*((a^7*c^2 + 3*a^5*c^4 +
3*a^3*c^6 + a*c^8)*cosh(1) + ((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*cosh(
1) + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*sinh(1))*cosh(x*cosh(1) + x*si
nh(1) + d) + (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*sinh(1))*sinh(x*cosh
(1) + x*sinh(1) + d)^3 + 2*((2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^
9)*cosh(1) + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*sinh(1))*cos
h(x*cosh(1) + x*sinh(1) + d)^2 + 2*(3*((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c
^9)*cosh(1) + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*sinh(1))*cosh(x*cosh(
1) + x*sinh(1) + d)^2 + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*c
osh(1) + 6*((a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*cosh(1) + (a^7*c^2 +
3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (
2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*sinh(1))*sinh(x*cosh(1) +
x*sinh(1) + d)^2 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*cosh(1) - 4*((a^
7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*cosh(1) + (a^7*c^2 + 3*a^5*c^4 + 3*a
^3*c^6 + a*c^8)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) + (a^6*c^3 + 3*a^4
*c^5 + 3*a^2*c^7 + c^9)*sinh(1) + 4*(((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^
9)*cosh(1) + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*sinh(1))*cosh(x*cosh(1
) + x*sinh(1) + d)^3 + 3*((a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*cosh(1)
 + (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*sinh(1))*cosh(x*cosh(1) + x*si
nh(1) + d)^2 - (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*cosh(1) + ((2*a^8*
c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*cosh(1) + (2*a^8*c + 5*a^6*c^3 +
 3*a^4*c^5 - a^2*c^7 - c^9)*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d) - (a^7
*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*sinh(1))*sinh(x*cosh(1) + x*sinh(1) +
 d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(170) = 340.
time = 0.45, size = 405, normalized size = 2.25

$$-\frac{\frac{\left(2\,Aa^2+3\,Cac-Ac^2\right)\log\left(\frac{\left|-2\,ce^{(ex+d)}-2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|-2\,ce^{(ex+d)}-2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^4+2\,a^2c^2+c^4\right)\sqrt{a^2+c^2}}-\frac{2\,\left(2\,Aa^2c^2e^{(3\,ex+3\,d)}+3\,Cac^3e^{(3\,ex+3\,d)}-Ac^4e^{(3\,ex+3\,d)}-2\,Ba^4e^{(2\,ex+2\,d)}-2\,Ca^4e^{(2\,ex+2\,d)}+6\,Aa^3ce^{(2\,ex+2\,d)}-4\,Ba^2c^2e^{(2\,ex+2\,d)}+5\,Ca^2c^2e^{(2\,ex+2\,d)}-3\,Aac^3e^{(2\,ex+2\,d)}-2\,Bc^4e^{(2\,ex+2\,d)}-2\,Cc^4e^{(2\,ex+2\,d)}+4\,Ca^3ce^{(ex+d)}-10\,Aa^2c^2e^{(ex+d)}-5\,Cac^3e^{(ex+d)}-Ac^4e^{(ex+d)}-Ca^2c^2+3\,Aac^3+2\,Cc^4\right)}{\left(a^4c+2\,a^2c^3+c^5\right)\left(ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right)^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] -1/2*((2*A*a^2 + 3*C*a*c - A*c^2)*log(abs(-2*c*e^(e*x + d) - 2*a - 2*sqrt(a
^2 + c^2))/abs(-2*c*e^(e*x + d) - 2*a + 2*sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c
^2 + c^4)*sqrt(a^2 + c^2)) - 2*(2*A*a^2*c^2*e^(3*e*x + 3*d) + 3*C*a*c^3*e^(
3*e*x + 3*d) - A*c^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2*d) - 2*C*a^4*e^
(2*e*x + 2*d) + 6*A*a^3*c*e^(2*e*x + 2*d) - 4*B*a^2*c^2*e^(2*e*x + 2*d) + 5
*C*a^2*c^2*e^(2*e*x + 2*d) - 3*A*a*c^3*e^(2*e*x + 2*d) - 2*B*c^4*e^(2*e*x +
 2*d) - 2*C*c^4*e^(2*e*x + 2*d) + 4*C*a^3*c*e^(e*x + d) - 10*A*a^2*c^2*e^(e
*x + d) - 5*C*a*c^3*e^(e*x + d) - A*c^4*e^(e*x + d) - C*a^2*c^2 + 3*A*a*c^3
 + 2*C*c^4)/((a^4*c + 2*a^2*c^3 + c^5)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d)
 - c)^2))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+c\sinh\left(d+e\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3,x)
```

```
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3, x)
```

3.256 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$

Optimal. Leaf size=250

$$-\frac{(2a^3A-3aAc^2+4a^2cC-c^3C)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{7/2}e}-\frac{B}{3ce(a+c\sinh(d+ex))^3}-\frac{(Ac-aC)\cosh}{3(a^2+c^2)e(a+c\sinh}$$

```
[Out] -(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2
+c^2)^(1/2))/(a^2+c^2)^(7/2)/e-1/3*B/c/e/(a+c*sinh(e*x+d))^3-1/3*(A*c-C*a)*
cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(e*x+d))^3-1/6*(5*A*a*c-2*C*a^2+3*C*c^2)*c
osh(e*x+d)/(a^2+c^2)^2/e/(a+c*sinh(e*x+d))^2-1/6*(11*A*a^2*c-4*A*c^3-2*C*a^
3+13*C*a*c^2)*cosh(e*x+d)/(a^2+c^2)^3/e/(a+c*sinh(e*x+d))
```

Rubi [A]
time = 0.33, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$-\frac{(-2a^2C+5aAc+3c^2C)\cosh(d+ex)}{6e(a^2+c^2)^2(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{3e(a^2+c^2)(a+c\sinh(d+ex))^3}-\frac{(2a^3A+4a^2cC-3aAc^2-c^3C)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e(a^2+c^2)^{7/2}}-\frac{(-2a^3C+11a^2Ac+13ac^2C-4Ac^3)\cosh(d+ex)}{6e(a^2+c^2)^3(a+c\sinh(d+ex))}-\frac{B}{3ce(a+c\sinh(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[(c - a*Tanh[(d + e*x)/
2])/Sqrt[a^2 + c^2]])/((a^2 + c^2)^(7/2)*e)) - B/(3*c*e*(a + c*Sinh[d + e*x
])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^
3) - ((5*a*A*c - 2*a^2*C + 3*c^2*C)*Cosh[d + e*x])/(6*(a^2 + c^2)^2*e*(a +
c*Sinh[d + e*x])^2) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*Cosh[d
 + e*x])/(6*(a^2 + c^2)^3*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{\int \frac{-3(aA+cC)+2(Ac-aC)\sinh}{(a+c\sinh(d+ex))^3}}{3(a^2+c^2)}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))}$$

$$= -\frac{(2a^3A-3aAc^2+4a^2cC-c^3C)\tanh^{-1}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{7/2}e}$$

Mathematica [**A**]
time = 1.13, size = 235, normalized size = 0.94

$$\frac{\frac{6(2a^3A-3aAc^2+4a^2cC-c^3C)\text{ArcTan}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{2(a^2+c^2)^2(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex))}{c(a+c\sinh(d+ex))^3} + \frac{(a^2+c^2)(-5aAc+2a^2C-3c^2C)\cosh(d+ex)}{(a+c\sinh(d+ex))^2} + \frac{(-11a^2Ac+4Ac^3+2a^3C-13ac^2C)\cosh(d+ex)}{a+c\sinh(d+ex)}}{6(a^2+c^2)^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTan[(c - a*Tanh[(d + e*x)/
2])/Sqrt[-a^2 - c^2]])/Sqrt[-a^2 - c^2] - (2*(a^2 + c^2)^2*(B*(a^2 + c^2) +
 c*(A*c - a*C)*Cosh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^3) + ((a^2 + c^2)*(
-5*a*A*c + 2*a^2*C - 3*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d + e*x])^2 + ((-1
1*a^2*A*c + 4*A*c^3 + 2*a^3*C - 13*a*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d +
e*x]))/(6*(a^2 + c^2)^3*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. 2(237) = 474.
time = 6.22, size = 844, normalized size = 3.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*(9*A*a^4*c^2+6*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*
c^4-2*B*c^6-4*C*a^5*c+C*a^3*c^3)/a/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e
*x+1/2*d)^5-1/2*(6*A*a^6*c-27*A*a^4*c^3-12*A*a^2*c^5-4*A*c^7+4*B*a^6*c+12*B
*a^4*c^3+12*B*a^2*c^5+4*B*c^7-2*C*a^7+14*C*a^5*c^2-11*C*a^3*c^4-2*C*a*c^6)/
(a^6+3*a^4*c^2+3*a^2*c^4+c^6)/a^2*tanh(1/2*e*x+1/2*d)^4+1/3/a^3*(54*A*a^6*c
^2-21*A*a^4*c^4-4*A*a^2*c^6-4*A*c^8-6*B*a^8-14*B*a^6*c^2-6*B*a^4*c^4+6*B*a^
2*c^6+4*B*c^8-18*C*a^7*c+42*C*a^5*c^3-17*C*a^3*c^5-2*C*a*c^7)/(a^6+3*a^4*c^
2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c-20*A*a^4*c^3-3*A*a^
2*c^5-2*A*c^7+2*B*a^6*c+6*B*a^4*c^3+6*B*a^2*c^5+2*B*c^7-2*C*a^7+10*C*a^5*c^
2-14*C*a^3*c^4-C*a*c^6)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^2
-1/2/a*(27*A*a^4*c^2+4*A*a^2*c^4+2*A*c^6-2*B*a^6-6*B*a^4*c^2-6*B*a^2*c^4-2*
B*c^6-8*C*a^5*c+19*C*a^3*c^3+2*C*a*c^5)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(
1/2*e*x+1/2*d)-1/6*(18*A*a^4*c+5*A*a^2*c^3+2*A*c^5-6*C*a^5+10*C*a^3*c^2+C*a
*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*
e*x+1/2*d)-a)^3+(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)/(a^6+3*a^4*c^2+3*a^2*c^
4+c^6)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^
(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(242) = 484$.
time = 0.53, size = 1290, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] 1/6*(3*(2*a^2 - 3*c^2)*a*e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/
(c*e^(-x*e - d) - a + sqrt(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6
)*sqrt(a^2 + c^2)) - 2*(11*a^2*c^3 - 4*c^5 + 15*(4*a^3*c^2 - a*c^4)*e^(-x*e
 - d) + 6*(17*a^4*c - 6*a^2*c^3 + 2*c^5)*e^(-2*x*e - 2*d) + 2*(22*a^5 - 41*
a^3*c^2 + 12*a*c^4)*e^(-3*x*e - 3*d) - 15*(2*a^4*c - 3*a^2*c^3)*e^(-4*x*e -
 4*d) + 3*(2*a^3*c^2 - 3*a*c^4)*e^(-5*x*e - 5*d))*e^(-1)/(a^6*c^3 + 3*a^4*c
^5 + 3*a^2*c^7 + c^9 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-x*e
- d) + 3*(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-2*x*e - 2*d
) + 4*(2*a^9 + 3*a^7*c^2 - 3*a^5*c^4 - 7*a^3*c^6 - 3*a*c^8)*e^(-3*x*e - 3*d
) - 3*(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-4*x*e - 4*d) +
 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-5*x*e - 5*d) - (a^6*c^3 +
3*a^4*c^5 + 3*a^2*c^7 + c^9)*e^(-6*x*e - 6*d)))*A + 1/6*(3*(4*a^2*c - c^3)*
e^(-1)*log((c*e^(-x*e - d) - a - sqrt(a^2 + c^2))/(c*e^(-x*e - d) - a + sqr
```

```
t(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)) + 2*(2
*a^3*c^3 - 13*a*c^5 + 3*(4*a^4*c^2 - 22*a^2*c^4 - c^6)*e^(-x*e - d) + 6*(4*
a^5*c - 17*a^3*c^3 + 4*a*c^5)*e^(-2*x*e - 2*d) + 2*(4*a^6 - 32*a^4*c^2 + 39
*a^2*c^4)*e^(-3*x*e - 3*d) + 15*(4*a^3*c^3 - a*c^5)*e^(-4*x*e - 4*d) - 3*(4
*a^2*c^4 - c^6)*e^(-5*x*e - 5*d))*e^(-1)/(a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8 +
 c^10 + 6*(a^7*c^3 + 3*a^5*c^5 + 3*a^3*c^7 + a*c^9)*e^(-x*e - d) + 3*(4*a^8
*c^2 + 11*a^6*c^4 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-2*x*e - 2*d) + 4*(2*a^9
*c + 3*a^7*c^3 - 3*a^5*c^5 - 7*a^3*c^7 - 3*a*c^9)*e^(-3*x*e - 3*d) - 3*(4*a
^8*c^2 + 11*a^6*c^4 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-4*x*e - 4*d) + 6*(a^7
*c^3 + 3*a^5*c^5 + 3*a^3*c^7 + a*c^9)*e^(-5*x*e - 5*d) - (a^6*c^4 + 3*a^4*c
^6 + 3*a^2*c^8 + c^10)*e^(-6*x*e - 6*d)))*C - 8/3*B*e^(-3*x*e - 3*d - 1)/(6
*a*c^3*e^(-x*e - d) + 6*a*c^3*e^(-5*x*e - 5*d) - c^4*e^(-6*x*e - 6*d) + c^4
 + 3*(4*a^2*c^2 - c^4)*e^(-2*x*e - 2*d) + 4*(2*a^3*c - 3*a*c^3)*e^(-3*x*e -
 3*d) - 3*(4*a^2*c^2 - c^4)*e^(-4*x*e - 4*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5966 vs. $2(242) = 484$.
time = 0.51, size = 5966, normalized size = 23.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] 1/6*(4*C*a^5*c^3 - 22*A*a^4*c^4 - 22*C*a^3*c^5 - 14*A*a^2*c^6 - 26*C*a*c^7
+ 8*A*c^8 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*
c^7 - C*c^8)*cosh(x*cosh(1) + x*sinh(1) + d)^5 + 6*(2*A*a^5*c^3 + 4*C*a^4*c
^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*sinh(x*cosh(1) + x*sinh(1
) + d)^5 + 30*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^
2*c^6 - C*a*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^4 + 30*(2*A*a^6*c^2 + 4*C*
a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7 + (2*A*a^5*c^3 +
4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(x*cosh(1) +
 x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^4 - 4*(4*(B + C)*a^8 - 22*
A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*
A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(x*cosh(1) +
x*sinh(1) + d)^3 - 4*(4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 +
19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6
- 12*A*a*c^7 + 4*B*c^8 - 15*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^
2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 30*(2*A*a^6*
c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(x
*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^3 + 12*(4*C*a^7*
c - 17*A*a^6*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6
 + 4*C*a*c^7 - 2*A*c^8)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 12*(4*C*a^7*c -
 17*A*a^6*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 +
```

4*C*a*c^7 - 2*A*c^8 + 5*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^
6 - 3*A*a*c^7 - C*c^8)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 15*(2*A*a^6*c^2
+ 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(x*cos
h(1) + x*sinh(1) + d)^2 - (4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c
^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16*B + 39*C)*a^2
*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1
) + x*sinh(1) + d)^2 + 3*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7 - (
2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(x*cosh(1) + x*sinh(1) +
d)^6 - (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*sinh(x*cosh(1) + x*
sinh(1) + d)^6 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh
(x*cosh(1) + x*sinh(1) + d)^5 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5
- C*a*c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(x*cosh(1)
+ x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^5 - 3*(8*A*a^5*c^2 + 16*C
*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(x*cosh(1) +
x*sinh(1) + d)^4 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*
c^5 + 3*A*a*c^6 + C*c^7 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)
*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 10*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^
2*c^5 - C*a*c^6)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1
) + d)^4 - 4*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a
^2*c^5 + 3*C*a*c^6)*cosh(x*cosh(1) + x*sinh(1) + d)^3 - 4*(4*A*a^6*c + 8*C*
a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6 + 5*(2*A*a^
3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^3
+ 15*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(x*cosh(1) + x
sinh(1) + d)^2 + 3(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^
5 + 3*A*a*c^6 + C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*
sinh(1) + d)^3 + 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5
+ 3*A*a*c^6 + C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + 3*(8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 - 5*(2*A*a^3*
c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^4 -
20*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(x*cosh(1) + x*s
inh(1) + d)^3 - 6*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5
+ 3*A*a*c^6 + C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 4*(4*A*a^6*c + 8*C
*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(x*co
sh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - 6*(2*A*a^4*c^3
+ 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(x*cosh(1) + x*sinh(1) + d) - 6*
(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6 + (2*A*a^3*c^4 + 4*C*a^2
*c^5 - 3*A*a*c^6 - C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^5 + 5*(2*A*a^4*c^
3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(x*cosh(1) + x*sinh(1) + d)^4
+ 2*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 +
C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + 2*(4*A*a^6*c + 8*C*a^5*c^2 - 12*
A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(x*cosh(1) + x*sinh
(1) + d)^2 - (8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A
*a*c^6 + C*c^7)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1)
+ d))*sqrt(a^2 + c^2)*log((c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + c^2*sin
h(x*cosh(1) + x*sinh(1) + d)^2 + 2*a*c*cosh(x*cosh(1) + x*sinh(1) + d) + 2*

```
a^2 + c^2 + 2*(c^2*cosh(x*cosh(1) + x*sinh(1) + d) + a*c)*sinh(x*cosh(1) +
x*sinh(1) + d) + 2*sqrt(a^2 + c^2)*(c*cosh(x*co...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**4,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(239) = 478.
time = 0.49, size = 685, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"giac")
[Out] 1/6*(3*(2*A*a^3 + 4*C*a^2*c - 3*A*a*c^2 - C*c^3)*log(abs(2*c*e^(e*x + d) +
2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/((
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)) + 2*(6*A*a^3*c^3*e^(5*e
*x + 5*d) + 12*C*a^2*c^4*e^(5*e*x + 5*d) - 9*A*a*c^5*e^(5*e*x + 5*d) - 3*C*
c^6*e^(5*e*x + 5*d) + 30*A*a^4*c^2*e^(4*e*x + 4*d) + 60*C*a^3*c^3*e^(4*e*x
+ 4*d) - 45*A*a^2*c^4*e^(4*e*x + 4*d) - 15*C*a*c^5*e^(4*e*x + 4*d) - 8*B*a^
6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*A*a^5*c*e^(3*e*x + 3*d) -
24*B*a^4*c^2*e^(3*e*x + 3*d) + 64*C*a^4*c^2*e^(3*e*x + 3*d) - 82*A*a^3*c^3*
e^(3*e*x + 3*d) - 24*B*a^2*c^4*e^(3*e*x + 3*d) - 78*C*a^2*c^4*e^(3*e*x + 3*
d) + 24*A*a*c^5*e^(3*e*x + 3*d) - 8*B*c^6*e^(3*e*x + 3*d) + 24*C*a^5*c*e^(2
*e*x + 2*d) - 102*A*a^4*c^2*e^(2*e*x + 2*d) - 102*C*a^3*c^3*e^(2*e*x + 2*d)
 + 36*A*a^2*c^4*e^(2*e*x + 2*d) + 24*C*a*c^5*e^(2*e*x + 2*d) - 12*A*c^6*e^(
2*e*x + 2*d) - 12*C*a^4*c^2*e^(e*x + d) + 60*A*a^3*c^3*e^(e*x + d) + 66*C*a
^2*c^4*e^(e*x + d) - 15*A*a*c^5*e^(e*x + d) + 3*C*c^6*e^(e*x + d) + 2*C*a^3
*c^3 - 11*A*a^2*c^4 - 13*C*a*c^5 + 4*A*c^6)/((a^6*c + 3*a^4*c^3 + 3*a^2*c^5
 + c^7)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)^3))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cosh(d + e\,x) + C\sinh(d + e\,x)}{\left(a + c\sinh\left(d + e\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4, x)
```

3.257 $\int \frac{x^3}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=439

$$\frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}}+\frac{3x^2\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

```
[Out] 1/2*x^3*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^3*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^
(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1
/2)/(a-b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/
a^(1/2)/(a-b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2))
)/a^(1/2)/(a-b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2
)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5748, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x^2\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}-\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3x\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}}+\frac{3\text{Li}_4\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{8\sqrt{a}\sqrt{a-b}}-\frac{3\text{Li}_4\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{8\sqrt{a}\sqrt{a-b}}+\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}-\frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sq
rt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a -
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(
4*Sqrt[a]*Sqrt[a - b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[
a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^3}{2a-b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{3\int x^2\log\left(1 \cdots\right.}{\cdots}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{\cdots}{2a\cdots}\right.}{4\sqrt{\cdots}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{\cdots}{2a\cdots}\right.}{4\sqrt{\cdots}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{\cdots}{2a\cdots}\right.}{4\sqrt{\cdots}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\text{Li}_2\left(-\frac{\cdots}{2a\cdots}\right.}{4\sqrt{\cdots}}$$

Mathematica [A]
time = 0.57, size = 319, normalized size = 0.73

$$\frac{-4x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)+4x^3\log\left(1-\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)-6x^2\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)+6x^2\text{PolyLog}\left(2,\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)+6x\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)-6x\text{PolyLog}\left(3,\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)-3\text{PolyLog}\left(4,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)+3\text{PolyLog}\left(4,\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)}{8\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (-4*x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 4*x^3*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 6*x^2*PolyLog[2, -((b
*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 6*x^2*PolyLog[2, (b*E^(2*x)
)/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + 6*x*PolyLog[3, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))] - 6*x*PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a
]*Sqrt[a - b] + b)] - 3*PolyLog[4, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a -
b] - b))] + 3*PolyLog[4, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)])/(
8*Sqrt[a]*Sqrt[a - b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(351) = 702$.
time = 0.86, size = 919, normalized size = 2.09

method	result
risch	$\frac{x^3 \ln\left(1-\frac{b\,e^{2x}}{2\sqrt{a(a-b)}-2a+b}\right)}{2\sqrt{a(a-b)}} - \frac{x^4}{4\sqrt{a(a-b)}} + \frac{3x^2\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{2\sqrt{a(a-b)}-2a+b}\right)}{4\sqrt{a(a-b)}} - \frac{3x\,\mathrm{polylog}\left(3,\frac{}{2\sqrt{a(a}\right.}}{4\sqrt{a(a-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*(a-b))^(1/2)*x^3*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/4/(a*(
a-b))^(1/2)*x^4+3/4/(a*(a-b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(
1/2)-2*a+b))-3/4/(a*(a-b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-
2*a+b))+3/8/(a*(a-b))^(1/2)*polylog(4,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
+1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x
^3+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))*a*x^3-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-
b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b*x^3-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)
*x^4-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^4+1/4/(a*(a-b))^(1/
2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^4+3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(
2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*x^2+3/2/(a*(a-b))^(1/2)/(-2*(a*(a-
b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x^2-3/4
/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-
b))^(1/2)-2*a+b))*b*x^2-3/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)
/(-2*(a*(a-b))^(1/2)-2*a+b))*x-3/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+
b)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x+3/4/(a*(a-b))^(1/2)
/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b)
)*b*x+3/4/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/
2)-2*a+b))+3/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(4,b*exp(2
*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-3/8/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2
*a+b)*polylog(4,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. 2(339) = 678.
time = 0.42, size = 1655, normalized size = 3.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^3*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 - a*b)
/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2
*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*
b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (
2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2
*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 - a*b)
/b^2)*polylog(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)
/b)/b) - 6*b*x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a
- b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*x*sqrt((a^2 - a*b)/b^2)*polylo
g(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
```

```
t((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 6*b*
x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*
b)/b^2) - 2*a + b)/b)/b) + 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*
cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*sqrt((a^2 - a*b
)/b^2)*polylog(4, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a -
b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4,
 -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b))/(a^2 - a
*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x**3/(a + b*sinh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*sinh(x)^2),x)
```

```
[Out] int(x^3/(a + b*sinh(x)^2), x)
```

3.258 $\int \frac{x^2}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=327

$$\frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\,\sqrt{a-b}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\,\sqrt{a-b}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}+\frac{x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\,\sqrt{a-b}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}-$$

```
[Out] 1/2*x^2*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^2*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
1/2*x*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1
/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b
)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a
-b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5748, 3401, 2296, 2221, 2611, 2320, 6724}

$$\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\,\sqrt{a}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{x\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\,\sqrt{a}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\,\sqrt{a}-b}\right)}{4\sqrt{a}\,\sqrt{a-b}}+\frac{\text{Li}_3\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\,\sqrt{a}-b}\right)}{4\sqrt{a}\,\sqrt{a-b}}+\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\,\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a
 - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
2*Sqrt[a]*Sqrt[a - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - PolyLog[3, -((b*E^(
2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = 2\int \frac{x^2}{2a-b+b\cosh(2x)}\,dx$$
$$= 4\int \frac{e^{2x}x^2}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$
$$= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int x\log\left(1+\right.}{}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{}{2a-2}\right.}{2\sqrt{a}}$$

$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{}{2a-2}\right.}{2\sqrt{a}}$$
$$= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\text{Li}_2\left(-\frac{}{2a-2}\right.}{2\sqrt{a}}$$

Mathematica [**A**]
time = 0.50, size = 240, normalized size = 0.73

$$\frac{-2x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)+2x^2\log\left(1-\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)-2x\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)+2x\text{PolyLog}\left(2,\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)+\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)-\text{PolyLog}\left(3,\frac{be^{2x}}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (-2*x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)] + 2*x^2*Log[
1 - (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] - 2*x*PolyLog[2, -((b*E
^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))] + 2*x*PolyLog[2, (b*E^(2*x))/(-
2*a + 2*Sqrt[a]*Sqrt[a - b] + b)] + PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[
a]*Sqrt[a - b] - b))] - PolyLog[3, (b*E^(2*x))/(-2*a + 2*Sqrt[a]*Sqrt[a - b
] + b)])/(4*Sqrt[a]*Sqrt[a - b])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(261) = 522$.
time = 0.85, size = 710, normalized size = 2.17

method	result
risch	$-\frac{2x^3}{3\left(-2\sqrt{a(a-b)}-2a+b\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)}{-2\sqrt{a(a-b)}-2a+b}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2a+b}\right)}{-2\sqrt{a(a-b)}-2a+b}-\frac{\mathrm{polylog}\cdots}{2(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-2*(a*(a-b))^(1/2)-2*a+b)*x^3+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2*ln(1-b
*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(-2*(a*(a-b))^(1/2)-2*a+b)*x*polylo
g(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*p
olylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-2/3/(a*(a-b))^(1/2)/(-2*(a*
(a-b))^(1/2)-2*a+b)*a*x^3+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^
2*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/(a*(a-b))^(1/2)/(-2*(a*(a-b
))^(1/2)-2*a+b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a
*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*polylog(3,b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))+1/3/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^3-1/2/(
a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x^2*ln(1-b*exp(2*x)/(-2*(a*(a-b
))^(1/2)-2*a+b))-1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*b*x*polylog
(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/4/(a*(a-b))^(1/2)/(-2*(a*(a-b))
^(1/2)-2*a+b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))-1/3/(a*(a-
b))^(1/2)*x^3+1/2/(a*(a-b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*
a+b))+1/2/(a*(a-b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
-1/4/(a*(a-b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(252) = 504$.
time = 0.44, size = 1247, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^2*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 2*b*x*sqrt((a^2 - a*b)/b
^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2
)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 2*b*sqrt((a^2 - a*b)/b^2)*poly
log(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) - 2
*b*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b)/b) + 2*b*sqrt((a^2 - a*b)/b^2)*polylog(3, ((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 2*b*sqrt((a^2 - a*
b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
b)/b)/b))/(a^2 - a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*sinh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*sinh(x)^2),x)
```

```
[Out] int(x^2/(a + b*sinh(x)^2), x)
```

3.259 $\int \frac{x}{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=215

$$\frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\,\sqrt{a-b}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\,\sqrt{a-b}-b}\right)}{2\sqrt{a}\,\sqrt{a-b}}+\frac{\mathrm{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\,\sqrt{a-b}-b}\right)}{4\sqrt{a}\,\sqrt{a-b}}-\text{Po}$$

```
[Out] 1/2*x*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
2*x*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+1/4*
polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
4*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5748, 3401, 2296, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a-2\sqrt{a-b}\,\sqrt{a}-b}\right)}{4\sqrt{a}\,\sqrt{a-b}}-\frac{\text{Li}_2\left(-\frac{be^{2x}}{2a+2\sqrt{a-b}\,\sqrt{a}-b}\right)}{4\sqrt{a}\,\sqrt{a-b}}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\,\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\,\sqrt{a-b}}-\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\,\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\,\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sinh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqrt[
a - b]) - (x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqr
t[a]*Sqrt[a - b]) + PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] -
 b))]/(4*Sqrt[a]*Sqrt[a - b]) - PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*S
qrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rubi steps

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = 2\int \frac{x}{2a-b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int\log\left(1+\frac{\quad}{-4\ldots}\right.}{2\ldots}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\text{Subst}\left(\int\frac{\log(\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{\mathrm{Li}_2\left(-\frac{\quad}{2a-2\sqrt{a}\ldots}\right.}{4\sqrt{a}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 576, normalized size = 2.68

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sinh[x]^2),x]
```

```
[Out] -1/4*(4*x*ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] - (2*I)*ArcCos[1 - (2*a)/b
]*ArcTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a] + (ArcCos[1 - (2*a)/b] + 2*(ArcTan[(
a*Coth[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sqrt[-a^2 + a*b]*Tanh[x])/a]))*Log
[(Sqrt[2]*Sqrt[a*(-a + b)])/(Sqrt[b]*E^x*Sqrt[2*a - b + b*Cosh[2*x]])] + (A
rcCos[1 - (2*a)/b] - 2*(ArcTan[(a*Coth[x])/Sqrt[-(a*(a - b))]] + ArcTan[(Sq
rt[-a^2 + a*b]*Tanh[x])/a]))*Log[(Sqrt[2]*Sqrt[a*(-a + b)]*E^x)/(Sqrt[b]*Sq
rt[2*a - b + b*Cosh[2*x]])] - (ArcCos[1 - (2*a)/b] + 2*ArcTan[(Sqrt[-a^2 +
a*b]*Tanh[x])/a])*Log[(2*a*((-I)*a + I*b + Sqrt[a*(-a + b)])*(-1 + Tanh[x])
)/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] - (ArcCos[1 - (2*a)/b] - 2*ArcTa
n[(Sqrt[-a^2 + a*b]*Tanh[x])/a])*Log[(2*a*(I*a - I*b + Sqrt[a*(-a + b)])*(1
 + Tanh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Tanh[x])] + I*(-PolyLog[2, ((-2
*a + b - (2*I)*Sqrt[a*(-a + b)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*
b + b*Sqrt[a*(-a + b)]*Tanh[x])] + PolyLog[2, ((-2*a + b + (2*I)*Sqrt[a*(-a
```

```
 + b)])*(I*a + Sqrt[a*(-a + b)]*Tanh[x]))/((-I)*a*b + b*Sqrt[a*(-a + b)]*Ta
nh[x])]))/Sqrt[a*(-a + b)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(171) = 342$.
time = 0.86, size = 505, normalized size = 2.35

method	result
risch	$\frac{x \ln\left(1-\frac{b\,e^{2x}}{2\sqrt{a(a-b)}-2a+b}\right)}{2\sqrt{a(a-b)}} - \frac{x^2}{2\sqrt{a(a-b)}} + \frac{\operatorname{polylog}\left(2, \frac{b\,e^{2x}}{2\sqrt{a(a-b)}-2a+b}\right)}{4\sqrt{a(a-b)}} + \frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a-b)}-2}\right.}{-2\sqrt{a(a-b)}-2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*(a-b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a-b))^(1/2)-2*a+b))-1/2/(a*(a-
b))^(1/2)*x^2+1/4/(a*(a-b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a-b))^(1/2)-2
*a+b))+1/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a
+b))*x-1/(-2*(a*(a-b))^(1/2)-2*a+b)*x^2+1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/
2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a*x-1/2/(a*(a-b))^(1/
2)/(-2*(a*(a-b))^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*b
*x-1/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*a*x^2+1/2/(a*(a-b))^(1/2)/(
-2*(a*(a-b))^(1/2)-2*a+b)*x^2*b+1/2/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*
exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))+1/2/(a*(a-b))^(1/2)/(-2*(a*(a-b))^(1/2
)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*a+b))*a-1/4/(a*(a-b))^(
1/2)/(-2*(a*(a-b))^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*(a*(a-b))^(1/2)-2*
a+b))*b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(165) = 330$.
time = 0.50, size = 837, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x*sqrt((a^
2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b
)/b) + b)/b) - b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) + (2*a -
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqr
t((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + b*sqrt((a^2 - a*b)/b^2)*dilog(-(
((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) + b)/b + 1) +
 b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/
b^2) + 2*a - b)/b) - b)/b + 1) - b*sqrt((a^2 - a*b)/b^2)*dilog(-(((2*a - b)
*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b
^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)/b + 1) - b*sqrt((a^
2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b) - b)/b + 1))/(a^2 - a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x/(a + b*sinh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{b\,{\sinh(x)}^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*sinh(x)^2),x)
```

```
[Out] int(x/(a + b*sinh(x)^2), x)
```

3.260 $\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{9}{4}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx)-\frac{9}{4}\sinh(a)\mathrm{Shi}(bx)+\frac{1}{4}\sinh(3a)\mathrm{Shi}(3bx)$$

```
[Out] -9/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-9/4*Shi(b*x)*sinh(a)+1/4*Shi
(3*b*x)*sinh(3*a)
```

Rubi [A]
time = 0.33, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6874, 3384, 3379, 3382, 5556}

$$-\frac{9}{4}\cosh(a)\mathrm{Chi}(bx)+\frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx)-\frac{9}{4}\sinh(a)\mathrm{Shi}(bx)+\frac{1}{4}\sinh(3a)\mathrm{Shi}(3bx)$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*S
inh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx &= \int \left(-\frac{2\cosh(a+bx)}{x}+\frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\right)dx \\
&= -\left(2\int \frac{\cosh(a+bx)}{x}\,dx\right)+\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx \\
&= -\left((2\cosh(a))\int \frac{\cosh(bx)}{x}\,dx\right)-(2\sinh(a))\int \frac{\sinh(bx)}{x}\,dx+\int \\
&= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx)-\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx+ \\
&= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx)-\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,d \\
&= -\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx)-\frac{9}{4}\sinh(a)\text{Shi}(bx)+
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.87

$$\frac{1}{4}(-9\cosh(a)\text{Chi}(bx)+\cosh(3a)\text{Chi}(3bx)-9\sinh(a)\text{Shi}(bx)+\sinh(3a)\text{Shi}(3bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] - 9*Sinh[a]*S
inhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A]
time = 9.79, size = 47, normalized size = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\text{expIntegral}(1,3bx)}{8}+\frac{9\,e^{-a}\,\text{expIntegral}(1,bx)}{8}+\frac{9\,e^{a}\,\text{expIntegral}(1,-bx)}{8}-\frac{e^{3a}\,\text{expIntegral}(1,-3bx)}{8}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)+9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Maxima [**A**]
time = 0.35, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-\frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)}+\frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-\frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

Fricas [**A**]
time = 0.42, size = 67, normalized size = 1.43

$$\frac{1}{8}\,(\mathrm{Ei}(3\,bx)+\mathrm{Ei}(-3\,bx))\cosh{(3\,a)}-\frac{9}{8}\,(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\cosh{(a)}+\frac{1}{8}\,(\mathrm{Ei}(3\,bx)-\mathrm{Ei}(-3\,bx))\sinh{(3\,a)}-\frac{9}{8}\,(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\sinh{(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 9/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sinh^{2}{\left(a+bx\right)}-2\right)\cosh{\left(a+bx\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)**2)/x,x)
```

```
[Out] Integral((sinh(a + b*x)**2 - 2)*cosh(a + b*x)/x, x)
```

Giac [**A**]
time = 0.41, size = 42, normalized size = 0.89

$$\frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-\frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)}+\frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-\frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a+b\,x)\,\left(\sinh(a+b\,x)^2-2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x,x)
```

```
[Out] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x, x)
```

3.261 $\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

[Out] 3/4*Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Shi(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3379}

$$\frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]

[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - SinhIntegral[(3*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(4*a)

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}+\frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= \frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 55, normalized size = 0.95

$$\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - SinhIntegral[(3*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/(4*a)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.262 $\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3382}

$$\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\mathrm{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\operatorname{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}\\ &= -\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.98

$$-\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[1 - a*x]/(4*a) -
 Log[1 + a*x]/(4*a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

```
[Out] -1/4*log(a*x + 1)/a + 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqr
t(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

```
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.263 $\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]

time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6813, 3379}

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$-\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.264 $\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$

Optimal. Leaf size=40

$$\operatorname{Int}\left(\frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)}, x\right)$$

```
[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Csch[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 6.94, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sinh \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \sinh{\left(\frac{\sqrt{- a x + 1}}{\sqrt{a x + 1}} \right)} - \sinh{\left(\frac{\sqrt{- a x + 1}}{\sqrt{a x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sinh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.265 $\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csch[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 20.77, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sinh \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

```
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - sqrt
(-a*x + 1)*a) - integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sq
rt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x) + integrate
(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sqrt(-a*x + 1)/sqrt(a*x + 1
)) - (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \sinh^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \sinh^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sinh(sqrt(-a
*x + 1)/sqrt(a*x + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\mathrm{sinh}\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}^2\,\left(a^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.266 $\int \sinh(a + b\log(cx^n))\ dx$

Optimal. Leaf size=54

$$-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}+\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

```
[Out] -b*n*x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5628}

$$\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]]
)/(1 - b^2*n^2)
```

Rule 5628

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\int \sinh(a+b\log(cx^n))\ dx=-\frac{bnx\cosh(a+b\log(cx^n))}{1-b^2n^2}+\frac{x\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.76

$$\frac{x(bn\cosh(a+b\log(cx^n))-\sinh(a+b\log(cx^n)))}{-1+b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x*(b*n*Cosh[a + b*Log[c*x^n]] - Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sinh \left(a + b \ln \left(c\,x^{n}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n)),x)
[Out] int(sinh(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.96

$$\frac{c^{b}xe^{(b\log(x^n)+a)}}{2\,(bn+1)}+\frac{xe^{(-b\log(x^n)-a)}}{2\,(bc^{b}n-c^{b})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) + 1/2*x*e^(-b*log(x^n) - a)/(b*c^b*n
 - c^b)
```

Fricas [A]
time = 0.49, size = 44, normalized size = 0.81

$$\frac{bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a) - x*sinh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases}\int \sinh\left(a-\frac{\log\left(cx^{n}\right)}{n}\right)\,dx & \text{for}\ b=-\frac{1}{n}\\ \int \sinh\left(a+\frac{\log\left(cx^{n}\right)}{n}\right)\,dx & \text{for}\ b=\frac{1}{n}\\ \frac{bnx\cosh\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}-1}-\frac{x\sinh\left(a+b\log\left(cx^{n}\right)\right)}{b^{2}n^{2}-1} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(si
nh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*cosh(a + b*log(c*x**n))/(b**
2*n**2 - 1) - x*sinh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

Giac [**A**]
time = 0.42, size = 47, normalized size = 0.87

$$\frac{c^b x x^{bn} e^a}{2\,(bn+1)} + \frac{x e^{(-a)}}{2\,(bn-1)c^b x^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) + 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

Mupad [**B**]
time = 0.65, size = 43, normalized size = 0.80

$$\frac{x\,\mathrm{e}^{-a}}{\left(c\,x^n\right)^b\,\left(2\,b\,n-2\right)} + \frac{x\,\mathrm{e}^{a}\,\left(c\,x^n\right)^b}{2\,b\,n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n)),x)
```

```
[Out] (x*exp(-a))/((c*x^n)^b*(2*b*n - 2)) + (x*exp(a)*(c*x^n)^b)/(2*b*n + 2)
```

3.267 $\int \sinh^2(a + b\log(cx^n))\,dx$

Optimal. Leaf size=88

$$\frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}$$

```
[Out] 2*b^2*n^2*x/(-4*b^2*n^2+1)-2*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/
(-4*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5630, 8}

$$\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*
Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(a+b\log(cx^n))\,dx &= -\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2} \\ &= \frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+}{1-4}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 0.62

$$-\frac{x(-1+4b^2n^2+\cosh(2(a+b\log(cx^n)))-2bn\sinh(2(a+b\log(cx^n))))}{-2+8b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2,x]
[Out] -((x*(-1 + 4*b^2*n^2 + Cosh[2*(a + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a + b*Log
[c*x^n])]))/(-2 + 8*b^2*n^2))
```

Maple [F]
time = 4.33, size = 0, normalized size = 0.00

$$\int \sinh^2(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2,x)
[Out] int(sinh(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.76

$$\frac{c^{2\,b}xe^{(2\,b\log(x^n)+2\,a)}}{4\,(2\,bn+1)}-\frac{1}{2}\,x-\frac{xe^{(-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})(x^n)^{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) - 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Fricas [A]
time = 0.49, size = 91, normalized size = 1.03

$$\frac{4\,bnx\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)-x\cosh(bn\log(x)+b\log(c)+a)^2-x\sinh(bn\log(x)+b\log(c)+a)^2-(4\,b^2n^2-1)x}{2\,(4\,b^2n^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 - (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^2\left(a - \frac{\log(cx^n)}{2n}\right) dx & \text{for } b = -\frac{1}{2n} \\ \int \sinh^2\left(a + \frac{\log(cx^n)}{2n}\right) dx & \text{for } b = \frac{1}{2n} \\ \frac{2b^2n^2x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1} - \frac{2b^2n^2x\cosh^2(a+b\log(cx^n))}{4b^2n^2-1} + \frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2n^2-1} - \frac{x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))), (
Integral(sinh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (2*b**2*n**2*
x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) - 2*b**2*n**2*x*cosh(a + b*l
og(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cosh(a +
 b*log(c*x**n))/(4*b**2*n**2 - 1) - x*sinh(a + b*log(c*x**n))**2/(4*b**2*n*
*2 - 1), True))
```

Giac [A]
time = 0.45, size = 169, normalized size = 1.92

$$\frac{bc^{2b}nxx^{2bn}e^{(2a)}}{2(4b^2n^2-1)} - \frac{2b^2n^2x}{4b^2n^2-1} - \frac{c^{2b}xx^{2bn}e^{(2a)}}{4(4b^2n^2-1)} - \frac{bnxe^{(-2a)}}{2(4b^2n^2-1)c^{2b}x^{2bn}} + \frac{x}{2(4b^2n^2-1)} - \frac{xe^{(-2a)}}{4(4b^2n^2-1)c^{2b}x^{2bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B]
time = 0.65, size = 53, normalized size = 0.60

$$\frac{x\,e^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4} - \frac{x\,e^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*exp(2*a)*(c*x^n)^(2*b))/(8*b*n + 4) - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*
n - 4)) - x/2
```

3.268 $\int \sinh^3(a+b\log(cx^n))\,dx$

Optimal. Leaf size=149

$$-\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}-\frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2}$$

```
[Out] -6*b^3*n^3*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+6*b^2*n^2*x*sinh(
a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+
b*ln(c*x^n))^2/(-9*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^3/(-9*b^2*n^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5630, 5628}

$$\frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-9b^2n^2}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}-\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2
*n^2*x*Sinh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2) + (x*Sinh[a +
b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2)
```

Rule 5628

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \sinh^3(a+b\log(cx^n))\,dx = -\frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2} + \frac{x\sinh^3(a+b\log(cx^n)}{1-9b^2n^2}$$
$$= -\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} - \frac{3bnx\cosh(}{}$$

Mathematica [A]
time = 0.40, size = 120, normalized size = 0.81

$$\frac{x(-3bn(-1+9b^2n^2)\cosh(a+b\log(cx^n))+3bn(-1+b^2n^2)\cosh(3(a+b\log(cx^n)))-2(1-13b^2n^2+(-1+b^2n^2)\cosh(2(a+b\log(cx^n))))\sinh(a+b\log(cx^n)))}{4-40b^2n^2+36b^4n^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(-3*b*n*(-1 + 9*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + 3*b*n*(-1 + b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 2*(1 - 13*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a +
b*Log[c*x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [F]
time = 2.84, size = 0, normalized size = 0.00

$$\int \sinh^3(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^3,x)
```

Maxima [A]
time = 0.31, size = 114, normalized size = 0.77

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+3\,a)}}{8\,(3\,bn+1)} - \frac{3\,c^bxe^{(b\log(x^n)+a)}}{8\,(bn+1)} - \frac{3\,xe^{(-b\log(x^n)-a)}}{8\,(bc^bn-c^b)} + \frac{xe^{(-3\,b\log(x^n)-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) - 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) + 1/8*x*e^(-3*b*
log(x^n) - 3*a)/(3*b*c^(3*b)*n - c^(3*b))
```

Fricas [A]
time = 0.45, size = 200, normalized size = 1.34

$$\frac{3\,(b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)^3+9\,(b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2-(b^2n^2-1)x\sinh(bn\log(x)+b\log(c)+a)^3-3\,(9\,b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)-3\left((b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^2-(9\,b^2n^2-1)x\right)\sinh(bn\log(x)+b\log(c)+a)}{4\,(9\,b^4n^4-10\,b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*(b^3*n^3 - b
*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - (
b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^3 - 3*(9*b^3*n^3 - b*n)*x*co
sh(b*n*log(x) + b*log(c) + a) - 3*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^2 - (9*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
- 10*b^2*n^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^{3}\left(a - \frac{\log\left(cx^{n}\right)}{n}\right)\, dx & \text{for}\: b = - \frac{1}{n} \\ \int \sinh^{3}\left(a - \frac{\log\left(cx^{n}\right)}{3n}\right)\, dx & \text{for}\: b = - \frac{1}{3n} \\ \int \sinh^{3}\left(a + \frac{\log\left(cx^{n}\right)}{3n}\right)\, dx & \text{for}\: b = \frac{1}{3n} \\ \int \sinh^{3}\left(a + \frac{\log\left(cx^{n}\right)}{n}\right)\, dx & \text{for}\: b = \frac{1}{n} \\ \frac{9b^{3}n^{3}x\sinh^{2}\left(a+b\log\left(cx^{n}\right)\right)\cosh\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} - \frac{6b^{3}n^{3}x\cosh^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} - \frac{7b^{2}n^{2}x\sinh^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} + \frac{6b^{2}n^{2}x\sinh\left(a+b\log\left(cx^{n}\right)\right)\cosh^{2}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} - \frac{3bnx\sinh^{2}\left(a+b\log\left(cx^{n}\right)\right)\cosh\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} + \frac{x\sinh^{3}\left(a+b\log\left(cx^{n}\right)\right)}{9b^{4}n^{4}-10b^{2}n^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integral
(sinh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(sinh(a + l
og(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(sinh(a + log(c*x**n)/n
)**3, x), Eq(b, 1/n)), (9*b**3*n**3*x*sinh(a + b*log(c*x**n))**2*cosh(a + b
*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 6*b**3*n**3*x*cosh(a + b*l
og(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 7*b**2*n**2*x*sinh(a + b*
log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sinh(a + b
*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) -
 3*b*n*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 -
10*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**
2 + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(150) = 300.
time = 0.47, size = 665, normalized size = 4.46

$$\frac{3\,b^{3}c^{3\,b}n^{3}xx^{3\,bn}e^{\left(3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} - \frac{27\,b^{3}c^{b}n^{3}xx^{bn}e^{a}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} - \frac{b^{2}c^{3\,b}n^{2}xx^{3\,bn}e^{\left(3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} + \frac{27\,b^{2}c^{b}n^{2}xx^{bn}e^{a}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} - \frac{3\,bc^{3\,b}nxx^{3\,bn}e^{\left(3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} - \frac{27\,b^{3}n^{3}xe^{\left(-a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{b}x^{bn}} + \frac{3\,b^{3}n^{3}xe^{\left(-3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{3\,b}x^{3\,bn}} + \frac{3\,bc^{b}nxx^{bn}e^{a}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} + \frac{c^{3\,b}xx^{3\,bn}e^{\left(3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} - \frac{27\,b^{2}n^{2}xe^{\left(-a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{b}x^{bn}} + \frac{b^{2}n^{2}xe^{\left(-3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{3\,b}x^{3\,bn}} - \frac{3\,c^{b}xx^{bn}e^{a}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}} + \frac{3\,bnxe^{\left(-a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{b}x^{bn}} - \frac{3\,bnxe^{\left(-3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{3\,b}x^{3\,bn}} + \frac{3\,xe^{\left(-a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{b}x^{bn}} - \frac{xe^{\left(-3\,a\right)}}{8\,{\left(9\,b^{4}n^{4}-10\,b^{2}n^{2}+1\right)}c^{3\,b}x^{3\,bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8*b^2*c^b*n^2*x*x^
```

```
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) + 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 - 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

Mupad [B]
time = 0.72, size = 93, normalized size = 0.62

$$\frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)}-\frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^{b}\,\left(8\,b\,n-8\right)}+\frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8}-\frac{3\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^{b}}{8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(-3*a))/((c*x^n)^(3*b)*(24*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*
n - 8)) + (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (3*x*exp(a)*(c*x^n)^b)/
(8*b*n + 8)
```

3.269 $\int \sinh^4(a+b\log(cx^n))\,dx$

Optimal. Leaf size=191

$$\frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}-$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-24*b^3*n^3*x*cosh(a+b*ln(c*x^n))*sin
h(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)+12*b^2*n^2*x*sinh(a+b*ln(c*x^n))
^2/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n)
)^3/(-16*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5630, 8}

$$\frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2}+\frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}-\frac{24b^3n^3x\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x}{64b^4n^4-20b^2n^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cosh[a + b*Log
[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^
2*x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2) + (x*Sinh[a +
 b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\int \sinh^4(a+b\log(cx^n))\,dx = -\frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n)}{1-16b^2n^2}$$
$$= -\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sinh^2(a+b}{1-20b^2n^2+6}$$
$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}$$

Mathematica [A]
time = 0.34, size = 167, normalized size = 0.87

$$\frac{x(3-60b^2n^2+192b^4n^4+(-4+64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))+8bn\sinh(2(a+b\log(cx^n)))-128b^3n^3\sinh(2(a+b\log(cx^n)))-4bn\sinh(4(a+b\log(cx^n)))+16b^3n^3\sinh(4(a+b\log(cx^n))))}{8(1-20b^2n^2+64b^4n^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (-4 + 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] + 8*b*n*Sinh[2*(a + b*Log
[c*x^n])] - 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Lo
g[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 6
4*b^4*n^4))
```

Maple [F]
time = 4.90, size = 0, normalized size = 0.00

$$\int \sinh^4(a+b\ln(c\,x^n))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^4,x)
```

Maxima [A]
time = 0.31, size = 129, normalized size = 0.68

$$\frac{c^{4\,b}xe^{(4\,b\log(x^n)+4\,a)}}{16\,(4\,bn+1)} - \frac{c^{2\,b}xe^{(2\,b\log(x^n)+2\,a)}}{4\,(2\,bn+1)} + \frac{3}{8}\,x + \frac{xe^{(-2\,b\log(x^n)-2\,a)}}{4\,(2\,bc^{2\,b}n-c^{2\,b})} - \frac{xe^{(-4\,a)}}{16\,(4\,bc^{4\,b}n-c^{4\,b})(x^n)^{4\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) - 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x + 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

Fricas [**A**]
time = 0.46, size = 294, normalized size = 1.54

$$-\frac{(4b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^4-16(4b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^3+(4b^2n^2-1)x\sinh(bn\log(x)+b\log(c)+a)^4-4(16b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^2+2\left(3(4b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^2-2(16b^2n^2-1)x\right)\sinh(bn\log(x)+b\log(c)+a)^2-3(64b^4n^4-20b^2n^2+1)x-16\left((4b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)^3-(16b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\right)\sinh(bn\log(x)+b\log(c)+a)}{8(64b^4n^4-20b^2n^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 - 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \sinh^4\left(a-\frac{\log(cx^n)}{2n}\right)dx & \text{for } b=-\frac{1}{2n} \\ \int \sinh^4\left(a-\frac{\log(cx^n)}{4n}\right)dx & \text{for } b=-\frac{1}{4n} \\ \int \sinh^4\left(a+\frac{\log(cx^n)}{4n}\right)dx & \text{for } b=\frac{1}{4n} \\ \int \sinh^4\left(a+\frac{\log(cx^n)}{2n}\right)dx & \text{for } b=\frac{1}{2n} \\ \frac{24b^4n^4x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}-\frac{48b^4n^4x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x\cosh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{40b^3n^3x\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}-\frac{24b^3n^3x\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}-\frac{16b^2n^2x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{12b^2n^2x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}-\frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1}+\frac{x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))), (
Integral(sinh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(si
nh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(sinh(a + log(c
*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c*x**
n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*log(c*
x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 24
*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) +
 40*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**4
*n**4 - 20*b**2*n**2 + 1) - 24*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a +
 b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 16*b**2*n**2*x*sinh(
a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n**2*x*si
nh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2
*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64
*b**4*n**4 - 20*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**4/(64*b**4*n**4
 - 20*b**2*n**2 + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(192) = 384.
time = 0.45, size = 777, normalized size = 4.07

$$\frac{b^3c^{4b}n^3xx^{4bn}e^{(4a)}}{64b^4n^4-20b^2n^2+1}-\frac{8b^3c^{2b}n^3xx^{2bn}e^{(2a)}}{64b^4n^4-20b^2n^2+1}+\frac{24b^4n^4x}{64b^4n^4-20b^2n^2+1}-\frac{b^2c^{4b}n^2xx^{4bn}e^{(4a)}}{4(64b^4n^4-20b^2n^2+1)}+\frac{4b^2c^{2b}n^2xx^{2bn}e^{(2a)}}{64b^4n^4-20b^2n^2+1}-\frac{bc^{4b}nxx^{4bn}e^{(4a)}}{4(64b^4n^4-20b^2n^2+1)}+\frac{bc^{2b}nxx^{2bn}e^{(2a)}}{2(64b^4n^4-20b^2n^2+1)}+\frac{8b^3n^3xe^{(-2a)}}{(64b^4n^4-20b^2n^2+1)c^{2b}x^{2bn}}-\frac{b^3n^3xe^{(-4a)}}{(64b^4n^4-20b^2n^2+1)c^{4b}x^{4bn}}-\frac{15b^2n^2x}{2(64b^4n^4-20b^2n^2+1)}+\frac{c^{4b}xx^{4bn}e^{(4a)}}{16(64b^4n^4-20b^2n^2+1)}-\frac{c^{2b}xx^{2bn}e^{(2a)}}{4(64b^4n^4-20b^2n^2+1)}+\frac{4b^2n^2xe^{(-2a)}}{(64b^4n^4-20b^2n^2+1)c^{2b}x^{2bn}}-\frac{b^2n^2xe^{(-4a)}}{4(64b^4n^4-20b^2n^2+1)c^{4b}x^{4bn}}-\frac{bnxe^{(-2a)}}{2(64b^4n^4-20b^2n^2+1)c^{2b}x^{2bn}}+\frac{bnxe^{(-4a)}}{4(64b^4n^4-20b^2n^2+1)c^{4b}x^{4bn}}+\frac{3x}{8(64b^4n^4-20b^2n^2+1)}-\frac{xe^{(-2a)}}{4(64b^4n^4-20b^2n^2+1)c^{2b}x^{2bn}}+\frac{xe^{(-4a)}}{16(64b^4n^4-20b^2n^2+1)c^{4b}x^{4bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) + 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

Mupad [B]
time = 0.69, size = 102, normalized size = 0.53

$$\frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}-\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 + (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) - (x*exp(2*a)*(c*x^n)^(
2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(4*
a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

3.270 $\int x^m \sinh(a + b \log(cx^n)) \, dx$

Optimal. Leaf size=73

$$-\frac{bnx^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}+\frac{(1+m)x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

```
[Out] -b*n*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*l
n(c*x^n))/((1+m)^2-b^2*n^2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5638}

$$\frac{(m+1)x^{m+1}\sinh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n))
```

Rule 5638

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\int x^m\sinh(a+b\log(cx^n))\,dx=-\frac{bnx^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}+\frac{(1+m)x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.74

$$\frac{x^{1+m}(-bn\cosh(a+b\log(cx^n))+(1+m)\sinh(a+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] (x^(1 + m)*(-(b*n*Cosh[a + b*Log[c*x^n]]) + (1 + m)*Sinh[a + b*Log[c*x^n]])
)/((1 + m - b*n)*(1 + m + b*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int x^{m} \sinh \left(a +b \ln \left(c \,x^{n}\right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.88

$$\frac{c^{b}xe^{\left(b\log\left(x^{n}\right)+m\log\left(x\right)+a\right)}}{2\left(bn+m+1\right)}+\frac{xe^{\left(-b\log\left(x^{n}\right)+m\log\left(x\right)-a\right)}}{2\left(bc^{b}n-c^{b}(m+1)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) + 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Fricas [A]
time = 0.37, size = 98, normalized size = 1.34

$$\frac{bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(m\log\left(x\right)\right)-\left((m+1)x\cosh\left(m\log\left(x\right)\right)+(m+1)x\sinh\left(m\log\left(x\right)\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^{2}n^{2}-m^{2}-2\,m-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + b*n*x*cosh(b*n*log(
x) + b*log(c) + a)*sinh(m*log(x)) - ((m + 1)*x*cosh(m*log(x)) + (m + 1)*x*s
inh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log\left(x\right)\sinh\left(a\right) & \text{for}\: b = 0 \wedge m = -1 \\ - \int x^{m} \sinh\left(- a + \frac{m \log\left(c x^{n}\right)}{n} + \frac{\log\left(c x^{n}\right)}{n}\right)\, dx & \text{for}\: b = \frac{- m - 1}{n} \\ \int x^{m} \sinh\left(a + \frac{m \log\left(c x^{n}\right)}{n} + \frac{\log\left(c x^{n}\right)}{n}\right)\, dx & \text{for}\: b = \frac{m + 1}{n} \\ \frac{b n x x^{m} \cosh\left(a + b \log\left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{m x x^{m} \sinh\left(a + b \log\left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} - \frac{x x^{m} \sinh\left(a + b \log\left(c x^{n}\right)\right)}{b^{2} n^{2} - m^{2} - 2 m - 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & Eq(m, -1)), (-Integral(x**m*sinh(-a +
 m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (-m - 1)/n)), (Integral(x**m*s
inh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x**
m*cosh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*sinh(a +
b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*sinh(a + b*log(c*x**n)
)/(b**2*n**2 - m**2 - 2*m - 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(75) = 150.
time = 0.43, size = 235, normalized size = 3.22

$$\frac{bc^bnxx^{bn}x^me^a}{2\,(b^2n^2-m^2-2\,m-1)}-\frac{c^bmxx^{bn}x^me^a}{2\,(b^2n^2-m^2-2\,m-1)}-\frac{c^bxx^{bn}x^me^a}{2\,(b^2n^2-m^2-2\,m-1)}+\frac{bnxx^me^{(-a)}}{2\,(b^2n^2-m^2-2\,m-1)c^bx^{bn}}+\frac{mxx^me^{(-a)}}{2\,(b^2n^2-m^2-2\,m-1)c^bx^{bn}}+\frac{xx^me^{(-a)}}{2\,(b^2n^2-m^2-2\,m-1)c^bx^{bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) + 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) + 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) + 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

Mupad [B]
time = 0.70, size = 56, normalized size = 0.77

$$\frac{x\,x^m\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,m+2\,b\,n+2}-\frac{x\,x^m\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,m-2\,b\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(a)*(c*x^n)^b)/(2*m + 2*b*n + 2) - (x*x^m*exp(-a))/((c*x^n)^b*(2*
m - 2*b*n + 2))
```

3.271 $\int x^m \sinh^2(a+b\log(cx^n))\,dx$

Optimal. Leaf size=120

$$\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)}-\frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}+\frac{(1+m)x^{1+m}\sinh^2(a+}{(1+m)^2-4b^2}$$

```
[Out] 2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n
))*sinh(a+b*ln(c*x^n))/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n)
)^2/((1+m)^2-4*b^2*n^2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5640, 30}

$$\frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}+\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*
Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((
1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Rubi steps

$$\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2} + \frac{(1+m)x^{1+m}\sinh}{(1+m)^2}$$

$$= \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} - \frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\,\mathrm{lo}\right.}{(1+m)^2-4b^2n^2}$$

Mathematica [A]
time = 0.22, size = 89, normalized size = 0.74

$$\frac{x^{1+m}\left(-1-2m-m^2+4b^2n^2+(1+m)^2\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2b(1+m)n\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)\right)}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(-1 - 2*m - m^2 + 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*
(1 + m + 2*b*n))
```

Maple [F]
time = 3.00, size = 0, normalized size = 0.00

$$\int x^m\left(\sinh^2\left(a+b\ln\left(c\,x^n\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

Maxima [A]
time = 0.27, size = 87, normalized size = 0.72

$$\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+m\log\left(x\right)+2\,a\right)}}{4\left(2\,bn+m+1\right)} - \frac{xe^{\left(-2\,b\log\left(x^n\right)+m\log\left(x\right)-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)} - \frac{x^{m+1}}{2\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(
-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/2*x^(
m + 1)/(m + 1)
```

Fricas [A]
time = 0.41, size = 248, normalized size = 2.07

$$\frac{(m^2+2m+1)x\cosh(bn\log(x)+b\log(c)+a)^2\cosh(m\log(x))+(4b^2n^2-m^2-2m-1)x\cosh(m\log(x))+((m^2+2m+1)x\cosh(m\log(x))+(m^2+2m+1)x\sinh(m\log(x)))\sinh(bn\log(x)+b\log(c)+a)^2-4((bm+b)nx\cosh(bn\log(x)+b\log(c)+a)\cosh(m\log(x))+(bm+b)nx\cosh(bn\log(x)+b\log(c)+a)\sinh(m\log(x)))\sinh(bn\log(x)+b\log(c)+a)+((m^2+2m+1)x\cosh(bn\log(x)+b\log(c)+a)^2+(4b^2n^2-m^2-2m-1)x)\sinh(m\log(x))}{2(m^3-4(b^2m+b^2)n^2+3m^2+3m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \log(x)\sinh^2(a) & \text{for}\ b=0 \wedge m=-1 \\ \int x^m \sinh^2\left(-a+\frac{m\log(cx^n)}{2n}+\frac{\log(cx^n)}{2n}\right)dx & \text{for}\ b=\frac{-m-1}{2n} \\ \int x^m \sinh^2\left(a+\frac{m\log(cx^n)}{2n}+\frac{\log(cx^n)}{2n}\right)dx & \text{for}\ b=\frac{m+1}{2n} \\ \int \frac{\sinh^2(a+b\log(cx^n))}{x}dx & \text{for}\ m=-1 \\ \frac{2b^2n^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{2b^2n^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2bmnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2bnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{m^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{2mxx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(-a
 + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, (-m - 1)/(2*n))),
 (Integral(x**m*sinh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), E
q(b, (m + 1)/(2*n))), (Integral(sinh(a + b*log(c*x**n))**2/x, x), Eq(m, -1)
), (2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n
**2 - m**3 - 3*m**2 - 3*m - 1) - 2*b**2*n**2*x*x**m*cosh(a + b*log(c*x**n))
**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m*n*x*x**
m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n
**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x**n))*cosh(
a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1)
- m**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**
3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**
2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*sinh(a + b*log(c*x**n))
**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 758 vs. 2(127) = 254.
time = 0.46, size = 758, normalized size = 6.32

$$\frac{bc^{2b}mnxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}-\frac{c^{2b}m^2xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}+\frac{bc^{2b}nxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}-\frac{2b^2n^2xx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{c^{2b}mxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}-\frac{c^{2b}xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}+\frac{m^2xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}-\frac{bmnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}+\frac{mxx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{m^2xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}-\frac{bnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}+\frac{xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)}-\frac{mxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}-\frac{xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="giac")

[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 2*b^2*n^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))

Mupad [B]
time = 0.75, size = 74, normalized size = 0.62

$$\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}-\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*sinh(a + b*log(c*x^n))^2,x)

[Out] (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) - (x*x^m)/(2*m + 2) + (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)

3.272 $\int x^m \sinh^3(a + b\log(cx^n))\,dx$

Optimal. Leaf size=203

$$-\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)}+\frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)}-\frac{3bnx^{1+m}\cosh(a+b\log}{(1+}$$

```
[Out] -6*b^3*n^3*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2
)+6*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-
b^2*n^2)-3*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^2/((1+m)^2-9
*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)
```

Rubi [A]
time = 0.07, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5640, 5638}

$$\frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}+\frac{6b^2(m+1)n^2x^{m+1}\sinh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)((m+1)^2-9b^2n^2)}-\frac{3bnx^{m+1}\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}-\frac{6b^3n^3x^{m+1}\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2(m+1)^2n^2+(m+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(1 + m)^2
*n^2 + 9*b^4*n^4) + (6*b^2*(1 + m)*n^2*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((
1 + m - b*n)*(1 + m + b*n)*((1 + m)^2 - 9*b^2*n^2)) - (3*b*n*x^(1 + m)*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 9*b^2*n^2) + ((1
+ m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/(1 + 2*m + m^2 - 9*b^2*n^2)
```

Rule 5638

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Rubi steps

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = -\frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}+\frac{(1+m)x^{1+m}\sin}{(1+m)}$$
$$= -\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4}+\frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log}{(1+m)^4-10b^2(1+m)^2n^2+9}$$

Mathematica [A]
time = 0.97, size = 292, normalized size = 1.44

$$\frac{1}{4}x^{1+m}\left(-\frac{3\cosh(bn\log(x))(-bn\cosh(a-bn\log(x)+b\log(cx^n))+(1+m)\sinh(a-bn\log(x)+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}-\frac{3\sinh(bn\log(x))((1+m)\cosh(a-bn\log(x)+b\log(cx^n))-bn\sinh(a-bn\log(x)+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}+\frac{\cosh(3bn\log(x))(-3bn\cosh(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(3(a-bn\log(x)+b\log(cx^n))))}{(1+m-3bn)(1+m+3bn)}+\frac{\sinh(3bn\log(x))((1+m)\cosh(3(a-bn\log(x)+b\log(cx^n)))-3bn\sinh(3(a-bn\log(x)+b\log(cx^n))))}{(1+m-3bn)(1+m+3bn)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((-3*Cosh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b
*n)) - (3*Sinh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b
*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (C
osh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)
*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n)
) + (Sinh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] -
3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n))))/4
```

Maple [F]
time = 2.15, size = 0, normalized size = 0.00

$$\int x^m \left(\sinh^3(a+b\ln(c\,x^n))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

Maxima [A]
time = 0.29, size = 138, normalized size = 0.68

$$\frac{c^{3\,b}xe^{(3\,b\log(x^n)+m\log(x)+3\,a)}}{8\,(3\,bn+m+1)}-\frac{3\,c^{b}xe^{(b\log(x^n)+m\log(x)+a)}}{8\,(bn+m+1)}-\frac{3\,xe^{(-b\log(x^n)+m\log(x)-a)}}{8\,(bc^{b}n-c^{b}(m+1))}+\frac{xe^{(-3\,b\log(x^n)+m\log(x)-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b}(m+1))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) - 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) + 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(214) = 428$.
time = 0.39, size = 585, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^
3*cosh(m*log(x)) - 3*(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x)
+ b*log(c) + a)*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m +
1)*x*cosh(m*log(x)) + (m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 9*((b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n
*log(x) + b*log(c) + a)^2 + 3*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (m^3 - 9*(b^2*m + b^2)
*n^2 + 3*m^2 + 3*m + 1)*x*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^
2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (m^3 - 9*(b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) +
3*((b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 -
(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a))*sinh
(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6
*m^2 + 4*m + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & Eq(m, -1)), (-Integral(x**m*sinh(-
a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n))**3, x), Eq(b, (-m - 1)/(3*n)))
, (-Integral(x**m*sinh(-a + m*log(c*x**n)/n + log(c*x**n)/n)**3, x), Eq(b,
(-m - 1)/n)), (Integral(x**m*sinh(a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*
n))**3, x), Eq(b, (m + 1)/(3*n))), (Integral(x**m*sinh(a + m*log(c*x**n)/n
```

```
+ log(c*x**n)/n)**3, x), Eq(b, (m + 1)/n)), (9*b**3*n**3*x*x**m*sinh(a + b*
log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 -
20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*b**3*
n**3*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 2
0*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*m
*n**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 -
20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**2*
m*n**2*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n*
*4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*
m**2 + 4*m + 1) - 7*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**
4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m
**2 + 4*m + 1) + 6*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(
c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**
2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b*m**2*n*x*x**m*sinh(a + b*log(c*
x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**
2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*b*m*n*x*x**
m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2
*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m +
 1) - 3*b*n*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b*
*4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3
 + 6*m**2 + 4*m + 1) + m**3*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4
- 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**
2 + 4*m + 1) + 3*m**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b
**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*
m + 1) + 3*m*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*
n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) +
x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**
2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(214) = 428$.
time = 0.50, size = 3225, normalized size = 15.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
```

3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*m^3*x*x^m*e^^

```
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 ...
```

Mupad [B]
time = 0.85, size = 118, normalized size = 0.58

$$\frac{3\,x\,x^m\,{\mathrm{e}}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,m-8\,b\,n+8\right)}-\frac{x\,x^m\,{\mathrm{e}}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(8\,m-24\,b\,n+8\right)}+\frac{x\,x^m\,{\mathrm{e}}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{8\,m+24\,b\,n+8}-\frac{3\,x\,x^m\,{\mathrm{e}}^{a}\,{\left(c\,x^n\right)}^b}{8\,m+8\,b\,n+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) - (x*x^m*exp(-3*a))/((c*x^n
)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*n
+ 8) - (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

3.273 $\int x^m \sinh^4(a + b\log(cx^n))\,dx$

Optimal. Leaf size=266

$$\frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)}-\frac{24b^3n^3x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)}+$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-24*b^3*n^
3*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+
m)^2-4*b^2*n^2)+12*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))^2/((1+m)^2-16*
b^2*n^2)/((1+m)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(
c*x^n))^3/((1+m)^2-16*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^4/((1+m)^2
-16*b^2*n^2)
```

Rubi [A]
time = 0.10, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5640, 30}

$$\frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}+\frac{12b^2(m+1)n^2x^{m+1}\sinh^2(a+b\log(cx^n))}{\left((m+1)^2-16b^2n^2\right)\left(-4b^2n^2+m^2+2m+1\right)}-\frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}-\frac{24b^3n^3x^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2(m+1)^2n^2+(m+1)^4}+\frac{24b^4n^4x^{m+1}}{(m+1)\left((m+1)^2-16b^2n^2\right)\left((m+1)^2-4b^2n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + (12*b^2*(1 + m)*n^2*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(((1 + m)^2 - 16*b^2*n^2)*(1 + 2*m + m^2
- 4*b^2*n^2)) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^
n]]^3)/((1 + m)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]
^4)/(1 + 2*m + m^2 - 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
```

```
, 0]
```

Rubi steps

$$\int x^m \sinh^4(a+b\log(cx^n))\, dx = -\frac{4bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} + \frac{(1+m)x^{1+m}\sin}{(1+m)^{}}$$

$$= -\frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} + \frac{12b^2(1+m)n^2}{(1+m)^4-}$$

$$= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} - \frac{24b^3n^3x^{1+m}\cosh(a+b}{(1+m)^4-20b}$$

Mathematica [A]
time = 2.40, size = 311, normalized size = 1.17

$$\frac{1}{8}x^{1+m}\left(\frac{3}{1+m}-\frac{4\sinh(2bn\log(x))\left(-2bn\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-2bn)(1+m+2bn)}-\frac{4\cosh(2bn\log(x))\left((1+m)\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)-2bn\sinh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-2bn)(1+m+2bn)}+\frac{\sinh(4bn\log(x))\left(-4bn\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-4bn)(1+m+4bn)}+\frac{\cosh(4bn\log(x))\left((1+m)\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)-4bn\sinh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)\right)}{(1+m-4bn)(1+m+4bn)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) - (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) - (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Maple [F]
time = 4.73, size = 0, normalized size = 0.00

$$\int x^m \left(\sinh^4\left(a+b\ln\left(c\,x^n\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

Maxima [A]
time = 0.29, size = 161, normalized size = 0.61

$$\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+m\log\left(x\right)+4\,a\right)}}{16\,\left(4\,bn+m+1\right)} - \frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+m\log\left(x\right)+2\,a\right)}}{4\,\left(2\,bn+m+1\right)} + \frac{xe^{\left(-2\,b\log\left(x^n\right)+m\log\left(x\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)} - \frac{xe^{\left(-4\,b\log\left(x^n\right)+m\log\left(x\right)-4\,a\right)}}{16\,\left(4\,bc^{4\,b}n-c^{4\,b}(m+1)\right)} + \frac{3\,x^{m+1}}{8\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) - 1/4*c^(2
*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) + 1/4*x*e^(-2*b*log
(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4*b*
log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m + 1
)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(283) = 566$.
time = 0.49, size = 1125, normalized size = 4.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) - 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) - (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 - (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 - 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
```

```
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6884 vs. 2(283) = 566.
time = 0.53, size = 6884, normalized size = 25.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) +
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
```

```
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
- 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*c^(2*b)*m^2*x*x^(
2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2
*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
 + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n...
```

<u>**Mupad**</u> [B]

time = 0.87, size = 136, normalized size = 0.51

$$\frac{3\,x\,x^m}{8\,m+8}-\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{\left(c\,x^n\right)^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}-\frac{x\,x^m\,\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}}{4\,m+8\,b\,n+4}+\frac{x\,x^m\,\mathrm{e}^{-4\,a}}{\left(c\,x^n\right)^{4\,b}\,\left(16\,m-64\,b\,n+16\right)}+\frac{x\,x^m\,\mathrm{e}^{4\,a}\,\left(c\,x^n\right)^{4\,b}}{16\,m+64\,b\,n+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) - (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) -
 (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c*x
^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m + 64
*b*n + 16)
```

3.274 $\int \frac{\sinh(a+b\log(cx^n))}{x} \, dx$

Optimal. Leaf size=18

$$\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2718\}$

$$\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+b\log\left(cx^n\right)\right)}{x} \, dx &= \frac{\text{Subst}\left(\int \sinh(a+bx) \, dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.
time = 0.01, size = 37, normalized size = 2.06

$$\frac{\cosh(a)\cosh\left(b\log\left(cx^n\right)\right)}{bn}+\frac{\sinh(a)\sinh\left(b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] (Cosh[a]*Cosh[b*Log[c*x^n]])/(b*n) + (Sinh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Maple [A]
time = 1.44, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\cosh(a+b\ln(c\,x^n))}{bn}$	19
default	$\frac{\cosh(a+b\ln(c\,x^n))}{bn}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Maxima [A]
time = 0.26, size = 18, normalized size = 1.00

$$\frac{\cosh\left(b\log\left(cx^n\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] cosh(b*log(c*x^n) + a)/(b*n)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 1.06

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] cosh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.26, size = 37, normalized size = 2.06

$$\begin{cases} \log\left(x\right)\sinh\left(a\right) & \text{for } b=0\wedge n=0 \\ \log\left(x\right)\sinh\left(a+b\log\left(c\right)\right) & \text{for } n=0 \\ \log\left(x\right)\sinh\left(a\right) & \text{for } b=0 \\ \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*sinh(a + b*log(c))
, Eq(n, 0)), (log(x)*sinh(a), Eq(b, 0)), (cosh(a + b*log(c*x**n))/(b*n), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.41, size = 40, normalized size = 2.22

$$\frac{\left(c^{2\,b}x^{bn}e^{(2\,a)} + \frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^{b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) + 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

Mupad [B]
time = 0.66, size = 18, normalized size = 1.00

$$\frac{\cosh(a + b\,\ln\left(c\,x^n\right))}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))/x,x)
```

```
[Out] cosh(a + b*log(c*x^n))/(b*n)
```

3.275 $\int \frac{\sinh^2(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=39

$$-\frac{\log(x)}{2}+\frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}$$

```
[Out] -1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2715, 8}

$$\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2bn}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -1/2*Log[x] + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^2(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}-\frac{\text{Subst}\left(\int 1\,dx,x,\log(cx^n)\right)}{2n}\\ &= -\frac{\log(x)}{2}+\frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.92

$$\frac{-2(a+b\log(cx^n))+\sinh(2(a+b\log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] (-2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A]
time = 1.87, size = 45, normalized size = 1.15

method	result	size
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}-\frac{b\ln(c\,x^{n})}{2}-\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}-\frac{b\ln(c\,x^{n})}{2}-\frac{a}{2}}{nb}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))-1/2*b*ln(c*x^n)-1/2*a)
```

Maxima [A]
time = 0.26, size = 49, normalized size = 1.26

$$\frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn}-\frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn}-\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) -
1/2*log(x)
```

Fricas [A]
time = 0.50, size = 40, normalized size = 1.03

$$-\frac{bn\log\left(x\right)-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(x) - cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^{2}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**2/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(35) = 70.
time = 0.42, size = 81, normalized size = 2.08

$$-\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log\left(x\right)-c^{4\,b}x^{2\,bn}e^{(4\,a)}-\frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] -1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) - c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)
*x^(2*b*n)*e^(2*a) - 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

Mupad [B]
time = 0.69, size = 32, normalized size = 0.82

$$\frac{\sinh(2\,a+2\,b\,\ln\left(c\,x^n\right))}{4\,b\,n}-\frac{\ln\left(x^n\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^2/x,x)
```

```
[Out] sinh(2*a + 2*b*log(c*x^n))/(4*b*n) - log(x^n)/(2*n)
```

3.276 $\int \frac{\sinh^3(a+b\log(cx^n))}{x} dx$

Optimal. Leaf size=43

$$-\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[Out] -cosh(a+b*ln(c*x^n))/b/n+1/3*cosh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2713}

$$\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}-\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Cosh[a + b*Log[c*x^n]]/(b*n)) + Cosh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^3(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\ &= -\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn}+\frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.05

$$-\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)}{12bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] (-3*Cosh[a + b*Log[c*x^n]])/(4*b*n) + Cosh[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{3}\left(a +b \ln \left(c \,x^{n}\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^3/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.
time = 0.26, size = 86, normalized size = 2.00

$$\frac{e^{\left(3\,b\log\left(cx^{n}\right)+3\,a\right)}}{24\,bn} - \frac{3\,e^{\left(b\log\left(cx^{n}\right)+a\right)}}{8\,bn} - \frac{3\,e^{\left(-b\log\left(cx^{n}\right)-a\right)}}{8\,bn} + \frac{e^{\left(-3\,b\log\left(cx^{n}\right)-3\,a\right)}}{24\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) - 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) + 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Fricas [A]
time = 0.39, size = 65, normalized size = 1.51

$$\frac{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-9\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{12\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^2 - 9*cosh(b*n*log(x) + b*log(c) + a))/(b*
n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.
time = 1.78, size = 76, normalized size = 1.77

$$\begin{cases} \log{\left(x \right)} \sinh^{3}{\left(a \right)} & \text{for}\: b = 0 \wedge n = 0 \\ \log{\left(x \right)} \sinh^{3}{\left(a + b \log{\left(c \right)} \right)} & \text{for}\: n = 0 \\ \log{\left(x \right)} \sinh^{3}{\left(a \right)} & \text{for}\: b = 0 \\ \frac{\sinh^{2}{\left(a + b \log{\left(c x^{n} \right)} \right)} \cosh{\left(a + b \log{\left(c x^{n} \right)} \right)}}{b n} - \frac{2 \cosh^{3}{\left(a + b \log{\left(c x^{n} \right)} \right)}}{3 b n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & Eq(n, 0)), (log(x)*sinh(a + b*log(
c))**3, Eq(n, 0)), (log(x)*sinh(a)**3, Eq(b, 0)), (sinh(a + b*log(c*x**n))*
*2*cosh(a + b*log(c*x**n))/(b*n) - 2*cosh(a + b*log(c*x**n))**3/(3*b*n), Tr
ue))
```

Giac [A]
time = 0.42, size = 81, normalized size = 1.88

$$\frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} - 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) - 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) - 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

Mupad [B]
time = 0.72, size = 37, normalized size = 0.86

$$-\frac{3\cosh(a + b\,\ln(c\,x^n)) - \cosh(a + b\,\ln(c\,x^n))^3}{3\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] -(3*cosh(a + b*log(c*x^n)) - cosh(a + b*log(c*x^n))^3)/(3*b*n)
```

3.277 $\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=73

$$\frac{3\log(x)}{8}-\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{4bn}$$

```
[Out] 3/8*ln(x)-3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(c
*x^n))*sinh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2715, 8}

$$\frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 - (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sinh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\frac{\text{Subst}\left(\int \sinh^4(a+bx)\,dx,x,\log\left(cx^n\right)\right)}{n}$$
$$=\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{4bn}-\frac{3\text{Subst}\left(\int \sinh^2(a+bx)\,dx,x,\text{l}\right.}{4n}$$
$$=-\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(\right.}{4bn}$$
$$=\frac{3\log(x)}{8}-\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{8bn}+\frac{\cosh\left(a+b\log\left(cx^n\right.\right.}{}$$

Mathematica [**A**]
time = 0.04, size = 51, normalized size = 0.70

$$\frac{12(a+b\log(cx^n))-8\sinh(2(a+b\log(cx^n)))+\sinh(4(a+b\log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) - 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(a+b\ln(c\,x^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^4/x,x)
```

Maxima [**A**]
time = 0.26, size = 93, normalized size = 1.27

$$\frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn}-\frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn}+\frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn}-\frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn}+\frac{3}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) - 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) +
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

Fricas [**A**]
time = 0.46, size = 84, normalized size = 1.15

$$\frac{\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^3+3\,bn\log(x)+\left(\cosh(bn\log(x)+b\log(c)+a)^3-4\,\cosh(bn\log(x)+b\log(c)+a)\right)\sinh(bn\log(x)+b\log(c)+a)}{8\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(a + b\log(cx^n))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**4/x, x)
```

Giac [A]
time = 0.44, size = 114, normalized size = 1.56

$$\frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log\left(x\right) + c^{8\,b}x^{4\,bn}e^{(8\,a)} - 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) - 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) - 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

Mupad [B]
time = 0.78, size = 51, normalized size = 0.70

$$\frac{3\,\ln\left(x^n\right)}{8\,n} - \frac{\frac{\sinh\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)}{4} - \frac{\sinh\left(4\,a+4\,b\,\ln\left(c\,x^n\right)\right)}{32}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) - (sinh(2*a + 2*b*log(c*x^n))/4 - sinh(4*a + 4*b*log(c*x
^n))/32)/(b*n)
```

3.278 $\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=65

$$\frac{\cosh(a+b\log(cx^n))}{bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh^5(a+b\log(cx^n))}{5bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n-2/3*cosh(a+b*ln(c*x^n))^3/b/n+1/5*cosh(a+b*ln(c*x^n
))^5/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2713}

$$\frac{\cosh^5(a+b\log(cx^n))}{5bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh(a+b\log(cx^n))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n) - (2*Cosh[a + b*Log[c*x^n]]^3)/(3*b*n) + Cosh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^5(a+b\log(cx^n))}{x}\,dx &= \frac{\text{Subst}\left(\int \sinh^5(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\ &= \frac{\text{Subst}\left(\int (1-2x^2+x^4)\,dx,x,\cosh(a+b\log(cx^n))\right)}{bn}\\ &= \frac{\cosh(a+b\log(cx^n))}{bn}-\frac{2\cosh^3(a+b\log(cx^n))}{3bn}+\frac{\cosh^5(a+b\log(cx^n))}{5bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.05

$$\frac{5\cosh(a+b\log(cx^n))}{8bn}-\frac{5\cosh(3(a+b\log(cx^n)))}{48bn}+\frac{\cosh(5(a+b\log(cx^n)))}{80bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (5*Cosh[a + b*Log[c*x^n]])/(8*b*n) - (5*Cosh[3*(a + b*Log[c*x^n])])/(48*b*n
) + Cosh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^5\left(a + b\ln\left(c\,x^n\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^5/x,x)
```

```
[Out] int(sinh(a+b*ln(c*x^n))^5/x,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.
time = 0.26, size = 130, normalized size = 2.00

$$\frac{e^{(5\,b\log(cx^n)+5\,a)}}{160\,bn} - \frac{5\,e^{(3\,b\log(cx^n)+3\,a)}}{96\,bn} + \frac{5\,e^{(b\log(cx^n)+a)}}{16\,bn} + \frac{5\,e^{(-b\log(cx^n)-a)}}{16\,bn} - \frac{5\,e^{(-3\,b\log(cx^n)-3\,a)}}{96\,bn} + \frac{e^{(-5\,b\log(cx^n)-5\,a)}}{160\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) - 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) + 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) + 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.
time = 0.39, size = 130, normalized size = 2.00

$$\frac{3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{5}+15\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-25\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+15\left(2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-5\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+150\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{240\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 15*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a)^4 - 25*cosh(b*n*log(x) + b*log(c) + a)
^3 + 15*(2*cosh(b*n*log(x) + b*log(c) + a)^3 - 5*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + 150*cosh(b*n*log(x) + b*log(c) +
 a))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.
time = 10.68, size = 110, normalized size = 1.69

$$\begin{cases} \log(x)\sinh^5(a) & \text{for } b = 0 \wedge n = 0 \\ \log(x)\sinh^5(a + b\log(c)) & \text{for } n = 0 \\ \log(x)\sinh^5(a) & \text{for } b = 0 \\ \frac{\sinh^4(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{bn} - \frac{4\sinh^2(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{3bn} + \frac{8\cosh^5(a+b\log(cx^n))}{15bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**5, Eq(b, 0) & Eq(n, 0)), (log(x)*sinh(a + b*log(
c))**5, Eq(n, 0)), (log(x)*sinh(a)**5, Eq(b, 0)), (sinh(a + b*log(c*x**n))*
*4*cosh(a + b*log(c*x**n))/(b*n) - 4*sinh(a + b*log(c*x**n))**2*cosh(a + b*
log(c*x**n))**3/(3*b*n) + 8*cosh(a + b*log(c*x**n))**5/(15*b*n), True))
```

Giac [A]
time = 0.43, size = 115, normalized size = 1.77

$$\frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)} - 25\,c^{8\,b}x^{3\,bn}e^{(8\,a)} + 150\,c^{6\,b}x^{bn}e^{(6\,a)} + \frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 25\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) - 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) + (150*c^(4*b)*x^(4*b*n)*e^(4*a) - 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

Mupad [B]
time = 0.84, size = 49, normalized size = 0.75

$$\frac{\frac{\cosh(a+b\,\ln(c\,x^n))^5}{5} - \frac{2\cosh(a+b\,\ln(c\,x^n))^3}{3} + \cosh(a + b\,\ln(c\,x^n))}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] (cosh(a + b*log(c*x^n)) - (2*cosh(a + b*log(c*x^n))^3)/3 + cosh(a + b*log(c
*x^n))^5/5)/(b*n)
```

3.279 $\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^(3/2)/b/n-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1
/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2715, 2721, 2719}

$$\frac{2\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{5bn}+\frac{6i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \sinh^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn} - \frac{3\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\right.}{5n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn} - \frac{\left(3\sqrt{\sinh(a+b\log(cx^n))}\right.}{5n\sqrt{}}$$

$$= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{5bn\sqrt{i\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log}{}$$

Mathematica [A]
time = 0.07, size = 96, normalized size = 0.86

$$\frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh(a+b\log(cx^n))\sinh(2(a+b\log(cx^n)))}{5bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]] + Sinh[a + b*Log[c*x^n]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n*Sq
rt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A]
time = 12.60, size = 227, normalized size = 2.05

method	result
derivativedivides	$-\frac{\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n}}{5}}{}$
default	$-\frac{\frac{6\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n}}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 331, normalized size = 2.98

$$\frac{12\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+6\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+12\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+6\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-1\right)\sqrt{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{10\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log
(x) + b*log(c) + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + (cosh(b*n
*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(
x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh(b*n*log(
x) + b*log(c) + a)^2 + 2)*sinh(b*n*log(x) + b*log(c) + a)^2 + 12*cosh(b*n*l
og(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n
*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*sqrt(sinh(b*n
*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*
n*log(x) + b*log(c) + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,\ln\left(c\,x^n\right))^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(5/2)/x,x)
[Out] int(sinh(a + b*log(c*x^n))^(5/2)/x, x)
```

3.280 $\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal. Leaf size=111

$$\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}+\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1
/2*I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*
(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2/3*cosh(a+b*ln
(c*x^n))*sinh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2715, 2721, 2720}

$$\frac{2\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\text{Subst}\left(\int \sinh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\right.}{3n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\sqrt{i\sinh(a+b\log(cx^n))}}{3n\sqrt{}}$$

$$= \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\,\mathrm{l}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 114, normalized size = 1.03

$$\frac{-2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+b\log(cx^n)))+\sinh(2(a+b\log(cx^n)))\right)\sqrt{1-\cosh(2(a+b\log(cx^n)))-\sinh(2(a+b\log(cx^n)))}+\sinh(2(a+b\log(cx^n)))}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a
 + b*Log[c*x^n])]]*Sqrt[1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[
c*x^n])]] + Sinh[2*(a + b*Log[c*x^n])])/(3*b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]
)
```

Maple [A]
time = 7.13, size = 143, normalized size = 1.29

method	result
derivativedivides	$-\frac{\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n}}{3}}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a}}$
default	$-\frac{\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n}}{3}}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n)))/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 171, normalized size = 1.54

$$-\frac{2\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\mathrm{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))-\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1\right)\sqrt{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n
*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)))/(b*n*
cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a + b \ln(c x^n))^{3/2}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(3/2)/x,x)
```

```
[Out] int(sinh(a + b*log(c*x^n))^(3/2)/x, x)
```

3.281 $\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sin
h(a+b*ln(c*x^n))^(1/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 2719}

$$-\frac{2i\sqrt{\sinh(a+b\log(cx^n))}\,E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c
*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\sqrt{\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n\sqrt{i\sinh(a+b\log(cx^n))}}$$
$$= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Mathematica [**A**]
time = 0.03, size = 68, normalized size = 0.94

$$\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+b\log(cx^n))\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [**A**]
time = 6.96, size = 146, normalized size = 2.03

method	result
derivativedivides	$\frac{\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\right.}}{n\cosh(a+b\ln(c\,x}$
default	$\frac{\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\right.}}{n\cosh(a+b\ln(c\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 58, normalized size = 0.81

$$-\frac{2\left(\sqrt{2}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+\sqrt{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x)
 + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(sinh(b*n*log(x)
 + b*log(c) + a)))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sinh(a + b*log(c*x**n)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*log(c*x^n))^(1/2)/x,x)
[Out] int(sinh(a + b*log(c*x^n))^(1/2)/x, x)
```

3.282 $\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx$

Optimal. Leaf size=72

$$-\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*
sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2721, 2720}

$$-\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,F\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log
[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n}$$

$$= \frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx,x,\log\left(cx^n\right)\right)}{n\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

$$= -\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [**A**]
time = 0.03, size = 66, normalized size = 0.92

$$-\frac{2F\left(\frac{1}{4}\left(-2ia+\pi-2ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Maple [**A**]
time = 6.57, size = 120, normalized size = 1.67

method	result
derivativedivides	$\frac{i\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^n\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^n\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\right.}}{n\cosh\left(a+b\ln\left(c\,x^n\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x\right.\right.}}$
default	$\frac{i\sqrt{-i\left(\sinh\left(a+b\ln\left(c\,x^n\right)\right)+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-\sinh\left(a+b\ln\left(c\,x^n\right)\right)+i\right)}\,\sqrt{i\sinh\left(a+b\right.}}{n\cosh\left(a+b\ln\left(c\,x^n\right)\right)\sqrt{\sinh\left(a+b\ln\left(c\,x\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 39, normalized size = 0.54

$$\frac{2\sqrt{2}\,\text{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(
b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(sinh(a + b*log(c*x**n)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{\sinh\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)), x)
```

3.283 $\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))} \, dx$

Optimal. Leaf size=107

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1/2)
/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2716, 2721, 2719}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}-\frac{2i\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\,E\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(b
*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{\sqrt{\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \sqrt{i\sinh(a}\right.}{n\sqrt{i\sinh(a+b\log(cx}}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.75

$$-\frac{2\left(\cosh(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n]
)/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A]
time = 8.80, size = 212, normalized size = 1.98

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}}{}$
default	$\frac{2\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 246, normalized size = 2.30

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-\sqrt{2}\right){\rm weierstrassZeta}(4,0,{\rm weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)))+2\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}\right)\sqrt{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
 b*log(c) + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + 2*(c
osh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b
*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(sinh(b*
n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b
*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*sinh(a + b*log(c*x**n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sinh\left(a + b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)), x)
```

3.284 $\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal. Leaf size=111

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(3/2)-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*sinh(a+b*ln(c*x^n)
))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2716, 2721, 2720}

$$-\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}+\frac{2i\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,F\left(\frac{1}{2}\left(ia+ib\log\left(cx^n\right)-\frac{\pi}{2}\right)\middle|2\right)}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\sqrt{i\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(}}\right.}{3n\sqrt{\sinh(a+b\log(cx}}$$

$$= -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2iF\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{i\sinh(a}}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.09, size = 122, normalized size = 1.10

$$-\frac{2\left(\cosh(a+b\log(cx^n)) + {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\cosh(2(a+b\log(cx^n))) + \sinh(2(a+b\log(cx^n)))\right)\sinh(a+b\log(cx^n))\sqrt{1-\cosh(2(a+b\log(cx^n))) - \sinh(2(a+b\log(cx^n)))}\right)}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a +
b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]*Sinh[a + b*Log[c*x^n]]*Sqrt[1
- Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Sinh[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [A]
time = 7.41, size = 144, normalized size = 1.30

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n)}}{3n\sinh(a+b\ln(cx^n))^{\frac{3}{2}}\cosh}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n)}}{3n\sinh(a+b\ln(cx^n))^{\frac{3}{2}}\cosh}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sinh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 504, normalized size = 4.54

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\sqrt{2}\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-\sqrt{2}\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-2\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+4\left(\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-\sqrt{2}\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sqrt{2}\right){\rm weierstrassPInverse}\left(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)+2\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+\left(3\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+1\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sqrt{\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}\right)}{3\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}+4bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}+bn\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-2bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\left(3bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}-bn\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+bn+4\left(bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{3}-bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")

[Out] -2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 - sqrt(2))*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 - sqrt(2)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a) + cosh(b*n*log(x) + b*log(c) + a))*sqrt(sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)), x)
```

3.285 $\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal. Leaf size=209

$$-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{5e^{-3a}x(cx$$

```
[Out] -1/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)-5/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(2*
a)/((c*x^n)^(4/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*sinh(a+2*ln(c*x^
n)/n)^(5/2)/(1-1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsc(exp(a)*(c*x^n)^(2/n
))*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1-1/exp(2*a)/((c*x
^n)^(4/n)))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5636, 5644, 360, 356, 352, 248, 283, 222}

$$-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^2}-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -1/4*(x*Sinh[a + (2*Log[c*x^n])/n]^(5/2)) - (5*x*Sinh[a + (2*Log[c*x^n])/n]
^(5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*
Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5
*x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(
c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 356

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 360

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1-e^{-2a}x^{-4/n}\right)^{5/2}dx\right.}{n\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{\left(5x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subs}}{2n\left(1-e^{-2a}\right.}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)\right.}{}$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}+$$

$$= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.21, size = 86, normalized size = 0.41

$$\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(-1+e^{2a}(cx^n)^{4/n}\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-e^{2a}(cx^n)^{4/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(-1 + E^(2*a)*(c*x^n)^(4/n))*Hypergeometric2F1[2,
7/2, 9/2, 1 - E^(2*a)*(c*x^n)^(4/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/14
```

Maple [F]
time = 2.21, size = 0, normalized size = 0.00

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Fricas [A]
time = 0.50, size = 162, normalized size = 0.78

$$\frac{\left(15\,\sqrt{2}\,x^{3}\arctan\left(\sqrt{2}\,\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log\left(c\right))}{n}\right)}-1}{x^{2}}}\right)e^{\left(\frac{3\,(an+2\,\log\left(c\right))}{2\,n}\right)}+2\,\sqrt{\frac{1}{2}}\left(2\,x^{8}e^{\left(\frac{4\,(an+2\,\log\left(c\right))}{n}\right)}-14\,x^{4}e^{\left(\frac{2\,(an+2\,\log\left(c\right))}{n}\right)}-3\right)\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log\left(c\right))}{n}\right)}-1}{x^{2}}}\,e^{\left(-\frac{an+2\,\log\left(c\right)}{2\,n}\right)}\right)e^{\left(-\frac{2\,(an+2\,\log\left(c\right))}{n}\right)}}{96\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log
(c))/n) - 1)/x^2))*e^(3/2*(a*n + 2*log(c))/n) + 2*sqrt(1/2)*(2*x^8*e^(4*(a*
n + 2*log(c))/n) - 14*x^4*e^(2*(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c
))/n)/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(5/2),x)
[Out] int(sinh(a + (2*log(c*x^n))/n)^(5/2), x)
```

3.286 $\int \sqrt{\sinh \left(a+\frac{2 \log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{e^{-a}x(cx^n)^{-2/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

```
[Out] 1/2*x*sinh(a+2*ln(c*x^n)/n)^(1/2)+1/2*x*arccsc(exp(a)*(c*x^n)^(2/n))*sinh(a
+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5636, 5644, 352, 248, 283, 222}

$$\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{e^{-a}x(cx^n)^{-2/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2 + (x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sqrt[
Sinh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a)*(c*x^
n)^(4/n))])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sinh\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1-e^{-2a}x^{-4/}}\right.}{n\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= \frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\right)\text{Subst}\left(\int\sqrt{1-\frac{e^{-2a}}{x^2}}\,dx,x,(c\right.}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= -\frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\right)\text{Subst}\left(\int\frac{\sqrt{1-e^{-2a}x^2}}{x^2}\,dx,x\right.}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)} + \frac{\left(e^{-2a}x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right.}\right.}{2\sqrt{1-e}}$$

$$= \frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)} + \frac{e^{-a}x(cx^n)^{-2/n}\sin^{-1}\left(e^{-a}(cx^n)^{-2/n}\right)\sqrt{\text{si}}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/}}}$$

<u>**Mathematica**</u> **[A]**
time = 0.18, size = 74, normalized size = 0.72

$$\frac{1}{2}x\left(1-\frac{\text{ArcTan}\left(\sqrt{-1+e^{2a}\left(cx^n\right)^{4/n}}\right)}{\sqrt{-1+e^{2a}\left(cx^n\right)^{4/n}}}\right)\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*(1 - ArcTan[Sqrt[-1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[-1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2
```

Maple [F]
time = 2.30, size = 0, normalized size = 0.00

$$\int \sqrt{\sinh}\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
[Out] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sinh(a + 2*log(c*x^n)/n)), x)
```

Fricas [A]
time = 0.51, size = 117, normalized size = 1.14

$$\frac{1}{4}\left(2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^{2}}}\,e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^{2}}}\right)e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\right)e^{\left(-\frac{an+2\,\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
[Out] 1/4*(2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) - sqrt(2)*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n +
2*log(c))/n) - 1)/x^2))*e^(1/2*(a*n + 2*log(c))/n))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(1/2),x)
[Out] Integral(sqrt(sinh(a + 2*log(c*x**n)/n)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(1/2),x)
```

```
[Out] int(sinh(a + (2*log(c*x^n))/n)^(1/2), x)
```

3.287 $\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/2*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5636, 5644, 270}

$$-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] -1/2*(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/Sinh[a + (2*Log[c*x^n])/n]^(3/2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{2/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{3/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 1.42

$$\frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])
```

Maple [F]
time = 2.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

```
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Fricas [A]
time = 0.45, size = 68, normalized size = 1.58

$$-\frac{2\sqrt{\frac{1}{2}}\,x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n +
2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(3/2),x)
```

```
[Out] Integral(sinh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Giac [A]
time = 0.60, size = 41, normalized size = 0.95

$$-\frac{\sqrt{2}}{\sqrt{c^{\frac{4}{n}}e^{(3\,a)}-\frac{e^a}{x^4}}\,c^{\left(\frac{1}{n}\right)}x^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)/(sqrt(c^(4/n)*e^(3*a) - e^a/x^4)*c^(1/n)*x^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2),x)
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2), x)
```

3.288 $\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/6*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(7/2)+1/15*x*(1
-1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/sinh(a+2*ln(c*x^n)/n)
^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5636, 5644, 277, 270}

$$\frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -1/6*(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/Sinh[a + (2*Log[c*x^n])/n]^(7/2) +
 (x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Sinh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left(x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,dx,x,cx^n\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{\left(2e^{-2a}x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\operatorname{Subst}\left(\int\right.}{3n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A]
time = 0.15, size = 121, normalized size = 1.17

$$\frac{\left((-2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-4\log(x)+\frac{4\log(cx^n)}{n}\right)+\sinh\left(2a-4\log(x)+\frac{4\log(cx^n)}{n}\right)\right)}{15x^5\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((-2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Sinh[a + (2*Log[c*x^n])/n
]^(5/2))
```

Maple [F]
time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh \left(a+\frac{2 \ln \left(c \,x^{n}\right)}{n}\right)^{\frac{7}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Fricas [A]
time = 0.44, size = 128, normalized size = 1.24

$$-\frac{8 \, \sqrt{\frac{1}{2}} {\left(5 \, x^{5} e^{\left(\frac{2 \, {\left(a n + 2 \, \log\left(c\right)\right)}}{n}\right)} - 2 \, x\right)} \sqrt{\frac{x^{4} e^{\left(\frac{2 \, {\left(a n + 2 \, \log\left(c\right)\right)}}{n}\right)} - 1}{x^{2}}} e^{\left(-\frac{a n + 2 \, \log\left(c\right)}{2 \, n}\right)}}{15 \, {\left(x^{12} e^{\left(\frac{6 \, {\left(a n + 2 \, \log\left(c\right)\right)}}{n}\right)} - 3 \, x^{8} e^{\left(\frac{4 \, {\left(a n + 2 \, \log\left(c\right)\right)}}{n}\right)} + 3 \, x^{4} e^{\left(\frac{2 \, {\left(a n + 2 \, \log\left(c\right)\right)}}{n}\right)} - 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) - 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) - 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
- 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(7/2),x)
```

[Out] Timed out

Giac [A]
time = 0.64, size = 81, normalized size = 0.79

$$-\frac{4\sqrt{2}\,c^{\frac{7}{n}}\left(\frac{5\,e^{a}}{c^{\frac{4}{n}}\,\mathrm{sgn}(x)}-\frac{2\,e^{(-a)}}{c^{\frac{8}{n}}\,x^{4}\mathrm{sgn}(x)}\right)e^{(3\,a)}}{15\left(c^{\frac{4}{n}}e^{(3\,a)}-\frac{e^{a}}{x^{4}}\right)^{\frac{5}{2}}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

```
[Out] -4/15*sqrt(2)*c^(7/n)*(5*e^a/(c^(4/n)*sgn(x)) - 2*e^(-a)/(c^(8/n)*x^4*sgn(x
)))*e^(3*a)/((c^(4/n)*e^(3*a) - e^a/x^4)^(5/2)*x^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(a+\frac{2\,\ln(c\,x^{n})}{n}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2),x)
```

```
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2), x)
```

3.289 $\int \sinh\left(\frac{a}{c+dx}\right)\,dx$

Optimal. Leaf size=36

$$-\frac{a\mathrm{Chi}\left(\frac{a}{c+dx}\right)}{d}+\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

[Out] -a*Chi(a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))/d

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5418, 5410, 3378, 3382}

$$\frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}-\frac{a\mathrm{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a/(c + d*x)],x]

[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sinh\left(\frac{a}{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \sinh\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{\sinh(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{\cosh(ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 1.00

$$-\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)],x]
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Maple [A]
time = 0.89, size = 38, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\text{hyperbolicCosineIntegral}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
default	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\text{hyperbolicCosineIntegral}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
risch	$-\frac{e^{-\frac{a}{dx+c}}x}{2}-\frac{e^{-\frac{a}{dx+c}}c}{2d}+\frac{a\,\text{expIntegral}\left(1,\frac{a}{dx+c}\right)}{2d}+\frac{e^{\frac{a}{dx+c}}x}{2}+\frac{e^{\frac{a}{dx+c}}c}{2d}+\frac{a\,\text{expIntegral}\left(1,-\frac{a}{dx+c}\right)}{2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*a*(-(d*x+c)/a*sinh(a/(d*x+c))+Chi(a/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*a*d
*integrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*x*e^(a/(d
*x + c)) - 1/2*x*e^(-a/(d*x + c))
```

Fricas [A]
time = 0.44, size = 48, normalized size = 1.33

$$-\frac{a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*Ei(a/(d*x + c)) + a*Ei(-a/(d*x + c)) - 2*(d*x + c)*sinh(a/(d*x + c)
))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(\frac{a}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x)
```

```
[Out] Integral(sinh(a/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(36) = 72.
time = 0.42, size = 102, normalized size = 2.83

$$-\frac{\left(\frac{a^3\mathrm{Ei}\left(\frac{a}{dx+c}\right)}{dx+c}-a^2e^{\left(\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d}-\frac{\left(\frac{a^3\mathrm{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c}+a^2e^{\left(-\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*Ei(a/(d*x + c))/(d*x + c) - a^2*e^(a/(d*x + c)))*(d*x + c)/(a^2*d
) - 1/2*(a^3*Ei(-a/(d*x + c))/(d*x + c) + a^2*e^(-a/(d*x + c)))*(d*x + c)/(
a^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh\left(\frac{a}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x)),x)
[Out] int(sinh(a/(c + d*x)), x)
```

3.290 $\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx$

Optimal. Leaf size=39

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[Out] -a*Shi(2*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5418, 5410, 3394, 12, 3379}

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d}-\frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2)/d - (a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \sinh^2\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{\sinh^2(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} + \frac{(2ia)\text{Subst}\left(\int \frac{i\sinh(2ax)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{\sinh(2ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.95

$$\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right) - a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^2,x]
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2 - a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Maple [A]
time = 1.34, size = 50, normalized size = 1.28

method	result	si
derivativedivides	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\text{hyperbolicSineIntegral}\left(\frac{2a}{dx+c}\right)\right)}{d}$	50
default	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\text{hyperbolicSineIntegral}\left(\frac{2a}{dx+c}\right)\right)}{d}$	50

risch	$-\frac{x}{2}+\frac{e^{-\frac{2a}{dx+c}}x}{4}+\frac{e^{-\frac{2a}{dx+c}}c}{4d}-\frac{a\,\text{expIntegral}\left(1,\frac{2a}{dx+c}\right)}{2d}+\frac{e^{\frac{2a}{dx+c}}x}{4}+\frac{e^{\frac{2a}{dx+c}}c}{4d}+\frac{a\,\text{expIntegral}\left(1,-\frac{2a}{dx+c}\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(1/2/a*(d*x+c)-1/2*(d*x+c)/a*cosh(2*a/(d*x+c))+Shi(2*a/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*a
*d*integrate(x*e^(-2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*x*e^(
2*a/(d*x + c)) + 1/4*x*e^(-2*a/(d*x + c)) - 1/2*x
```

Fricas [A]
time = 0.43, size = 73, normalized size = 1.87

$$\frac{(dx+c)\cosh\left(\frac{a}{dx+c}\right)^2+(dx+c)\sinh\left(\frac{a}{dx+c}\right)^2-dx-a\text{Ei}\left(\frac{2a}{dx+c}\right)+a\text{Ei}\left(-\frac{2a}{dx+c}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((d*x + c)*cosh(a/(d*x + c))^2 + (d*x + c)*sinh(a/(d*x + c))^2 - d*x -
a*Ei(2*a/(d*x + c)) + a*Ei(-2*a/(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh^2\left(\frac{a}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**2,x)
```

```
[Out] Integral(sinh(a/(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.
time = 0.44, size = 97, normalized size = 2.49

$$-\frac{\left(\frac{2a^3\text{Ei}\left(\frac{2a}{dx+c}\right)}{dx+c}-\frac{2a^3\text{Ei}\left(-\frac{2a}{dx+c}\right)}{dx+c}-a^2e^{\left(\frac{2a}{dx+c}\right)}-a^2e^{\left(-\frac{2a}{dx+c}\right)}+2a^2\right)(dx+c)}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3*Ei(2*a/(d*x + c))/(d*x + c) - 2*a^3*Ei(-2*a/(d*x + c))/(d*x + c
) - a^2*e^(2*a/(d*x + c)) - a^2*e^(-2*a/(d*x + c)) + 2*a^2)*(d*x + c)/(a^2*
d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh\left(\frac{a}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x))^2,x)
```

```
[Out] int(sinh(a/(c + d*x))^2, x)
```

3.291 $\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx$

Optimal. Leaf size=59

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] 3/4*a*Chi(a/(d*x+c))/d-3/4*a*Chi(3*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5418, 5410, 3394, 3382}

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)])/(4*d) - (3*a*CoshIntegral[(3*a)/(c + d*x)])
/(4*d) + ((c + d*x)*Sinh[a/(c + d*x)]^3)/d
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
```

```
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \frac{\text{Subst}\left(\int \sinh^3\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\text{Subst}\left(\int \left(\frac{\cosh(ax)}{4x} - \frac{\cosh(3ax)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d}$$
$$= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\text{Subst}\left(\int \frac{\cosh(ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d} - \frac{(3a)\text{Subst}\left(\int \frac{\cosh(3ax)}{x}\,d\right.}{4d}$$
$$= \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.92

$$\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right) - 3a\text{Chi}\left(\frac{3a}{c+dx}\right) + 4(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a/(c + d*x)]^3,x]
[Out] (3*a*CoshIntegral[a/(c + d*x)] - 3*a*CoshIntegral[(3*a)/(c + d*x)] + 4*(c +
 d*x)*Sinh[a/(c + d*x)]^3)/(4*d)
```

Maple [A]
time = 1.30, size = 74, normalized size = 1.25

method	result
derivativedivides	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a} - \frac{3\,\text{hyperbolicCosineIntegral}\left(\frac{a}{dx+c}\right)}{4} - \frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a} + \frac{3\,\text{hyperbolicCosineIntegral}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
default	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a} - \frac{3\,\text{hyperbolicCosineIntegral}\left(\frac{a}{dx+c}\right)}{4} - \frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a} + \frac{3\,\text{hyperbolicCosineIntegral}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
risch	$-\frac{e^{-\frac{3a}{dx+c}}x}{8} - \frac{e^{-\frac{3a}{dx+c}}c}{8d} + \frac{3a\,\text{expIntegral}\left(1, \frac{3a}{dx+c}\right)}{8d} + \frac{3\,e^{-\frac{a}{dx+c}}x}{8} + \frac{3\,e^{-\frac{a}{dx+c}}c}{8d} - \frac{3a\,\text{expIntegral}\left(1, \frac{a}{dx+c}\right)}{8d} + \text{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(3/4*(d*x+c)/a*sinh(a/(d*x+c))-3/4*Chi(a/(d*x+c))-1/4*(d*x+c)/a*sinh
(3*a/(d*x+c))+3/4*Chi(3*a/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*d*integrate(x*e^(3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a
*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a*d*inte
grate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 3/8*a*d*integrate(
x*e^(-3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/8*x*e^(3*a/(d*x + c)
) - 3/8*x*e^(a/(d*x + c)) + 3/8*x*e^(-a/(d*x + c)) - 1/8*x*e^(-3*a/(d*x + c
))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.
time = 0.50, size = 118, normalized size = 2.00

$$\frac{2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)^{3}-3\,a\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)+3\,a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+3\,a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-3\,a\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)+6\left((dx+c)\cosh\left(\frac{a}{dx+c}\right)^{2}-dx-c\right)\sinh\left(\frac{a}{dx+c}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(d*x + c)*sinh(a/(d*x + c))^3 - 3*a*Ei(3*a/(d*x + c)) + 3*a*Ei(a/(d*
x + c)) + 3*a*Ei(-a/(d*x + c)) - 3*a*Ei(-3*a/(d*x + c)) + 6*((d*x + c)*cosh
(a/(d*x + c))^2 - d*x - c)*sinh(a/(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(55) = 110.
time = 0.44, size = 167, normalized size = 2.83

$$-\frac{\left(\frac{3\,a^{3}\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)}{dx+c}-\frac{3\,a^{3}\mathrm{Ei}\left(\frac{a}{dx+c}\right)}{dx+c}-\frac{3\,a^{3}\mathrm{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c}+\frac{3\,a^{3}\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)}{dx+c}-a^{2}e^{\left(\frac{3\,a}{dx+c}\right)}+3\,a^{2}e^{\left(\frac{a}{dx+c}\right)}-3\,a^{2}e^{\left(-\frac{a}{dx+c}\right)}+a^{2}e^{\left(-\frac{3\,a}{dx+c}\right)}\right)(dx+c)}{8\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3*Ei(3*a/(d*x + c))/(d*x + c) - 3*a^3*Ei(a/(d*x + c))/(d*x + c) -
 3*a^3*Ei(-a/(d*x + c))/(d*x + c) + 3*a^3*Ei(-3*a/(d*x + c))/(d*x + c) - a^
2*e^(3*a/(d*x + c)) + 3*a^2*e^(a/(d*x + c)) - 3*a^2*e^(-a/(d*x + c)) + a^2*
e^(-3*a/(d*x + c)))*(d*x + c)/(a^2*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sinh\left(\frac{a}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a/(c + d*x))^3,x)
```

```
[Out] int(sinh(a/(c + d*x))^3, x)
```

3.292 $\int \sinh\left(\frac{bx}{c+dx}\right) dx$

Optimal. Leaf size=74

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}-\frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

```
[Out] b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2-b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2+(
d*x+c)*sinh(b*x/(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {5726, 3378, 3384, 3379, 3382}

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}-\frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/d^2 + ((c + d*x)*Sinh[(b*
x)/(c + d*x)])/d - (b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh\left(\frac{bx}{c+dx}\right) dx &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{\left(bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left(bc\sinh\left(\frac{b}{d}\right)\right)}{} \\
&= \frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 70, normalized size = 0.95

$$\frac{bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right) + d(c+dx)\sinh\left(\frac{bx}{c+dx}\right) - bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + d*(c + d*x)*Sinh[(b*x)/(
c + d*x)] - b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Maple [A]
time = 1.14, size = 113, normalized size = 1.53

method	result	size

risch	$-\frac{e^{-\frac{bx}{dx+c}}(dx+c)}{2d}-\frac{bc\,e^{-\frac{b}{d}}\operatorname{expIntegral}\left(1,-\frac{bc}{d(dx+c)}\right)}{2d^2}+\frac{e^{\frac{bx}{dx+c}}x}{2}+\frac{c\,e^{\frac{bx}{dx+c}}}{2d}-\frac{bc\,e^{\frac{b}{d}}\operatorname{expIntegral}\left(1,\frac{bc}{d(dx+c)}\right)}{2d^2}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-b*x/(d*x+c))*(d*x+c)-1/2*b*c/d^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))
+1/2*exp(b*x/(d*x+c))*x+1/2*c/d*exp(b*x/(d*x+c))-1/2*b*c/d^2*exp(b/d)*Ei(1,
b*c/d/(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/
d) + c^2*e^(b/d)), x) - 1/2*b*c*integrate(x*e^(-b*c/(d^2*x + c*d) + b/d)/(d
^2*x^2 + 2*c*d*x + c^2), x) - 1/2*(x*e^(b*c/(d^2*x + c*d)) - x*e^(-b*c/(d^2
*x + c*d) + 2*b/d))*e^(-b/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(74) = 148.
time = 0.53, size = 253, normalized size = 3.42

$$-\frac{bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{bx}{dx+c}\right)^2-\left(bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2+bc\operatorname{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\sinh\left(\frac{bx}{dx+c}\right)-\left(bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2-bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\sinh\left(\frac{bx}{dx+c}\right)^2-bc\operatorname{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\sinh\left(\frac{b}{d}\right)}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^2-d^2\sinh\left(\frac{bx}{dx+c}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b/d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(
-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh
(b/d) - 2*(d^2*x + c*d)*sinh(b*x/(d*x + c)) - (b*c*Ei(-b*c/(d^2*x + c*d))*c
osh(b*x/(d*x + c))^2 - b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2 - b
*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(
b*x/(d*x + c))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(\frac{bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x)
```

```
[Out] Integral(sinh(b*x/(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x)/(c + d*x)),x)
```

```
[Out] int(sinh((b*x)/(c + d*x)), x)
```

3.293 $\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx$

Optimal. Leaf size=80

$$\frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}-\frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}$$

```
[Out] -b*c*cosh(2*b/d)*Shi(2*b*c/d/(d*x+c))/d^2+b*c*Chi(2*b*c/d/(d*x+c))*sinh(2*b
/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5726, 3394, 12, 3384, 3379, 3382}

$$\frac{bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}-\frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (b*c*CoshIntegral[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 + ((c + d*x)*Si
nh[(b*x)/(c + d*x)]^2)/d - (b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(d*(c +
d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2\left(\frac{bx}{c+dx}\right)dx &= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{(2ibc)\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{2x}dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(bc\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2bcx}{d}\right)}{x}dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left(bc\sinh\left(\frac{2}{\cdot}\right.\right.}{}\\
&= \frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 85, normalized size = 1.06

$$\frac{d\left(-dx+(c+dx)\cosh\left(\frac{2bx}{c+dx}\right)\right)+2bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)-2bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*b*x)/(c + d*x)]) + 2*b*c*CoshIntegral[(2*b*c
)/(d*(c + d*x))]*Sinh[(2*b)/d] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(
d*(c + d*x))])/(2*d^2)
```

Maple [A]
time = 18.35, size = 120, normalized size = 1.50

method	result	size
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{-\frac{2 b x}{d x+c}} \left(d x+c\right)}{4 d}+\frac{b c \,{\mathrm e}^{-\frac{2 b}{d}} \operatorname{expIntegral}\left(1,-\frac{2 b c}{d \left(d x+c\right)}\right)}{2 d^{2}}+\frac{{\mathrm e}^{\frac{2 b x}{d x+c}} x}{4}+\frac{c \,{\mathrm e}^{\frac{2 b x}{d x+c}}}{4 d}-\frac{b c \,{\mathrm e}^{\frac{2 b}{d}} \operatorname{expIntegral}\left(1,\frac{2 b c}{d \left(d x+c\right)}\right)}{2 d^{2}}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4/d*exp(-2*b*x/(d*x+c))*(d*x+c)+1/2*b*c/d^2*exp(-2*b/d)*Ei(1,-2*b*
c/d/(d*x+c))+1/4*exp(2*b*x/(d*x+c))*x+1/4*c/d*exp(2*b*x/(d*x+c))-1/2*b*c/d^
2*exp(2*b/d)*Ei(1,2*b*c/d/(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*integrate(x*e^(2*b*c/(d^2*x + c*d))/(d^2*x^2*e^(2*b/d) + 2*c*d*x*e^
(2*b/d) + c^2*e^(2*b/d)), x) - 1/2*b*c*integrate(x*e^(-2*b*c/(d^2*x + c*d)
+ 2*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*(x*e^(2*b*c/(d^2*x + c*d)) + x
*e^(-2*b*c/(d^2*x + c*d) + 4*b/d))*e^(-2*b/d) - 1/2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(80) = 160$.
time = 0.52, size = 277, normalized size = 3.46

$$-\frac{d^{2} x - \left(d^{2} x + c d\right) \cosh\left(\frac{b x}{d x + c}\right)^{2} + \left(b c \operatorname{Ei}\left(-\frac{2 \, b c}{d^{2} x + c d}\right) \cosh\left(\frac{2 \, b}{d}\right) - d^{2} x - c d\right) \sinh\left(\frac{b x}{d x + c}\right)^{2} - \left(b c \operatorname{Ei}\left(-\frac{2 \, b c}{d^{2} x + c d}\right) \cosh\left(\frac{b x}{d x + c}\right)^{2} - b c \operatorname{Ei}\left(\frac{2 \, b c}{d^{2} x + c d}\right)\right) \cosh\left(\frac{2 \, b}{d}\right) - \left(b c \operatorname{Ei}\left(-\frac{2 \, b c}{d^{2} x + c d}\right) \cosh\left(\frac{b x}{d x + c}\right)^{2} - b c \operatorname{Ei}\left(-\frac{2 \, b c}{d^{2} x + c d}\right) \sinh\left(\frac{b x}{d x + c}\right)^{2} + b c \operatorname{Ei}\left(\frac{2 \, b c}{d^{2} x + c d}\right)\right) \sinh\left(\frac{2 \, b}{d}\right)}{2 \, {\left(d^{2} \cosh\left(\frac{b x}{d x + c}\right)^{2} - d^{2} \sinh\left(\frac{b x}{d x + c}\right)^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh(b*x/(d*x + c))^2 + (b*c*Ei(-2*b*c/(d^2*x +
 c*d))*cosh(2*b/d) - d^2*x - c*d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-2*b*c/(d
```

```
^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(2*b*c/(d^2*x + c*d)))*cosh(2*b/
d) - (b*c*Ei(-2*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-2*b*c/(d
^2*x + c*d))*sinh(b*x/(d*x + c))^2 + b*c*Ei(2*b*c/(d^2*x + c*d)))*sinh(2*b/
d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**2,x)
```

```
[Out] Integral(sinh(b*x/(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{sinh}\left(\frac{b\,x}{c+d\,x}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x)/(c + d*x))^2,x)
```

```
[Out] int(sinh((b*x)/(c + d*x))^2, x)
```

3.294 $\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx$

Optimal. Leaf size=143

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d}+\frac{3bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}$$

```
[Out] -3/4*b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2+3/4*b*c*Chi(3*b*c/d/(d*x+c))*cosh
(3*b/d)/d^2+3/4*b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2-3/4*b*c*Shi(3*b*c/d/(d
*x+c))*sinh(3*b/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5726, 3394, 3384, 3379, 3382}

$$-\frac{3bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\cosh\left(\frac{3b}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{3bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2}-\frac{3bc\sinh\left(\frac{3b}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/(4*d^2) + (3*b*c*Cosh[
(3*b)/d]*CoshIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(b*
x)/(c + d*x)]^3)/d + (3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/(4
*d^2) - (3*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{(3bc)\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{4x} + \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{4x}\right) dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{(3bc)\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3bc)\text{Subst}\left(\int \frac{\text{co}}{}\right.}{}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(3bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3bc\cosh(}{}$$

$$= -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} +$$

Mathematica [A]
time = 0.35, size = 172, normalized size = 1.20

$$\frac{-3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right) + 3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right) - 3cd\sinh\left(\frac{bx}{c+dx}\right) - 3d^2x\sinh\left(\frac{bx}{c+dx}\right) + cd\sinh\left(\frac{3bx}{c+dx}\right) + d^2x\sinh\left(\frac{3bx}{c+dx}\right) + 3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right) - 3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))] + 3*b*c*Cosh[(3*b)/d]*C
oshIntegral[(3*b*c)/(d*(c + d*x))] - 3*c*d*Sinh[(b*x)/(c + d*x)] - 3*d^2*x*
```

```
Sinh[(b*x)/(c + d*x)] + c*d*Sinh[(3*b*x)/(c + d*x)] + d^2*x*Sinh[(3*b*x)/(c
 + d*x)] + 3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))] - 3*b*c*Sinh[(
3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Maple [A]
time = 8.99, size = 228, normalized size = 1.59

method	result
risch	$-\frac{e^{-\frac{3bx}{dx+c}}(dx+c)}{8d}-\frac{3bc\,e^{-\frac{3b}{d}}\operatorname{expIntegral}\left(1,-\frac{3bc}{d(dx+c)}\right)}{8d^2}+\frac{3\,e^{-\frac{bx}{dx+c}}(dx+c)}{8d}+\frac{3bc\,e^{-\frac{b}{d}}\operatorname{expIntegral}\left(1,-\frac{bc}{d(dx+c)}\right)}{8d^2}+\frac{e^{\frac{3bx}{dx+c}}x}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*exp(-3*b*x/(d*x+c))*(d*x+c)-3/8*b*c/d^2*exp(-3*b/d)*Ei(1,-3*b*c/d/(d
*x+c))+3/8/d*exp(-b*x/(d*x+c))*(d*x+c)+3/8*b*c/d^2*exp(-b/d)*Ei(1,-b*c/d/(d
*x+c))+1/8*exp(3*b*x/(d*x+c))*x+1/8*c/d*exp(3*b*x/(d*x+c))-3/8*b*c/d^2*exp(
3*b/d)*Ei(1,3*b*c/d/(d*x+c))-3/8*exp(b*x/(d*x+c))*x-3/8*c/d*exp(b*x/(d*x+c)
)+3/8*b*c/d^2*exp(b/d)*Ei(1,b*c/d/(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*b*c*integrate(x*e^(3*b*c/(d^2*x + c*d))/(d^2*x^2*e^(3*b/d) + 2*c*d*x*e
^(3*b/d) + c^2*e^(3*b/d)), x) + 3/8*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(
d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) + 3/8*b*c*integrate(x*
e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*b*c*integr
ate(x*e^(-3*b*c/(d^2*x + c*d) + 3*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/8*
(x*e^(3*b*c/(d^2*x + c*d)) - 3*x*e^(b*c/(d^2*x + c*d) + 2*b/d) + 3*x*e^(-b*
c/(d^2*x + c*d) + 4*b/d) - x*e^(-3*b*c/(d^2*x + c*d) + 6*b/d))*e^(-3*b/d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 701 vs. 2(135) = 270.
time = 0.47, size = 701, normalized size = 4.90

$$\frac{3\left(bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{3b}{d}\right)-bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\right)\sinh\left(\frac{bx}{dx+c}\right)^4+2\left(d^2x+cd\right)\sinh\left(\frac{bx}{dx+c}\right)^3-6\left(bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2\cosh\left(\frac{3b}{d}\right)-bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2\cosh\left(\frac{b}{d}\right)\right)\sinh\left(\frac{bx}{dx+c}\right)^2+3\left(bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^4+bc\operatorname{Ei}\left(\frac{3bc}{d^2x+cd}\right)\right)\cosh\left(\frac{3b}{d}\right)-3\left(bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^4+bc\operatorname{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)-6\left(d^2x-\left(d^2x+cd\right)\cosh\left(\frac{bx}{dx+c}\right)^2+cd\right)\sinh\left(\frac{bx}{dx+c}\right)+3\left(bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^4-2\,bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2\sinh\left(\frac{bx}{dx+c}\right)^2+bc\operatorname{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\sinh\left(\frac{bx}{dx+c}\right)^4-bc\operatorname{Ei}\left(\frac{3bc}{d^2x+cd}\right)\right)\sinh\left(\frac{3b}{d}\right)-3\left(bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^4-2\,bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2\sinh\left(\frac{bx}{dx+c}\right)^2+bc\operatorname{Ei}\left(-\frac{bc}{d^2x+cd}\right)\sinh\left(\frac{bx}{dx+c}\right)^4-bc\operatorname{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\sinh\left(\frac{b}{d}\right)}{8\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^4-2\,d^2\cosh\left(\frac{bx}{dx+c}\right)^2\sinh\left(\frac{bx}{dx+c}\right)^2+d^2\sinh\left(\frac{bx}{dx+c}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(3*b/d) - b*c*Ei(-b*c/(d^2*x + c*d
))*cosh(b/d))*sinh(b*x/(d*x + c))^4 + 2*(d^2*x + c*d)*sinh(b*x/(d*x + c))^3
 - 6*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(3*b/d) - b*c*
Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(b/d))*sinh(b*x/(d*x + c))
^2 + 3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 + b*c*Ei(3*b*c/(
d^2*x + c*d)))*cosh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
c))^4 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 6*(d^2*x - (d^2*x + c*d)*cos
h(b*x/(d*x + c))^2 + c*d)*sinh(b*x/(d*x + c)) + 3*(b*c*Ei(-3*b*c/(d^2*x + c
*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-3*b*c/(d^2*x + c*d))*sinh(b*x/(d*x +
 c))^4 - b*c*Ei(3*b*c/(d^2*x + c*d)))*sinh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x +
 c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c
))^4 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^4 - 2
*d^2*cosh(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + d^2*sinh(b*x/(d*x + c))^
4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x/(d*x + c))^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{b\,x}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x)/(c + d*x))^3,x)
```

```
[Out] int(sinh((b*x)/(c + d*x))^3, x)
```

3.295 $\int \sinh\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

```
[Out] (-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2-(-a*d+b*c)*Shi((-a*d+b*c
)/d/(d*x+c))*sinh(b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))/d
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5726, 3378, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}-\frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c +
 d*x)*Sinh[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*Sinh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{((\ldots}{}$$

$$= \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc\ldots}{d(c\ldots}\right.}{d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(101) = 202.
time = 0.51, size = 373, normalized size = 3.69

$$\frac{(bc-ad)\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right)\left(\cosh\left(\frac{b}{d}\right)-\sinh\left(\frac{b}{d}\right)\right)+(bc-ad)\text{Chi}\left(\frac{-bc+ad}{d(c+dx)}\right)\left(\cosh\left(\frac{b}{d}\right)+\sinh\left(\frac{b}{d}\right)\right)+2cd\sinh\left(\frac{a+bx}{c+dx}\right)+2d^2x\sinh\left(\frac{a+bx}{c+dx}\right)+bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right)-ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right)+bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right)-ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right)+bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)-ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)-bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)+ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*(Cosh[b/d] - Sinh[b/d]
) + (b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/d] + Sin
h[b/d]) + 2*c*d*Sinh[(a + b*x)/(c + d*x)] + 2*d^2*x*Sinh[(a + b*x)/(c + d*x
)] + b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Cosh[b/
d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + b*c*Sinh[b/d]*SinhIntegral[
(-(b*c) + a*d)/(d*(c + d*x))] - a*d*Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(
```

```
d*(c + d*x))] + b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - a*d
*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - b*c*Sinh[b/d]*SinhInte
gral[(b*c - a*d)/(c*d + d^2*x)] + a*d*Sinh[b/d]*SinhIntegral[(b*c - a*d)/(c
*d + d^2*x)])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(101) = 202$.
time = 1.19, size = 347, normalized size = 3.44

method	result
risch	$-\frac{e^{-\frac{bx+a}{dx+c}} a}{2\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{bx+a}{dx+c}} bc}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{b}{d}} \operatorname{expIntegral}\left(1,\frac{ad-bc}{d(dx+c)}\right) a}{2d}-\frac{e^{-\frac{b}{d}} \operatorname{expIntegral}\left(1,\frac{ad-bc}{d(dx+c)}\right) bc}{2d^2}+\frac{d\, e^{\frac{bx+a}{dx+c}} xa}{2ad-2bc}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/2/d*exp(-(b*x+a)/(
d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c+1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x
+c))*a-1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x+a)/(d
*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/2*exp((b*x+
a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*
exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d
*x+c))*b*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 0.46, size = 171, normalized size = 1.69

$$\frac{\left((bc-ad)\operatorname{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\operatorname{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-\left((bc-ad)\operatorname{Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-ad)\operatorname{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sinh\left(\frac{b}{d}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*cosh(b/d) + 2*(d^2*x + c*d)*sinh((b*x + a)/(d*x + c)) -
```

```
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x)
[Out] Integral(sinh((a + b*x)/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(101) = 202.
time = 1.82, size = 764, normalized size = 7.56

$$\frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - 2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c} + a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} + \frac{2\,(bx+a)abcd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c} - \frac{(bx+a)a^2d^3\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c} + b^2c^2de^{\left(\frac{bx+a}{dx+c}\right)} - 2\,abcd^2e^{\left(\frac{bx+a}{dx+c}\right)} + a^2d^3e^{\left(\frac{bx+a}{dx+c}\right)}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)}{2\left(bd^2 - \frac{(bx+a)d^3}{dx+c}\right)} + \frac{\left(b^3c^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - 2\,ab^2cd\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c} + a^2bd^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} + \frac{2\,(bx+a)abcd^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c} - \frac{(bx+a)a^2d^3\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c} - b^2c^2de^{\left(-\frac{bx+a}{dx+c}\right)} + 2\,abcd^2e^{\left(-\frac{bx+a}{dx+c}\right)} - a^2d^3e^{\left(-\frac{bx+a}{dx+c}\right)}\right)\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)}{2\left(bd^2 - \frac{(bx+a)d^3}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-(
b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x +
 a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x
 + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d
)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*
e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^((b*x
 + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) + 1/2*(b^3*c^2*Ei((b - (
b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(
-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 2*
(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) -
(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) - a
^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b
*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x)),x)
[Out] int(sinh((a + b*x)/(c + d*x)), x)
```

3.296 $\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=107

$$\frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

```
[Out] -(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-
a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5726, 3394, 12, 3384, 3379, 3382}

$$\frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2
 + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(2i(bc-ad))\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} +\\
&= \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}(}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 112, normalized size = 1.05

$$\frac{d\left(-dx+(c+dx)\cosh\left(\frac{2(a+bx)}{c+dx}\right)\right)+2(bc-ad)\text{Chi}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)+2(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)*CoshI
ntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] + 2*(b*c - a*d)*Cos
h[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(2*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(107) = 214$.
time = 22.79, size = 358, normalized size = 3.35

method	result
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{-\frac{2(bx+a)}{dx+c}} a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{{\mathrm e}^{-\frac{2(bx+a)}{dx+c}} bc}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{{\mathrm e}^{-\frac{2b}{d}} \operatorname{expIntegral}\left(1,\frac{2ad-2bc}{(dx+c)d}\right) a}{2d}+\frac{{\mathrm e}^{-\frac{2b}{d}} \operatorname{expIntegral}\left(1,\frac{2ad-2bc}{(dx+c)d}\right) bc}{2d^{2}}+\frac{d\,{\mathrm e}^{\frac{2bx+2a}{dx+c}}}{4ad-4bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-1/4/d*exp(-2
*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c-1/2/d*exp(-2*b/d)*Ei(1,2*(a
*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*b*c+1/
4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*
c)*x*b*c+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+
c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2
*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(107) = 214$.
time = 0.43, size = 370, normalized size = 3.46

$$-\frac{d^{2} x-\left(d^{2} x+c d\right) \cosh\left(\frac{b x+a}{d x+c}\right)^{2}-\left(d^{2} x-(b c-a d) \operatorname{Ei}\left(-\frac{2\,(b c-a d)}{d^{2} x+c d}\right) \cosh\left(\frac{2\,b}{d}\right)+c d\right) \sinh\left(\frac{b x+a}{d x+c}\right)^{2}-\left((b c-a d) \operatorname{Ei}\left(-\frac{2\,(b c-a d)}{d^{2} x+c d}\right) \cosh\left(\frac{b x+a}{d x+c}\right)^{2}-(b c-a d) \operatorname{Ei}\left(\frac{2\,(b c-a d)}{d^{2} x+c d}\right)\right) \cosh\left(\frac{2\,b}{d}\right)-\left((b c-a d) \operatorname{Ei}\left(-\frac{2\,(b c-a d)}{d^{2} x+c d}\right) \cosh\left(\frac{b x+a}{d x+c}\right)^{2}-(b c-a d) \operatorname{Ei}\left(-\frac{2\,(b c-a d)}{d^{2} x+c d}\right) \sinh\left(\frac{b x+a}{d x+c}\right)^{2}+(b c-a d) \operatorname{Ei}\left(\frac{2\,(b c-a d)}{d^{2} x+c d}\right)\right) \sinh\left(\frac{2\,b}{d}\right)}{2\,\left(d^{2} \cosh\left(\frac{b x+a}{d x+c}\right)^{2}-d^{2} \sinh\left(\frac{b x+a}{d x+c}\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 - (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
 + c))^2 - ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*
x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) - ((b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*
c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/
(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(107) = 214.
time = 7.22, size = 749, normalized size = 7.00

$$\frac{\left(2\,b^{3}c^{2}\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}-4\,ab^{2}cd\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}-\frac{2\,(bx+a)b^{2}c^{2}d\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}}{dx+c}+2\,a^{2}bd^{2}\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}+\frac{4\,(bx+a)abcd^{2}\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}}{dx+c}-\frac{2\,(bx+a)a^{2}d^{3}\operatorname{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2b}{d}\right)}}{dx+c}-2\,b^{3}c^{2}\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}+4\,ab^{2}cd\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}+\frac{2\,(bx+a)b^{2}c^{2}d\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}}{dx+c}-2\,a^{2}bd^{2}\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}-\frac{4\,(bx+a)abcd^{2}\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}}{dx+c}+\frac{2\,(bx+a)a^{2}d^{3}\operatorname{Ei}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(-\frac{2b}{d}\right)}}{dx+c}+b^{2}c^{2}de^{\left(\frac{2(bx+a)}{dx+c}\right)}-2\,abcd^{2}e^{\left(\frac{2(bx+a)}{dx+c}\right)}+a^{2}d^{3}e^{\left(\frac{2(bx+a)}{dx+c}\right)}+b^{2}c^{2}de^{\left(-\frac{2(bx+a)}{dx+c}\right)}-2\,abcd^{2}e^{\left(-\frac{2(bx+a)}{dx+c}\right)}+a^{2}d^{3}e^{\left(-\frac{2(bx+a)}{dx+c}\right)}-2\,b^{2}c^{2}d+4\,abcd^{2}-2\,a^{2}d^{3}\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)}{4\left(bd^{2}-\frac{(bx+a)d^{3}}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*d
*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*Ei(
-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(b -
(b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b - (b*
x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*d/(d*x
+ c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(d*x + c)
)/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)*e^(
-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) + b^2*c^2*d*e^
(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x + c)) + a^2*d^3
*e^(-2*(b*x + a)/(d*x + c)) - 2*b^2*c^2*d + 4*a*b*c*d^2 - 2*a^2*d^3)*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x))^2,x)
[Out] int(sinh((a + b*x)/(c + d*x))^2, x)
```

3.297 $\int \sinh^3\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=194

$$-\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\operatorname{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\operatorname{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{3(bc-ad)\mathrm{s}}{}$$

```
[Out] -3/4*(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2+3/4*(-a*d+b*c)*Chi(
3*(-a*d+b*c)/d/(d*x+c))*cosh(3*b/d)/d^2+3/4*(-a*d+b*c)*Shi((-a*d+b*c)/d/(d*
x+c))*sinh(b/d)/d^2-3/4*(-a*d+b*c)*Shi(3*(-a*d+b*c)/d/(d*x+c))*sinh(3*b/d)/
d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5726, 3394, 3384, 3379, 3382}

$$-\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\operatorname{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}+\frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\operatorname{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\operatorname{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sinh\left(\frac{3b}{d}\right)(bc-ad)\operatorname{Shi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2)
+ (3*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])
/(4*d^2) + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sinh
[(3*b)/d]*SinhIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x} + \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)}{d}\right.}{4x}\right.\right.}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(}{}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$= -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+d}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 599 vs. $2(194) = 388$.
time = 0.93, size = 599, normalized size = 3.09

$$\frac{6(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(-bc+ad)}{d(c+dx)}\right) - 3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right) + 3ad\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right) + 3bc\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right)\sinh\left(\frac{b}{d}\right) - 3ad\text{Chi}\left(\frac{bc-ad}{cd+d^2x}\right)\sinh\left(\frac{b}{d}\right) - 3(bc-ad)\text{Chi}\left(\frac{-bc+ad}{d(c+dx)}\right)\left(\cosh\left(\frac{b}{d}\right)+\sinh\left(\frac{b}{d}\right)\right) - 6cd\sinh\left(\frac{a+bx}{c+dx}\right) - 6d^2x\sinh\left(\frac{a+bx}{c+dx}\right) + 2cd\sinh\left(\frac{3(a+bx)}{c+dx}\right) + 2d^2x\sinh\left(\frac{3(a+bx)}{c+dx}\right) - 3bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) + 3ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) - 3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) + 3ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{-bc+ad}{d(c+dx)}\right) + 6bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3(-bc+ad)}{d(c+dx)}\right) - 6ad\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3(-bc+ad)}{d(c+dx)}\right) - 3bc\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) + 3ad\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) + 3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right) - 3ad\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{cd+d^2x}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (6*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]
 - 3*b*c*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*a*d*Cosh[b/d
]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*CoshIntegral[(b*c - a*d)/
(c*d + d^2*x)]*Sinh[b/d] - 3*a*d*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)]*Si
nh[b/d] - 3*(b*c - a*d)*CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cosh[b/
d] + Sinh[b/d]) - 6*c*d*Sinh[(a + b*x)/(c + d*x)] - 6*d^2*x*Sinh[(a + b*x)/
(c + d*x)] + 2*c*d*Sinh[(3*(a + b*x))/(c + d*x)] + 2*d^2*x*Sinh[(3*(a + b*x
))/(c + d*x)] - 3*b*c*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))]
+ 3*a*d*Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - 3*b*c*Sinh[b
/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + 3*a*d*Sinh[b/d]*SinhIntegr
al[(-(b*c) + a*d)/(d*(c + d*x))] + 6*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b
*c) + a*d))/(d*(c + d*x))] - 6*a*d*Sinh[(3*b)/d]*SinhIntegral[(3*(-(b*c) +
a*d))/(d*(c + d*x))] - 3*b*c*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*
x)] + 3*a*d*Cosh[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + 3*b*c*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] - 3*a*d*Sinh[b/d]*SinhIntegral
[(b*c - a*d)/(c*d + d^2*x)])/(8*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(186) = 372$.

time = 8.94, size = 700, normalized size = 3.61

method	result
risch	$-\frac{e^{-\frac{3(bx+a)}{dx+c}}a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{3(bx+a)}{dx+c}}bc}{8d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{3\,e^{-\frac{3b}{d}}\operatorname{expIntegral}\left(1,\frac{3ad-3bc}{d(dx+c)}\right)a}{8d}-\frac{3\,e^{-\frac{3b}{d}}\operatorname{expIntegral}\left(1,\frac{3ad-3bc}{d(dx+c)}\right)bc}{8d^2}+\frac{3\,e^{-\frac{bx+}{dx+}}}{8\left(\frac{da}{dx+c}-\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*exp(-3*(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a+1/8/d*exp(-3*(b*x+
a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*b*c+3/8/d*exp(-3*b/d)*Ei(1,3*(a*d-b*c
)/d/(d*x+c))*a-3/8/d^2*exp(-3*b/d)*Ei(1,3*(a*d-b*c)/d/(d*x+c))*b*c+3/8*exp(
-(b*x+a)/(d*x+c))/(d*a/(d*x+c)-b*c/(d*x+c))*a-3/8/d*exp(-(b*x+a)/(d*x+c))/(
d*a/(d*x+c)-b*c/(d*x+c))*b*c-3/8/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*a+3/
8/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/8*d*exp(3*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*a-1/8*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/8*exp(3*(b*x+a)/
(d*x+c))/(a*d-b*c)*c*a-1/8/d*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+3/8/d*e
xp(3*b/d)*Ei(1,-3*(a*d-b*c)/d/(d*x+c))*a-3/8/d^2*exp(3*b/d)*Ei(1,-3*(a*d-b*
c)/d/(d*x+c))*b*c-3/8*d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*a+3/8*exp((b*x+a)/
(d*x+c))/(a*d-b*c)*x*b*c-3/8*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a+3/8/d*exp((
b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b-3/8/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a
+3/8/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*b*c
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(186) = 372$.
time = 0.49, size = 717, normalized size = 3.70

$$-\frac{6\,(bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2\cosh\left(\frac{3\,b}{d}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-3\,(bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{3\,b}{d}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^4-2\,(d^2x+cd)\sinh\left(\frac{bx+a}{dx+c}\right)^3-3\left((bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^4+(bc-ad){\rm Ei}\left(\frac{3\,(bc-ad)}{d^2x+cd}\right)\right)\cosh\left(\frac{3\,b}{d}\right)+3\left((bc-ad){\rm Ei}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad){\rm Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right)+6\left(d^2x-(d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-3\left((bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^4-2\,(bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2\sinh\left(\frac{bx+a}{dx+c}\right)^2+(bc-ad){\rm Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^4-(bc-ad){\rm Ei}\left(\frac{3\,(bc-ad)}{d^2x+cd}\right)\right)\sinh\left(\frac{3\,b}{d}\right)-3\left((bc-ad){\rm Ei}\left(\frac{bc-ad}{d^2x+cd}\right)-(bc-ad){\rm Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sinh\left(\frac{b}{d}\right)}{8\left(d^2\cosh\left(\frac{bx+a}{dx+c}\right)^4-2\,d^2\cosh\left(\frac{bx+a}{dx+c}\right)^2\sinh\left(\frac{bx+a}{dx+c}\right)^2+d^2\sinh\left(\frac{bx+a}{dx+c}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x +
c))^2*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^2 - 3*(b*c - a*d)*Ei(-3*(b*c -
a*d)/(d^2*x + c*d))*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^4 - 2*(d^2*x + c*
d)*sinh((b*x + a)/(d*x + c))^3 - 3*((b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x +
c*d))*cosh((b*x + a)/(d*x + c))^4 + (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c
*d)))*cosh(3*b/d) + 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a
*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) + 6*(d^2*x - (d^2*x + c*d)*co
sh((b*x + a)/(d*x + c))^2 + c*d)*sinh((b*x + a)/(d*x + c)) - 3*((b*c - a*d)
*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^4 - 2*(b*c - a*
d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2*sinh((b*x +
 a)/(d*x + c))^2 + (b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x +
 a)/(d*x + c))^4 - (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c*d)))*sinh(3*b/d)
 - 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh((b*x + a)/(d*x + c))^4 - 2*d^2*cosh
((b*x + a)/(d*x + c))^2*sinh((b*x + a)/(d*x + c))^2 + d^2*sinh((b*x + a)/(d
*x + c))^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(186) = 372$.

time = 8.62, size = 1383, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c^2*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 6*a*b^2*c*d
*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 3*(b*x + a)*b^2*c^2*d*Ei(
-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) + 3*a^2*b*d^2*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) + 6*(b*x + a)*a*b*c*d^2*Ei(-3*(b -
(b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*b^3*c^2*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d) + 6*a*b^2*c*d*Ei(-(b - (b*x + a)*d/(d*x + c))
/d)*e^(b/d) + 3*(b*x + a)*b^2*c^2*d*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b
/d)/(d*x + c) - 3*a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 6*
(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) +
3*(b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) -
3*b^3*c^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 6*a*b^2*c*d*Ei((b -
(b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 3*(b*x + a)*b^2*c^2*d*Ei((b - (b*x + a
)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - 3*a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x
 + c))/d)*e^(-b/d) - 6*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*b^3*c^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*
b/d) - 6*a*b^2*c*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) - 3*(b*x
+ a)*b^2*c^2*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + 3
*a^2*b*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) + 6*(b*x + a)*a*b
*c*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) - 3*(b*x +
a)*a^2*d^3*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + b^2*c
^2*d*e^(3*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(3*(b*x + a)/(d*x + c)) + a^
2*d^3*e^(3*(b*x + a)/(d*x + c)) - 3*b^2*c^2*d*e^((b*x + a)/(d*x + c)) + 6*a
*b*c*d^2*e^((b*x + a)/(d*x + c)) - 3*a^2*d^3*e^((b*x + a)/(d*x + c)) + 3*b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) - 6*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) + 3
*a^2*d^3*e^(-(b*x + a)/(d*x + c)) - b^2*c^2*d*e^(-3*(b*x + a)/(d*x + c)) +
2*a*b*c*d^2*e^(-3*(b*x + a)/(d*x + c)) - a^2*d^3*e^(-3*(b*x + a)/(d*x + c))
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)/(c + d*x))^3,x)
[Out] int(sinh((a + b*x)/(c + d*x))^3, x)
```

3.298 $\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=121

$$\frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

```
[Out] (-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2-(-a*d+b*c)*f*Shi
((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/d
```

Rubi [A]
time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5728, 5726, 3378, 3384, 3379, 3382}

$$\frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)])/d - ((b*c
- a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx &= \int \sinh\left(\frac{ce+af+(de+bf)x}{c+dx}\right)dx \\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\text{Subst}\left(\int \frac{\cosh\left(\frac{de+bf}{d}-\frac{(bc-ad)fx}{d}\right)}{x}\,d\right.}{d^2} \\
&= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{\left((bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc}{x}\right.}{}\right.}{d^2} \\
&= \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 449 vs. 2(121) = 242.

time = 1.03, size = 449, normalized size = 3.71

$$\frac{(bc-ad)f\,\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)\left(\cosh\left(e+\frac{bf}{d}\right)-\sinh\left(e+\frac{bf}{d}\right)\right)+(bc-ad)f\,\text{Chi}\left(\frac{-bcf+adf}{d(c+dx)}\right)\left(\cosh\left(e+\frac{bf}{d}\right)+\sinh\left(e+\frac{bf}{d}\right)\right)+2cd\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)+2d^2x\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)+bcf\cosh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)-adf\cosh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)-bcf\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+adf\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+bcf\cosh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)-adf\cosh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)+bcf\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)-adf\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/
d] - Sinh[e + (b*f)/d]) + (b*c - a*d)*f*CoshIntegral[(-(b*c*f) + a*d*f)/(d*
(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) + 2*c*d*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*d^2*x*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c
+ d*x)] + b*c*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x)
)] - a*d*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] -
b*c*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + a*d*f
*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + b*c*f*Cosh
[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Cosh[e
 + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + b*c*f*Sinh[e +
 (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - a*d*f*Sinh[e + (
b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(121) = 242$.
time = 1.44, size = 459, normalized size = 3.79

method	result
risch	$-\frac{f\,\mathrm{e}^{-\frac{bfx+dex+fa+ce}{dx+c}}a}{2\left(\frac{dfa}{dx+c}-\frac{fbc}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{bfx+dex+fa+ce}{dx+c}}bc}{2d\left(\frac{dfa}{dx+c}-\frac{fbc}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{bf+de}{d}}\,\mathrm{expIntegral}\left(1,\frac{(ad-bc)f}{d(dx+c)}\right)a}{2d}-\frac{f\,\mathrm{e}^{-\frac{bf+de}{d}}\,\mathrm{expIntegral}\left(1,\frac{(ad-bc)f}{d(dx+c)}\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*c)*a+
1/2/d*f*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*c)*b
*c+1/2/d*f*exp(-(b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a-1/2/d^2*f*exp(-(
b*f+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*b*c+1/2/d*f*exp((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*a-1/2/d^2*f*exp((b*f*x+d*e*x+a*f+
c*e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*b*c+1/2/d*f*exp((b*f+d*e)/d)*Ei
(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a-1/2/d^2*f*exp((b*f+d*
e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*b*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)*f/(d*x + c) + e), x)
```

Fricas [A]
time = 0.49, size = 220, normalized size = 1.82

$$\frac{\left(\left(bc-ad\right)f\mathrm{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right)+\left(bc-ad\right)f\mathrm{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{bf+d\cosh(1)+d\sinh(1)}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)-\left(\left(bc-ad\right)f\mathrm{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right)-\left(bc-ad\right)f\mathrm{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\sinh\left(\frac{bf+d\cosh(1)+d\sinh(1)}{d}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b*
c - a*d)*f/(d^2*x + c*d)))*cosh((b*f + d*cosh(1) + d*sinh(1))/d) + 2*(d^2*x
 + c*d)*sinh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c
)) - ((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei(-(b*
c - a*d)*f/(d^2*x + c*d)))*sinh((b*f + d*cosh(1) + d*sinh(1))/d))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(sinh(e + f*(a + b*x)/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1624 vs. $2(121) = 242$.
time = 4.57, size = 1624, normalized size = 13.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x +
c))/d)*e^((d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*
x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*d^3*e*f^2*Ei(-(d*e +
 b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + b^3*
c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*
e + b*f)/d) - 2*a*b^2*c*d*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*
d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*b*d^2*f^3*Ei(-(d*e + b*f - (d*e*x +
 b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - (d*e*x + b*f*x + c*
e + a*f)*b^2*c^2*d*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x
+ c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a*f)*a*b*c*
d^2*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*
```

```
e + b*f)/d)/(d*x + c) - (d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-(d*e +
b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x +
c) + b^2*c^2*d*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*
f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^((d*e*x + b*f
*x + c*e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*
d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e + a*f)*d^3/(d*
x + c)) + 1/2*(b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*
d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*Ei((d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + a^2*d^3*e*f^2*
Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)
/d) + b^3*c^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/
d)*e^(-(d*e + b*f)/d) - 2*a*b^2*c*d*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*
e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + a^2*b*d^2*f^3*Ei((d*e + b*f -
 (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - (d*e*x +
b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f
)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a
*f)*a*b*c*d^2*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/
d)*e^(-(d*e + b*f)/d)/(d*x + c) - (d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*E
i((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/
d)/(d*x + c) - b^2*c^2*d*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 2
*a*b*c*d^2*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - a^2*d^3*f^2*e^(
-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 -
 (c*e + a*f)*d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e +
 a*f)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e + \frac{f\,(a + b\,x)}{c + d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x)),x)
```

```
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x)), x)
```

3.299 $\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=129

$$\frac{(bc-ad)f\operatorname{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{(bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\operatorname{Sh}}{d^2}$$

[Out] -(-a*d+b*c)*f*cosh(2*e+2*b*f/d)*Shi(2*(-a*d+b*c)*f/d/(d*x+c))/d^2+(-a*d+b*c)*f*Chi(2*(-a*d+b*c)*f/d/(d*x+c))*sinh(2*e+2*b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*e)/(d*x+c))^2/d

Rubi [A]
time = 0.22, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5728, 5726, 3394, 12, 3384, 3379, 3382}

$$\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\operatorname{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\operatorname{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e + (b*f)/d)])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)]^2)/d - ((b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh^2\left(\frac{ce+af+(de+bf)x}{c+dx}\right)\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}dx,x,\frac{1}{c+dx}\right)}{d}$$
$$=\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{(2i(bc-ad)f)\text{Subst}\left(\int \frac{i\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-}{}\right.}{2x}\right.}{d^2}$$
$$=\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}+\frac{((bc-ad)f)\text{Subst}\left(\int \frac{\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)}{d}\right.}{x}\right.}{d^2}$$
$$=\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{\left((bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int \right.}{d^2}$$
$$=\frac{(bc-ad)f\,\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right.}{d}$$

Mathematica [A]
time = 1.46, size = 136, normalized size = 1.05

$$\frac{d\left(-dx+(c+dx)\cosh\left(\frac{2(ce+af+dex+bfx)}{c+dx}\right)\right)+2(bc-ad)f\,\text{Chi}\left(\frac{2(-bcf+adf)}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)+2(bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{2(-bcf+adf)}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] (d*(-(d*x) + (c + d*x)*Cosh[(2*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)]) + 2
*(b*c - a*d)*f*CoshIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*Sinh[2*(e
 + (b*f)/d)] + 2*(b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c
*f) + a*d*f))/(d*(c + d*x))])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(131) = 262$.
time = 31.03, size = 468, normalized size = 3.63

method	result
risch	$-\frac{x}{2}+\frac{f\,e^{-\frac{2(bfx+dex+fa+ce)}{dx+c}}a}{\frac{4dfa}{dx+c}-\frac{4fbc}{dx+c}}-\frac{f\,e^{-\frac{2(bfx+dex+fa+ce)}{dx+c}}bc}{4d\left(\frac{dfa}{dx+c}-\frac{fbc}{dx+c}\right)}-\frac{f\,e^{-\frac{2(bf+de)}{d}}\text{expIntegral}\left(1,\frac{2(ad-bc)f}{d(dx+c)}\right)a}{2d}+\frac{f\,e^{-\frac{2(bf+de)}{d}}\text{expI}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)
*b*c)*a-1/4/d*f*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x
+c)*b*c)*b*c-1/2/d*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*a+1/
2/d^2*f*exp(-2*(b*f+d*e)/d)*Ei(1,2*(a*d-b*c)*f/d/(d*x+c))*b*c+1/4/d*f*exp(2
*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*a-1/4/d^2*f*e
xp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*b*c+1/2/d
*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(b*f+d*e)/d-2*(-b*f-d
*e)/d)*a-1/2/d^2*f*exp(2*(b*f+d*e)/d)*Ei(1,-2*(a*d-b*c)*f/d/(d*x+c)-2*(b*f+
d*e)/d-2*(-b*f-d*e)/d)*b*c
```

<u>Maxima</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c*f/(d^2*x + c*d) - 2*a*f/(d*x + c) - 2*b*f/d
 - 2*e), x) + 1/4*integrate(e^(-2*b*c*f/(d^2*x + c*d) + 2*a*f/(d*x + c) + 2
*b*f/d + 2*e), x)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(135) = 270$.
time = 0.51, size = 548, normalized size = 4.25

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2+\left((bc-ad)f\operatorname{Ei}\left(-\frac{2(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{2(bf+d\cosh(1)+d\sinh(1))}{d}\right)-d^2x-cd\right)\sinh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2-\left((bc-ad)f\operatorname{Ei}\left(-\frac{2(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2-(bc-ad)f\operatorname{Ei}\left(\frac{2(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{2(bf+d\cosh(1)+d\sinh(1))}{d}\right)-\left((bc-ad)f\operatorname{Ei}\left(-\frac{2(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2-(bc-ad)f\operatorname{Ei}\left(-\frac{2(bc-ad)f}{d^2x+cd}\right)\sinh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2+(bc-ad)f\operatorname{Ei}\left(\frac{2(bc-ad)f}{d^2x+cd}\right)\right)\sinh\left(\frac{2(bf+d\cosh(1)+d\sinh(1))}{d}\right)}{2\left(d^2\cosh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2-d^2\sinh\left(\frac{bfx+af+(dx+c)\cosh(1)+(dx+c)\sinh(1)}{dx+c}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x +
c)*sinh(1))/(d*x + c))^2 + ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d)
)*cosh(2*(b*f + d*cosh(1) + d*sinh(1))/d) - d^2*x - c*d)*sinh((b*f*x + a*f
+ (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2 - ((b*c - a*d)*f*Ei(-
2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x
 + c)*sinh(1))/(d*x + c))^2 - (b*c - a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d
)))*cosh(2*(b*f + d*cosh(1) + d*sinh(1))/d) - ((b*c - a*d)*f*Ei(-2*(b*c - a
*d)*f/(d^2*x + c*d))*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh
(1))/(d*x + c))^2 - (b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*sinh((
b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2 + (b*c -
a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(2*(b*f + d*cosh(1) + d*sinh(
1))/d))/(d^2*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*
x + c))^2 - d^2*sinh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/
(d*x + c))^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. 2(131) = 262.
time = 20.37, size = 1596, normalized size = 12.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*c^2*d*e*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*
x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b*c*d^2*e*f^2*Ei(-2*(d*e + b*f - (d*e*
x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*d^3*e*f^
2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e
+ b*f)/d) + 2*b^3*c^2*f^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/
(d*x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b^2*c*d*f^3*Ei(-2*(d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*b*d^2*f
^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e
 + b*f)/d) - 2*b^2*c^2*d*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f
)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b*c*d^2*e*f^2*Ei(2*(d*e + b*f
- (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) - 2*a^2*
d^3*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(
-2*(d*e + b*f)/d) - 2*b^3*c^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e +
a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b^2*c*d*f^3*Ei(2*(d*e + b*f
 - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) - 2*a^2
*b*d^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^
(-2*(d*e + b*f)/d) - 2*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-2*(d*e
 + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d
*x + c) + 4*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-2*(d*e + b*f - (d
*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d*x + c) - 2
*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x
+ c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f
*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)
*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d)/(d*x + c) - 4*(d*e*x + b*f*x + c*e +
a*f)*a*b*c*d^2*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c
))/d)*e^(-2*(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a*f)*a^2*d^
```

```
3*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(
d*e + b*f)/d)/(d*x + c) + b^2*c^2*d*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d
*x + c)) - 2*a*b*c*d^2*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^
2*d^3*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + b^2*c^2*d*f^2*e^(-2
*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(-2*(d*e*x + b*
f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^(-2*(d*e*x + b*f*x + c*e + a*f)
/(d*x + c)) - 2*b^2*c^2*d*f^2 + 4*a*b*c*d^2*f^2 - 2*a^2*d^3*f^2)*((d*e + b*
f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f
- (d*e*x + b*f*x + c*e + a*f)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^2,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^2, x)
```

3.300 $\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal. Leaf size=226

$$-\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\right.}{d}$$

```
[Out] -3/4*(-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2+3/4*(-a*d+b
*c)*f*Chi(3*(-a*d+b*c)*f/d/(d*x+c))*cosh(3*e+3*b*f/d)/d^2+3/4*(-a*d+b*c)*f*
Shi((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2-3/4*(-a*d+b*c)*f*Shi(3*(-a*d+
b*c)*f/d/(d*x+c))*sinh(3*e+3*b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*e)/
(d*x+c))^3/d
```

Rubi [A]

time = 0.34, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5728, 5726, 3394, 3384, 3379, 3382}

$$-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}-\frac{3f(bc-ad)\sinh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*
x))])/(4*d^2) + (3*(b*c - a*d)*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c
 - a*d)*f)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b
*f*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b
*c - a*d)*f)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*f*Sinh[3*(e + (b*f)/d
)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \int \sinh^3\left(\frac{ce+af+(de+bf)x}{c+dx}\right)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(3(bc-ad)f)\text{Subst}\left(\int\left(-\frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc}{}\right.}{4x}\right.\right.}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{(3(bc-ad)f)\text{Subst}\left(\int \frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)f}{d}\right.}{x}\right.}{4d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh(}{}\right.}{4d^2}$$

$$= -\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{C}}{4d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 671 vs. 2(226) = 452.
time = 4.21, size = 671, normalized size = 2.97

$$\frac{6bcf\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\mathrm{Chi}\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)-6adf\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\mathrm{Chi}\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)+3(bc-ad)f\mathrm{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)\left(-\cosh\left(e+\frac{bf}{d}\right)+\sinh\left(e+\frac{bf}{d}\right)\right)-3(bc-ad)f\mathrm{Chi}\left(\frac{-bcf+adf}{d(c+dx)}\right)\left(\cosh\left(e+\frac{bf}{d}\right)+\sinh\left(e+\frac{bf}{d}\right)\right)-6cd\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)-6d^2x\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)+2cd\sinh\left(\frac{3(ce+af+dex+bfx)}{c+dx}\right)+2d^2x\sinh\left(\frac{3(ce+af+dex+bfx)}{c+dx}\right)-3bcf\cosh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+3adf\cosh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)+3bcf\sinh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)-3adf\sinh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)-3bcf\cosh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)+3adf\cosh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)-3bcf\sinh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)+3adf\sinh\left(e+\frac{bf}{d}\right)\mathrm{Shi}\left(\frac{-bcf+adf}{d(c+dx)}\right)+6bcf\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\mathrm{Shi}\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)-6adf\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\mathrm{Shi}\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (6*b*c*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c +
d*x))] - 6*a*d*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(-(b*c*f) + a*d*f))/
(d*(c + d*x))] + 3*(b*c - a*d)*f*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
]*(-Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) - 3*(b*c - a*d)*f*CoshIntegral[(
-(b*c*f) + a*d*f)/(d*(c + d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) -
6*c*d*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)] - 6*d^2*x*Sinh[(c*e + a*f
 + d*e*x + b*f*x)/(c + d*x)] + 2*c*d*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(
c + d*x)] + 2*d^2*x*Sinh[(3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)] - 3*b*c
*f*Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*a*d*f*
Cosh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] + 3*b*c*f*Sin
h[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*a*d*f*Sinh[e
 + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))] - 3*b*c*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Cosh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] - 3*b*c*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 3*a*d*f*Sinh[e +
(b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(c + d*x))] + 6*b*c*f*Sinh[3*(e
 + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))] - 6*a*d*f*S
inh[3*(e + (b*f)/d)]*SinhIntegral[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))])/(8
*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. 2(220) = 440.
time = 21.26, size = 922, normalized size = 4.08

method	result
risch	$-\frac{f\,\mathrm{e}^{-\frac{3(bfx+dex+fa+ce)}{dx+c}}a}{8\left(\frac{dfa}{dx+c}-\frac{fbc}{dx+c}\right)}+\frac{f\,\mathrm{e}^{-\frac{3(bfx+dex+fa+ce)}{dx+c}}bc}{8d\left(\frac{dfa}{dx+c}-\frac{fbc}{dx+c}\right)}+\frac{3f\,\mathrm{e}^{-\frac{3(bf+de)}{d}}\,\mathrm{expIntegral}\left(1,\frac{3(ad-bc)f}{d(dx+c)}\right)a}{8d}-\frac{3f\,\mathrm{e}^{-\frac{3(bf+de)}{d}}\,\mathrm{expInte}}{8d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*c)*
a+1/8/d*f*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*
c)*b*c+3/8/d*f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*a-3/8/d^2*
f*exp(-3*(b*f+d*e)/d)*Ei(1,3*(a*d-b*c)*f/d/(d*x+c))*b*c+3/8*f*exp(-(b*f*x+d
```

```
*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*c)*a-3/8/d*f*exp(-(b*f*x+
d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-f/(d*x+c)*b*c)*b*c-3/8/d*f*exp(-(b*f
+d*e)/d)*Ei(1,(a*d-b*c)*f/d/(d*x+c))*a+3/8/d^2*f*exp(-(b*f+d*e)/d)*Ei(1,(a*
d-b*c)*f/d/(d*x+c))*b*c+1/8/d*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*
x+c)*a-f/d/(d*x+c)*b*c)*a-1/8/d^2*f*exp(3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f
/(d*x+c)*a-f/d/(d*x+c)*b*c)*b*c+3/8/d*f*exp(3*(b*f+d*e)/d)*Ei(1,-3*(a*d-b*c
)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*a-3/8/d^2*f*exp(3*(b*f+d*e)/d)*
Ei(1,-3*(a*d-b*c)*f/d/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*b*c-3/8/d*f*exp
((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*a+3/8/d^2*f*e
xp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-f/d/(d*x+c)*b*c)*b*c-3/8/d*f
*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a+3
/8/d^2*f*exp((b*f+d*e)/d)*Ei(1,-(a*d-b*c)*f/d/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e
)/d)*b*c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sinh((b*x + a)*f/(d*x + c) + e)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1092 vs. 2(226) = 452.
time = 0.52, size = 1092, normalized size = 4.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((b*f*x + a*f
 + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2*cosh(3*(b*f + d*cosh(
1) + d*sinh(1))/d)*sinh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1
))/(d*x + c))^2 - 3*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh(3
*(b*f + d*cosh(1) + d*sinh(1))/d)*sinh((b*f*x + a*f + (d*x + c)*cosh(1) + (
d*x + c)*sinh(1))/(d*x + c))^4 - 2*(d^2*x + c*d)*sinh((b*f*x + a*f + (d*x +
 c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^3 - 3*((b*c - a*d)*f*Ei(-3*(b*c
 - a*d)*f/(d^2*x + c*d))*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*
sinh(1))/(d*x + c))^4 + (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*co
sh(3*(b*f + d*cosh(1) + d*sinh(1))/d) + 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(
d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*cosh((b*f +
 d*cosh(1) + d*sinh(1))/d) + 6*(d^2*x - (d^2*x + c*d)*cosh((b*f*x + a*f + (
d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2 + c*d)*sinh((b*f*x + a*f
```

```
 + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c)) - 3*((b*c - a*d)*f*Ei(
-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*
x + c)*sinh(1))/(d*x + c))^4 - 2*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x +
 c*d))*cosh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c)
)^2*sinh((b*f*x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2
 + (b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*sinh((b*f*x + a*f + (d*
x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^4 - (b*c - a*d)*f*Ei(3*(b*c
- a*d)*f/(d^2*x + c*d)))*sinh(3*(b*f + d*cosh(1) + d*sinh(1))/d) - 3*((b*c
- a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/
(d^2*x + c*d)))*sinh((b*f + d*cosh(1) + d*sinh(1))/d))/(d^2*cosh((b*f*x + a
*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^4 - 2*d^2*cosh((b*f*
x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2*sinh((b*f*x +
 a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^2 + d^2*sinh((b*f*
x + a*f + (d*x + c)*cosh(1) + (d*x + c)*sinh(1))/(d*x + c))^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3021 vs. $2(220) = 440$.
time = 25.07, size = 3021, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2*d*e*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*
x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b*c*d^2*e*f^2*Ei(-3*(d*e + b*f - (d*e*
x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*d^3*e*f^
2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e
+ b*f)/d) + 3*b^3*c^2*f^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/
(d*x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b^2*c*d*f^3*Ei(-3*(d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*b*d^2*f
^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e
 + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)
*d/(d*x + c))/d)*e^((d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei(-(d*e + b*f - (d*
e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a^2*d^3*e*f^
2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*
```

f)/d) - 3*b^3*c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + 6*a*b^2*c*d*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a^2*b*d^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*a^2*d^3*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*b^3*c^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 6*a*b^2*c*d*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*a^2*b*d^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 3*b^2*c^2*d*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 6*a*b*c*d^2*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*a^2*d^3*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*b^3*c^2*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 6*a*b^2*c*d*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*a^2*b*d^2*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) + 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) - 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) - 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) + 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) + b^2*c^2*d*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 3*b^2*c^2*d*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 6*a*b*c*d^

```
2*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 3*a^2*d^3*f^2*e^((d*e*x +
 b*f*x + c*e + a*f)/(d*x + c)) + 3*b^2*c^2*d*f^2*e^(-(d*e*x + b*f*x + c*e +
 a*f)/(d*x + c)) - 6*a*b*c*d^2*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c
)) + 3*a^2*d^3*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - b^2*c^2*d*f
^2*e^(-3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 2*a*b*c*d^2*f^2*e^(-3*(d*
e*x + b*f*x + c*e + a*f)/(d*x + c)) - a^2*d^3*f^2*e^(-3*(d*e*x + b*f*x + c*
e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*d/(b*c*
f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e + a*f)*d^3/(d*x + c))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^3,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^3, x)
```

3.301 $\int e^{a+bx}\sinh^4(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b+1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/12*exp(3*b*x+3
*a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 276}

$$-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] -1/48*E^(-3*a - 3*b*x)/b + E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) - E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh^4(a+bx)\,dx=\frac{\text{Subst}\left(\int\frac{(1-x^2)^4}{16x^4}\,dx,x,e^{a+bx}\right)}{b}$$
$$=\frac{\text{Subst}\left(\int\frac{(1-x^2)^4}{x^4}\,dx,x,e^{a+bx}\right)}{16b}$$
$$=\frac{\text{Subst}\left(\int\left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)\,dx,x,e^{a+bx}\right)}{16b}$$
$$=-\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.75

$$\frac{e^{-3(a+bx)}\left(-5+60e^{2(a+bx)}+90e^{4(a+bx)}-20e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] (-5 + 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) - 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A]
time = 0.91, size = 80, normalized size = 0.96

method	result	size
risch	$-\frac{e^{-3bx-3a}}{48b}+\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$	69
default	$\frac{\sinh(bx+a)}{8b}-\frac{\sinh(3bx+3a)}{16b}+\frac{\sinh(5bx+5a)}{80b}+\frac{5\cosh(bx+a)}{8b}-\frac{5\cosh(3bx+3a)}{48b}+\frac{\cosh(5bx+5a)}{80b}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sinh(b*x+a)/b-1/16*sinh(3*b*x+3*a)/b+1/80*sinh(5*b*x+5*a)/b+5/8*cosh(b*
x+a)/b-5/48/b*cosh(3*b*x+3*a)+1/80/b*cosh(5*b*x+5*a)
```

Maxima [A]
time = 0.26, size = 68, normalized size = 0.82

$$\frac{e^{(5\,bx+5\,a)}}{80\,b}-\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}+\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b - 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Fricas [A]
time = 0.46, size = 113, normalized size = 1.36

$$-\frac{\cosh(bx+a)^4-16\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-10\right)\sinh(bx+a)^2-20\cosh(bx+a)^2-16\left(\cosh(bx+a)^3-5\cosh(bx+a)\right)\sinh(bx+a)-45}{120\left(b\cosh(bx+a)-b\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b*x + a)^2 - 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(65) = 130$.
time = 3.25, size = 139, normalized size = 1.67

$$\begin{cases} \frac{e^{a}e^{bx}\sinh^{4}(a+bx)}{5b}+\frac{4e^{a}e^{bx}\sinh^{3}(a+bx)\cosh(a+bx)}{5b}-\frac{4e^{a}e^{bx}\sinh^{2}(a+bx)\cosh^{2}(a+bx)}{5b}-\frac{8e^{a}e^{bx}\sinh(a+bx)\cosh^{3}(a+bx)}{15b}+\frac{8e^{a}e^{bx}\cosh^{4}(a+bx)}{15b} & \text{for } b\neq 0 \\ xe^{a}\sinh^{4}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh(
a + b*x)**3*cosh(a + b*x)/(5*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b)
+ 8*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**
4, True))
```

Giac [A]
time = 0.43, size = 60, normalized size = 0.72

$$\frac{5\left(12\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}+3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}+90\,e^{(bx+a)}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/240*(5*(12*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) - 20
*e^(3*b*x + 3*a) + 90*e^(b*x + a))/b
```

Mupad [B]
time = 0.53, size = 58, normalized size = 0.70

$$\frac{90\,\mathrm{e}^{a+b\,x}+60\,\mathrm{e}^{-a-b\,x}-5\,\mathrm{e}^{-3\,a-3\,b\,x}-20\,\mathrm{e}^{3\,a+3\,b\,x}+3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x)^4,x)
```

```
[Out] (90*exp(a + b*x) + 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) - 20*exp(3*a +
3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

3.302 $\int e^{a+bx}\sinh^3(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] 1/16*exp(-2*b*x-2*a)/b-3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 272, 45}

$$\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) - (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32*
b) + (3*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\sinh^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{16b}\\ &= \frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.79

$$\frac{e^{-2(a+bx)}-3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^3,x]
[Out] (E^(-2*(a + b*x)) - 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Maple [A]
time = 0.73, size = 61, normalized size = 1.07

method	result	size
risch	$\frac{e^{-2bx-2a}}{16b}-\frac{3\,e^{2bx+2a}}{16b}+\frac{e^{4bx+4a}}{32b}+\frac{3x}{8}$	47
default	$\frac{3x}{8}-\frac{\sinh(2bx+2a)}{4b}+\frac{\sinh(4bx+4a)}{32b}-\frac{\cosh(2bx+2a)}{8b}+\frac{\cosh(4bx+4a)}{32b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 3/8*x-1/4*sinh(2*b*x+2*a)/b+1/32*sinh(4*b*x+4*a)/b-1/8/b*cosh(2*b*x+2*a)+1/
32/b*cosh(4*b*x+4*a)
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.93

$$\frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}-\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}+\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b - 3/16*e^(2*b*x + 2*a)/b + 1/16*e^
(-2*b*x - 2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.
time = 0.53, size = 95, normalized size = 1.67

$$\frac{3\,\cosh\left(bx+a\right)^{3}+9\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-\sinh\left(bx+a\right)^{3}+6\,(2\,bx-1)\cosh\left(bx+a\right)-3\,\left(4\,bx+\cosh\left(bx+a\right)^{2}+2\right)\sinh\left(bx+a\right)}{32\,(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 + 6*(2*b*x - 1)*cosh(b*x + a) - 3*(4*b*x + cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(48) = 96$.
time = 1.23, size = 207, normalized size = 3.63

$$\begin{cases}\frac{3xe^{a}e^{bx}\sinh^{3}\left(a+bx\right)}{8}-\frac{3xe^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{8}-\frac{3xe^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{8}+\frac{3xe^{a}e^{bx}\cosh^{3}\left(a+bx\right)}{8}-\frac{3e^{a}e^{bx}\sinh^{3}\left(a+bx\right)}{8b}+\frac{e^{a}e^{bx}\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{b}+\frac{e^{a}e^{bx}\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{4b}-\frac{5e^{a}e^{bx}\cosh^{3}\left(a+bx\right)}{8b} & \text{for}\: b\neq 0\\xe^{a}\sinh^{3}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 3*exp(a)*exp(b*x)*sinh
(a + b*x)**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/b + exp
(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/(4*b) - 5*exp(a)*exp(b*x)*cosh(
a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3, True))
```

Giac [A]
time = 0.43, size = 57, normalized size = 1.00

$$\frac{12\,bx-2\,\left(3\,e^{(2\,bx+2\,a)}-1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}-6\,e^{(2\,bx+2\,a)}}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) - 6*e^(2*b*x + 2*a))/b
```

Mupad [B]
time = 0.80, size = 42, normalized size = 0.74

$$\frac{3\,x}{8}+\frac{\frac{{\mathrm{e}}^{-2\,a-2\,b\,x}}{16}-\frac{3\,{\mathrm{e}}^{2\,a+2\,b\,x}}{16}+\frac{{\mathrm{e}}^{4\,a+4\,b\,x}}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*x)/8 + (exp(- 2*a - 2*b*x)/16 - (3*exp(2*a + 2*b*x))/16 + exp(4*a + 4*b*
x)/32)/b
```

3.303 $\int e^{a+bx} \sinh^2(a+bx)\, dx$

Optimal. Leaf size=49

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

```
[Out] -1/4*exp(-b*x-a)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 276}

$$-\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*E^(-a - b*x)/b - E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b}$$
$$= -\frac{e^{-a-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$\frac{e^{-a-bx}\left(-3-6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^2,x]
[Out] (E^(-a - b*x)*(-3 - 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Maple [A]
time = 0.52, size = 52, normalized size = 1.06

method	result	size
risch	$-\frac{e^{-bx-a}}{4b}-\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$	41
default	$-\frac{\sinh(bx+a)}{4b}+\frac{\sinh(3bx+3a)}{12b}-\frac{3\cosh(bx+a)}{4b}+\frac{\cosh(3bx+3a)}{12b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/4*sinh(b*x+a)/b+1/12*sinh(3*b*x+3*a)/b-3/4*cosh(b*x+a)/b+1/12/b*cosh(3*b
*x+3*a)
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.82

$$\frac{e^{(3\,bx+3\,a)}}{12\,b} - \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Fricas [A]
time = 0.51, size = 54, normalized size = 1.10

$$-\frac{\cosh\left(bx+a\right)^{2}-4\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+3}{6\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.
time = 0.49, size = 78, normalized size = 1.59

$$\begin{cases} \frac{e^{a}e^{bx}\sinh^{2}\left(a+bx\right)}{3b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}-\frac{2e^{a}e^{bx}\cosh^{2}\left(a+bx\right)}{3b} & \text{for } b\neq 0 \\ xe^{a}\sinh^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh(
a + b*x)*cosh(a + b*x)/(3*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne
(b, 0)), (x*exp(a)*sinh(a)**2, True))
```

Giac [A]
time = 0.40, size = 34, normalized size = 0.69

$$\frac{e^{(3\,bx+3\,a)}-6\,e^{(bx+a)}-3\,e^{(-bx-a)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*(e^(3*b*x + 3*a) - 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

Mupad [B]
time = 0.61, size = 36, normalized size = 0.73

$$-\frac{6\,e^{a+b\,x}+3\,e^{-a-b\,x}-e^{3\,a+3\,b\,x}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x)^2,x)
```

```
[Out] -(6*exp(a + b*x) + 3*exp(- a - b*x) - exp(3*a + 3*b*x))/(12*b)
```

3.304 $\int e^{a+bx} \sinh(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 12, 14}

$$\frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Maple [A]
time = 0.34, size = 37, normalized size = 1.61

method	result	size
risch	$\frac{e^{2bx+2a}}{4b}-\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\left(\cosh^2(bx+a)\right)}{2}}{b}$	37
default	$\frac{\frac{\sinh(bx+a)\cosh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}+\frac{\left(\cosh^2(bx+a)\right)}{2}}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*sinh(b*x+a)*cosh(b*x+a)-1/2*b*x-1/2*a+1/2*cosh(b*x+a)^2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.04

$$-\frac{1}{2}\,x - \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.
time = 0.40, size = 50, normalized size = 2.17

$$-\frac{(2\,bx-1)\cosh{(bx+a)}-(2\,bx+1)\sinh{(bx+a)}}{4\,(b\cosh{(bx+a)}-b\sinh{(bx+a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*x - 1)*cosh(b*x + a) - (2*b*x + 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.
time = 0.21, size = 63, normalized size = 2.74

$$\begin{cases} \frac{xe^{a}e^{bx}\sinh{(a+bx)}}{2}-\frac{xe^{a}e^{bx}\cosh{(a+bx)}}{2}+\frac{e^{a}e^{bx}\cosh{(a+bx)}}{2b} & \text{for}\ b\neq 0\\ xe^{a}\sinh{(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)/2 - x*exp(a)*exp(b*x)*cosh(a + b
*x)/2 + exp(a)*exp(b*x)*cosh(a + b*x)/(2*b), Ne(b, 0)), (x*exp(a)*sinh(a),
True))
```

Giac [A]
time = 0.42, size = 24, normalized size = 1.04

$$-\frac{2\,bx+2\,a-e^{(2\,bx+2\,a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a))/b
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.78

$$\frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*sinh(a + b*x),x)
```

```
[Out] exp(2*a + 2*b*x)/(4*b) - x/2
```

3.305 $\int e^{a+bx}\operatorname{csch}(a+bx)\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

[Out] ln(1-exp(2*b*x+2*a))/b

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 12, 266}

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(a + b*x)*Csch[a + b*x],x]

[Out] Log[1 - E^(2*a + 2*b*x)]/b

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{2\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx,x,e^{a+bx}\right)}{b}$$
$$= \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x],x]
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Maple [A]
time = 0.40, size = 17, normalized size = 0.89

method	result	size
derivativedivides	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
default	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(b*x+a+ln(sinh(b*x+a)))
```

Maxima [A]
time = 0.26, size = 27, normalized size = 1.42

$$\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Fricas [A]
time = 0.48, size = 30, normalized size = 1.58

$$\frac{\log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x), x)
```

Giac [A]
time = 0.41, size = 24, normalized size = 1.26

$$\frac{\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] (log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.84

$$\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x),x)
[Out] log(exp(2*a + 2*b*x) - 1)/b
```

3.306 $\int e^{a+bx}\mathbf{csch}^2(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 294, 212}

$$\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^2,x]
```

```
[Out] (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{4\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.90

$$\frac{2\left(-\frac{e^{a+bx}}{-1+e^{2(a+bx)}} - \tanh^{-1}\left(e^{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^2,x]
[Out] (2*(-(E^(a + b*x)/(-1 + E^(2*(a + b*x)))) - ArcTanh[E^(a + b*x)]))/b
```

Maple [A]
time = 0.35, size = 25, normalized size = 0.60

method	result	size
derivativedivides	$\frac{-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{\sinh(bx+a)}}{b}$	25
default	$\frac{-2\,\mathrm{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{\sinh(bx+a)}}{b}$	25
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-2*arctanh(exp(b*x+a))-1/sinh(b*x+a))
```

Maxima [A]
time = 0.26, size = 52, normalized size = 1.24

$$-\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b
*x + 2*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(37) = 74.
time = 0.51, size = 157, normalized size = 3.74

$$-\frac{\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+2\,\cosh\left(bx+a\right)+2\,\sinh\left(bx+a\right)}{b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 2*cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**2, x)
```

Giac [A]
time = 0.42, size = 48, normalized size = 1.14

$$-\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) + log(e^(b*x + a) + 1) - log(abs(e^(b
*x + a) - 1)))/b
```

Mupad [B]
time = 0.62, size = 52, normalized size = 1.24

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^2,x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x)
)/(b*(exp(2*a + 2*b*x) - 1))
```

3.307 $\int e^{a+bx}\text{csch}^3(a+bx)\,dx$

Optimal. Leaf size=31

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] -2*exp(4*b*x+4*a)/b/(1-exp(2*b*x+2*a))^2
```

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 270}

$$-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{8\text{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}$$
$$= -\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.94

$$-\frac{2e^{4a+4bx}}{b\left(-1+e^{2a+2bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^3,x]
[Out] (-2*E^(4*a + 4*b*x))/(b*(-1 + E^(2*a + 2*b*x))^2)
```

Maple [A]
time = 1.13, size = 32, normalized size = 1.03

method	result	size
risch	$-\frac{2\left(2\,e^{2bx+2a}-1\right)}{b\left(e^{2bx+2a}-1\right)^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -2*(2*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 0.27, size = 68, normalized size = 2.19

$$-\frac{4\,e^{(2\,bx+2\,a)}}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}+\frac{2}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1)) + 2/(b*(e^
(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.
time = 0.45, size = 88, normalized size = 2.84

$$-\frac{2\left(\cosh\left(bx+a\right)+3\,\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^{3}+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}+b\sinh\left(bx+a\right)^{3}-b\cosh\left(bx+a\right)+3\,\left(b\cosh\left(bx+a\right)^{2}-b\right)\sinh\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**3, x)
```

Giac [A]
time = 0.43, size = 31, normalized size = 1.00

$$-\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B]
time = 0.58, size = 31, normalized size = 1.00

$$-\frac{2\left(2\,e^{2\,a+2\,b\,x}-1\right)}{b\left(e^{2\,a+2\,b\,x}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^3,x)
```

```
[Out] -(2*(2*exp(2*a + 2*b*x) - 1))/(b*(exp(2*a + 2*b*x) - 1)^2)
```

3.308 $\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

```
[Out] 8/3*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))
^2+exp(b*x+a)/b/(1-exp(2*b*x+2*a))+arctanh(exp(b*x+a))/b
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2320, 12, 294, 205, 212}

$$\frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3}+\frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (8*E^(3*a + 3*b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1 -
 E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{16x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\operatorname{Subst}\left(\int \frac{x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{8\operatorname{Subst}\left(\int \frac{x^2}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{2\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2} + \frac{e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} + \frac{\tanh^{-1}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 75, normalized size = 0.74

$$
\frac{3e^{a+bx} - 8e^{3(a+bx)} - 3e^{5(a+bx)} + 3\left(-1+e^{2(a+bx)}\right)^3\tanh^{-1}\left(e^{a+bx}\right)}{3b\left(-1+e^{2(a+bx)}\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (3*E^(a + b*x) - 8*E^(3*(a + b*x)) - 3*E^(5*(a + b*x)) + 3*(-1 + E^(2*(a +
b*x)))^3*ArcTanh[E^(a + b*x)])/(3*b*(-1 + E^(2*(a + b*x)))^3)
```

Maple [A]
time = 0.96, size = 78, normalized size = 0.77

method	result	size
risch	$-\frac{e^{bx+a}\left(3\,e^{4bx+4a}+8\,e^{2bx+2a}-3\right)}{3b\left(e^{2bx+2a}-1\right)^{3}}+\frac{\ln\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}-1\right)}{2b}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*exp(b*x+a)*(3*exp(4*b*x+4*a)+8*exp(2*b*x+2*a)-3)/b/(exp(2*b*x+2*a)-1)^
3+1/2/b*ln(exp(b*x+a)+1)-1/2/b*ln(exp(b*x+a)-1)
```

Maxima [A]
time = 0.27, size = 100, normalized size = 0.99

$$\frac{\log\left(e^{(bx+a)}+1\right)}{2\,b}-\frac{\log\left(e^{(bx+a)}-1\right)}{2\,b}-\frac{3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{3\,b(e^{(6\,bx+6\,a)}-3\,e^{(4\,bx+4\,a)}+3\,e^{(2\,bx+2\,a)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/2*log(e^(b*x + a) + 1)/b - 1/2*log(e^(b*x + a) - 1)/b - 1/3*(3*e^(5*b*x +
 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(6*b*x + 6*a) - 3*e^(4*b*x
 + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(87) = 174$.
time = 0.57, size = 705, normalized size = 6.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^3 + 16*cosh(b*x + a)^3 + 12*
(5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
```

```
3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 -
 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 8*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a) - 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b
)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x +
a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b
*x + a))*sinh(b*x + a) - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{4}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**4, x)
```

Giac [A]
time = 0.42, size = 75, normalized size = 0.74

$$-\frac{\frac{2\left(3\,e^{(5\,bx+5\,a)}+8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^{3}}-3\,\log\left(e^{(bx+a)}+1\right)+3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*e^(5*b*x + 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x +
 2*a) - 1)^3 - 3*log(e^(b*x + a) + 1) + 3*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.60, size = 135, normalized size = 1.34

$$\frac{\operatorname{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{8\,e^{3\,a+3\,b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1\right)}-\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^4,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*
exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a + b*
x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.309 $\int e^{a+bx}\mathbf{csch}^5(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] -4/b/(1-exp(2*b*x+2*a))^4+32/3/b/(1-exp(2*b*x+2*a))^3-8/b/(1-exp(2*b*x+2*a)
)^2
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2320, 12, 272, 45}

$$-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 - E^(2*a + 2*b*x))^4) + 32/(3*b*(1 - E^(2*a + 2*b*x))^3) - 8/(b*(1
 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{32\operatorname{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{16\operatorname{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b} \\
&= \frac{16\operatorname{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)dx, x, e^{2a+2bx}\right)}{b} \\
&= -\frac{4}{b\,(1-e^{2a+2bx})^4}+\frac{32}{3b\,(1-e^{2a+2bx})^3}-\frac{8}{b\,(1-e^{2a+2bx})^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.67

$$-\frac{4\left(1-4e^{2(a+bx)}+6e^{4(a+bx)}\right)}{3b\left(-1+e^{2(a+bx)}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^5,x]
[Out] (-4*(1 - 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(-1 + E^(2*(a + b*x))
)^4)
```

Maple [A]
time = 0.99, size = 43, normalized size = 0.65

method	result	size
risch	$-\frac{4\left(6\,\mathrm{e}^{4bx+4a}-4\,\mathrm{e}^{2bx+2a}+1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*csch(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -4/3*(6*exp(4*b*x+4*a)-4*exp(2*b*x+2*a)+1)/b/(exp(2*b*x+2*a)-1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.
time = 0.26, size = 172, normalized size = 2.61

$$-\frac{8\, e^{(4\, bx+4\, a)}}{b(e^{(8\, bx+8\, a)} - 4\, e^{(6\, bx+6\, a)} + 6\, e^{(4\, bx+4\, a)} - 4\, e^{(2\, bx+2\, a)} + 1)} + \frac{16\, e^{(2\, bx+2\, a)}}{3\, b(e^{(8\, bx+8\, a)} - 4\, e^{(6\, bx+6\, a)} + 6\, e^{(4\, bx+4\, a)} - 4\, e^{(2\, bx+2\, a)} + 1)} - \frac{4}{3\, b(e^{(8\, bx+8\, a)} - 4\, e^{(6\, bx+6\, a)} + 6\, e^{(4\, bx+4\, a)} - 4\, e^{(2\, bx+2\, a)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) - 4*e^(2*b*x + 2*a) + 1)) + 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) -
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a)
+ 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(55) = 110$.
time = 0.39, size = 233, normalized size = 3.53

$$-\frac{4\,\left(7\,\cosh\left(bx+a\right)^{2} + 10\,\cosh\left(bx+a\right)\sinh\left(bx+a\right) + 7\,\sinh\left(bx+a\right)^{2} - 4\right)}{3\,\left(b\cosh\left(bx+a\right)^{6} + 6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5} + b\sinh\left(bx+a\right)^{6} - 4\,b\cosh\left(bx+a\right)^{4} + \left(15\,b\cosh\left(bx+a\right)^{2} - 4\,b\right)\sinh\left(bx+a\right)^{4} + 4\,\left(5\,b\cosh\left(bx+a\right)^{3} - 4\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^{3} + 7\,b\cosh\left(bx+a\right)^{2} + \left(15\,b\cosh\left(bx+a\right)^{4} - 24\,b\cosh\left(bx+a\right)^{2} + 7\,b\right)\sinh\left(bx+a\right)^{2} + 2\,\left(3\,b\cosh\left(bx+a\right)^{5} - 8\,b\cosh\left(bx+a\right)^{3} + 5\,b\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right) - 4\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 - 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
+ a)^6 - 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 - 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 - 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 - 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) - 4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{a}\int e^{bx}\operatorname{csch}^{5}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**5,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**5, x)
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.64

$$-\frac{4\,\left(6\,e^{(4\,bx+4\,a)} - 4\,e^{(2\,bx+2\,a)} + 1\right)}{3\,b(e^{(2\,bx+2\,a)} - 1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^4
)
```

Mupad [B]
time = 0.60, size = 42, normalized size = 0.64

$$-\frac{4\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)/sinh(a + b*x)^5,x)
```

```
[Out] -(4*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) -
 1)^4)
```

3.310 $\int e^x \sinh^2(2x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[Out] -1/12/exp(3*x)-1/2*exp(x)+1/20*exp(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh^2(2x)\,dx &= \text{Subst}\left(\int \frac{(1-x^4)^2}{4x^4}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^4)^2}{x^4}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)\\ &= -\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

Maple [A]
time = 0.51, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{5x}}{20}-\frac{e^x}{2}-\frac{e^{-3x}}{12}$	18
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}-\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sinh(x)+1/12*sinh(3*x)+1/20*sinh(5*x)-1/2*cosh(x)-1/12*cosh(3*x)+1/20*
cosh(5*x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.65

$$\frac{1}{20}\,e^{(5\,x)}-\frac{1}{12}\,e^{(-3\,x)}-\frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="maxima")
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.
time = 0.55, size = 47, normalized size = 1.81

$$-\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 15}{30(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="fricas")
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 + 15)/(cosh(x) - sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.19, size = 42, normalized size = 1.62

$$\frac{7e^x\sinh^2(2x)}{15} + \frac{4e^x\sinh(2x)\cosh(2x)}{15} - \frac{8e^x\cosh^2(2x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)**2,x)
[Out] 7*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 - 8*exp(x)*cosh(
2*x)**2/15
```

Giac [A]
time = 0.41, size = 17, normalized size = 0.65

$$\frac{1}{20}e^{(5x)} - \frac{1}{12}e^{(-3x)} - \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="giac")
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Mupad [B]
time = 0.09, size = 17, normalized size = 0.65

$$\frac{e^{5x}}{20} - \frac{e^{-3x}}{12} - \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)^2*exp(x),x)
[Out] exp(5*x)/20 - exp(-3*x)/12 - exp(x)/2
```

3.311 $\int e^x \sinh(2x)\,dx$

Optimal. Leaf size=19

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

```
[Out] 1/2/exp(x)+1/6*exp(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {2320, 12, 14}

$$\frac{e^{-x}}{2}+\frac{e^{3x}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[2*x],x]
```

```
[Out] 1/(2*E^x) + E^(3*x)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(2x)\,dx &= \text{Subst}\left(\int \frac{-1+x^4}{2x^2}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx, x, e^x\right)\\ &= \frac{e^{-x}}{2}+\frac{e^{3x}}{6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{6}e^{-x}\left(3+e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[2*x],x]
[Out] (3 + E^(4*x))/(6*E^x)
```

Maple [A]
time = 0.25, size = 22, normalized size = 1.16

method	result	size
risch	$\frac{e^{3x}}{6}+\frac{e^{-x}}{2}$	14
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{6}+\frac{\cosh(x)}{2}+\frac{\cosh(3x)}{6}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(2*x),x,method=_RETURNVERBOSE)
[Out] -1/2*sinh(x)+1/6*sinh(3*x)+1/2*cosh(x)+1/6*cosh(3*x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="maxima")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

Fricas [A]
time = 0.43, size = 26, normalized size = 1.37

$$\frac{2\left(\cosh\left(x\right)^{2}-\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}{3\left(\cosh\left(x\right)-\sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="fricas")
[Out] 2/3*(cosh(x)^2 - cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A]
time = 0.10, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh\left(2x\right)}{3}+\frac{2e^{x}\cosh\left(2x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x)
[Out] -exp(x)*sinh(2*x)/3 + 2*exp(x)*cosh(2*x)/3
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.68

$$\frac{1}{6}\,e^{(3\,x)}+\frac{1}{2}\,e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

Mupad [B]
time = 0.56, size = 12, normalized size = 0.63

$$\frac{\mathrm{e}^{-x}\,\left(\mathrm{e}^{4\,x}+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)*exp(x),x)
[Out] (exp(-x)*(exp(4*x) + 3))/6
```

3.312 $\int e^x \mathbf{csch}(2x)\,dx$

Optimal. Leaf size=11

$$\operatorname{ArcTan}(e^x) - \tanh^{-1}(e^x)$$

```
[Out] arctan(exp(x))-arctanh(exp(x))
```

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2320, 12, 304, 209, 212}

$$\operatorname{ArcTan}(e^x) - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x\text{csch}(2x)\,dx &= \text{Subst}\left(\int \frac{2x^2}{-1+x^4}\,dx,x,e^x\right)\\ &= 2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,e^x\right)\\ &= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)\\ &= \tan^{-1}\left(e^x\right)-\tanh^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\text{ArcTan}(e^x)-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.58, size = 34, normalized size = 3.09

method	result	size
risch	$\frac{\ln(e^x-1)}{2}-\frac{\ln(e^x+1)}{2}+\frac{i\ln(e^x+i)}{2}-\frac{i\ln(e^x-i)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(exp(x)-1)-1/2*ln(exp(x)+1)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)
```

Maxima [A]
time = 0.48, size = 18, normalized size = 1.64

$$\arctan\left(e^x\right)-\frac{1}{2}\,\log\left(e^x+1\right)+\frac{1}{2}\,\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="maxima")
```

```
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(9) = 18.
time = 0.39, size = 25, normalized size = 2.27

$$\arctan(\cosh(x) + \sinh(x)) - \frac{1}{2}\log(\cosh(x) + \sinh(x) + 1) + \frac{1}{2}\log(\cosh(x) + \sinh(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="fricas")
```

```
[Out] arctan(cosh(x) + sinh(x)) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x
) + sinh(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \operatorname{csch}(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x)
```

```
[Out] Integral(exp(x)*csch(2*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.40, size = 19, normalized size = 1.73

$$\arctan(e^x) - \frac{1}{2}\log(e^x + 1) + \frac{1}{2}\log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="giac")
```

```
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B]
time = 0.17, size = 26, normalized size = 2.36

$$\frac{\ln(4\,e^x - 4)}{2} - \frac{\ln(-4\,e^x - 4)}{2} - \operatorname{atan}\left(e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(2*x),x)
```

```
[Out] log(4*exp(x) - 4)/2 - log(- 4*exp(x) - 4)/2 - atan(exp(-x))
```

3.313 $\int e^x \operatorname{csch}^2(2x)\,dx$

Optimal. Leaf size=32

$$\frac{e^x}{1-e^{4x}}-\frac{\operatorname{ArcTan}(e^x)}{2}-\frac{1}{2}\tanh^{-1}(e^x)$$

```
[Out] exp(x)/(1-exp(4*x))-1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2320, 12, 294, 218, 212, 209}

$$-\frac{1}{2}\operatorname{ArcTan}(e^x)+\frac{e^x}{1-e^{4x}}-\frac{1}{2}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int e^x \operatorname{csch}^2(2x)\,dx &= \text{Subst}\left(\int \frac{4x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{e^x}{1-e^{4x}} - \frac{\text{ArcTan}(e^x)}{2} - \frac{1}{2}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[2*x]^2,x]
```

```
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.65, size = 46, normalized size = 1.44

method	result	size
risch	$-\frac{e^{x}}{e^{4x}-1}-\frac{\ln(e^{x}+1)}{4}+\frac{i\ln(e^{x}-i)}{4}-\frac{i\ln(e^{x}+i)}{4}+\frac{\ln(e^{x}-1)}{4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
+1/4*ln(exp(x)-1)
```

Maxima [A]
time = 0.47, size = 32, normalized size = 1.00

$$-\frac{e^{x}}{e^{(4x)}-1}-\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="maxima")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(23) = 46.
time = 0.51, size = 182, normalized size = 5.69

$$-\frac{2\left(\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\sinh(x)^{2}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}-1\right)\arctan\left(\cosh(x)+\sinh(x)\right)+\left(\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\sinh(x)^{2}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}-1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\sinh(x)^{2}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}-1\right)\log\left(\cosh(x)+\sinh(x)-1\right)+4\cosh(x)+4\sinh(x)}{4\left(\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\sinh(x)^{2}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x)
- 1) + 4*cosh(x) + 4*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}^{2}\left(2x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(2*x)**2, x)
```

Giac [A]
time = 0.41, size = 33, normalized size = 1.03

$$-\frac{e^x}{e^{(4x)}-1}-\frac{1}{2}\arctan\left(e^x\right)-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(\left|e^x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="giac")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e^x -
 1))
```

Mupad [B]
time = 0.72, size = 36, normalized size = 1.12

$$\frac{\ln\left(1-e^x\right)}{4}-\frac{\ln\left(-e^x-1\right)}{4}-\frac{\mathrm{atan}(e^x)}{2}-\frac{e^x}{e^{4x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(2*x)^2,x)
```

```
[Out] log(1 - exp(x))/4 - log(- exp(x) - 1)/4 - atan(exp(x))/2 - exp(x)/(exp(4*x)
 - 1)
```

3.314 $\int e^x \sinh^2(3x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

```
[Out] -1/20/exp(5*x)-1/2*exp(x)+1/28*exp(7*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \sinh^2(3x)\,dx = \text{Subst}\left(\int \frac{(1-x^6)^2}{4x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^6)^2}{x^6}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^6}+x^6\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x]^2,x]
[Out] -1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

Maple [A]
time = 0.43, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{7x}}{28}-\frac{e^{x}}{2}-\frac{e^{-5x}}{20}$	18
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}-\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*sinh(x)+1/20*sinh(5*x)+1/28*sinh(7*x)-1/2*cosh(x)-1/20*cosh(5*x)+1/28*
cosh(7*x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.65

$$\frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} - \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.
time = 0.51, size = 67, normalized size = 2.58

$$-\frac{\cosh(x)^{6} - 36\cosh(x)^{5}\sinh(x) + 15\cosh(x)^{4}\sinh(x)^{2} - 120\cosh(x)^{3}\sinh(x)^{3} + 15\cosh(x)^{2}\sinh(x)^{4} - 36\cosh(x)\sinh(x)^{5} + \sinh(x)^{6} + 35}{70(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 35)/(cosh(x) - sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.19, size = 42, normalized size = 1.62

$$\frac{17e^{x}\sinh^{2}(3x)}{35} + \frac{6e^{x}\sinh(3x)\cosh(3x)}{35} - \frac{18e^{x}\cosh^{2}(3x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)**2,x)
```

```
[Out] 17*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 - 18*exp(x)*cos
h(3*x)**2/35
```

Giac [A]
time = 0.42, size = 17, normalized size = 0.65

$$\frac{1}{28}e^{(7x)} - \frac{1}{20}e^{(-5x)} - \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Mupad [B]
time = 0.58, size = 17, normalized size = 0.65

$$\frac{e^{7x}}{28} - \frac{e^{-5x}}{20} - \frac{e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)^2*exp(x),x)
```

```
[Out] exp(7*x)/28 - exp(-5*x)/20 - exp(x)/2
```

3.315 $\int e^x \sinh(3x)\, dx$

Optimal. Leaf size=19

$$\frac{e^{-2x}}{4}+\frac{e^{4x}}{8}$$

[Out] 1/4/exp(2*x)+1/8*exp(4*x)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\frac{e^{-2x}}{4}+\frac{e^{4x}}{8}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[3*x],x]

[Out] 1/(4*E^(2*x)) + E^(4*x)/8

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(3x)\,dx &= \text{Subst}\left(\int \frac{-1+x^6}{2x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^6}{x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+x^3\right)\,dx, x, e^x\right)\\ &= \frac{e^{-2x}}{4}+\frac{e^{4x}}{8}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{8}e^{-2x}\left(2+e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[3*x],x]
[Out] (2 + E^(6*x))/(8*E^(2*x))
```

Maple [A]
time = 0.50, size = 26, normalized size = 1.37

method	result	size
risch	$\frac{e^{4x}}{8}+\frac{e^{-2x}}{4}$	14
default	$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}+\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(3*x),x,method=_RETURNVERBOSE)
[Out] -1/4*sinh(2*x)+1/8*sinh(4*x)+1/4*cosh(2*x)+1/8*cosh(4*x)
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.68

$$\frac{1}{8}\,e^{(4\,x)}+\frac{1}{4}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="maxima")
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.
time = 0.38, size = 40, normalized size = 2.11

$$\frac{3 \cosh(x)^3 - 3 \cosh(x)^2 \sinh(x) + 9 \cosh(x) \sinh(x)^2 - \sinh(x)^3}{8 (\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*cosh(x)^3 - 3*cosh(x)^2*sinh(x) + 9*cosh(x)*sinh(x)^2 - sinh(x)^3)/(
cosh(x) - sinh(x))
```

Sympy [A]
time = 0.09, size = 20, normalized size = 1.05

$$-\frac{e^x \sinh(3x)}{8} + \frac{3e^x \cosh(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x)
```

```
[Out] -exp(x)*sinh(3*x)/8 + 3*exp(x)*cosh(3*x)/8
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.68

$$\frac{1}{8} e^{(4x)} + \frac{1}{4} e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="giac")
```

```
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

Mupad [B]
time = 0.05, size = 12, normalized size = 0.63

$$\frac{e^{-2x} (e^{6x} + 2)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(3*x)*exp(x),x)
```

```
[Out] (exp(-2*x)*(exp(6*x) + 2))/8
```

3.316 $\int e^x \text{csch}(3x) \, dx$

Optimal. Leaf size=54

$$\frac{\text{ArcTan}\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)$$

```
[Out] 1/3*ln(1-exp(2*x))-1/6*ln(1+exp(2*x)+exp(4*x))+1/3*arctan(1/3*(1+2*exp(2*x)
)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {2320, 12, 281, 298, 31, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(1-e^{2x}\right)-\frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x],x]
```

```
[Out] ArcTan[(1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] + Log[1 - E^(2*x)]/3 - Log[1 + E^(2
*x) + E^(4*x)]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{csch}(3x)\,dx = \operatorname{Subst}\left(\int \frac{2x^3}{-1+x^6}\,dx, x, e^x\right)$$
$$= 2\operatorname{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, e^x\right)$$
$$= \operatorname{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, e^{2x}\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx, x, e^{2x}\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, e^{2x}\right)$$
$$= \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^{2x}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^{2x}\right)$$
$$= \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right) - \operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^{2x}\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 22, normalized size = 0.41

$$-\frac{1}{2}e^{4x}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x],x]
```

```
[Out] -1/2*(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, E^(6*x)])
```

Maple [C] Result contains complex when optimal does not.
time = 0.57, size = 79, normalized size = 1.46

method	result	si
risch	$\frac{\ln(e^{2x}-1)}{3} - \frac{\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6} + \frac{i\ln\left(e^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6} - \frac{\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6} - \frac{i\ln\left(e^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(exp(2*x)-1)-1/6*ln(exp(2*x)+1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)+1/2
+1/2*I*3^(1/2))*3^(1/2)-1/6*ln(exp(2*x)+1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x
)+1/2-1/2*I*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.47, size = 73, normalized size = 1.35

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,e^{x}+1)\right)+\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,e^{x}-1)\right)-\frac{1}{6}\,\log\left(e^{(2\,x)}+e^{x}+1\right)-\frac{1}{6}\,\log\left(e^{(2\,x)}-e^{x}+1\right)+\frac{1}{3}\,\log\left(e^{x}+1\right)+\frac{1}{3}\,\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) +
 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Fricas [A]
time = 0.49, size = 83, normalized size = 1.54

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(-\frac{3\,\sqrt{3}\,\cosh\left(x\right)+\sqrt{3}\,\sinh\left(x\right)}{3\,(\cosh\left(x\right)-\sinh\left(x\right))}\right)-\frac{1}{6}\,\log\left(\frac{2\,\cosh\left(x\right)^{2}+2\,\sinh\left(x\right)^{2}+1}{\cosh\left(x\right)^{2}-2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}\right)+\frac{1}{3}\,\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) - 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\,\mathrm{csch}\,(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x)
```

```
[Out] Integral(exp(x)*csch(3*x), x)
```

Giac [A]
time = 0.41, size = 43, normalized size = 0.80

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,e^{(2\,x)}+1\right)\right)-\frac{1}{6}\,\log\left(e^{(4\,x)}+e^{(2\,x)}+1\right)+\frac{1}{3}\,\log\left(\left|e^{(2\,x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1)) - 1/6*log(e^(4*x) + e^(2*x)
 + 1) + 1/3*log(abs(e^(2*x) - 1))
```

Mupad [B]
time = 0.18, size = 65, normalized size = 1.20

$$\frac{\ln\left(8\,\mathrm{e}^{2\,x}-8\right)}{3}+\ln\left(24\,\mathrm{e}^{2\,x}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(-24\,\mathrm{e}^{2\,x}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(3*x),x)
```

```
[Out] log(8*exp(2*x) - 8)/3 + log(24*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) - 8)*((3^(1/
2)*1i)/6 - 1/6) - log(- 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6) - 8)*((3^(1/2)*1
i)/6 + 1/6)
```

3.317 $\int e^x \textbf{csch}^2(3x)\,dx$

Optimal. Leaf size=105

$$\frac{2e^x}{3(1-e^{6x})}+\frac{\text{ArcTan}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\tanh^{-1}(e^x)+\frac{1}{18}\log\left(1-e^x+e^{2x}\right)-\frac{1}{18}\log\left(1+e^x+e^{2x}\right)$$

```
[Out] 2/3*exp(x)/(1-exp(6*x))-2/9*arctanh(exp(x))+1/18*ln(1-exp(x)+exp(2*x))-1/18
*ln(1+exp(x)+exp(2*x))+1/9*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-1/9*arc
tan(1/3*(1+2*exp(x))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {2320, 12, 294, 216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e^x}{3(1-e^{6x})}+\frac{1}{18}\log\left(-e^x+e^{2x}+1\right)-\frac{1}{18}\log\left(e^x+e^{2x}+1\right)-\frac{2}{9}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - ArcTa
n[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x + E^(
2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{csch}^2(3x)\,dx = \text{Subst}\left(\int \frac{4x^6}{(1-x^6)^2}\,dx, x, e^x\right)$$
$$= 4\text{Subst}\left(\int \frac{x^6}{(1-x^6)^2}\,dx, x, e^x\right)$$
$$= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)$$
$$= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Su}$$
$$= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right) - \frac{1}{18}\text{Subst}\left(\int \frac{1+}{1+x}\right.$$
$$= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right) + \frac{1}{3}\text{Subst}\Big($$
$$= \frac{2e^x}{3(1-e^{6x})} + \frac{\tan^{-1}\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2}{9}\tanh^{-1}(e^x) + \frac{1}{18}\log\left(1-e^x+e^{2x}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 34, normalized size = 0.32

$$\frac{2}{3}e^x\left(\frac{1}{1-e^{6x}} - {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; e^{6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x*((1 - E^(6*x))^(-1) - Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]))/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.66, size = 148, normalized size = 1.41

method	result
risch	$-\frac{2\,e^x}{3(e^{6x}-1)} + \frac{\ln\left(e^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18} + \frac{i\ln\left(e^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{18} + \frac{\ln\left(e^x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18} - \frac{i\ln\left(e^x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{18} + \frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*exp(x)/(exp(6*x)-1)+1/18*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*I*ln(exp(x)
-1/2-1/2*I*3^(1/2))*3^(1/2)+1/18*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*ln(exp
```

```
(x)-1/2+1/2*I*3^(1/2))*3^(1/2)+1/9*ln(exp(x)-1)-1/9*ln(exp(x)+1)-1/18*ln(ex
p(x)+1/2-1/2*I*3^(1/2))+1/18*I*ln(exp(x)+1/2-1/2*I*3^(1/2))*3^(1/2)-1/18*ln
(exp(x)+1/2+1/2*I*3^(1/2))-1/18*I*ln(exp(x)+1/2+1/2*I*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 85, normalized size = 0.81

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2e^{x}+1)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2e^{x}-1)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{9}\log\left(e^{x}+1\right)+\frac{1}{9}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(76) = 152.
time = 0.48, size = 560, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh
(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sin
h(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(
2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) + 2*(sqrt(3)*cosh(
x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 + 20*sq
rt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3)*cosh
(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh(x) + 2
/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) -
 (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*lo
g((2*cosh(x) - 1)/(cosh(x) - sinh(x))) + 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) - 2*(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cos
h(x) + sinh(x) - 1) + 12*cosh(x) + 12*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}^{2}(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(3*x)**2, x)
```

Giac [**A**]
time = 0.42, size = 86, normalized size = 0.82

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{x}-1\right)\right)-\frac{2e^{x}}{3\left(e^{(6x)}-1\right)}-\frac{1}{18}\log\left(e^{(2x)}+e^{x}+1\right)+\frac{1}{18}\log\left(e^{(2x)}-e^{x}+1\right)-\frac{1}{9}\log\left(e^{x}+1\right)+\frac{1}{9}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

Mupad [**B**]
time = 0.40, size = 91, normalized size = 0.87

$$\frac{\ln\left(\frac{2}{3}-\frac{2e^{x}}{3}\right)}{9}-\frac{\ln\left(-\frac{2e^{x}}{3}-\frac{2}{3}\right)}{9}+\frac{\ln\left(\left(\frac{2e^{x}}{3}-\frac{1}{3}\right)^{2}+\frac{1}{3}\right)}{18}-\frac{\ln\left(\left(\frac{2e^{x}}{3}+\frac{1}{3}\right)^{2}+\frac{1}{3}\right)}{18}-\frac{2e^{x}}{3\left(e^{6x}-1\right)}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2e^{x}}{3}-\frac{1}{3}\right)\right)}{9}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2e^{x}}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(3*x)^2,x)
```

```
[Out] log(2/3 - (2*exp(x))/3)/9 - log(- (2*exp(x))/3 - 2/3)/9 + log(((2*exp(x))/3
 - 1/3)^2 + 1/3)/18 - log(((2*exp(x))/3 + 1/3)^2 + 1/3)/18 - (2*exp(x))/(3*
(exp(6*x) - 1)) - (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 - 1/3)))/9 - (3^(1/2)
*atan(3^(1/2)*((2*exp(x))/3 + 1/3)))/9
```

3.318 $\int e^x \sinh^2(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

```
[Out] -1/28/exp(7*x)-1/2*exp(x)+1/36*exp(9*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$-\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \sinh^2(4x)\,dx = \text{Subst}\left(\int \frac{(1-x^8)^2}{4x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^8)^2}{x^8}\,dx, x, e^x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^8}+x^8\right)\,dx, x, e^x\right)$$
$$= -\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$-\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

Maple [A]
time = 0.42, size = 34, normalized size = 1.31

method	result	size
risch	$\frac{e^{9x}}{36} - \frac{e^x}{2} - \frac{e^{-7x}}{28}$	18
default	$-\frac{\sinh(x)}{2} + \frac{\sinh(7x)}{28} + \frac{\sinh(9x)}{36} - \frac{\cosh(x)}{2} - \frac{\cosh(7x)}{28} + \frac{\cosh(9x)}{36}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sinh(x)+1/28*sinh(7*x)+1/36*sinh(9*x)-1/2*cosh(x)-1/28*cosh(7*x)+1/36*
cosh(9*x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.65

$$\frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.
time = 0.43, size = 87, normalized size = 3.35

$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh(x)^4 - 448\cosh(x)^3\sinh(x)^5 + 28\cosh(x)^2\sinh(x)^6 - 64\cosh(x)\sinh(x)^7 + \sinh(x)^8 + 63}{126(\cosh(x) - \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 + 63)/(cosh(x) - sinh(
x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.17, size = 42, normalized size = 1.62

$$\frac{31e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} - \frac{32e^x\cosh^2(4x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)**2,x)
```

```
[Out] 31*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 - 32*exp(x)*cos
h(4*x)**2/63
```

Giac [A]
time = 0.40, size = 17, normalized size = 0.65

$$\frac{1}{36}e^{(9x)} - \frac{1}{28}e^{(-7x)} - \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="giac")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Mupad [B]
time = 0.08, size = 17, normalized size = 0.65

$$\frac{e^{9x}}{36} - \frac{e^{-7x}}{28} - \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)^2*exp(x),x)
```

```
[Out] exp(9*x)/36 - exp(-7*x)/28 - exp(x)/2
```

3.319 $\int e^x \sinh(4x)\, dx$

Optimal. Leaf size=19

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

[Out] 1/6/exp(3*x)+1/10*exp(5*x)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

[In] Int[E^x*Sinh[4*x],x]

[Out] 1/(6*E^(3*x)) + E^(5*x)/10

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh(4x)\,dx &= \text{Subst}\left(\int \frac{-1+x^8}{2x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^8}{x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)\\ &= \frac{e^{-3x}}{6}+\frac{e^{5x}}{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[4*x],x]
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Maple [A]
time = 0.42, size = 26, normalized size = 1.37

method	result	size
risch	$\frac{e^{5x}}{10}+\frac{e^{-3x}}{6}$	14
default	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(4*x),x,method=_RETURNVERBOSE)
[Out] -1/6*sinh(3*x)+1/10*sinh(5*x)+1/6*cosh(3*x)+1/10*cosh(5*x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{(5\,x)}+\frac{1}{6}\,e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.
time = 0.45, size = 46, normalized size = 2.42

$$\frac{4\left(\cosh\left(x\right)^{4} - \cosh\left(x\right)^{3}\sinh\left(x\right) + 6\,\cosh\left(x\right)^{2}\sinh\left(x\right)^{2} - \cosh\left(x\right)\sinh\left(x\right)^{3} + \sinh\left(x\right)^{4}\right)}{15\left(\cosh\left(x\right) - \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 4/15*(cosh(x)^4 - cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - cosh(x)*sinh(
x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A]
time = 0.10, size = 20, normalized size = 1.05

$$-\frac{e^{x}\sinh\left(4x\right)}{15} + \frac{4e^{x}\cosh\left(4x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x)
```

```
[Out] -exp(x)*sinh(4*x)/15 + 4*exp(x)*cosh(4*x)/15
```

Giac [A]
time = 0.42, size = 13, normalized size = 0.68

$$\frac{1}{10}\,e^{\left(5\,x\right)} + \frac{1}{6}\,e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

Mupad [B]
time = 0.06, size = 13, normalized size = 0.68

$$\frac{\mathrm{e}^{-3\,x}}{6} + \frac{\mathrm{e}^{5\,x}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(4*x)*exp(x),x)
```

```
[Out] exp(-3*x)/6 + exp(5*x)/10
```

3.320 $\int e^x \operatorname{csch}(4x)\,dx$

Optimal. Leaf size=113

$$-\frac{1}{2}\text{ArcTan}(e^x)-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}(e^x)-\frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}}+\frac{\log\left(\right.}{}$$

```
[Out] -1/2*arctan(exp(x))-1/2*arctanh(exp(x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/
2)+1/4*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2
^(1/2)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {2320, 12, 307, 217, 1179, 642, 1176, 631, 210, 218, 212, 209}

$$-\frac{1}{2}\text{ArcTan}(e^x)-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,e^x+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x],x]
```

```
[Out] -1/2*ArcTan[E^x] - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]
*E^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt
[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \operatorname{csch}(4x)\,dx = \operatorname{Subst}\left(\int \frac{2x^4}{-1+x^8}\,dx, x, e^x\right)$$
$$= 2\operatorname{Subst}\left(\int \frac{x^4}{-1+x^8}\,dx, x, e^x\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) + \operatorname{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.$$
$$= -\frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{}{1+}\right.$$
$$= -\frac{1}{2}\tan^{-1}(e^x) - \frac{1}{2}\tanh^{-1}(e^x) - \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Sub}}{}$$
$$= -\frac{1}{2}\tan^{-1}(e^x) - \frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{2\sqrt{2}} - \frac{1}{2}\tanh^{-1}(e^x) - \frac{\log\left(1-\right.}{4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 22, normalized size = 0.19

$$-\frac{2}{5}e^{5x}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};e^{8x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x],x]
```

```
[Out] (-2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, E^(8*x)])/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.67, size = 56, normalized size = 0.50

method	result	size
risch	$\frac{\ln(e^{x}-1)}{4}-\frac{\ln(e^{x}+1)}{4}+2\left(\sum_{\_R=\mathrm{RootOf}(4096\_Z^{4}+1)}\_R\ln\left(e^{x}+8\_R\right)\right)+\frac{i\ln(e^{x}-i)}{4}-\frac{i\ln(e^{x}+i)}{4}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(exp(x)-1)-1/4*ln(exp(x)+1)+2*sum(_R*ln(exp(x)+8*_R),_R=RootOf(4096*_
Z^4+1))+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
```

Maxima [A]
time = 0.48, size = 95, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

Fricas [A]
time = 0.46, size = 132, normalized size = 1.17

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\sqrt{2}\sqrt{\sqrt{2}e^{x}+e^{(2x)}+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}e^{x}+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^{x}+4e^{(2x)}+4}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}e^{x}+4e^{(2x)}+4\right)-\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}e^{x}+4e^{(2x)}+4\right)-\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt(2)*e^x + e^(2*x) + 1)
- 1) - 1/2*sqrt(2)*arctan(-sqrt(2)*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x +
4*e^(2*x) + 4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/8*
sqrt(2)*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\operatorname{csch}(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x)
```

```
[Out] Integral(exp(x)*csch(4*x), x)
```

Giac [A]
time = 0.44, size = 96, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(abs(e^x - 1))
```

Mupad [B]
time = 0.37, size = 106, normalized size = 0.94

$$\frac{\ln\left(128-128e^{x}\right)}{4}-\frac{\ln\left(-128e^{x}-128\right)}{4}-\frac{\mathrm{atan}(e^{x})}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(128e^{x}-64\sqrt{2}\right)}{128}\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(128e^{x}+64\sqrt{2}\right)}{128}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\left(128e^{x}-64\sqrt{2}\right)^{2}+8192\right)}{8}+\frac{\sqrt{2}\,\ln\left(\left(128e^{x}+64\sqrt{2}\right)^{2}+8192\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(4*x),x)
```

```
[Out] log(128 - 128*exp(x))/4 - log(- 128*exp(x) - 128)/4 - atan(exp(x))/2 + (2^(
1/2)*atan((2^(1/2)*(128*exp(x) - 64*2^(1/2)))/128))/4 + (2^(1/2)*atan((2^(1
/2)*(128*exp(x) + 64*2^(1/2)))/128))/4 - (2^(1/2)*log((128*exp(x) - 64*2^(1
/2))^2 + 8192))/8 + (2^(1/2)*log((128*exp(x) + 64*2^(1/2))^2 + 8192))/8
```

3.321 $\int e^x \mathbf{csch}^2(4x)\,dx$

Optimal. Leaf size=131

$$\frac{e^x}{2(1-e^{8x})}-\frac{\mathrm{ArcTan}(e^x)}{8}+\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}}-\frac{\mathrm{ArcTan}\left(1+\sqrt{2}\,e^x\right)}{8\sqrt{2}}-\frac{1}{8}\tanh^{-1}(e^x)+\frac{\log\left(1-\sqrt{2}\,e^x+\right.}{16\sqrt{2}}$$

```
[Out] 1/2*exp(x)/(1-exp(8*x))-1/8*arctan(exp(x))-1/8*arctanh(exp(x))-1/16*arctan(
-1+exp(x)*2^(1/2))*2^(1/2)-1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+1/32*ln(1+
exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$,
Rules used = {2320, 12, 294, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$-\frac{1}{8}\mathrm{ArcTan}(e^x)+\frac{\mathrm{ArcTan}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}}-\frac{\mathrm{ArcTan}\left(\sqrt{2}\,e^x+1\right)}{8\sqrt{2}}+\frac{e^x}{2(1-e^{8x})}+\frac{\log\left(-\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{\log\left(\sqrt{2}\,e^x+e^{2x}+1\right)}{16\sqrt{2}}-\frac{1}{8}\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csch[4*x]^2,x]
```

```
[Out] E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sqrt[2])
 - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqrt[2]*E
^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int e^x \text{csch}^2(4x)\,dx = \text{Subst}\left(\int \frac{4x^8}{(1-x^8)^2}\,dx, x, e^x\right)$$

$$= 4\text{Subst}\left(\int \frac{x^8}{(1-x^8)^2}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\Big($$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, e^x\right) -$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) - \frac{1}{8}\tanh^{-1}(e^x) + \frac{\log\left(1-\sqrt{2}\,e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,e^x\right.}{16\sqrt{2}}$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\tan^{-1}(e^x) + \frac{\tan^{-1}\left(1-\sqrt{2}\,e^x\right)}{8\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,e^x\right)}{8\sqrt{2}} - \frac{1}{8}\tanh^{-1}(e^{\cdot}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 34, normalized size = 0.26

$$\frac{1}{2}e^x\left(\frac{1}{1-e^{8x}}-{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{8x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csch[4*x]^2,x]
```

```
[Out] (E^x*((1 - E^(8*x))^(-1) - Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.77, size = 68, normalized size = 0.52

method	result
risch	$-\frac{e^x}{2(e^{8x}-1)}-\frac{\ln(e^x+1)}{16}+\frac{i\ln(e^x-i)}{16}-\frac{i\ln(e^x+i)}{16}+4\left(\sum_{\_R=\mathrm{RootOf}(16777216\_Z^4+1)}\_R\ln(e^x-64\_R)\right)+\frac{\ln(e}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csch(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(x)/(exp(8*x)-1)-1/16*ln(exp(x)+1)+1/16*I*ln(exp(x)-I)-1/16*I*ln(ex
p(x)+I)+4*sum(_R*ln(exp(x)-64*_R),_R=RootOf(16777216*_Z^4+1))+1/16*ln(exp(x
)-1)
```

Maxima [A]
time = 0.48, size = 107, normalized size = 0.82

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{e^x}{2\left(e^{(8x)}-1\right)}-\frac{1}{8}\arctan\left(e^x\right)-\frac{1}{16}\log\left(e^x+1\right)+\frac{1}{16}\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(88) = 176.
time = 0.40, size = 207, normalized size = 1.58

$$\frac{4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\sqrt{2}\sqrt{\sqrt{2}e^x+e^{(2x)}+1}-1\right)+4\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\arctan\left(-\sqrt{2}e^x+\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}e^x+4e^{(2x)}+4}+1\right)-4\left(e^{(8x)}-1\right)\arctan\left(e^x\right)-\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\log\left(4\sqrt{2}e^x+4e^{(2x)}+4\right)+\left(\sqrt{2}e^{(8x)}-\sqrt{2}\right)\log\left(-4\sqrt{2}e^x+4e^{(2x)}+4\right)-2\left(e^{(8x)}-1\right)\log\left(e^x+1\right)+2\left(e^{(8x)}-1\right)\log\left(e^x-1\right)-16e^x}{32\left(e^{(8x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)*e^x + sqrt(2)*sqrt(sqrt
(2)*e^x + e^(2*x) + 1) - 1) + 4*(sqrt(2)*e^(8*x) - sqrt(2))*arctan(-sqrt(2)
*e^x + 1/2*sqrt(2)*sqrt(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) + 1) - 4*(e^(8*x) -
 1)*arctan(e^x) - (sqrt(2)*e^(8*x) - sqrt(2))*log(4*sqrt(2)*e^x + 4*e^(2*x)
 + 4) + (sqrt(2)*e^(8*x) - sqrt(2))*log(-4*sqrt(2)*e^x + 4*e^(2*x) + 4) - 2
*(e^(8*x) - 1)*log(e^x + 1) + 2*(e^(8*x) - 1)*log(e^x - 1) - 16*e^x)/(e^(8*
x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x} \operatorname{csch}^{2}(4x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)**2,x)
```

```
[Out] Integral(exp(x)*csch(4*x)**2, x)
```

Giac [A]
time = 0.43, size = 108, normalized size = 0.82

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)+\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{e^{x}}{2\left(e^{(8x)}-1\right)}-\frac{1}{8}\arctan\left(e^{x}\right)-\frac{1}{16}\log\left(e^{x}+1\right)+\frac{1}{16}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

Mupad [B]
time = 0.92, size = 120, normalized size = 0.92

$$\frac{\ln\left(\frac{1}{2}-\frac{e^{x}}{2}\right)}{16}-\frac{\ln\left(-\frac{e^{x}}{2}-\frac{1}{2}\right)}{16}-\frac{\mathrm{atan}(e^{x})}{8}-\frac{e^{x}}{2\left(e^{8x}-1\right)}-\frac{\sqrt{2}\,\mathrm{atan}\left(2\sqrt{2}\left(\frac{e^{x}}{2}-\frac{\sqrt{2}}{4}\right)\right)}{16}-\frac{\sqrt{2}\,\mathrm{atan}\left(2\sqrt{2}\left(\frac{e^{x}}{2}+\frac{\sqrt{2}}{4}\right)\right)}{16}+\frac{\sqrt{2}\,\ln\left(\left(\frac{e^{x}}{2}-\frac{\sqrt{2}}{4}\right)^{2}+\frac{1}{8}\right)}{32}-\frac{\sqrt{2}\,\ln\left(\left(\frac{e^{x}}{2}+\frac{\sqrt{2}}{4}\right)^{2}+\frac{1}{8}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/sinh(4*x)^2,x)
```

```
[Out] log(1/2 - exp(x)/2)/16 - log(- exp(x)/2 - 1/2)/16 - atan(exp(x))/8 - exp(x)
/(2*(exp(8*x) - 1)) - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 - 2^(1/2)/4)))/16 -
 (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 + 2^(1/2)/4)))/16 + (2^(1/2)*log((exp(x)
/2 - 2^(1/2)/4)^2 + 1/8))/32 - (2^(1/2)*log((exp(x)/2 + 2^(1/2)/4)^2 + 1/8)
)/32
```

3.322 $\int F^{c(a+bx)}\sinh^3(d+ex)\,dx$

Optimal. Leaf size=202

$$-\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}+\frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh}{9e^2-b^2c^2\log^2(F}$$

```
[Out] -6*e^3*F^(c*(b*x+a))*cosh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^
2+b^4*c^4*ln(F)^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2/(9*e^2-b^2*c
^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)^3/(9*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A]

time = 0.06, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5584, 5582}

$$-\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}-\frac{6e^3\cosh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cosh[d + e*x])/(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b
^4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(9*e^4
- 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d
 + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*
Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F]^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)} - (
$$
$$= -\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+e}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4}$$

Mathematica [A]
time = 0.45, size = 157, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(-9e^3+b^2c^2e\log^2(F)\right)+2bc\log(F)\left(13e^2-b^2c^2\log^2(F)+\cosh(2(d+ex))\left(-e^2+b^2c^2\log^2(F)\right)\right)\sinh(d+ex)\right)}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[d
 + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Log[
F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(9*e
^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A]
time = 1.18, size = 326, normalized size = 1.61

method	result
risch	$\frac{\left(\ln(F)^3b^3c^3e^{6ex+6d}-3\ln(F)^3b^3c^3e^{4ex+4d}-3\ln(F)^2b^2c^2e\,e^{6ex+6d}+3\ln(F)^3b^3c^3e^{2ex+2d}+3\ln(F)^2b^2c^2e\,e^{4ex+4d}-\ln(F)bc\,e^2e^{6ex+6}\right.}{8(bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(ln(F)^3*b^3*c^3*exp(6*e*x+6*d)-3*ln(F)^3*b^3*c^3*exp(4*e*x+4*d)-3*ln(F
)^2*b^2*c^2*e*exp(6*e*x+6*d)+3*ln(F)^3*b^3*c^3*exp(2*e*x+2*d)+3*ln(F)^2*b^2
*c^2*e*exp(4*e*x+4*d)-ln(F)*b*c*e^2*exp(6*e*x+6*d)-ln(F)^3*b^3*c^3+3*ln(F)^
2*b^2*c^2*e*exp(2*e*x+2*d)+27*ln(F)*b*c*e^2*exp(4*e*x+4*d)+3*e^3*exp(6*e*x+
6*d)-3*ln(F)^2*b^2*c^2*e-27*ln(F)*b*c*e^2*exp(2*e*x+2*d)-27*e^3*exp(4*e*x+4
*d)+ln(F)*b*c*e^2-27*e^3*exp(2*e*x+2*d)+3*e^3)/(b*c*ln(F)-e)*exp(-3*e*x-3*d
)/(b*c*ln(F)-3*e)/(e+b*c*ln(F))/(b*c*ln(F)+3*e)*F^(c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 142, normalized size = 0.70

$$\frac{F^{ac}e^{(bcx\log(F)+3\,xe+3\,d)}}{8\,(bc\log(F)+3\,e)} - \frac{3\,F^{ac}e^{(bcx\log(F)+xe+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-xe)}}{8\,(bce^d\log(F)-e^{(d+1)})} - \frac{F^{ac}e^{(bcx\log(F)-3\,xe)}}{8\,(bce^{(3\,d)}\log(F)-3\,e^{(3\,d+1)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*x*e + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c)
*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- x*e)/(b*c*e^d*log(F) - e^(d + 1)) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*x*e)/
(b*c*e^(3*d)*log(F) - 3*e^(3*d + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4705 vs. $2(197) = 394$.
time = 0.40, size = 4705, normalized size = 23.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)
*cosh(x*cosh(1) + x*sinh(1) + d)^6 + (b^3*c^3*log(F)^3 + 3*cosh(1)^3 - 3*(b
^2*c^2*cosh(1) + b^2*c^2*sinh(1))*log(F)^2 + 9*cosh(1)^2*sinh(1) + 9*cosh(1
)*sinh(1)^2 + 3*sinh(1)^3 - (b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*si
nh(1)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^6 + 6*(b^3*c^3*cosh(x*cosh
(1) + x*sinh(1) + d)*log(F)^3 - 3*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(
x*cosh(1) + x*sinh(1) + d)*log(F)^2 - (b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1
) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)*log(F) + 3*(cosh(1)^3 +
3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*s
inh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d)^5 - 27*(cosh(1)^3 + 3*cosh(1)^
2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d
)^4 + 3*((5*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 - b^3*c^3)*log(F)^3 -
 9*cosh(1)^3 + 15*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 +
sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (b^2*c^2*cosh(1) + b^2*c^2*s
inh(1) - 15*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1)
+ d)^2)*log(F)^2 - 27*cosh(1)^2*sinh(1) - 27*cosh(1)*sinh(1)^2 - 9*sinh(1)^
3 + (9*b*c*cosh(1)^2 + 18*b*c*cosh(1)*sinh(1) + 9*b*c*sinh(1)^2 - 5*(b*c*co
sh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
 + d)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + (b^3*c^3*cosh(x*cosh(1
) + x*sinh(1) + d)^6 - 3*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^4 + 3*b^3*
c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 - b^3*c^3)*log(F)^3 + 4*(15*(cosh(1)^
3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) +
 x*sinh(1) + d)^3 + (5*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^3 - 3*b^3*c^
3*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^3 - 3*(5*(b^2*c^2*cosh(1) + b^2*c
^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3 - (b^2*c^2*cosh(1) + b^2*c^2*
sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^2 - 27*(cosh(1)^3 + 3*cosh
(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1)
 + d) - (5*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*c
```

```
osh(1) + x*sinh(1) + d)^3 - 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c
*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*log(F))*sinh(x*cosh(1) + x*sin
h(1) + d)^3 + 3*cosh(1)^3 - 27*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)
*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 - 3*((b^2*c^2*cos
h(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^6 + b^2*c^2*cosh(1)
 - (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 +
b^2*c^2*sinh(1) - (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*si
nh(1) + d)^2)*log(F)^2 + 9*cosh(1)^2*sinh(1) + 9*cosh(1)*sinh(1)^2 + 3*sinh
(1)^3 + 3*(15*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh
(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^4 + (5*b^3*c^3*cosh(x*cosh(1) + x*si
nh(1) + d)^4 - 6*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^3*c^3)*log(F
)^3 - 9*cosh(1)^3 - 54*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)
^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (b^2*c^2*cosh(1) - 15*(
b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 + b^2*
c^2*sinh(1) + 6*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*cosh(1) + x*sinh
(1) + d)^2)*log(F)^2 - 27*cosh(1)^2*sinh(1) - 27*cosh(1)*sinh(1)^2 - 9*sinh
(1)^3 - (5*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*c
osh(1) + x*sinh(1) + d)^4 + 9*b*c*cosh(1)^2 + 18*b*c*cosh(1)*sinh(1) + 9*b*
c*sinh(1)^2 - 54*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*co
sh(x*cosh(1) + x*sinh(1) + d)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^2
- ((b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) +
 x*sinh(1) + d)^6 - 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)
^2)*cosh(x*cosh(1) + x*sinh(1) + d)^4 - b*c*cosh(1)^2 - 2*b*c*cosh(1)*sinh(
1) - b*c*sinh(1)^2 + 27*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1
)^2)*cosh(x*cosh(1) + x*sinh(1) + d)^2)*log(F) + 6*(3*(cosh(1)^3 + 3*cosh(1
)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) +
 d)^5 - 18*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3*cosh(1)*sinh(1)^2 + sinh(1)
^3)*cosh(x*cosh(1) + x*sinh(1) + d)^3 + (b^3*c^3*cosh(x*cosh(1) + x*sinh(1)
 + d)^5 - 2*b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^3 + b^3*c^3*cosh(x*cosh
(1) + x*sinh(1) + d))*log(F)^3 - (3*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cos
h(x*cosh(1) + x*sinh(1) + d)^5 - 2*(b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh
(x*cosh(1) + x*sinh(1) + d)^3 - (b^2*c^2*cosh(1) + b^2*c^2*sinh(1))*cosh(x*
cosh(1) + x*sinh(1) + d))*log(F)^2 - 9*(cosh(1)^3 + 3*cosh(1)^2*sinh(1) + 3
*cosh(1)*sinh(1)^2 + sinh(1)^3)*cosh(x*cosh(1) + x*sinh(1) + d) - ((b*c*cos
h(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
+ d)^5 - 18*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*
cosh(1) + x*sinh(1) + d)^3 + 9*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c
*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*log(F))*sinh(x*cosh(1) + x*sin
h(1) + d))*cosh((b*c*x + a*c)*log(F)) + (3*(cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 0.47, size = 1211, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*
b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*
c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3
*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F)
- 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e
) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e))*
e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b*c*log(abs
(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2
*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c
*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(
F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*
e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(-I*e^(1/2*I*pi*b*c
*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*
c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + I*e^(-1/2*I*pi*b*c*x*s
gn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*s
gn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*
log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*
b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) - e)*x - d) + 3*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/
2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*
log(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*
pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log
(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/4*(
2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) -
 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log
(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d) +
```

```
I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e) +
I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e))*e^(a
*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d)
```

Mupad [B]
time = 1.56, size = 166, normalized size = 0.82

$$-\frac{F^{ac+bcx}\left(-b^3c^3\sinh(d+ex)^3\ln(F)^3+3b^2c^2e\cosh(d+ex)\sinh(d+ex)^2\ln(F)^2-6bce^2\cosh(d+ex)^2\sinh(d+ex)\ln(F)+7bce^2\sinh(d+ex)^3\ln(F)+6e^3\cosh(d+ex)^3-9e^3\cosh(d+ex)\sinh(d+ex)^2\right)}{b^4c^4\ln(F)^4-10b^2c^2e^2\ln(F)^2+9e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^3,x)
```

```
[Out] -(F^(a*c + b*c*x)*(6*e^3*cosh(d + e*x)^3 - 9*e^3*cosh(d + e*x)*sinh(d + e*x
)^2 - b^3*c^3*sinh(d + e*x)^3*log(F)^3 + 7*b*c*e^2*sinh(d + e*x)^3*log(F) +
 3*b^2*c^2*e*cosh(d + e*x)*sinh(d + e*x)^2*log(F)^2 - 6*b*c*e^2*cosh(d + e*
x)^2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(
F)^2)
```

3.323 $\int F^{c(a+bx)} \sinh^2(d+ex)\, dx$

Optimal. Leaf size=132

$$-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

```
[Out] -2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*co
sh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(
e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5584, 2225}

$$-\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}-\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)} - $$

$$= -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - $$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.65

$$\frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Lo
g[F]^3)
```

Maple [A]
time = 1.08, size = 143, normalized size = 1.08

method	result	size
risch	$\frac{\left(\ln(F)^2 b^2c^2 e^{4ex+4d}-2\ln(F)^2b^2c^2e^{2ex+2d}-2\ln(F)bce\,e^{4ex+4d}+b^2c^2\ln(F)^2+2\ln(F)bce+8e^2e^{2ex+2d}\right)e^{-2ex-2d}F^{c(bx+a)}}{4bc\ln(F)(bc\ln(F)-2e)(bc\ln(F)+2e)}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(ln(F)^2*b^2*c^2*exp(4*e*x+4*d)-2*ln(F)^2*b^2*c^2*exp(2*e*x+2*d)-2*ln(F
)*b*c*e*exp(4*e*x+4*d)+b^2*c^2*ln(F)^2+2*ln(F)*b*c*e+8*e^2*exp(2*e*x+2*d))/
b/c/ln(F)/(b*c*ln(F)-2*e)*exp(-2*e*x-2*d)/(b*c*ln(F)+2*e)*F^(c*(b*x+a))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 0.74

$$\frac{F^{ac}e^{(bcx\log(F)+2\,xe+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,xe)}}{4\,(bce^{(2\,d)}\log(F)-2\,e^{(2\,d+1)})} - \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*x*e + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*x*e)/(b*c*e^(2*d)*log(F) - 2*e^(2*d + 1)) - 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(128) = 256$.
time = 0.39, size = 1115, normalized size = 8.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*(b*c*cosh(1) + b*c*sinh(1))*log(F))*sinh(x*cosh
(1) + x*sinh(1) + d)^4 + 4*(b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)*log(F)^
2 - 2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*s
inh(x*cosh(1) + x*sinh(1) + d)^3 + 8*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(
1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)^2 + (b^2*c^2*cosh(x*cosh(1) + x*sinh(
1) + d)^4 - 2*b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 + b^2*c^2)*log(F)^2
 - 2*(6*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F
) - (3*b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^2 - b^2*c^2)*log(F)^2 - 4*co
sh(1)^2 - 8*cosh(1)*sinh(1) - 4*sinh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d)^
2 - 2*((b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^4 - b*c*
cosh(1) - b*c*sinh(1))*log(F) - 4*(2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cos
h(1) + x*sinh(1) + d)^3*log(F) - (b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^3
 - b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d))*log(F)^2 - 4*(cosh(1)^2 + 2*cos
h(1)*sinh(1) + sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) +
 x*sinh(1) + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*(b*c*c
osh(1) + b*c*sinh(1))*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^4 + 4*(b^2*c^
2*cosh(x*cosh(1) + x*sinh(1) + d)*log(F)^2 - 2*(b*c*cosh(1) + b*c*sinh(1))*
cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^3 +
 8*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) +
 d)^2 + (b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^4 - 2*b^2*c^2*cosh(x*cosh(
1) + x*sinh(1) + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*(b*c*cosh(1) + b*c*sinh(1)
)*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F) - (3*b^2*c^2*cosh(x*cosh(1) + x*
sinh(1) + d)^2 - b^2*c^2)*log(F)^2 - 4*cosh(1)^2 - 8*cosh(1)*sinh(1) - 4*si
nh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d)^2 - 2*((b*c*cosh(1) + b*c*sinh(1))
*cosh(x*cosh(1) + x*sinh(1) + d)^4 - b*c*cosh(1) - b*c*sinh(1))*log(F) - 4*
(2*(b*c*cosh(1) + b*c*sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^3*log(F) - (
b^2*c^2*cosh(x*cosh(1) + x*sinh(1) + d)^3 - b^2*c^2*cosh(x*cosh(1) + x*sinh
(1) + d))*log(F)^2 - 4*(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2)*cosh(x*c
osh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh(1) + d))*sinh((b*c*x + a*c
)*log(F)))/(b^3*c^3*cosh(x*cosh(1) + x*sinh(1) + d)^2*log(F)^3 - 4*(b*c*cos
h(1)^2 + 2*b*c*cosh(1)*sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1)
+ d)^2*log(F) + (b^3*c^3*log(F)^3 - 4*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*sinh(1
```

```
) + b*c*sinh(1)^2)*log(F))*sinh(x*cosh(1) + x*sinh(1) + d)^2 + 2*(b^3*c^3*c
osh(x*cosh(1) + x*sinh(1) + d)*log(F)^3 - 4*(b*c*cosh(1)^2 + 2*b*c*cosh(1)*
sinh(1) + b*c*sinh(1)^2)*cosh(x*cosh(1) + x*sinh(1) + d)*log(F))*sinh(x*cos
h(1) + x*sinh(1) + d))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(119) = 238$.

time = 14.04, size = 1052, normalized size = 7.97

$$\begin{cases} \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} & \text{for}\ F=1 \\ \frac{b^2c^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\log\left(e^{-\frac{2e}{bc}}\right)^2\sinh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)} - \frac{2bce\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\log\left(e^{-\frac{2e}{bc}}\right)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)} - \frac{2e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)} + \frac{2e^2\left(e^{-\frac{2e}{bc}}\right)^{ac}\left(e^{-\frac{2e}{bc}}\right)^{bcx}\cosh^2(d+ex)}{b^3c^3\log\left(e^{-\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{-\frac{2e}{bc}}\right)} & \text{for}\ F=e^{-\frac{2e}{bc}} \\ \frac{b^2c^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\log\left(e^{\frac{2e}{bc}}\right)^2\sinh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)} - \frac{2bce\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\log\left(e^{\frac{2e}{bc}}\right)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)} - \frac{2e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\sinh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)} + \frac{2e^2\left(e^{\frac{2e}{bc}}\right)^{ac}\left(e^{\frac{2e}{bc}}\right)^{bcx}\cosh^2(d+ex)}{b^3c^3\log\left(e^{\frac{2e}{bc}}\right)^3-4bce^2\log\left(e^{\frac{2e}{bc}}\right)} & \text{for}\ F=e^{\frac{2e}{bc}} \\ F^{ac}\left(\frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) & \text{for}\ b=0 \\ \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} & \text{for}\ c=0 \\ \frac{F^{ac}F^{bcx}b^2c^2\log(F)^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac}F^{bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} + \frac{2F^{ac}F^{bcx}e^2\cosh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh
(d + e*x)/(2*e), Eq(F, 1)), (b**2*c**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c
))**(b*c*x)*log(exp(-2*e/(b*c)))**2*sinh(d + e*x)**2/(b**3*c**3*log(exp(-2*
e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))) - 2*b*c*e*exp(-2*e/(b*c))**
(a*c)*exp(-2*e/(b*c))**(b*c*x)*log(exp(-2*e/(b*c)))*sinh(d + e*x)*cosh(d +
e*x)/(b**3*c**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c))))
- 2*e**2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2/(
b**3*c**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))) + 2*e*
*2*exp(-2*e/(b*c))**(a*c)*exp(-2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2/(b**3*c
**3*log(exp(-2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(-2*e/(b*c)))), Eq(F, exp(-
2*e/(b*c)))), (b**2*c**2*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))**(b*c*x)*log(
exp(2*e/(b*c)))**2*sinh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*c)))**3 - 4*b
*c*e**2*log(exp(2*e/(b*c)))) - 2*b*c*e*exp(2*e/(b*c))**(a*c)*exp(2*e/(b*c))
**(b*c*x)*log(exp(2*e/(b*c)))*sinh(d + e*x)*cosh(d + e*x)/(b**3*c**3*log(ex
p(2*e/(b*c)))**3 - 4*b*c*e**2*log(exp(2*e/(b*c)))) - 2*e**2*exp(2*e/(b*c))*
*(a*c)*exp(2*e/(b*c))**(b*c*x)*sinh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*c
)))**3 - 4*b*c*e**2*log(exp(2*e/(b*c)))) + 2*e**2*exp(2*e/(b*c))**(a*c)*exp
(2*e/(b*c))**(b*c*x)*cosh(d + e*x)**2/(b**3*c**3*log(exp(2*e/(b*c)))**3 - 4
*b*c*e**2*log(exp(2*e/(b*c)))), Eq(F, exp(2*e/(b*c)))), (F**(a*c)*(x*sinh(d
 + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e)), E
q(b, 0)), (x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh
(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*sinh(d
 + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x
)*b*c*e*log(F)*sinh(d + e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**
2*log(F)) - 2*F**(a*c)*F**(b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**
```

```
3 - 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**2*cosh(d + e*x)**2/(b**3*
c**3*log(F)**3 - 4*b*c*e**2*log(F)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.46, size = 890, normalized size = 6.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c
)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn
(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(
F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi
*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2
*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(
abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) +
I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - I*e
^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*
a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c*log(abs(F)) - 2
*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c*sgn
(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2))*e^
(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) + I*(I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c
*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn
(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log
(abs(F)) - 2*e)*x - 2*d)
```

Mupad [B]
time = 0.92, size = 97, normalized size = 0.73

$$-\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2-\frac{b^2\,c^2\,\ln(F)^2}{2}+\frac{b^2\,c^2\,\ln(F)^2\,\cosh(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sinh(2\,d+2\,e\,x)\right)}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\,{\ln\left(F\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F^(c*(a + b*x))*sinh(d + e*x)^2,x)

[Out] -(F^(a*c + b*c*x)*(2*e^2 - (b^2*c^2*log(F)^2)/2 + (b^2*c^2*log(F)^2*cosh(2*
d + 2*e*x))/2 - b*c*e*log(F)*sinh(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 - b^2*c
^2*log(F)^2))

3.324 $\int F^{c(a+bx)}\sinh(d+ex)\,dx$

Optimal. Leaf size=75

$$\frac{eF^{c(a+bx)}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

```
[Out] e*F^(c*(b*x+a))*cosh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*s
inh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5582}

$$\frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a +
 b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx=\frac{eF^{c(a+bx)}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)}-\frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.67

$$\frac{F^{c(a+bx)}(e\cosh(d+ex)-bc\log(F)\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*Lo
g[F])*(e + b*c*Log[F]))
```

Maple [A]
time = 0.18, size = 77, normalized size = 1.03

method	result	size
risch	$\frac{\left(\ln \left(F\right) b c \,{\mathrm e}^{2 e x +2 d}-b c \ln \left(F\right)-e \,{\mathrm e}^{2 e x +2 d}-e\right) {\mathrm e}^{-e x -d} F^{c \left(b x +a \right)}}{2 \left(b c \ln \left(F\right)-e\right) \left(e +b c \ln \left(F\right)\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(F)*b*c*exp(2*e*x+2*d)-b*c*ln(F)-e*exp(2*e*x+2*d)-e)/(b*c*ln(F)-e)*e
xp(-e*x-d)/(e+b*c*ln(F))*F^(c*(b*x+a))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.89

$$\frac{F^{ac}e^{\left(bcx\log\left(F\right)+xe+d\right)}}{2\left(bc\log\left(F\right)+e\right)}-\frac{F^{ac}e^{\left(bcx\log\left(F\right)-xe\right)}}{2\left(bce^{d}\log\left(F\right)-e^{\left(d+1\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b*
c*x*log(F) - x*e)/(b*c*e^d*log(F) - e^(d + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(78) = 156.
time = 0.39, size = 376, normalized size = 5.01

$$-\frac{\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\log\left(F\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-bc\right)\log\left(F\right)-2\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)-\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)+\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(\left(bcx+ac\right)\log\left(F\right)\right)+\left(\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\log\left(F\right)-\cosh\left(1\right)-\sinh\left(1\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)^{2}-bc\right)\log\left(F\right)-2\left(bc\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)-\left(\cosh\left(1\right)+\sinh\left(1\right)\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)+\cosh\left(1\right)+\sinh\left(1\right)\right)\sinh\left(\left(bcx+ac\right)\log\left(F\right)\right)}{2\left(b^{2}c^{2}\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\log\left(F\right)^{2}-\left(\cosh\left(1\right)^{2}+2\cosh\left(1\right)\sinh\left(1\right)+\sinh\left(1\right)^{2}\right)\cosh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)+\left(b^{2}c^{2}\log\left(F\right)^{2}-\cosh\left(1\right)^{2}-2\cosh\left(1\right)\sinh\left(1\right)-\sinh\left(1\right)^{2}\right)\sinh\left(x\cosh\left(1\right)+x\sinh\left(1\right)+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*(((cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*log(F)
- cosh(1) - sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*cosh(x*cosh(1
) + x*sinh(1) + d)^2 - b*c)*log(F) - 2*(b*c*cosh(x*cosh(1) + x*sinh(1) + d)
*log(F) - (cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(
1) + x*sinh(1) + d) + cosh(1) + sinh(1))*cosh((b*c*x + a*c)*log(F)) + ((cos
h(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*log(F) - cosh(1) -
 sinh(1))*sinh(x*cosh(1) + x*sinh(1) + d)^2 - (b*c*cosh(x*cosh(1) + x*sinh(
1) + d)^2 - b*c)*log(F) - 2*(b*c*cosh(x*cosh(1) + x*sinh(1) + d)*log(F) - (
```

```
cosh(1) + sinh(1))*cosh(x*cosh(1) + x*sinh(1) + d))*sinh(x*cosh(1) + x*sinh
(1) + d) + cosh(1) + sinh(1))*sinh((b*c*x + a*c)*log(F)))/(b^2*c^2*cosh(x*c
osh(1) + x*sinh(1) + d)*log(F)^2 - (cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)
^2)*cosh(x*cosh(1) + x*sinh(1) + d) + (b^2*c^2*log(F)^2 - cosh(1)^2 - 2*cos
h(1)*sinh(1) - sinh(1)^2)*sinh(x*cosh(1) + x*sinh(1) + d))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.47, size = 416, normalized size = 5.55

$$\begin{cases} \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\sinh(d+ex)}{2} - \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}x\cosh(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{-\frac{iex}{\pi}}\cosh(d+ex)}{2e} & \text{for } F=-1 \wedge b=-\frac{ie}{\pi c} \\ x\sinh(d) & \text{for } F=1 \wedge e=0 \\ \frac{bc\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\log\left(e^{-\frac{e}{bc}}\right)\sinh(d+ex)}{b^2c^2\log\left(e^{-\frac{e}{bc}}\right)^2-e^2} - \frac{e\left(e^{-\frac{e}{bc}}\right)^{ac}\left(e^{-\frac{e}{bc}}\right)^{bcx}\cosh(d+ex)}{b^2c^2\log\left(e^{-\frac{e}{bc}}\right)^2-e^2} & \text{for } F=e^{-\frac{e}{bc}} \\ \frac{bc\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\log\left(e^{\frac{e}{bc}}\right)\sinh(d+ex)}{b^2c^2\log\left(e^{\frac{e}{bc}}\right)^2-e^2} - \frac{e\left(e^{\frac{e}{bc}}\right)^{ac}\left(e^{\frac{e}{bc}}\right)^{bcx}\cosh(d+ex)}{b^2c^2\log\left(e^{\frac{e}{bc}}\right)^2-e^2} & \text{for } F=e^{\frac{e}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} - \frac{F^{ac}F^{bcx}e\cosh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] Piecewise(((-1)**(a*c)*x*sinh(d + e*x)/(2*(-1)**(I*e*x/pi)) - (-1)**(a*c)*x
*cosh(d + e*x)/(2*(-1)**(I*e*x/pi)) + (-1)**(a*c)*cosh(d + e*x)/(2*(-1)**(I
*e*x/pi)*e), Eq(F, -1) & Eq(b, -I*e/(pi*c))), (x*sinh(d), Eq(F, 1) & Eq(e,
0)), (b*c*exp(-e/(b*c))**(a*c)*exp(-e/(b*c))**(b*c*x)*log(exp(-e/(b*c)))*si
nh(d + e*x)/(b**2*c**2*log(exp(-e/(b*c)))**2 - e**2) - e*exp(-e/(b*c))**(a*
c)*exp(-e/(b*c))**(b*c*x)*cosh(d + e*x)/(b**2*c**2*log(exp(-e/(b*c)))**2 -
e**2), Eq(F, exp(-e/(b*c)))), (b*c*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x
)*log(exp(e/(b*c)))*sinh(d + e*x)/(b**2*c**2*log(exp(e/(b*c)))**2 - e**2) -
 e*exp(e/(b*c))**(a*c)*exp(e/(b*c))**(b*c*x)*cosh(d + e*x)/(b**2*c**2*log(e
xp(e/(b*c)))**2 - e**2), Eq(F, exp(e/(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log
(F)*sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2) - F**(a*c)*F**(b*c*x)*e*cosh
(d + e*x)/(b**2*c**2*log(F)**2 - e**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 598, normalized size = 7.97

$$\left(\frac{2\,(bc\log(|F|)+e)\cos\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)+e)^2} - \frac{(\pi bc\mathrm{sgn}(F)-\pi bc)\sin\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)+e)^2}\right)e^{(ac\log(|F|)+(bc\log(|F|)+e)x+d)} + \frac{1}{2}i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac\right)}}{i\,\pi bc\mathrm{sgn}(F)-i\,\pi bc+2\,bc\log(|F|)+2\,e} - \frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac\right)}}{-i\,\pi bc\mathrm{sgn}(F)+i\,\pi bc+2\,bc\log(|F|)+2\,e}\right)e^{(ac\log(|F|)+(bc\log(|F|)+e)x+d)} - \left(\frac{2\,(bc\log(|F|)-e)\cos\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)-e)^2} - \frac{(\pi bc\mathrm{sgn}(F)-\pi bc)\sin\left(-\frac{1}{2}\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}\,\pi bcx-\frac{1}{2}\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}\,\pi ac\right)}{(\pi bc\mathrm{sgn}(F)-\pi bc)^2+4\,(bc\log(|F|)-e)^2}\right)e^{(ac\log(|F|)+(bc\log(|F|)-e)x-d)} + \frac{1}{2}i\left(-\frac{i\,e^{\left(\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)-\frac{1}{2}i\,\pi bcx+\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)-\frac{1}{2}i\,\pi ac\right)}}{i\,\pi bc\mathrm{sgn}(F)-i\,\pi bc+2\,bc\log(|F|)-2\,e} + \frac{i\,e^{\left(-\frac{1}{2}i\,\pi bcx\mathrm{sgn}(F)+\frac{1}{2}i\,\pi bcx-\frac{1}{2}i\,\pi ac\mathrm{sgn}(F)+\frac{1}{2}i\,\pi ac\right)}}{-i\,\pi bc\mathrm{sgn}(F)+i\,\pi bc+2\,bc\log(|F|)-2\,e}\right)e^{(ac\log(|F|)+(bc\log(|F|)-e)x-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
```

```
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2*I*(I
*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(-1/2*I*
pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*
pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*
b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) - e)*x - d) + 1/2*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*lo
g(abs(F)) - 2*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*
a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))
 - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

Mupad [B]
time = 0.66, size = 73, normalized size = 0.97

$$\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e+e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)-b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,{\ln\left(F\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x),x)
```

```
[Out] (F^(a*c + b*c*x)*exp(- d - e*x)*(e + e*exp(2*d + 2*e*x) + b*c*log(F) - b*c*
exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

3.325 $\int F^{c(a+bx)}\mathbf{csch}(d+ex)\,dx$

Optimal. Leaf size=66

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)}{e+bc\log(F)}$$

[Out] -2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))/(e+b*c*ln(F))

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5601}

$$-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{bc\log(F)+e}$$

Antiderivative was successfully verified.

[In] Int[F^(c*(a + b*x))*Csch[d + e*x],x]

[Out] (-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\mathrm{csch}(d+ex)\,dx=-\frac{2e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right);e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A]
time = 3.43, size = 93, normalized size = 1.41

$$\frac{F^{c(a+bx)}\left({}_2F_1\left(1,\frac{bc\log(F)}{e};1+\frac{bc\log(F)}{e};-\cosh(d+ex)-\sinh(d+ex)\right)-{}_2F_1\left(1,\frac{bc\log(F)}{e};1+\frac{bc\log(F)}{e};\cosh(d+ex)+\sinh(d+ex)\right)\right)}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
-Cosh[d + e*x] - Sinh[d + e*x]] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 +
(b*c*Log[F])/e, Cosh[d + e*x] + Sinh[d + e*x]]))/(b*c*Log[F])
```

Maple [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

```
[Out] -4*F^(a*c)*integrate(-e^(b*c*x*log(F) + x*e + d + 1)/(b*c*log(F) + (b*c*e^(
4*d)*log(F) - e^(4*d + 1))*e^(4*x*e) - 2*(b*c*e^(2*d)*log(F) - e^(2*d + 1))
*e^(2*x*e) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + x*e + d)/(b*c*log(F) - (b
*c*e^(2*d)*log(F) - e^(2*d + 1))*e^(2*x*e) - e)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x),x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x), x)
```

3.326 $\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx$

Optimal. Leaf size=68

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

```
[Out] 4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*
ln(F)/e],exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {5601}

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx=\frac{4e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};2+\frac{bc\log(F)}{2e};e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A]
time = 1.79, size = 87, normalized size = 1.28

$$-\frac{2F^{c(a+bx)}\left(\left(-1+e^{2d}\right)\,{}_2F_1\left(1,\frac{bc\log(F)}{2e};1+\frac{bc\log(F)}{2e};e^{2(d+ex)}\right)+\text{csch}(d+ex)\sinh(d)(\cosh(ex)-\sinh(ex))\right)}{e\left(-1+e^{2d}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[e*
x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\mathrm{csch}(ex+d)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 16*F^(a*c)*b*c*integrate(-e^(b*c*x*log(F) + 1)/(b^2*c^2*log(F)^2 - 6*b*c*e*
log(F) - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e^(6*d + 1)*log(F) + 8*e^(6*d +
2))*e^(6*x*e) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e^(4*d + 1)*log(F) + 8*
e^(4*d + 2))*e^(4*x*e) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e^(2*d + 1)*lo
g(F) + 8*e^(2*d + 2))*e^(2*x*e) + 8*e^2), x)*log(F) + 4*(4*F^(a*c)*e + (F^(
a*c)*b*c*e^(2*d)*log(F) - 4*F^(a*c)*e^(2*d + 1))*e^(2*x*e))*F^(b*c*x)/(b^2*
c^2*log(F)^2 - 6*b*c*e*log(F) + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e^(4*d +
1)*log(F) + 8*e^(4*d + 2))*e^(4*x*e) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*
e^(2*d + 1)*log(F) + 8*e^(2*d + 2))*e^(2*x*e) + 8*e^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(x*e + d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\sinh\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^2,x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^2, x)
```

3.327 $\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\,dx$

Optimal. Leaf size=122

$$-\frac{F^{c(a+bx)}\coth(d+ex)\mathrm{csch}(d+ex)}{2e}-\frac{bcF^{c(a+bx)}\mathrm{csch}(d+ex)\log(F)}{2e^2}+\frac{e^{d+ex}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(3+\frac{b}{}\right.\right.}{e^2}$$

[Out] -1/2*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csch(e*x+d)*ln(F)/e^2+exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5599, 5601}

$$\frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,{}_2F_1\left(1,\frac{e+bc\log(F)}{2e};\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right);e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\mathrm{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\mathrm{csch}(d+ex)F^{c(a+bx)}}{2e}$$

Antiderivative was successfully verified.

[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]

[Out] -1/2*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/e - (b*c*F^(c*(a + b*x))*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(e - b*c*Log[F]))/e^2

Rule 5599

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n - 1)*(n - 2))), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a + b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hypergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\operatorname{csch}(d+ex)\log(F)}{2e^2} - \frac{1}{2}\Big(1$$

$$= -\frac{F^{c(a+bx)}\coth(d+ex)\operatorname{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\operatorname{csch}(d+ex)\log(F)}{2e^2} + \frac{e^{d+e}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 299 vs. 2(122) = 244.
time = 12.95, size = 299, normalized size = 2.45

$$\frac{F^{c(a+bx)}\left(-e\operatorname{csch}^2\left(\frac{1}{2}(d+ex)\right) - 4bc\operatorname{csch}(d)\log(F) + \operatorname{csch}(d)\left(-\frac{4e^2}{bc\log(F)} + 4bc\log(F)\right) - e\operatorname{sech}^2\left(\frac{1}{2}(d+ex)\right) + \frac{4(e^2-b^2c^2\log^2(F))\left(1+{}_2F_1\left(1,\frac{bc\log(F)}{e};1+\frac{bc\log(F)}{e};\cosh(d+ex)+\sinh(d+ex)\right)(-1+\cosh(d)+\sinh(d))\right)}{bc\log(F)(-1+\cosh(d)+\sinh(d))} + \frac{4(e^2-b^2c^2\log^2(F))\left(1-{}_2F_1\left(1,\frac{bc\log(F)}{e};1+\frac{bc\log(F)}{e};-\cosh(d+ex)-\sinh(d+ex)\right)(1+\cosh(d)+\sinh(d))\right)}{bc\log(F)(1+\cosh(d)+\sinh(d))} + 2bc\operatorname{csch}\left(\frac{d}{2}\right)\operatorname{csch}\left(\frac{1}{2}(d+ex)\right)\log(F)\sinh\left(\frac{ex}{2}\right) + 2bc\log(F)\operatorname{sech}\left(\frac{d}{2}\right)\operatorname{sech}\left(\frac{1}{2}(d+ex)\right)\sinh\left(\frac{ex}{2}\right)\right)}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d]
*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) - e*Sech[(d + e*x)/2]^2 + (4*(e^2 -
 b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F
])/e, Cosh[d + e*x] + Sinh[d + e*x]]*(-1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]
*(-1 + Cosh[d] + Sinh[d])) + (4*(e^2 - b^2*c^2*Log[F]^2)*(1 - Hypergeometri
c2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -Cosh[d + e*x] - Sinh[d + e*x]]
*(1 + Cosh[d] + Sinh[d])))/(b*c*Log[F]*(1 + Cosh[d] + Sinh[d])) + 2*b*c*Csc
h[d/2]*Csch[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech
[(d + e*x)/2]*Sinh[(e*x)/2]))/(8*e^2)
```

Maple [**F**]
time = 0.41, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 48*(F^(a*c)*b*c*e^(d + 1)*log(F) + F^(a*c)*e^(d + 2))*integrate(e^(b*c*x*lo
g(F) + x*e)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + (b^2*c^2*e^(8*d)*log(F)^2
- 8*b*c*e^(8*d + 1)*log(F) + 15*e^(8*d + 2))*e^(8*x*e) - 4*(b^2*c^2*e^(6*d)
*log(F)^2 - 8*b*c*e^(6*d + 1)*log(F) + 15*e^(6*d + 2))*e^(6*x*e) + 6*(b^2*c
^2*e^(4*d)*log(F)^2 - 8*b*c*e^(4*d + 1)*log(F) + 15*e^(4*d + 2))*e^(4*x*e)
- 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e^(2*d + 1)*log(F) + 15*e^(2*d + 2))*
e^(2*x*e) + 15*e^2), x) - 8*((F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^(a*c)*e^(3*d
 + 1))*e^(3*x*e) + 6*F^(a*c)*e^(x*e + d + 1))*F^(b*c*x)/(b^2*c^2*log(F)^2 -
 8*b*c*e*log(F) - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e^(6*d + 1)*log(F) + 15
*e^(6*d + 2))*e^(6*x*e) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e^(4*d + 1)*l
og(F) + 15*e^(4*d + 2))*e^(4*x*e) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e^(
2*d + 1)*log(F) + 15*e^(2*d + 2))*e^(2*x*e) + 15*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csch(x*e + d)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\mathrm{sinh}\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^3,x)
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^3, x)
```

3.328 $\int F^{c(a+bx)}\text{csch}^4(d+ex)\,dx$

Optimal. Leaf size=131

$$-\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e}-\frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2}-\frac{2e^{2(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1+\frac{bc\log(F)}{2e};\right.}{3}$$

```
[Out] -1/3*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csch(e
*x+d)^2*ln(F)/e^2-2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*
ln(F)/e],[2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5599, 5601}

$$-\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,{}_2F_1\left(2,\frac{bc\log(F)}{2e}+1;\frac{bc\log(F)}{2e}+2;e^{2(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\text{csch}^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\coth(d+ex)\text{csch}^2(d+ex)F^{c(a+bx)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] -1/3*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/e - (b*c*F^(c*(a + b*x
))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hyp
ergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d +
 e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
```

Rule 5599

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(
n - 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a +
 b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F,
a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
 && NeQ[n, 2]
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\text{csch}^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} - \frac{1}{6}\Big($$

$$= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} - \frac{2e}{}$$

Mathematica [A]
time = 5.21, size = 202, normalized size = 1.54

$$\frac{F^{c(a+bx)}\left(-1+\coth(d)+2\,{}_2F_1\left(1,\frac{bc\log(F)}{2e};1+\frac{bc\log(F)}{2e};\cosh(2(d+ex))+\sinh(2(d+ex))\right)\right)\left(4e^2-b^2c^2\log^2(F)\right)}{6e^3} - \frac{F^{ac+bcx}\text{csch}(d)\text{csch}^2(d+ex)(2e\cosh(d)+bc\log(F)\sinh(d))}{6e^2} + \frac{F^{ac+bcx}\text{csch}(d)\text{csch}^3(d+ex)\sinh(ex)}{3e} - \frac{F^{ac+bcx}\text{csch}(d)\text{csch}(d+ex)\left(4e^2-b^2c^2\log^2(F)\right)\sinh(ex)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-1 + Coth[d] + 2*Hypergeometric2F1[1, (b*c*Log[F])/(2*e),
 1 + (b*c*Log[F])/(2*e), Cosh[2*(d + e*x)] + Sinh[2*(d + e*x)]])*(4*e^2 - b
^2*c^2*Log[F]^2))/(6*e^3) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]^2*(2*e*C
osh[d] + b*c*Log[F]*Sinh[d]))/(6*e^2) + (F^(a*c + b*c*x)*Csch[d]*Csch[d + e
*x]^3*Sinh[e*x])/(3*e) - (F^(a*c + b*c*x)*Csch[d]*Csch[d + e*x]*(4*e^2 - b^
2*c^2*Log[F]^2)*Sinh[e*x])/(6*e^3)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int F^{c(bx+a)}\text{csch}(ex+d)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - (b^3
*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e^(10*d + 1)*log(F)^2 + 104*b*c*e^(10*d
```

```
 + 2)*log(F) - 192*e^(10*d + 3))*e^(10*x*e) + 5*(b^3*c^3*e^(8*d)*log(F)^3 -
 18*b^2*c^2*e^(8*d + 1)*log(F)^2 + 104*b*c*e^(8*d + 2)*log(F) - 192*e^(8*d
+ 3))*e^(8*x*e) - 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e^(6*d + 1)*log
(F)^2 + 104*b*c*e^(6*d + 2)*log(F) - 192*e^(6*d + 3))*e^(6*x*e) + 10*(b^3*c
^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e^(4*d + 1)*log(F)^2 + 104*b*c*e^(4*d + 2)
*log(F) - 192*e^(4*d + 3))*e^(4*x*e) - 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2
*c^2*e^(2*d + 1)*log(F)^2 + 104*b*c*e^(2*d + 2)*log(F) - 192*e^(2*d + 3))*e
^(2*x*e) - 192*e^3), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^
(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e^(4*d + 1)*log(F) + 48*F^(
a*c)*e^(4*d + 2))*e^(4*x*e) + 8*(F^(a*c)*b*c*e^(2*d + 1)*log(F) - 8*F^(a*c)
*e^(2*d + 2))*e^(2*x*e))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^
2 + 104*b*c*e^2*log(F) + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e^(8*d + 1)
*log(F)^2 + 104*b*c*e^(8*d + 2)*log(F) - 192*e^(8*d + 3))*e^(8*x*e) - 4*(b^
3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e^(6*d + 1)*log(F)^2 + 104*b*c*e^(6*d +
 2)*log(F) - 192*e^(6*d + 3))*e^(6*x*e) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*
b^2*c^2*e^(4*d + 1)*log(F)^2 + 104*b*c*e^(4*d + 2)*log(F) - 192*e^(4*d + 3)
)*e^(4*x*e) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e^(2*d + 1)*log(F)^2
 + 104*b*c*e^(2*d + 2)*log(F) - 192*e^(2*d + 3))*e^(2*x*e) - 192*e^3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(x*e + d)^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int F^{c(a+bx)} \operatorname{csch}^4(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**4,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{F^{c\,(a+b\,x)}}{{\mathrm{sinh}\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^4,x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^4, x)
```

3.329 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2} \, dx$

Optimal. Leaf size=250

$$\frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{64bc}+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac$$

```
[Out] 1/128*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*c
sch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/16*x*csch(b*c*x+a*c)*(sinh(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{128bc}-\frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{64bc}+\frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{32bc}-\frac{5e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{128bc}+\frac{e^{6c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{192bc}-\frac{5}{16}x\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128*b*c*E^(4*c*(a + b*x))) -
 (5*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(64*b*c*E^(2*c*(a + b*x)))
 + (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*
c) - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128
*b*c) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(19
2*b*c) - (5*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+\right.\right.}{64bc}$$
$$= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc} - \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\,}{64bc}$$

Mathematica [A]
time = 0.08, size = 106, normalized size = 0.42

$$\frac{\left(\frac{1}{2}e^{-4c(a+bx)}-5e^{-2c(a+bx)}+10e^{2c(a+bx)}-\frac{5}{2}e^{4c(a+bx)}+\frac{1}{3}e^{6c(a+bx)}-20bcx\right)\operatorname{csch}^5(c(a+bx))\sinh^2(c(a+bx))^{5/2}}{64bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(2*E^(4*c*(a + b*x))) - 5/E^(2*c*(a + b*x)) + 10*E^(2*c*(a + b*x)) - (5
*E^(4*c*(a + b*x)))/2 + E^(6*c*(a + b*x))/3 - 20*b*c*x)*Csch[c*(a + b*x)]^5
*(Sinh[c*(a + b*x)]^2)^(5/2))/(64*b*c)
```

Maple [A]
time = 3.37, size = 184, normalized size = 0.74

method	result
default	$-\frac{-8\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\,\cosh(c(bx+a))(\sinh^5(c(bx+a)))-8\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\,(\sinh^6(c(bx+a)))+10\sqrt{-\frac{1}{2}+}}{}$
risch	$-\frac{5x\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,e^{c(bx+a)}}{16\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,e^{7c(bx+a)}}{192cb\left(e^{2c(bx+a)}-1\right)}-\frac{5\sqrt{\left(e^{2c(bx+a)}-1\right)}}{128cb(e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/sinh(c*(b*x+a))*(-8*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))*sinh(c*
(b*x+a))^5-8*(sinh(c*(b*x+a))^2)^(1/2)*sinh(c*(b*x+a))^6+10*(sinh(c*(b*x+a)
)^2)^(1/2)*cosh(c*(b*x+a))*sinh(c*(b*x+a))^3-15*cosh(c*(b*x+a))*(sinh(c*(b*
x+a))^2)^(1/2)*sinh(c*(b*x+a))+15*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1
/2))*sinh(c*(b*x+a))-8*(sinh(c*(b*x+a))^2)^(1/2))/b/c
```

Maxima [A]
time = 0.49, size = 90, normalized size = 0.36

$$\frac{\left(2\,e^{(10\,bcx+10\,ac)}-15\,e^{(8\,bcx+8\,ac)}+60\,e^{(6\,bcx+6\,ac)}-30\,e^{(2\,bcx+2\,ac)}+3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc}-\frac{5\,(bcx+ac)}{16\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Fricas [A]
time = 0.41, size = 218, normalized size = 0.87

$$\frac{5\cosh(bcx+ac)^5+25\cosh(bcx+ac)\sinh(bcx+ac)^4-\sinh(bcx+ac)^5-5\left(2\cosh(bcx+ac)^2-3\right)\sinh(bcx+ac)^3-45\cosh(bcx+ac)^3+5\left(10\cosh(bcx+ac)^3-27\cosh(bcx+ac)\right)\sinh(bcx+ac)^2-60\,(2\,bcx-1)\cosh(bcx+ac)-5\left(\cosh(bcx+ac)^4-24\,bcx-9\cosh(bcx+ac)^2-12\right)\sinh(bcx+ac)}{384\left(bc\cosh(bcx+ac)-bc\sinh(bcx+ac)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 269, normalized size = 1.08

$$-\frac{120\,bcx\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-3\,\left(30\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-10\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-4\,bcx-4\,ac)}-\left(2\,e^{(6\,bcx+18\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-15\,e^{(4\,bcx+16\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+60\,e^{(2\,bcx+14\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-12\,ac)}}{384\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*e^(4*b*c*
x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x + 2*a*c)
*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*b*c*x - 4*a*c) - (2*e^(6*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)) + 60*e^(2*b*c*x + 14*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c
)))*e^(-12*a*c))/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{c\,\left(a+b\,x\right)}\,{\left({\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2), x)
```

3.330 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} \, dx$

Optimal. Leaf size=162

$$\frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+$$

```
[Out] 1/16*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*csch(b*c*x+a*c)
*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}-\frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc}+\frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{32bc}+\frac{3}{8}x\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c*E^(2*c*(a + b*x))) -
(3*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c)
+ (E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*c)
+ (3*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x^2)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x^2)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc}\\ &= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx,x,\right.}{16bc}\\ &= \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{}}{16bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.47

$$\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{csch}^3(c(a+bx))\sinh^2(c(a+bx))^{3/2}}{16bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3*(Sinh[c*(a + b*x)]^2)^(3/2))/(16*b*c)
```

Maple [A]
time = 1.95, size = 128, normalized size = 0.79

method	result
default	$\frac{2\left(-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}\right)^{\frac{5}{2}}+2\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\,\cosh(c(bx+a))\left(\sinh^{3}(c(bx+a))\right)-3\cosh(c(bx+a))\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+\cdots}{2}}}{8\sinh(c(bx+a))bc}$
risch	$\frac{3x\sqrt{\left(e^{2c(bx+a)}-1\right)^{2}e^{-2c(bx+a)}}\,e^{c(bx+a)}}{8\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^{2}e^{-2c(bx+a)}}\,e^{5c(bx+a)}}{32cb\left(e^{2c(bx+a)}-1\right)}-\frac{3\sqrt{\left(e^{2c(bx+a)}-1\right)^{2}\cdots}}{16cb\left(e^{2c(b\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*(sinh(c*(b*x+a))^2)^(5/2)+2*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a)
)*sinh(c*(b*x+a))^3-3*cosh(c*(b*x+a))*(sinh(c*(b*x+a))^2)^(1/2)*sinh(c*(b*x
+a))+3*ln(cosh(c*(b*x+a))+(sinh(c*(b*x+a))^2)^(1/2))*sinh(c*(b*x+a)))/sinh(
c*(b*x+a))/b/c
```

Maxima [A]
time = 0.49, size = 62, normalized size = 0.38

$$\frac{\left(e^{(6\,bcx+6\,ac)}-6\,e^{(4\,bcx+4\,ac)}+2\right)e^{(-2\,bcx-2\,ac)}}{32\,bc}+\frac{3\,(bcx+ac)}{8\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Fricas [A]
time = 0.38, size = 126, normalized size = 0.78

$$\frac{3\,\cosh\left(bcx+ac\right)^{3}+9\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}-\sinh\left(bcx+ac\right)^{3}+6\,(2\,bcx-1)\cosh\left(bcx+ac\right)-3\,\left(4\,bcx+\cosh\left(bcx+ac\right)^{2}+2\right)\sinh\left(bcx+ac\right)}{32\,\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**A**]
time = 0.43, size = 195, normalized size = 1.20

$$\frac{12\,bcx\,\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-2\,bcx-2\,ac)}+\left(e^{(4\,bcx+8\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-6\,e^{(2\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)e^{(-4\,ac)}}{32\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2*b*c*x +
2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e^(-
b*c*x - a*c)))*e^(-2*b*c*x - 2*a*c) + (e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b
*c*x - a*c)))*e^(-4*a*c))/(b*c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{c\,(a+b\,x)}\,{\left({\mathrm{sinh}\left(a\,c+b\,c\,x\right)}^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2), x)
```

3.331 $\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\, dx$

Optimal. Leaf size=74

$$\frac{e^{2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc}-\frac{1}{2}x\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-1/2*x*cs
ch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 14}

$$\frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)}{4bc}-\frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\,\text{csch}(ac+bcx)$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(4*b*c) - (
x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc}$$
$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc}$$
$$= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc} - \frac{1}{2}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(a$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.65

$$\frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))\sqrt{\sinh^2(c(a+bx))}}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)]*Sqrt[Sinh[c*(a + b*x)]^2])
/(4*b*c)
```

Maple [A]
time = 1.88, size = 89, normalized size = 1.20

method	result
default	$\frac{\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\cosh(c(bx+a))}{2}-\frac{\ln\left(\cosh(c(bx+a))+\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\right)}{2}+\frac{\sqrt{-\frac{1}{2}+\frac{\cosh(2bcx+2ac)}{2}}\left(\cos\right.}{2\sinh(c(bx+a))}}{bc}$

risch	$-\frac{x\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{c(bx+a)}}{2\left(\mathrm{e}^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\mathrm{e}^{3c(bx+a)}}{4cb\left(\mathrm{e}^{2c(bx+a)}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))-1/2*ln(cosh(c*(b*x+a))+(sinh
(c*(b*x+a))^2)^(1/2))+1/2*(sinh(c*(b*x+a))^2)^(1/2)*cosh(c*(b*x+a))^2/sinh(
c*(b*x+a)))/b/c
```

Maxima [A]
time = 0.50, size = 36, normalized size = 0.49

$$-\frac{bcx+ac}{2\,bc}+\frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 0.89

$$-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(68) = 136.
time = 2.83, size = 275, normalized size = 3.72

$$\begin{cases}\frac{\sqrt{\sinh^{2}\left(bcx+\log\left(-e^{-bcx}\right)\right)}\log\left(-e^{-bcx}\right)}{bc} & \text{for}\ a=\frac{\log\left(-e^{-bcx}\right)}{c}\\ -\frac{\sqrt{\sinh^{2}\left(bcx+\log\left(e^{-bcx}\right)\right)}\log\left(e^{-bcx}\right)}{bc} & \text{for}\ a=\frac{\log\left(e^{-bcx}\right)}{c}\\ 0 & \text{for}\ c=0\\ x\sqrt{\sinh^{2}\left(ac\right)}\,e^{ac} & \text{for}\ b=0\\ \frac{x\sqrt{\sinh^{2}\left(ac+bcx\right)}\,e^{ac}e^{bcx}}{2}-\frac{x\sqrt{\sinh^{2}\left(ac+bcx\right)}\,e^{ac}e^{bcx}\cosh\left(ac+bcx\right)}{2\sinh\left(ac+bcx\right)}-\frac{\sqrt{\sinh^{2}\left(ac+bcx\right)}\,e^{ac}e^{bcx}}{2bc}+\frac{\sqrt{\sinh^{2}\left(ac+bcx\right)}\,e^{ac}e^{bcx}\cosh\left(ac+bcx\right)}{bc\sinh\left(ac+bcx\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((sqrt(sinh(b*c*x + log(-exp(-b*c*x)))**2)*log(-exp(-b*c*x))/(b*c)
, Eq(a, log(-exp(-b*c*x))/c)), (-sqrt(sinh(b*c*x + log(exp(-b*c*x)))**2)*lo
g(exp(-b*c*x))/(b*c), Eq(a, log(exp(-b*c*x))/c)), (0, Eq(c, 0)), (x*sqrt(si
nh(a*c)**2)*exp(a*c), Eq(b, 0)), (x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp
(b*c*x)/2 - x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*cosh(a*c + b*c
*x)/(2*sinh(a*c + b*c*x)) - sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)/
(2*b*c) + sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*cosh(a*c + b*c*x)/
(b*c*sinh(a*c + b*c*x)), True))
```

Giac [A]
time = 0.43, size = 71, normalized size = 0.96

$$-\frac{1}{2}\,x\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\frac{e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

Mupad [B]
time = 0.63, size = 77, normalized size = 1.04

$$-\frac{\left(x\,{\mathrm{e}}^{a\,c+b\,c\,x}-\frac{{\mathrm{e}}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] -((x*exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
- exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) - 1)
```

3.332 $\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)/b/c/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
```

```
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(2\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.96

$$\frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b*c*Sqrt[Sinh[c*(a + b*x)]^
2])
```

Maple [A]
time = 9.83, size = 68, normalized size = 1.48

method	result	size
risch	$\frac{\ln\left(e^{2bcx}-e^{-2ac}\right)\left(e^{2c(bx+a)}-1\right)e^{-c(bx+a)}}{cb\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(2*b*c*x)-exp(-2*a*c))/c/b*(exp(2*c*(b*x+a))-1)/((exp(2*c*(b*x+a))-1)
^2*exp(-2*c*(b*x+a)))^(1/2)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.49, size = 39, normalized size = 0.85

$$\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.91

$$\frac{\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\sqrt{\sinh^2\left(ac+bcx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sinh(a*c + b*c*x)**2), x)
```

Giac [A]
time = 0.43, size = 85, normalized size = 1.85

$$\frac{\log\left(e^{(bcx)}+e^{(-ac)}\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\log\left(\left|e^{(bcx)}-e^{(-ac)}\right|\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (log(e^(b*c*x) + e^(-a*c))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + log(ab
s(e^(b*c*x) - e^(-a*c)))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{e^{c\,(a+b\,x)}}{\sqrt{\sinh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.333 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+
a*c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$-\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqr
t[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}} \\
&= \frac{\sinh(ac+bcx)\operatorname{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}} \\
&= \frac{(8\sinh(ac+bcx))\operatorname{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}} \\
&= -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.79

$$-\frac{4e^{5c(a+bx)}\sqrt{\sinh^2(c(a+bx))}}{bc\left(-1+e^{2c(a+bx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
[Out] (-4*E^(5*c*(a + b*x))*Sqrt[Sinh[c*(a + b*x)]^2])/(b*c*(-1 + E^(2*c*(a + b*x
)))^3)
```

Maple [A]
time = 9.78, size = 69, normalized size = 1.19

method	result	size
risch	$-\frac{2\left(2\,e^{2c(bx+a)}-1\right)e^{-c(bx+a)}}{cb\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,\left(e^{2c(bx+a)}-1\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/b*(2*exp(2*c*(b*x+a))-1)/((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1
/2)/(exp(2*c*(b*x+a))-1)*exp(-c*(b*x+a))
```

Maxima [A]
time = 0.48, size = 84, normalized size = 1.45

$$-\frac{4\,e^{(2\,bcx+2\,ac)}}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}+\frac{2}{bc\left(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.
time = 0.35, size = 121, normalized size = 2.09

$$-\frac{2\,(\cosh(bcx+ac)+3\,\sinh(bcx+ac))}{bc\cosh(bcx+ac)^3+3\,bc\cosh(bcx+ac)\sinh(bcx+ac)^2+bc\sinh(bcx+ac)^3-bc\cosh(bcx+ac)+3\,\left(bc\cosh(bcx+ac)^2-bc\right)\sinh(bcx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int\frac{e^{bcx}}{\left(\sinh^2(ac+bcx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(sinh(a*c + b*c*x)**2)**(3/2), x)
```

Giac [A]
time = 0.43, size = 87, normalized size = 1.50

$$-\frac{2\left(2\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{bc\left(e^{(2\,bcx+2\,ac)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2)
```

Mupad [B]
time = 0.59, size = 76, normalized size = 1.31

$$-\frac{4\,{\mathrm{e}}^{a\,c+b\,c\,x}\,\left(2\,{\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{{\left(\frac{{\mathrm{e}}^{a\,c+b\,c\,x}}{2}-\frac{{\mathrm{e}}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left({\mathrm{e}}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*((exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1)^3)
```

3.334 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(a$$

```
[Out] -4*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+32/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)-8*si
nh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$-\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Sinh[a*c + b*c*x
]^2]) + (32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3*Sqrt[Sinh[a
*c + b*c*x]^2]) - (8*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt
[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}^5(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\ &= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\ &= \frac{(32\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\ &= \frac{(16\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx,x,e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\ &= \frac{(16\sinh(ac+bcx))\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)\,dx,x,e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\ &= -\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.49

$$-\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sinh(c(a+bx))}{3bc\left(-1+e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sinh[c*(a + b*x)])/(3*b
*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Sinh[c*(a + b*x)]^2])
```

Maple [**A**]
time = 12.62, size = 80, normalized size = 0.54

method	result	size
risch	$-\frac{4\left(6\,{\mathrm e}^{4 c \left(b x +a \right)}-4\,{\mathrm e}^{2 c \left(b x +a \right)}+1\right) {\mathrm e}^{-c \left(b x +a \right)}}{3 c b \sqrt{\left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{2} {\mathrm e}^{-2 c \left(b x +a \right)}}\, \left({\mathrm e}^{2 c \left(b x +a \right)}-1\right)^{3}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/c/b*(6*exp(4*c*(b*x+a))-4*exp(2*c*(b*x+a))+1)/((exp(2*c*(b*x+a))-1)^2*
exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^3*exp(-c*(b*x+a))
```

Maxima [**A**]
time = 0.50, size = 209, normalized size = 1.42

$$-\frac{8 \, e^{\left(4 \, b c x + 4 \, a c\right)}}{b c {\left(e^{\left(8 \, b c x + 8 \, a c\right)} - 4 \, e^{\left(6 \, b c x + 6 \, a c\right)} + 6 \, e^{\left(4 \, b c x + 4 \, a c\right)} - 4 \, e^{\left(2 \, b c x + 2 \, a c\right)} + 1\right)}} + \frac{16 \, e^{\left(2 \, b c x + 2 \, a c\right)}}{3 \, b c {\left(e^{\left(8 \, b c x + 8 \, a c\right)} - 4 \, e^{\left(6 \, b c x + 6 \, a c\right)} + 6 \, e^{\left(4 \, b c x + 4 \, a c\right)} - 4 \, e^{\left(2 \, b c x + 2 \, a c\right)} + 1\right)}} - \frac{4}{3 \, b c {\left(e^{\left(8 \, b c x + 8 \, a c\right)} - 4 \, e^{\left(6 \, b c x + 6 \, a c\right)} + 6 \, e^{\left(4 \, b c x + 4 \, a c\right)} - 4 \, e^{\left(2 \, b c x + 2 \, a c\right)} + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.
time = 0.36, size = 315, normalized size = 2.14

$$-\frac{4 \, {\left(7 \, \cosh\left(b c x + a c\right)^{2} + 10 \, \cosh\left(b c x + a c\right) \sinh\left(b c x + a c\right) + 7 \, \sinh\left(b c x + a c\right)^{2} - 4\right)}}{3 \, {\left(b c \cosh\left(b c x + a c\right)^{6} + 6 \, b c \cosh\left(b c x + a c\right) \sinh\left(b c x + a c\right)^{5} + b c \sinh\left(b c x + a c\right)^{6} - 4 \, b c \cosh\left(b c x + a c\right)^{4} + {\left(15 \, b c \cosh\left(b c x + a c\right)^{2} - 4 \, b c\right)} \sinh\left(b c x + a c\right)^{4} + 7 \, b c \cosh\left(b c x + a c\right)^{2} + 4 \, {\left(5 \, b c \cosh\left(b c x + a c\right)^{3} - 4 \, b c \cosh\left(b c x + a c\right)\right)} \sinh\left(b c x + a c\right)^{3} + {\left(15 \, b c \cosh\left(b c x + a c\right)^{4} - 24 \, b c \cosh\left(b c x + a c\right)^{2} + 7 \, b c\right)} \sinh\left(b c x + a c\right)^{2} - 4 \, b c + 2 \, {\left(3 \, b c \cosh\left(b c x + a c\right)^{5} - 8 \, b c \cosh\left(b c x + a c\right)^{3} + 5 \, b c \cosh\left(b c x + a c\right)\right)} \sinh\left(b c x + a c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
```

```
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**A**]
time = 0.45, size = 122, normalized size = 0.83

$$-\frac{4\left(6\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-4\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 4*e^(
2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^4)
```

Mupad [**B**]
time = 0.62, size = 89, normalized size = 0.61

$$-\frac{8\,\mathrm{e}^{a\,c+b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}\,\left(6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}-4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*
(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*c +
2*b*c*x) - 1)^5)
```

3.335 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(}}$$

```
[Out] -32/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^6/(sinh(b*c*x+a*c)^2)^(1/2)+
192/5*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^5/(sinh(b*c*x+a*c)^2)^(1/2)-
48*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+64/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^6*Sqrt[Sinh[a*c + b*
c*x]^2]) + (192*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^(2*c*(a + b*x)))^5*Sqrt[Si
nh[a*c + b*c*x]^2]) - (48*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4
*Sqrt[Sinh[a*c + b*c*x]^2]) + (64*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a
+ b*x)))^3*Sqrt[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{128x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(128\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^7}{(-1+x^2)^7}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(64\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(64\sinh(ac+bcx))\text{Subst}\left(\int \left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right)\,dx, x, e^{2c(a}\right.}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= -\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}} + \frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx}}$$

Mathematica [A]

time = 0.06, size = 84, normalized size = 0.42

$$-\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(c(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b*x)))^6*Sqrt[Sinh[c*(a + b
*x)]^2])
```

Maple [A]
time = 10.59, size = 91, normalized size = 0.46

method	result	size
risch	$-\frac{16\left(20\,\mathrm{e}^{6c(bx+a)}-15\,\mathrm{e}^{4c(bx+a)}+6\,\mathrm{e}^{2c(bx+a)}-1\right)\mathrm{e}^{-c(bx+a)}}{15cb\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2\mathrm{e}^{-2c(bx+a)}}\,\left(\mathrm{e}^{2c(bx+a)}-1\right)^5}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/c/b*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)/(
(exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)/(exp(2*c*(b*x+a))-1)^5*exp(
-c*(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(173) = 346.
time = 0.49, size = 386, normalized size = 1.94

$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}+\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}+\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
```

```
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(173) = 346$.
time = 0.37, size = 592, normalized size = 2.97

$$-\frac{16 \left(19 \cosh\left(bcx+ac\right)^{3}+57 \cosh\left(bcx+ac\right) \sinh\left(bcx+ac\right)^{2}+21 \sinh\left(bcx+ac\right)^{3}+21 \left(3 \cosh\left(bcx+ac\right)^{2}-1\right) \sinh\left(bcx+ac\right)-9 \cosh\left(bcx+ac\right)\right)}{15 \left(bc \cosh\left(bcx+ac\right)^{9}+9 bc \cosh\left(bcx+ac\right) \sinh\left(bcx+ac\right)^{8}+bc \sinh\left(bcx+ac\right)^{9}-6 bc \cosh\left(bcx+ac\right)^{7}+6 \left(6 bc \cosh\left(bcx+ac\right)^{2}-bc\right) \sinh\left(bcx+ac\right)^{7}+15 bc \cosh\left(bcx+ac\right)^{5}+42 \left(2 bc \cosh\left(bcx+ac\right)^{3}-bc \cosh\left(bcx+ac\right)\right) \sinh\left(bcx+ac\right)^{6}+3 \left(42 bc \cosh\left(bcx+ac\right)^{4}-42 bc \cosh\left(bcx+ac\right)^{2}+5 bc\right) \sinh\left(bcx+ac\right)^{5}-19 bc \cosh\left(bcx+ac\right)^{3}+3 \left(42 bc \cosh\left(bcx+ac\right)^{5}-70 bc \cosh\left(bcx+ac\right)^{3}+25 bc \cosh\left(bcx+ac\right)\right) \sinh\left(bcx+ac\right)^{4}+3 \left(28 bc \cosh\left(bcx+ac\right)^{6}-70 bc \cosh\left(bcx+ac\right)^{4}+50 bc \cosh\left(bcx+ac\right)^{2}-7 bc\right) \sinh\left(bcx+ac\right)^{3}+9 bc \cosh\left(bcx+ac\right)+3 \left(12 bc \cosh\left(bcx+ac\right)^{7}-42 bc \cosh\left(bcx+ac\right)^{5}+50 bc \cosh\left(bcx+ac\right)^{3}-19 bc \cosh\left(bcx+ac\right)\right) \sinh\left(bcx+ac\right)^{2}+3 \left(3 bc \cosh\left(bcx+ac\right)^{8}-14 bc \cosh\left(bcx+ac\right)^{6}+25 bc \cosh\left(bcx+ac\right)^{4}-21 bc \cosh\left(bcx+ac\right)^{2}+7 bc\right) \sinh\left(bcx+ac\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 161, normalized size = 0.81

$$-\frac{16 \left(20 e^{\left(6 bcx+6 ac\right)} \mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)-15 e^{\left(4 bcx+4 ac\right)} \mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)+6 e^{\left(2 bcx+2 ac\right)} \mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)-\mathrm{sgn}\left(e^{\left(bcx+ac\right)}-e^{\left(-bcx-ac\right)}\right)\right)}{15 bc\left(e^{\left(2 bcx+2 ac\right)}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 15
*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 6*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6)
```

Mupad [B]
time = 0.62, size = 353, normalized size = 1.77

$$\frac{128\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}+\frac{96\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}^4}+\frac{384\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{5\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}^5}+\frac{64\,e^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}-e^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(
1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x
) - 1)^3) + (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c
*x)/2)^2)^(1/2))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c
+ 2*b*c*x) - 1)^4) + (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(-
 a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))
*(exp(2*a*c + 2*b*c*x) - 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x
)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^6)
```

3.336 $\int e^x \sinh(a+bx)\,dx$

Optimal. Leaf size=41

$$-\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

```
[Out] -b*exp(x)*cosh(b*x+a)/(-b^2+1)+exp(x)*sinh(b*x+a)/(-b^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{5582\}$

$$\frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x],x]
[Out] -((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sinh(a+bx)\,dx=-\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.68

$$\frac{e^x(b\cosh(a+bx)-\sinh(a+bx))}{-1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x],x]
[Out] (E^x*(b*Cosh[a + b*x] - Sinh[a + b*x]))/(-1 + b^2)
```

Maple [A]
time = 0.51, size = 62, normalized size = 1.51

method	result	size
risch	$\frac{e^{bx+a+x}}{2+2b}+\frac{e^{-bx-a+x}}{2b-2}$	33
default	$-\frac{\sinh((b-1)x+a)}{2(b-1)}+\frac{\sinh((1+b)x+a)}{2+2b}+\frac{\cosh((b-1)x+a)}{2b-2}+\frac{\cosh((1+b)x+a)}{2+2b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(b-1)*sinh((b-1)*x+a)+1/2/(1+b)*sinh((1+b)*x+a)+1/2*cosh((b-1)*x+a)/(b
-1)+1/2*cosh((1+b)*x+a)/(1+b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-b>0)', see `assume?` for more deta
ils)Is
```

Fricas [A]
time = 0.36, size = 42, normalized size = 1.02

$$\frac{b\cosh(bx+a)\cosh(x)+b\cosh(bx+a)\sinh(x)-(\cosh(x)+\sinh(x))\sinh(bx+a)}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*cosh(b*x + a)*cosh(x) + b*cosh(b*x + a)*sinh(x) - (cosh(x) + sinh(x))*si
nh(b*x + a))/(b^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(31) = 62$.
time = 0.20, size = 99, normalized size = 2.41

$$\begin{cases} \frac{xe^x\sinh(a-x)}{2}+\frac{xe^x\cosh(a-x)}{2}+\frac{e^x\sinh(a-x)}{2} & \text{for } b=-1 \\ \frac{xe^x\sinh(a+x)}{2}-\frac{xe^x\cosh(a+x)}{2}+\frac{e^x\cosh(a+x)}{2} & \text{for } b=1 \\ \frac{be^x\cosh(a+bx)}{b^2-1}-\frac{e^x\sinh(a+bx)}{b^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 + exp(x)*sinh(a
- x)/2, Eq(b, -1)), (x*exp(x)*sinh(a + x)/2 - x*exp(x)*cosh(a + x)/2 + exp(
x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*cosh(a + b*x)/(b**2 - 1) - exp(x)*si
nh(a + b*x)/(b**2 - 1), True))
```

Giac [**A**]
time = 0.40, size = 32, normalized size = 0.78

$$\frac{e^{(bx+a+x)}}{2\,(b+1)}+\frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) + 1/2*e^(-b*x - a + x)/(b - 1)
```

Mupad [**B**]
time = 0.09, size = 45, normalized size = 1.10

$$\frac{\mathrm{e}^{x-a-b\,x}\,\left(b-\mathrm{e}^{2\,a+2\,b\,x}+b\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(a + b*x),x)
```

```
[Out] (exp(x - a - b*x)*(b - exp(2*a + 2*b*x) + b*exp(2*a + 2*b*x) + 1))/(2*(b^2
- 1))
```

3.337 $\int e^x \sinh(a + cx^2)\, dx$

Optimal. Leaf size=85

$$\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-1/
4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5623, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{\frac{1}{4c}-a}\mathrm{Erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,e^{a-\frac{1}{4c}}\mathrm{Erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^(
a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^x \sinh\left(a+cx^2\right)\,dx &= \int \left(-\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a+x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+x+cx^2}\,dx\\ &= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}\,dx-\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}\,dx\\ &= \frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.94

$$\frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(-e^{\frac{1}{2}/c}\mathrm{Erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + c*x^2],x]
[Out] (Sqrt[Pi]*(-(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]))
 + Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c
)))
```

Maple [A]
time = 1.20, size = 72, normalized size = 0.85

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\,\mathrm{erf}\left(x\sqrt{-c}-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf(x*(-c)^(1/2)-1/2/(-c)^(1/2))
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.76

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) - 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Fricas [A]
time = 0.36, size = 103, normalized size = 1.21

$$-\frac{\sqrt{\pi}\,\sqrt{-c}\left(\cosh\left(\frac{4ac-1}{4c}\right)+\sinh\left(\frac{4ac-1}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+1)\sqrt{-c}}{2c}\right)+\sqrt{\pi}\,\sqrt{c}\left(\cosh\left(\frac{4ac-1}{4c}\right)-\sinh\left(\frac{4ac-1}{4c}\right)\right)\operatorname{erf}\left(\frac{2cx-1}{2\sqrt{c}}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) + sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x}\sinh\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + c*x**2), x)
```

Giac [A]
time = 0.41, size = 73, normalized size = 0.86

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^x \sinh\left(c\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(a + c*x^2),x)
```

```
[Out] int(exp(x)*sinh(a + c*x^2), x)
```

3.338 $\int e^x \sinh(a+bx+cx^2)\,dx$

Optimal. Leaf size=101

$$\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/
4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5623, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{\frac{(1-b)^2}{4c}-a}\mathrm{Erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,e^{a-\frac{(b+1)^2}{4c}}\mathrm{Erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sqr
t[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]
)/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x \sinh\left(a+bx+cx^2\right) dx &= \int \left(-\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right) dx \\
&= -\left(\frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx \\
&= -\left(\frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx\right)+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx \\
&= \frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 92, normalized size = 0.91

$$\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(-e^{\frac{1+b^2}{2c}}\mathrm{Erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
 - Sinh[a])) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*S
qrt[c]*E^((1 + b)^2/(4*c)))
```

Maple [A]
time = 3.78, size = 97, normalized size = 0.96

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2+2b-1}{4c}}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2-2b-1}{4c}}\,\mathrm{erf}\left(-x\sqrt{-c}+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-x*(-c)^
(1/2)+1/2*(1+b)/(-c)^(1/2))
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.80

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\,x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4\,c}\right)}}{4\sqrt{-c}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c}\,x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4\,c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) - 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

Fricas [A]
time = 0.38, size = 129, normalized size = 1.28

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)-\sinh\left(-\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2\,cx+b-1}{2\sqrt{c}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) + sqrt(pi)*s
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^x \sinh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.42, size = 91, normalized size = 0.90

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2\,x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2\,x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4\,ac-2\,b+1}{4\,c}\right)}}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x + c*x^2)*exp(x),x)
```

```
[Out] int(sinh(a + b*x + c*x^2)*exp(x), x)
```

3.339 $\int e^{x^2} \sinh(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}(b+2x)\right)$$

```
[Out] 1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+x)
*Pi^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5623, 2266, 2235}

$$\frac{1}{4}\sqrt{\pi}\,e^{a-\frac{b^2}{4}}\,\text{Erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}\,e^{-a-\frac{b^2}{4}}\,\text{Erfi}\left(\frac{1}{2}(2x-b)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x],x]
```

```
[Out] -1/4*(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2]) + (E^(a - b^2/4)*Sqrt[Pi]
*Erfi[(b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\sinh(a+bx)\,dx = \int \left(-\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right)dx$$
$$= -\left(\frac{1}{2}\int e^{-a-bx+x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+x^2}\,dx$$
$$= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx$$
$$= -\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(b+2x)\right)$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.78

$$\frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{b}{2}-x\right)(\cosh(a)-\sinh(a))+\mathrm{Erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x],x]
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(Cosh[a] - Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Sin
h[a])))/(4*E^(b^2/4))
```

Maple [C] Result contains complex when optimal does not.
time = 1.15, size = 52, normalized size = 0.80

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^2}{4}}\,\mathrm{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4}}\,\mathrm{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.27, size = 45, normalized size = 0.69

$$-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Fricas [A]
time = 0.39, size = 45, normalized size = 0.69

$$\frac{1}{4}\,\sqrt{\pi}\,\left(\mathrm{erfi}\left(\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2+a\right)}-\mathrm{erfi}\left(-\frac{1}{2}\,b+x\right)e^{\left(\frac{1}{4}\,b^2-a\right)}\right)e^{\left(-\frac{1}{2}\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erfi(1/2*b + x)*e^(1/4*b^2 + a) - erfi(-1/2*b + x)*e^(1/4*b^2
 - a))*e^(-1/2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 45, normalized size = 0.69

$$\frac{1}{4}i\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\,\sinh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(a + b*x),x)
```

```
[Out] int(exp(x^2)*sinh(a + b*x), x)
```

3.340 $\int e^{x^2} \sinh\left(a+cx^2\right) \, dx$

Optimal. Leaf size=65

$$-\frac{e^{-a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

```
[Out] -1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c
)^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5623, 2235}

$$\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{c+1}\,x\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[
Sqrt[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\sinh\left(a+cx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-a+(1-c)x^2}+\frac{1}{2}e^{a+(1+c)x^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-a+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+c)x^2}\,dx\\ &= -\frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 1.11

$$\frac{\sqrt{\pi}\left(-\sqrt{-1+c}\,(1+c)\text{Erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{Erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\sinh(a))\right)}{4\,(-1+c^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]))
+ (-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c
^2))
```

Maple [A]
time = 6.68, size = 48, normalized size = 0.74

method	result	size
risch	$-\frac{\sqrt{\pi}\,e^{-a}\,\text{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,e^{a}\,\text{erf}\left(\sqrt{-c-1}\,x\right)}{4\sqrt{-c-1}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
c-1)^(1/2)*erf((-c-1)^(1/2)*x)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.72

$$-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{c-1}\,x\right)e^{(-a)}}{4\,\sqrt{c-1}}+\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{-c-1}\,x\right)e^{a}}{4\,\sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.15

$$-\frac{\sqrt{\pi}\,((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\,\text{erf}\left(\sqrt{c-1}\,x\right)+\sqrt{\pi}\,((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{-c-1}\,\text{erf}\left(\sqrt{-c-1}\,x\right)}{4\,(c^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) + sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt
(-c - 1)*x))/(c^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2} \sinh\left(a + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + c*x**2), x)
```

Giac [A]
time = 0.42, size = 49, normalized size = 0.75

$$\frac{\sqrt{\pi}\, \operatorname{erf}\left(-\sqrt{c-1}\, x\right) e^{(-a)}}{4\, \sqrt{c-1}} - \frac{\sqrt{\pi}\, \operatorname{erf}\left(-\sqrt{-c-1}\, x\right) e^{a}}{4\, \sqrt{-c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sqr
t(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{x^2}\, \sinh\left(c\, x^2 + a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(a + c*x^2),x)
```

```
[Out] int(exp(x^2)*sinh(a + c*x^2), x)
```

3.341 $\int e^{x^2} \sinh(a+bx+cx^2)\,dx$

Optimal. Leaf size=115

$$\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

```
[Out] 1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1-c
)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^(1/
2)/(1+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5623, 2266, 2235}

$$\frac{\sqrt{\pi}\,e^{-a-\frac{b^2}{4(1-c)}}\mathrm{Erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{\sqrt{\pi}\,e^{a-\frac{b^2}{4(c+1)}}\mathrm{Erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])
/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)
/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx=\int\left(-\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)\,dx$$
$$=-\left(\frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx$$
$$=-\left(\frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx$$
$$=\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

Mathematica [A]
time = 0.29, size = 123, normalized size = 1.07

$$\frac{e^{-\frac{b^2}{4+4c}}\sqrt{\pi}\left(-\sqrt{-1+c}\,(1+c)e^{\frac{b^2c}{2(-1+c^2)}}\mathrm{Erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\mathrm{Erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)(\cosh(a)+\sinh(a))\right)}{4(-1+c^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
 + c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a])) + (-1 + c)*Sqrt[1 + c]*Erfi
[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

Maple [A]
time = 6.53, size = 105, normalized size = 0.91

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2-4a}{4(c-1)}}\,\mathrm{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\,\mathrm{erf}\left(-\sqrt{-c-1}\,x+\frac{b}{2\sqrt{-c-1}}\right)}{4\sqrt{-c-1}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-c-1)^(1/2
)*erf(-(-c-1)^(1/2)*x+1/2*b/(-c-1)^(1/2))
```

Maxima [A]
time = 0.26, size = 89, normalized size = 0.77

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c-1}\,x-\frac{b}{2\,\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\,\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Fricas [A]
time = 0.41, size = 164, normalized size = 1.43

$$-\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\operatorname{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)+\sqrt{\pi}\left((c-1)\cosh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)+(c-1)\sinh\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)\right)\sqrt{-c-1}\,\operatorname{erf}\left(\frac{(2\,(c+1)x+b)\sqrt{-c-1}}{2\,(c+1)}\right)}{4\,(c^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)
/sqrt(c - 1)) + sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) +
(c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c
+ 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x + c*x**2), x)
```

Giac [A]
time = 0.41, size = 101, normalized size = 0.88

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\,\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\,\sqrt{c-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(c\,x^2+b\,x+a\right)\,{\mathrm{e}}^{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x + c*x^2)*exp(x^2),x)
```

```
[Out] int(sinh(a + b*x + c*x^2)*exp(x^2), x)
```

3.342 $\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx$

Optimal. Leaf size=110

$$-\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f
^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Lo
g[f])/(2*Sqrt[f])]) + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*E
rfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx = \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)\,dx$$
$$= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx$$
$$= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx$$
$$= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,d$$
$$= -\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\Big($$

Mathematica [A]
time = 0.10, size = 103, normalized size = 0.94

$$\frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-e^{\frac{b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(-(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Co
sh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A]
time = 0.78, size = 100, normalized size = 0.91

method	result	size

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}} \operatorname{erf}\left(-\sqrt{-f}\, x+\frac{\ln(f) b}{2 \sqrt{-f}}\right)}{4 \sqrt{-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-4 d f}{4 f}} \operatorname{erf}\left(-\sqrt{f}\, x+\frac{\ln(f) b}{2 \sqrt{f}}\right)}{4 \sqrt{f}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)/(-f)^(1/2)*erf(-(-f)^(1/2
)*x+1/2*ln(f)*b/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)
/f^(1/2)*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.82

$$-\frac{1}{4}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log\left(f\right)}{2\,\sqrt{f}}\right)e^{\left(\frac{b^{2}\log\left(f\right)^{2}}{4\,f}-d\right)}+\frac{\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log\left(f\right)}{2\,\sqrt{-f}}\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}}{4\,f}+d\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*
e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(84) = 168.
time = 0.37, size = 213, normalized size = 1.94

$$-\frac{\sqrt{\pi}\,\sqrt{-f}\,\cosh\left(\frac{b^{2}\log\left(f\right)^{2}-4\,af\log\left(f\right)-4\,df}{4\,f}\right)\operatorname{erf}\left(\frac{\left(2\,fx+b\log\left(f\right)\right)\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\,\sqrt{f}\,\cosh\left(\frac{b^{2}\log\left(f\right)^{2}+4\,af\log\left(f\right)-4\,df}{4\,f}\right)\operatorname{erf}\left(-\frac{2\,fx-b\log\left(f\right)}{2\,\sqrt{f}}\right)-\sqrt{\pi}\,\sqrt{f}\,\operatorname{erf}\left(-\frac{2\,fx-b\log\left(f\right)}{2\,\sqrt{f}}\right)\sinh\left(\frac{b^{2}\log\left(f\right)^{2}+4\,af\log\left(f\right)-4\,df}{4\,f}\right)-\sqrt{\pi}\,\sqrt{-f}\,\operatorname{erf}\left(\frac{\left(2\,fx+b\log\left(f\right)\right)\sqrt{-f}}{2\,f}\right)\sinh\left(\frac{b^{2}\log\left(f\right)^{2}-4\,af\log\left(f\right)-4\,df}{4\,f}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) - sqrt
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh\left(d+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2), x)
```

Giac [A]
time = 0.41, size = 106, normalized size = 0.96

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\,\left(2\,x-\frac{b\log\left(f\right)}{f}\right)\right)e^{\left(\frac{b^{2}\log\left(f\right)^{2}+4\,af\log\left(f\right)-4\,df}{4\,f}\right)}}{4\,\sqrt{f}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\,\left(2\,x+\frac{b\log\left(f\right)}{f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,af\log\left(f\right)-4\,df}{4\,f}\right)}}{4\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*
a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*lo
g(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\mathrm{sinh}\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2), x)
```

3.343 $\int f^{a+bx} \sinh^2(d+fx^2)\,dx$

Optimal. Leaf size=148

$$\frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*
(4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(f
)^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2
)
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5623, 2225, 2325, 2266, 2236, 2235}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\mathrm{Erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\,\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x)/(
2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx &= \int\left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx\\ &= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\\ &= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int\\ &= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{er}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 149, normalized size = 1.01

$$\frac{1}{16}f^a\left(-\frac{8f^{bx}}{b\log(f)}+\frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}}+\frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((-8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (S
```

qrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16

Maple [A]
time = 2.94, size = 126, normalized size = 0.85

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-16 d f}{8 f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{b \ln(f) \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-16 d f}{8 f}} \mathrm{erf}\left(-\sqrt{-2 f}\, x+\frac{b \ln(f)}{2 \sqrt{-2 f}}\right)}{8 \sqrt{-2 f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)*2^(1/2)/f^(1/2)*erf(-2^(
1/2)*f^(1/2)*x+1/4*b*ln(f)*2^(1/2)/f^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/8*(b^2*
ln(f)^2-16*d*f)/f)/(-2*f)^(1/2)*erf(-(-2*f)^(1/2)*x+1/2*b*ln(f)/(-2*f)^(1/2
))-1/2*f^a*f^(b*x)/b/ln(f)
```

Maxima [A]
time = 0.48, size = 127, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{f}\,x-\frac{\sqrt{2}\,b\log(f)}{4\,\sqrt{f}}\right)e^{\left(\frac{b^{2}\log(f)^{2}}{8\,f}-2\,d\right)}}{16\,\sqrt{f}}+\frac{\sqrt{2}\,\sqrt{\pi}\,f^{a}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{-f}\,x-\frac{\sqrt{2}\,b\log(f)}{4\,\sqrt{-f}}\right)e^{\left(-\frac{b^{2}\log(f)^{2}}{8\,f}+2\,d\right)}}{16\,\sqrt{-f}}-\frac{f^{bx+a}}{2\,b\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) - 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(114) = 228.
time = 0.37, size = 278, normalized size = 1.88

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,b\sqrt{-f}\,\cosh\left(\frac{b^{2}\log(f)^{2}-8\,af\log(f)-16\,df}{8\,f}\right)\mathrm{erf}\left(\frac{\sqrt{2}\,(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log\left(f\right)+\sqrt{2}\,\sqrt{\pi}\,b\sqrt{f}\,\cosh\left(\frac{b^{2}\log(f)^{2}+8\,af\log(f)-16\,df}{8\,f}\right)\mathrm{erf}\left(-\frac{\sqrt{2}\,(4\,fx-b\log(f))}{4\,\sqrt{f}}\right)\log\left(f\right)+\sqrt{2}\,\sqrt{\pi}\,b\sqrt{f}\,\mathrm{erf}\left(-\frac{\sqrt{2}\,(4\,fx-b\log(f))}{4\,\sqrt{f}}\right)\log\left(f\right)\sinh\left(\frac{b^{2}\log(f)^{2}+8\,af\log(f)-16\,df}{8\,f}\right)-\sqrt{2}\,\sqrt{\pi}\,b\sqrt{-f}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log\left(f\right)\sinh\left(\frac{b^{2}\log(f)^{2}-8\,af\log(f)-16\,df}{8\,f}\right)+8\,f\cosh\left((bx+a)\log\left(f\right)\right)+8\,f\sinh\left((bx+a)\log\left(f\right)\right)}{16\,bf\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
```

```
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx} \sinh^2\left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 356, normalized size = 2.41

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{f}\left(4\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-16\,df}{8\,f}\right)}}{16\,\sqrt{f}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{4}\,\sqrt{2}\,\sqrt{-f}\left(4\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)}}{16\,\sqrt{-f}}-\left(\frac{2\,b\cos\left(-\frac{1}{2}\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\operatorname{sgn}(f)+\frac{1}{2}\,\pi a\right)\log\left(|f|\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\operatorname{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}\,\pi bx-\frac{1}{2}\,\pi a\operatorname{sgn}(f)+\frac{1}{2}\,\pi a\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log\left(|f|\right)+a\log\left(|f|\right)\right)}+i\left(-\frac{i\,e^{\left(\frac{1}{2}i\,\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\,\pi bx+\frac{1}{2}i\,\pi a\operatorname{sgn}(f)-\frac{1}{2}i\,\pi a\right)}}{2i\,\pi b\operatorname{sgn}\left(f\right)-2i\,\pi b+4\,b\log\left(|f|\right)}+\frac{i\,e^{\left(-\frac{1}{2}i\,\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\,\pi bx-\frac{1}{2}i\,\pi a\operatorname{sgn}(f)+\frac{1}{2}i\,\pi a\right)}}{-2i\,\pi b\operatorname{sgn}\left(f\right)+2i\,\pi b+4\,b\log\left(|f|\right)}\right)e^{\left(bx\log\left(|f|\right)+a\log\left(|f|\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi
*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*lo
g(abs(f))) + I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) +
 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(ab
s(f)) + a*log(abs(f)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^2, x)
```

3.344 $\int f^{a+bx} \sinh^3(d+fx^2)\,dx$

Optimal. Leaf size=239

$$\frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}e^{d-\frac{b^2 l}{}}$$

```
[Out] -1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/
2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*er
fi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*b^
2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*exp(
d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\text{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{Erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\text{Erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx &= \int\left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}-\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)\\ &= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx\\ &= -\left(\frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\\ &= -\left(\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(}{12f}}\\ &= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 287, normalized size = 1.20

$$\frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}\cosh(d)\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\mathrm{Erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))-3\sqrt{3}\,\mathrm{Erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)-e^{\frac{b^2\log^2(f)}{3f}}\mathrm{Erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\mathrm{Erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt
[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d]) - 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])]*Sinh[d] - E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*
Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x +
b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A]
time = 3.41, size = 207, normalized size = 0.87

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-36 d f}{12 f}} \operatorname{erf}\left(-\sqrt{-3 f}\, x+\frac{\ln(f) b}{2 \sqrt{-3 f}}\right)}{16 \sqrt{-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-36 d f}{12 f}} \sqrt{3}\, \operatorname{erf}\left(-\sqrt{3}\, \sqrt{f}\, x+\frac{\ln(f) b \sqrt{3}}{6 \sqrt{f}}\right)}{48 \sqrt{f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2-36*d*f)/f)/(-3*f)^(1/2)*erf(-(-3*
f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(1/2))+1/48*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^
2-36*d*f)/f)*3^(1/2)/f^(1/2)*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(
1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)/f^(1/2)*erf(-f^(1/2)
*x+1/2*ln(f)*b/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)/(
-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
```

Maxima [A]
time = 0.49, size = 200, normalized size = 0.84

$$\frac{3}{16}\,\sqrt{\pi} f^{a-\frac{1}{2}} \operatorname{erf}\left(\sqrt{f} x-\frac{b \log(f)}{2 \sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{4 f}-d\right)}-\frac{\sqrt{3} \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{3} \sqrt{f} x-\frac{\sqrt{3} b \log(f)}{6 \sqrt{f}}\right) e^{\left(\frac{b^{2} \log(f)^{2}}{12 f}-3 d\right)}}{48 \sqrt{f}}+\frac{\sqrt{3} \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{3} \sqrt{-f} x-\frac{\sqrt{3} b \log(f)}{6 \sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{12 f}+3 d\right)}}{48 \sqrt{-f}}-\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-f} x-\frac{b \log(f)}{2 \sqrt{-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4 f}+d\right)}}{16 \sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(181) = 362.
time = 0.37, size = 445, normalized size = 1.86

$$-\frac{\sqrt{3} \sqrt{\pi} \sqrt{-f} \cosh\left(\frac{b^{2} \log(f)^{2}-12 a f \log(f)-36 d f}{12 f}\right) \operatorname{erf}\left(\frac{\sqrt{3} (6 f x+b \log(f)) \sqrt{-f}}{6 f}\right)-\sqrt{3} \sqrt{\pi} \sqrt{f} \cosh\left(\frac{b^{2} \log(f)^{2}+12 a f \log(f)-36 d f}{12 f}\right) \operatorname{erf}\left(-\frac{\sqrt{3} (6 f x-b \log(f))}{6 \sqrt{f}}\right)-\sqrt{3} \sqrt{\pi} \sqrt{f} \operatorname{erf}\left(-\frac{\sqrt{3} (6 f x-b \log(f))}{6 \sqrt{f}}\right) \sinh\left(\frac{b^{2} \log(f)^{2}+12 a f \log(f)-36 d f}{12 f}\right)-\sqrt{3} \sqrt{\pi} \sqrt{-f} \operatorname{erf}\left(\frac{\sqrt{3} (6 f x+b \log(f)) \sqrt{-f}}{6 f}\right) \sinh\left(\frac{b^{2} \log(f)^{2}-12 a f \log(f)-36 d f}{12 f}\right)-9 \sqrt{\pi} \sqrt{-f} \cosh\left(\frac{b^{2} \log(f)^{2}-4 a f \log(f)-4 d f}{4 f}\right) \operatorname{erf}\left(\frac{(2 f x+b \log(f)) \sqrt{-f}}{2 f}\right)+9 \sqrt{\pi} \sqrt{f} \cosh\left(\frac{b^{2} \log(f)^{2}+4 a f \log(f)-4 d f}{4 f}\right) \operatorname{erf}\left(-\frac{2 f x-b \log(f)}{2 \sqrt{f}}\right)+9 \sqrt{\pi} \sqrt{f} \operatorname{erf}\left(-\frac{2 f x-b \log(f)}{2 \sqrt{f}}\right) \sinh\left(\frac{b^{2} \log(f)^{2}+4 a f \log(f)-4 d f}{4 f}\right)+9 \sqrt{\pi} \sqrt{-f} \operatorname{erf}\left(\frac{(2 f x+b \log(f)) \sqrt{-f}}{2 f}\right) \sinh\left(\frac{b^{2} \log(f)^{2}-4 a f \log(f)-4 d f}{4 f}\right)}{48 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) - sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
```

```
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) - 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**3, x)
```

Giac [A]
time = 0.42, size = 223, normalized size = 0.93

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{f}\left(6\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{6}\,\sqrt{3}\,\sqrt{-f}\left(6\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{-f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{f}\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{f}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-f}\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12*
(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*
erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a
*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*l
og(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) + 3/16*sq
rt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*
log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\mathrm{sinh}\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^3, x)
```

3.345 $\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=115

$$-\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/
2))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*
ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}\,f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x
- b*Log[f])/(2*Sqrt[f])]) + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sq
rt[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx\\ &= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\\ &= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\\ &= -\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f\end{aligned}$$

Mathematica [A]
time = 0.22, size = 124, normalized size = 1.08

$$\frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{e^2+b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e
+ 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

Maple [A]
time = 0.76, size = 126, normalized size = 1.10

method	result

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln(f) b e-4 d f+e^{2}}{4 f}} \operatorname{erf}\left(-\sqrt{-f}\, x+\frac{e+b \ln(f)}{2 \sqrt{-f}}\right)}{4 \sqrt{-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-2 \ln(f) b e-4 d f+e^{2}}{4 f}} \operatorname{erf}\left(-\sqrt{f}\, x+\frac{b \ln(}{2 \sqrt{}}\right.}{4 \sqrt{f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)/(-f)^(1/2
)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(1/4*(b
^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*ln(f)-e)
/f^(1/2))
```

Maxima [A]
time = 0.27, size = 106, normalized size = 0.92

$$-\frac{1}{4}\sqrt{\pi}\, f^{a-\frac{1}{2}} \operatorname{erf}\left(\sqrt{f}\, x-\frac{b \log(f)-e}{2 \sqrt{f}}\right) e^{\left(-d+\frac{(b \log(f)-e)^{2}}{4 f}\right)}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-f}\, x-\frac{b \log(f)+e}{2 \sqrt{-f}}\right) e^{\left(d-\frac{(b \log(f)+e)^{2}}{4 f}\right)}}{4 \sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f
) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(94) = 188.
time = 0.42, size = 337, normalized size = 2.93

$$-\frac{\sqrt{\pi}\sqrt{-f} \cosh\left(\frac{b^{2} \log(f)^{2}-4 d f+\cosh(1)^{2}-2 (2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 f}\right) \operatorname{erf}\left(\frac{(2 f x+b \log(f)+\cosh(1)+\sinh(1)) \sqrt{-f}}{2 f}\right)-\sqrt{\pi}\sqrt{f} \cosh\left(\frac{b^{2} \log(f)^{2}-4 d f+\cosh(1)^{2}+2 (2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 f}\right) \operatorname{erf}\left(-\frac{2 f x-b \log(f)+\cosh(1)+\sinh(1)}{2 \sqrt{f}}\right)-\sqrt{\pi}\sqrt{f} \operatorname{erf}\left(-\frac{2 f x-b \log(f)+\cosh(1)+\sinh(1)}{2 \sqrt{f}}\right) \sinh\left(\frac{b^{2} \log(f)^{2}-4 d f+\cosh(1)^{2}+2 (2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 f}\right)-\sqrt{\pi}\sqrt{-f} \operatorname{erf}\left(\frac{(2 f x+b \log(f)+\cosh(1)+\sinh(1)) \sqrt{-f}}{2 f}\right) \sinh\left(\frac{b^{2} \log(f)^{2}-4 d f+\cosh(1)^{2}-2 (2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4 f}\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a
*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(
1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*c
osh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*a*f - b*cosh(1) - b*sinh(1
))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(-1/2*(2*f*x - b*log(f) +
cosh(1) + sinh(1))/sqrt(f)) - sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) +
 cosh(1) + sinh(1))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2
*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)
 - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f
```

```
)/f)*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a*f - b*cosh(1) - b*
sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f))/f
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2), x)
```

Giac [**A**]
time = 0.44, size = 132, normalized size = 1.15

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(
f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) + 1/4*sqrt(pi
)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*e*l
og(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2), x)
```

3.346 $\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=161

$$\frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{2e+4fx+b\log(\,}{2\sqrt{2}\,\sqrt{f}}\right.$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(
1/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*
(2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))
*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5623, 2225, 2325, 2266, 2236, 2235}

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\mathrm{Erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\,\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\mathrm{Erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\,\sqrt{f}}\right)-\frac{f^{a+bx}}{2b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 - f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx &= \int \left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx}\right)\,dx \\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx \\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int \exp\left(-2d-2fx^2+a\log(f)-x(2e-b\log(f))\right)\,dx \\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{2d-}\right. \\
&= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}e^{2d-\frac{(2e+b\log(}{8f}}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 220, normalized size = 1.37

$$\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}}f^{a-\frac{be+f}{2f}}\left(-4\sqrt{2}\,e^{\frac{4e^2+b^2\log^2(f)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\operatorname{Erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)-\sinh(2d))+b\sqrt{\pi}\operatorname{Erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{2}\,b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(-4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
 + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) +
b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[
2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

Maple [A]
time = 3.01, size = 158, normalized size = 0.98

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-4 \ln(f) b e-16 d f+4 e^{2}}{8 f}} \sqrt{2}\, \mathrm{erf}\left(-\sqrt{2}\, \sqrt{f}\, x+\frac{(b \ln(f)-2 e) \sqrt{2}}{4 \sqrt{f}}\right)}{16 \sqrt{f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+4 \ln(f) b e-16 d f+4 e^{2}}{8 f}}}{8 \sqrt{-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)*2^(1/2
)/f^(1/2)*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))-1/8*Pi^
(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)/(-2*f)^(1/2)*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))-1/2*f^a*f^(b*x)/b/ln(f)
```

Maxima [A]
time = 0.48, size = 147, normalized size = 0.91

$$\frac{\sqrt{2}\, \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\, \sqrt{-f}\, x-\frac{\sqrt{2}\, (b \log(f)+2 e)}{4 \sqrt{-f}}\right) e^{\left(2 d-\frac{(b \log(f)+2 e)^{2}}{8 f}\right)}}{16 \sqrt{-f}}+\frac{\sqrt{2}\, \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{2}\, \sqrt{f}\, x-\frac{\sqrt{2}\, (b \log(f)-2 e)}{4 \sqrt{f}}\right) e^{\left(-2 d+\frac{(b \log(f)-2 e)^{2}}{8 f}\right)}}{16 \sqrt{f}}-\frac{f^{b x+a}}{2 b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) - 1/2*f^(b*x + a)/(b*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(130) = 260.
time = 0.38, size = 434, normalized size = 2.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 16*d*f + 4*cosh
(1)^2 - 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*si
nh(1)^2)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt
(-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 - 16*d*f
 + 4*cosh(1)^2 + 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(
1) + 4*sinh(1)^2)/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2*cosh(1) + 2*sin
```

```
h(1))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*(4*f*x
- b*log(f) + 2*cosh(1) + 2*sinh(1))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
- 16*d*f + 4*cosh(1)^2 + 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(
1)*sinh(1) + 4*sinh(1)^2)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2
*log(f)^2 - 16*d*f + 4*cosh(1)^2 - 4*(2*a*f - b*cosh(1) - b*sinh(1))*log(f)
 + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*s
inh((b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**2, x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.43, size = 388, normalized size = 2.41

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)+2e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+4be\log(f)-8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{-f}}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)-2e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-4be\log(f)+8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{f}}-\left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)\log\left(|f|\right)}{4b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\operatorname{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)}{4b^2\log\left(|f|\right)^2+\left(\pi b\operatorname{sgn}\left(f\right)-\pi b\right)^2}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}+i\left(-\frac{ie^{\left(\frac{1}{2}i\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\operatorname{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\operatorname{sgn}\left(f\right)-2i\pi b+4b\log\left(|f|\right)}+\frac{ie^{\left(-\frac{1}{2}i\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\operatorname{sgn}\left(f\right)+2i\pi b+4b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f)
)*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f)/
sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f)
- 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*
d*f)/f)/sqrt(f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*
pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) +
I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/
(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log
(abs(f)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{a+b\,x}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2, x)
```

3.347 $\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=257

$$\frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{3e+6fx-b\log}{2\sqrt{3}\sqrt{f}}\right.$$

```
[Out] -1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f
))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*f
^(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/
16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2)
)*Pi^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*l
n(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\mathrm{Erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\mathrm{Erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\mathrm{Erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\mathrm{Erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx &= \int \left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx^2\right)\right.\right. \\
&= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx}\,dx\right)+\frac{1}{8}\int \exp\left(6d+6ex+6fx^2-3\left(d+ex\right.\right. \\
&= -\left(\frac{1}{8}\int \exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)\,dx\right)+\frac{1}{8}\int \exp \\
&= \frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx-\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right) \\
&= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f)}{12f}}
\end{aligned}$$

Mathematica [A]
time = 0.54, size = 354, normalized size = 1.38

$$\frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}\,e^{\frac{e^2}{2f}}\cosh(d)\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\mathrm{Erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}\,e^{\frac{2e^2+b^2\log^2(f)}{2f}}\mathrm{Erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))-3\sqrt{3}\,e^{\frac{e^2}{2f}}\mathrm{Erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)-e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\mathrm{Erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\mathrm{Erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(-3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^
2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[
d]) - 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh
```

```
[d] - E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sq
rt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*
e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*
Log[f]^2)/(4*f)))
```

Maple [A]
time = 3.31, size = 265, normalized size = 1.03

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+6 \ln(f) b e-36 d f+9 e^{2}}{12 f}} \operatorname{erf}\left(-\sqrt{-3 f}\, x+\frac{3 e+b \ln(f)}{2 \sqrt{-3 f}}\right)}{16 \sqrt{-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2} \ln(f)^{2}-6 \ln(f) b e-36 d f+9 e^{2}}{12 f}} \sqrt{3}\, \operatorname{erf}\left(\right.}{48 \sqrt{f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)/(-3*
f)^(1/2)*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))+1/48*Pi^(1/2)*
f^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)*3^(1/2)/f^(1/2)*erf(
-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))-3/16*Pi^(1/2)*f^a*exp
(1/4*(b^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)/f^(1/2)*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+
e^2)/f)/(-f)^(1/2)*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))
```

Maxima [A]
time = 0.51, size = 236, normalized size = 0.92

$$\frac{\sqrt{3}\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{3}\,\sqrt{-f}\,x-\frac{\sqrt{3}\,(b\log(f)+3\,e)}{6\,\sqrt{-f}}\right)e^{\left(3\,d-\frac{(b\log(f)+3\,e)^{2}}{12\,f}\right)}}{48\,\sqrt{-f}}+\frac{3}{16}\,\sqrt{\pi}\,f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}\,x-\frac{b\log(f)-e}{2\,\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^{2}}{4\,f}\right)}-\frac{\sqrt{3}\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{3}\,\sqrt{f}\,x-\frac{\sqrt{3}\,(b\log(f)-3\,e)}{6\,\sqrt{f}}\right)e^{\left(-3\,d+\frac{(b\log(f)-3\,e)^{2}}{12\,f}\right)}}{48\,\sqrt{f}}-\frac{3\,\sqrt{\pi}\,f^{a}\operatorname{erf}\left(\sqrt{-f}\,x-\frac{b\log(f)+e}{2\,\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^{2}}{4\,f}\right)}}{16\,\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) -
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(207) = 414$.
time = 0.41, size = 725, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(
1)^2 - 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*si
nh(1)^2)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*cosh(1) + 3*sinh(1))*sqrt
(-f)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cos
h(1)^2 + 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*
sinh(1)^2)/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*cosh(1) + 3*sinh(1))/s
qrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*c
osh(1) + 3*sinh(1))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(1)^2
 + 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1
)^2)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*c
osh(1) + 3*sinh(1))*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 36*d*f + 9*cosh(1
)^2 - 6*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sin
h(1)^2)/f) - 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2
 - 2*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2
)/f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + sinh(1))*sqrt(-f)/f) + 9*sqrt(pi
)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*a*f - b*cosh(1)
 - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f)*erf(-1/2*(2*f*x -
b*log(f) + cosh(1) + sinh(1))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x
 - b*log(f) + cosh(1) + sinh(1))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 - 4*d*f +
cosh(1)^2 + 2*(2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) +
sinh(1)^2)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f) + cosh(1) + s
inh(1))*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*a*f -
 b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/f))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**3, x)
```

Giac [A]
time = 0.45, size = 281, normalized size = 1.09

$$-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+6be\log(f)-12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{-f}}+\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-6be\log(f)+12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{f}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{-f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f)
)*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/f
)/sqrt(-f) + 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f
) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 -
 36*d*f)/f)/sqrt(f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)
/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/
sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*
(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{a+b\,x}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3, x)
```

3.348 $\int f^{a+cx^2}\sinh(d+ex)\,dx$

Optimal. Leaf size=133

$$\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {5623, 2325, 2266, 2235}

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\text{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*Sq
rt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh(d+ex)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\ &= -\left(\frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,d\\ &= \frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 104, normalized size = 0.78

$$\frac{e^{-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(-\cosh(d)+\sinh(d))+\mathrm{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x],x]
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[d]
 + Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 0.81, size = 117, normalized size = 0.88

method	result
risch	$-\frac{\sqrt{\pi}\, f^a \mathrm{e}^{\frac{4d\ln(f)c-e^2}{4\ln(f)c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)}{4\sqrt{-c\ln(f)}}-\frac{\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{4d\ln(f)c+e^2}{4\ln(f)c}}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{}{2\sqrt{}}\right)}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(4*d*l
n(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f
))^(1/2))
```

Maxima [A]
time = 0.27, size = 105, normalized size = 0.79

$$\frac{\sqrt{\pi}\, f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\, x - \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(d - \frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\, f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\, x + \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(-d - \frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(101) = 202.
time = 0.36, size = 277, normalized size = 2.08

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-\cosh(1)^2-2\,\cosh(1)\sinh(1)-\sinh(1)^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-\cosh(1)^2-2\,\cosh(1)\sinh(1)-\sinh(1)^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+\cosh(1)+\sinh(1))\sqrt{-c\log(f)}}{2\,c\log(f)}\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-\cosh(1)^2-2\,\cosh(1)\sinh(1)-\sinh(1)^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-\cosh(1)^2-2\,\cosh(1)\sinh(1)-\sinh(1)^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)-\cosh(1)-\sinh(1))\sqrt{-c\log(f)}}{2\,c\log(f)}\right)}{4\,c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - c
osh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(
4*a*c*log(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/
(c*log(f))))*erf(1/2*(2*c*x*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f))/(c*
log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(
1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1
)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) - cosh(1) - sinh(1))*sqrt(-c*log(f)
)/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x), x)
```

Giac [A]
time = 0.42, size = 132, normalized size = 0.99

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\,\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{sinh}\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x), x)
```

3.349 $\int f^{a+cx^2}\sinh^2(d+ex)\,dx$

Optimal. Leaf size=161

$$-\frac{f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/c^
(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^a*erfi(x*c^(1/2)*ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5623, 2235, 2325, 2266}

$$-\frac{\sqrt{\pi}\,f^a e^{-\frac{e^2}{c\log(f)}-2d}\mathrm{Erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}-\frac{\sqrt{\pi}\,f^a\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) - (
E^(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[
Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh^2(d+ex)\,dx &= \int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2ex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2d+2e}\\ &= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\Big(\\ &= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{2d}}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 131, normalized size = 0.81

$$\frac{e^{-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{c\log(f)}}\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)+\text{Erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\text{Erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
 c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E
^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.31, size = 139, normalized size = 0.86

method	result

risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 d \ln(f) c+e^{2}}{\ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\frac{e}{\sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 d \ln(f) c-e^{2}}{\ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{e}{\sqrt{-c \ln}}\right.}{8 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*l
n(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))-1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.81

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(2 d-\frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{e}{\sqrt{-c \log(f)}}\right) e^{\left(-2 d-\frac{e^{2}}{c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(124) = 248$.
time = 0.35, size = 304, normalized size = 1.89

$$\frac{2 \sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh(a \log(f))+\sqrt{\pi} \sinh(a \log(f))\right) \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)-\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(\frac{a c \log(f)^{2}+2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{a c \log(f)^{2}+2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(c x \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)}}{c \log(f)}\right)-\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(\frac{a c \log(f)^{2}-2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)+\sqrt{\pi} \sinh\left(\frac{a c \log(f)^{2}-2 c d \log(f)-\cosh(1)^{2}-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{c \log(f)}\right)\right) \operatorname{erf}\left(\frac{(c x \log(f)-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)}}{c \log(f)}\right)}{8 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))*
erf(sqrt(-c*log(f))*x) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*c
*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(p
i)*sinh((a*c*log(f)^2 + 2*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh
(1)^2)/(c*log(f))))*erf((c*x*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f))/(c
*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - c
osh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*
log(f)^2 - 2*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log
```

```
(f)))*erf((c*x*log(f) - cosh(1) - sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(c
*log(f))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**2,x)
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**2, x)
```

Giac [A]

time = 0.42, size = 150, normalized size = 0.93

$$\frac{\sqrt{\pi}\,f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="giac")
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^2
)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(c
*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f
))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\sinh(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^2,x)
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^2, x)
```

3.350 $\int f^{a+cx^2} \sinh^3(d+ex)\,dx$

Optimal. Leaf size=271

$$-\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Er}}{16\sqrt{c}\ \sqrt{}}$$

```
[Out] 3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi(1
/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/1
6*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/2*(3*
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5623, 2325, 2266, 2235}

$$-\frac{3\sqrt{\pi}\,f^a e^{-\frac{e^2}{4c\log(f)}-d}\mathrm{Erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\mathrm{Erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{3d-\frac{9e^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh^3(d+ex)\,dx &= \int\left(-\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}-\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)\,dx\\
&=-\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx-\\
&=-\left(\frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int\\
&=-\left(\frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(}{4c\log(f)}}\\
&=-\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-2cx\log}{2\sqrt{c}\sqrt{\log}}\right)}{16\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 214, normalized size = 0.79

$$\frac{e^{-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)+\mathrm{Erfi}\left(\frac{-3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(-\cosh(3d)+\sinh(3d))\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
 + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[
(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[3*d] + Sinh[3*d])))/
(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.70, size = 234, normalized size = 0.86

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 d \ln(f) c-\frac{9 e^{2}}{4}}{c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{3 e}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 d \ln(f) c+3 e^{2}\right)}{4 \ln(f) c}} \operatorname{erf}\left(\sqrt{-c \ln(f)}\, x+\right.}{16 \sqrt{-c \ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-3/4*(4
*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+3/2*e/(-
c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)/(-c*l
n(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^
a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)
*x+1/2*e/(-c*ln(f))^(1/2))
```

<u>Maxima</u> [A]
time = 0.28, size = 211, normalized size = 0.78

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(3 d-\frac{9 e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}-\frac{3 \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d-\frac{e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x+\frac{3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-3 d-\frac{9 e^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(205) = 410$.
time = 0.37, size = 549, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f))) + sqrt(pi)*sin
h(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9*cosh(1)^2 - 18*cosh(1)*sinh(1) -
```

```
9*sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*cosh(1) + 3*sinh(1))*sq
rt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log
(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f)
)) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - cosh(1)^2 - 2*cosh(
1)*sinh(1) - sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + cosh(1) + sinh
(1))*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*
a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c
*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - cosh(1)^2 -
2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) - cosh(1)
 - sinh(1))*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(1/
4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*si
nh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) -
9*cosh(1)^2 - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f))))*erf(1/2*(2*c*x
*log(f) - 3*cosh(1) - 3*sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**3,x)
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**3, x)
```

Giac [**A**]
time = 0.40, size = 264, normalized size = 0.97

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x-\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="giac")
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^3, x)
```

3.351 $\int f^{a+cx^2} \sinh(d+fx^2)\,dx$

Optimal. Leaf size=81

$$-\frac{e^{-d}f^a\sqrt{\pi}\,\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^d f^a\sqrt{\pi}\,\text{Erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5623, 2325, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^d f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(E^d*Sqrt[f - c*Log[f]]) + (E
^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\ &= -\frac{e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 76, normalized size = 0.94

$$\frac{1}{4}f^a\sqrt{\pi}\left(-\frac{\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}}+\frac{\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c
*Log[f]]) + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log
[f]]))/4
```

Maple [A]
time = 0.74, size = 70, normalized size = 0.86

method	result	size
risch	$\frac{\sqrt{\pi}\,f^a e^d\,\mathrm{erf}\left(\sqrt{-c\ln(f)-f}\,x\right)}{4\sqrt{-c\ln(f)-f}}-\frac{\sqrt{\pi}\,f^a e^{-d}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{4\sqrt{f-c\ln(f)}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)-1/4*Pi
^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.85

$$-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f}\, x\right) e^{\left(-d\right)}}{4 \sqrt{-c \log\left(f\right) + f}} + \frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f}\, x\right) e^{d}}{4 \sqrt{-c \log\left(f\right) - f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(63) = 126.
time = 0.39, size = 146, normalized size = 1.80

$$\frac{\left(\sqrt{\pi} \left(c \log\left(f\right) + f\right) \cosh\left(a \log\left(f\right) - d\right) + \sqrt{\pi} \left(c \log\left(f\right) + f\right) \sinh\left(a \log\left(f\right) - d\right)\right) \sqrt{-c \log\left(f\right) + f} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f}\, x\right) - \left(\sqrt{\pi} \left(c \log\left(f\right) - f\right) \cosh\left(a \log\left(f\right) + d\right) + \sqrt{\pi} \left(c \log\left(f\right) - f\right) \sinh\left(a \log\left(f\right) + d\right)\right) \sqrt{-c \log\left(f\right) - f} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f}\, x\right)}{4 \left(c^{2} \log\left(f\right)^{2} - f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)*
sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) - (sqrt(
pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log(
f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a + c x^{2}} \sinh\left(d + f x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2), x)
```

Giac [A]
time = 0.41, size = 75, normalized size = 0.93

$$-\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right) - f}\, x\right) e^{\left(a \log\left(f\right) + d\right)}}{4 \sqrt{-c \log\left(f\right) - f}} + \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log\left(f\right) + f}\, x\right) e^{\left(a \log\left(f\right) - d\right)}}{4 \sqrt{-c \log\left(f\right) + f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) + 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{sinh}\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2), x)
```

3.352 $\int f^{a+cx^2} \sinh^2(d+fx^2)\,dx$

Optimal. Leaf size=128

$$-\frac{f^a\sqrt{\pi}\,\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a\sqrt{\pi}\,\text{Erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*e
rf(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d
)*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5623, 2235, 2325, 2236}

$$\frac{\sqrt{\pi}\,e^{-2d}f^a\text{Erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,e^{2d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\,f^a\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
f^a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]])
+ (E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f
]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx &= \int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx+\frac{1}{4}\int e^{2d+}\\ &= -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d}f^a}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 179, normalized size = 1.40

$$\frac{f^a\sqrt{\pi}\left(\mathrm{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)\left(8f^2-2c^2\log^2(f)\right)+\sqrt{c}\,\sqrt{\log(f)}\left(\mathrm{Erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(-\cosh(2d)+\sinh(2d))-\mathrm{Erfi}\left(x\sqrt{2f+c\log(f)}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}\,\left(-4f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(8*f^2 - 2*c^2*Log[f]^2) + Sqrt
[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*L
og[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[
f]]*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A]
time = 1.43, size = 101, normalized size = 0.79

method	result
risch	$\frac{\sqrt{\pi}\,f^a e^{-2d}\,\mathrm{erf}\left(x\sqrt{2f-c\ln(f)}\right)}{8\sqrt{2f-c\ln(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-2f}\,x\right)}{8\sqrt{-c\ln(f)-2f}}-\frac{f^a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x\right)}{4\sqrt{-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*d)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*Pi^(1/2)*f^a*exp(2*d)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)-1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [**A**]
time = 0.28, size = 100, normalized size = 0.78

$$\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)-2f}\,x\right)e^{(2d)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)+2f}\,x\right)e^{(-2d)}}{8\sqrt{-c\log(f)+2f}}-\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(98) = 196.
time = 0.35, size = 254, normalized size = 1.98

$$-\frac{\left(\sqrt{\pi}\left(c^2\log(f)^2+2cf\log(f)\right)\cosh\left(a\log(f)-2d\right)+\sqrt{\pi}\left(c^2\log(f)^2+2cf\log(f)\right)\sinh\left(a\log(f)-2d\right)\right)\sqrt{-c\log(f)+2f}\operatorname{erf}\left(\sqrt{-c\log(f)+2f}\,x\right)+\left(\sqrt{\pi}\left(c^2\log(f)^2-2cf\log(f)\right)\cosh\left(a\log(f)+2d\right)+\sqrt{\pi}\left(c^2\log(f)^2-2cf\log(f)\right)\sinh\left(a\log(f)+2d\right)\right)\sqrt{-c\log(f)-2f}\operatorname{erf}\left(\sqrt{-c\log(f)-2f}\,x\right)-2\left(\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\cosh\left(a\log(f)\right)+\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\sqrt{-c\log(f)}\,x\right)}{8\left(c^3\log(f)^3-4cf^2\log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) - 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**2, x)
```

Giac [A]
time = 0.44, size = 107, normalized size = 0.84

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(-\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log(f) - 2 f}\, x\right) e^{(a \log(f) + 2 d)}}{8 \sqrt{-c \log(f) - 2 f}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c \log(f) + 2 f}\, x\right) e^{(a \log(f) - 2 d)}}{8 \sqrt{-c \log(f) + 2 f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*s
qrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) + 2
*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c x^2 + a} \sinh\left(f x^2 + d\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^2, x)
```

3.353 $\int f^{a+cx^2} \sinh^3(d+fx^2)\, dx$

Optimal. Leaf size=171

$$\frac{3e^{-d}f^a\sqrt{\pi}\,\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\,\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d}f^a\sqrt{\pi}\,\text{Erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}$$

```
[Out] 3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)-1/16*f^
a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)-3/16*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d)*
f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5623, 2325, 2236, 2235}

$$\frac{3\sqrt{\pi}\,e^{-d}f^a\text{Erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}\,e^{-3d}f^a\text{Erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,e^{d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,e^{3d}f^a\text{Erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
- (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx &= \int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right.\\ &= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}\,dx\\ &= -\left(\frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}\,dx\right)+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}\,dx+\frac{3}{8}\\ &= \frac{3e^{-d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e}{}\end{aligned}$$

Mathematica [A]
time = 0.84, size = 272, normalized size = 1.59

$$\frac{f^a\sqrt{\pi}\left(3\mathrm{Erf}\left(x\sqrt{f-c\log(f)}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh(d))-(f-c\log(f))\left(\mathrm{Erf}\left(x\sqrt{3f-c\log(f)}\right)\sqrt{3f-c\log(f)}\left(3f^2+4cf\log(f)+c^2\log^2(f)\right)(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3\mathrm{Erfi}\left(x\sqrt{f+c\log(f)}\right)\sqrt{f+c\log(f)}(3f+c\log(f))(\cosh(d)+\sinh(d))-\mathrm{Erfi}\left(x\sqrt{3f+c\log(f)}\right)(f+c\log(f))\sqrt{3f+c\log(f)}(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) -
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 1.82, size = 144, normalized size = 0.84

method	result
risch	$\frac{\sqrt{\pi}\,f^a e^{3d}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}-\frac{\sqrt{\pi}\,f^a e^{-3d}\,\mathrm{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{3\sqrt{\pi}\,f^a e^{-d}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(3*d)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
-1/16*Pi^(1/2)*f^a*exp(-3*d)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+3/16*Pi^(1/2)*f^a*exp(-d)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))-3/16*
Pi^(1/2)*f^a*exp(d)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Maxima [A]
time = 0.27, size = 143, normalized size = 0.84

$$\frac{\sqrt{\pi}\,f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-3f}\,x\right)e^{(3d)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}\,f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+f}\,x\right)e^{(-d)}}{16\sqrt{-c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+3f}\,x\right)e^{(-3d)}}{16\sqrt{-c\log(f)+3f}}-\frac{3\sqrt{\pi}\,f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-f}\,x\right)e^{d}}{16\sqrt{-c\log(f)-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(135) = 270.
time = 0.37, size = 492, normalized size = 2.88

$$\frac{\left(\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-cf^2\log(f)-3f^3\right)\cosh\left(a\log(f)-3d\right)+\sqrt{\pi}\left(c^3\log(f)^3+3c^2f\log(f)^2-cf^2\log(f)-3f^3\right)\sinh\left(a\log(f)-3d\right)\right)\sqrt{-c\log(f)+3f}\,\mathrm{erf}\left(\sqrt{-c\log(f)+3f}\,x\right)-3\left(\sqrt{\pi}\left(c^3\log(f)^3+c^2f\log(f)^2-9cf^2\log(f)-9f^3\right)\cosh\left(a\log(f)-d\right)+\sqrt{\pi}\left(c^3\log(f)^3+c^2f\log(f)^2-9cf^2\log(f)-9f^3\right)\sinh\left(a\log(f)-d\right)\right)\sqrt{-c\log(f)+f}\,\mathrm{erf}\left(\sqrt{-c\log(f)+f}\,x\right)+3\left(\sqrt{\pi}\left(c^3\log(f)^3-c^2f\log(f)^2-9cf^2\log(f)+9f^3\right)\cosh\left(a\log(f)+d\right)+\sqrt{\pi}\left(c^3\log(f)^3-c^2f\log(f)^2-9cf^2\log(f)+9f^3\right)\sinh\left(a\log(f)+d\right)\right)\sqrt{-c\log(f)-f}\,\mathrm{erf}\left(\sqrt{-c\log(f)-f}\,x\right)-\left(\sqrt{\pi}\left(c^3\log(f)^3-3c^2f\log(f)^2-cf^2\log(f)+3f^3\right)\cosh\left(a\log(f)+3d\right)+\sqrt{\pi}\left(c^3\log(f)^3-3c^2f\log(f)^2-cf^2\log(f)+3f^3\right)\sinh\left(a\log(f)+3d\right)\right)\sqrt{-c\log(f)-3f}\,\mathrm{erf}\left(\sqrt{-c\log(f)-3f}\,x\right)}{16\left(c^4\log(f)^4-10c^2f^2\log(f)^2+9f^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(
f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
+ 3*f)*x) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9
*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^
2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f
) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9
*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^
2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f
) - f)*x) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^
```

```
2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*l
og(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**3, x)
```

Giac [A]
time = 0.43, size = 155, normalized size = 0.91

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-3f}\,x\right)e^{(a\log(f)+3d)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)-f}\,x\right)e^{(a\log(f)+d)}}{16\sqrt{-c\log(f)-f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)+f}\,x\right)e^{(a\log(f)-d)}}{16\sqrt{-c\log(f)+f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)+3f}\,x\right)e^{(a\log(f)-3d)}}{16\sqrt{-c\log(f)+3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) + 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{sinh}\left(f\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^3, x)
```

3.354 $\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=140

$$-\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi
(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\,\mathrm{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\,\mathrm{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[
f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(d - e^2/(4*(f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,d\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\frac{1}{2}\\
&= -\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\right.}{4\sqrt{f+c\,\mathrm{lo}}}
\end{aligned}$$

Mathematica [A]
time = 0.52, size = 166, normalized size = 1.19

$$\frac{e^{-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(-e^{\frac{e^2f}{2f^2-2c^2\log^2(f)}}\mathrm{Erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}\,(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{e+2fx+2cx\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f-c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\sqrt{f-c\log(f)}\,\sqrt{f+c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*
x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d])) +
 Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqr
t[f + c*Log[f]])
```

Maple [A]
time = 0.81, size = 147, normalized size = 1.05

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4 d \ln(f) c+4 d f-e^{2}}{4 c \ln(f)+4 f}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-f}\, x+\frac{e}{2 \sqrt{-c \ln(f)-f}}\right)}{4 \sqrt{-c \ln(f)-f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4 d \ln(f) c-4 d f+e^{2}}{4(-f+c \ln(f))}} \operatorname{erf}\left(x \sqrt{\right.}{4 \sqrt{f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)
^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))-1/4*Pi^(1/2)*f^a
*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(x*(f-
c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 127, normalized size = 0.91

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{e}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(d-\frac{e^{2}}{4(c \log(f)+f)}\right)}}{4 \sqrt{-c \log(f)-f}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x+\frac{e}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-d-\frac{e^{2}}{4(c \log(f)-f)}\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(119) = 238$.
time = 0.38, size = 382, normalized size = 2.73

$$\frac{\left(\sqrt{\pi}(c \log(f)+f) \cosh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}-4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)-f)}\right)+\sqrt{\pi}(c \log(f)+f) \sinh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}-4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f} \operatorname{erf}\left(\frac{(2 c x \log(f)-2 f x-\cosh(1)-\sinh(1)) \sqrt{-c \log(f)+f}}{2(c \log(f)-f)}\right)-\left(\sqrt{\pi}(c \log(f)-f) \cosh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}+4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)+f)}\right)+\sqrt{\pi}(c \log(f)-f) \sinh\left(\frac{4 a c \log(f)^{2}+4 d f-\cosh(1)^{2}+4(c d+a f) \log(f)-2 \cosh(1) \sinh(1)-\sinh(1)^{2}}{4(c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f} \operatorname{erf}\left(\frac{(2 c x \log(f)+2 f x+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)-f}}{2(c \log(f)+f)}\right)}{4\left(c^{2} \log(f)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2
- 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)) + s
qrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 - 4*(c*
d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*l
og(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqrt(-c*log(f
) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2
+ 4*d*f - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)
```

```
/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*f
 - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log
(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + cosh(1) + si
nh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2} \sinh \left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Giac [A]
time = 0.42, size = 172, normalized size = 1.23

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f
))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^
2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2), x)
```

3.355 $\int f^{a+cx^2} \sinh^2(d+ex+fx^2)\, dx$

Optimal. Leaf size=183

$$-\frac{f^a\sqrt{\pi}\,\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\,\text{Erfi}\left(\frac{e\ldots}{\sqrt{\ldots}}\right.}{8\sqrt{2f+c\log(\ldots}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^
(1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2*f
+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5623, 2235, 2325, 2266, 2236}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\text{Erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\text{Erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\,f^a\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
E^(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/S
qrt[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log
[f]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*
Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx &= \int\left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,d\\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2d-2ex+a\log(f)-x^2(2\right.\\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e+}{4(}\right.\\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{e+x(2f-c\log(}{\sqrt{2f-c\log(}}\right.}{8\sqrt{2f-c\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 1.02, size = 258, normalized size = 1.41

$$\frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(2e^{\frac{e^2}{-2f+c\log(f)}}\text{Erfi}\left(\sqrt{c}\,x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\,\sqrt{\log(f)}\left(\text{Erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+e^{\frac{4e^2f}{-4f^2+c^2\log^2(f)}}\text{Erfi}\left(\frac{e+2fx+cx\log(f)}{\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)\right)}{8\sqrt{c}\,\sqrt{\log(f)}\left(-4f^2+c^2\log^2(f)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(2*E^(e^2/(-2*f + c*Log[f]))*Erfi[Sq
rt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(e
 + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c*
Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Erf
i[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
```

```
+ c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c^
2*Log[f]^2))
```

Maple [A]
time = 1.56, size = 177, normalized size = 0.97

method	result
risch	$\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2 d \ln(f) c-4 d f+e^{2}}{-2 f+c \ln(f)}} \operatorname{erf}\left(x \sqrt{2 f-c \ln(f)}+\frac{e}{\sqrt{2 f-c \ln(f)}}\right)}{8 \sqrt{2 f-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2 d \ln(f) c+4 d f-e^{2}}{2 f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln\right.}{8 \sqrt{-c \ln}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))
^(1/2)*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f
)-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*er
f((-c*ln(f))^(1/2)*x)
```

Maxima [A]
time = 0.28, size = 161, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{e}{\sqrt{-c \log(f)-2 f}}\right) e^{\left(2 d-\frac{e^{2}}{c \log(f)+2 f}\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x+\frac{e}{\sqrt{-c \log(f)+2 f}}\right) e^{\left(-2 d-\frac{e^{2}}{c \log(f)-2 f}\right)}}{8 \sqrt{-c \log(f)+2 f}}-\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x\right)}{4 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(155) = 310$.
time = 0.41, size = 482, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(
f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) - (sqr
```

```
t(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 + 4*d*f - cosh(1)^2
- 2*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh((a*c*log(f)^2 + 4*d*f - cosh(1
)^2 - 2*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - 2*f
)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqrt
(-c*log(f) + 2*f)/(c*log(f) - 2*f)) - (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f
))*cosh((a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 2*(c*d + a*f)*log(f) - 2*cosh(1
)*sinh(1) - sinh(1)^2)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*l
og(f))*sinh((a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 2*(c*d + a*f)*log(f) - 2*co
sh(1)*sinh(1) - sinh(1)^2)/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*
x*log(f) + 2*f*x + cosh(1) + sinh(1))*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f
)))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Giac [A]
time = 0.44, size = 198, normalized size = 1.08

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}\, x\right)}{4\,\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\, \operatorname{erf}\left(-\sqrt{-c\log\left(f\right) - 2\,f}\,\left(x + \frac{e}{c\log\left(f\right)+2\,f}\right)\right) e^{\left(\frac{ac\log\left(f\right)^{2}+2\,cd\log\left(f\right)+2\,af\log\left(f\right)-e^{2}+4\,df}{c\log\left(f\right)+2\,f}\right)}}{8\,\sqrt{-c\log\left(f\right) - 2\,f}} - \frac{\sqrt{\pi}\, \operatorname{erf}\left(-\sqrt{-c\log\left(f\right) + 2\,f}\,\left(x - \frac{e}{c\log\left(f\right)-2\,f}\right)\right) e^{\left(\frac{ac\log\left(f\right)^{2}-2\,cd\log\left(f\right)-2\,af\log\left(f\right)-e^{2}+4\,df}{c\log\left(f\right)-2\,f}\right)}}{8\,\sqrt{-c\log\left(f\right) + 2\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d*
log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f
) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((a
*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - 2*f))/
sqrt(-c*log(f) + 2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

3.356 $\int f^{a+cx^2} \sinh^3(d+ex+fx^2)\,dx$

Optimal. Leaf size=300

$$\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{e^2}{4(f+c\log}}}{}$$

```
[Out] 3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))*f
^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(
f))^(1/2)-3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f)))/
(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*f+c*l
n(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(
3*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\mathrm{Erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\mathrm{Erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\mathrm{Erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+ex+fx\right.\right.\right.$$

$$=-\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-3\left(d+\right.\right.$$

$$=-\left(\frac{1}{8}\int\exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)\,dx\right)+\frac{1}{8}\int\text{ex}$$

$$=\frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)\,dx-\frac{1}{8}\left(e\right.$$

$$=\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\text{er}}{16\sqrt{3f-}}$$

Mathematica [A]
time = 4.02, size = 480, normalized size = 1.60

$$\frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\text{Erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh(d))-(f-c\log(f))\left(e^{\frac{1}{4}e^2\left(\frac{9}{3f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\text{Erf}\left(\frac{3e+6fx-2cx\log(f)}{2\sqrt{3f-c\log(f)}}\right)\sqrt{3f-c\log(f)}\left(3f^2+4cf\log(f)+c^2\log^2(f)\right)(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3e^{\frac{9e^2}{4(3f+c\log(f))}}\text{Erfi}\left(\frac{e+2fx+2cx\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f+c\log(f)}(3f+c\log(f))(\cosh(d)+\sinh(d))-e^{\frac{e^2}{4(f+c\log(f))}}\text{Erfi}\left(\frac{3e+6fx+2cx\log(f)}{2\sqrt{3f+c\log(f)}}\right)(f+c\log(f))\sqrt{3f+c\log(f)}(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) - E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

Maple [A]
time = 1.98, size = 302, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{3 d \ln(f) c+9 d f-\frac{9 e^{2}}{4}}{3 f+c \ln(f)}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-3 f}\, x+\frac{3 e}{2 \sqrt{-c \ln(f)-3 f}}\right)}{16 \sqrt{-c \ln(f)-3 f}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{3\left(4 d \ln(f) c-12 d f+3 e^{2}\right)}{4(-3 f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))/(-c*ln
(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))-1/16
*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln
(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))+3/16*Pi^(1/
2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf
(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(1/4*(4*
d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2
)*x+1/2*e/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.29, size = 263, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 \, f} x - \frac{3 \, e}{2 \, \sqrt{-c \log\left(f\right) - 3 \, f}}\right) e^{\left(3 \, d - \frac{9 \, e^{2}}{4 \, {\left(c \log\left(f\right) + 3 \, f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) - 3 \, f}} - \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{e}{2 \, \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(d - \frac{e^{2}}{4 \, {\left(c \log\left(f\right) + f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) - f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f} x + \frac{e}{2 \, \sqrt{-c \log\left(f\right) + f}}\right) e^{\left(-d - \frac{e^{2}}{4 \, {\left(c \log\left(f\right) - f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) + f}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 3 \, f} x + \frac{3 \, e}{2 \, \sqrt{-c \log\left(f\right) + 3 \, f}}\right) e^{\left(-3 \, d - \frac{9 \, e^{2}}{4 \, {\left(c \log\left(f\right) - 3 \, f\right)}}\right)}}{16 \, \sqrt{-c \log\left(f\right) + 3 \, f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
```

```
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(253) = 506$.
time = 0.39, size = 970, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(1/4*(4*a*c*log(f)^2 + 36*d*f - 9*cosh(1)^2 - 12*(c*d + a*f)*log(f) - 18*
cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 +
 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 36*d*f
 - 9*cosh(1)^2 - 12*(c*d + a*f)*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/
(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*
cosh(1) - 3*sinh(1))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*
(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*lo
g(f)^2 + 4*d*f - cosh(1)^2 - 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sin
h(1)^2)/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2
*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 - 4*(c*d + a*
f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*log(f)
+ f)*erf(1/2*(2*c*x*log(f) - 2*f*x - cosh(1) - sinh(1))*sqrt(-c*log(f) + f)
/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log
(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 + 4*d*f - cosh(1)^2 + 4*(c*d + a*f)*l
og(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c^3*log(
f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 +
4*d*f - cosh(1)^2 + 4*(c*d + a*f)*log(f) - 2*cosh(1)*sinh(1) - sinh(1)^2)/(
c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + cosh(1)
 + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)*(c^3*log(f)^3 -
 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*log(f)^2 + 36*d*f
 - 9*cosh(1)^2 + 12*(c*d + a*f)*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/
(c*log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f
) + 3*f^3)*sinh(1/4*(4*a*c*log(f)^2 + 36*d*f - 9*cosh(1)^2 + 12*(c*d + a*f)
*log(f) - 18*cosh(1)*sinh(1) - 9*sinh(1)^2)/(c*log(f) + 3*f)))*sqrt(-c*log(
f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*cosh(1) + 3*sinh(1))*sqrt(-c*lo
g(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 352, normalized size = 1.17

$$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\,\left(2\,x+\frac{3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+3\,f}\,\left(2\,x-\frac{3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-
c*log(f) + f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) - 9*
e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

3.357 $\int f^{a+bx+cx^2} \sinh(d+ex)\,dx$

Optimal. Leaf size=153

$$\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f
))^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^
(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5623, 2325, 2266, 2235}

$$\frac{\sqrt{\pi} f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
(e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx = \int \left(-\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)\,dx$$
$$= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx$$
$$= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)\,dx\right)+\frac{1}{2}\int \exp$$
$$= -\left(\frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx\right)+\frac{1}{2}$$
$$= \frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e+b}{2\sqrt{}}\right.}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Mathematica [A]
time = 0.21, size = 135, normalized size = 0.88

$$\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-e^{\frac{be}{c}}\text{Erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\text{Erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d])) + Erfi[(e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 0.88, size = 156, normalized size = 1.02

method	result

risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln(f) b e-4 d \ln(f) c+e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{e+b \ln(f)}{2 \sqrt{-c \ln(f)}}\right)}{4 \sqrt{-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-2 \ln(f) b e+4 d \ln(}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2
))+1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c+e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1
/2))
```

Maxima [A]
time = 0.28, size = 133, normalized size = 0.87

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right) e^{\left(d-\frac{(b\log(f)+e)^{2}}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c\log(f)}\, x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right) e^{\left(-d-\frac{(b\log(f)-e)^{2}}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(125) = 250$.
time = 0.38, size = 339, normalized size = 2.22

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^{2}-4\,ac)\log(f)^{2}+\cosh(1)^{2}-2\,(2\,cd-b\cosh(1)-b\sinh(1))\log(f)+2\,\cosh(1)\sinh(1)+\sinh(1)^{2}}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^{2}-4\,ac)\log(f)^{2}+\cosh(1)^{2}-2\,(2\,cd-b\cosh(1)-b\sinh(1))\log(f)+2\,\cosh(1)\sinh(1)+\sinh(1)^{2}}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)+\cosh(1)+\sinh(1))\sqrt{-c\log(f)}}{2\,c\log(f)}\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^{2}-4\,ac)\log(f)^{2}+\cosh(1)^{2}+2\,(2\,cd-b\cosh(1)-b\sinh(1))\log(f)+2\,\cosh(1)\sinh(1)+\sinh(1)^{2}}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^{2}-4\,ac)\log(f)^{2}+\cosh(1)^{2}+2\,(2\,cd-b\cosh(1)-b\sinh(1))\log(f)+2\,\cosh(1)\sinh(1)+\sinh(1)^{2}}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{((2\,cx+b)\log(f)-\cosh(1)-\sinh(1))\sqrt{-c\log(f)}}{2\,c\log(f)}\right)}{4\,c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)
^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)
^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 -
2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(
c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-c*log(f)
)/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1)
```

```
 + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + co
sh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + si
nh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - cosh(1) - sinh(1))*sqrt
(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x), x)
```

Giac [A]
time = 0.42, size = 167, normalized size = 1.09

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e^
(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(
c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (
b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*l
og(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x), x)
```

3.358 $\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$

Optimal. Leaf size=219

$$-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}}{8\sqrt{c}}$$

```
[Out] 1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*(2
*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)
*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5623, 2266, 2235, 2325}

$$-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\mathrm{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\mathrm{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/
(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx=\int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$=\frac{1}{4}\int\exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int\exp\left(\frac{(-2e\right.$$

$$=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e-b\log(f)\cdots}{2\sqrt{c}\,\sqrt{\cdots}}\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Mathematica [A]
time = 0.39, size = 183, normalized size = 0.84

$$\frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{e(e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\mathrm{Erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{2e+(b+2cx)\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)}{8\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x
)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*S
qrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.46, size = 211, normalized size = 0.96

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{b^{2} \ln(f)^{2}-4 \ln(f) b e+8 d \ln(f) c+4 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{b \ln(f)-2 e}{2 \sqrt{-c \ln(f)}}\right)}{8 \sqrt{-c \ln(f)}}-\frac{\sqrt{\pi}\, f^{a} e^{-\frac{b^{2} \ln(f)^{2}+4 \ln(f) b e-8 d \ln}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*c+4*e^2)/ln(f
)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^
(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*ln(f)*b*e-8*d*ln(f)*c+4*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*l
n(f))^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 189, normalized size = 0.86

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(2 d-\frac{(b \log(f)+2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}+\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-2 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-2 d-\frac{(b \log(f)-2 e)^{2}}{4 c \log(f)}\right)}}{8 \sqrt{-c \log(f)}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)}}\right)}{4 \sqrt{-c \log(f)}\, f^{\frac{b^{2}}{4 c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(171) = 342$.
time = 0.38, size = 429, normalized size = 1.96

$$\frac{2 \sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)\right) \operatorname{erf}\left(\frac{(2 c x+b) \sqrt{-c \log(f)}}{2 c}\right)-\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}-4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}-4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)+2 \cosh(1)+2 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)-\sqrt{-c \log(f)} \left(\sqrt{\pi} \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}+4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)+\sqrt{\pi} \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}+4 \cosh(1)^{2}+4 (2 c d-b \cosh(1)-b \sinh(1)) \log(f)+8 \cosh(1) \sinh(1)+4 \sinh(1)^{2}}{4 c \log(f)}\right)\right) \operatorname{erf}\left(\frac{((2 c x+b) \log(f)-2 \cosh(1)-2 \sinh(1)) \sqrt{-c \log(f)}}{2 c \log(f)}\right)}{8 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(p
i)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c
) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*cosh(1)
^2 - 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*sinh(
```

```
1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*cosh(1)^
2 - 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1) + 4*sinh(1
)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt
(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c
)*log(f)^2 + 4*cosh(1)^2 + 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cos
h(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 4*cosh(1)^2 + 4*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh
(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*cos
h(1) - 2*sinh(1))*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**2, x)
```

Giac [A]
time = 0.43, size = 223, normalized size = 1.02

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*
a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8
*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*(
b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*log
(f)))/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2, x)
```

3.359 $\int f^{a+bx+cx^2} \sinh^3(d+ex)\,dx$

Optimal. Leaf size=315

$$-\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\,\text{Erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}} f^a \sqrt{\pi}\,\text{Erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}} \cdots}{\cdots}$$

```
[Out] 3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-1/4*(3*e-
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))*f^
a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln
(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e+b*ln(
f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$,
Rules used = {5623, 2325, 2266, 2235}

$$-\frac{3\sqrt{\pi}\, f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}-\frac{3\sqrt{\pi}\, f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\, f^a e^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\,\sqrt{\log(f)}}\right)}{16\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = \int \left(-\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}-\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3e}\right.$$

$$= -\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-ex}f^{a+}$$

$$= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)\,dx\right)+\frac{1}{8}\int e$$

$$= \frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx-\frac{1}{8}\left(e^{-}\right.$$

$$= -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{er}}{16\sqrt{c}\sqrt{}}$$

Mathematica [A]
time = 0.67, size = 263, normalized size = 0.83

$$\frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\mathrm{Erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\mathrm{Erfi}\left(\frac{3e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)+\sinh(2d))\right)-e^{\frac{3be}{c}}\mathrm{Erfi}\left(\frac{-3e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(3d)-\sinh(3d))\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((e*(2*e + b*Log[f])
)/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E
^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) - E^((3*b*e)/c)*Erfi[(-3
*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d]))
)/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A]
time = 1.85, size = 316, normalized size = 1.00

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+6 \ln(f) b e-12 d \ln(f) c+9 e^{2}}{4 \ln(f) c}} \operatorname{erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{3 e+b \ln(f)}{2 \sqrt{-c \ln(f)}}\right)}{16 \sqrt{-c \ln(f)}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-6 \ln(f) b e+12 d}{4 \ln(f) c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(f)*c+9*e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)
)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f)*c+9
*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln
(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e
)/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d
*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln
(f))/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 271, normalized size = 0.86

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(3 d-\frac{(b \log(f)+3 e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}-\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)+e}{2 \sqrt{-c \log(f)}}\right) e^{\left(d-\frac{(b \log(f)+e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}+\frac{3 \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-d-\frac{(b \log(f)-e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}-\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)-3 e}{2 \sqrt{-c \log(f)}}\right) e^{\left(-3 d-\frac{(b \log(f)-3 e)^{2}}{4 c \log(f)}\right)}}{16 \sqrt{-c \log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(255) = 510$.
time = 0.38, size = 689, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh
(1)^2 - 6*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*s
inh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh
(1)^2 - 6*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*s
inh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + 3*cosh(1) + 3*sinh(1))
*sqrt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + cosh(1)^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) +
2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a
*c)*log(f)^2 + cosh(1)^2 - 2*(2*c*d - b*cosh(1) - b*sinh(1))*log(f) + 2*cos
h(1)*sinh(1) + sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + cosh(1
) + sinh(1))*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh
(-1/4*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1
))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))) + sqrt(pi)*sinh(-1/4
*((b^2 - 4*a*c)*log(f)^2 + cosh(1)^2 + 2*(2*c*d - b*cosh(1) - b*sinh(1))*lo
g(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f))))*erf(1/2*((2*c*x + b)*log
(f) - cosh(1) - sinh(1))*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqr
t(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh(1)^2 + 6*(2*c*d - b*cosh(1
) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f))) + sqr
t(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*cosh(1)^2 + 6*(2*c*d - b*cosh(1
) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f))))*erf(
1/2*((2*c*x + b)*log(f) - 3*cosh(1) - 3*sinh(1))*sqrt(-c*log(f))/(c*log(f))
))/(c*log(f))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**3, x)
```

Giac [**A**]
time = 0.44, size = 339, normalized size = 1.08

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9*
```

```
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^
2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f))
)/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f)
 + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f)
 + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3, x)
```

3.360 $\int f^{a+bx+cx^2}\sinh(d+fx^2)\,dx$

Optimal. Leaf size=154

$$\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] 1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(f)
))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/
(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(
1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d -
 (b^2*Log[f]^2)/(4*(f + c*Log[f]))))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f +
c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx &= \int\left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)dx\\ &= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\ &= -\left(\frac{1}{2}\int\exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx\right)+\frac{1}{2}\int \mathrm{ex}\\ &= -\left(\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right)+\\ &= \frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\mathrm{l}}{2}\right.}{4\sqrt{f+c\log}}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 179, normalized size = 1.16

$$\frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(-e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\mathrm{Erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}\,(\cosh(d)-\sinh(d))+\mathrm{Erfi}\left(\frac{2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f-c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] -
Sinh[d])) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[
f - c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
)*Sqrt[f - c*Log[f]]*Sqrt[f + c*Log[f]])
```

Maple [A]
time = 0.87, size = 160, normalized size = 1.04

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-4 d \ln(f) c-4 d f}{4(f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-f}\, x+\frac{\ln(f) b}{2 \sqrt{-c \ln(f)-f}}\right)}{4 \sqrt{-c \ln(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+4 d \ln(f) c-}{4(-f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))/(-c
*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))+1
/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(-f+c*ln(f)))/(f-c
*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.27, size = 139, normalized size = 0.90

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)+f)}+d\right)}}{4 \sqrt{-c \log(f)-f}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)-f)}-d\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) - 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(131) = 262$.
time = 0.40, size = 325, normalized size = 2.11

$$\frac{\left(\sqrt{\pi}\,(c \log(f)+f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+4(c d+a f) \log(f)}{4(c \log(f)-f)}\right)+\sqrt{\pi}\,(c \log(f)+f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+4(c d+a f) \log(f)}{4(c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f}\, \operatorname{erf}\left(-\frac{(2 f x-(2 c x+b) \log(f)) \sqrt{-c \log(f)+f}}{2(c \log(f)-f)}\right)-\left(\sqrt{\pi}\,(c \log(f)-f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f-4(c d+a f) \log(f)}{4(c \log(f)+f)}\right)+\sqrt{\pi}\,(c \log(f)-f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f-4(c d+a f) \log(f)}{4(c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f}\, \operatorname{erf}\left(\frac{(2 f x+(2 c x+b) \log(f)) \sqrt{-c \log(f)-f}}{2(c \log(f)+f)}\right)}{4(c^{2} \log(f)^{2}-f^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((
b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt
(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(
c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
```

```
 + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2), x)
```

Giac [A]
time = 0.43, size = 181, normalized size = 1.18

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\mathrm{sinh}\left(f\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2), x)
```

3.361 $\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=225

$$-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}}{8\sqrt{2}}$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(
f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1/8
*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*ln(f
)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5623, 2266, 2235, 2325, 2236}

$$-\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\text{Erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*
f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])
])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*
f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]]
)])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx &= \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)dx+\frac{1}{4}\int \exp\left(\right.\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log\right.\right.\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2}{2\sqrt{2f-}}\right.}{8\sqrt{2f-c\log(f)}}
\end{aligned}$$

Mathematica [A]
time = 1.59, size = 257, normalized size = 1.14

$$\frac{1}{8}f^a\sqrt{\pi}\left(-\frac{2f^{-\frac{b^2}{4c}}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\text{Erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+\text{Erfi}\left(\frac{4fx+(b+2cx)\log(f)}{2\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)}{-4f^2+c^2\log^2(f)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*E
```

```
rf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[
f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Lo
g[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh
[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Lo
g[f]^2))))/8
```

Maple [A]
time = 1.49, size = 217, normalized size = 0.96

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+8 d \ln(f) c-16 d f}{4(-2 f+c \ln(f))}} \operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f) b}{2\sqrt{2f-c\ln(f)}}\right)}{8\sqrt{2f-c\ln(f)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}-8 d \ln(f) c-}{4(2 f+c \ln(f))}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(-2*f+c*ln(f)))
/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)/(2*f+c*ln(
f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)
-2*f)^(1/2))+1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 199, normalized size = 0.88

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)-2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)+2 f)}+2 d\right)}}{8 \sqrt{-c \log(f)-2 f}}+\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+2 f}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)+2 f}}\right) e^{\left(-\frac{b^{2} \log(f)^{2}}{4(c \log(f)-2 f)}-2 d\right)}}{8 \sqrt{-c \log(f)+2 f}}-\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)}\, x-\frac{b \log(f)}{2 \sqrt{-c \log(f)}}\right)}{4 \sqrt{-c \log(f)}\, f^{\frac{b^{2}}{4 c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(185) = 370$.
time = 0.37, size = 466, normalized size = 2.07

$$-\frac{\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}+2 c f \log(f)\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f+8(c d+a f) \log(f)}{4(c \log(f)-2 f)}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}+2 c f \log(f)\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f+8(c d+a f) \log(f)}{4(c \log(f)-2 f)}\right)\right) \sqrt{-c \log(f)+2 f} \operatorname{erf}\left(-\frac{(4 f x-(2 c x+b) \log(f)) \sqrt{-c \log(f)+2 f}}{2(c \log(f)-2 f)}\right)+\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}-2 c f \log(f)\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f-8(c d+a f) \log(f)}{4(c \log(f)+2 f)}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}-2 c f \log(f)\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-16 d f-8(c d+a f) \log(f)}{4(c \log(f)+2 f)}\right)\right) \sqrt{-c \log(f)-2 f} \operatorname{erf}\left(\frac{(4 f x+(2 c x+b) \log(f)) \sqrt{-c \log(f)-2 f}}{2(c \log(f)+2 f)}\right)-2\left(\sqrt{\pi}\left(c^{2} \log(f)^{2}-4 f^{2}\right) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)+\sqrt{\pi}\left(c^{2} \log(f)^{2}-4 f^{2}\right) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)}{4 c}\right)\right) \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2 c x+b) \sqrt{-c \log(f)}}{2 c}\right)}{8\left(c^{3} \log(f)^{3}-4 c f^{2} \log(f)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**2, x)
```

Giac [A]
time = 0.42, size = 239, normalized size = 1.06

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log(f)}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)-8af\log(f)-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2f}\left(2x+\frac{b\log(f)}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)+8af\log(f)-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2, x)
```

3.362 $\int f^{a+bx+cx^2} \sinh^3(d+fx^2)\,dx$

Optimal. Leaf size=323

$$-\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{b^2}{4(f-}}}{}$$

```
[Out] -3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+b^2*ln(f)^
2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f)))*
f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(
f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)
+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$-\frac{3\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\mathrm{Erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\mathrm{Erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}\,f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}\,f^a e^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) -
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx &= \int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}+\frac{1}{8}\right. \\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-d-} \\
&= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)\,dx\right)+\frac{1}{8} \\
&= \frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)\,dx-\frac{1}{8} \\
&= -\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}}{16\sqrt{3f}}
\end{aligned}$$

Mathematica [A]
time = 4.74, size = 503, normalized size = 1.56

$$\frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh(d))-(f-c\log(f))\left(e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{3f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\mathrm{Erf}\left(\frac{6fx-(b+2cx)\log(f)}{2\sqrt{3f-c\log(f)}}\right)\sqrt{3f-c\log(f)}\left(3f^2+4cf\log(f)+c^2\log^2(f)\right)(\cosh(3d)-\sinh(3d))+(3f-c\log(f))\left(3e^{\frac{b^2\log^2(f)}{12f+4c\log(f)}}\mathrm{Erfi}\left(\frac{2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)\sqrt{f+c\log(f)}(3f+c\log(f))(\cosh(d)+\sinh(d))-e^{\frac{b^2\log^2(f)}{4(f+c\log(f))}}\mathrm{Erfi}\left(\frac{6fx+(b+2cx)\log(f)}{2\sqrt{3f+c\log(f)}}\right)(f+c\log(f))\sqrt{3f+c\log(f)}(\cosh(3d)+\sinh(3d))\right)\right)\right)}{16\left(9f^4-10c^2f^2\log^2(f)+c^4\log^4(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1
) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f -
 c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 -
c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((b^2*Log[f]^2*((3*f
- c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(6*
f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3
*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[
f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] +
 Sinh[d]) - E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (b + 2*c*x)
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/(2*(f + c*Lo
g[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A]
time = 3.15, size = 326, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln\left(f\right)^{2}-12 d \ln\left(f\right) c-36 d f}{4\left(3 f+c \ln\left(f\right)\right)}} \operatorname{erf}\left(-\sqrt{-c \ln\left(f\right)-3 f}\, x+\frac{\ln\left(f\right) b}{2 \sqrt{-c \ln\left(f\right)-3 f}}\right)}{16 \sqrt{-c \ln\left(f\right)-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln\left(f\right)^{2}+12 d \ln}{4\left(-3 f+c \ln\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*d*f)/(3*f+c*ln(f))
)/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*
f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*d*f)/(-3*
f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f
-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)
/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*
ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-f)^(1/2))
```

Maxima [A]
time = 0.30, size = 287, normalized size = 0.89

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 3 \, f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) - 3 \, f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) + 3 \, f\right)}} + 3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) - 3 \, f}} - \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) - f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) + f\right)}} + d\right)}}{16 \, \sqrt{-c \log\left(f\right) - f}} + \frac{3 \, \sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) + f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) - f\right)}} - d\right)}}{16 \, \sqrt{-c \log\left(f\right) + f}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 3 \, f} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right) + 3 \, f}}\right) e^{\left(-\frac{b^{2} \log\left(f\right)^{2}}{4 \, {\left(c \log\left(f\right) - 3 \, f\right)}} - 3 \, d\right)}}{16 \, \sqrt{-c \log\left(f\right) + 3 \, f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
```

```
 - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(275) = 550$.
time = 0.39, size = 852, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*lo
g(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*s
qrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*
f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*l
og(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log(
f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^
3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*l
og(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/
(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f
))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(
f)^2 + 9*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2} \sinh^3 \left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**3, x)
```

Giac [A]
time = 0.45, size = 369, normalized size = 1.14

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{b\log\left(f\right)}{c\log\left(f\right)+3\,f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}-12\,cd\log\left(f\right)-12\,af\log\left(f\right)-36\,df}{4\,\left(c\log\left(f\right)+3\,f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log\left(f\right)}{c\log\left(f\right)+f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}-4\,cd\log\left(f\right)-4\,af\log\left(f\right)-4\,df}{4\,\left(c\log\left(f\right)+f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log\left(f\right)}{c\log\left(f\right)-f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}+4\,cd\log\left(f\right)+4\,af\log\left(f\right)-4\,df}{4\,\left(c\log\left(f\right)-f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x+\frac{b\log\left(f\right)}{c\log\left(f\right)-3\,f}\right)\right)e^{\left(-\frac{b^{2}\log\left(f\right)^{2}-4\,ac\log\left(f\right)^{2}+12\,cd\log\left(f\right)+12\,af\log\left(f\right)-36\,df}{4\,\left(c\log\left(f\right)-3\,f\right)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3, x)
```

3.363 $\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=161

$$-\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*ln
(f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1
/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\frac{\sqrt{\pi}\,f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}\,f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\mathrm{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*
Log[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] +
(E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f
] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx &= \int \left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx\right.\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f))}{4(-f+c\log(f))}\right.\right.\\
&= -\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\ }}{4}
\end{aligned}$$

Mathematica [A]
time = 1.06, size = 252, normalized size = 1.57

$$\frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(-e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\mathrm{Erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}\,(f+c\log(f))(\cosh(d)-\sinh(d))+f^{\frac{be}{2f-2c\log(f)}}\mathrm{Erfi}\left(\frac{e+2fx+(b+2cx)\log(f)}{2\sqrt{f+c\log(f)}}\right)(f-c\log(f))\sqrt{f+c\log(f)}\,(\cosh(d)+\sinh(d))\right)}{4\left(f^2-c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^((f*(e^2 + b^2*Log[f]^
2))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d])) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]
]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2
 - c^2*Log[f]^2))
```

Maple [A]
time = 0.92, size = 186, normalized size = 1.16

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+2 \ln(f) b e-4 d \ln(f) c-4 d f+e^{2}}{4(f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-f}\, x+\frac{e+b \ln(f)}{2 \sqrt{-c \ln(f)-f}}\right)}{4 \sqrt{-c \ln(f)-f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*f+e^2)/
(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(
-c*ln(f)-f)^(1/2))+1/4*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*l
n(f)*c-4*d*f+e^2)/(-f+c*ln(f)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+
1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))
```

Maxima [A]
time = 0.27, size = 155, normalized size = 0.96

$$\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)-f}\, x-\frac{b \log(f)+e}{2 \sqrt{-c \log(f)-f}}\right) e^{\left(-\frac{(b \log(f)+e)^{2}}{4(c \log(f)+f)}+d\right)}}{4 \sqrt{-c \log(f)-f}}-\frac{\sqrt{\pi}\, f^{a} \operatorname{erf}\left(\sqrt{-c \log(f)+f}\, x-\frac{b \log(f)-e}{2 \sqrt{-c \log(f)+f}}\right) e^{\left(-\frac{(b \log(f)-e)^{2}}{4(c \log(f)-f)}-d\right)}}{4 \sqrt{-c \log(f)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
- 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(143) = 286.
time = 0.40, size = 437, normalized size = 2.71

$$\frac{\left(\sqrt{\pi}(c \log(f)+f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}+2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)-f)}\right)+\sqrt{\pi}(c \log(f)+f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}+2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)-f)}\right)\right) \sqrt{-c \log(f)+f} \operatorname{erf}\left(-\frac{(2 f x-(2 c x+b) \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)+f}}{2(c \log(f)-f)}\right)-\left(\sqrt{\pi}(c \log(f)-f) \cosh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}-2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)+f)}\right)+\sqrt{\pi}(c \log(f)-f) \sinh\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-4 d f+\cosh(1)^{2}-2(2 c d+2 a f-b \cosh(1)-b \sinh(1)) \log(f)+2 \cosh(1) \sinh(1)+\sinh(1)^{2}}{4(c \log(f)+f)}\right)\right) \sqrt{-c \log(f)-f} \operatorname{erf}\left(\frac{(2 f x+(2 c x+b) \log(f)+\cosh(1)+\sinh(1)) \sqrt{-c \log(f)-f}}{2(c \log(f)+f)}\right)}{4\left(c^{2} \log(f)^{2}-f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + c
osh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sin
h(1) + sinh(1)^2)/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((b^2
```

```
 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*s
inh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*lo
g(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-c
*log(f) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4
*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(
1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c*l
og(f) - f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 - 2*(2*c*d
 + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(
c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + c
osh(1) + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Giac [A]
time = 0.41, size = 207, normalized size = 1.29

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\,\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{4\,\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\,\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{4\,\sqrt{-c\log\left(f\right)+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f)
 - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + 4*a*
f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2), x)
```

3.364 $\int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=239

$$-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\text{Erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(2
*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(
1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln(f)
+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5623, 2266, 2235, 2325, 2236}

$$-\frac{\sqrt{\pi}\,f^{a-\frac{b^2}{4c}}\,\text{Erfi}\left(\frac{\sqrt{\log(f)}\,(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\sqrt{\pi}\,f^a\exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right)\text{Erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}\,f^a\exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f
]))*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f -
c*Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f +
 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*
Sqrt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx &= \int\left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx\right.\\&=\frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int\\&=\frac{1}{4}\int\exp\left(-2d+a\log(f)-x(2e-b\log(f))-x^2(2f-c\log(f))\right)\,dx\\&=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2d+\frac{(2e-b\log(f)}{8f-4c\log(f}\right.\right.\\&=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}+\frac{\exp\left(-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right)f^a\sqrt{}}{8\sqrt{2f-}}\end{aligned}$$

Mathematica [A]

time = 4.08, size = 339, normalized size = 1.42

$$-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\,\sqrt{\log(f)}}-\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\mathrm{Erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}\,(2f+c\log(f))(\cosh(2d)-\sinh(2d))+f^{\frac{be}{2f-c\log(f)}}\mathrm{Erfi}\left(\frac{2(e+2fx)+(b+2cx)\log(f)}{2\sqrt{2f+c\log(f)}}\right)(2f-c\log(f))\sqrt{2f+c\log(f)}\,(\cosh(2d)+\sinh(2d))\right)}{8\left(-4f^2+c^2\log^2(f)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqr
t[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f
```

```
+ c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f
]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*
e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f
+ c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])
))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2)
)
```

Maple [A]
time = 1.76, size = 249, normalized size = 1.04

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln \left(f\right)^{2}-4 \ln \left(f\right) b e+8 d \ln \left(f\right) c-16 d f+4 e^{2}}{4 \left(-2 f+c \ln \left(f\right)\right)}} \operatorname{erf}\left(-x \sqrt{2 f-c \ln \left(f\right)}+\frac{b \ln \left(f\right)-2 e}{2 \sqrt{2 f-c \ln \left(f\right)}}\right)}{8 \sqrt{2 f-c \ln \left(f\right)}}-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*c-16*d*f+4*e^
2)/(-2*f+c*ln(f)))/(2*f-c*ln(f))^(1/2)*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-2*e)/(2*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*ln(f
)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))/(-c*ln(f)-2*f)^(1/2)*erf(-(-
c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f)-2*f)^(1/2))+1/4*Pi^(1/2)*f
^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*
ln(f))^(1/2))
```

Maxima [A]
time = 0.28, size = 219, normalized size = 0.92

$$\frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) - 2 \, f} x - \frac{b \log\left(f\right) + 2 \, e}{2 \, \sqrt{-c \log\left(f\right) - 2 \, f}}\right) e^{\left(-\frac{{\left(b \log\left(f\right) + 2 \, e\right)}^{2}}{4 \, {\left(c \log\left(f\right) + 2 \, f\right)}} + 2 \, d\right)}}{8 \, \sqrt{-c \log\left(f\right) - 2 \, f}} + \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right) + 2 \, f} x - \frac{b \log\left(f\right) - 2 \, e}{2 \, \sqrt{-c \log\left(f\right) + 2 \, f}}\right) e^{\left(-\frac{{\left(b \log\left(f\right) - 2 \, e\right)}^{2}}{4 \, {\left(c \log\left(f\right) - 2 \, f\right)}} - 2 \, d\right)}}{8 \, \sqrt{-c \log\left(f\right) + 2 \, f}} - \frac{\sqrt{\pi} f^{a} \operatorname{erf}\left(\sqrt{-c \log\left(f\right)} x - \frac{b \log\left(f\right)}{2 \, \sqrt{-c \log\left(f\right)}}\right)}{4 \, \sqrt{-c \log\left(f\right)} f^{\frac{b^{2}}{4 \, c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(201) = 402$.
time = 0.40, size = 602, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 4*cosh(1)^2 + 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log
(f) + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*lo
g(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 4*cosh(
1)^2 + 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*sinh(1)
 + 4*sinh(1)^2)/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x -
(2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-c*log(f) + 2*f)/(c*log(f)
 - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)
*log(f)^2 - 16*d*f + 4*cosh(1)^2 - 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1)
)*log(f) + 8*cosh(1)*sinh(1) + 4*sinh(1)^2)/(c*log(f) + 2*f)) + sqrt(pi)*(c
^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 4*
cosh(1)^2 - 4*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 8*cosh(1)*si
nh(1) + 4*sinh(1)^2)/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*
x + (2*c*x + b)*log(f) + 2*cosh(1) + 2*sinh(1))*sqrt(-c*log(f) - 2*f)/(c*lo
g(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*l
og(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c
))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c))/(c^3*log(f)^3 -
4*c*f^2*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Giac [A]
time = 0.46, size = 271, normalized size = 1.13

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2f}\left(2x+\frac{b\log(f)+2e}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)-8af\log(f)+4e^2-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log\left(f\right)-2f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2f}\left(2x+\frac{b\log(f)-2e}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+8af\log(f)+4e^2-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*
log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) -
```

```
4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

3.365 $\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=344

$$\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}} f^a \sqrt{\pi}\,\mathrm{Erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{Erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d}}{}$$

```
[Out] 3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+(3*e-b*
ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(3*f
-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/
(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi
^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f)))*f^
a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3
*f+c*ln(f))^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$-\frac{\sqrt{\pi} f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\mathrm{Erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right)\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}-\frac{3\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\mathrm{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
- (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = \int\left(-\frac{1}{8}e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3\left(d+e\right.\right.\right.$$

$$= -\left(\frac{1}{8}\int e^{-3\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int\exp\left(6d+6ex+6fx^2-\right.$$

$$= -\left(\frac{1}{8}\int\exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f)\right.\right.$$

$$= -\left(\frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)}{4(-3}\right.\right.$$

$$= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\exp\left(-3d+\frac{(3e}{12}\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.
time = 6.43, size = 2991, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]

[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c*f^2*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (27*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr

```
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 + (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Erfi[(e + 2*f*x +
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) + (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) - (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]...
```

Maple [A]
time = 2.08, size = 384, normalized size = 1.12

method	result
risch	$-\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2} \ln(f)^{2}+6 \ln(f) b e-12 d \ln(f) c-36 d f+9 e^{2}}{4(3 f+c \ln(f))}} \operatorname{erf}\left(-\sqrt{-c \ln(f)-3 f}\, x+\frac{3 e+b \ln(f)}{2 \sqrt{-c \ln(f)-3 f}}\right)}{16 \sqrt{-c \ln(f)-3 f}}+\frac{\sqrt{\pi}\, f^{a} \mathrm{e}^{\cdots}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(f)*c-36*d*f+9*
e^2)/(3*f+c*ln(f)))/(-c*ln(f)-3*f)^(1/2)*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3
*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))+1/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6
*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*e^2)/(-3*f+c*ln(f)))/(3*f-c*ln(f))^(1/2)*e
rf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))-3/16*Pi^(1
/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(-f+c*ln(f
)))/(f-c*ln(f))^(1/2)*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^
(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*
```

```
f+e^2)/(f+c*ln(f)))/(-c*ln(f)-f)^(1/2)*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*l
n(f))/(-c*ln(f)-f)^(1/2))
```

Maxima [A]
time = 0.29, size = 323, normalized size = 0.94

$$\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}\,x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log(f)-f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log(f)+f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}\,x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-\frac{(b\log(f)-3e)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log(f)+3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. 2(303) = 606.
time = 0.47, size = 1100, normalized size = 3.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 + 6*(2*c*d + 2*a*f -
 b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f
) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^
3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 + 6*(2*c*d + 2*
a*f - b*cosh(1) - b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*
log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f)
+ 3*cosh(1) + 3*sinh(1))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(
pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^
2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*
sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)) + sqrt(pi)
*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 + 2*(2*c*d + 2*a*f - b*cosh(1) - b*sin
h(1))*log(f) + 2*cosh(1)*sinh(1) + sinh(1)^2)/(c*log(f) - f)))*sqrt(-c*log(
f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f) + cosh(1) + sinh(1))*sqrt(-c*l
og(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9
```

```
*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^
2 - 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) +
sinh(1)^2)/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*
f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + cosh(1)^2 -
 2*(2*c*d + 2*a*f - b*cosh(1) - b*sinh(1))*log(f) + 2*cosh(1)*sinh(1) + sin
h(1)^2)/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*l
og(f) + cosh(1) + sinh(1))*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)*
(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 -
4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 - 6*(2*c*d + 2*a*f - b*cosh(1) - b*s
inh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f) + 3*f)) + sqrt
(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b
^2 - 4*a*c)*log(f)^2 - 36*d*f + 9*cosh(1)^2 - 6*(2*c*d + 2*a*f - b*cosh(1)
- b*sinh(1))*log(f) + 18*cosh(1)*sinh(1) + 9*sinh(1)^2)/(c*log(f) + 3*f)))*
sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) + 3*cosh(1) + 3*s
inh(1))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2
*log(f)^2 + 9*f^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 427, normalized size = 1.24

$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)-12af\log(f)+9e^2-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log\left(f\right)-3f}}+\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log\left(f\right)-f}}-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3f}\left(2x+\frac{b\log(f)-3e}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+12af\log(f)+9e^2-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log\left(f\right)+3f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*b*
e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*
b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*lo
g(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*l
```

```
og(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int f^{c x^2+b x+a} \sinh\left(f x^2+e x+d\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

3.366 $\int (x+\sinh(x))^2\,dx$

Optimal. Leaf size=30

$$-\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

```
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6874, 3377, 2717, 2715, 8}

$$\frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^2,x]
```

```
[Out] -1/2*x + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (x+\sinh(x))^2\,dx &= \int \left(x^2+2x\sinh(x)+\sinh^2(x)\right)\,dx\\
&= \frac{x^3}{3}+2\int x\sinh(x)\,dx+\int \sinh^2(x)\,dx\\
&= \frac{x^3}{3}+2x\cosh(x)+\frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-2\int \cosh(x)\,dx\\
&= -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{1}{6}x\left(-3+2x^2\right)+2x\cosh(x)-2\sinh(x)+\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^2,x]
```

```
[Out] (x*(-3 + 2*x^2))/6 + 2*x*Cosh[x] - 2*Sinh[x] + Sinh[2*x]/4
```

Maple [A]
time = 0.29, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{x}{2}+\frac{x^3}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$	25
risch	$\frac{x^3}{3}-\frac{x}{2}+\frac{e^{2x}}{8}+\left(x-1\right)e^{x}+\left(1+x\right)e^{-x}-\frac{e^{-2x}}{8}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.17

$$\frac{1}{3}\,x^3+(x+1)e^{(-x)}+(x-1)e^{x}-\frac{1}{2}\,x+\frac{1}{8}\,e^{(2\,x)}-\frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A]
time = 0.40, size = 22, normalized size = 0.73

$$\frac{1}{3}\,x^3 + 2\,x\cosh\left(x\right) + \frac{1}{2}\left(\cosh\left(x\right) - 4\right)\sinh\left(x\right) - \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 2*x*cosh(x) + 1/2*(cosh(x) - 4)*sinh(x) - 1/2*x
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sinh^2\left(x\right)}{2} - \frac{x\cosh^2\left(x\right)}{2} + 2x\cosh\left(x\right) + \frac{\sinh\left(x\right)\cosh\left(x\right)}{2} - 2\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**2,x)
[Out] x**3/3 + x*sinh(x)**2/2 - x*cosh(x)**2/2 + 2*x*cosh(x) + sinh(x)*cosh(x)/2
- 2*sinh(x)
```

Giac [A]
time = 0.41, size = 35, normalized size = 1.17

$$\frac{1}{3}\,x^3 + (x+1)e^{(-x)} + (x-1)e^x - \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Mupad [B]
time = 0.60, size = 24, normalized size = 0.80

$$\frac{\cosh(x)\,\sinh(x)}{2} - 2\sinh(x) - \frac{x}{2} + 2\,x\cosh(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sinh(x))^2,x)
[Out] (cosh(x)*sinh(x))/2 - 2*sinh(x) - x/2 + 2*x*cosh(x) + x^3/3
```

3.367 $\int (x+\sinh(x))^3\,dx$

Optimal. Leaf size=56

$$-\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}$$

```
[Out] -3/4*x^2+1/4*x^4+5*cosh(x)+3*x^2*cosh(x)+1/3*cosh(x)^3-6*x*sinh(x)+3/2*x*co
sh(x)*sinh(x)-3/4*sinh(x)^2
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6874, 3377, 2718, 3391, 30, 2713}

$$\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x)+\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sinh[x])^3,x]
```

```
[Out] (-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
+ (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\sinh(x))^3\,dx &= \int \left(x^3+3x^2\sinh(x)+3x\sinh^2(x)+\sinh^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\sinh(x)\,dx+3\int x\sinh^2(x)\,dx+\int \sinh^3(x)\,dx\\ &= \frac{x^4}{4}+3x^2\cosh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}-\frac{3\int x\,dx}{2}-6\int x\cosh(x)\,dx\\ &= -\frac{3x^2}{4}+\frac{x^4}{4}-\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)\\ &= -\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(18\left(7+4x^2\right)\cosh(x)-9\cosh(2x)+2\cosh(3x)+6x\left(-3x+x^3-24\sinh(x)+3\sinh(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sinh[x])^3,x]
```

```
[Out] (18*(7 + 4*x^2)*Cosh[x] - 9*Cosh[2*x] + 2*Cosh[3*x] + 6*x*(-3*x + x^3 - 24*
Sinh[x] + 3*Sinh[2*x]))/24
```

Maple [A]
time = 0.42, size = 73, normalized size = 1.30

method	result
risch	$\frac{x^4}{4}-\frac{3x^2}{4}+\frac{9}{16}+\frac{e^{3x}}{24}+\left(-\frac{3}{16}+\frac{3x}{8}\right)e^{2x}+\left(\frac{21}{8}-3x+\frac{3}{2}x^2\right)e^{x}+\left(\frac{21}{8}+3x+\frac{3}{2}x^2\right)e^{-x}+\left(-\frac{3}{16}-\frac{3x}{8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4-3/4*x^2+9/16+1/24*exp(3*x)+(-3/16+3/8*x)*exp(2*x)+(21/8-3*x+3/2*x^2
)*exp(x)+(21/8+3*x+3/2*x^2)*exp(-x)+(-3/16-3/8*x)*exp(-2*x)+1/24*exp(-3*x)
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.45

$$\frac{1}{4}x^4-\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}+\frac{3}{2}(x^2+2x+2)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{2}(x^2-2x+2)e^x+\frac{1}{24}e^{(3x)}-\frac{3}{8}e^{(-x)}+\frac{1}{24}e^{(-3x)}-\frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/2*(x^2 + 2*x + 2)*e^(-x) - 3
/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x
) + 1/24*e^(-3*x) - 3/8*e^x
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.04

$$\frac{1}{4}x^4+\frac{1}{12}\cosh(x)^3+\frac{1}{8}(2\cosh(x)-3)\sinh(x)^2-\frac{3}{4}x^2+\frac{3}{4}(4x^2+7)\cosh(x)-\frac{3}{8}\cosh(x)^2+\frac{3}{2}(x\cosh(x)-4x)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/12*cosh(x)^3 + 1/8*(2*cosh(x) - 3)*sinh(x)^2 - 3/4*x^2 + 3/4*(4
*x^2 + 7)*cosh(x) - 3/8*cosh(x)^2 + 3/2*(x*cosh(x) - 4*x)*sinh(x)
```

Sympy [A]
time = 0.09, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sinh^2(x)}{4}-\frac{3x^2\cosh^2(x)}{4}+3x^2\cosh(x)+\frac{3x\sinh(x)\cosh(x)}{2}-6x\sinh(x)+\sinh^2(x)\cosh(x)-\frac{2\cosh^3(x)}{3}-\frac{3\cosh^2(x)}{4}+6\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sinh(x)**2/4 - 3*x**2*cosh(x)**2/4 + 3*x**2*cosh(x) + 3*x*s
inh(x)*cosh(x)/2 - 6*x*sinh(x) + sinh(x)**2*cosh(x) - 2*cosh(x)**3/3 - 3*co
sh(x)**2/4 + 6*cosh(x)
```

Giac [A]
time = 0.41, size = 75, normalized size = 1.34

$$\frac{1}{4}x^4-\frac{3}{4}x^2+\frac{3}{16}(2x-1)e^{(2x)}+\frac{3}{8}(4x^2+8x+7)e^{(-x)}-\frac{3}{16}(2x+1)e^{(-2x)}+\frac{3}{8}(4x^2-8x+7)e^x+\frac{1}{24}e^{(3x)}+\frac{1}{24}e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/8*(4*x^2 + 8*x + 7)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 7)*e^x + 1/24*e^(3*x) + 1/24*
e^(-3*x)
```

Mupad [B]
time = 0.08, size = 46, normalized size = 0.82

$$5\cosh(x)+3\,x^2\cosh(x)-\frac{3\cosh(x)^2}{4}+\frac{\cosh(x)^3}{3}-6\,x\sinh(x)-\frac{3\,x^2}{4}+\frac{x^4}{4}+\frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sinh(x))^3,x)
```

```
[Out] 5*cosh(x) + 3*x^2*cosh(x) - (3*cosh(x)^2)/4 + cosh(x)^3/3 - 6*x*sinh(x) - (
3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

3.368 $\int \frac{\sinh(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=213

$$-\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}}{2\sqrt{-c}\sqrt{d}}$$

```
[Out] 1/2*cosh(a+b*(-c)^(1/2)/d^(1/2))*Shi(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d
^(1/2)-1/2*cosh(a-b*(-c)^(1/2)/d^(1/2))*Shi(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^
(1/2)/d^(1/2)-1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^(1/2)
)/(-c)^(1/2)/d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^(1/2)
/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5388, 3384, 3379, 3382}

$$-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/2*(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[d
]])/(Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a +
 (b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sqrt[
d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[
a - (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt
[-c]*Sqrt[d])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(a+bx)}{c+dx^2}\,dx &= \int \left(\frac{\sqrt{-c}\,\sinh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\sinh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\\
&= -\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}\\
&= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}}{2\sqrt{-c}}\\
&= -\frac{\operatorname{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\operatorname{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 180, normalized size = 0.85

$$\frac{i\left(\operatorname{CosIntegral}\left(-\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)\sinh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)-\operatorname{CosIntegral}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)\sinh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)+i\left(\cosh\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\operatorname{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+\cosh\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\operatorname{Si}\left(\frac{b\sqrt{c}}{\sqrt{d}}+ibx\right)\right)\right)}{2\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[-((b*Sqrt[c])/Sqrt[d]) + I*b*x]*Sinh[a - (I*b*Sqrt[c])/
Sqrt[d]] - CosIntegral[(b*Sqrt[c])/Sqrt[d] + I*b*x]*Sinh[a + (I*b*Sqrt[c])/
Sqrt[d]] + I*(Cosh[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] - I*b*x] + Cosh[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(b*Sqrt[c])/Sqrt[
d] + I*b*x])))/(Sqrt[c]*Sqrt[d])
```

Maple [A]
time = 0.50, size = 212, normalized size = 1.00

method	result
risch	$\frac{e^{-\frac{b\sqrt{-cd}+ad}{d}}\operatorname{expIntegral}\left(1,-\frac{b\sqrt{-cd}-d(bx+a)+ad}{d}\right)}{4\sqrt{-cd}}-\frac{e^{-\frac{-b\sqrt{-cd}+ad}{d}}\operatorname{expIntegral}\left(1,\frac{b\sqrt{-cd}+d(bx+a)-ad}{d}\right)}{4\sqrt{-cd}}-\frac{e^{\frac{b\sqrt{-c}}{d}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+a*d)/d)*Ei(1,-(b*(-c*d)^(1/2)-d*(b*x+
a)+a*d)/d)-1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*(-c*d)^(1
/2)+d*(b*x+a)-a*d)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*
(-c*d)^(1/2)-d*(b*x+a)+a*d)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+a*d)/d
)*Ei(1,-(b*(-c*d)^(1/2)+d*(b*x+a)-a*d)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.
time = 0.37, size = 316, normalized size = 1.48

$$-\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx-\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(-a+\sqrt{-\frac{b^2c}{d}}\right)+\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\sinh\left(a+\sqrt{-\frac{b^2c}{d}}\right)+\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx-\sqrt{-\frac{b^2c}{d}}\right)\right)\sinh\left(-a+\sqrt{-\frac{b^2c}{d}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x^2), x)
```

3.369 $\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}-\frac{\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\right.}{}\right.}{\sqrt{d^2-4ce}}$$

```
[Out] cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))
/e)/(-4*c*e+d^2)^(1/2)-cosh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Chi(b*x+1/2*b*(d-(-4*c*e+d^2)
^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Chi(b*
x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-
4*c*e+d^2)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6860, 3384, 3379, 3382}

$$\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sqr
t[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[
d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)$$

$$= \frac{(2e)\int \frac{\sinh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sinh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cosh\left(a\right.\right.}{}$$

$$= \frac{\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right.}{2e}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 248, normalized size = 0.92

$$\frac{\mathrm{CosIntegral}\left(\frac{ib\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)\sinh\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right) - \mathrm{CosIntegral}\left(\frac{ib\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right) - \cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right) + i\cosh\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Si}\left(\frac{ib\left(-d+\sqrt{d^2-4ce}\right)}{2e}-ibx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[((I/2)*b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e]*Sinh[a + (b*(-d +
 Sqrt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] +
 2*e*x))/e]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cosh[a - (b*(d +
```

```
Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))
/(2*e)] + I*Cosh[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[((I/2)
*b*(-d + Sqrt[d^2 - 4*c*e]))/e - I*b*x])/Sqrt[d^2 - 4*c*e]
```

Maple [A]
time = 0.51, size = 376, normalized size = 1.39

method	result
risch	$\frac{b\,\mathrm{e}^{-\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}\left(1,-\frac{-2(bx+a)e+2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}} - \frac{b\,\mathrm{e}^{-\frac{2ea-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expInt}}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,-1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1
/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)^
(1/2)))*Ei(1,1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*
b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
)))*Ei(1,1/2*(-2*(b*x+a)*e+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*b/(
-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)^(1/2)))
*Ei(1,-1/2*(2*(b*x+a)*e-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*%e*c>0)', see `assume?` for m
ore det
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. $2(235) = 470$.
time = 0.41, size = 1346, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*co
sh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b
*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh
(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sin
h(1))) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*
cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) +
2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(c
osh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) +
sinh(1))))*cosh(1/2*(b*d - 2*a*cosh(1) - 2*a*sinh(1) + sqrt((b^2*d^2 - 4*b^
2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))
*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*cosh(
1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1)
 + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2 - 4
*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^
2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - sqrt((b^2*d^2 - 4*b^2*c*cos
h(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(
1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2
- 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(
1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))))*cosh(-1/2*(b*d - 2*a*cosh
(1) - 2*a*sinh(1) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cos
h(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + si
nh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 +
2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) +
 2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(
cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) +
 sinh(1))) + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2
+ 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1
) + 2*b*x*sinh(1) + b*d + sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1)
)/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1
) + sinh(1))))*sinh(1/2*(b*d - 2*a*cosh(1) - 2*a*sinh(1) + sqrt((b^2*d^2 -
4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)
^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) - (sqrt((b^2*d^2 - 4*b^2*c*c
osh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cos
h(1) + sinh(1))*Ei(1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*d^2
 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh
(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))) + sqrt((b^2*d^2 - 4*b^2*c
*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(c
osh(1) + sinh(1))*Ei(-1/2*(2*b*x*cosh(1) + 2*b*x*sinh(1) + b*d - sqrt((b^2*
d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/(cosh(1)^2 + 2*cosh(1)*sinh(1) + s
inh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1) + sinh(1))))*sinh(-1/2*(b*d - 2*a*
cosh(1) - 2*a*sinh(1) - sqrt((b^2*d^2 - 4*b^2*c*cosh(1) - 4*b^2*c*sinh(1))/
(cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2))*(cosh(1) + sinh(1)))/(cosh(1)
+ sinh(1))))/(b*d^2 - 4*b*c*cosh(1) - 4*b*c*sinh(1))
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x**2+d*x+c),x)
[Out] Integral(sinh(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)/(c + d*x + e*x^2),x)
[Out] int(sinh(a + b*x)/(c + d*x + e*x^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
       if expr.args[1] == Rational(1,2):
          return True
       else:
          return False
    else:
       return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
       if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
       else:
         return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```