Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.5-Inverse-secant/157-5.5.2-Inverse-secant-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [50]. This is test number [157].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (50)	0.00 (0)
Mathematica	98.00 (49)	2.00 (1)
Maple	74.00 (37)	26.00 (13)
Fricas	56.00 (28)	44.00 (22)
Giac	54.00 (27)	46.00 (23)
Maxima	36.00 (18)	64.00 (32)
Sympy	26.00 (13)	74.00 (37)
Mupad	20.00 (10)	80.00 (40)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

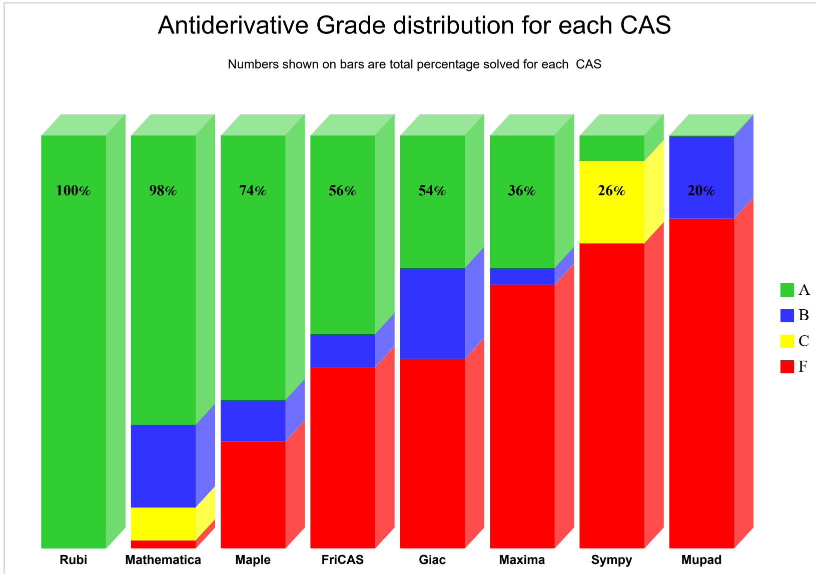
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

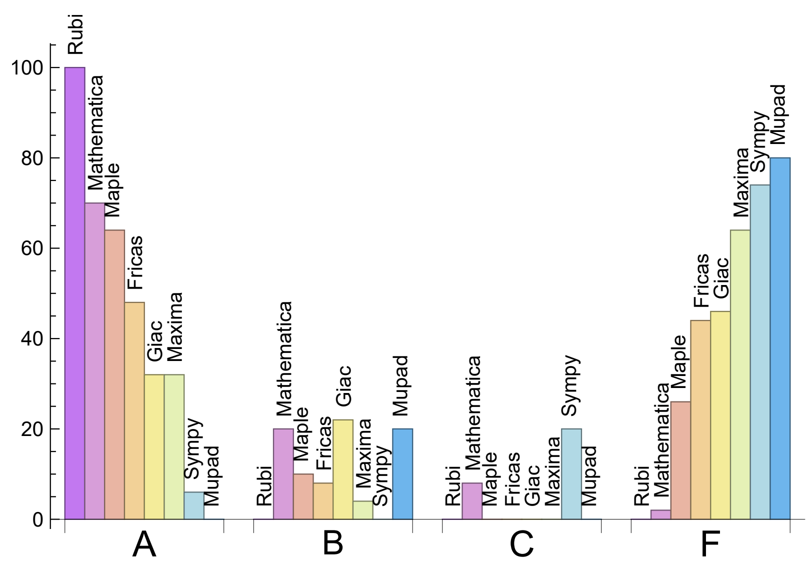
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.00	20.00	8.00	2.00
Maple	64.00	10.00	0.00	26.00
Fricas	48.00	8.00	0.00	44.00
Giac	32.00	22.00	0.00	46.00
Maxima	32.00	4.00	0.00	64.00
Sympy	6.00	0.00	20.00	74.00
Mupad	N/A	20.00	0.00	80.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	13	100.00 %	0.00 %	0.00 %
Fricas	22	90.91 %	0.00 %	9.09 %
Giac	23	95.65 %	0.00 %	4.35 %
Maxima	32	100.00 %	0.00 %	0.00 %
Sympy	37	89.19 %	8.11 %	2.70 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

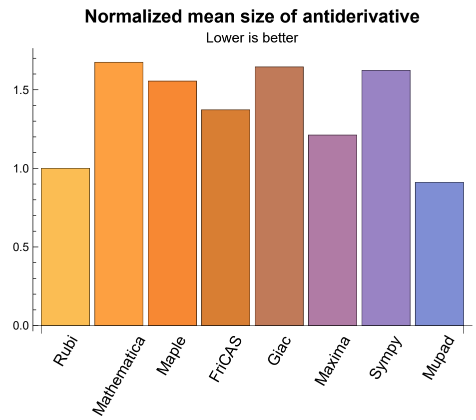
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

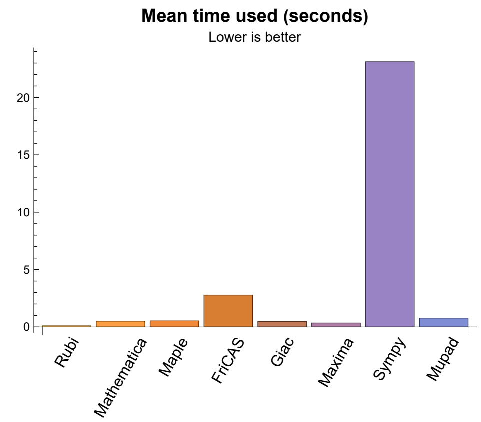
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	120.76	1.00	69.00	1.00
Mathematica	0.50	186.86	1.67	107.00	1.00
Maple	0.53	194.19	1.55	91.00	1.39
Maxima	0.34	56.67	1.21	54.50	1.20
Fricas	2.78	102.61	1.37	56.50	0.92
Sympy	23.11	76.23	1.62	58.00	1.61
Giac	0.48	120.67	1.65	82.00	1.72
Mupad	0.77	37.50	0.91	36.50	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{27, 28, 31, 36\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

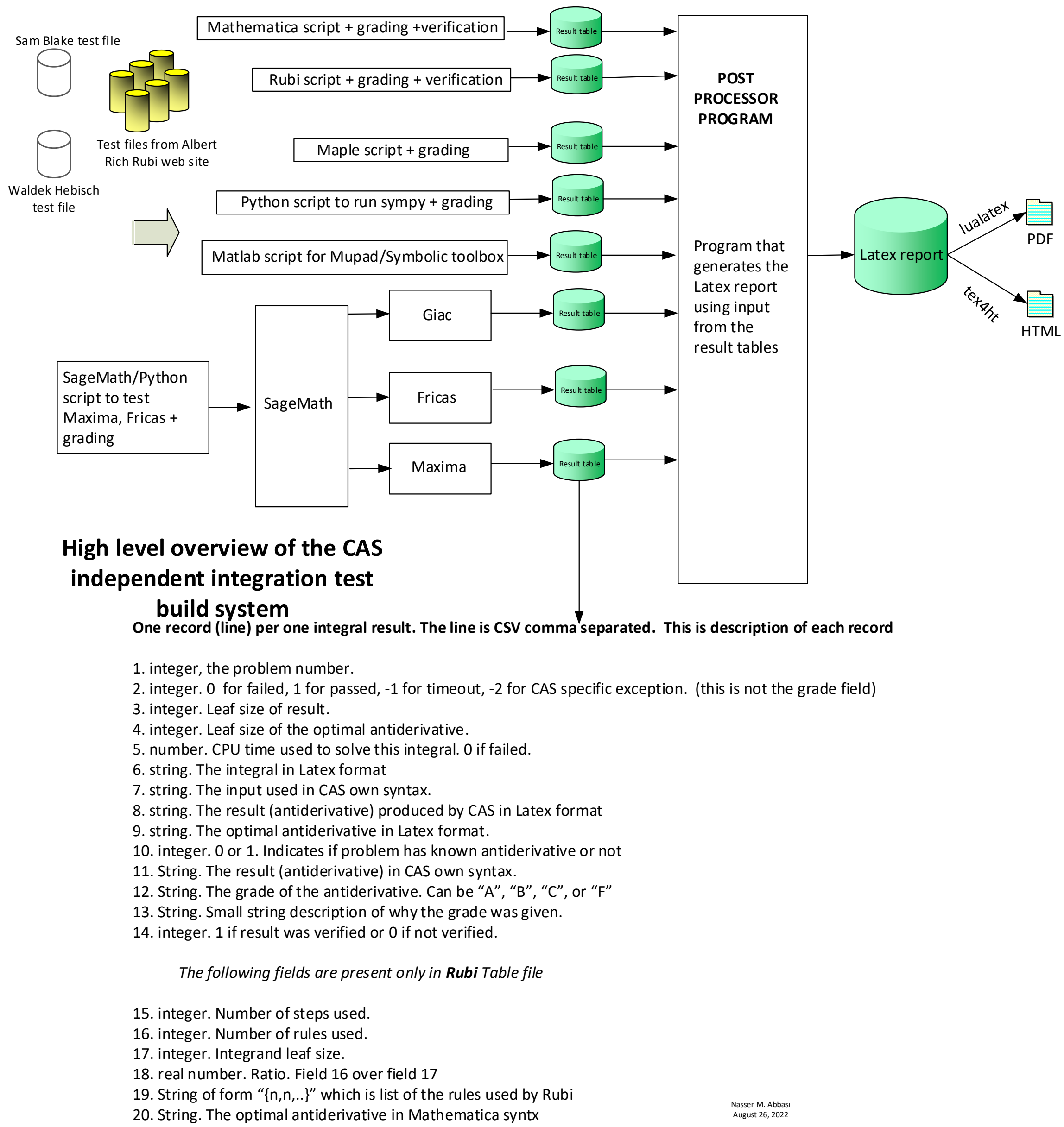
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20, 21, 23, 27, 28, 29, 30, 33, 34, 35, 43, 44, 45, 46, 47, 48, 49, 50 }

B grade: { 14, 22, 31, 32, 36, 38, 39, 40, 41, 42 }

C grade: { 17, 24, 25, 26 }

F grade: { 37 }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 32, 33, 34, 38, 39, 40, 42, 50 }

B grade: { 11, 12, 24, 25, 26 }

C grade: { }

F grade: { 1, 31, 35, 36, 37, 41, 43, 44, 45, 46, 47, 48, 49 }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 7, 10, 11, 12, 14, 15, 16, 22, 38, 39, 40, 41 }

B grade: { 8, 9 }

C grade: { }

F grade: { 1, 6, 13, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 18, 19, 20, 21, 25, 26, 38, 39, 40, 41, 47, 48, 49 }

B grade: { 14, 16, 22, 24 }

C grade: { }

F grade: { 1, 6, 13, 17, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 50 }

2.1.6 Sympy

A grade: { 10, 11, 12 }

B grade: { }

C grade: { 2, 3, 4, 5, 7, 8, 9, 14, 15, 16 }

F grade: { 1, 6, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.1.7 Giac

A grade: { 7, 8, 9, 10, 11, 12, 15, 16, 21, 24, 38, 39, 40, 41, 47, 50 }

B grade: { 2, 3, 4, 5, 14, 18, 19, 20, 22, 25, 26 }

C grade: { }

F grade: { 1, 6, 13, 17, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 48, 49 }

2.1.8 Mupad

A grade: { }

B grade: { 5, 7, 11, 12, 14, 22, 38, 39, 40, 41 }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.027	0.056	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	43	66	32	119	152	-1
N.S.	1	1.00	0.69	0.74	1.14	0.55	2.05	2.62	-0.02
time (sec)	N/A	0.015	0.020	0.102	0.266	1.297	63.071	0.449	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	38	52	27	90	116	-1
N.S.	1	1.00	0.74	0.81	1.11	0.57	1.91	2.47	-0.02
time (sec)	N/A	0.013	0.017	0.112	0.265	1.686	20.962	0.468	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	31	38	20	58	80	-1
N.S.	1	1.00	0.78	0.86	1.06	0.56	1.61	2.22	-0.03
time (sec)	N/A	0.009	0.014	0.099	0.253	1.516	6.736	0.466	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	21	14	29	41	21
N.S.	1	1.00	1.00	1.39	1.17	0.78	1.61	2.28	1.17
time (sec)	N/A	0.003	0.007	0.096	0.252	0.992	2.845	0.451	1.149

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	63	0	0	0	0	-1
N.S.	1	1.00	0.96	1.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.022	0.299	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	46	51	19	75	30	28
N.S.	1	1.00	0.84	1.21	1.34	0.50	1.97	0.79	0.74
time (sec)	N/A	0.013	0.017	0.096	0.470	3.877	15.317	0.438	0.684

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	57	80	29	144	44	-1
N.S.	1	1.00	1.02	1.06	1.48	0.54	2.67	0.81	-0.02
time (sec)	N/A	0.015	0.023	0.098	0.474	7.028	48.187	0.470	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	67	106	35	180	58	-1
N.S.	1	1.00	0.66	0.99	1.56	0.51	2.65	0.85	-0.01
time (sec)	N/A	0.017	0.035	0.102	0.468	5.123	138.714	0.532	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	42	66	54	39	51	47	-1
N.S.	1	1.00	0.75	1.18	0.96	0.70	0.91	0.84	-0.02
time (sec)	N/A	0.033	0.023	0.736	0.471	2.463	0.137	0.612	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	91	46	38	41	39	38
N.S.	1	1.00	0.94	1.94	0.98	0.81	0.87	0.83	0.81
time (sec)	N/A	0.016	0.018	0.721	0.469	2.587	0.094	0.435	0.628

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	51	24	27	22	28	24
N.S.	1	1.00	1.00	1.96	0.92	1.04	0.85	1.08	0.92
time (sec)	N/A	0.007	0.009	0.717	0.260	2.279	0.104	0.444	0.109

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	76	0	0	0	0	-1
N.S.	1	1.00	1.00	1.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.019	0.934	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	93	44	52	107	29	61	29
N.S.	1	1.00	3.00	1.42	1.68	3.45	0.94	1.97	0.94
time (sec)	N/A	0.022	0.092	0.030	0.252	1.612	1.472	0.458	0.605

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	54	32	39	53	61	-1
N.S.	1	1.00	0.95	1.42	0.84	1.03	1.39	1.61	-0.03
time (sec)	N/A	0.016	0.017	0.721	0.462	2.428	0.823	0.457	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	91	64	142	100	80	-1
N.S.	1	1.00	1.15	1.52	1.07	2.37	1.67	1.33	-0.02
time (sec)	N/A	0.030	0.029	0.727	0.466	3.442	1.977	0.442	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	93	0	0	0	0	-1
N.S.	1	1.00	0.87	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.061	0.475	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	173	329	0	152	0	409	-1
N.S.	1	1.00	0.88	1.67	0.00	0.77	0.00	2.08	-0.01
time (sec)	N/A	0.180	0.120	0.136	0.000	2.599	0.000	0.466	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	150	249	0	130	0	299	-1
N.S.	1	1.00	0.97	1.61	0.00	0.84	0.00	1.93	-0.01
time (sec)	N/A	0.111	0.187	0.150	0.000	2.598	0.000	0.452	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	131	190	0	117	0	204	-1
N.S.	1	1.00	1.13	1.64	0.00	1.01	0.00	1.76	-0.01
time (sec)	N/A	0.067	0.120	0.146	0.000	3.179	0.000	0.454	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	110	108	0	104	0	133	-1
N.S.	1	1.00	1.41	1.38	0.00	1.33	0.00	1.71	-0.01
time (sec)	N/A	0.040	0.076	0.135	0.000	1.906	0.000	0.455	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	336	45	55	73	0	82	35
N.S.	1	1.00	9.08	1.22	1.49	1.97	0.00	2.22	0.95
time (sec)	N/A	0.023	0.542	0.038	0.281	1.452	0.000	0.453	0.862

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	284	374	0	0	0	0	-1
N.S.	1	1.00	1.42	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.217	0.764	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	112	126	0	281	0	94	-1
N.S.	1	1.00	1.60	1.80	0.00	4.01	0.00	1.34	-0.01
time (sec)	N/A	0.078	0.209	0.269	0.000	2.904	0.000	0.466	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	198	456	0	427	0	216	-1
N.S.	1	1.00	1.58	3.65	0.00	3.42	0.00	1.73	-0.01
time (sec)	N/A	0.154	0.739	0.274	0.000	2.136	0.000	0.454	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	241	970	0	548	0	451	-1
N.S.	1	1.00	1.33	5.36	0.00	3.03	0.00	2.49	-0.01
time (sec)	N/A	0.222	0.287	0.292	0.000	3.623	0.000	0.478	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	667	673	0	0	0	0	-1
N.S.	1	1.00	1.75	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	8.278	5.172	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	473	498	0	0	0	0	-1
N.S.	1	1.00	1.64	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	4.199	1.162	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	142	227	0	0	0	0	-1
N.S.	1	1.00	0.92	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.096	0.473	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	111	162	0	0	0	0	-1
N.S.	1	1.00	1.18	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.077	0.255	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	813	0	0	0	0	0	-1
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	1.554	0.852	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	686	336	0	0	0	0	-1
N.S.	1	1.00	2.81	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.505	0.800	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	442	716	0	0	0	0	-1
N.S.	1	1.00	0.89	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.364	1.099	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	248	379	0	0	0	0	-1
N.S.	1	1.00	0.89	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.294	1.009	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	160	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.075	0.242	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	1058	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	2.659	0.905	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	180.003	0.576	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	385	67	71	96	0	100	52
N.S.	1	1.00	6.64	1.16	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.053	0.653	0.080	0.254	1.974	0.000	0.542	1.031

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	385	67	71	96	0	100	52
N.S.	1	1.00	6.64	1.16	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.055	0.195	0.076	0.264	2.768	0.000	0.561	0.774

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	137	67	71	96	0	100	52
N.S.	1	1.00	2.36	1.16	1.22	1.66	0.00	1.72	0.90
time (sec)	N/A	0.062	0.274	0.078	0.274	2.717	0.000	0.554	0.777

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	130	0	66	92	0	75	44
N.S.	1	1.00	2.65	0.00	1.35	1.88	0.00	1.53	0.90
time (sec)	N/A	0.055	0.233	0.050	0.253	3.879	0.000	0.501	1.084

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	280	111	0	0	0	0	-1
N.S.	1	1.00	3.29	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.599	0.509	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.222	0.028	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	107	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.141	0.020	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	54	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.046	0.018	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.041	0.020	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	0	0	23	0	43	-1
N.S.	1	1.00	0.87	0.00	0.00	0.59	0.00	1.10	-0.03
time (sec)	N/A	0.021	0.033	0.020	0.000	3.065	0.000	0.474	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	30	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.032	0.019	0.000	2.905	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	54	0	0	40	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.104	0.019	0.000	3.752	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	99	0	0	0	115	-1
N.S.	1	1.00	0.86	1.43	0.00	0.00	0.00	1.67	-0.01
time (sec)	N/A	0.067	0.039	0.457	0.000	0.000	0.000	0.608	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [19]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	10	0.600
2	A	4	3	1.00	10	0.300
3	A	4	3	1.00	10	0.300
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	7	6	1.00	10	0.600
7	A	5	5	1.00	10	0.500
8	A	6	5	1.00	10	0.500
9	A	7	5	1.00	10	0.500
10	A	5	4	1.00	10	0.400
11	A	4	4	1.00	8	0.500
12	A	3	3	1.00	6	0.500
13	A	6	6	1.00	10	0.600
14	A	5	5	1.00	10	0.500
15	A	3	3	1.00	10	0.300
16	A	6	6	1.00	10	0.600
17	A	7	6	1.00	10	0.600
18	A	9	8	1.00	10	0.800
19	A	8	7	1.00	10	0.700
20	A	7	6	1.00	10	0.600
21	A	6	6	1.00	8	0.750
22	A	5	5	1.00	6	0.833
23	A	14	8	1.00	10	0.800
24	A	5	5	1.00	10	0.500
25	A	7	7	1.00	10	0.700

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	8	1.00	10	0.800
27	A	20	9	1.00	12	0.750
28	A	17	9	1.00	12	0.750
29	A	11	8	1.00	10	0.800
30	A	8	6	1.00	8	0.750
31	A	17	9	1.00	12	0.750
32	A	12	8	1.00	12	0.667
33	A	25	14	1.00	12	1.167
34	A	16	12	1.00	10	1.200
35	A	10	7	1.00	8	0.875
36	A	20	10	1.00	12	0.833
37	A	14	9	1.00	12	0.750
38	A	7	6	1.00	14	0.429
39	A	7	6	1.00	16	0.375
40	A	7	6	1.00	16	0.375
41	A	6	6	1.00	14	0.429
42	A	7	7	1.00	10	0.700
43	A	6	4	1.00	10	0.400
44	A	6	4	1.00	8	0.500
45	A	5	3	1.00	6	0.500
46	A	6	5	1.00	10	0.500
47	A	3	3	1.00	10	0.300
48	A	5	4	1.00	10	0.400
49	A	6	4	1.00	10	0.400
50	A	8	8	1.00	19	0.421

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\sec^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=62

$$\frac{1}{10}i\sec^{-1}\left(ax^5\right)^2-\frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}(ax^5)}\right)+\frac{1}{10}i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(ax^5)}\right)$$

```
[Out] 1/10*I*arcsec(a*x^5)^2-1/5*arcsec(a*x^5)*ln(1+(1/a/x^5+I*(1-1/a^2/x^10)^(1/
2))^2)+1/10*I*polylog(2,-(1/a/x^5+I*(1-1/a^2/x^10)^(1/2))^2)
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5326, 4722, 3800, 2221, 2317, 2438}

$$\frac{1}{10}i\text{Li}_2\left(-e^{2i\sec^{-1}(ax^5)}\right)+\frac{1}{10}i\sec^{-1}\left(ax^5\right)^2-\frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a*x^5]/x,x]
```

```
[Out] (I/10)*ArcSec[a*x^5]^2 - (ArcSec[a*x^5]*Log[1 + E^((2*I)*ArcSec[a*x^5])])/5
 + (I/10)*PolyLog[2, -E^((2*I)*ArcSec[a*x^5])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\sec^{-1}(ax)}{x}\,dx, x, x^5\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, \frac{1}{x^5}\right)\right)\\
&= \frac{1}{5}\text{Subst}\left(\int x\tan(x)\,dx, x, \sec^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{2}{5}i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \sec^{-1}\left(ax^5\right)\right)\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) + \frac{1}{5}\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x\right.\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) - \frac{1}{10}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.\\
&= \frac{1}{10}i\sec^{-1}\left(ax^5\right)^2 - \frac{1}{5}\sec^{-1}\left(ax^5\right)\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right) + \frac{1}{10}i\text{Li}_2\left(-e^{2i\sec^{-1}\left(ax^5\right)}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.90

$$\frac{1}{10}i\left(\sec^{-1}\left(ax^5\right)\left(\sec^{-1}\left(ax^5\right)+2i\log\left(1+e^{2i\sec^{-1}\left(ax^5\right)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\sec^{-1}\left(ax^5\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a*x^5]/x,x]
```

```
[Out] (I/10)*(ArcSec[a*x^5]*(ArcSec[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcSec[a*x^5]
)]) + PolyLog[2, -E^((2*I)*ArcSec[a*x^5])])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a*x^5)/x,x)
```

```
[Out] int(arcsec(a*x^5)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -5*a^2*integrate(sqrt(a*x^5 + 1)*sqrt(a*x^5 - 1)*log(x)/(a^4*x^11 - a^2*x),
 x) - 5*I*a^2*integrate(log(x)/(a^4*x^11 - a^2*x), x) + arctan(sqrt(a*x^5 +
 1)*sqrt(a*x^5 - 1))*log(x) - 1/2*I*log(a^2*x^10)*log(x) + 1/2*I*log(a*x^5
+ 1)*log(x) + 1/2*I*log(-a*x^5 + 1)*log(x) + I*log(a)*log(x) + 5/2*I*log(x)
^2 + 1/10*I*dilog(a*x^5) + 1/10*I*dilog(-a*x^5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(ax^{5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a*x**5)/x,x)
```

```
[Out] Integral(asec(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{1}{a\,x^5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a*x^5))/x,x)
```

```
[Out] int(acos(1/(a*x^5))/x, x)
```

3.2 $\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=58

$$-\frac{1}{4}\sqrt{-1+x}-\frac{1}{4}(-1+x)^{3/2}-\frac{3}{20}(-1+x)^{5/2}-\frac{1}{28}(-1+x)^{7/2}+\frac{1}{4}x^4\sec^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/4*(-1+x)^(3/2)-3/20*(-1+x)^(5/2)-1/28*(-1+x)^(7/2)+1/4*x^4*arcsec(x^(1/2
))-1/4*(-1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5378, 12, 45}

$$\frac{1}{4}x^4\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{28}(x-1)^{7/2}-\frac{3}{20}(x-1)^{5/2}-\frac{1}{4}(x-1)^{3/2}-\frac{\sqrt{x-1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[Sqrt[x]],x]
```

```
[Out] -1/4*Sqrt[-1 + x] - (-1 + x)^(3/2)/4 - (3*(-1 + x)^(5/2))/20 - (-1 + x)^(7/
2)/28 + (x^4*ArcSec[Sqrt[x]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x^3 \sec^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{x^3}{2\sqrt{-1+x}}\,dx \\ &= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \frac{x^3}{\sqrt{-1+x}}\,dx \\ &= \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right) - \frac{1}{8}\int \left(\frac{1}{\sqrt{-1+x}} + 3\sqrt{-1+x} + 3(-1+x)^{3/2} + (-1+x)^{5/2}\right) \\ &= -\frac{1}{4}\sqrt{-1+x} - \frac{1}{4}(-1+x)^{3/2} - \frac{3}{20}(-1+x)^{5/2} - \frac{1}{28}(-1+x)^{7/2} + \frac{1}{4}x^4 \sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 40, normalized size = 0.69

$$-\frac{1}{140}\sqrt{-1+x}\left(16+8x+6x^2+5x^3\right)+\frac{1}{4}x^4\sec^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSec[Sqrt[x]],x]
[Out] -1/140*(Sqrt[-1 + x]*(16 + 8*x + 6*x^2 + 5*x^3)) + (x^4*ArcSec[Sqrt[x]])/4
```

Maple [**A**]
time = 0.10, size = 43, normalized size = 0.74

method	result	size
derivativedivides	$\frac{x^4 \operatorname{arcsec}\left(\sqrt{x}\right)}{4}-\frac{(x-1)\left(5x^3+6x^2+8x+16\right)}{140\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	43
default	$\frac{x^4 \operatorname{arcsec}\left(\sqrt{x}\right)}{4}-\frac{(x-1)\left(5x^3+6x^2+8x+16\right)}{140\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arcsec(x^(1/2))-1/140*(x-1)*(5*x^3+6*x^2+8*x+16)/((x-1)/x)^(1/2)/x^
(1/2)
```

Maxima [**A**]
time = 0.27, size = 66, normalized size = 1.14

$$-\frac{1}{28}x^{\frac{7}{2}}\left(-\frac{1}{x}+1\right)^{\frac{7}{2}}-\frac{3}{20}x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+\frac{1}{4}x^4\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{4}x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/28*x^(7/2)*(-1/x + 1)^(7/2) - 3/20*x^(5/2)*(-1/x + 1)^(5/2) + 1/4*x^4*ar
csec(sqrt(x)) - 1/4*x^(3/2)*(-1/x + 1)^(3/2) - 1/4*sqrt(x)*sqrt(-1/x + 1)
```

Fricas [A]
time = 1.30, size = 32, normalized size = 0.55

$$\frac{1}{4}\,x^4\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{140}\left(5\,x^3+6\,x^2+8\,x+16\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*arcsec(sqrt(x)) - 1/140*(5*x^3 + 6*x^2 + 8*x + 16)*sqrt(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 63.07, size = 119, normalized size = 2.05

$$\frac{x^4\operatorname{asec}\left(\sqrt{x}\right)}{4}-\frac{\begin{cases}\frac{2x^3\sqrt{x-1}}{7}+\frac{12x^2\sqrt{x-1}}{35}+\frac{16x\sqrt{x-1}}{35}+\frac{32\sqrt{x-1}}{35} & \text{for}\ |x|>1\\ \frac{2ix^3\sqrt{1-x}}{7}+\frac{12ix^2\sqrt{1-x}}{35}+\frac{16ix\sqrt{1-x}}{35}+\frac{32i\sqrt{1-x}}{35} & \text{otherwise}\end{cases}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(x**(1/2)),x)
```

```
[Out] x**4*asec(sqrt(x))/4 - Piecewise((2*x**3*sqrt(x - 1)/7 + 12*x**2*sqrt(x - 1
)/35 + 16*x*sqrt(x - 1)/35 + 32*sqrt(x - 1)/35, Abs(x) > 1), (2*I*x**3*sqrt
(1 - x)/7 + 12*I*x**2*sqrt(1 - x)/35 + 16*I*x*sqrt(1 - x)/35 + 32*I*sqrt(1
- x)/35, True))/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(38) = 76.
time = 0.45, size = 152, normalized size = 2.62

$$-\frac{1}{3584}\,x^{\frac{7}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^7-\frac{7}{2560}\,x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5+\frac{1}{4}\,x^4\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{7}{512}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3-\frac{35}{512}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)+\frac{1225\,x^3\left(\sqrt{-\frac{1}{x}+1}-1\right)^6+245\,x^2\left(\sqrt{-\frac{1}{x}+1}-1\right)^4+49\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+5}{17920\,x^{\frac{7}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3584*x^(7/2)*(sqrt(-1/x + 1) - 1)^7 - 7/2560*x^(5/2)*(sqrt(-1/x + 1) - 1
)^5 + 1/4*x^4*arccos(1/sqrt(x)) - 7/512*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 - 35
/512*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/17920*(1225*x^3*(sqrt(-1/x + 1) - 1)^
```

```
6 + 245*x^2*(sqrt(-1/x + 1) - 1)^4 + 49*x*(sqrt(-1/x + 1) - 1)^2 + 5)/(x^(7
/2)*(sqrt(-1/x + 1) - 1)^7)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/x^(1/2)),x)
[Out] int(x^3*acos(1/x^(1/2)), x)
```

3.3 $\int x^2 \sec^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=47

$$-\frac{1}{3}\sqrt{-1+x}-\frac{2}{9}(-1+x)^{3/2}-\frac{1}{15}(-1+x)^{5/2}+\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)$$

```
[Out] -2/9*(-1+x)^(3/2)-1/15*(-1+x)^(5/2)+1/3*x^3*arcsec(x^(1/2))-1/3*(-1+x)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5378, 12, 45}

$$\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{15}(x-1)^{5/2}-\frac{2}{9}(x-1)^{3/2}-\frac{\sqrt{x-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[Sqrt[x]],x]
```

```
[Out] -1/3*Sqrt[-1 + x] - (2*(-1 + x)^(3/2))/9 - (-1 + x)^(5/2)/15 + (x^3*ArcSec[
Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sec^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x^2}{2\sqrt{-1+x}}\,dx\\ &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{x^2}{\sqrt{-1+x}}\,dx\\ &= \frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\\ &= -\frac{1}{3}\sqrt{-1+x}-\frac{2}{9}(-1+x)^{3/2}-\frac{1}{15}(-1+x)^{5/2}+\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.74

$$-\frac{1}{45}\sqrt{-1+x}\left(8+4x+3x^2\right)+\frac{1}{3}x^3\sec^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[Sqrt[x]],x]
```

```
[Out] -1/45*(Sqrt[-1 + x]*(8 + 4*x + 3*x^2)) + (x^3*ArcSec[Sqrt[x]])/3
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$\frac{x^3\operatorname{arcsec}\left(\sqrt{x}\right)}{3}-\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38
default	$\frac{x^3\operatorname{arcsec}\left(\sqrt{x}\right)}{3}-\frac{(x-1)\left(3x^2+4x+8\right)}{45\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arcsec(x^(1/2))-1/45*(x-1)*(3*x^2+4*x+8)/((x-1)/x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.27, size = 52, normalized size = 1.11

$$-\frac{1}{15}\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+\frac{1}{3}\,x^3\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{2}{9}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/15*x^(5/2)*(-1/x + 1)^(5/2) + 1/3*x^3*arcsec(sqrt(x)) - 2/9*x^(3/2)*(-1/
x + 1)^(3/2) - 1/3*sqrt(x)*sqrt(-1/x + 1)
```

Fricas [A]
time = 1.69, size = 27, normalized size = 0.57

$$\frac{1}{3}\,x^3\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2+4\,x+8\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsec(sqrt(x)) - 1/45*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.96, size = 90, normalized size = 1.91

$$\frac{x^3\operatorname{asec}\left(\sqrt{x}\right)}{3}-\frac{\begin{cases}\frac{2x^2\sqrt{x-1}}{5}+\frac{8x\sqrt{x-1}}{15}+\frac{16\sqrt{x-1}}{15} & \text{for } |x|>1\\ \frac{2ix^2\sqrt{1-x}}{5}+\frac{8ix\sqrt{1-x}}{15}+\frac{16i\sqrt{1-x}}{15} & \text{otherwise}\end{cases}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(x**(1/2)),x)
```

```
[Out] x**3*asec(sqrt(x))/3 - Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15
 + 16*sqrt(x - 1)/15, Abs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 -
 x)/15 + 16*I*sqrt(1 - x)/15, True))/6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(31) = 62.
time = 0.47, size = 116, normalized size = 2.47

$$-\frac{1}{480}\,x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5-\frac{5}{288}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3+\frac{1}{3}\,x^3\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{5}{48}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)+\frac{150\,x^2\left(\sqrt{-\frac{1}{x}+1}-1\right)^4+25\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+3}{1440\,x^{\frac{5}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] -1/480*x^(5/2)*(sqrt(-1/x + 1) - 1)^5 - 5/288*x^(3/2)*(sqrt(-1/x + 1) - 1)^
3 + 1/3*x^3*arccos(1/sqrt(x)) - 5/48*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/1440*
(150*x^2*(sqrt(-1/x + 1) - 1)^4 + 25*x*(sqrt(-1/x + 1) - 1)^2 + 3)/(x^(5/2)
*(sqrt(-1/x + 1) - 1)^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/x^(1/2)),x)
```

```
[Out] int(x^2*acos(1/x^(1/2)), x)
```

3.4 $\int x \sec^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\sqrt{-1+x}-\frac{1}{6}(-1+x)^{3/2}+\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/6*(-1+x)^(3/2)+1/2*x^2*arcsec(x^(1/2))-1/2*(-1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5378, 12, 45}

$$\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{6}(x-1)^{3/2}-\frac{\sqrt{x-1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[Sqrt[x]],x]
```

```
[Out] -1/2*Sqrt[-1 + x] - (-1 + x)^(3/2)/6 + (x^2*ArcSec[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int x\sec^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x}{2\sqrt{-1+x}}\,dx\\ &= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x}{\sqrt{-1+x}}\,dx\\ &= \frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)dx\\ &= -\frac{1}{2}\sqrt{-1+x}-\frac{1}{6}(-1+x)^{3/2}+\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$-\frac{1}{6}\sqrt{-1+x}\,(2+x)+\frac{1}{2}x^2\sec^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[Sqrt[x]],x]
[Out] -1/6*(Sqrt[-1 + x]*(2 + x)) + (x^2*ArcSec[Sqrt[x]])/2
```

Maple [A]
time = 0.10, size = 31, normalized size = 0.86

method	result	size
derivativedivides	$\frac{x^2\,\mathrm{arcsec}\left(\sqrt{x}\right)}{2}-\frac{(x-1)(x+2)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31
default	$\frac{x^2\,\mathrm{arcsec}\left(\sqrt{x}\right)}{2}-\frac{(x-1)(x+2)}{6\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arcsec(x^(1/2))-1/6*(x-1)*(x+2)/((x-1)/x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.25, size = 38, normalized size = 1.06

$$-\frac{1}{6}\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+\frac{1}{2}\,x^2\,\mathrm{arcsec}\left(\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/6*x^(3/2)*(-1/x + 1)^(3/2) + 1/2*x^2*arcsec(sqrt(x)) - 1/2*sqrt(x)*sqrt(
-1/x + 1)
```

Fricas [**A**]
time = 1.52, size = 20, normalized size = 0.56

$$\frac{1}{2}\,x^2\operatorname{arcsec}\left(\sqrt{x}\right)-\frac{1}{6}\,(x+2)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arcsec(sqrt(x)) - 1/6*(x + 2)*sqrt(x - 1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 6.74, size = 58, normalized size = 1.61

$$\frac{x^2\operatorname{asec}\left(\sqrt{x}\right)}{2}-\frac{\begin{cases}\frac{x\sqrt{x-1}}{3}+\frac{2\sqrt{x-1}}{3} & \text{for}\ |x|>1\\ \frac{ix\sqrt{1-x}}{3}+\frac{2i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(x**(1/2)),x)
```

```
[Out] x**2*asec(sqrt(x))/2 - Piecewise((x*sqrt(x - 1)/3 + 2*sqrt(x - 1)/3, Abs(x)
 > 1), (I*x*sqrt(1 - x)/3 + 2*I*sqrt(1 - x)/3, True))/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(24) = 48.
time = 0.47, size = 80, normalized size = 2.22

$$-\frac{1}{48}\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3+\frac{1}{2}\,x^2\arccos\left(\frac{1}{\sqrt{x}}\right)-\frac{3}{16}\,\sqrt{x}\left(\sqrt{-\frac{1}{x}+1}-1\right)+\frac{9\,x\left(\sqrt{-\frac{1}{x}+1}-1\right)^2+1}{48\,x^{\frac{3}{2}}\left(\sqrt{-\frac{1}{x}+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] -1/48*x^(3/2)*(sqrt(-1/x + 1) - 1)^3 + 1/2*x^2*arccos(1/sqrt(x)) - 3/16*sqr
t(x)*(sqrt(-1/x + 1) - 1) + 1/48*(9*x*(sqrt(-1/x + 1) - 1)^2 + 1)/(x^(3/2)*
(sqrt(-1/x + 1) - 1)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/x^(1/2)),x)
[Out] int(x*acos(1/x^(1/2)), x)
```

3.5 $\int \sec^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=18

$$-\sqrt{-1+x}+x\sec^{-1}\left(\sqrt{x}\right)$$

```
[Out] x*arcsec(x^(1/2))-(-1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5376, 12, 32}

$$x\sec^{-1}\left(\sqrt{x}\right)-\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]],x]
```

```
[Out] -Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5376

```
Int[ArcSec[u_], x_Symbol] :> Simp[x*ArcSec[u], x] - Dist[u/Sqrt[u^2], Int[S
implifyIntegrand[x*(D[u, x]/(u*Sqrt[u^2 - 1])), x], x], x] /; InverseFuncti
onFreeQ[u, x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{-1}\left(\sqrt{x}\right)\,dx &= x\sec^{-1}\left(\sqrt{x}\right)-\int \frac{1}{2\sqrt{-1+x}}\,dx\\ &= x\sec^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{1}{\sqrt{-1+x}}\,dx\\ &= -\sqrt{-1+x}+x\sec^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\sqrt{-1+x}+x\sec^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]],x]
[Out] -Sqrt[-1 + x] + x*ArcSec[Sqrt[x]]
```

Maple [A]
time = 0.10, size = 25, normalized size = 1.39

method	result	size
derivativedivides	$x\,\mathrm{arcsec}\left(\sqrt{x}\right)-\frac{x-1}{\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	25
default	$x\,\mathrm{arcsec}\left(\sqrt{x}\right)-\frac{x-1}{\sqrt{\frac{x-1}{x}}\,\sqrt{x}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arcsec(x^(1/2))-1/((x-1)/x)^(1/2)/x^(1/2)*(x-1)
```

Maxima [A]
time = 0.25, size = 21, normalized size = 1.17

$$x\,\mathrm{arcsec}\left(\sqrt{x}\right)-\sqrt{x}\,\sqrt{-\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="maxima")
[Out] x*arcsec(sqrt(x)) - sqrt(x)*sqrt(-1/x + 1)
```

Fricas [A]
time = 0.99, size = 14, normalized size = 0.78

$$x\,\mathrm{arcsec}\left(\sqrt{x}\right)-\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="fricas")
[Out] x*arcsec(sqrt(x)) - sqrt(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.85, size = 29, normalized size = 1.61

$$x \operatorname{asec}\left(\sqrt{x}\right) - \frac{\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2)),x)
```

```
[Out] x*asec(sqrt(x)) - Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x),
True))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.
time = 0.45, size = 41, normalized size = 2.28

$$x \arccos\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{2}\sqrt{x}\left(\sqrt{-\frac{1}{x}+1} - 1\right) + \frac{1}{2\sqrt{x}\left(\sqrt{-\frac{1}{x}+1} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arccos(1/sqrt(x)) - 1/2*sqrt(x)*(sqrt(-1/x + 1) - 1) + 1/2/(sqrt(x)*(sqrt
(-1/x + 1) - 1))
```

Mupad [B]
time = 1.15, size = 21, normalized size = 1.17

$$x \operatorname{acos}\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x}\,\sqrt{1-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2)),x)
```

```
[Out] x*acos(1/x^(1/2)) - x^(1/2)*(1 - 1/x)^(1/2)
```

3.6 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=56

$$i\sec^{-1}\left(\sqrt{x}\right)^2-2\sec^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)+i\mathrm{PolyLog}\left(2,-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+I
*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5326, 4722, 3800, 2221, 2317, 2438}

$$i\mathrm{Li}_2\left(-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)+i\sec^{-1}\left(\sqrt{x}\right)^2-2\sec^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x,x]
```

```
[Out] I*ArcSec[Sqrt[x]]^2 - 2*ArcSec[Sqrt[x]]*Log[1 + E^((2*I)*ArcSec[Sqrt[x]])]
+ I*PolyLog[2, -E^((2*I)*ArcSec[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\text{Subst}\left(\int \frac{\sec^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= 2\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 4i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\right)\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + 2\text{Subst}\left(\int \log\left(1+e^{2i
\right.\right.\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) - i\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\
&= i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)^2 - 2\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right) + i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.96

$$
i\left(\sec^{-1}\left(\sqrt{x}\right)\left(\sec^{-1}\left(\sqrt{x}\right)+2i\log\left(1+e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\sec^{-1}\left(\sqrt{x}\right)}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x,x]
```

```
[Out] I*(ArcSec[Sqrt[x]]*(ArcSec[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcSec[Sqrt[x]
])]) + PolyLog[2, -E^((2*I)*ArcSec[Sqrt[x]])])
```

Maple [A]
time = 0.30, size = 63, normalized size = 1.12

method	result
derivativedivides	$i\operatorname{arcsec}\left(\sqrt{x}\right)^{2}-2\operatorname{arcsec}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+i\sqrt{1-\frac{1}{x}}\right)^{2}\right)+i\operatorname{polylog}\left(2,-\left(\frac{1}{\sqrt{x}}\right.\right.$
default	$i\operatorname{arcsec}\left(\sqrt{x}\right)^{2}-2\operatorname{arcsec}\left(\sqrt{x}\right)\ln\left(1+\left(\frac{1}{\sqrt{x}}+i\sqrt{1-\frac{1}{x}}\right)^{2}\right)+i\operatorname{polylog}\left(2,-\left(\frac{1}{\sqrt{x}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] I*arcsec(x^(1/2))^2-2*arcsec(x^(1/2))*ln(1+(1/x^(1/2)+I*(1-1/x)^(1/2))^2)+I
*polylog(2,-(1/x^(1/2)+I*(1-1/x)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(sqrt(x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asec}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x,x)
```

```
[Out] Integral(asec(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x,x)
```

```
[Out] int(acos(1/x^(1/2))/x, x)
```

3.7 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\mathrm{ArcTan}\left(\sqrt{-1+x}\right)$$

```
[Out] -arcsec(x^(1/2))/x+1/2*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5378, 12, 44, 65, 209}

$$\frac{1}{2}\mathrm{ArcTan}\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^2,x]
```

```
[Out] Sqrt[-1 + x]/(2*x) - ArcSec[Sqrt[x]]/x + ArcTan[Sqrt[-1 + x]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2\sqrt{-1+x}\,x^2}\,dx\\ &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\ &= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\ &= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= \frac{\sqrt{-1+x}}{2x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\tan^{-1}\left(\sqrt{-1+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.84

$$\frac{\sqrt{-1+x}-2\sec^{-1}\left(\sqrt{x}\right)-x\text{ArcSin}\left(\frac{1}{\sqrt{x}}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^2,x]
[Out] (Sqrt[-1 + x] - 2*ArcSec[Sqrt[x]] - x*ArcSin[1/Sqrt[x]])/(2*x)
```

Maple [A]
time = 0.10, size = 46, normalized size = 1.21

method	result	size

derivativedivides	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x-1}\,\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46
default	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x-1}\,\left(\arctan\left(\frac{1}{\sqrt{x-1}}\right)x-\sqrt{x-1}\right)}{2\sqrt{\frac{x-1}{x}}\,x^{\frac{3}{2}}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsec(x^(1/2))/x-1/2*(x-1)^(1/2)*(arctan(1/(x-1)^(1/2))*x-(x-1)^(1/2))/((
x-1)/x)^(1/2)/x^(3/2)
```

Maxima [A]
time = 0.47, size = 51, normalized size = 1.34

$$-\frac{\sqrt{x}\,\sqrt{-\frac{1}{x}+1}}{2\,\left(x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\,\arctan\left(\sqrt{x}\,\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -1/2*sqrt(x)*sqrt(-1/x + 1)/(x*(1/x - 1) - 1) - arcsec(sqrt(x))/x + 1/2*arc
tan(sqrt(x)*sqrt(-1/x + 1))
```

Fricas [A]
time = 3.88, size = 19, normalized size = 0.50

$$\frac{(x-2)\operatorname{arcsec}\left(\sqrt{x}\right)+\sqrt{x-1}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/2*((x - 2)*arcsec(sqrt(x)) + sqrt(x - 1))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 15.32, size = 75, normalized size = 1.97

$$\frac{\begin{cases} i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)-\frac{i}{\sqrt{x}\,\sqrt{-1+\frac{1}{x}}}+\frac{i}{x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|}>1\\ -\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)+\frac{\sqrt{1-\frac{1}{x}}}{\sqrt{x}} & \text{otherwise}\end{cases}}{2}-\frac{\operatorname{asec}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**2,x)
```

```
[Out] Piecewise((I*acosh(1/sqrt(x)) - I/(sqrt(x)*sqrt(-1 + 1/x)) + I/(x**(3/2)*sq
rt(-1 + 1/x)), 1/Abs(x) > 1), (-asin(1/sqrt(x)) + sqrt(1 - 1/x)/sqrt(x), Tr
ue))/2 - asec(sqrt(x))/x
```

Giac [A]
time = 0.44, size = 30, normalized size = 0.79

$$\frac{\sqrt{-\frac{1}{x}+1}}{2\sqrt{x}}-\frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{x}+\frac{1}{2}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-1/x + 1)/sqrt(x) - arccos(1/sqrt(x))/x + 1/2*arccos(1/sqrt(x))
```

Mupad [B]
time = 0.68, size = 28, normalized size = 0.74

$$\frac{\sqrt{1-\frac{1}{x}}}{2\sqrt{x}}-\frac{\mathrm{acos}\left(\frac{1}{\sqrt{x}}\right)\left(\frac{2}{x}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^2,x)
```

```
[Out] (1 - 1/x)^(1/2)/(2*x^(1/2)) - (acos(1/x^(1/2))*(2/x - 1))/2
```

3.8 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\text{ArcTan}\left(\sqrt{-1+x}\right)$$

```
[Out] -1/2*arcsec(x^(1/2))/x^2+3/16*arctan((-1+x)^(1/2))+1/8*(-1+x)^(1/2)/x^2+3/1
6*(-1+x)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5378, 12, 44, 65, 209}

$$\frac{3}{16}\text{ArcTan}\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{8x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3\sqrt{x-1}}{16x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^3,x]
```

```
[Out] Sqrt[-1 + x]/(8*x^2) + (3*Sqrt[-1 + x])/(16*x) - ArcSec[Sqrt[x]]/(2*x^2) +
(3*ArcTan[Sqrt[-1 + x]])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2\sqrt{-1+x}\,x^3}\,dx\\
&= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{32}\int \frac{1}{\sqrt{-1+x}\,x}\,dx\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= \frac{\sqrt{-1+x}}{8x^2}+\frac{3\sqrt{-1+x}}{16x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{3}{16}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.02

$$\left(\frac{1}{8x^{3/2}}+\frac{3}{16\sqrt{x}}\right)\sqrt{\frac{-1+x}{x}}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{3}{16}\mathrm{ArcSin}\left(\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^3,x]
[Out] (1/(8*x^(3/2)) + 3/(16*Sqrt[x]))*Sqrt[(-1 + x)/x] - ArcSec[Sqrt[x]]/(2*x^2)
 - (3*ArcSin[1/Sqrt[x]])/16
```

Maple [A]
time = 0.10, size = 57, normalized size = 1.06

method	result	size
derivativedivides	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{x-1}\,\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{2}-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57
default	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\sqrt{x-1}\,\left(3\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{2}-3\sqrt{x-1}\,x-2\sqrt{x-1}\right)}{16\sqrt{\frac{x-1}{x}}\,x^{\frac{5}{2}}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsec(x^(1/2))/x^2-1/16*(x-1)^(1/2)*(3*arctan(1/(x-1)^(1/2))*x^2-3*(x
-1)^(1/2)*x-2*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(38) = 76$.
time = 0.47, size = 80, normalized size = 1.48

$$\frac{3\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+5\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}}{16\left(x^{2}\left(\frac{1}{x}-1\right)^{2}-2\,x\left(\frac{1}{x}-1\right)+1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{2\,x^{2}}+\frac{3}{16}\arctan\left(\sqrt{x}\,\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*x^(3/2)*(-1/x + 1)^(3/2) + 5*sqrt(x)*sqrt(-1/x + 1))/(x^2*(1/x - 1)
^2 - 2*x*(1/x - 1) + 1) - 1/2*arcsec(sqrt(x))/x^2 + 3/16*arctan(sqrt(x)*sqr
t(-1/x + 1))
```

Fricas [A]
time = 7.03, size = 29, normalized size = 0.54

$$\frac{\left(3\,x^{2}-8\right)\operatorname{arcsec}\left(\sqrt{x}\right)+\left(3\,x+2\right)\sqrt{x-1}}{16\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*((3*x^2 - 8)*arcsec(sqrt(x)) + (3*x + 2)*sqrt(x - 1))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 48.19, size = 144, normalized size = 2.67

$$\frac{\begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{4} - \frac{\operatorname{asec}\left(\sqrt{x}\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**3,x)
```

```
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x
**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3
*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1
/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))/4 - asec(sqrt(x))/(2*x**2)
```

Giac [A]
time = 0.47, size = 44, normalized size = 0.81

$$\frac{3\sqrt{-\frac{1}{x}+1}}{16\sqrt{x}} + \frac{\sqrt{-\frac{1}{x}+1}}{8x^{\frac{3}{2}}} - \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{2x^2} + \frac{3}{16}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(-1/x + 1)/sqrt(x) + 1/8*sqrt(-1/x + 1)/x^(3/2) - 1/2*arccos(1/sqr
t(x))/x^2 + 3/16*arccos(1/sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^3,x)
```

```
[Out] int(acos(1/x^(1/2))/x^3, x)
```

3.9 $\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4} dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\text{ArcTan}\left(\sqrt{-1+x}\right)$$

```
[Out] -1/3*arcsec(x^(1/2))/x^3+5/48*arctan((-1+x)^(1/2))+1/18*(-1+x)^(1/2)/x^3+5/
72*(-1+x)^(1/2)/x^2+5/48*(-1+x)^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5378, 12, 44, 65, 209}

$$\frac{5}{48}\text{ArcTan}\left(\sqrt{x-1}\right)+\frac{\sqrt{x-1}}{18x^3}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5\sqrt{x-1}}{72x^2}+\frac{5\sqrt{x-1}}{48x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[Sqrt[x]]/x^4,x]
```

```
[Out] Sqrt[-1 + x]/(18*x^3) + (5*Sqrt[-1 + x])/(72*x^2) + (5*Sqrt[-1 + x])/(48*x)
 - ArcSec[Sqrt[x]]/(3*x^3) + (5*ArcTan[Sqrt[-1 + x]])/48
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5378

```
Int[((a_.) + ArcSec[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcSec[u])/(d*(m + 1))), x] - Dist[b*(u/(d*(m +
 1)*Sqrt[u^2])), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(u*Sqrt[u
^2 - 1])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && Inverse
FunctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !Function
OfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{3}\int \frac{1}{2\sqrt{-1+x}\,x^4}\,dx \\
&= -\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{6}\int \frac{1}{\sqrt{-1+x}\,x^4}\,dx \\
&= \frac{\sqrt{-1+x}}{18x^3}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{36}\int \frac{1}{\sqrt{-1+x}\,x^3}\,dx \\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx \\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{96}\int \frac{1}{\sqrt{-1+x}\,x}\,dx \\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x,\right. \\
&= \frac{\sqrt{-1+x}}{18x^3}+\frac{5\sqrt{-1+x}}{72x^2}+\frac{5\sqrt{-1+x}}{48x}-\frac{\sec^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{5}{48}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.66

$$\frac{\sqrt{-1+x}\,(8+10x+15x^2)-48\sec^{-1}\left(\sqrt{x}\right)-15x^3\,\mathrm{ArcSin}\left(\frac{1}{\sqrt{x}}\right)}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[-1 + x]*(8 + 10*x + 15*x^2) - 48*ArcSec[Sqrt[x]] - 15*x^3*ArcSin[1/Sq
rt[x]])/(144*x^3)
```

Maple [A]
time = 0.10, size = 67, normalized size = 0.99

method	result	s
derivativedivides	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{x-1}\,\left(15\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{3}-15\sqrt{x-1}\,x^{2}-10\sqrt{x-1}\,x-8\sqrt{x-1}\right)}{144\sqrt{\frac{x-1}{x}}\,x^{\frac{7}{2}}}$	6
default	$-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{x-1}\,\left(15\arctan\left(\frac{1}{\sqrt{x-1}}\right)x^{3}-15\sqrt{x-1}\,x^{2}-10\sqrt{x-1}\,x-8\sqrt{x-1}\right)}{144\sqrt{\frac{x-1}{x}}\,x^{\frac{7}{2}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsec(x^(1/2))/x^3-1/144*(x-1)^(1/2)*(15*arctan(1/(x-1)^(1/2))*x^3-15
*(x-1)^(1/2)*x^2-10*(x-1)^(1/2)*x-8*(x-1)^(1/2))/((x-1)/x)^(1/2)/x^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(48) = 96.
time = 0.47, size = 106, normalized size = 1.56

$$-\frac{15\,x^{\frac{5}{2}}\left(-\frac{1}{x}+1\right)^{\frac{5}{2}}+40\,x^{\frac{3}{2}}\left(-\frac{1}{x}+1\right)^{\frac{3}{2}}+33\,\sqrt{x}\,\sqrt{-\frac{1}{x}+1}}{144\left(x^{3}\left(\frac{1}{x}-1\right)^{3}-3\,x^{2}\left(\frac{1}{x}-1\right)^{2}+3\,x\left(\frac{1}{x}-1\right)-1\right)}-\frac{\operatorname{arcsec}\left(\sqrt{x}\right)}{3\,x^{3}}+\frac{5}{48}\arctan\left(\sqrt{x}\,\sqrt{-\frac{1}{x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/144*(15*x^(5/2)*(-1/x + 1)^(5/2) + 40*x^(3/2)*(-1/x + 1)^(3/2) + 33*sqrt
(x)*sqrt(-1/x + 1))/(x^3*(1/x - 1)^3 - 3*x^2*(1/x - 1)^2 + 3*x*(1/x - 1) -
1) - 1/3*arcsec(sqrt(x))/x^3 + 5/48*arctan(sqrt(x)*sqrt(-1/x + 1))
```

Fricas [A]
time = 5.12, size = 35, normalized size = 0.51

$$\frac{3\,(5\,x^{3}-16)\operatorname{arcsec}\left(\sqrt{x}\right)+(15\,x^{2}+10\,x+8)\sqrt{x-1}}{144\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/144*(3*(5*x^3 - 16)*arcsec(sqrt(x)) + (15*x^2 + 10*x + 8)*sqrt(x - 1))/x^
3
```

Sympy [C] Result contains complex when optimal does not.
time = 138.71, size = 180, normalized size = 2.65

$$\frac{\begin{cases} \frac{5i\operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{8} - \frac{5i}{8\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{5i}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{12x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{3x^{\frac{7}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for } \frac{1}{|x|} > 1 \\ -\frac{5\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{8} + \frac{5}{8\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{5}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{12x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{3x^{\frac{7}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}}{6} - \frac{\operatorname{asec}\left(\sqrt{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x**(1/2))/x**4,x)
```

```
[Out] Piecewise((5*I*acosh(1/sqrt(x))/8 - 5*I/(8*sqrt(x)*sqrt(-1 + 1/x)) + 5*I/(2
4*x**(3/2)*sqrt(-1 + 1/x)) + I/(12*x**(5/2)*sqrt(-1 + 1/x)) + I/(3*x**(7/2)
*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-5*asin(1/sqrt(x))/8 + 5/(8*sqrt(x)*sqrt(
1 - 1/x)) - 5/(24*x**(3/2)*sqrt(1 - 1/x)) - 1/(12*x**(5/2)*sqrt(1 - 1/x)) -
 1/(3*x**(7/2)*sqrt(1 - 1/x)), True))/6 - asec(sqrt(x))/(3*x**3)
```

Giac [A]
time = 0.53, size = 58, normalized size = 0.85

$$\frac{5\sqrt{-\frac{1}{x}+1}}{48\sqrt{x}} + \frac{5\sqrt{-\frac{1}{x}+1}}{72\,x^{\frac{3}{2}}} + \frac{\sqrt{-\frac{1}{x}+1}}{18\,x^{\frac{5}{2}}} - \frac{\arccos\left(\frac{1}{\sqrt{x}}\right)}{3\,x^3} + \frac{5}{48}\arccos\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 5/48*sqrt(-1/x + 1)/sqrt(x) + 5/72*sqrt(-1/x + 1)/x^(3/2) + 1/18*sqrt(-1/x
+ 1)/x^(5/2) - 1/3*arccos(1/sqrt(x))/x^3 + 5/48*arccos(1/sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{\sqrt{x}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x^(1/2))/x^4,x)
```

```
[Out] int(acos(1/x^(1/2))/x^4, x)
```

3.10 $\int x^2 \sec^{-1}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=56

$$-\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2}+\frac{1}{3}x^3\text{ArcCos}\left(\frac{x}{a}\right)$$

```
[Out] 1/9*a^3*(1-x^2/a^2)^(3/2)+1/3*x^3*arccos(x/a)-1/3*a^3*(1-x^2/a^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5372, 4724, 272, 45}

$$\frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2}-\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{3}x^3\text{ArcCos}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a/x],x]
```

```
[Out] -1/3*(a^3*Sqrt[1 - x^2/a^2]) + (a^3*(1 - x^2/a^2)^(3/2))/9 + (x^3*ArcCos[x/
a])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sec^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \cos^{-1}\left(\frac{x}{a}\right) dx \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x^3}{\sqrt{1-\frac{x^2}{a^2}}} dx}{3a} \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{a^2}}} dx, x, x^2\right)}{6a} \\
&= \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\text{Subst}\left(\int \left(\frac{a^2}{\sqrt{1-\frac{x}{a^2}}} - a^2\sqrt{1-\frac{x}{a^2}}\right) dx, x, x^2\right)}{6a} \\
&= -\frac{1}{3}a^3\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{9}a^3\left(1-\frac{x^2}{a^2}\right)^{3/2} + \frac{1}{3}x^3 \cos^{-1}\left(\frac{x}{a}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.75

$$
-\frac{1}{9}a\left(2a^2+x^2\right)\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{3}x^3 \sec^{-1}\left(\frac{a}{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a/x],x]
[Out] -1/9*(a*(2*a^2 + x^2)*Sqrt[1 - x^2/a^2]) + (x^3*ArcSec[a/x])/3
```

Maple [A]
time = 0.74, size = 66, normalized size = 1.18

method	result	size
derivativedivides	$-a^3\left(-\frac{x^3 \operatorname{arcsec}\left(\frac{a}{x}\right)}{3a^3} + \frac{\left(\frac{a^2}{x^2}-1\right)\left(\frac{2a^2}{x^2}+1\right)x^4}{9\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\, a^4}\right)$	66

default	$-a^{3}\left(-\frac{x^{3}\operatorname{arcsec}\left(\frac{a}{x}\right)}{3a^{3}}+\frac{\left(\frac{a^{2}}{x^{2}}-1\right)\left(\frac{2a^{2}}{x^{2}}+1\right)x^{4}}{9\sqrt{\frac{\left(\frac{a^{2}}{x^{2}}-1\right)x^{2}}{a^{2}}}\, a^{4}}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-1/3*x^3/a^3*arcsec(a/x)+1/9*(a^2/x^2-1)*(2*a^2/x^2+1)/((a^2/x^2-1)*x
^2/a^2)^(1/2)*x^4/a^4)
```

Maxima [A]
time = 0.47, size = 54, normalized size = 0.96

$$\frac{1}{3}\,x^{3}\operatorname{arcsec}\left(\frac{a}{x}\right)-\frac{2\,a^{4}\sqrt{-\frac{x^{2}}{a^{2}}+1}+a^{2}x^{2}\sqrt{-\frac{x^{2}}{a^{2}}+1}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsec(a/x) - 1/9*(2*a^4*sqrt(-x^2/a^2 + 1) + a^2*x^2*sqrt(-x^2/a^2
 + 1))/a
```

Fricas [A]
time = 2.46, size = 39, normalized size = 0.70

$$\frac{1}{3}\,x^{3}\operatorname{arcsec}\left(\frac{a}{x}\right)-\frac{1}{9}\left(2\,a^{2}x+x^{3}\right)\sqrt{\frac{a^{2}-x^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsec(a/x) - 1/9*(2*a^2*x + x^3)*sqrt((a^2 - x^2)/x^2)
```

Sympy [A]
time = 0.14, size = 51, normalized size = 0.91

$$\begin{cases} -\frac{2a^{3}\sqrt{1-\frac{x^{2}}{a^{2}}}}{9}-\frac{ax^{2}\sqrt{1-\frac{x^{2}}{a^{2}}}}{9}+\frac{x^{3}\operatorname{asec}\left(\frac{a}{x}\right)}{3} & \text{for}\ a\neq 0\\ \tilde{\infty}x^{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(a/x),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(1 - x**2/a**2)/9 - a*x**2*sqrt(1 - x**2/a**2)/9 + x
**3*asec(a/x)/3, Ne(a, 0)), (zoo*x**3, True))
```

Giac [A]
time = 0.61, size = 47, normalized size = 0.84

$$\frac{1}{3}\,x^3\arccos\left(\frac{x}{a}\right) - \frac{2}{9}\,a^3\sqrt{-\frac{x^2}{a^2}+1} - \frac{1}{9}\,ax^2\sqrt{-\frac{x^2}{a^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(a/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(x/a) - 2/9*a^3*sqrt(-x^2/a^2 + 1) - 1/9*a*x^2*sqrt(-x^2/a^2
+ 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{x^3\,\mathrm{acos}\left(\frac{x}{a}\right)}{3} - \frac{\sqrt{a^2-x^2}\,\left(2\,a^2+x^2\right)}{9} & \text{if}\ \ 0<a \\ \int x^2\,\mathrm{acos}\left(\frac{x}{a}\right)\,dx & \text{if}\ \ \neg 0<a \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(x/a),x)
```

```
[Out] piecewise(0 < a, (x^3*acos(x/a))/3 - ((a^2 - x^2)^(1/2)*(2*a^2 + x^2))/9, ~
0 < a, int(x^2*acos(x/a), x))
```

3.11 $\int x \sec^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=47

$$-\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{2}x^2\text{ArcCos}\left(\frac{x}{a}\right)+\frac{1}{4}a^2\text{ArcSin}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arccos(x/a)+1/4*a^2*arcsin(x/a)-1/4*a*x*(1-x^2/a^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5372, 4724, 327, 222}

$$\frac{1}{4}a^2\text{ArcSin}\left(\frac{x}{a}\right)-\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}}+\frac{1}{2}x^2\text{ArcCos}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a/x],x]
```

```
[Out] -1/4*(a*x*Sqrt[1 - x^2/a^2]) + (x^2*ArcCos[x/a])/2 + (a^2*ArcSin[x/a])/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int x \sec^{-1}\left(\frac{a}{x}\right) dx = \int x \cos^{-1}\left(\frac{x}{a}\right) dx$$
$$= \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}} dx}{2a}$$
$$= -\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{1}{4}a \int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} dx$$
$$= -\frac{1}{4}ax\sqrt{1-\frac{x^2}{a^2}} + \frac{1}{2}x^2 \cos^{-1}\left(\frac{x}{a}\right) + \frac{1}{4}a^2 \sin^{-1}\left(\frac{x}{a}\right)$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.94

$$\frac{1}{4}\left(-ax\sqrt{1-\frac{x^2}{a^2}} + 2x^2 \sec^{-1}\left(\frac{a}{x}\right) + a^2 \text{ArcSin}\left(\frac{x}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a/x],x]
[Out] (-(a*x*Sqrt[1 - x^2/a^2]) + 2*x^2*ArcSec[a/x] + a^2*ArcSin[x/a])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.
time = 0.72, size = 91, normalized size = 1.94

method	result	size

derivativedivides	$-a^2\left(-\frac{x^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2a^2}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^2}{x^2}-1}}\right)a^2}{x^2}-\sqrt{\frac{a^2}{x^2}-1}\right)x^3}{4\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a^3}\right)$	91
default	$-a^2\left(-\frac{x^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2a^2}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{\arctan\left(\frac{1}{\sqrt{\frac{a^2}{x^2}-1}}\right)a^2}{x^2}-\sqrt{\frac{a^2}{x^2}-1}\right)x^3}{4\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a^3}\right)$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(-1/2*x^2/a^2*arcsec(a/x)-1/4*(a^2/x^2-1)^(1/2)*(arctan(1/(a^2/x^2-1)^
(1/2))/x^2*a^2-(a^2/x^2-1)^(1/2))/((a^2/x^2-1)*x^2/a^2)^(1/2)*x^3/a^3)
```

Maxima [A]
time = 0.47, size = 46, normalized size = 0.98

$$\frac{1}{2}\,x^2\operatorname{arcsec}\left(\frac{a}{x}\right)+\frac{a^3\arcsin\left(\frac{x}{a}\right)-a^2x\sqrt{-\frac{x^2}{a^2}+1}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsec(a/x) + 1/4*(a^3*arcsin(x/a) - a^2*x*sqrt(-x^2/a^2 + 1))/a
```

Fricas [A]
time = 2.59, size = 38, normalized size = 0.81

$$-\frac{1}{4}\,x^{2}\sqrt{\frac{a^{2}-x^{2}}{x^{2}}}-\frac{1}{4}\,\left(a^{2}-2\,x^{2}\right)\operatorname{arcsec}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="fricas")
[Out] -1/4*x^2*sqrt((a^2 - x^2)/x^2) - 1/4*(a^2 - 2*x^2)*arcsec(a/x)
```

Sympy [A]
time = 0.09, size = 41, normalized size = 0.87

$$\begin{cases} -\frac{a^{2}\operatorname{asec}\left(\frac{a}{x}\right)}{4}-\frac{ax\sqrt{1-\frac{x^{2}}{a^{2}}}}{4}+\frac{x^{2}\operatorname{asec}\left(\frac{a}{x}\right)}{2} & \text{for}\ a\neq 0\\ \tilde{\infty}x^{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(a/x),x)
[Out] Piecewise((-a**2*asec(a/x)/4 - a*x*sqrt(1 - x**2/a**2)/4 + x**2*asec(a/x)/2
, Ne(a, 0)), (zoo*x**2, True))
```

Giac [A]
time = 0.44, size = 39, normalized size = 0.83

$$-\frac{1}{4}\,a^{2}\arccos\left(\frac{x}{a}\right)+\frac{1}{2}\,x^{2}\arccos\left(\frac{x}{a}\right)-\frac{1}{4}\,ax\sqrt{-\frac{x^{2}}{a^{2}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(a/x),x, algorithm="giac")
[Out] -1/4*a^2*arccos(x/a) + 1/2*x^2*arccos(x/a) - 1/4*a*x*sqrt(-x^2/a^2 + 1)
```

Mupad [B]
time = 0.63, size = 38, normalized size = 0.81

$$\frac{a^{2}\operatorname{acos}\left(\frac{x}{a}\right)\,\left(\frac{2\,x^{2}}{a^{2}}-1\right)}{4}-\frac{a\,x\,\sqrt{1-\frac{x^{2}}{a^{2}}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(x/a),x)
[Out] (a^2*acos(x/a)*((2*x^2)/a^2 - 1))/4 - (a*x*(1 - x^2/a^2)^(1/2))/4
```

3.12 $\int \sec^{-1}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=26

$$-a\sqrt{1-\frac{x^2}{a^2}}+x\text{ArcCos}\left(\frac{x}{a}\right)$$

[Out] x*arccos(x/a)-a*(1-x^2/a^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5372, 4716, 267}

$$x\text{ArcCos}\left(\frac{x}{a}\right)-a\sqrt{1-\frac{x^2}{a^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcSec[a/x],x]

[Out] -(a*Sqrt[1 - x^2/a^2]) + x*ArcCos[x/a]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int \sec^{-1}\left(\frac{a}{x}\right)\,dx = \int \cos^{-1}\left(\frac{x}{a}\right)\,dx$$
$$= x\cos^{-1}\left(\frac{x}{a}\right) + \frac{\int \frac{x}{\sqrt{1-\frac{x^2}{a^2}}}\,dx}{a}$$
$$= -a\sqrt{1-\frac{x^2}{a^2}} + x\cos^{-1}\left(\frac{x}{a}\right)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-a\sqrt{1-\frac{x^2}{a^2}} + x\sec^{-1}\left(\frac{a}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x],x]
[Out] -(a*Sqrt[1 - x^2/a^2]) + x*ArcSec[a/x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.
time = 0.72, size = 51, normalized size = 1.96

method	result	size
derivativedivides	$-a\left(-\frac{x\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{a}+\frac{x^2\left(\frac{a^2}{x^2}-1\right)}{\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a^2}\right)$	51
default	$-a\left(-\frac{x\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{a}+\frac{x^2\left(\frac{a^2}{x^2}-1\right)}{\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a^2}\right)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x),x,method=_RETURNVERBOSE)
[Out] -a*(-x/a*arcsec(a/x)+1/((a^2/x^2-1)*x^2/a^2)^(1/2)*x^2/a^2*(a^2/x^2-1))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.92

$$x\,\mathrm{arcsec}\left(\frac{a}{x}\right) - a\sqrt{-\frac{x^2}{a^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="maxima")
[Out] x*arcsec(a/x) - a*sqrt(-x^2/a^2 + 1)
```

Fricas [A]
time = 2.28, size = 27, normalized size = 1.04

$$x\,\mathrm{arcsec}\left(\frac{a}{x}\right)-x\sqrt{\frac{a^2-x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="fricas")
[Out] x*arcsec(a/x) - x*sqrt((a^2 - x^2)/x^2)
```

Sympy [A]
time = 0.10, size = 22, normalized size = 0.85

$$\begin{cases}-a\sqrt{1-\frac{x^2}{a^2}}+x\,\mathrm{asec}\left(\frac{a}{x}\right) & \text{for } a\neq 0\\ \tilde{\infty}x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x),x)
[Out] Piecewise((-a*sqrt(1 - x**2/a**2) + x*asec(a/x), Ne(a, 0)), (zoo*x, True))
```

Giac [A]
time = 0.44, size = 28, normalized size = 1.08

$$a\left(\frac{x\arccos\left(\frac{x}{a}\right)}{a}-\sqrt{-\frac{x^2}{a^2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x),x, algorithm="giac")
[Out] a*(x*arccos(x/a)/a - sqrt(-x^2/a^2 + 1))
```

Mupad [B]
time = 0.11, size = 24, normalized size = 0.92

$$x\,\mathrm{acos}\left(\frac{x}{a}\right)-a\,\sqrt{1-\frac{x^2}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a),x)
[Out] x*acos(x/a) - a*(1 - x^2/a^2)^(1/2)
```

3.13 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}i\text{ArcCos}\left(\frac{x}{a}\right)^2+\text{ArcCos}\left(\frac{x}{a}\right)\log\left(1+e^{2i\text{ArcCos}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}\left(\frac{x}{a}\right)}\right)$$

```
[Out] -1/2*I*arccos(x/a)^2+arccos(x/a)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*po
lylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5372, 4722, 3800, 2221, 2317, 2438}

$$-\frac{1}{2}i\text{Li}_2\left(-e^{2i\text{ArcCos}\left(\frac{x}{a}\right)}\right)-\frac{1}{2}i\text{ArcCos}\left(\frac{x}{a}\right)^2+\text{ArcCos}\left(\frac{x}{a}\right)\log\left(1+e^{2i\text{ArcCos}\left(\frac{x}{a}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x,x]
```

```
[Out] (-1/2*I)*ArcCos[x/a]^2 + ArcCos[x/a]*Log[1 + E^((2*I)*ArcCos[x/a])] - (I/2)
*PolyLog[2, -E^((2*I)*ArcCos[x/a])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x}\,dx &= \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx\\
&= -\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{x}{a}\right)\right)\\
&= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + 2i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{x}{a}\right)\right)\\
&= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) - \text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}\right.\\
&= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-}}\right.\\
&= -\frac{1}{2}i\cos^{-1}\left(\frac{x}{a}\right)^2 + \cos^{-1}\left(\frac{x}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right) - \frac{1}{2}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{x}{a}\right)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 1.00

$$-\frac{1}{2}i\sec^{-1}\left(\frac{a}{x}\right)^2 + \sec^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\sec^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i\text{PolyLog}\left(2, -e^{2i\sec^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x,x]
```

```
[Out] (-1/2*I)*ArcSec[a/x]^2 + ArcSec[a/x]*Log[1 + E^((2*I)*ArcSec[a/x])] - (I/2)
*PolyLog[2, -E^((2*I)*ArcSec[a/x])]
```

Maple [A]
time = 0.93, size = 76, normalized size = 1.29

method	result
derivativedivides	$-\frac{i\operatorname{arcsec}\left(\frac{a}{x}\right)^{2}}{2}+\operatorname{arcsec}\left(\frac{a}{x}\right)\ln\left(1+\left(\frac{x}{a}+i\sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)-\frac{i\operatorname{polylog}\left(2,-\left(\frac{x}{a}+i\sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)}{2}$
default	$-\frac{i\operatorname{arcsec}\left(\frac{a}{x}\right)^{2}}{2}+\operatorname{arcsec}\left(\frac{a}{x}\right)\ln\left(1+\left(\frac{x}{a}+i\sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)-\frac{i\operatorname{polylog}\left(2,-\left(\frac{x}{a}+i\sqrt{1-\frac{x^{2}}{a^{2}}}\right)^{2}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*I*arcsec(a/x)^2+arcsec(a/x)*ln(1+(x/a+I*(1-x^2/a^2)^(1/2))^2)-1/2*I*po
lylog(2,-(x/a+I*(1-x^2/a^2)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="maxima")
[Out] integrate(arcsec(a/x)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="fricas")
[Out] integral(arcsec(a/x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x,x)
```

```
[Out] Integral(asec(a/x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(a/x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{x}{a}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x,x)
```

```
[Out] int(acos(x/a)/x, x)
```

3.14 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2} dx$

Optimal. Leaf size=31

$$-\frac{\text{ArcCos}\left(\frac{x}{a}\right)}{x}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}$$

```
[Out] -arccos(x/a)/x+arctanh((1-x^2/a^2)^(1/2))/a
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5372, 4724, 272, 65, 214}

$$\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}-\frac{\text{ArcCos}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^2,x]
```

```
[Out] -(ArcCos[x/a]/x) + ArcTanh[Sqrt[1 - x^2/a^2]]/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx &= \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} - \frac{\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx}{a} \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{2a} \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} + a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right) \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{x} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(31) = 62.
time = 0.09, size = 93, normalized size = 3.00

$$-\frac{\sec^{-1}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{-1+\frac{a^2}{x^2}}\,x\left(-\log\left(1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right)+\log\left(1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\,x}\right)\right)}{2a^2\sqrt{1-\frac{x^2}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^2,x]
```

```
[Out] -(ArcSec[a/x]/x) + (Sqrt[-1 + a^2/x^2]*x*(-Log[1 - a/(Sqrt[-1 + a^2/x^2]*x)
] + Log[1 + a/(Sqrt[-1 + a^2/x^2]*x)]))/(2*a^2*Sqrt[1 - x^2/a^2])
```

Maple [A]
time = 0.03, size = 44, normalized size = 1.42

method	result	size
derivativedivides	$-\frac{\frac{a\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{x}-\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^2}{a^2}}}{x}\right)}{a}$	44
default	$-\frac{\frac{a\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{x}-\ln\left(\frac{a}{x}+\frac{a\sqrt{1-\frac{x^2}{a^2}}}{x}\right)}{a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(a/x*arcsec(a/x)-ln(a/x+a/x*(1-x^2/a^2)^(1/2)))
```

Maxima [A]
time = 0.25, size = 52, normalized size = 1.68

$$-\frac{\frac{2\,a\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{x}-\log\left(\sqrt{-\frac{x^2}{a^2}+1}+1\right)+\log\left(-\sqrt{-\frac{x^2}{a^2}+1}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*arcsec(a/x)/x - log(sqrt(-x^2/a^2 + 1) + 1) + log(-sqrt(-x^2/a^2
+ 1) + 1))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(29) = 58.
time = 1.61, size = 107, normalized size = 3.45

$$-\frac{2\,ax\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right)-2\,(ax-a)\,\mathrm{arcsec}\left(\frac{a}{x}\right)-x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right)+x\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - 2*(a*x - a)*ar
csec(a/x) - x*log(x*sqrt((a^2 - x^2)/x^2) + a) + x*log(x*sqrt((a^2 - x^2)/x
^2) - a))/(a*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.47, size = 29, normalized size = 0.94

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{x}-\frac{\begin{cases}-\operatorname{acosh}\left(\frac{a}{x}\right) & \text{for}\ \left|\frac{a^2}{x^2}\right|>1\\ i\operatorname{asin}\left(\frac{a}{x}\right) & \text{otherwise}\end{cases}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**2,x)
```

```
[Out] -asec(a/x)/x - Piecewise((-acosh(a/x), Abs(a**2/x**2) > 1), (I*asin(a/x), T
rue))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(29) = 58.
time = 0.46, size = 61, normalized size = 1.97

$$\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a}\right)}{2\left|a\right|}-\frac{\arccos\left(\frac{x}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*(log(abs(a + sqrt(a^2 - x^2)))/a - log(abs(-a + sqrt(a^2 - x^2)))/a)/
abs(a) - arccos(x/a)/x
```

Mupad [B]
time = 0.60, size = 29, normalized size = 0.94

$$\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)}{a}-\frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^2,x)
```

```
[Out] atanh(1/(1 - x^2/a^2)^(1/2))/a - acos(x/a)/x
```

3.15 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\operatorname{ArcCos}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/2*arccos(x/a)/x^2+1/2*(1-x^2/a^2)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5372, 4724, 270}

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax}-\frac{\operatorname{ArcCos}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^3,x]
```

```
[Out] Sqrt[1 - x^2/a^2]/(2*a*x) - ArcCos[x/a]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx = \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx$$
$$= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx}{2a}$$
$$= \frac{\sqrt{1-\frac{x^2}{a^2}}}{2ax} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.95

$$\frac{x\sqrt{1-\frac{x^2}{a^2}} - a\sec^{-1}\left(\frac{a}{x}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^3,x]
[Out] (x*Sqrt[1 - x^2/a^2] - a*ArcSec[a/x])/(2*a*x^2)
```

Maple [A]
time = 0.72, size = 54, normalized size = 1.42

method	result	size
derivativedivides	$-\frac{\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2}-\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54
default	$-\frac{\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)}{2x^2}-\frac{x\left(\frac{a^2}{x^2}-1\right)}{2\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/a^2*(1/2/x^2*a^2*arcsec(a/x)-1/2/((a^2/x^2-1)*x^2/a^2)^(1/2)*x/a*(a^2/x^
2-1))
```

Maxima [A]
time = 0.46, size = 32, normalized size = 0.84

$$-\frac{\operatorname{arcsec}\left(\frac{a}{x}\right)}{2\,x^2}+\frac{\sqrt{-\frac{x^2}{a^2}+1}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arcsec(a/x)/x^2 + 1/2*sqrt(-x^2/a^2 + 1)/(a*x)
```

Fricas [A]
time = 2.43, size = 39, normalized size = 1.03

$$-\frac{a^2\operatorname{arcsec}\left(\frac{a}{x}\right)-x^2\sqrt{\frac{a^2-x^2}{x^2}}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*arcsec(a/x) - x^2*sqrt((a^2 - x^2)/x^2))/(a^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 53, normalized size = 1.39

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{2x^2}-\frac{\begin{cases}-\frac{\sqrt{\frac{a^2}{x^2}-1}}{a} & \text{for}\ \left|\frac{a^2}{x^2}\right|>1\\ -\frac{i\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{otherwise}\end{cases}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**3,x)
```

```
[Out] -asec(a/x)/(2*x**2) - Piecewise((-sqrt(a**2/x**2 - 1)/a, Abs(a**2/x**2) > 1
), (-I*sqrt(-a**2/x**2 + 1)/a, True))/(2*a)
```

Giac [A]
time = 0.46, size = 61, normalized size = 1.61

$$\frac{a\left(\frac{a+\sqrt{a^2-x^2}}{a^2x}-\frac{x}{\left(a+\sqrt{a^2-x^2}\right)a^2}\right)}{4\,|a|}-\frac{\arccos\left(\frac{x}{a}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*a*((a + sqrt(a^2 - x^2))/(a^2*x) - x/((a + sqrt(a^2 - x^2))*a^2))/abs(a
) - 1/2*arccos(x/a)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^3,x)
```

```
[Out] int(acos(x/a)/x^3, x)
```

3.16 $\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2}-\frac{\text{ArcCos}\left(\frac{x}{a}\right)}{3x^3}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}$$

```
[Out] -1/3*arccos(x/a)/x^3+1/6*arctanh((1-x^2/a^2)^(1/2))/a^3+1/6*(1-x^2/a^2)^(1/
2)/a/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5372, 4724, 272, 44, 65, 214}

$$\frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}-\frac{\text{ArcCos}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a/x]/x^4,x]
```

```
[Out] Sqrt[1 - x^2/a^2]/(6*a*x^2) - ArcCos[x/a]/(3*x^3) + ArcTanh[Sqrt[1 - x^2/a^
2]]/(6*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5372

```
Int[ArcSec[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcCos[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx &= \int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\int \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx}{3a} \\
&= -\frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{6a} \\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, x^2\right)}{12a^3} \\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{x^2}{a^2}}\right)}{6a} \\
&= \frac{\sqrt{1-\frac{x^2}{a^2}}}{6ax^2} - \frac{\cos^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 1.15

$$\frac{a^2x\sqrt{1-\frac{x^2}{a^2}}-2a^3\sec^{-1}\left(\frac{a}{x}\right)-x^3\log(x)+x^3\log\left(1+\sqrt{1-\frac{x^2}{a^2}}\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a/x]/x^4,x]
```

```
[Out] (a^2*x*Sqrt[1 - x^2/a^2] - 2*a^3*ArcSec[a/x] - x^3*Log[x] + x^3*Log[1 + Sqr
t[1 - x^2/a^2]])/(6*a^3*x^3)
```

Maple [A]
time = 0.73, size = 91, normalized size = 1.52

method	result	size
derivativedivides	$-\frac{\frac{a^3\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x}+\ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91
default	$-\frac{\frac{a^3\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\sqrt{\frac{a^2}{x^2}-1}\left(\frac{a\sqrt{\frac{a^2}{x^2}-1}}{x}+\ln\left(\frac{a}{x}+\sqrt{\frac{a^2}{x^2}-1}\right)\right)x}{6\sqrt{\frac{\left(\frac{a^2}{x^2}-1\right)x^2}{a^2}}\,a}}{a^3}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(1/3/x^3*a^3*arcsec(a/x)-1/6*(a^2/x^2-1)^(1/2)*(1/x*a*(a^2/x^2-1)^(1
/2)+ln(a/x+(a^2/x^2-1)^(1/2)))/((a^2/x^2-1)*x^2/a^2)^(1/2)*x/a)
```

Maxima [A]
time = 0.47, size = 64, normalized size = 1.07

$$\frac{\frac{\log\left(\frac{2\sqrt{-\frac{x^2}{a^2}+1}}{|x|}+\frac{2}{|x|}\right)}{a^2}+\frac{\sqrt{-\frac{x^2}{a^2}+1}}{x^2}}{6\,a}-\frac{\mathrm{arcsec}\left(\frac{a}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(log(2*sqrt(-x^2/a^2 + 1)/abs(x) + 2/abs(x))/a^2 + sqrt(-x^2/a^2 + 1)/x
^2)/a - 1/3*arcsec(a/x)/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(50) = 100$.
time = 3.44, size = 142, normalized size = 2.37

$$-\frac{4\,a^3x^3\arctan\left(-\frac{x^2\sqrt{\frac{a^2-x^2}{x^2}}}{a^2-x^2}\right)-x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}+a\right)+x^3\log\left(x\sqrt{\frac{a^2-x^2}{x^2}}-a\right)-2\,ax^2\sqrt{\frac{a^2-x^2}{x^2}}-4\,(a^3x^3-a^3)\,\mathrm{arcsec}\left(\frac{a}{x}\right)}{12\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^3*x^3*arctan(-x^2*sqrt((a^2 - x^2)/x^2)/(a^2 - x^2)) - x^3*log(x
*sqrt((a^2 - x^2)/x^2) + a) + x^3*log(x*sqrt((a^2 - x^2)/x^2) - a) - 2*a*x^
2*sqrt((a^2 - x^2)/x^2) - 4*(a^3*x^3 - a^3)*arcsec(a/x))/(a^3*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 100, normalized size = 1.67

$$-\frac{\operatorname{asec}\left(\frac{a}{x}\right)}{3x^3}-\frac{\begin{cases}-\frac{\sqrt{\frac{a^2}{x^2}-1}}{2ax}-\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^2} & \text{for}\ \left|\frac{a^2}{x^2}\right|>1\\ \frac{ia}{2x^3\sqrt{-\frac{a^2}{x^2}+1}}-\frac{i}{2ax\sqrt{-\frac{a^2}{x^2}+1}}+\frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{2a^2} & \text{otherwise}\end{cases}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a/x)/x**4,x)
```

```
[Out] -asec(a/x)/(3*x**3) - Piecewise((-sqrt(a**2/x**2 - 1)/(2*a*x) - acosh(a/x)/
(2*a**2), Abs(a**2/x**2) > 1), (I*a/(2*x**3*sqrt(-a**2/x**2 + 1)) - I/(2*a*
x*sqrt(-a**2/x**2 + 1)) + I*asin(a/x)/(2*a**2), True))/(3*a)
```

Giac [A]
time = 0.44, size = 80, normalized size = 1.33

$$\frac{a\left(\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{a^3}-\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{a^3}+\frac{2\sqrt{a^2-x^2}}{a^2x^2}\right)}{12\,|a|}-\frac{\arccos\left(\frac{x}{a}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*a*(log(abs(a + sqrt(a^2 - x^2)))/a^3 - log(abs(-a + sqrt(a^2 - x^2)))/
a^3 + 2*sqrt(a^2 - x^2)/(a^2*x^2))/abs(a) - 1/3*arccos(x/a)/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\frac{x}{a}\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x/a)/x^4,x)
```

```
[Out] int(acos(x/a)/x^4, x)
```

3.17 $\int \frac{\sec^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=69

$$\frac{i\sec^{-1}(ax^n)^2}{2n} - \frac{\sec^{-1}(ax^n)\log\left(1+e^{2i\sec^{-1}(ax^n)}\right)}{n} + \frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(ax^n)}\right)}{2n}$$

```
[Out] 1/2*I*arcsec(a*x^n)^2/n-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(
1/2))^2)/n+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2)/n
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5326, 4722, 3800, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}(ax^n)}\right)}{2n} + \frac{i\sec^{-1}(ax^n)^2}{2n} - \frac{\sec^{-1}(ax^n)\log\left(1+e^{2i\sec^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a*x^n]/x,x]
```

```
[Out] ((I/2)*ArcSec[a*x^n]^2)/n - (ArcSec[a*x^n]*Log[1 + E^((2*I)*ArcSec[a*x^n])]
)/n + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[a*x^n])])/n
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}(ax^n)}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sec^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{a}\right)}{x}\,dx, x, x^{-n}\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}\\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{(2i)\operatorname{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{x^{-n}}{a}\right)\right)}{n}\\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{\operatorname{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x,\right.}{n}\\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} - \frac{i\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos}\right.}{2n}\\
&= \frac{i\cos^{-1}\left(\frac{x^{-n}}{a}\right)^2}{2n} - \frac{\cos^{-1}\left(\frac{x^{-n}}{a}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{n} + \frac{i\operatorname{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{x^{-n}}{a}\right)}\right)}{2n}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.06, size = 60, normalized size = 0.87

$$\frac{x^{-n}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{x^{-2n}}{a^2}\right)}{an}+\left(\sec^{-1}(ax^n)+\text{ArcSin}\left(\frac{x^{-n}}{a}\right)\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a*x^n]/x,x]
```

```
[Out] HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, 1/(a^2*x^(2*n))]/(a*n*x^n) +
 (ArcSec[a*x^n] + ArcSin[1/(a*x^n)])*Log[x]
```

Maple [**A**]
time = 0.48, size = 93, normalized size = 1.35

method	result	size
derivativedivides	$\frac{\frac{i\,\mathrm{arcsec}(a\,x^n)^2}{2}-\mathrm{arcsec}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)}{2}}{n}$	93
default	$\frac{\frac{i\,\mathrm{arcsec}(a\,x^n)^2}{2}-\mathrm{arcsec}(a\,x^n)\ln\left(1+\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{x^{-n}}{a}+i\sqrt{1-\frac{x^{-2n}}{a^2}}\right)^2\right)}{2}}{n}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/2*I*arcsec(a*x^n)^2-arcsec(a*x^n)*ln(1+(1/a/(x^n)+I*(1-1/a^2/(x^n)^2
)^(1/2))^2)+1/2*I*polylog(2,-(1/a/(x^n)+I*(1-1/a^2/(x^n)^2)^(1/2))^2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a^2*n*integrate(sqrt(a*x^n + 1)*sqrt(a*x^n - 1)*log(x)/(a^4*x*x^(2*n) - a^
2*x), x) + arctan(sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(a*x**n)/x,x)
```

```
[Out] Integral(asec(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{a\,x^n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a*x^n))/x,x)
```

```
[Out] int(acos(1/(a*x^n))/x, x)
```

3.18 $\int x^4 \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=197

$$\frac{a(20+53a^2)\,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5}+\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3}-\frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2}-\frac{(9\ \cdots}{}$$

```
[Out] 1/5*a^5*arcsec(b*x+a)/b^5+1/5*x^5*arcsec(b*x+a)-1/40*(40*a^4+40*a^2+3)*arct
anh((1-1/(b*x+a)^2)^(1/2))/b^5+1/30*a*(53*a^2+20)*(b*x+a)*(1-1/(b*x+a)^2)^(
1/2)/b^5+11/60*a*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/20*x^3*(b*x+a)*(1-
1/(b*x+a)^2)^(1/2)/b^2-1/120*(58*a^2+9)*(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^5
```

Rubi [A]
time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5366, 4511, 3867, 4141, 4133, 3855, 3852, 8}

$$\frac{a^5\sec^{-1}(a+bx)}{5b^5}+\frac{(53a^2+20)\,a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5}-\frac{(58a^2+9)\,(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{120b^5}-\frac{(40a^4+40a^2+3)\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{40b^5}+\frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3}-\frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2}+\frac{1}{5}x^5\sec^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSec[a + b*x],x]
```

```
[Out] (a*(20 + 53*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(30*b^5) + (11*a*x^2*(
a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(60*b^3) - (x^3*(a + b*x)*Sqrt[1 - (a +
b*x)^(-2)])/(20*b^2) - ((9 + 58*a^2)*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)])/
(120*b^5) + (a^5*ArcSec[a + b*x])/(5*b^5) + (x^5*ArcSec[a + b*x])/5 - ((3 +
 40*a^2 + 40*a^4)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(40*b^5)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^4\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^5}$$

$$= \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^5\,dx, x, \sec^{-1}(a+bx)\right)}{5b^5}$$

$$= -\frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^2\,(-4a\right.}{}$$

$$= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} + \frac{1}{5}x^5\sec^{-1}(a+bx$$

$$= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{(9+58a^2)(a+b}{1}$$

$$= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{(9+58a^2)(a+b}{1}$$

$$= \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{20b^2} - \frac{(9+58a^2)(a+b}{1}$$

$$= \frac{a(20+53a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{30b^5} + \frac{11ax^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{60b^3} - \frac{x^3(a+}{}$$

Mathematica [A]
time = 0.12, size = 173, normalized size = 0.88

$$\frac{\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(a^2(71+154a^2)+2a(31+48a^2)\,bx-9(1+4a^2)\,b^2x^2+16ab^3x^3-6b^4x^4\right)+24b^5x^5\sec^{-1}(a+bx)-24a^5\,\text{ArcSin}\left(\frac{1}{a+bx}\right)-3(3+40a^2+40a^4)\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)\right)}{120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcSec[a + b*x],x]
```

```
[Out] (Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^2*(71 + 154*a^2) + 2*a
*(31 + 48*a^2)*b*x - 9*(1 + 4*a^2)*b^2*x^2 + 16*a*b^3*x^3 - 6*b^4*x^4) + 24
*b^5*x^5*ArcSec[a + b*x] - 24*a^5*ArcSin[(a + b*x)^(-1)] - 3*(3 + 40*a^2 +
40*a^4)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]
)])/(120*b^5)
```

Maple [A]
time = 0.14, size = 329, normalized size = 1.67

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^5}{5}+\operatorname{arcsec}(bx+a)a^4(bx+a)-2\operatorname{arcsec}(bx+a)a^3(bx+a)^2+2\operatorname{arcsec}(bx+a)a^2(bx+a)^3-\operatorname{arcsec}(bx+a)a(bx+a}{}$
default	$\frac{-\frac{\operatorname{arcsec}(bx+a)a^5}{5}+\operatorname{arcsec}(bx+a)a^4(bx+a)-2\operatorname{arcsec}(bx+a)a^3(bx+a)^2+2\operatorname{arcsec}(bx+a)a^2(bx+a)^3-\operatorname{arcsec}(bx+a)a(bx+a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(-1/5*arcsec(b*x+a)*a^5+arcsec(b*x+a)*a^4*(b*x+a)-2*arcsec(b*x+a)*a^3
*(b*x+a)^2+2*arcsec(b*x+a)*a^2*(b*x+a)^3-arcsec(b*x+a)*a*(b*x+a)^4+1/5*arcs
ec(b*x+a)*(b*x+a)^5-1/120*((b*x+a)^2-1)^(1/2)*(24*a^5*arctan(1/((b*x+a)^2-1
)^(1/2))+120*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))-240*a^3*((b*x+a)^2-1)^(1/2)+
120*a^2*(b*x+a)*((b*x+a)^2-1)^(1/2)-40*a*(b*x+a)^2*((b*x+a)^2-1)^(1/2)+6*(b
*x+a)^3*((b*x+a)^2-1)^(1/2)+120*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))-80*a*((b*
x+a)^2-1)^(1/2)+9*(b*x+a)*((b*x+a)^2-1)^(1/2)+9*ln(b*x+a+((b*x+a)^2-1)^(1/2
)))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/5*(b^2*x^
6 + a*b*x^5)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

Fricas [A]
time = 2.60, size = 152, normalized size = 0.77

$$\frac{24\,b^5x^5\operatorname{arcsec}\left(bx+a\right)+48\,a^5\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+3\left(40\,a^4+40\,a^2+3\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(6\,b^3x^3-22\,ab^2x^2-154\,a^3+\left(58\,a^2+9\right)bx-71\,a\right)\sqrt{b^2x^2+2\,abx+a^2-1}}{120\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5*arcsec(b*x + a) + 48*a^5*arctan(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)) + 3*(40*a^4 + 40*a^2 + 3)*log(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1)) - (6*b^3*x^3 - 22*a*b^2*x^2 - 154*a^3 + (58*a^2 + 9)*
b*x - 71*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{asec}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asec(b*x+a),x)
```

```
[Out] Integral(x**4*asec(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(173) = 346.
time = 0.47, size = 409, normalized size = 2.08

$$-\frac{1}{960}\, b \left(\frac{192\,(bx+a)^5\left(\frac{5a}{bx+a} - \frac{10a^2}{(bx+a)^2} + \frac{10a^3}{(bx+a)^3} - \frac{5a^4}{(bx+a)^4} - 1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^6} - \frac{3\,(bx+a)^4\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^4 + 40\,(bx+a)^3 a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3 + 240\,(bx+a)^2 a^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2 + 960\,(bx+a)a^3\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right) + 24\,(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2 + 360\,(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right) + 24\,(40a^4+40a^2+3)\log\left(-\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)|bx+a|\right) - \frac{120\,(8a^3+3a)(bx+a)^3\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3 + 24\,(10a^2+1)(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2 + 40\,(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+3}{(bx+a)^4\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^4}}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/960*b*(192*(b*x + a)^5*(5*a/(b*x + a) - 10*a^2/(b*x + a)^2 + 10*a^3/(b*x
 + a)^3 - 5*a^4/(b*x + a)^4 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a
))/b^6 - (3*(b*x + a)^4*(sqrt(-1/(b*x + a)^2 + 1) - 1)^4 + 40*(b*x + a)^3*a
*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 240*(b*x + a)^2*a^2*(sqrt(-1/(b*x + a)^
2 + 1) - 1)^2 + 960*(b*x + a)*a^3*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 24*(b*x
+ a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 360*(b*x + a)*a*(sqrt(-1/(b*x + a
)^2 + 1) - 1) + 24*(40*a^4 + 40*a^2 + 3)*log(-(sqrt(-1/(b*x + a)^2 + 1) - 1
)*abs(b*x + a)) - (120*(8*a^3 + 3*a)*(b*x + a)^3*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^3 + 24*(10*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 40*
(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 3)/((b*x + a)^4*(sqrt(-1/(b*x
+ a)^2 + 1) - 1)^4))/b^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acos}\left(\frac{1}{a + b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acos(1/(a + b*x)),x)
[Out] int(x^4*acos(1/(a + b*x)), x)
```

3.19 $\int x^3 \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{(2+17a^2)\,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4}-\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}+\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}-\frac{a^4\sec^{-1}}{4}$$

```
[Out] -1/4*a^4*arcsec(b*x+a)/b^4+1/4*x^4*arcsec(b*x+a)+1/2*a*(2*a^2+1)*arctanh((1
-1/(b*x+a)^2)^(1/2))/b^4-1/12*(17*a^2+2)*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-
1/12*x^2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2+1/3*a*(b*x+a)^2*(1-1/(b*x+a)^2)^
(1/2)/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5366, 4511, 3867, 4133, 3855, 3852, 8}

$$-\frac{a^4\sec^{-1}(a+bx)}{4b^4}-\frac{(17a^2+2)\,(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4}+\frac{(2a^2+1)\,a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{2b^4}+\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4}-\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2}+\frac{1}{4}x^4\sec^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[a + b*x],x]
```

```
[Out] -1/12*((2 + 17*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/b^4 - (x^2*(a + b*x
)*Sqrt[1 - (a + b*x)^(-2)])/(12*b^2) + (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-
2)])/(3*b^4) - (a^4*ArcSec[a + b*x])/(4*b^4) + (x^4*ArcSec[a + b*x])/4 + (a
*(1 + 2*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(2*b^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^3\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^4}$$
$$= \frac{1}{4}x^4\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^4\,dx, x, \sec^{-1}(a+bx)\right)}{4b^4}$$
$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{1}{4}x^4\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))\left(-3a^3\right.\right.}{}$$
$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} + \frac{1}{4}x^4\sec^{-1}(a+bx) -$$
$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\sec^{-1}(a+bx)}{4b^4} +$$
$$= -\frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}}{3b^4} - \frac{a^4\sec^{-1}(a+bx)}{4b^4} +$$
$$= -\frac{(2+17a^2)(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^4} - \frac{x^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{12b^2} + \frac{a(a+bx)^2}{}$$

Mathematica [A]
time = 0.19, size = 150, normalized size = 0.97

$$\frac{-\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\,(2a+13a^3+2bx+9a^2bx-3ab^2x^2+b^3x^3)+3b^4x^4\sec^{-1}(a+bx)+3a^4\,\text{ArcSin}\left(\frac{1}{a+bx}\right)+6a(1+2a^2)\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSec[a + b*x],x]
```

```
[Out] (-(Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(2*a + 13*a^3 + 2*b*x +
 9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)) + 3*b^4*x^4*ArcSec[a + b*x] + 3*a^4*Ar
cSin[(a + b*x)^(-1)] + 6*a*(1 + 2*a^2)*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 +
2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(12*b^4)
```

Maple [A]
time = 0.15, size = 249, normalized size = 1.61

method	result

derivativedivides	$\frac{\frac{\operatorname{arcsec}(bx+a)a^4}{4}-\operatorname{arcsec}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(b \ldots}}{\ldots}}{\ldots}$
default	$\frac{\frac{\operatorname{arcsec}(bx+a)a^4}{4}-\operatorname{arcsec}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)(bx+a)^4}{4}+\frac{\sqrt{(b \ldots}}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arcsec(b*x+a)*a^4-arcsec(b*x+a)*a^3*(b*x+a)+3/2*arcsec(b*x+a)*a^
2*(b*x+a)^2-arcsec(b*x+a)*a*(b*x+a)^3+1/4*arcsec(b*x+a)*(b*x+a)^4+1/12*((b*
x+a)^2-1)^(1/2)*(3*a^4*arctan(1/((b*x+a)^2-1)^(1/2))+12*a^3*ln(b*x+a+((b*x+
a)^2-1)^(1/2))-18*a^2*((b*x+a)^2-1)^(1/2)+6*a*(b*x+a)*((b*x+a)^2-1)^(1/2)-(
b*x+a)^2*((b*x+a)^2-1)^(1/2)+6*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))-2*((b*x+a)^2
-1)^(1/2))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/4*(b^2*x^
5 + a*b*x^4)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

Fricas [A]
time = 2.60, size = 130, normalized size = 0.84

$$\frac{3\,b^4x^4\operatorname{arcsec}(bx+a)-6\,a^4\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-6\,(2\,a^3+a)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}\,(b^2x^2-4\,abx+13\,a^2+2)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*arcsec(b*x + a) - 6*a^4*arctan(-b*x - a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1)) - 6*(2*a^3 + a)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a
```

```
^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b^2*x^2 - 4*a*b*x + 13*a^2 +
2))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asec}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(b*x+a),x)
```

```
[Out] Integral(x**3*asec(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(135) = 270.
time = 0.45, size = 299, normalized size = 1.93

$$-\frac{1}{96}\, b \left(\frac{24\,(bx+a)^4 \left(\frac{4\,a}{bx+a} - \frac{6\,a^2}{(bx+a)^2} + \frac{4\,a^3}{(bx+a)^3} - 1\right) \arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a} - 1\right) - a}\right)}{b^5} + \frac{(bx+a)^3 \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right)^3 + 12\,(bx+a)^2 a \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right)^2 + 72\,(bx+a) a^2 \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right) + 9\,(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right) + 48\,(2\,a^3 + a) \log\left(-\left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right) |bx+a|\right) - \frac{9\,(8\,a^2+1)(bx+a)^2 \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right)^2 + 12\,(bx+a) a \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right) + 1}{(bx+a)^3 \left(\sqrt{-\frac{1}{(bx+a)^2} + 1} - 1\right)^3}}{b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/96*b*(24*(b*x + a)^4*(4*a/(b*x + a) - 6*a^2/(b*x + a)^2 + 4*a^3/(b*x + a
)^3 - 1)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^5 + ((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3 + 12*(b*x + a)^2*a*(sqrt(-1/(b*x + a)^2 + 1)
 - 1)^2 + 72*(b*x + a)*a^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 9*(b*x + a)*(sq
rt(-1/(b*x + a)^2 + 1) - 1) + 48*(2*a^3 + a)*log(-(sqrt(-1/(b*x + a)^2 + 1)
 - 1)*abs(b*x + a)) - (9*(8*a^2 + 1)*(b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1)
- 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^3*(s
qrt(-1/(b*x + a)^2 + 1) - 1)^3))/b^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acos}\left(\frac{1}{a + b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/(a + b*x)),x)
```

```
[Out] int(x^3*acos(1/(a + b*x)), x)
```

3.20 $\int x^2 \sec^{-1}(a+bx)\, dx$

Optimal. Leaf size=116

$$\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}-\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{a^3\sec^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\sec^{-1}(a+bx)-\frac{(1+6a^2)\,\mathrm{t}}{}$$

```
[Out] 1/3*a^3*arcsec(b*x+a)/b^3+1/3*x^3*arcsec(b*x+a)-1/6*(6*a^2+1)*arctanh((1-1/
(b*x+a)^2)^(1/2))/b^3+5/6*a*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/6*x*(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5366, 4511, 3867, 3855, 3852, 8}

$$\frac{a^3\sec^{-1}(a+bx)}{3b^3}-\frac{(6a^2+1)\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{6b^3}+\frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3}-\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2}+\frac{1}{3}x^3\sec^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x],x]
```

```
[Out] (5*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*b^3) - (x*(a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)])/(6*b^2) + (a^3*ArcSec[a + b*x])/(3*b^3) + (x^3*ArcSec[a + b*
x])/3 - ((1 + 6*a^2)*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/(6*b^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
```

```
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))^2\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3}$$

$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int\left(-2a^3+(1+6a^2)\text{sec}\right.\right.}{}$$

$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx) + \frac{(5a)\text{Subst}\left(\right.}{}$$

$$= -\frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx) - \frac{(1+6a^2)\,\text{t}}{}$$

$$= \frac{5a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^3} - \frac{x(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6b^2} + \frac{a^3\sec^{-1}(a+bx)}{3b^3} + \frac{1}{3}$$

Mathematica [A]

time = 0.12, size = 131, normalized size = 1.13

$$\frac{(5a^2+4abx-b^2x^2)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}+2b^3x^3\sec^{-1}(a+bx)-2a^3\text{ArcSin}\left(\frac{1}{a+bx}\right)-(1+6a^2)\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a + b*x],x]
```

```
[Out] ((5*a^2 + 4*a*b*x - b^2*x^2)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^
2] + 2*b^3*x^3*ArcSec[a + b*x] - 2*a^3*ArcSin[(a + b*x)^(-1)] - (1 + 6*a^2)
*Log[(a + b*x)*(1 + Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(6*
b^3)
```

Maple [A]
time = 0.15, size = 190, normalized size = 1.64

method	result
derivativedivides	$-\frac{\operatorname{arcsec}(bx+a)a^3}{3}+\operatorname{arcsec}(bx+a)a^2(bx+a)-\operatorname{arcsec}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsec}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(2a^3\right.}{}$
default	$-\frac{\operatorname{arcsec}(bx+a)a^3}{3}+\operatorname{arcsec}(bx+a)a^2(bx+a)-\operatorname{arcsec}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsec}(bx+a)(bx+a)^3}{3}-\frac{\sqrt{(bx+a)^2-1}\left(2a^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arcsec(b*x+a)*a^3+arcsec(b*x+a)*a^2*(b*x+a)-arcsec(b*x+a)*a*(b*
x+a)^2+1/3*arcsec(b*x+a)*(b*x+a)^3-1/6*((b*x+a)^2-1)^(1/2)*(2*a^3*arctan(1/
((b*x+a)^2-1)^(1/2))+6*a^2*ln(b*x+a+((b*x+a)^2-1)^(1/2))-6*a*((b*x+a)^2-1)^
(1/2)+(b*x+a)*((b*x+a)^2-1)^(1/2)+ln(b*x+a+((b*x+a)^2-1)^(1/2)))/((((b*x+a)^
2-1)/(b*x+a)^2)^(1/2)/(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/3*(b^2*x^
4 + a*b*x^3)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

Fricas [A]
time = 3.18, size = 117, normalized size = 1.01

$$\frac{2\,b^3x^3\operatorname{arcsec}\left(bx+a\right)+4\,a^3\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\left(6\,a^2+1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}\,\left(bx-5\,a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arcsec(b*x + a) + 4*a^3*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)) + (6*a^2 + 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2
- 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 5*a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{asec}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a),x)
```

```
[Out] Integral(x**2*asec(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(100) = 200.
time = 0.45, size = 204, normalized size = 1.76

$$-\frac{1}{24}\,b\left(\frac{8\,(bx+a)^3\left(\frac{3\,a}{bx+a}-\frac{3\,a^2}{(bx+a)^2}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^4}-\frac{(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+12\,(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+4\,(6\,a^2+1)\log\left(-\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)|bx+a|\right)-\frac{12\,(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+1}{(bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2}}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*b*(8*(b*x + a)^3*(3*a/(b*x + a) - 3*a^2/(b*x + a)^2 - 1)*arccos(-1/((
b*x + a)*(a/(b*x + a) - 1) - a))/b^4 - ((b*x + a)^2*(sqrt(-1/(b*x + a)^2 +
1) - 1)^2 + 12*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*(6*a^2 + 1)*l
og(-(sqrt(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - (12*(b*x + a)*a*(sqrt(-1
/(b*x + a)^2 + 1) - 1) + 1)/((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2))
/b^4)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x)),x)
[Out] int(x^2*acos(1/(a + b*x)), x)
```

3.21 $\int x \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=78

$$-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}-\frac{a^2\sec^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)+\frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}$$

```
[Out] -1/2*a^2*arcsec(b*x+a)/b^2+1/2*x^2*arcsec(b*x+a)+a*arctanh((1-1/(b*x+a)^2)^
(1/2))/b^2-1/2*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5366, 4511, 3858, 3855, 3852, 8}

$$-\frac{a^2\sec^{-1}(a+bx)}{2b^2}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2}+\frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x],x]
```

```
[Out] -1/2*((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/b^2 - (a^2*ArcSec[a + b*x])/(2*b^
2) + (x^2*ArcSec[a + b*x])/2 + (a*ArcTanh[Sqrt[1 - (a + b*x)^(-2)]])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\sec^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int x\sec(x)(-a+\sec(x))\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int(-a+\sec(x))^2\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2}$$

$$= -\frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) - \frac{\text{Subst}\left(\int\sec^2(x)\,dx, x, \sec^{-1}(a+bx)\right)}{2b^2} +$$

$$= -\frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) + \frac{a\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b^2} + \frac{\text{Subst}\big(}{}$$

$$= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2b^2} - \frac{a^2\sec^{-1}(a+bx)}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx) + \frac{a\tanh^{-1}\big(}{}$$

Mathematica [A]
time = 0.08, size = 110, normalized size = 1.41

$$\frac{-\left((a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right) + b^2x^2\sec^{-1}(a+bx) + a^2\text{ArcSin}\left(\frac{1}{a+bx}\right) + 2a\log\left((a+bx)\left(1+\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x],x]
```

```
[Out] (-((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]) + b^2*x^2*Ar
cSec[a + b*x] + a^2*ArcSin[(a + b*x)^(-1)] + 2*a*Log[(a + b*x)*(1 + Sqrt[(-
1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])])/(2*b^2)
```

Maple [A]
time = 0.14, size = 108, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsec}(bx+a)(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)+\frac{\sqrt{(bx+a)^2-1}\left(-\sqrt{(bx+a)^2-1}+2a\ln\left(bx+a+\sqrt{(bx+\right.\right.}{2(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}}{b^2}$
default	$\frac{\frac{\operatorname{arcsec}(bx+a)(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)a(bx+a)+\frac{\sqrt{(bx+a)^2-1}\left(-\sqrt{(bx+a)^2-1}+2a\ln\left(bx+a+\sqrt{(bx+\right.\right.}{2(bx+a)\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arcsec(b*x+a)*(b*x+a)^2-arcsec(b*x+a)*a*(b*x+a)+1/2*((b*x+a)^2-1
)^(1/2)*(-((b*x+a)^2-1)^(1/2)+2*a*ln(b*x+a+((b*x+a)^2-1)^(1/2)))/(b*x+a)/((
(b*x+a)^2-1)/(b*x+a)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) - integrate(1/2*(b^2*x^
3 + a*b*x^2)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/(b^2*x^2 + 2*a
*b*x + a^2 + (b^2*x^2 + 2*a*b*x + a^2 - 1)*e^(log(b*x + a + 1) + log(b*x +
a - 1)) - 1), x)
```

Fricas [A]
time = 1.91, size = 104, normalized size = 1.33

$$\frac{b^2x^2\operatorname{arcsec}(bx+a)-2a^2\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-2a\log\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-\sqrt{b^2x^2+2abx+a^2-1}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2*arcsec(b*x + a) - 2*a^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - 2*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a),x)
```

```
[Out] Integral(x*asec(a + b*x), x)
```

Giac [**A**]
time = 0.45, size = 133, normalized size = 1.71

$$-\frac{1}{4}\,b\left(\frac{2\,(bx+a)^2\left(\frac{2\,a}{bx+a}-1\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^3}+\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+4\,a\log\left(-\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)|bx+a|\right)-\frac{1}{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*(b*x + a)^2*(2*a/(b*x + a) - 1)*arccos(-1/((b*x + a)*(a/(b*x + a)
 - 1) - a))/b^3 + ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 4*a*log(-(sqr
t(-1/(b*x + a)^2 + 1) - 1)*abs(b*x + a)) - 1/((b*x + a)*(sqrt(-1/(b*x + a)^
2 + 1) - 1)))/b^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x)),x)
```

```
[Out] int(x*acos(1/(a + b*x)), x)
```

3.22 $\int \sec^{-1}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arcsec(b*x+a)/b-arctanh((1-1/(b*x+a)^2)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5358, 379, 272, 65, 212}

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x])/b - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{-1}(a+bx)\,dx &= \frac{(a+bx)\sec^{-1}(a+bx)}{b}-\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx\\ &= \frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)\sec^{-1}(a+bx)}{b}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{(a+bx)^2}\right)}{2b}\\ &= \frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}\\ &= \frac{(a+bx)\sec^{-1}(a+bx)}{b}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx)^2}}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 336 vs. 2(37) = 74.
time = 0.54, size = 336, normalized size = 9.08

$$x\sec^{-1}(a+bx)-\frac{(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\left(2a\left(b+\sqrt{b^2}\right)\text{ArcTan}\left(a+\sqrt{b^2}\,x-\sqrt{-1+a^2+2abx+b^2x^2}\right)+2a\left(-b+\sqrt{b^2}\right)\text{ArcTan}\left(a-\sqrt{b^2}\,x+\sqrt{-1+a^2+2abx+b^2x^2}\right)-\sqrt{b^2}\log\left(-a-\sqrt{b^2}\,x+\sqrt{-1+a^2+2abx+b^2x^2}\right)+b\log\left(a-\sqrt{b^2}\,x+\sqrt{-1+a^2+2abx+b^2x^2}\right)-\sqrt{b^2}\log\left(a-\sqrt{b^2}\,x+\sqrt{-1+a^2+2abx+b^2x^2}\right)-b\log\left(ab^2+(b^2)^{3/2}x-b^2\sqrt{-1+a^2+2abx+b^2x^2}\right)\right)}{2b^2\sqrt{-1+a^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x],x]
```

```
[Out] x*ArcSec[a + b*x] - ((a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x
)^2]*(2*a*(b + Sqrt[b^2])*ArcTan[a + Sqrt[b^2]*x - Sqrt[-1 + a^2 + 2*a*b*x
```

```
+ b^2*x^2]] + 2*a*(-b + Sqrt[b^2])*ArcTan[a - Sqrt[b^2]*x + Sqrt[-1 + a^2 +
 2*a*b*x + b^2*x^2]] - Sqrt[b^2]*Log[-a - Sqrt[b^2]*x + Sqrt[-1 + a^2 + 2*a
*b*x + b^2*x^2]] + b*Log[a - Sqrt[b^2]*x + Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^
2]] - Sqrt[b^2]*Log[a - Sqrt[b^2]*x + Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]] -
 b*Log[a*b^2 + (b^2)^(3/2)*x - b^2*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2]]))/(2
*b^2*Sqrt[-1 + a^2 + 2*a*b*x + b^2*x^2])
```

Maple [A]
time = 0.04, size = 45, normalized size = 1.22

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arcsec}(bx+a)-\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$	45
default	$\frac{(bx+a)\operatorname{arcsec}(bx+a)-\ln\left(bx+a+(bx+a)\sqrt{1-\frac{1}{(bx+a)^2}}\right)}{b}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arcsec(b*x+a)-ln(b*x+a+(b*x+a)*(1-1/(b*x+a)^2)^(1/2)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.49

$$\frac{2\,(bx+a)\operatorname{arcsec}(bx+a)-\log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arcsec(b*x + a) - log(sqrt(-1/(b*x + a)^2 + 1) + 1) + log(
-sqrt(-1/(b*x + a)^2 + 1) + 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 1.45, size = 73, normalized size = 1.97

$$\frac{bx\operatorname{arcsec}(bx+a)+2\,a\arctan\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*arcsec(b*x + a) + 2*a*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)) + log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a),x)
```

```
[Out] Integral(asec(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.
time = 0.45, size = 82, normalized size = 2.22

$$\frac{1}{2}\,b\left(\frac{2\,(bx+a)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2}-\frac{\log\left(\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(bx+a)^2}+1}+1\right)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x + a)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/b^2 - (log(
sqrt(-1/(b*x + a)^2 + 1) + 1) - log(-sqrt(-1/(b*x + a)^2 + 1) + 1))/b^2)
```

Mupad [B]
time = 0.86, size = 35, normalized size = 0.95

$$-\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x)^2}}}\right)-\operatorname{acos}\left(\frac{1}{a+b\,x}\right)\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x)),x)
```

```
[Out] -(atanh(1/(1 - 1/(a + b*x)^2)^(1/2)) - acos(1/(a + b*x))*(a + b*x))/b
```

3.23 $\int \frac{\sec^{-1}(a+bx)}{x}\,dx$

<u>**Optimal**</u>. Leaf size=200

$$\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}\right.$$

```
[Out] -arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)*ln
(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x+a)*
ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+1/2*I*polylo
g(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-I*polylog(2,a*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-I*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x
+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

<u>**Rubi**</u> [A]
time = 0.24, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5366, 4647, 4626, 3800, 2221, 2317, 2438, 4616}

$$-i\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-i\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\frac{1}{2}i\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)-\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x,x]
```

```
[Out] ArcSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + Ar
cSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ArcS
ec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] - I*PolyLog[2, (a*E^(I*ArcSe
c[a + b*x]))/(1 - Sqrt[1 - a^2])] - I*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/
(1 + Sqrt[1 - a^2])] + (I/2)*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

<u>Rule 2438</u>

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4626

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n -
1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4647

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)}{x}\,dx = \text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= \text{Subst}\left(\int \frac{x\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= a\text{Subst}\left(\int \frac{x\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x\tan(x)\,dx, x, \sec^{-1}(a+b$$
$$= -\left(2i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\text{Subst}\left(\int \frac{e^{ix}x}{1-\sqrt{1-a^2}-ae^{ix}}$$
$$= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{se}$$
$$= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{se}$$
$$= \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) - \text{se}$$

Mathematica [A]
time = 0.22, size = 284, normalized size = 1.42

$$-4i\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\text{ArcTan}\left(\frac{(1+a)\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a^2}}\right) + \left(\sec^{-1}(a+bx) - 2\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + \left(\sec^{-1}(a+bx) + 2\text{ArcSin}\left(\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right)\right)\log\left(1-\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) - \sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right) - i\left(\text{PolyLog}\left(2,-\frac{\left(-1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right) + \text{PolyLog}\left(2,\frac{\left(1+\sqrt{1-a^2}\right)e^{i\sec^{-1}(a+bx)}}{a}\right)\right) + \frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x,x]
```

```
[Out] (-4*I)*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*ArcTan[((1 + a)*Tan[ArcSec[a + b*x]
/2])/Sqrt[1 - a^2]] + (ArcSec[a + b*x] - 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]
)*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + (ArcSec[a + b*x
] + 2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]])*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*A
rcSec[a + b*x]))/a] - ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])] -
I*(PolyLog[2, -(((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a)] + PolyLog[
2, ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]) + (I/2)*PolyLog[2, -E^((
2*I)*ArcSec[a + b*x])]
```

Maple [A]
time = 0.76, size = 374, normalized size = 1.87

method	result

derivativedivides	$-\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+\right.\right.$
default	$-\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-arcsec(b*x+a)*ln
(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+arcsec(b*x+a)*ln((-a*(1/(b*x+a)+I
*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))+arcsec(b*x+a)
*ln((a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(
1/2)))+I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+I*dilog(1-I*(1/(b*x
+a)+I*(1-1/(b*x+a)^2)^(1/2)))-I*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^(1/2)))-I*dilog((a*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(b*x + a)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x,x)
```

```
[Out] Integral(asec(a + b*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x,x)
```

```
[Out] int(acos(1/(a + b*x))/x, x)
```

3.24 $\int \frac{\sec^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=70

$$-\frac{b\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{x}+\frac{2b\text{ArcTan}\left(\frac{\sqrt{1+a}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsec(b*x+a)/a-arcsec(b*x+a)/x+2*b*arctan((1+a)^(1/2)*tan(1/2*arcsec(b*
x+a))/(1-a)^(1/2))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5366, 4511, 3868, 2738, 211}

$$\frac{2b\text{ArcTan}\left(\frac{\sqrt{a+1}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}-\frac{b\sec^{-1}(a+bx)}{a}-\frac{\sec^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x])/a) - ArcSec[a + b*x]/x + (2*b*ArcTan[(Sqrt[1 + a]*Tan
[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a*Sqrt[1 - a^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{-1}(a+bx)}{x^2}\,dx &= b\text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= -\frac{\sec^{-1}(a+bx)}{x} + b\text{Subst}\left(\int \frac{1}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{b\text{Subst}\left(\int \frac{1}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1-a+(1+a)x^2}\,dx, x, \tan\left(\frac{1}{2}\sec^{-1}(a+b\right.\right.}{a}\\
&= -\frac{b\sec^{-1}(a+bx)}{a} - \frac{\sec^{-1}(a+bx)}{x} + \frac{2b\tan^{-1}\left(\frac{\sqrt{1+a}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 112, normalized size = 1.60

$$-\frac{\sec^{-1}(a+bx)}{x} + \frac{b\left(\text{ArcSin}\left(\frac{1}{a+bx}\right) - \frac{i\log\left(\frac{2\left(\frac{ia\left(-1+a^2+abx\right)}{\sqrt{1-a^2}} + a(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^2,x]
```

```
[Out] -(ArcSec[a + b*x]/x) + (b*(ArcSin[(a + b*x)^(-1)] - (I*Log[(2*((I*a*(-1 + a
^2 + a*b*x))/Sqrt[1 - a^2] + a*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2
)/(a + b*x)^2]))/(b*x)])/Sqrt[1 - a^2]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.
time = 0.27, size = 126, normalized size = 1.80

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsec}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2-1}\left(-\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}+\ln\left(\frac{2\sqrt{a^2-1}\right.\right.}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)a\sqrt{a^2-1}}\right.$
default	$b\left(-\frac{\operatorname{arcsec}(bx+a)}{bx}-\frac{\sqrt{(bx+a)^2-1}\left(-\arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)\sqrt{a^2-1}+\ln\left(\frac{2\sqrt{a^2-1}\right.\right.}{\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)a\sqrt{a^2-1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arcsec(b*x+a)-((b*x+a)^2-1)^(1/2)*(-arctan(1/((b*x+a)^2-1)^(1/2))
*(a^2-1)^(1/2)+ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x))/(
((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a/(a^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] (x*integrate((b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*x + a - 1))/
(b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 - 1)*x)*e^
(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*x + a + 1)*sqrt(
b*x + a - 1)))/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.
time = 2.90, size = 281, normalized size = 4.01

$$\left[-\frac{2\left(a^{2}-1\right)bx\arctan\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\right)-\sqrt{a^{2}-1}\,bx\log\left(\frac{a^{2}bx+a^{3}+\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\left(a^{2}-\sqrt{a^{2}-1}\,a-1\right)-\left(abx+a^{2}-1\right)\sqrt{a^{2}-1}-a}{x}\right)+\left(a^{3}-a\right)\operatorname{arcsec}\left(bx+a\right)}{\left(a^{3}-a\right)x},\ -\frac{2\left(a^{2}-1\right)bx\arctan\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\right)-2\sqrt{-a^{2}+1}\,bx\arctan\left(-\frac{\sqrt{-a^{2}+1}\,bx-\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\,\sqrt{-a^{2}+1}}{a^{2}-1}\right)+\left(a^{3}-a\right)\operatorname{arcsec}\left(bx+a\right)}{\left(a^{3}-a\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - s
qrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^
2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^3 -
 a)*arcsec(b*x + a))/((a^3 - a)*x), -(2*(a^2 - 1)*b*x*arctan(-b*x - a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 +
 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a
^3 - a)*arcsec(b*x + a))/((a^3 - a)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}\left(a+bx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**2,x)
```

```
[Out] Integral(asec(a + b*x)/x**2, x)
```

Giac [A]
time = 0.47, size = 94, normalized size = 1.34

$$b\left(\frac{2\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^{2}}+1}-1\right)+a}{\sqrt{-a^{2}+1}}\right)}{\sqrt{-a^{2}+1}\,a}+\frac{\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a\left(\frac{a}{bx+a}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*(2*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/
(sqrt(-a^2 + 1)*a) + arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a*(a/(b*
x + a) - 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))/x^2, x)
```

3.25 $\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x}+\frac{b^2\sec^{-1}(a+bx)}{2a^2}-\frac{\sec^{-1}(a+bx)}{2x^2}-\frac{\left(1-2a^2\right)b^2\mathrm{ArcTan}\left(\frac{\sqrt{1+a}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}$$

```
[Out] 1/2*b^2*arcsec(b*x+a)/a^2-1/2*arcsec(b*x+a)/x^2-(-2*a^2+1)*b^2*arctan((1+a)
^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^2/(-a^2+1)^(3/2)+1/2*b*(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x
```

Rubi [A]
time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5366, 4511, 3870, 4004, 3916, 2738, 211}

$$-\frac{\left(1-2a^2\right)b^2\mathrm{ArcTan}\left(\frac{\sqrt{a+1}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{a^2\left(1-a^2\right)^{3/2}}+\frac{b^2\sec^{-1}(a+bx)}{2a^2}+\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\left(1-a^2\right)x}-\frac{\sec^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^3,x]
```

```
[Out] (b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(2*a*(1 - a^2)*x) + (b^2*ArcSec[a +
b*x])/(2*a^2) - ArcSec[a + b*x]/(2*x^2) - ((1 - 2*a^2)*b^2*ArcTan[(Sqrt[1 +
 a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(a^2*(1 - a^2)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)}{x^3}\,dx = b^2\text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^3}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= -\frac{\sec^{-1}(a+bx)}{2x^2} + \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{b^2\text{Subst}\left(\int \frac{1-a^2-a\sec(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+\right.}{2a\,(1-a^2)}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)\,b^2)\,\text{Subs}}{}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)\,b^2)\,\text{Subs}}{}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{((1-2a^2)\,b^2)\,\text{Subs}}{}$$

$$= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{2a\,(1-a^2)\,x} + \frac{b^2\sec^{-1}(a+bx)}{2a^2} - \frac{\sec^{-1}(a+bx)}{2x^2} - \frac{(1-2a^2)\,b^2\tan^{-1}\left(\right.}{a^2\,(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.74, size = 198, normalized size = 1.58

$$-\frac{\frac{bx(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}}{a(-1+a^2)} + \sec^{-1}(a+bx) + \frac{b^2x^2\text{ArcSin}\left(\frac{1}{a+bx}\right)}{a^2} + \frac{i(-1+2a^2)b^2x^2\log\left(\frac{4(-1+a)a^2(1+a)\left(-\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}-(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(-1+2a^2)b^2x}\right)}{a^2(1-a^2)^{3/2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^3,x]
```

```
[Out] -1/2*((b*x*(a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2])/(a*(
-1 + a^2)) + ArcSec[a + b*x] + (b^2*x^2*ArcSin[(a + b*x)^(-1)])/a^2 + (I*(-
1 + 2*a^2)*b^2*x^2*Log[(4*(-1 + a)*a^2*(1 + a)*(((-I)*(-1 + a^2 + a*b*x))/S
qrt[1 - a^2] - (a + b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))
/((-1 + 2*a^2)*b^2*x)])/(a^2*(1 - a^2)^(3/2)))/x^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(109) = 218$.
time = 0.27, size = 456, normalized size = 3.65

method	result
derivativedivides	$b^2 \left(-\frac{\operatorname{arcsec}(bx+a)}{2b^2x^2} - \frac{\sqrt{(bx+a)^2-1}\left(-(a^2-1)^{\frac{3}{2}} \arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)a^3+(a^2-1)^{\frac{3}{2}} \arctan\right.}{}\right.$
default	$b^2 \left(-\frac{\operatorname{arcsec}(bx+a)}{2b^2x^2} - \frac{\sqrt{(bx+a)^2-1}\left(-(a^2-1)^{\frac{3}{2}} \arctan\left(\frac{1}{\sqrt{(bx+a)^2-1}}\right)a^3+(a^2-1)^{\frac{3}{2}} \arctan\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arcsec(b*x+a)-1/2*((b*x+a)^2-1)^(1/2)*(-(a^2-1)^(3/2)*arc
tan(1/((b*x+a)^2-1)^(1/2))*a^3+(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*
a^2*(b*x+a)+2*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^5
-2*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^4*(b*x+a)+((
b*x+a)^2-1)^(1/2)*(a^2-1)^(3/2)*a+(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2
))*a-(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*(b*x+a)-3*ln(2*((a^2-1)^(1
/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^3+3*ln(2*((a^2-1)^(1/2)*((b*x+a
)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^2*(b*x+a)+a*ln(2*((a^2-1)^(1/2)*((b*x+a)^2
-1)^(1/2)+a*(b*x+a)-1)/b/x)-ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+
a)-1)/b/x)*(b*x+a))/(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^2/(a^2-1)^(5/
2)/b/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*integrate(1/2*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*
x + a - 1))/(b^2*x^4 + 2*a*b*x^3 + (a^2 - 1)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (
```

```
a^2 - 1)*x^2)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)))/x^2
```

Fricas [A]
time = 2.14, size = 427, normalized size = 3.42

$$\left[\frac{(2a^2-1)\sqrt{a^2-1}\,b^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2-1}\,(a^2+\sqrt{a^2-1}\,a-1)+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+2(a^4-2a^2+1)b^2x^2\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-(a^3-a)b^2x^2-\sqrt{b^2x^2+2abx+a^2-1}\,(a^3-a)bx-(a^6-2a^4+a^2)\,\mathrm{arcsec}\,(bx+a)}{2(a^6-2a^4+a^2)x^2},\ -\frac{2(2a^2-1)\sqrt{-a^2+1}\,b^2x^2\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{b^2x^2+2abx+a^2-1}\,\sqrt{-a^2+1}}{a^2-1}\right)-2(a^4-2a^2+1)b^2x^2\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)+(a^3-a)b^2x^2+\sqrt{b^2x^2+2abx+a^2-1}\,(a^3-a)bx+(a^6-2a^4+a^2)\,\mathrm{arcsec}\,(bx+a)}{2(a^6-2a^4+a^2)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*((2*a^2 - 1)*sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) + 2*(a^4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1)) - (a^3 - a)*b^2*x^2 - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*(a^3 - a)*b*x - (a^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4 + a^
2)*x^2), -1/2*(2*(2*a^2 - 1)*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)
*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) - 2*(a^
4 - 2*a^2 + 1)*b^2*x^2*arctan(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))
 + (a^3 - a)*b^2*x^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^3 - a)*b*x + (a
^6 - 2*a^4 + a^2)*arcsec(b*x + a))/((a^6 - 2*a^4 + a^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**3,x)
```

```
[Out] Integral(asec(a + b*x)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(106) = 212.
time = 0.45, size = 216, normalized size = 1.73

$$-\frac{1}{2}\,b\left(\frac{2(2a^2b-b)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^4-a^2)\sqrt{-a^2+1}}+\frac{2\left((bx+a)ab\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+b\right)}{\left((bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+2(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+1\right)(a^3-a)}+\frac{\left(\frac{2ab}{bx+a}-b\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a^2\left(\frac{a}{bx+a}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*(2*(2*a^2*b - b)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) +
a)/sqrt(-a^2 + 1))/((a^4 - a^2)*sqrt(-a^2 + 1)) + 2*((b*x + a)*a*b*(sqrt(-1
/(b*x + a)^2 + 1) - 1) + b)/((((b*x + a)^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2
+ 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)*(a^3 - a)) + (2*a*b/(b*
x + a) - b)*arccos(-1/((b*x + a)*(a/(b*x + a) - 1) - a))/(a^2*(a/(b*x + a)
- 1)^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^3,x)
```

```
[Out] int(acos(1/(a + b*x))/x^3, x)
```

3.26 $\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=181

$$\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2}-\frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x}-\frac{b^3\sec^{-1}(a+bx)}{3a^3}-\frac{\sec^{-1}(a+bx)}{3x^3}+\frac{(2-5\cdots}{}$$

```
[Out] -1/3*b^3*arcsec(b*x+a)/a^3-1/3*arcsec(b*x+a)/x^3+1/3*(6*a^4-5*a^2+2)*b^3*ar
ctan((1+a)^(1/2)*tan(1/2*arcsec(b*x+a))/(1-a)^(1/2))/a^3/(-a^2+1)^(5/2)+1/6
*b*(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/a/(-a^2+1)/x^2-1/6*(-5*a^2+2)*b^2*(b*x+a)*
(1-1/(b*x+a)^2)^(1/2)/a^2/(-a^2+1)^2/x
```

Rubi [A]
time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5366, 4511, 3870, 4145, 4004, 3916, 2738, 211}

$$-\frac{b^3\sec^{-1}(a+bx)}{3a^3}-\frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x}+\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2}+\frac{(6a^4-5a^2+2)\,b^3\text{ArcTan}\left(\frac{\sqrt{a+1}\,\tan\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\sqrt{1-a}}\right)}{3a^3\,(1-a^2)^{5/2}}-\frac{\sec^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/x^4,x]
```

```
[Out] (b*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*a*(1 - a^2)*x^2) - ((2 - 5*a^2)*b
^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)])/(6*a^2*(1 - a^2)^2*x) - (b^3*ArcSec[
a + b*x])/(3*a^3) - ArcSec[a + b*x]/(3*x^3) + ((2 - 5*a^2 + 6*a^4)*b^3*ArcT
an[(Sqrt[1 + a]*Tan[ArcSec[a + b*x]/2])/Sqrt[1 - a]])/(3*a^3*(1 - a^2)^(5/2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
```

```
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{-1}(a+bx)}{x^4}\,dx &= b^3\text{Subst}\left(\int \frac{x\sec(x)\tan(x)}{(-a+\sec(x))^4}\,dx, x, \sec^{-1}(a+bx)\right) \\
&= -\frac{\sec^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\text{Subst}\left(\int \frac{1}{(-a+\sec(x))^3}\,dx, x, \sec^{-1}(a+bx)\right) \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{\sec^{-1}(a+bx)}{3x^3} - \frac{b^3\text{Subst}\left(\int \frac{2(1-a^2)-2a\sec(x)-\sec^2(x)}{(-a+\sec(x))^2}\,dx, x, \right.}{6a\,(1-a^2)} \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x} - \frac{\sec^{-1}(a+bx)}{3x^3} \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\sec^{-1}(a+bx}{3a^3} \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\sec^{-1}(a+bx}{3a^3} \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\sec^{-1}(a+bx}{3a^3} \\
&= \frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a\,(1-a^2)\,x^2} - \frac{(2-5a^2)\,b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2\,(1-a^2)^2\,x} - \frac{b^3\sec^{-1}(a+bx}{3a^3}
\end{aligned}
$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 0.29, size = 241, normalized size = 1.33

$$
\frac{1}{6}\left(-\frac{b\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\,(a^4+abx-4a^3bx+2b^2x^2-a^2(1+5b^2x^2))}{a^2\,(-1+a^2)^2\,x^2} - \frac{2\sec^{-1}(a+bx)}{x^3} + \frac{2b^3\text{ArcSin}\left(\frac{1}{a+bx}\right)}{a^3} - \frac{i(2-5a^2+6a^4)\,b^3\log\left(\frac{12a^3(-1+a^2)^2\left(\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}+(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(2-5a^2+6a^4)b^3x}\right)}{a^3\,(1-a^2)^{5/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/x^4,x]
```

```
[Out] (-((b*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*(a^4 + a*b*x - 4*a^3
*b*x + 2*b^2*x^2 - a^2*(1 + 5*b^2*x^2)))/(a^2*(-1 + a^2)^2*x^2)) - (2*ArcSe
```

```
c[a + b*x])/x^3 + (2*b^3*ArcSin[(a + b*x)^(-1)])/a^3 - (I*(2 - 5*a^2 + 6*a^
4)*b^3*Log[(12*a^3*(-1 + a^2)^2*((I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2] + (a
+ b*x)*Sqrt[(-1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]))/((2 - 5*a^2 + 6*a
^4)*b^3*x)])/(a^3*(1 - a^2)^(5/2)))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(159) = 318$.

time = 0.29, size = 970, normalized size = 5.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arcsec(b*x+a)-1/6*((b*x+a)^2-1)^(1/2)*(-8*(a^2-1)^(3/2)*a
rctan(1/((b*x+a)^2-1)^(1/2))*a^3*(b*x+a)+4*(a^2-1)^(3/2)*arctan(1/((b*x+a)^
2-1)^(1/2))*a*(b*x+a)+4*(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*a^5*(b*
x+a)-5*((b*x+a)^2-1)^(1/2)*(a^2-1)^(3/2)*a^3*(b*x+a)+6*ln(2*((a^2-1)^(1/2)*
((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^6*(b*x+a)^2+6*((b*x+a)^2-1)^(1/2)*(
a^2-1)^(3/2)*a^4+4*(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*a^4+22*ln(2*
((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^5*(b*x+a)-11*ln(2*((
a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^4*(b*x+a)^2-3*((b*x+a)
^2-1)^(1/2)*(a^2-1)^(3/2)*a^2-2*(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))
*a^2-2*(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*(b*x+a)^2-2*(a^2-1)^(3/2
)*arctan(1/((b*x+a)^2-1)^(1/2))*a^6+4*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/
2)+a*(b*x+a)-1)/b/x)*a*(b*x+a)-14*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a
*(b*x+a)-1)/b/x)*a^3*(b*x+a)+7*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b
*x+a)-1)/b/x)*a^2*(b*x+a)^2-12*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b
*x+a)-1)/b/x)*a^7*(b*x+a)-11*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x
+a)-1)/b/x)*a^6+7*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)
*a^4-2*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^2-2*ln(2
*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*(b*x+a)^2+6*ln(2*((a^
2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)*a^8-2*(a^2-1)^(3/2)*arctan
(1/((b*x+a)^2-1)^(1/2))*a^4*(b*x+a)^2+2*((b*x+a)^2-1)^(1/2)*(a^2-1)^(3/2)*a
*(b*x+a)+4*(a^2-1)^(3/2)*arctan(1/((b*x+a)^2-1)^(1/2))*a^2*(b*x+a)^2)/(((b*
x+a)^2-1)/(b*x+a)^2)^(1/2)/(b*x+a)/a^3/(a^2-1)^(7/2)/b^2/x^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^3*integrate(1/3*(b^2*x + a*b)*e^(1/2*log(b*x + a + 1) + 1/2*log(b*
x + a - 1))/(b^2*x^5 + 2*a*b*x^4 + (a^2 - 1)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (
a^2 - 1)*x^3)*e^(log(b*x + a + 1) + log(b*x + a - 1))), x) - arctan(sqrt(b*
x + a + 1)*sqrt(b*x + a - 1)))/x^3
```

Fricas [A]
time = 3.62, size = 548, normalized size = 3.03

$$\left[\frac{(6a^4-5a^2+2)\sqrt{a^2-1}\,b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-4(a^6-3a^4+3a^2-1)b^3x^3\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)+(5a^5-7a^3+2a)b^3x^3-2(a^9-3a^7+3a^5-a^3)\operatorname{arcsec}(bx+a)+\left((5a^5-7a^3+2a)b^2x^2-(a^6-2a^4+a^2)bx\right)\sqrt{b^2x^2+2abx+a^2-1}}{6(a^9-3a^7+3a^5-a^3)x^3}, \frac{2(6a^4-5a^2+2)\sqrt{-a^2+1}\,b^3x^3\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{b^2x^2+2abx+a^2-1}\sqrt{-a^2+1}}{a^2-1}\right)-4(a^6-3a^4+3a^2-1)b^3x^3\arctan\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)+(5a^5-7a^3+2a)b^3x^3-2(a^9-3a^7+3a^5-a^3)\operatorname{arcsec}(bx+a)+\left((5a^5-7a^3+2a)b^2x^2-(a^6-2a^4+a^2)bx\right)\sqrt{b^2x^2+2abx+a^2-1}}{6(a^9-3a^7+3a^5-a^3)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((6*a^4 - 5*a^2 + 2)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)
*sqrt(a^2 - 1) - a)/x) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x -
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 - 2*
(a^9 - 3*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2*x^
2 - (a^6 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a
^7 + 3*a^5 - a^3)*x^3), 1/6*(2*(6*a^4 - 5*a^2 + 2)*sqrt(-a^2 + 1)*b^3*x^3*a
rctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2 +
1))/(a^2 - 1)) - 4*(a^6 - 3*a^4 + 3*a^2 - 1)*b^3*x^3*arctan(-b*x - a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (5*a^5 - 7*a^3 + 2*a)*b^3*x^3 - 2*(a^9 - 3
*a^7 + 3*a^5 - a^3)*arcsec(b*x + a) + ((5*a^5 - 7*a^3 + 2*a)*b^2*x^2 - (a^6
 - 2*a^4 + a^2)*b*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^9 - 3*a^7 + 3*a
^5 - a^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/x**4,x)
```

```
[Out] Integral(asec(a + b*x)/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(155) = 310.
time = 0.48, size = 451, normalized size = 2.49

$$\frac{1}{3}b\left(\frac{(6a^4b^2-5a^2b^2+2b^2)\arctan\left(\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+a}{\sqrt{-a^2+1}}\right)}{(a^7-2a^5+a^3)\sqrt{-a^2+1}}+\frac{4(bx+a)^3a^3b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3+10(bx+a)^2a^4b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2-(bx+a)^3ab^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^3+(bx+a)^2a^2b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+16(bx+a)a^3b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)-2(bx+a)^2b^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2-7(bx+a)ab^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+5a^2b^2-2b^2}{(a^6-2a^4+a^2)\left((bx+a)^2\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)^2+2(bx+a)a\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)+1\right)^2}-\frac{\left(\frac{3ab^2}{bx+a}-\frac{3a^2b^2}{(bx+a)^2}-b^2\right)\arccos\left(-\frac{1}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{a^3\left(\frac{a}{bx+a}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b*((6*a^4*b^2 - 5*a^2*b^2 + 2*b^2)*arctan(((b*x + a)*(sqrt(-1/(b*x + a)
^2 + 1) - 1) + a)/sqrt(-a^2 + 1))/((a^7 - 2*a^5 + a^3)*sqrt(-a^2 + 1)) + (4
```

```
*(b*x + a)^3*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^3 + 10*(b*x + a)^2*a^4*
b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 - (b*x + a)^3*a*b^2*(sqrt(-1/(b*x + a)
^2 + 1) - 1)^3 + (b*x + a)^2*a^2*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1)^2 + 16*
(b*x + a)*a^3*b^2*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 2*(b*x + a)^2*b^2*(sqrt(
-1/(b*x + a)^2 + 1) - 1)^2 - 7*(b*x + a)*a*b^2*(sqrt(-1/(b*x + a)^2 + 1) -
1) + 5*a^2*b^2 - 2*b^2)/((a^6 - 2*a^4 + a^2)*((b*x + a)^2*(sqrt(-1/(b*x + a
)^2 + 1) - 1)^2 + 2*(b*x + a)*a*(sqrt(-1/(b*x + a)^2 + 1) - 1) + 1)^2) - (3
*a*b^2/(b*x + a) - 3*a^2*b^2/(b*x + a)^2 - b^2)*arccos(-1/((b*x + a)*(a/(b*
x + a) - 1) - a))/(a^3*(a/(b*x + a) - 1)^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/x^4,x)
```

```
[Out] int(acos(1/(a + b*x))/x^4, x)
```

3.27 $\int x^3 \sec^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=381

$$-\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4}-\frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4}+\frac{a(a}{}$$

```
[Out] -a*x/b^3+1/12*(b*x+a)^2/b^4-1/4*a^4*arcsec(b*x+a)^2/b^4+1/4*x^4*arcsec(b*x+
a)^2-2*I*a^3*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4-I*a*polyl
og(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4+1/3*ln(b*x+a)/b^4+3*a^2*ln(
b*x+a)/b^4-4*I*a^3*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/
b^4-2*I*a*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^4+2*I*a
^3*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4+I*a*polylog(2,-I*(
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^4-1/3*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x
+a)^2)^(1/2)/b^4-3*a^2*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4+a*(b
*x+a)^2*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^4-1/6*(b*x+a)^3*arcsec(b*x+a)
*(1-1/(b*x+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.23, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5366, 4511, 4275, 4266, 2317, 2438, 4269, 3556, 4270}

$$-\frac{a^4\sec^{-1}(a+bx)^2}{4b^4}-\frac{4ia^3\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{2ia^3\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{2ia^3\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{3a^2\log(a+bx)}{b^4}-\frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4}-\frac{2ia\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{ia\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^4}-\frac{ia\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^4}+\frac{(a+bx)^2}{12b^4}+\frac{\log(a+bx)}{3b^4}+\frac{a(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^4}-\frac{(a+bx)^3\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{6b^4}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{3b^4}-\frac{ax}{b^3}+\frac{1}{4}x^4\sec^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSec[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) - ((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcSec[a + b*x])/(3*b^4) - (3*a^2*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[
a + b*x])/b^4 + (a*(a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^
4 - ((a + b*x)^3*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/(6*b^4) - (a^4*A
rcSec[a + b*x]^2)/(4*b^4) + (x^4*ArcSec[a + b*x]^2)/4 - ((2*I)*a*ArcSec[a +
 b*x]*ArcTan[E^(I*ArcSec[a + b*x])])/b^4 - ((4*I)*a^3*ArcSec[a + b*x]*ArcTa
n[E^(I*ArcSec[a + b*x])])/b^4 + Log[a + b*x]/(3*b^4) + (3*a^2*Log[a + b*x])
/b^4 + (I*a*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b^4 + ((2*I)*a^3*PolyLo
g[2, (-I)*E^(I*ArcSec[a + b*x])])/b^4 - (I*a*PolyLog[2, I*E^(I*ArcSec[a + b
*x])])/b^4 - ((2*I)*a^3*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^4
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^3 \sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int x^2 \sec(x)(-a+\sec(x))^3 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{1}{4}x^4 \sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\sec(x))^4\,dx, x, \sec^{-1}(a+bx)\right)}{2b^4}$$

$$= \frac{1}{4}x^4 \sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \left(a^4x - 4a^3x\sec(x) + 6a^2x\sec^2(x) - 4ax\sec^3(x) + \cdots\right.\right.}{2b^4}$$

$$= -\frac{a^4\sec^{-1}(a+bx)^2}{4b^4} + \frac{1}{4}x^4\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\sec^4(x)\,dx, x, \sec^{-1}(a+bx)\right.}{2b^4}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{3a^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{b^4} + \frac{a(a+bx)^2\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} - \frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{3b^4} - \frac{3a^2(a+bx)\sqrt{1-\cdots}}{\cdots}$$

Mathematica [A]
time = 8.28, size = 667, normalized size = 1.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*ArcSec[a + b*x]^2,x]
```

```
[Out] ((1 - a/(a + b*x))^3*(24*a*(2 + (1 + 2*a^2)*ArcSec[a + b*x]^2) + (2 + (-2 +
 24*a)*ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(-1 + Sqrt
```

```
[1 - (a + b*x)^(-2)]) + 16*(1 + 9*a^2)*Log[(a + b*x)^(-1)] - 24*a*(1 + 2*a^
2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x])] - Log[1 + I/
E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*x])/4]] + (2*I)*(
PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(I*ArcSec[a + b*x])
])) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2
])^4 + (4*ArcSec[a + b*x]*(1 + 6*a*ArcSec[a + b*x])*Sin[ArcSec[a + b*x]/2])
/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (8*(2*ArcSec[a + b*x
] + 18*a^2*ArcSec[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + ArcSec[a +
b*x]^2))*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] - Sin[ArcSec[a + b
*x]/2]) - (3*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*
x]/2])^4 + (4*ArcSec[a + b*x]*(1 - 6*a*ArcSec[a + b*x])*Sin[ArcSec[a + b*x]
/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 - (2 + (2 - 24*a)*
ArcSec[a + b*x] + 3*(1 - 4*a + 12*a^2)*ArcSec[a + b*x]^2)/(Cos[ArcSec[a + b
*x]/2] + Sin[ArcSec[a + b*x]/2])^2 - (8*(-2*ArcSec[a + b*x] - 18*a^2*ArcSec
[a + b*x] + 6*a^3*ArcSec[a + b*x]^2 + 3*a*(2 + ArcSec[a + b*x]^2))*Sin[ArcS
ec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])))/(48*b^4
*(-1 + a/(a + b*x))^3)
```

Maple [A]
time = 5.17, size = 673, normalized size = 1.77

method	result
derivativedivides	$\frac{-\operatorname{arcsec}(bx+a)^2 a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)^2 a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)^2 a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)^2(bx+a)^4}{4}-3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}{}$
default	$\frac{-\operatorname{arcsec}(bx+a)^2 a^3(bx+a)+\frac{3\operatorname{arcsec}(bx+a)^2 a^2(bx+a)^2}{2}-\operatorname{arcsec}(bx+a)^2 a(bx+a)^3+\frac{\operatorname{arcsec}(bx+a)^2(bx+a)^4}{4}-3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(-arcsec(b*x+a)^2*a^3*(b*x+a)+3/2*arcsec(b*x+a)^2*a^2*(b*x+a)^2-arcse
c(b*x+a)^2*a*(b*x+a)^3+1/4*arcsec(b*x+a)^2*(b*x+a)^4-3*(((b*x+a)^2-1)/(b*x+
a)^2)^(1/2)*arcsec(b*x+a)*a^2*(b*x+a)+(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*arcse
c(b*x+a)*a*(b*x+a)^2-1/6*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*(b*x
+a)^3-3*I*a^2*arcsec(b*x+a)-1/3*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*arcsec(b*x+
a)*(b*x+a)-2*I*a^3*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-a*(b*x+a)
+1/12*(b*x+a)^2-1/3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2/3*ln(1/(b
*x+a)+I*(1-1/(b*x+a)^2)^(1/2))-3*a^2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
))^2)+6*a^2*ln(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))-2*a^3*arcsec(b*x+a)*ln(1+
I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*a^3*arcsec(b*x+a)*ln(1-I*(1/(b*x+a
)+I*(1-1/(b*x+a)^2)^(1/2)))-I*a*dilog(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2
)))-1/3*I*arcsec(b*x+a)-a*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
```

```
^(1/2)))+a*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+I*a*di
log(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*a^3*dilog(1+I*(1/(b*x+a)+I
*(1-1/(b*x+a)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/16*x^4*log(b^2*x^
2 + 2*a*b*x + a^2)^2 - integrate(1/4*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
*b*x^4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 4*(b^3*x^6 + 3*a*b^2*x
^5 + (3*a^2 - 1)*b*x^4 + (a^3 - a)*x^3)*log(b*x + a)^2 - (b^3*x^6 + 2*a*b^2
*x^5 + (a^2 - 1)*b*x^4 + 4*(b^3*x^6 + 3*a*b^2*x^5 + (3*a^2 - 1)*b*x^4 + (a^
3 - a)*x^3)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsec(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{asec}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(b*x+a)**2,x)
```

```
[Out] Integral(x**3*asec(a + b*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARa+sageVAR
b*sageVAR
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(1/(a + b*x))^2,x)
```

```
[Out] int(x^3*acos(1/(a + b*x))^2, x)
```

3.28 $\int x^2 \sec^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=288

$$\frac{x}{3b^2}+\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)}{b^3}-\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)}{3b^3}+\frac{a^3\sec^{-1}(a+bx)}{3b^3}$$

```
[Out] 1/3*x/b^2+1/3*a^3*arcsec(b*x+a)^2/b^3+1/3*x^3*arcsec(b*x+a)^2+2/3*I*arcsec(
b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3+4*I*a^2*arcsec(b*x+a)*
arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^3-2*a*ln(b*x+a)/b^3-1/3*I*polyl
og(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-2*I*a^2*polylog(2,-I*(1/(b
*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+1/3*I*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2)))/b^3+2*I*a^2*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)/b^3+2*a*(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a)^2*arc
sec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]

time = 0.18, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5366, 4511, 4275, 4266, 2317, 2438, 4269, 3556, 4270}

$$\frac{a^3\sec^{-1}(a+bx)^2}{3b^3}+\frac{4ia^2\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^3}-\frac{2ia^2\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^3}+\frac{2ia^2\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^3}+\frac{2i\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{3b^3}-\frac{i\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{3b^3}+\frac{i\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{3b^3}-\frac{2a\log(a+bx)}{b^3}+\frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)}{b^3}-\frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\sec^{-1}(a+bx)^2+\frac{x}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x]^2,x]
```

```
[Out] x/(3*b^2) + (2*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^3 -
((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/(3*b^3) + (a^3*ArcSe
c[a + b*x]^2)/(3*b^3) + (x^3*ArcSec[a + b*x]^2)/3 + (((2*I)/3)*ArcSec[a + b
*x]*ArcTan[E^(I*ArcSec[a + b*x])])/b^3 + ((4*I)*a^2*ArcSec[a + b*x]*ArcTan[
E^(I*ArcSec[a + b*x])])/b^3 - (2*a*Log[a + b*x])/b^3 - ((I/3)*PolyLog[2, (-
I)*E^(I*ArcSec[a + b*x])])/b^3 - ((2*I)*a^2*PolyLog[2, (-I)*E^(I*ArcSec[a +
 b*x])])/b^3 + ((I/3)*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^3 + ((2*I)*a^2
*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sec^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int x^2 \sec(x)(-a+\sec(x))^2 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3} \\
&= \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3} \\
&= \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int \left(-a^3x + 3a^2x\sec(x) - 3ax\sec^2(x) + x\sec^3(x)\right)\,dx \right.}{3b^3} \\
&= \frac{a^3 \sec^{-1}(a+bx)^2}{3b^3} + \frac{1}{3}x^3 \sec^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int x\sec^3(x)\,dx, x, \sec^{-1}(a+bx)\right)}{3b^3} \\
&= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec}{3b^3} \\
&= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec}{3b^3} \\
&= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec}{3b^3} \\
&= \frac{x}{3b^2} + \frac{2a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)}{b^3} - \frac{(a+bx)^2\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec}{3b^3}
\end{aligned}$$

Mathematica [A]
time = 4.20, size = 473, normalized size = 1.64

$$\frac{4+2(1+6a^2)\sec^{-1}(a+bx)^2+\frac{\sec^{-1}(a+bx)(2+(-1+6a)\sec^{-1}(a+bx))}{-1+\sqrt{1-\frac{1}{(a+bx)^2}}}+24a\log\left(\frac{1}{a+bx}\right)+2(-1-6a^2)\left((\pi-2\sec^{-1}(a+bx))\left(\log\left(1-ie^{-i\sec^{-1}(a+bx)}\right)-\log\left(1+ie^{-i\sec^{-1}(a+bx)}\right)\right)-\pi\log\left(\cot\left(\frac{1}{4}(\pi+2\sec^{-1}(a+bx))\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-i\sec^{-1}(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-i\sec^{-1}(a+bx)}\right)\right)\right)+\frac{2\sec^{-1}(a+bx)^2\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\left(\cos\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)\right)^3}+\frac{2(2+12a\sec^{-1}(a+bx)+(1+6a^2)\sec^{-1}(a+bx)^2)\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\cos\left(\frac{1}{2}\sec^{-1}(a+bx)\right)-\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}-\frac{2\sec^{-1}(a+bx)^2\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\left(\cos\left(\frac{1}{2}\sec^{-1}(a+bx)\right)+\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)\right)^3}+\frac{\sec^{-1}(a+bx)(2+(1-6a)\sec^{-1}(a+bx))}{\left(\cos\left(\frac{1}{2}\sec^{-1}(a+bx)\right)+\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)\right)^2}-\frac{2(2-12a\sec^{-1}(a+bx)+(1+6a^2)\sec^{-1}(a+bx)^2)\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}{\cos\left(\frac{1}{2}\sec^{-1}(a+bx)\right)+\sin\left(\frac{1}{2}\sec^{-1}(a+bx)\right)}}{12b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcSec[a + b*x]^2,x]
```

```
[Out] (4 + 2*(1 + 6*a^2)*ArcSec[a + b*x]^2 + (ArcSec[a + b*x]*(2 + (-1 + 6*a)*Arc
Sec[a + b*x]))/(-1 + Sqrt[1 - (a + b*x)^(-2)]) + 24*a*Log[(a + b*x)^(-1)] +
 2*(-1 - 6*a^2)*((Pi - 2*ArcSec[a + b*x])*(Log[1 - I/E^(I*ArcSec[a + b*x])]
 - Log[1 + I/E^(I*ArcSec[a + b*x])]) - Pi*Log[Cot[(Pi + 2*ArcSec[a + b*x])/
4]] + (2*I)*(PolyLog[2, (-I)/E^(I*ArcSec[a + b*x])] - PolyLog[2, I/E^(I*Arc
Sec[a + b*x])])) + (2*ArcSec[a + b*x]^2*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec
```

```
[a + b*x]/2] - Sin[ArcSec[a + b*x]/2])^3 + (2*(2 + 12*a*ArcSec[a + b*x] + (
1 + 6*a^2)*ArcSec[a + b*x]^2)*Sin[ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/
2] - Sin[ArcSec[a + b*x]/2]) - (2*ArcSec[a + b*x]^2*Sin[ArcSec[a + b*x]/2])
/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2])^3 + (ArcSec[a + b*x]*(2
+ (1 - 6*a)*ArcSec[a + b*x]))/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]
/2])^2 - (2*(2 - 12*a*ArcSec[a + b*x] + (1 + 6*a^2)*ArcSec[a + b*x]^2)*Sin[
ArcSec[a + b*x]/2])/(Cos[ArcSec[a + b*x]/2] + Sin[ArcSec[a + b*x]/2]))/(12*
b^3)
```

Maple [A]

time = 1.16, size = 498, normalized size = 1.73

method	result
derivativedivides	$\frac{\operatorname{arcsec}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsec}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsec}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsec}(bx+a) \sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\, a (bx+a) - \cdots}{}$
default	$\frac{\operatorname{arcsec}(bx+a)^2 a^2 (bx+a) - \operatorname{arcsec}(bx+a)^2 a (bx+a)^2 + \frac{\operatorname{arcsec}(bx+a)^2 (bx+a)^3}{3} + 2 \operatorname{arcsec}(bx+a) \sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\, a (bx+a) - \cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(arcsec(b*x+a)^2*a^2*(b*x+a)-arcsec(b*x+a)^2*a*(b*x+a)^2+1/3*arcsec(b
*x+a)^2*(b*x+a)^3+2*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*a*(b*x+a)
-1/3*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)^2+1/3*I*dilog(1-
I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+1/3*b*x+1/3*a+1/3*arcsec(b*x+a)*ln(1
+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-1/3*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))-2*I*a^2*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(
1/2)))-1/3*I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*a*ln(1+(1/(b*
x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-4*a*ln(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+
2*a^2*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2*a^2*arcse
c(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*a^2*dilog(1-I*(1/(
b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*arcsec(b*x+a)*a)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/12*x^3*log(b^2*x^
2 + 2*a*b*x + a^2)^2 - integrate(1/3*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
```

```
*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 3*(b^3*x^5 + 3*a*b^2*x
^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3*x^5 + 2*a*b^2
*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^
3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*
x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^2*arcsec(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asec}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a)**2,x)
[Out] Integral(x**2*asec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^2*arcsec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acos}\left(\frac{1}{a + b\,x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x))^2,x)
[Out] int(x^2*acos(1/(a + b*x))^2, x)
```

3.29 $\int x \sec^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=154

$$-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2}-\frac{a^2\sec^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)^2-\frac{4ia\sec^{-1}(a+bx)\mathrm{ArcTan}}{b^2}$$

```
[Out] -1/2*a^2*arcsec(b*x+a)^2/b^2+1/2*x^2*arcsec(b*x+a)^2-4*I*a*arcsec(b*x+a)*ar
ctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2+ln(b*x+a)/b^2+2*I*a*polylog(2,-
I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-2*I*a*polylog(2,I*(1/(b*x+a)+I*(
1-1/(b*x+a)^2)^(1/2)))/b^2-(b*x+a)*arcsec(b*x+a)*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5366, 4511, 4275, 4266, 2317, 2438, 4269, 3556}

$$-\frac{a^2\sec^{-1}(a+bx)^2}{2b^2}-\frac{4ia\sec^{-1}(a+bx)\mathrm{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{2ia\mathrm{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{2ia\mathrm{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{\log(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x]^2,x]
```

```
[Out] -(((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x])/b^2) - (a^2*ArcSec[a
 + b*x]^2)/(2*b^2) + (x^2*ArcSec[a + b*x]^2)/2 - ((4*I)*a*ArcSec[a + b*x]*A
rcTan[E^(I*ArcSec[a + b*x])])/b^2 + Log[a + b*x]/b^2 + ((2*I)*a*PolyLog[2,
(-I)*E^(I*ArcSec[a + b*x])])/b^2 - ((2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*
x])])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int x\sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int x^2\sec(x)(-a+\sec(x))\tan(x)\,dx,x,\sec^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x(-a+\sec(x))^2\,dx,x,\sec^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \left(a^2x-2ax\sec(x)+x\sec^2(x)\right)\,dx,x,\sec^{-1}(a+bx)\right)}{b^2}$$

$$= -\frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int x\sec^2(x)\,dx,x,\sec^{-1}(a+bx)\right)}{b^2}$$

$$= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)$$

$$= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)$$

$$= -\frac{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)}{b^2} - \frac{a^2\sec^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)$$

Mathematica [A]
time = 0.10, size = 142, normalized size = 0.92

$$\frac{-\left((a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)\right)-\frac{1}{2}a^2\sec^{-1}(a+bx)^2+\frac{1}{2}b^2x^2\sec^{-1}(a+bx)^2-4ia\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)+\log(a+bx)+2ia\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)-2ia\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x]^2,x]
```

```
[Out] (-((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]) - (a^2*ArcSec[a + b*
x]^2)/2 + (b^2*x^2*ArcSec[a + b*x]^2)/2 - (4*I)*a*ArcSec[a + b*x]*ArcTan[E^
(I*ArcSec[a + b*x])] + Log[a + b*x] + (2*I)*a*PolyLog[2, (-I)*E^(I*ArcSec[a
 + b*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2
```

Maple [A]
time = 0.47, size = 227, normalized size = 1.47

method	result
derivativedivides	$-a\text{arcsec}(bx+a)^2(bx+a)-2a\,\text{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)+2a\,\text{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+\right.\right.$

default	$\frac{-a\text{arcsec}(bx+a)^2(bx+a)-2a\,\text{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)+2a\,\text{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-a*arcsec(b*x+a)^2*(b*x+a)-2*a*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2)))+2*a*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2)))+2*I*a*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2*I*a*dilog(1-
I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+1/2*arcsec(b*x+a)^2*(b*x+a)^2-(((b*x
+a)^2-1)/(b*x+a)^2)^(1/2)*arcsec(b*x+a)*(b*x+a)-ln(1/(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/8*x^2*log(b^2*x^2
 + 2*a*b*x + a^2)^2 - integrate(1/2*(2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*
b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)) + 2*(b^3*x^4 + 3*a*b^2*x^
3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (b^3*x^4 + 2*a*b^2*x^
3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 -
 a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arcsec(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a)**2,x)
```

```
[Out] Integral(x*asec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x))^2,x)
```

```
[Out] int(x*acos(1/(a + b*x))^2, x)
```

3.30 $\int \sec^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=94

$$\frac{(a+bx)\sec^{-1}(a+bx)^2}{b}+\frac{4i\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{2i\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{2i\text{PolyLog}}{}$$

```
[Out] (b*x+a)*arcsec(b*x+a)^2/b+4*I*arcsec(b*x+a)*arctan(1/(b*x+a)+I*(1-1/(b*x+a)
^2)^(1/2))/b-2*I*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b+2*I*po
lylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5360, 5324, 3842, 4266, 2317, 2438}

$$\frac{4i\sec^{-1}(a+bx)\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{2i\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{2i\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sec^{-1}(a+bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^2)/b + ((4*I)*ArcSec[a + b*x]*ArcTan[E^(I*ArcSec
[a + b*x])])/b - ((2*I)*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b + ((2*I)*
PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5324

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rule 5360

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \sec^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sec^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int x^2 \sec(x)\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{2\text{Subst}\left(\int \log(1 -\right.}{}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{(2i)\text{Subst}\left(\int \frac{\log(1}{}\right.}{}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)^2}{b} + \frac{4i\sec^{-1}(a+bx)\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{2i\text{Li}_2\left(-ie^{i\sec^{-1}(a\right.}{b}$$

Mathematica [A]

time = 0.08, size = 111, normalized size = 1.18

$$\frac{\sec^{-1}(a+bx)\left((a+bx)\sec^{-1}(a+bx)-2\log\left(1-ie^{i\sec^{-1}(a+bx)}\right)+2\log\left(1+ie^{i\sec^{-1}(a+bx)}\right)\right)-2i\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)+2i\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]^2,x]
```

```
[Out] (ArcSec[a + b*x]*((a + b*x)*ArcSec[a + b*x] - 2*Log[1 - I*E^(I*ArcSec[a + b
*x])] + 2*Log[1 + I*E^(I*ArcSec[a + b*x])]) - (2*I)*PolyLog[2, (-I)*E^(I*Ar
cSec[a + b*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b
```

Maple [A]
time = 0.26, size = 162, normalized size = 1.72

method	result
derivativedivides	$\frac{\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\right.\right.}{b}$
default	$\frac{\operatorname{arcsec}(bx+a)^2(bx+a)+2\operatorname{arcsec}(bx+a)\ln\left(1+i\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)\right)-2\operatorname{arcsec}(bx+a)\ln\left(1-i\left(\frac{1}{bx+a}+i\sqrt{1-\right.\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsec(b*x+a)^2*(b*x+a)+2*arcsec(b*x+a)*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2)))-2*arcsec(b*x+a)*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-2
*I*dilog(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*dilog(1-I*(1/(b*x+a)+
I*(1-1/(b*x+a)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 1/4*x*log(b^2*x^2 + 2*a*b
*x + a^2)^2 - integrate((2*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan(s
qrt(b*x + a + 1)*sqrt(b*x + a - 1)) + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2
 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x + (b
^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^2*x^2
 + 2*a*b*x + a^2))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2,x)
[Out] Integral(asec(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(arcsec(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2,x)
[Out] int(acos(1/(a + b*x))^2, x)
```

3.31 $\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=310

$$\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\sec^{-1}(a+bx)^2\log\left(1+e^{2i\,\mathrm{se}}\right.$$

```
[Out] -arcsec(b*x+a)^2*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)^
2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x
+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+I*arcs
ec(b*x+a)*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-2*I*arcsec(b*x+
a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))-2*I*
arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(
1/2)))-1/2*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+2*polylog(3,a*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+2*polylog(3,a*(1/(b
*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
```

Rubi [A]
time = 0.36, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5366, 4647, 4626, 3800, 2221, 2611, 2320, 6724, 4616}

$$-2i\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-2i\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+2\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+2\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+i\sec^{-1}(a+bx)\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)-\frac{1}{2}\text{Li}_3\left(-e^{2i\sec^{-1}(a+bx)}\right)-\sec^{-1}(a+bx)^2\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2/x,x]
```

```
[Out] ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] +
ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] -
ArcSec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a + b*x])] - (2*I)*ArcSec[a + b*x
]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] - (2*I)*ArcSec[
a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + I*ArcS
ec[a + b*x]*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 2*PolyLog[3, (a*E^(I*A
rcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + 2*PolyLog[3, (a*E^(I*ArcSec[a + b*x
]))/(1 + Sqrt[1 - a^2])] - PolyLog[3, -E^((2*I)*ArcSec[a + b*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4626

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n -
1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4647

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^2}{x}\,dx = \text{Subst}\left(\int \frac{x^2\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \text{Subst}\left(\int \frac{x^2\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= a\text{Subst}\left(\int \frac{x^2\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x^2\tan(x)\,dx, x, \sec^{-1}(a+\right.$$

$$= -\left(2i\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\text{Subst}\left(\int \frac{e^{ix}x^2}{1-\sqrt{1-a^2}-ae^{i\cdots}}\right.$$

$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) -$$

$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) -$$

$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) -$$

$$= \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right) -$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 813 vs. 2(310) = 620.
time = 1.55, size = 813, normalized size = 2.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^2/x,x]
```

```
[Out] ArcSec[a + b*x]^2*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])] +
 ArcSec[a + b*x]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]
- 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt[1
- a^2])*E^(I*ArcSec[a + b*x]))/a] + ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSe
c[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^2*Log[1 - ((1 + Sqrt[1
- a^2])*E^(I*ArcSec[a + b*x]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/
a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] - 2*ArcS
ec[a + b*x]^2*Log[1 + E^((2*I)*ArcSec[a + b*x])] + ArcSec[a + b*x]^2*Log[(2
*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x)] - ArcSec[a + b*x
]^2*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-
2)]))/a] + 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1
+ Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] - ArcSec
[a + b*x]^2*Log[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a +
b*x)^(-2)]))/a] - 4*ArcSec[a + b*x]*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1
- ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] -
(2*I)*ArcSec[a + b*x]*PolyLog[2, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 -
 a^2]))] - (2*I)*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 +
Sqrt[1 - a^2])] + I*ArcSec[a + b*x]*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]
+ 2*PolyLog[3, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] + 2*PolyL
og[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - PolyLog[3, -E^((2*I)
*ArcSec[a + b*x])]/2
```

Maple [**F**]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx + a)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2/x,x)
```

```
[Out] int(arcsec(b*x+a)^2/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(b*x + a)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^{2}(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2/x,x)
```

```
[Out] Integral(asec(a + b*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2/x,x)
```

```
[Out] int(acos(1/(a + b*x))^2/x, x)
```

3.32 $\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=244

$$-\frac{b\sec^{-1}(a+bx)^2}{a}-\frac{\sec^{-1}(a+bx)^2}{x}-\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsec(b*x+a)^2/a-arcsec(b*x+a)^2/x-2*I*b*arcsec(b*x+a)*ln(1-a*(1/(b*x+a
)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+2*I*b*arcse
c(b*x+a)*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(
-a^2+1)^(1/2)-2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+
1)^(1/2)))/a/(-a^2+1)^(1/2)+2*b*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1
/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5366, 4511, 4276, 3402, 2296, 2221, 2317, 2438}

$$-\frac{2b\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2b\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{b\sec^{-1}(a+bx)^2}{a}-\frac{\sec^{-1}(a+bx)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^2/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x]^2)/a) - ArcSec[a + b*x]^2/x - ((2*I)*b*ArcSec[a + b*x]
*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
+ ((2*I)*b*ArcSec[a + b*x]*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
a^2])])/(a*Sqrt[1 - a^2]) - (2*b*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 -
Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (2*b*PolyLog[2, (a*E^(I*ArcSec[a + b*x
]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^2}{x^2}\,dx = b\text{Subst}\left(\int \frac{x^2\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \frac{x}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \left(-\frac{x}{a} + \frac{x}{a(1-a\cos(x))}\right)dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} + \frac{(2b)\text{Subst}\left(\int \frac{x}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} + \frac{(4b)\text{Subst}\left(\int \frac{e^{ix}x}{-a+2e^{ix}-ae^{2ix}}\,dx, x, \sec^{-1}(a+b\right.}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{(4b)\text{Subst}\left(\int \frac{e^{ix}x}{2-2\sqrt{1-a^2}-2ae^{ix}}\,dx, x, \sec^{-1}(\right.}{\sqrt{1-a^2}}$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$

$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$
$$= -\frac{b\sec^{-1}(a+bx)^2}{a} - \frac{\sec^{-1}(a+bx)^2}{x} - \frac{2ib\sec^{-1}(a+bx)\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 686 vs. 2(244) = 488.
time = 1.51, size = 686, normalized size = 2.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]^2/x^2,x]
```

```
[Out] -((((a + b*x)*ArcSec[a + b*x]^2)/x + (2*b*(2*ArcSec[a + b*x]*ArcTanh[((-1 +
 a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] - 2*ArcCos[a^(-1)]*ArcTanh[((1
+ a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a^(-1)] - (2*I)*ArcT
anh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] + (2*I)*ArcTanh[((1 +
 a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[Sqrt[-1 + a^2]/(Sqrt[2]*Sq
```

```
rt[a]*E^((I/2)*ArcSec[a + b*x])*Sqrt[-((b*x)/(a + b*x))])] + (ArcCos[a^(-1)
] + (2*I)*(ArcTanh[((-1 + a)*Cot[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]] - ArcT
anh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]]))*Log[(Sqrt[-1 + a^2]*
E^((I/2)*ArcSec[a + b*x]))/(Sqrt[2]*Sqrt[a]*Sqrt[-((b*x)/(a + b*x))])] - (A
rcCos[a^(-1)] - (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^
2]])*Log[((-1 + a)*(I + I*a + Sqrt[-1 + a^2])*(-I + Tan[ArcSec[a + b*x]/2])
)/(a*(-1 + a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] - (ArcCos[a^(-1)] +
 (2*I)*ArcTanh[((1 + a)*Tan[ArcSec[a + b*x]/2])/Sqrt[-1 + a^2]])*Log[((-1 +
 a)*(-I - I*a + Sqrt[-1 + a^2])*(I + Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a +
Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + I*(-PolyLog[2, ((1 - I*Sqrt[-1 +
 a^2])*(1 - a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a + Sqrt[-
1 + a^2]*Tan[ArcSec[a + b*x]/2]))] + PolyLog[2, ((1 + I*Sqrt[-1 + a^2])*(1
- a + Sqrt[-1 + a^2]*Tan[ArcSec[a + b*x]/2]))/(a*(-1 + a + Sqrt[-1 + a^2]*T
an[ArcSec[a + b*x]/2]))])))/Sqrt[-1 + a^2])/a)
```

Maple [A]
time = 0.80, size = 336, normalized size = 1.38

method	result
derivativedivides	$b\left(-\frac{(bx+a)\operatorname{arcsec}(bx+a)^2}{abx}-\frac{2i\sqrt{-a^2+1}\,\operatorname{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}\right.$
default	$b\left(-\frac{(bx+a)\operatorname{arcsec}(bx+a)^2}{abx}-\frac{2i\sqrt{-a^2+1}\,\operatorname{arcsec}(bx+a)\ln\left(\frac{-a\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)+\sqrt{-a^2+1}+1}{1+\sqrt{-a^2+1}}\right)}{a(a^2-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-(b*x+a)*arcsec(b*x+a)^2/a/b/x-2*I*(-a^2+1)^(1/2)/a/(a^2-1)*arcsec(b*x+a
)*ln((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1+(-a^2+1)^
(1/2)))+2*I*(-a^2+1)^(1/2)/a/(a^2-1)*arcsec(b*x+a)*ln((a*(1/(b*x+a)+I*(1-1/
(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2)))-2*(-a^2+1)^(1/2)/a
/(a^2-1)*dilog((-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+(-a^2+1)^(1/2)+1)/(1
+(-a^2+1)^(1/2)))+2*(-a^2+1)^(1/2)/a/(a^2-1)*dilog((a*(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2))+(-a^2+1)^(1/2)-1)/(-1+(-a^2+1)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - 4*x*integrate((2*sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1)) - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2
 - (b^3*x^3 + 2*a*b^2*x^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 +
(3*a^2 - 1)*b*x - a)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))/(b^3*x^5 +
 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x) - log(b^2*x^2 + 2*a*b
*x + a^2)^2)/x
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^2/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**2/x**2,x)
```

```
[Out] Integral(asec(a + b*x)**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^2/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))^2/x^2, x)
```

3.33 $\int x^2 \sec^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=494

$$\frac{(a+bx)\sec^{-1}(a+bx)}{b^3}-\frac{3ia\sec^{-1}(a+bx)^2}{b^3}+\frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\,\sec^{-1}(a+bx)^2}{b^3}-\frac{(a+bx)^2\sqrt{1-}}{}$$

```
[Out] (b*x+a)*arcsec(b*x+a)/b^3-6*I*a^2*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(
1-1/(b*x+a)^2)^(1/2)))/b^3+1/3*a^3*arcsec(b*x+a)^3/b^3+1/3*x^3*arcsec(b*x+a
)^3-I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+6
*I*a^2*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-a
rctanh((1-1/(b*x+a)^2)^(1/2))/b^3+6*a*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/
(b*x+a)^2)^(1/2))^2)/b^3+I*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2))/b^3-3*I*a*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^3+I
*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-3*I*a*a
rcsec(b*x+a)^2/b^3+6*I*a^2*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2))/b^3+polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3+6*a^2*p
olylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-polylog(3,I*(1/(b*x+a)
+I*(1-1/(b*x+a)^2)^(1/2)))/b^3-6*a^2*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^
2)^(1/2)))/b^3+3*a*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3-1/2*(b
*x+a)^2*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.31, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {5366, 4511, 4275, 4266, 2611, 2320, 6724, 4269, 3800, 2221, 2317, 2438, 4271, 3855}

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x])/b^3 - ((3*I)*a*ArcSec[a + b*x]^2)/b^3 + (3*a*(a
 + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/b^3 - ((a + b*x)^2*Sqrt
[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/(2*b^3) + (a^3*ArcSec[a + b*x]^3)/(
3*b^3) + (x^3*ArcSec[a + b*x]^3)/3 + (I*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSe
c[a + b*x])])/b^3 + ((6*I)*a^2*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x
])])/b^3 - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]]/b^3 + (6*a*ArcSec[a + b*x]*Log
[1 + E^((2*I)*ArcSec[a + b*x])])/b^3 - (I*ArcSec[a + b*x]*PolyLog[2, (-I)*E
^(I*ArcSec[a + b*x])])/b^3 - ((6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, (-I)*E^(
I*ArcSec[a + b*x])])/b^3 + (I*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a +
b*x])])/b^3 + ((6*I)*a^2*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])
])/b^3 - ((3*I)*a*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])/b^3 + PolyLog[3,
```

```
(-I)*E^(I*ArcSec[a + b*x])]/b^3 + (6*a^2*PolyLog[3, (-I)*E^(I*ArcSec[a + b*
x])])/b^3 - PolyLog[3, I*E^(I*ArcSec[a + b*x])]/b^3 - (6*a^2*PolyLog[3, I*E
^(I*ArcSec[a + b*x])])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \sec^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int x^3 \sec(x)(-a+\sec(x))^2 \tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int x^2(-a+\sec(x))^3\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{1}{3}x^3 \sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int \left(-a^3x^2+3a^2x^2\sec(x)-3ax^2\sec^2(x)+x^2\sec^3(x)\right)\right.}{b^3}$$

$$= \frac{a^3\sec^{-1}(a+bx)^3}{3b^3} + \frac{1}{3}x^3\sec^{-1}(a+bx)^3 - \frac{\text{Subst}\left(\int x^2\sec^3(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)^2}{b^3} - \frac{(a+bx)^2}{}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(}{b^3}$$

$$= \frac{(a+bx)\sec^{-1}(a+bx)}{b^3} - \frac{3ia\sec^{-1}(a+bx)^2}{b^3} + \frac{3a(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(}{b^3}$$

Mathematica [A]
time = 0.36, size = 442, normalized size = 0.89

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x] + 3*a*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[
a + b*x]^2 - ((a + b*x)^2*Sqrt[1 - (a + b*x)^(-2)]*ArcSec[a + b*x]^2)/2 + (
a^3*ArcSec[a + b*x]^3)/3 + (b^3*x^3*ArcSec[a + b*x]^3)/3 + I*ArcSec[a + b*x
]^2*ArcTan[E^(I*ArcSec[a + b*x])] + (6*I)*a^2*ArcSec[a + b*x]^2*ArcTan[E^(I
*ArcSec[a + b*x])] - ArcTanh[Sqrt[1 - (a + b*x)^(-2)]] - I*ArcSec[a + b*x]*
PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + I*ArcSec[a + b*x]*PolyLog[2, I*E^(
I*ArcSec[a + b*x])] - (3*I)*a*(ArcSec[a + b*x]*(ArcSec[a + b*x] + (2*I)*Log
[1 + E^((2*I)*ArcSec[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])
+ PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] + 6*a^2*((-I)*ArcSec[a + b*x]*Poly
Log[2, (-I)*E^(I*ArcSec[a + b*x])] + PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])]
) + (6*I)*a^2*(ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])] + I*Poly
Log[3, I*E^(I*ArcSec[a + b*x])]) - PolyLog[3, I*E^(I*ArcSec[a + b*x])])/b^3
```

Maple [A]
time = 1.10, size = 716, normalized size = 1.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/6*arcsec(b*x+a)*(6*arcsec(b*x+a)^2*a^2*(b*x+a)-6*arcsec(b*x+a)^2*a
*(b*x+a)^2+2*arcsec(b*x+a)^2*(b*x+a)^3+18*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x
+a)^2)^(1/2)*a*(b*x+a)-3*arcsec(b*x+a)*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x
+a)^2+18*I*a*arcsec(b*x+a)+6*b*x+6*a)-I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+
a)+I*(1-1/(b*x+a)^2)^(1/2)))+2*I*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))+
3*a^2*arcsec(b*x+a)^2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+6*a^2*pol
ylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-6*I*arcsec(b*x+a)^2*a-6*I*a^
2*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-3*a^2*arc
sec(b*x+a)^2*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-6*a^2*polylog(3,I*
(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+6*a*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1
-1/(b*x+a)^2)^(1/2))^2)+6*I*a^2*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1
/(b*x+a)^2)^(1/2)))+I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2)))+1/2*arcsec(b*x+a)^2*ln(1+I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-3
*I*a*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+polylog(3,-I*(1/(b*x
+a)+I*(1-1/(b*x+a)^2)^(1/2)))-1/2*arcsec(b*x+a)^2*ln(1-I*(1/(b*x+a)+I*(1-1/
(b*x+a)^2)^(1/2)))-polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 1/4*x^3*arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate
(1/4*((4*b*x^3*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^3*log(b^
2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(3*(b^3*x
^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a)^2 - (b^3
*x^5 + 2*a*b^2*x^4 + (a^2 - 1)*b*x^3 + 3*(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 -
1)*b*x^3 + (a^3 - a)*x^2)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arcta
n(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a
^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^2*arcsec(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asec}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(b*x+a)**3,x)
[Out] Integral(x**2*asec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2*arcsec(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(1/(a + b*x))^3,x)
[Out] int(x^2*acos(1/(a + b*x))^3, x)
```

3.34 $\int x \sec^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=278

$$\frac{3i\sec^{-1}(a+bx)^2}{2b^2}-\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)^2}{2b^2}-\frac{a^2\sec^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)^3-\frac{6ia\,\mathrm{s}}{}$$

```
[Out] 3/2*I*arcsec(b*x+a)^2/b^2-1/2*a^2*arcsec(b*x+a)^3/b^2+1/2*x^2*arcsec(b*x+a)
^3-6*I*a*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/b^2-3*ar
csec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2+6*I*a*arcsec(b*
x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2-6*I*a*arcsec(b*x
+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b^2+3/2*I*polylog(2,-(
1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/b^2-6*a*polylog(3,-I*(1/(b*x+a)+I*(1-
1/(b*x+a)^2)^(1/2)))/b^2+6*a*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)
))/b^2-3/2*(b*x+a)*arcsec(b*x+a)^2*(1-1/(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.20, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5366, 4511, 4275, 4266, 2611, 2320, 6724, 4269, 3800, 2221, 2317, 2438}

$$-\frac{a^2\sec^{-1}(a+bx)^3}{2b^2}-\frac{6ia\sec^{-1}(a+bx)^2\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{6ia\sec^{-1}(a+bx)\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{6ia\sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{3i\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)}{2b^2}-\frac{6a\text{Li}_3\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{6a\text{Li}_3\left(ie^{i\sec^{-1}(a+bx)}\right)}{b^2}-\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\ \sec^{-1}(a+bx)^2}{2b^2}+\frac{3i\sec^{-1}(a+bx)^2}{2b^2}-\frac{3\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{b^2}+\frac{1}{2}x^2\sec^{-1}(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*ArcSec[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*A
rcSec[a + b*x]^2)/(2*b^2) - (a^2*ArcSec[a + b*x]^3)/(2*b^2) + (x^2*ArcSec[a
 + b*x]^3)/2 - ((6*I)*a*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])])/b^
2 - (3*ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a + b*x])])/b^2 + ((6*I)*a*A
rcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])])/b^2 - ((6*I)*a*ArcSe
c[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b^2 + (((3*I)/2)*PolyLog[2,
 -E^((2*I)*ArcSec[a + b*x])])/b^2 - (6*a*PolyLog[3, (-I)*E^(I*ArcSec[a + b*
x])])/b^2 + (6*a*PolyLog[3, I*E^(I*ArcSec[a + b*x])])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\sec^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int x^3\sec(x)(-a+\sec(x))\tan(x)\,dx,x,\sec^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int x^2(-a+\sec(x))^2\,dx,x,\sec^{-1}(a+bx)\right)}{2b^2}$$

$$= \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int \left(a^2x^2-2ax^2\sec(x)+x^2\sec^2(x)\right)\,dx,x,\sec^{-1}(a+b\right.}{2b^2}$$

$$= -\frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+bx)^3 - \frac{3\text{Subst}\left(\int x^2\sec^2(x)\,dx,x,\sec^{-1}(a+bx\right.}{2b^2}$$

$$= -\frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2} + \frac{1}{2}x^2\sec^{-1}(a+b$$

$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2}$$

$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2}$$

$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2}$$

$$= \frac{3i\sec^{-1}(a+bx)^2}{2b^2} - \frac{3(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2}{2b^2} - \frac{a^2\sec^{-1}(a+bx)^3}{2b^2}$$

Mathematica [A]
time = 0.29, size = 248, normalized size = 0.89

$$\frac{1}{2}\left(x^2\sec^{-1}(a+bx)^3 - \frac{3\left(-i\sec^{-1}(a+bx)^2+(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}\sec^{-1}(a+bx)^2+\frac{1}{3}a^2\sec^{-1}(a+bx)^3+4ia\sec^{-1}(a+bx)^2\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)+2\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)+4ia\sec^{-1}(a+bx)\text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)-i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)+4a\left(-i\sec^{-1}(a+bx)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right)\right)-4a\text{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)\right)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSec[a + b*x]^3,x]
```

```
[Out] (x^2*ArcSec[a + b*x]^3 - (3*((-I)*ArcSec[a + b*x]^2 + (a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)]*ArcSec[a + b*x]^2 + (a^2*ArcSec[a + b*x]^3)/3 + (4*I)*a*ArcSe
c[a + b*x]^2*ArcTan[E^(I*ArcSec[a + b*x])] + 2*ArcSec[a + b*x]*Log[1 + E^((
2*I)*ArcSec[a + b*x])] + (4*I)*a*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a
 + b*x])] - I*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 4*a*((-I)*ArcSec[a +
```

```
 b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b*x])] + PolyLog[3, (-I)*E^(I*ArcSec[
a + b*x])]) - 4*a*PolyLog[3, I*E^(I*ArcSec[a + b*x])]))/b^2)/2
```

Maple [A]
time = 1.01, size = 379, normalized size = 1.36

method	result
derivativedivides	$-\frac{\operatorname{arcsec}(bx+a)^2\left(2\operatorname{arcsec}(bx+a)a(bx+a)-\operatorname{arcsec}(bx+a)(bx+a)^2+3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}+6ia\operatorname{arcsec}(bx+a)\operatorname{polylog}\Big(2$
default	$-\frac{\operatorname{arcsec}(bx+a)^2\left(2\operatorname{arcsec}(bx+a)a(bx+a)-\operatorname{arcsec}(bx+a)(bx+a)^2+3\sqrt{\frac{(bx+a)^2-1}{(bx+a)^2}}\,(bx+a)+3i\right)}{2}+6ia\operatorname{arcsec}(bx+a)\operatorname{polylog}\Big(2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2*arcsec(b*x+a)^2*(2*arcsec(b*x+a)*a*(b*x+a)-arcsec(b*x+a)*(b*x+a
)^2+3*(((b*x+a)^2-1)/(b*x+a)^2)^(1/2)*(b*x+a)+3*I)+6*I*a*arcsec(b*x+a)*poly
log(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-3*a*arcsec(b*x+a)^2*ln(1+I*(1
/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))-6*a*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2)))-6*I*a*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2)))+3*a*arcsec(b*x+a)^2*ln(1-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+6*a
*polylog(3,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))+3*I*arcsec(b*x+a)^2-3*arc
sec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+3/2*I*polylog(2,-(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/8*x^2*arctan(sqrt
(b*x + a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate
(3/8*((4*b*x^2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x^2*log(b^
2*x^2 + 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*(2*(b^3*x
^4 + 3*a*b^2*x^3 + (3*a^2 - 1)*b*x^2 + (a^3 - a)*x)*log(b*x + a)^2 - (b^3*x
^4 + 2*a*b^2*x^3 + (a^2 - 1)*b*x^2 + 2*(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2 - 1)
*b*x^2 + (a^3 - a)*x)*log(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sq
rt(b*x + a + 1)*sqrt(b*x + a - 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 -
 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x*arcsec(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asec}^{3}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(b*x+a)**3,x)
[Out] Integral(x*asec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x*arcsec(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(1/(a + b*x))^3,x)
[Out] int(x*acos(1/(a + b*x))^3, x)
```

3.35 $\int \sec^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=154

$$\frac{(a+bx)\sec^{-1}(a+bx)^3}{b}+\frac{6i\sec^{-1}(a+bx)^2\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{6i\sec^{-1}(a+bx)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right)}{b}$$

```
[Out] (b*x+a)*arcsec(b*x+a)^3/b+6*I*arcsec(b*x+a)^2*arctan(1/(b*x+a)+I*(1-1/(b*x+
a)^2)^(1/2))/b-6*I*arcsec(b*x+a)*polylog(2,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2)))/b+6*I*arcsec(b*x+a)*polylog(2,I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))
)/b+6*polylog(3,-I*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b-6*polylog(3,I*(1/
(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2)))/b
```

Rubi [A]

time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5360, 5324, 3842, 4266, 2611, 2320, 6724}

$$\frac{6i\sec^{-1}(a+bx)^2\text{ArcTan}\left(e^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6i\sec^{-1}(a+bx)\text{Li}_2\left(ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{6\text{Li}_3\left(-ie^{i\sec^{-1}(a+bx)}\right)}{b}-\frac{6\text{Li}_3\left(ie^{i\sec^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sec^{-1}(a+bx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^3)/b + ((6*I)*ArcSec[a + b*x]^2*ArcTan[E^(I*ArcS
ec[a + b*x])])/b - ((6*I)*ArcSec[a + b*x]*PolyLog[2, (-I)*E^(I*ArcSec[a + b
*x])])/b + ((6*I)*ArcSec[a + b*x]*PolyLog[2, I*E^(I*ArcSec[a + b*x])])/b +
(6*PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])])/b - (6*PolyLog[3, I*E^(I*ArcSec[
a + b*x])])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5324

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/c, Subst[I
nt[(a + b*x)^n*Sec[x]*Tan[x], x], x, ArcSec[c*x]], x] /; FreeQ[{a, b, c, n}
, x] && IGtQ[n, 0]
```

Rule 5360

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcSec[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \sec^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sec^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int x^3\sec(x)\tan(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} - \frac{3\text{Subst}\left(\int x^2\sec(x)\,dx, x, \sec^{-1}(a+bx)\right)}{b}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} + \frac{6\text{Subst}\left(\int x\log\right.}{}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx}{}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx}{}$$
$$= \frac{(a+bx)\sec^{-1}(a+bx)^3}{b} + \frac{6i\sec^{-1}(a+bx)^2\tan^{-1}\left(e^{i\sec^{-1}(a+bx)}\right)}{b} - \frac{6i\sec^{-1}(a+bx}{}$$

Mathematica [A]
time = 0.07, size = 160, normalized size = 1.04

$$\frac{(a+bx)\sec^{-1}(a+bx)^3 - 3\sec^{-1}(a+bx)^2\left(\log\left(1-ie^{i\sec^{-1}(a+bx)}\right) - \log\left(1+ie^{i\sec^{-1}(a+bx)}\right)\right) - 6i\sec^{-1}(a+bx)\left(\text{PolyLog}\left(2,-ie^{i\sec^{-1}(a+bx)}\right) - \text{PolyLog}\left(2,ie^{i\sec^{-1}(a+bx)}\right)\right) + 6\left(\text{PolyLog}\left(3,-ie^{i\sec^{-1}(a+bx)}\right) - \text{PolyLog}\left(3,ie^{i\sec^{-1}(a+bx)}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]^3,x]
```

```
[Out] ((a + b*x)*ArcSec[a + b*x]^3 - 3*ArcSec[a + b*x]^2*(Log[1 - I*E^(I*ArcSec[a
 + b*x])] - Log[1 + I*E^(I*ArcSec[a + b*x])]) - (6*I)*ArcSec[a + b*x]*(Poly
Log[2, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[2, I*E^(I*ArcSec[a + b*x])]) +
 6*(PolyLog[3, (-I)*E^(I*ArcSec[a + b*x])] - PolyLog[3, I*E^(I*ArcSec[a + b
*x])]))/b
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \operatorname{arcsec}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3,x)
```

```
[Out] int(arcsec(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3/4*x*arctan(sqrt(b*x + a
 + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - integrate(3/4*((4
*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2 + 2*a*
b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*((b^3*x^3 + 3*a*b^2*x
^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 - (b^3*x^3 + 2*a*b^2*x^2 + (
a^2 - 1)*b*x + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x
+ a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a -
 1)))/(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}^{3}(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3,x)
```

```
[Out] Integral(asec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3,x)
```

```
[Out] int(acos(1/(a + b*x))^3, x)
```

3.36 $\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx$

Optimal. Leaf size=430

$$\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)-\sec^{-1}(a+bx)^3\log\left(1+e^{2i\,\mathrm{se}}\right.$$

```
[Out] -arcsec(b*x+a)^3*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)+arcsec(b*x+a)^
3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))+arcsec(b*x
+a)^3*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))+3/2*I*
arcsec(b*x+a)^2*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)-3*I*arcse
c(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2
)))-3*I*arcsec(b*x+a)^2*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+
(-a^2+1)^(1/2)))-3/2*arcsec(b*x+a)*polylog(3,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^
(1/2))^2)+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(
1-(-a^2+1)^(1/2)))+6*arcsec(b*x+a)*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)
^(1/2))/(1+(-a^2+1)^(1/2)))-3/4*I*polylog(4,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(
1/2))^2)+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1
/2)))+6*I*polylog(4,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)
))
```

Rubi [A]
time = 0.40, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5366, 4647, 4626, 3800, 2221, 2611, 6744, 2320, 6724, 4616}

$$-3i\sec^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)-3i\sec^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+6\sec^{-1}(a+bx)\mathrm{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6\sec^{-1}(a+bx)\mathrm{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+6i\mathrm{Li}_4\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+6i\mathrm{Li}_4\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)+\sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)+\frac{3}{2}i\sec^{-1}(a+bx)^2\mathrm{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)-\frac{3}{2}\sec^{-1}(a+bx)\mathrm{Li}_3\left(-e^{2i\sec^{-1}(a+bx)}\right)-\frac{3}{4}i\mathrm{Li}_4\left(-e^{2i\sec^{-1}(a+bx)}\right)-\sec^{-1}(a+bx)^3\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3/x,x]
```

```
[Out] ArcSec[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] +
ArcSec[a + b*x]^3*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] -
ArcSec[a + b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] - (3*I)*ArcSec[a + b*x
]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] - (3*I)*ArcSe
c[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ((
3*I)/2)*ArcSec[a + b*x]^2*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])] + 6*ArcSec
[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + 6*Arc
Sec[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - (3
*ArcSec[a + b*x]*PolyLog[3, -E^((2*I)*ArcSec[a + b*x])])/2 + (6*I)*PolyLog[
4, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])] + (6*I)*PolyLog[4, (a*E^(
I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ((3*I)/4)*PolyLog[4, -E^((2*I)*A
rcSec[a + b*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4626

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tan[(c_.) + (d_.)*(x_)]^(n_.))/(Cos[(c_.) +
 (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Tan[c +
d*x]^n, x], x] - Dist[b/a, Int[(e + f*x)^m*Sin[c + d*x]*(Tan[c + d*x]^(n -
1)/(a + b*Cos[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && IGtQ[n, 0]
```

Rule 4647

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*(F_)[(c_.) + (d_.)*(x_)]^(n_.)*(G_)[(c_.) +
 (d_.)*(x_)]^(p_.))/((a_) + (b_.)*Sec[(c_.) + (d_.)*(x_)]), x_Symbol] :> In
t[(e + f*x)^m*Cos[c + d*x]*F[c + d*x]^n*(G[c + d*x]^p/(b + a*Cos[c + d*x]))
, x] /; FreeQ[{a, b, c, d, e, f}, x] && TrigQ[F] && TrigQ[G] && IntegersQ[m
, n, p]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^3}{x}\,dx = \text{Subst}\left(\int \frac{x^3\sec(x)\tan(x)}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= \text{Subst}\left(\int \frac{x^3\tan(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$

$$= a\text{Subst}\left(\int \frac{x^3\sin(x)}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right) + \text{Subst}\left(\int x^3\tan(x)\,dx, x, \sec^{-1}(a\right.$$

$$= -\left(2i\text{Subst}\left(\int \frac{e^{2ix}x^3}{1+e^{2ix}}\,dx, x, \sec^{-1}(a+bx)\right)\right) - (ia)\text{Subst}\left(\int \frac{e^{ix}x^3}{1-\sqrt{1-a^2}-ae\ldots}\right.$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

$$= \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right) + \sec^{-1}(a+bx)^3\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1+\sqrt{1-a^2}}\right)$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1058 vs. 2(430) = 860.
time = 2.66, size = 1058, normalized size = 2.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSec[a + b*x]^3/x,x]
```

```
[Out] 2*ArcSec[a + b*x]^3*Log[1 + (a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2])]
 + ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a
] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqr
t[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 2*ArcSec[a + b*x]^3*Log[1 - (a*E^(I
*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] + ArcSec[a + b*x]^3*Log[1 - ((1 + S
qrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a] + 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(
-1 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt[1 - a^2])*E^(I*ArcSec[a + b*x]))/a]
```

```
- 3*ArcSec[a + b*x]^3*Log[1 + E^((2*I)*ArcSec[a + b*x])] + 2*ArcSec[a + b*x
]^3*Log[(2*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(a + b*x)] - ArcS
ec[a + b*x]^3*Log[1 + (a*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/(-1
 + Sqrt[1 - a^2])] - ArcSec[a + b*x]^3*Log[1 + ((-1 + Sqrt[1 - a^2])*((a +
b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] + 6*ArcSec[a + b*x]^2*ArcSin[Sq
rt[(-1 + a)/a]/Sqrt[2]]*Log[1 + ((-1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*S
qrt[1 - (a + b*x)^(-2)]))/a] - ArcSec[a + b*x]^3*Log[1 - (a*((a + b*x)^(-1)
 + I*Sqrt[1 - (a + b*x)^(-2)]))/(1 + Sqrt[1 - a^2])] - ArcSec[a + b*x]^3*Lo
g[1 - ((1 + Sqrt[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a
] - 6*ArcSec[a + b*x]^2*ArcSin[Sqrt[(-1 + a)/a]/Sqrt[2]]*Log[1 - ((1 + Sqrt
[1 - a^2])*((a + b*x)^(-1) + I*Sqrt[1 - (a + b*x)^(-2)]))/a] - (3*I)*ArcSec
[a + b*x]^2*PolyLog[2, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] -
 (3*I)*ArcSec[a + b*x]^2*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 -
 a^2])] + ((3*I)/2)*ArcSec[a + b*x]^2*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])
] + 6*ArcSec[a + b*x]*PolyLog[3, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 -
 a^2]))] + 6*ArcSec[a + b*x]*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt
[1 - a^2])] - (3*ArcSec[a + b*x]*PolyLog[3, -E^((2*I)*ArcSec[a + b*x])])/2
+ (6*I)*PolyLog[4, -((a*E^(I*ArcSec[a + b*x]))/(-1 + Sqrt[1 - a^2]))] + (6*
I)*PolyLog[4, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])] - ((3*I)/4)*Po
lyLog[4, -E^((2*I)*ArcSec[a + b*x])]
```

Maple [F]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3/x,x)
```

```
[Out] int(arcsec(b*x+a)^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsec(b*x + a)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^3(a + bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3/x,x)
```

```
[Out] Integral(asec(a + b*x)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acos}\left(\frac{1}{a+b\,x}\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3/x,x)
```

```
[Out] int(acos(1/(a + b*x))^3/x, x)
```

3.37 $\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=362

$$-\frac{b\sec^{-1}(a+bx)^3}{a}-\frac{\sec^{-1}(a+bx)^3}{x}-\frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\right.}{a\sqrt{1-a^2}}$$

```
[Out] -b*arcsec(b*x+a)^3/a-arcsec(b*x+a)^3/x-3*I*b*arcsec(b*x+a)^2*ln(1-a*(1/(b*x
+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+3*I*b*arc
sec(b*x+a)^2*ln(1-a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))
/a/(-a^2+1)^(1/2)-6*b*arcsec(b*x+a)*polylog(2,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2
)^(1/2))/(1-(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)+6*b*arcsec(b*x+a)*polylog(2,a
*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a^2+1)^(1/2)))/a/(-a^2+1)^(1/2)-6
*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1-(-a^2+1)^(1/2)))/a/
(-a^2+1)^(1/2)+6*I*b*polylog(3,a*(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))/(1+(-a
^2+1)^(1/2)))/a/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$,
Rules used = {5366, 4511, 4276, 3402, 2296, 2221, 2611, 2320, 6724}

$$-\frac{6b\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6b\sec^{-1}(a+bx)\text{Li}_2\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{6ib\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{6ib\text{Li}_3\left(\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}}+\frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{\sqrt{1-a^2}+1}\right)}{a\sqrt{1-a^2}}-\frac{b\sec^{-1}(a+bx)^3}{a}-\frac{\sec^{-1}(a+bx)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]^3/x^2,x]
```

```
[Out] -((b*ArcSec[a + b*x]^3)/a) - ArcSec[a + b*x]^3/x - ((3*I)*b*ArcSec[a + b*x]
^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]
) + ((3*I)*b*ArcSec[a + b*x]^2*Log[1 - (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[
1 - a^2])])/(a*Sqrt[1 - a^2]) - (6*b*ArcSec[a + b*x]*PolyLog[2, (a*E^(I*Arc
Sec[a + b*x]))/(1 - Sqrt[1 - a^2])])/(a*Sqrt[1 - a^2]) + (6*b*ArcSec[a + b*
x]*PolyLog[2, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt[1 - a^2])])/(a*Sqrt[1 - a
^2]) - ((6*I)*b*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 - Sqrt[1 - a^2])])/
(a*Sqrt[1 - a^2]) + ((6*I)*b*PolyLog[3, (a*E^(I*ArcSec[a + b*x]))/(1 + Sqrt
[1 - a^2])])/(a*Sqrt[1 - a^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4511

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*Sec[(c
_.) + (d_.)*(x_)])^(n_.)*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*
x)^m*((a + b*Sec[c + d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n +
 1))), Int[(e + f*x)^(m - 1)*(a + b*Sec[c + d*x])^(n + 1), x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5366

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d^(m + 1), Subst[Int[(a + b*x)^p*Sec[x]*Tan[x]*(d*
e - c*f + f*Sec[x])^m, x], x, ArcSec[c + d*x]], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && IGtQ[p, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)^3}{x^2}\,dx = b\text{Subst}\left(\int \frac{x^3\sec(x)\tan(x)}{(-a+\sec(x))^2}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \frac{x^2}{-a+\sec(x)}\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{\sec^{-1}(a+bx)^3}{x} + (3b)\text{Subst}\left(\int \left(-\frac{x^2}{a} + \frac{x^2}{a(1-a\cos(x))}\right)\,dx, x, \sec^{-1}(a+bx)\right)$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{1-a\cos(x)}\,dx, x, \sec^{-1}(a+bx)\right)}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} + \frac{(6b)\text{Subst}\left(\int \frac{e^{ix}x^2}{-a+2e^{ix}-ae^{2ix}}\,dx, x, \sec^{-1}(a+b\right.}{a}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{(6b)\text{Subst}\left(\int \frac{e^{ix}x^2}{2-2\sqrt{1-a^2}-2ae^{ix}}\,dx, x, \sec^{-1}(\right.}{\sqrt{1-a^2}}$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$
$$= -\frac{b\sec^{-1}(a+bx)^3}{a} - \frac{\sec^{-1}(a+bx)^3}{x} - \frac{3ib\sec^{-1}(a+bx)^2\log\left(1-\frac{ae^{i\sec^{-1}(a+bx)}}{1-\sqrt{1-a^2}}\right)}{a\sqrt{1-a^2}} +$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[ArcSec[a + b*x]^3/x^2,x]
[Out] $Aborted
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsec}(bx+a)^3}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsec(b*x+a)^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^3 - 3*arctan(sqrt(b*x +
 a + 1)*sqrt(b*x + a - 1))*log(b^2*x^2 + 2*a*b*x + a^2)^2 - 4*x*integrate(3
/4*((4*b*x*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))^2 - b*x*log(b^2*x^2
+ 2*a*b*x + a^2)^2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 4*((b^3*x^3 + 3*a
*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*log(b*x + a)^2 + (b^3*x^3 + 2*a*b^2*x
^2 + (a^2 - 1)*b*x - (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2 - 1)*b*x - a)*lo
g(b*x + a))*log(b^2*x^2 + 2*a*b*x + a^2))*arctan(sqrt(b*x + a + 1)*sqrt(b*x
 + a - 1)))/(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2 - 1)*b*x^3 + (a^3 - a)*x^2), x)
)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsec(b*x + a)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asec}^3(a+bx)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)**3/x**2,x)
```

```
[Out] Integral(asec(a + b*x)**3/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsec(b*x + a)^3/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))^3/x^2,x)
```

```
[Out] int(acos(1/(a + b*x))^3/x^2, x)
```

3.38 $\int x(a+b\sec^{-1}(c+dx^2))\,dx$

Optimal. Leaf size=58

$$\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}(c+dx^2)}{2d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^2)^2}}\right)}{2d}$$

[Out] 1/2*a*x^2+1/2*b*(d*x^2+c)*arcsec(d*x^2+c)/d-1/2*b*arctanh((1-1/(d*x^2+c)^2)^(1/2))/d

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6847, 5358, 379, 272, 65, 212}

$$\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}(c+dx^2)}{2d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^2)^2}}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*ArcSec[c + d*x^2]),x]

[Out] (a*x^2)/2 + (b*(c + d*x^2)*ArcSec[c + d*x^2])/(2*d) - (b*ArcTanh[Sqrt[1 - (c + d*x^2)^(-2)]])/(2*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x\left(a+b\sec^{-1}\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\text{Subst}\left(\int\left(a+b\sec^{-1}(c+dx)\right)\,dx,x,x^2\right)$$

$$=\frac{ax^2}{2}+\frac{1}{2}b\text{Subst}\left(\int\sec^{-1}(c+dx)\,dx,x,x^2\right)$$

$$=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{1}{2}b\text{Subst}\left(\int\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right.$$

$$=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx,x,c+dx^2\right)}{2d}$$

$$=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{(c+dx^2)^2}\right)}{4d}$$

$$=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{(c+dx^2)}}\right.}{2d}$$

$$=\frac{ax^2}{2}+\frac{b(c+dx^2)\sec^{-1}\left(c+dx^2\right)}{2d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^2)^2}}\right)}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 385 vs. $2(58) = 116$.
time = 0.65, size = 385, normalized size = 6.64

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\sec^{-1}\left(c+dx^2\right)-\frac{b(c+dx^2)\sqrt{\frac{-1+c^2+2cdx^2+d^2x^4}{(c+dx^2)^2}}\left(2c\left(d+\sqrt{d^2}\right)\text{ArcTan}\left(c+\sqrt{d^2}\,x^2-\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)+2c\left(-d+\sqrt{d^2}\right)\text{ArcTan}\left(c-\sqrt{d^2}\,x^2+\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)-\sqrt{d^2}\log\left(-c-\sqrt{d^2}\,x^2+\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)+d\log\left(c-\sqrt{d^2}\,x^2+\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)-\sqrt{d^2}\log\left(c-\sqrt{d^2}\,x^2+\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)-d\log\left(cd^2+(d^2)^{3/2}x^2-d^2\sqrt{-1+c^2+2cdx^2+d^2x^4}\right)\right)}{4d^2\sqrt{-1+c^2+2cdx^2+d^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSec[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcSec[c + d*x^2])/2 - (b*(c + d*x^2)*Sqrt[(-1 + c^2 + 2
*c*d*x^2 + d^2*x^4)/(c + d*x^2)^2]*(2*c*(d + Sqrt[d^2])*ArcTan[c + Sqrt[d^2
]*x^2 - Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] + 2*c*(-d + Sqrt[d^2])*ArcTan
[c - Sqrt[d^2]*x^2 + Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] - Sqrt[d^2]*Log[
-c - Sqrt[d^2]*x^2 + Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] + d*Log[c - Sqrt
[d^2]*x^2 + Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] - Sqrt[d^2]*Log[c - Sqrt[
d^2]*x^2 + Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]] - d*Log[c*d^2 + (d^2)^(3/2
)*x^2 - d^2*Sqrt[-1 + c^2 + 2*c*d*x^2 + d^2*x^4]]))/(4*d^2*Sqrt[-1 + c^2 +
2*c*d*x^2 + d^2*x^4])
```

Maple [A]
time = 0.08, size = 67, normalized size = 1.16

method	result	size
derivativedivides	$\frac{(d\,x^2+c)a+\operatorname{arcsec}(d\,x^2+c)b(d\,x^2+c)-\ln\left(d\,x^2+c+(d\,x^2+c)\sqrt{1-\frac{1}{(d\,x^2+c)^2}}\right)b}{2d}$	67
default	$\frac{(d\,x^2+c)a+\operatorname{arcsec}(d\,x^2+c)b(d\,x^2+c)-\ln\left(d\,x^2+c+(d\,x^2+c)\sqrt{1-\frac{1}{(d\,x^2+c)^2}}\right)b}{2d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsec(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*((d*x^2+c)*a+arcsec(d*x^2+c)*b*(d*x^2+c)-ln(d*x^2+c+(d*x^2+c)*(1-1/(d
*x^2+c)^2)^(1/2))*b)
```

Maxima [A]
time = 0.25, size = 71, normalized size = 1.22

$$\frac{1}{2}\,ax^2+\frac{\left(2\,(dx^2+c)\,\operatorname{arcsec}\left(dx^2+c\right)-\log\left(\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{\left(dx^2+c\right)^2}+1}+1\right)\right)b}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*(d*x^2 + c)*arcsec(d*x^2 + c) - log(sqrt(-1/(d*x^2 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^2 + c)^2 + 1) + 1))*b/d
```

Fricas [A]
time = 1.97, size = 96, normalized size = 1.66

$$\frac{bdx^2\operatorname{arcsec}(dx^2+c)+adx^2+2\,bc\arctan\left(-dx^2-c+\sqrt{d^2x^4+2\,cdx^2+c^2-1}\right)+b\log\left(-dx^2-c+\sqrt{d^2x^4+2\,cdx^2+c^2-1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x^2*arcsec(d*x^2 + c) + a*d*x^2 + 2*b*c*arctan(-d*x^2 - c + sqrt(d
^2*x^4 + 2*c*d*x^2 + c^2 - 1)) + b*log(-d*x^2 - c + sqrt(d^2*x^4 + 2*c*d*x^
2 + c^2 - 1)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asec(d*x**2+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 100, normalized size = 1.72

$$\frac{1}{2}\,ax^2+\frac{1}{4}\,bd\left(\frac{2\,(dx^2+c)\arccos\left(-\frac{1}{(dx^2+c)\left(\frac{c}{dx^2+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(dx^2+c)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsec(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2 + 1/4*b*d*(2*(d*x^2 + c)*arccos(-1/((d*x^2 + c)*(c/(d*x^2 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^2 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^2 +
 c)^2 + 1) + 1))/d^2)
```

Mupad [B]
time = 1.03, size = 52, normalized size = 0.90

$$\frac{a\,x^2}{2}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{{\left(d\,x^2+c\right)}^2}}}\right)}{2\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^2+c}\right)\,\left(d\,x^2+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acos(1/(c + d*x^2))),x)
```

```
[Out] (a*x^2)/2 - (b*atanh(1/(1 - 1/(c + d*x^2)^2)^(1/2)))/(2*d) + (b*acos(1/(c +
 d*x^2))*(c + d*x^2))/(2*d)
```

3.39 $\int x^2(a+b\sec^{-1}(c+dx^3))\,dx$

Optimal. Leaf size=58

$$\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}(c+dx^3)}{3d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^3)^2}}\right)}{3d}$$

```
[Out] 1/3*a*x^3+1/3*b*(d*x^3+c)*arcsec(d*x^3+c)/d-1/3*b*arctanh((1-1/(d*x^3+c)^2)
^(1/2))/d
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6847, 5358, 379, 272, 65, 212}

$$\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}(c+dx^3)}{3d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^3)^2}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*(c + d*x^3)*ArcSec[c + d*x^3])/(3*d) - (b*ArcTanh[Sqrt[1 - (
c + d*x^3)^(-2)]])/(3*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^2\left(a+b\sec^{-1}\left(c+dx^3\right)\right)\,dx=\frac{1}{3}\text{Subst}\left(\int\left(a+b\sec^{-1}(c+dx)\right)\,dx,x,x^3\right)$$

$$=\frac{ax^3}{3}+\frac{1}{3}b\text{Subst}\left(\int\sec^{-1}(c+dx)\,dx,x,x^3\right)$$

$$=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{1}{3}b\text{Subst}\left(\int\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right.$$

$$=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx,x,c+dx^3\right)}{3d}$$

$$=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}\left(c+dx^3\right)}{3d}+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{(c+dx^3)^2}\right)}{6d}$$

$$=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{(c+dx^3)}}\right.}{3d}$$

$$=\frac{ax^3}{3}+\frac{b(c+dx^3)\sec^{-1}\left(c+dx^3\right)}{3d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^3)^2}}\right)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 385 vs. 2(58) = 116.
time = 0.19, size = 385, normalized size = 6.64

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\sec^{-1}\left(c+dx^3\right)-\frac{b(c+dx^3)\sqrt{\frac{-1+c^2+2cdx^3+d^2x^6}{(c+dx^3)^2}}\left(2c\left(d+\sqrt{d^2}\right)\text{ArcTan}\left(c+\sqrt{d^2}\,x^3-\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)+2c\left(-d+\sqrt{d^2}\right)\text{ArcTan}\left(c-\sqrt{d^2}\,x^3+\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)-\sqrt{d^2}\log\left(-c-\sqrt{d^2}\,x^3+\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)+d\log\left(c-\sqrt{d^2}\,x^3+\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)-\sqrt{d^2}\log\left(c-\sqrt{d^2}\,x^3+\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)-d\log\left(cd^2+(d^2)^{3/2}x^3-d^2\sqrt{-1+c^2+2cdx^3+d^2x^6}\right)\right)}{6d^2\sqrt{-1+c^2+2cdx^3+d^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcSec[c + d*x^3]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*ArcSec[c + d*x^3])/3 - (b*(c + d*x^3)*Sqrt[(-1 + c^2 + 2
*c*d*x^3 + d^2*x^6)/(c + d*x^3)^2]*(2*c*(d + Sqrt[d^2])*ArcTan[c + Sqrt[d^2
]*x^3 - Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] + 2*c*(-d + Sqrt[d^2])*ArcTan
[c - Sqrt[d^2]*x^3 + Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] - Sqrt[d^2]*Log[
-c - Sqrt[d^2]*x^3 + Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] + d*Log[c - Sqrt
[d^2]*x^3 + Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] - Sqrt[d^2]*Log[c - Sqrt[
d^2]*x^3 + Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]] - d*Log[c*d^2 + (d^2)^(3/2
)*x^3 - d^2*Sqrt[-1 + c^2 + 2*c*d*x^3 + d^2*x^6]]))/(6*d^2*Sqrt[-1 + c^2 +
2*c*d*x^3 + d^2*x^6])
```

Maple [A]
time = 0.08, size = 67, normalized size = 1.16

method	result	size
derivativedivides	$\frac{(d\,x^3+c)a+\mathrm{arcsec}(d\,x^3+c)b(d\,x^3+c)-\ln\left(d\,x^3+c+(d\,x^3+c)\sqrt{1-\frac{1}{(d\,x^3+c)^2}}\right)b}{3d}$	67
default	$\frac{(d\,x^3+c)a+\mathrm{arcsec}(d\,x^3+c)b(d\,x^3+c)-\ln\left(d\,x^3+c+(d\,x^3+c)\sqrt{1-\frac{1}{(d\,x^3+c)^2}}\right)b}{3d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsec(d*x^3+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*((d*x^3+c)*a+arcsec(d*x^3+c)*b*(d*x^3+c)-ln(d*x^3+c+(d*x^3+c)*(1-1/(d
*x^3+c)^2)^(1/2))*b)
```

Maxima [A]
time = 0.26, size = 71, normalized size = 1.22

$$\frac{1}{3}\,ax^3+\frac{\left(2\,(dx^3+c)\,\mathrm{arcsec}\left(dx^3+c\right)-\log\left(\sqrt{-\frac{1}{(dx^3+c)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(dx^3+c)^2}+1}+1\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*(d*x^3 + c)*arcsec(d*x^3 + c) - log(sqrt(-1/(d*x^3 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^3 + c)^2 + 1) + 1))*b/d
```

Fricas [A]
time = 2.77, size = 96, normalized size = 1.66

$$\frac{bdx^3 \operatorname{arcsec}(dx^3+c) + adx^3 + 2\,bc\arctan\left(-dx^3-c+\sqrt{d^2x^6+2\,cdx^3+c^2-1}\right) + b\log\left(-dx^3-c+\sqrt{d^2x^6+2\,cdx^3+c^2-1}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d*x^3*arcsec(d*x^3 + c) + a*d*x^3 + 2*b*c*arctan(-d*x^3 - c + sqrt(d
^2*x^6 + 2*c*d*x^3 + c^2 - 1)) + b*log(-d*x^3 - c + sqrt(d^2*x^6 + 2*c*d*x^
3 + c^2 - 1)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asec(d*x**3+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 100, normalized size = 1.72

$$\frac{1}{3}\,ax^3 + \frac{1}{6}\,bd\left(\frac{2\,(dx^3+c)\arccos\left(-\frac{1}{(dx^3+c)\left(\frac{c}{dx^3+c}-1\right)-c}\right)}{d^2} - \frac{\log\left(\sqrt{-\frac{1}{(dx^3+c)^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{(dx^3+c)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsec(d*x^3+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/6*b*d*(2*(d*x^3 + c)*arccos(-1/((d*x^3 + c)*(c/(d*x^3 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^3 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^3 +
 c)^2 + 1) + 1))/d^2)
```

Mupad [B]
time = 0.77, size = 52, normalized size = 0.90

$$\frac{a\,x^3}{3} - \frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{{\left(d\,x^3+c\right)}^2}}}\right)}{3\,d} + \frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^3+c}\right)\,\left(d\,x^3+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acos(1/(c + d*x^3))),x)
```

```
[Out] (a*x^3)/3 - (b*atanh(1/(1 - 1/(c + d*x^3)^2)^(1/2)))/(3*d) + (b*acos(1/(c +
 d*x^3))*(c + d*x^3))/(3*d)
```

3.40 $\int x^3(a+b\sec^{-1}(c+dx^4))\,dx$

Optimal. Leaf size=58

$$\frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}(c+dx^4)}{4d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^4)^2}}\right)}{4d}$$

[Out] 1/4*a*x^4+1/4*b*(d*x^4+c)*arcsec(d*x^4+c)/d-1/4*b*arctanh((1-1/(d*x^4+c)^2)^(1/2))/d

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6847, 5358, 379, 272, 65, 212}

$$\frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}(c+dx^4)}{4d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^4)^2}}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*ArcSec[c + d*x^4]),x]

[Out] (a*x^4)/4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/(4*d) - (b*ArcTanh[Sqrt[1 - (c + d*x^4)^(-2)]])/(4*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\sec^{-1}\left(c+dx^4\right)\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int\left(a+b\sec^{-1}(c+dx)\right)\,dx, x, x^4\right)\\
&= \frac{ax^4}{4}+\frac{1}{4}b\text{Subst}\left(\int\sec^{-1}(c+dx)\,dx, x, x^4\right)\\
&= \frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{1}{4}b\text{Subst}\left(\int\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\,dx, x, x^4\right)\\
&= \frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx, x, c+dx^4\right)}{4d}\\
&= \frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}\left(c+dx^4\right)}{4d}+\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{(c+dx^4)^2}\right)}{8d}\\
&= \frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\text{Subst}\left(\int\frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{(c+dx^4)^2}}\right)}{4d}\\
&= \frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}\left(c+dx^4\right)}{4d}-\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{(c+dx^4)^2}}\right)}{4d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.
time = 0.27, size = 137, normalized size = 2.36

$$\frac{ax^4}{4}+\frac{b(c+dx^4)\sec^{-1}(c+dx^4)}{4d}-\frac{b\sqrt{-1+(c+dx^4)^2}\left(-\log\left(1-\frac{c+dx^4}{\sqrt{-1+(c+dx^4)^2}}\right)+\log\left(1+\frac{c+dx^4}{\sqrt{-1+(c+dx^4)^2}}\right)\right)}{8d(c+dx^4)\sqrt{1-\frac{1}{(c+dx^4)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcSec[c + d*x^4]),x]
```

```
[Out] (a*x^4)/4 + (b*(c + d*x^4)*ArcSec[c + d*x^4])/(4*d) - (b*Sqrt[-1 + (c + d*x
^4)^2]*(-Log[1 - (c + d*x^4)/Sqrt[-1 + (c + d*x^4)^2]] + Log[1 + (c + d*x^4
)/Sqrt[-1 + (c + d*x^4)^2]]))/(8*d*(c + d*x^4)*Sqrt[1 - (c + d*x^4)^(-2)])
```

Maple [A]
time = 0.08, size = 67, normalized size = 1.16

method	result	size
derivativedivides	$\frac{(d\,x^4+c)a+\mathrm{arcsec}(d\,x^4+c)b(d\,x^4+c)-\ln\left(d\,x^4+c+(d\,x^4+c)\sqrt{1-\frac{1}{(d\,x^4+c)^2}}\right)b}{4d}$	67
default	$\frac{(d\,x^4+c)a+\mathrm{arcsec}(d\,x^4+c)b(d\,x^4+c)-\ln\left(d\,x^4+c+(d\,x^4+c)\sqrt{1-\frac{1}{(d\,x^4+c)^2}}\right)b}{4d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsec(d*x^4+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*((d*x^4+c)*a+arcsec(d*x^4+c)*b*(d*x^4+c)-ln(d*x^4+c+(d*x^4+c)*(1-1/(d
*x^4+c)^2)^(1/2))*b)
```

Maxima [A]
time = 0.27, size = 71, normalized size = 1.22

$$\frac{1}{4}\,ax^4+\frac{\left(2\,(dx^4+c)\,\mathrm{arcsec}\,(dx^4+c)-\log\left(\sqrt{-\frac{1}{(dx^4+c)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(dx^4+c)^2}+1}+1\right)\right)b}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/8*(2*(d*x^4 + c)*arcsec(d*x^4 + c) - log(sqrt(-1/(d*x^4 + c)^
2 + 1) + 1) + log(-sqrt(-1/(d*x^4 + c)^2 + 1) + 1))*b/d
```

Fricas [A]
time = 2.72, size = 96, normalized size = 1.66

$$\frac{bdx^4\operatorname{arcsec}\left(dx^4+c\right)+adx^4+2\,bc\arctan\left(-dx^4-c+\sqrt{d^2x^8+2\,cdx^4+c^2-1}\right)+b\log\left(-dx^4-c+\sqrt{d^2x^8+2\,cdx^4+c^2-1}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x^4*arcsec(d*x^4 + c) + a*d*x^4 + 2*b*c*arctan(-d*x^4 - c + sqrt(d
^2*x^8 + 2*c*d*x^4 + c^2 - 1)) + b*log(-d*x^4 - c + sqrt(d^2*x^8 + 2*c*d*x^
4 + c^2 - 1)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*asec(d*x**4+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 100, normalized size = 1.72

$$\frac{1}{4}\,ax^4+\frac{1}{8}\,bd\left(\frac{2\,\left(dx^4+c\right)\arccos\left(-\frac{1}{\left(dx^4+c\right)\left(\frac{c}{dx^4+c}-1\right)-c}\right)}{d^2}-\frac{\log\left(\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{\left(dx^4+c\right)^2}+1}+1\right)}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsec(d*x^4+c)),x, algorithm="giac")
```

```
[Out] 1/4*a*x^4 + 1/8*b*d*(2*(d*x^4 + c)*arccos(-1/((d*x^4 + c)*(c/(d*x^4 + c) -
1) - c))/d^2 - (log(sqrt(-1/(d*x^4 + c)^2 + 1) + 1) - log(-sqrt(-1/(d*x^4 +
 c)^2 + 1) + 1))/d^2)
```

Mupad [B]
time = 0.78, size = 52, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{{\left(d\,x^4+c\right)}^2}}}\right)}{4\,d}+\frac{b\,\mathrm{acos}\left(\frac{1}{d\,x^4+c}\right)\,\left(d\,x^4+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acos(1/(c + d*x^4))),x)
```

```
[Out] (a*x^4)/4 - (b*atanh(1/(1 - 1/(c + d*x^4)^2)^(1/2)))/(4*d) + (b*acos(1/(c +
 d*x^4))*(c + d*x^4))/(4*d)
```

3.41 $\int x^{-1+n} \sec^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=49

$$\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

```
[Out] (a+b*x^n)*arcsec(a+b*x^n)/b/n-arctanh((1-1/(a+b*x^n)^2)^(1/2))/b/n
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {6847, 5358, 379, 272, 65, 212}

$$\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSec[a + b*x^n])/(b*n) - ArcTanh[Sqrt[1 - (a + b*x^n)^(-2)]]
/(b*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\sec^{-1}(a+bx^n)\,dx=\frac{\text{Subst}\left(\int \sec^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$

$$=\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\,dx,x,x^n\right)}{n}$$

$$=\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx,x,a+bx^n\right)}{bn}$$

$$=\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{(a+bx^n)^2}\right)}{2bn}$$

$$=\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

$$=\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{(a+bx^n)^2}}\right)}{bn}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 130 vs. $2(49)=98$.

time = 0.23, size = 130, normalized size = 2.65

$$\frac{(a+bx^n)\sec^{-1}(a+bx^n)}{bn}-\frac{\sqrt{-1+(a+bx^n)^2}\left(-\log\left(1-\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)+\log\left(1+\frac{a+bx^n}{\sqrt{-1+(a+bx^n)^2}}\right)\right)}{2bn\,(a+bx^n)\sqrt{1-\frac{1}{(a+bx^n)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSec[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcSec[a + b*x^n])/(b*n) - (Sqrt[-1 + (a + b*x^n)^2]*(-Log[1 -
 (a + b*x^n)/Sqrt[-1 + (a + b*x^n)^2]] + Log[1 + (a + b*x^n)/Sqrt[-1 + (a +
 b*x^n)^2]]))/(2*b*n*(a + b*x^n)*Sqrt[1 - (a + b*x^n)^(-2)])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1+n}\operatorname{arcsec}(a+b\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arcsec(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arcsec(a+b*x^n),x)
```

Maxima [A]
time = 0.25, size = 66, normalized size = 1.35

$$\frac{2\,(bx^n+a)\,\operatorname{arcsec}(bx^n+a)-\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)+\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arcsec(b*x^n + a) - log(sqrt(-1/(b*x^n + a)^2 + 1) + 1)
+ log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/(b*n)
```

Fricas [A]
time = 3.88, size = 92, normalized size = 1.88

$$\frac{bx^n\,\operatorname{arcsec}(bx^n+a)+2\,a\arctan\left(-bx^n-a+\sqrt{b^2x^{2\,n}+2\,abx^n+a^2-1}\right)+\log\left(-bx^n-a+\sqrt{b^2x^{2\,n}+2\,abx^n+a^2-1}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*arcsec(b*x^n + a) + 2*a*arctan(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b
*x^n + a^2 - 1)) + log(-b*x^n - a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
))/(b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asec(a+b*x**n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [A]
time = 0.50, size = 75, normalized size = 1.53

$$\frac{b\left(\frac{2\,(bx^n+a)\arccos\left(\frac{1}{bx^n+a}\right)}{b^2}-\frac{\log\left(\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{(bx^n+a)^2}+1}+1\right)}{b^2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsec(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x^n + a)*arccos(1/(b*x^n + a))/b^2 - (log(sqrt(-1/(b*x^n + a)^2
 + 1) + 1) - log(-sqrt(-1/(b*x^n + a)^2 + 1) + 1))/b^2)/n
```

Mupad [B]
time = 1.08, size = 44, normalized size = 0.90

$$-\frac{\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{(a+b\,x^n)^2}}}\right)-\mathrm{acos}\left(\frac{1}{a+b\,x^n}\right)\,(a+b\,x^n)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acos(1/(a + b*x^n)),x)
```

```
[Out] -(atanh(1/(1 - 1/(a + b*x^n)^2)^(1/2)) - acos(1/(a + b*x^n))*(a + b*x^n))/(
b*n)
```

3.42 $\int \sec^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=85

$$\frac{i\sec^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\sec^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{b}+\frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] 1/2*I*arcsec(c*exp(b*x+a))^2/b-arcsec(c*exp(b*x+a))*ln(1+(1/c/exp(b*x+a)+I*
(1-1/c^2/exp(b*x+a)^2)^(1/2))^2)/b+1/2*I*polylog(2,-(1/c/exp(b*x+a)+I*(1-1/
c^2/exp(b*x+a)^2)^(1/2))^2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2320, 5326, 4722, 3800, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{2b}+\frac{i\sec^{-1}\left(ce^{a+bx}\right)^2}{2b}-\frac{\sec^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\sec^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[c*E^(a + b*x)],x]
```

```
[Out] ((I/2)*ArcSec[c*E^(a + b*x)]^2)/b - (ArcSec[c*E^(a + b*x)]*Log[1 + E^((2*I)
*ArcSec[c*E^(a + b*x)])])/b + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[c*E^(a + b
*x)])])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{-1}\left(ce^{a+bx}\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\sec^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, e^{-a-bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\right)}{b} \\
&= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{\text{Subst}\left(\int \log\left(1+e\right.\right.}{} \\
&= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} - \frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,d\right.}{} \\
&= \frac{i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)^2}{2b} - \frac{\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{e^{-a-bx}}{c}\right)}\right)}{b} + \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{e^{-a}}{c}\right.}\right.}{2b}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(85) = 170.
time = 0.60, size = 280, normalized size = 3.29

$$
x\sec^{-1}\left(ce^{a+bx}\right) - \frac{e^{-a-bx}\left(4\sqrt{-1+c^2e^{2(a+bx)}}\,\text{ArcTan}\left(\sqrt{-1+c^2e^{2(a+bx)}}\right)\left(2bx-\log\left(c^2e^{2(a+bx)}\right)\right)+\sqrt{1-c^2e^{2(a+bx)}}\left(\log^2\left(c^2e^{2(a+bx)}\right)-4\log\left(c^2e^{2(a+bx)}\right)\log\left(\frac{1}{2}\left(1+\sqrt{1-c^2e^{2(a+bx)}}\right)\right)+2\log^2\left(\frac{1}{2}\left(1+\sqrt{1-c^2e^{2(a+bx)}}\right)\right)\right)-4\sqrt{1-c^2e^{2(a+bx)}}\,\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{1-c^2e^{2(a+bx)}}\right)\right)\right)}{8bc\sqrt{1-\frac{e^{-2(a+bx)}}{c^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[c*E^(a + b*x)],x]
```

```
[Out] x*ArcSec[c*E^(a + b*x)] - (E^(-a - b*x)*(4*Sqrt[-1 + c^2*E^(2*(a + b*x))]*A
rcTan[Sqrt[-1 + c^2*E^(2*(a + b*x))]]*(2*b*x - Log[c^2*E^(2*(a + b*x))]) +
Sqrt[1 - c^2*E^(2*(a + b*x))]*(Log[c^2*E^(2*(a + b*x))]^2 - 4*Log[c^2*E^(2*
(a + b*x))]*Log[(1 + Sqrt[1 - c^2*E^(2*(a + b*x))])/2] + 2*Log[(1 + Sqrt[1
- c^2*E^(2*(a + b*x))])/2]^2) - 4*Sqrt[1 - c^2*E^(2*(a + b*x))]*PolyLog[2,
(1 - Sqrt[1 - c^2*E^(2*(a + b*x))])/2]))/(8*b*c*Sqrt[1 - 1/(c^2*E^(2*(a + b
*x)))])
```

Maple [A]
time = 0.51, size = 111, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{i\,\mathrm{arcsec}\left(\mathrm{e}^{bx+a}c\right)^{2}}{2}-\mathrm{arcsec}\left(\mathrm{e}^{bx+a}c\right)\ln\left(1+\left(\frac{\mathrm{e}^{-bx-a}}{c}+i\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^{2}}}\right)^{2}\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{\mathrm{e}^{-bx-a}}{c}+i\sqrt{1-\frac{\mathrm{e}^{-2b}}{c}}\right.\right.}{2}}{b}$
default	$\frac{\frac{i\,\mathrm{arcsec}\left(\mathrm{e}^{bx+a}c\right)^{2}}{2}-\mathrm{arcsec}\left(\mathrm{e}^{bx+a}c\right)\ln\left(1+\left(\frac{\mathrm{e}^{-bx-a}}{c}+i\sqrt{1-\frac{\mathrm{e}^{-2bx-2a}}{c^{2}}}\right)^{2}\right)+\frac{i\,\mathrm{polylog}\left(2,-\left(\frac{\mathrm{e}^{-bx-a}}{c}+i\sqrt{1-\frac{\mathrm{e}^{-2b}}{c}}\right.\right.}{2}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(exp(b*x+a)*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*I*arcsec(exp(b*x+a)*c)^2-arcsec(exp(b*x+a)*c)*ln(1+(1/exp(b*x+a)/c
+I*(1-1/exp(b*x+a)^2/c^2)^(1/2))^2)+1/2*I*polylog(2,-(1/exp(b*x+a)/c+I*(1-1
/exp(b*x+a)^2/c^2)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*c^2*integrate(x*e^(2*b*x + 2*a + 1/2*log(c*e^(b*x + a) + 1) + 1
/2*log(c*e^(b*x + a) - 1))/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)
*e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) + 2*I*b^2*c^2
*integrate(x*e^(2*b*x + 2*a)/(c^2*e^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) -
1)*e^(log(c*e^(b*x + a) + 1) + log(c*e^(b*x + a) - 1)) - 1), x) - I*b^2*x^2
 - 2*b*x*arctan(sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x + a) - 1)) + I*b*x*lo
g(c^2*e^(2*b*x + 2*a)) - I*b*x*log(c*e^(b*x + a) + 1) - I*b*x*log(-c*e^(b*x
 + a) + 1) - 2*(I*a*b + I*b*log(c))*x - I*dilog(c*e^(b*x + a)) - I*dilog(-c
*e^(b*x + a)))/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asec}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(c*exp(b*x+a)),x)
```

```
[Out] Integral(asec(c*exp(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsec(c*e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(\frac{\mathrm{e}^{-a-b\,x}}{c}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(exp(- a - b*x)/c),x)
```

```
[Out] int(acos(exp(- a - b*x)/c), x)
```

3.43 $\int e^{\sec^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=99

$$-\frac{\left(\frac{12}{5}+\frac{4i}{5}\right) e^{(1+3i)\sec^{-1}(ax)} {}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}+\frac{\left(\frac{24}{5}+\frac{8i}{5}\right) e^{(1+3i)\sec^{-1}(ax)} {}_2F_1\left(\frac{3}{2}-\frac{i}{2},4;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}$$

```
[Out] (-12/5-4/5*I)*exp((1+3*I)*arcsec(a*x))*hypergeom([3, 3/2-1/2*I],[5/2-1/2*I]
,-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^3+(24/5+8/5*I)*exp((1+3*I)*arcsec(a*x)
)*hypergeom([4, 3/2-1/2*I],[5/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^
3
```

Rubi [A]
time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5374, 12, 4559, 2283}

$$\frac{\left(\frac{24}{5}+\frac{8i}{5}\right) e^{(1+3i)\sec^{-1}(ax)} {}_2F_1\left(\frac{3}{2}-\frac{i}{2},4;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}-\frac{\left(\frac{12}{5}+\frac{4i}{5}\right) e^{(1+3i)\sec^{-1}(ax)} {}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]*x^2,x]
```

```
[Out] ((-12/5 - (4*I)/5)*E^((1 + 3*I)*ArcSec[a*x])*Hypergeometric2F1[3/2 - I/2, 3
, 5/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a^3 + ((24/5 + (8*I)/5)*E^((1 + 3*I)*
ArcSec[a*x])*Hypergeometric2F1[3/2 - I/2, 4, 5/2 - I/2, -E^((2*I)*ArcSec[a*
x])])/a^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
```

```
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\sec^{-1}(ax)}x^2\,dx &= \frac{\text{Subst}\left(\int \frac{e^x\sec^3(x)\tan(x)}{a^2}\,dx, x, \sec^{-1}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int e^x\sec^3(x)\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a^3}\\ &= \frac{\text{Subst}\left(\int \left(\frac{16ie^{(1+3i)x}}{(1+e^{2ix})^4}-\frac{8ie^{(1+3i)x}}{(1+e^{2ix})^3}\right)dx, x, \sec^{-1}(ax)\right)}{a^3}\\ &= -\frac{(8i)\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(1+e^{2ix})^3}\,dx, x, \sec^{-1}(ax)\right)}{a^3}+\frac{(16i)\text{Subst}\left(\int \frac{e^{(1+3i)x}}{(1+e^{2ix})^4}\,dx, x, \sec^{-1}(ax)\right)}{a^3}\\ &= -\frac{\left(\frac{12}{5}+\frac{4i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{3}{2}-\frac{i}{2},3;\frac{5}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^3}+\frac{\left(\frac{24}{5}+\frac{8i}{5}\right)e^{(1+3i)\sec^{-1}(ax)}\,{}_2\ldots}{\ldots}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 95, normalized size = 0.96

$$\frac{e^{\sec^{-1}(ax)}\left((-4-4i)\left(-i+a\sqrt{1-\frac{1}{a^2x^2}}\,x\right){}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)+a^4x^4(5+\cos\left(2\sec^{-1}(ax)\right)-\sin\left(2\sec^{-1}(ax)\right))\right)}{12a^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]*x^2,x]
```

```
[Out] (E^ArcSec[a*x]*((-4 - 4*I)*(-I + a*Sqrt[1 - 1/(a^2*x^2)]*x)*Hypergeometric2
F1[1/2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])] + a^4*x^4*(5 + Cos[2*Ar
cSec[a*x]] - Sin[2*ArcSec[a*x]])))/(12*a^4*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arcsec}(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))*x^2,x)
[Out] int(exp(arcsec(a*x))*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(arcsec(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="fricas")
[Out] integral(x^2*e^(arcsec(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\operatorname{asec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))*x**2,x)
[Out] Integral(x**2*exp(asec(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x^2,x, algorithm="giac")
[Out] integrate(x^2*e^(arcsec(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(acos(1/(a*x))),x)
[Out] int(x^2*exp(acos(1/(a*x))), x)
```

3.44 $\int e^{\sec^{-1}(ax)}x\,dx$

Optimal. Leaf size=91

$$-\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2}+\frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},3;2-\frac{i}{2};-e^{2}\right.}{a^2}$$

```
[Out] (-8/5-4/5*I)*exp((1+2*I)*arcsec(a*x))*hypergeom([2, 1-1/2*I],[2-1/2*I],-(1/
a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2+(16/5+8/5*I)*exp((1+2*I)*arcsec(a*x))*hyp
ergeom([3, 1-1/2*I],[2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a^2
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5374, 12, 4559, 2283}

$$\frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},3;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2}-\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2},2;2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]*x,x]
```

```
[Out] ((-8/5 - (4*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
 - I/2, -E^((2*I)*ArcSec[a*x])])/a^2 + ((16/5 + (8*I)/5)*E^((1 + 2*I)*ArcSe
c[a*x])*Hypergeometric2F1[1 - I/2, 3, 2 - I/2, -E^((2*I)*ArcSec[a*x])])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\sec^{-1}(ax)}x\,dx &= \frac{\text{Subst}\left(\int \frac{e^x\sec^2(x)\tan(x)}{a}\,dx, x, \sec^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int e^x\sec^2(x)\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{8ie^{(1+2i)x}}{(1+e^{2ix})^3} - \frac{4ie^{(1+2i)x}}{(1+e^{2ix})^2}\right)dx, x, \sec^{-1}(ax)\right)}{a^2}\\
&= -\frac{(4i)\text{Subst}\left(\int \frac{e^{(1+2i)x}}{(1+e^{2ix})^2}\,dx, x, \sec^{-1}(ax)\right)}{a^2} + \frac{(8i)\text{Subst}\left(\int \frac{e^{(1+2i)x}}{(1+e^{2ix})^3}\,dx, x, \sec^{-1}(ax)\right)}{a^2}\\
&= -\frac{\left(\frac{8}{5}+\frac{4i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1-\frac{i}{2}, 2; 2-\frac{i}{2}; -e^{2i\sec^{-1}(ax)}\right)}{a^2} + \frac{\left(\frac{16}{5}+\frac{8i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 107, normalized size = 1.18

$$\frac{\left(\frac{1}{5}+\frac{i}{10}\right)e^{\sec^{-1}(ax)}\left((-2+i)ax\left(\sqrt{1-\frac{1}{a^2x^2}}-ax\right)+(1+2i)\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)-e^{2i\sec^{-1}(ax)}\,{}_2F_1\left(1,1-\frac{i}{2};2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]*x,x]
```

```
[Out] ((1/5 + I/10)*E^ArcSec[a*x]*((-2 + I)*a*x*(Sqrt[1 - 1/(a^2*x^2)] - a*x) + (
1 + 2*I)*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcSec[a*x])] - E^
((2*I)*ArcSec[a*x])*Hypergeometric2F1[1, 1 - I/2, 2 - I/2, -E^((2*I)*ArcSec
[a*x])]))/a^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arcsec}(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))*x,x)
```

```
[Out] int(exp(arcsec(a*x))*x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arcsec(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arcsec(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\operatorname{asec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))*x,x)
```

```
[Out] Integral(x*exp(asec(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(arcsec(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(acos(1/(a*x))),x)
```

```
[Out] int(x*exp(acos(1/(a*x))), x)
```

3.45 $\int e^{\sec^{-1}(ax)}\,dx$

Optimal. Leaf size=91

$$-\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}+\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}}\right.}{a}$$

```
[Out] (-1-I)*exp((1+I)*arcsec(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],-(1/a/x+
I*(1-1/a^2/x^2)^(1/2))^2)/a+(2+2*I)*exp((1+I)*arcsec(a*x))*hypergeom([2, 1/
2-1/2*I],[3/2-1/2*I],-(1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)/a
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5374, 4559, 2283}

$$\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}-\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x],x]
```

```
[Out] ((-1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, -E^((2*I)*ArcSec[a*x])])/a + ((2 + 2*I)*E^((1 + I)*ArcSec[a*x])*Hypergeom
etric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sec^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int e^x\sec(x)\tan(x)\,dx,x,\sec^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int\left(\frac{4ie^{(1+i)x}}{(1+e^{2ix})^2}-\frac{2ie^{(1+i)x}}{1+e^{2ix}}\right)\,dx,x,\sec^{-1}(ax)\right)}{a}$$
$$= -\frac{(2i)\text{Subst}\left(\int\frac{e^{(1+i)x}}{1+e^{2ix}}\,dx,x,\sec^{-1}(ax)\right)}{a}+\frac{(4i)\text{Subst}\left(\int\frac{e^{(1+i)x}}{(1+e^{2ix})^2}\,dx,x,\sec^{-1}(ax)\right)}{a}$$
$$= -\frac{(1+i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}+\frac{(2+2i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\right.}{a}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.59

$$e^{\sec^{-1}(ax)}x-\frac{(1-i)e^{(1+i)\sec^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x],x]
[Out] E^ArcSec[a*x]*x - ((1 - I)*E^((1 + I)*ArcSec[a*x])*Hypergeometric2F1[1/2 -
I/2, 1, 3/2 - I/2, -E^((2*I)*ArcSec[a*x])])/a
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arcsec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x)),x)
[Out] int(exp(arcsec(a*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="fricas")
[Out] integral(e^(arcsec(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{asec}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x)),x)
[Out] Integral(exp(asec(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x)),x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x))),x)
[Out] int(exp(acos(1/(a*x))), x)
```

3.46 $\int \frac{e^{\sec^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=45

$$-ie^{\sec^{-1}(ax)} + 2ie^{\sec^{-1}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\sec^{-1}(ax)}\right)$$

```
[Out] -I*exp(arcsec(a*x))+2*I*exp(arcsec(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],-(
1/a/x+I*(1-1/a^2/x^2)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5374, 12, 4527, 2225, 2283}

$$2ie^{\sec^{-1}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\sec^{-1}(ax)}\right) - ie^{\sec^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x,x]
```

```
[Out] (-I)*E^ArcSec[a*x] + (2*I)*E^ArcSec[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I
/2, -E^((2*I)*ArcSec[a*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
```

```
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\sec^{-1}(ax)}}{x}\,dx &= \frac{\text{Subst}\left(\int ae^x\tan(x)\,dx, x, \sec^{-1}(ax)\right)}{a}\\
&= \text{Subst}\left(\int e^x\tan(x)\,dx, x, \sec^{-1}(ax)\right)\\
&= i\text{Subst}\left(\int\left(-e^x+\frac{2e^x}{1+e^{2ix}}\right)\,dx, x, \sec^{-1}(ax)\right)\\
&= -\left(i\text{Subst}\left(\int e^x\,dx, x, \sec^{-1}(ax)\right)\right)+2i\text{Subst}\left(\int \frac{e^x}{1+e^{2ix}}\,dx, x, \sec^{-1}(ax)\right)\\
&= -ie^{\sec^{-1}(ax)}+2ie^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.76

$$-i\left(-e^{\sec^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)+\left(\frac{1}{5}-\frac{2i}{5}\right)e^{(1+2i)\sec^{-1}(ax)}\,{}_2F_1\left(1,1-\frac{i}{2};2-\frac{i}{2};-e^{2i\sec^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x,x]
[Out] (-I)*(-(E^ArcSec[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcSe
c[a*x])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcSec[a*x])*Hypergeometric2F1[1,
1 - I/2, 2 - I/2, -E^((2*I)*ArcSec[a*x])])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsec}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x,x)
[Out] int(exp(arcsec(a*x))/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="maxima")
[Out] integrate(e^(arcsec(a*x))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="fricas")
[Out] integral(e^(arcsec(a*x))/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x,x)
[Out] Integral(exp(asec(a*x))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x,x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x))/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x,x)
[Out] int(exp(acos(1/(a*x)))/x, x)
```

3.47 $\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}ae^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{e^{\sec^{-1}(ax)}}{2x}$$

```
[Out] -1/2*exp(arcsec(a*x))/x+1/2*a*exp(arcsec(a*x))*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5374, 12, 4517}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}-\frac{e^{\sec^{-1}(ax)}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^2,x]
```

```
[Out] (a*E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/2 - E^ArcSec[a*x]/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{e^{\sec^{-1}(ax)}}{x^2}\,dx = \frac{\text{Subst}\left(\int a^2 e^x \sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}$$
$$= a\text{Subst}\left(\int e^x \sin(x)\,dx, x, \sec^{-1}(ax)\right)$$
$$= \frac{1}{2}ae^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}} - \frac{e^{\sec^{-1}(ax)}}{2x}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.87

$$\frac{1}{2}ae^{\sec^{-1}(ax)}\left(\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^2,x]
[Out] (a*E^ArcSec[a*x]*(Sqrt[1 - 1/(a^2*x^2)] - 1/(a*x)))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsec}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^2,x)
[Out] int(exp(arcsec(a*x))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="maxima")
[Out] integrate(e^(arcsec(a*x))/x^2, x)
```

Fricas [A]
time = 3.07, size = 23, normalized size = 0.59

$$\frac{\left(\sqrt{a^2x^2-1}-1\right)e^{(\operatorname{arcsec}(ax))}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2*x^2 - 1) - 1)*e^(arcsec(a*x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**2,x)
```

```
[Out] Integral(exp(asec(a*x))/x**2, x)
```

Giac [A]
time = 0.47, size = 43, normalized size = 1.10

$$\frac{1}{2}\left(\sqrt{-\frac{1}{a^2x^2}+1}\,e^{\left(\arccos\left(\frac{1}{ax}\right)\right)}-\frac{e^{\left(\arccos\left(\frac{1}{ax}\right)\right)}}{ax}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-1/(a^2*x^2) + 1)*e^(arccos(1/(a*x))) - e^(arccos(1/(a*x)))/(a*x)
)*a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^2,x)
```

```
[Out] int(exp(acos(1/(a*x)))/x^2, x)
```

3.48 $\int \frac{e^{\sec^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=41

$$-\frac{1}{5}a^2e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)+\frac{1}{10}a^2e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)$$

```
[Out] -1/5*a^2*exp(arcsec(a*x))*cos(2*arcsec(a*x))+1/10*a^2*exp(arcsec(a*x))*sin(
2*arcsec(a*x))
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5374, 12, 4557, 4517}

$$\frac{1}{10}a^2e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)-\frac{1}{5}a^2e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^3,x]
```

```
[Out] -1/5*(a^2*E^ArcSec[a*x]*Cos[2*ArcSec[a*x]]) + (a^2*E^ArcSec[a*x]*Sin[2*ArcS
ec[a*x]])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
```

```
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\sec^{-1}(ax)}}{x^3}\,dx &= \frac{\text{Subst}\left(\int a^3e^x\cos(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}\\
&= a^2\text{Subst}\left(\int e^x\cos(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)\\
&= a^2\text{Subst}\left(\int \frac{1}{2}e^x\sin(2x)\,dx, x, \sec^{-1}(ax)\right)\\
&= \frac{1}{2}a^2\text{Subst}\left(\int e^x\sin(2x)\,dx, x, \sec^{-1}(ax)\right)\\
&= -\frac{1}{5}a^2e^{\sec^{-1}(ax)}\cos\left(2\sec^{-1}(ax)\right)+\frac{1}{10}a^2e^{\sec^{-1}(ax)}\sin\left(2\sec^{-1}(ax)\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.73

$$\frac{1}{10}a^2e^{\sec^{-1}(ax)}\left(-2\cos\left(2\sec^{-1}(ax)\right)+\sin\left(2\sec^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^3,x]
[Out] (a^2*E^ArcSec[a*x]*(-2*Cos[2*ArcSec[a*x]] + Sin[2*ArcSec[a*x]]))/10
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsec}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^3,x)
[Out] int(exp(arcsec(a*x))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsec(a*x))/x^3, x)
```

Fricas [A]
time = 2.91, size = 30, normalized size = 0.73

$$\frac{\left(a^{2} x^{2}+\sqrt{a^{2} x^{2}-1}-2\right) e^{(\operatorname{arcsec}(a x))}}{5 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/5*(a^2*x^2 + sqrt(a^2*x^2 - 1) - 2)*e^(arcsec(a*x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asec}(ax)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**3,x)
```

```
[Out] Integral(exp(asec(a*x))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsec(a*x))/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{\operatorname{acos}\left(\frac{1}{a\,x}\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^3,x)
```

```
[Out] int(exp(acos(1/(a*x)))/x^3, x)
```

3.49 $\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{1}{8}a^3e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^2e^{\sec^{-1}(ax)}}{8x}-\frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right)+\frac{1}{40}a^3e^{\sec^{-1}(ax)}\sin\left(3\sec^{-1}(ax)\right)$$

```
[Out] -1/8*a^2*exp(arcsec(a*x))/x-3/40*a^3*exp(arcsec(a*x))*cos(3*arcsec(a*x))+1/
40*a^3*exp(arcsec(a*x))*sin(3*arcsec(a*x))+1/8*a^3*exp(arcsec(a*x))*(1-1/a^
2/x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5374, 12, 4557, 4517}

$$-\frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right)+\frac{1}{40}a^3e^{\sec^{-1}(ax)}\sin\left(3\sec^{-1}(ax)\right)-\frac{a^2e^{\sec^{-1}(ax)}}{8x}+\frac{1}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\,e^{\sec^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSec[a*x]/x^4,x]
```

```
[Out] (a^3*E^ArcSec[a*x]*Sqrt[1 - 1/(a^2*x^2)])/8 - (a^2*E^ArcSec[a*x])/(8*x) - (
3*a^3*E^ArcSec[a*x]*Cos[3*ArcSec[a*x]])/40 + (a^3*E^ArcSec[a*x]*Sin[3*ArcSe
c[a*x]])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5374

```
Int[(u_.)*(f_)^(ArcSec[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sec[x]/b)*f^(c*x^n)*Sec[x]*Tan[x], x], x,
ArcSec[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sec^{-1}(ax)}}{x^4}\,dx &= \frac{\text{Subst}\left(\int a^4 e^x \cos^2(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)}{a}\\
&= a^3\text{Subst}\left(\int e^x \cos^2(x)\sin(x)\,dx, x, \sec^{-1}(ax)\right)\\
&= a^3\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(x)+\frac{1}{4}e^x\sin(3x)\right)\,dx, x, \sec^{-1}(ax)\right)\\
&= \frac{1}{4}a^3\text{Subst}\left(\int e^x\sin(x)\,dx, x, \sec^{-1}(ax)\right)+\frac{1}{4}a^3\text{Subst}\left(\int e^x\sin(3x)\,dx, x, \sec^{-1}(ax)\right)\\
&= \frac{1}{8}a^3e^{\sec^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^2e^{\sec^{-1}(ax)}}{8x}-\frac{3}{40}a^3e^{\sec^{-1}(ax)}\cos\left(3\sec^{-1}(ax)\right)+\frac{1}{40}a^3e^{\sec^{-1}(ax)}\text{s}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.64

$$\frac{1}{40}a^3e^{\sec^{-1}(ax)}\left(5\sqrt{1-\frac{1}{a^2x^2}}-\frac{5}{ax}-3\cos\left(3\sec^{-1}(ax)\right)+\sin\left(3\sec^{-1}(ax)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSec[a*x]/x^4,x]
[Out] (a^3*E^ArcSec[a*x]*(5*Sqrt[1 - 1/(a^2*x^2)] - 5/(a*x) - 3*Cos[3*ArcSec[a*x]
] + Sin[3*ArcSec[a*x]]))/40
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\mathrm{arcsec}(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsec(a*x))/x^4,x)
[Out] int(exp(arcsec(a*x))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="maxima")
[Out] integrate(e^(arcsec(a*x))/x^4, x)
```

Fricas [A]
time = 3.75, size = 40, normalized size = 0.48

$$\frac{\left(a^2x^2 + (a^2x^2+1)\sqrt{a^2x^2-1} - 3\right)e^{(\mathrm{arcsec}(ax))}}{10\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="fricas")
[Out] 1/10*(a^2*x^2 + (a^2*x^2 + 1)*sqrt(a^2*x^2 - 1) - 3)*e^(arcsec(a*x))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\mathrm{asec}\,(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asec(a*x))/x**4,x)
[Out] Integral(exp(asec(a*x))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsec(a*x))/x^4,x, algorithm="giac")
[Out] integrate(e^(arcsec(a*x))/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{\mathrm{acos}\left(\frac{1}{a\,x}\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(1/(a*x)))/x^4,x)
[Out] int(exp(acos(1/(a*x)))/x^4, x)
```

3.50 $\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=69

$$\frac{i\sec^{-1}(a+bx)^2}{2d}-\frac{\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{d}+\frac{i\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)}{2d}$$

```
[Out] 1/2*I*arcsec(b*x+a)^2/d-arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/
2))^2)/d+1/2*I*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.421$, Rules used = {5364, 12, 5326, 4722, 3800, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{2i\sec^{-1}(a+bx)}\right)}{2d}+\frac{i\sec^{-1}(a+bx)^2}{2d}-\frac{\sec^{-1}(a+bx)\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*ArcSec[a + b*x]^2)/d - (ArcSec[a + b*x]*Log[1 + E^((2*I)*ArcSec[a +
b*x])])/d + ((I/2)*PolyLog[2, -E^((2*I)*ArcSec[a + b*x])])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 5326

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcCos[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rule 5364

```
Int[((a_.) + ArcSec[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcSec[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\sec^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sec^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, \frac{1}{a+bx}\right)}{d}$$
$$= \frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}$$
$$= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{1}{a+bx}\right)\right)}{d}$$
$$= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x\right.}{d}$$
$$= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} - \frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i}\right.}{2d}$$
$$= \frac{i\cos^{-1}\left(\frac{1}{a+bx}\right)^2}{2d} - \frac{\cos^{-1}\left(\frac{1}{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{d} + \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{1}{a+bx}\right)}\right)}{2d}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.86

$$\frac{i\left(\sec^{-1}(a+bx)\left(\sec^{-1}(a+bx)+2i\log\left(1+e^{2i\sec^{-1}(a+bx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\sec^{-1}(a+bx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*(ArcSec[a + b*x]*(ArcSec[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcSec[a
+ b*x])]) + PolyLog[2, -E^((2*I)*ArcSec[a + b*x])]))/d
```

Maple [A]
time = 0.46, size = 99, normalized size = 1.43

method	result

derivativedivides	$\frac{\frac{ib\,\mathrm{arcsec}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsec}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{2d}}{b}$
default	$\frac{\frac{ib\,\mathrm{arcsec}(bx+a)^2}{2d}-\frac{b\,\mathrm{arcsec}(bx+a)\ln\left(1+\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{d}+\frac{ib\,\mathrm{polylog}\left(2,-\left(\frac{1}{bx+a}+i\sqrt{1-\frac{1}{(bx+a)^2}}\right)^2\right)}{2d}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*I*b/d*arcsec(b*x+a)^2-b/d*arcsec(b*x+a)*ln(1+(1/(b*x+a)+I*(1-1/(b*
x+a)^2)^(1/2))^2)+1/2*I*b/d*polylog(2,-(1/(b*x+a)+I*(1-1/(b*x+a)^2)^(1/2))^
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*integrate(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*log(b*x + a)/(b^3
*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 - a)*d), x) + 2*I*b*d*int
egrate(log(b*x + a)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + (3*a^2 - 1)*b*d*x + (a^3 -
 a)*d), x) - 2*arctan(sqrt(b*x + a + 1)*sqrt(b*x + a - 1))*log(b*x + a) + I
*log(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) - I*log(b*x + a + 1)*log(b*x + a
) - I*log(b*x + a)^2 - I*log(b*x + a)*log(-b*x - a + 1) - I*dilog(b*x + a)
- I*dilog(-b*x - a))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsec(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{asec}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asec(a + b*x)/(a + b*x), x)/d
```

Giac [A]
time = 0.61, size = 115, normalized size = 1.67

$$-\frac{1}{4}\,b^2\left(\frac{2\,(bx+a)^2\arccos\left(\frac{1}{\left((bx+a)\left(\frac{a}{bx+a}-1\right)-a\right)\left(\frac{a}{bx+a}-1\right)+a}\right)}{b^3d}-\frac{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)-\frac{1}{(bx+a)\left(\sqrt{-\frac{1}{(bx+a)^2}+1}-1\right)}}{b^3d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*(b*x + a)^2*arccos(1/(((b*x + a)*(a/(b*x + a) - 1) - a)*(a/(b*x
 + a) - 1) + a))/(b^3*d) - ((b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1) - 1/((
b*x + a)*(sqrt(-1/(b*x + a)^2 + 1) - 1)))/(b^3*d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\frac{1}{a+b\,x}\right)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/(a + b*x))/(d*x + (a*d)/b),x)
```

```
[Out] int(acos(1/(a + b*x))/(d*x + (a*d)/b), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):    #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):    #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```