Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.4-Inverse-cotangent/154-5.4.1-Inverse-cotangent-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [234]. This is test number [154].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (234)	0.00 (0)
Mathematica	97.44 (228)	2.56 (6)
Maple	97.44 (228)	2.56 (6)
Fricas	71.79 (168)	28.21 (66)
Maxima	61.11 (143)	38.89 (91)
Giac	47.44 (111)	52.56 (123)
Mupad	46.15 (108)	53.85 (126)
Sympy	35.04 (82)	64.96 (152)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

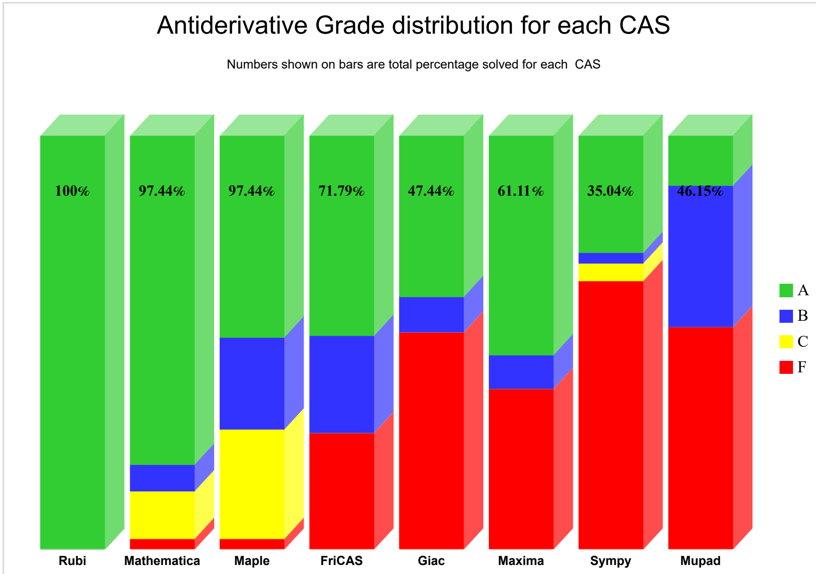
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

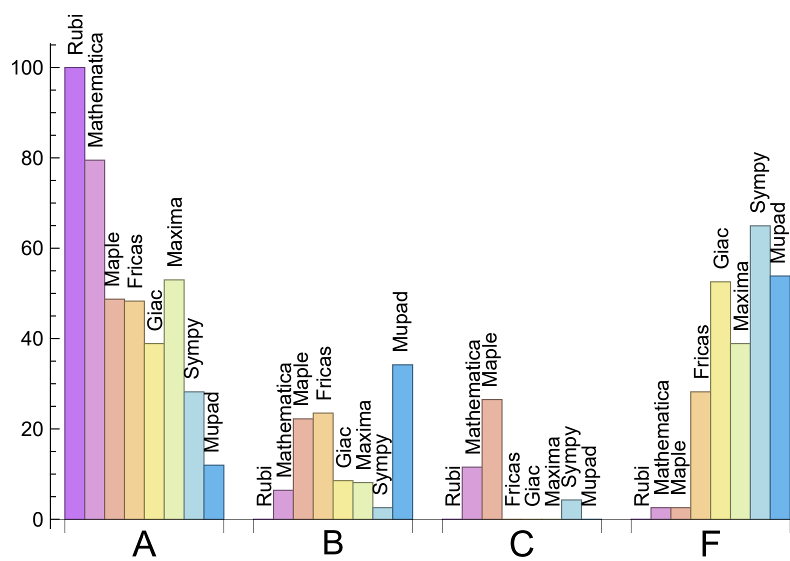
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	79.49	6.41	11.54	2.56
Maxima	52.99	8.12	0.00	38.89
Maple	48.72	22.22	26.50	2.56
Fricas	48.29	23.50	0.00	28.21
Giac	38.89	8.55	0.00	52.56
Sympy	28.21	2.56	4.27	64.96
Mupad	N/A	34.19	0.00	53.85

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	83.33 %	16.67 %	0.00 %
Maple	6	100.00 %	0.00 %	0.00 %
Fricas	66	98.48 %	0.00 %	1.52 %
Giac	123	92.68 %	5.69 %	1.63 %
Maxima	91	79.12 %	2.20 %	18.68 %
Sympy	152	53.29 %	29.61 %	17.11 %
Mupad	126	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

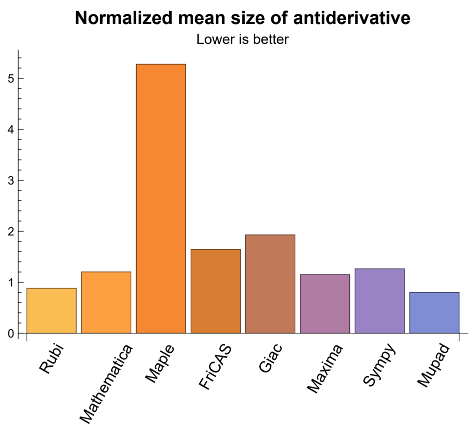
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

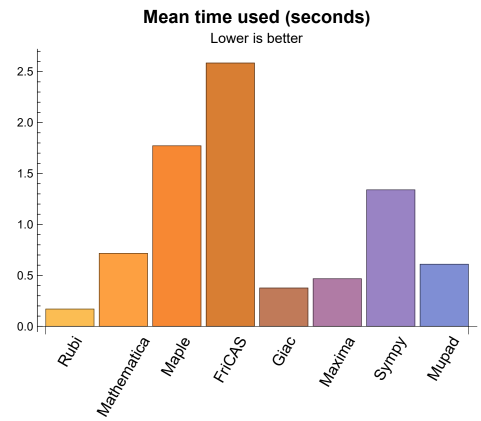
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	133.02	0.88	91.00	1.00
Mathematica	0.72	175.84	1.20	90.00	0.90
Maple	1.77	1087.11	5.28	153.00	1.54
Maxima	0.47	212.05	1.15	53.00	0.93
Fricas	2.58	262.84	1.64	64.50	1.04
Sympy	1.34	91.23	1.26	39.00	0.95
Giac	0.38	210.61	1.93	38.00	1.00
Mupad	0.61	60.05	0.80	31.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{34, 35, 59, 60, 116, 117, 120, 121, 128, 147, 148, 151, 155, 156, 161, 165, 169, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {116, 117, 120, 121}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{109, 110, 116, 117, 120, 121, 140, 141, 160, 173\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

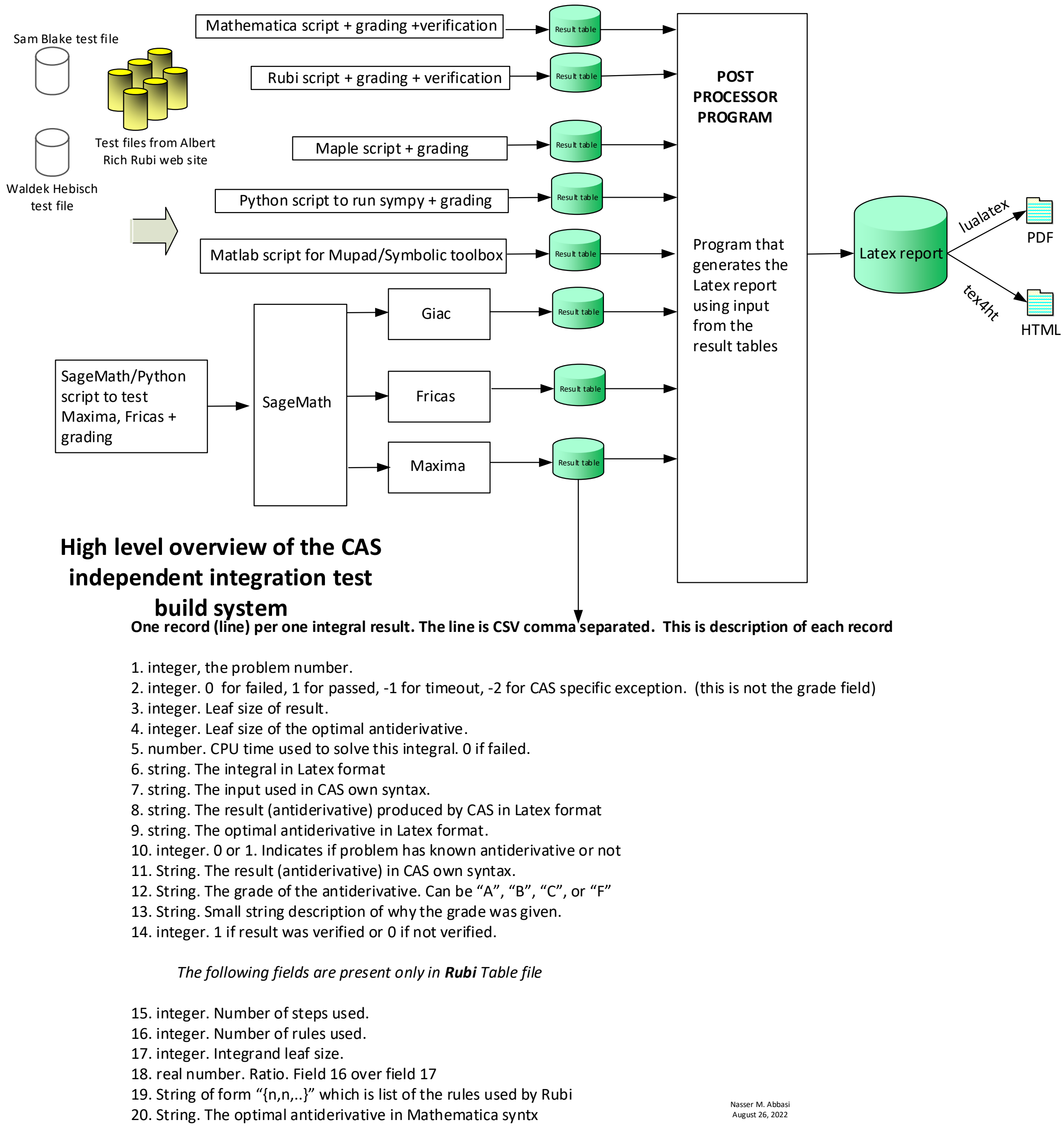
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 102, 107, 109, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 132, 136, 137, 138, 140, 142, 143, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234 }

B grade: { 38, 40, 42, 46, 103, 108, 110, 133, 141, 160, 173, 183, 190, 200, 207 }

C grade: { 9, 11, 44, 61, 62, 63, 64, 79, 89, 90, 99, 100, 101, 104, 105, 106, 123, 129, 130, 131, 134, 135, 217, 230, 231, 232, 233 }

F grade: { 113, 139, 144, 145, 152, 153 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 20, 22, 34, 35, 37, 39, 41, 43, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 59, 60, 65, 66, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 128, 131, 132, 133, 134, 138, 147, 148, 149, 151, 155, 156, 157, 161, 165, 169, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216, 219, 220, 224, 227, 228 }

B grade: { 7, 15, 17, 19, 21, 28, 38, 40, 42, 44, 48, 49, 57, 58, 88, 97, 107, 113, 124, 126, 127, 129, 130, 135, 136, 137, 140, 143, 152, 153, 154, 160, 164, 168, 170, 173, 177, 181, 186, 190, 194, 198, 203, 207, 211, 215, 218, 221, 222, 223, 225, 226 }

C grade: { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 67, 68, 69, 77, 98, 110, 111, 112, 139, 141, 142, 144, 145, 150, 158, 159, 162, 163, 166, 167, 171, 172, 175, 176, 179, 180, 183, 184, 185, 188, 189, 192, 193, 196, 197, 200, 201, 202, 205, 206, 209, 210, 213, 214, 217, 229, 230, 231, 232, 233, 234 }

F grade: { 36, 61, 62, 63, 64, 146 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 34, 35, 37, 39, 41, 43, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 109, 116, 117, 120, 121, 123, 125, 129, 130, 131, 132, 134, 135, 147, 148, 149, 150, 151, 155, 156, 157, 161, 170, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 221, 224, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade: { 7, 77, 88, 98, 107, 108, 110, 122, 124, 126, 127, 160, 162, 163, 164, 166, 167, 168, 173 }

C grade: { }

F grade: { 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 47, 49, 59, 61, 62, 63, 64, 65, 66, 97, 111, 112, 113, 114, 115, 118, 119, 128, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 165, 169, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 200, 201, 202, 203, 205, 206, 207, 217, 219, 220, 222, 223, 225, 226 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 34, 35, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 116, 117, 120, 121, 122, 123, 125, 129, 130, 131, 132, 134, 147, 148, 149, 150, 151, 155, 156, 157, 161, 165, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 187, 191, 195, 199, 204, 208, 212, 216, 217, 219, 220, 223, 224, 225, 226, 227, 228, 234 }

B grade: { 61, 62, 63, 64, 80, 81, 83, 84, 135, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 218, 221, 222, 229, 230, 231, 232, 233 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 77, 88, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 128, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 34, 35, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 59, 60, 70, 74, 75, 76, 78, 79, 80, 81, 83, 85, 86, 87, 92, 93, 94, 96, 99, 100, 101, 102, 116, 117, 120, 121, 123, 128, 132, 149, 151, 155, 156, 161, 170, 187, 227, 228 }

B grade: { 71, 89, 90, 91, 95, 157 }

C grade: { 82, 84, 104, 105, 106, 122, 125, 129, 130, 131 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 77, 88, 97, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 124, 126, 127, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 152, 153, 154, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 20, 22, 31, 34, 35, 41, 43, 45, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 116, 117, 120, 121, 126, 128, 147, 148, 150, 151, 155, 156, 157, 161, 165, 169, 170, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216, 227, 228, 229, 230, 233, 234 }

B grade: { 6, 99, 100, 101, 102, 104, 105, 106, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 149 }

C grade: { }

F grade: { 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 36, 37, 38, 39, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 65, 66, 72, 73, 77, 97, 98, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 231, 232 }

2.1.8 Mupad

A grade: { 34, 35, 59, 60, 116, 117, 120, 121, 128, 147, 148, 151, 155, 156, 161, 165, 169, 174, 178, 182, 187, 191, 195, 199, 204, 208, 212, 216 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 17, 20, 22, 37, 39, 41, 43, 45, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 99, 100, 101, 102, 104, 105, 106, 122, 123, 125, 129, 130, 131, 132, 134, 135, 138, 149, 150, 157, 170, 227, 228, 229, 230, 231, 232, 233, 234 }

C grade: { }

F grade: { 7, 13, 15, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 38, 40, 42, 44, 46, 47, 48, 49, 50, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 88, 92, 97, 98, 103, 107, 108, 109, 110, 111, 112,

$113, 114, 115, 118, 119, 124, 126, 127, 133, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 152, 153, 154, 158, 159, 160, 162, 163, 164, 166, 167, 168, 171, 172, 173, 175, 176, 177, 179, 180, 181, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 196, 197, 198, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226\ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	47	41	48	59	55
N.S.	1	1.00	1.00	0.86	0.92	0.80	0.94	1.16	1.08
time (sec)	N/A	0.017	0.004	0.059	0.491	3.341	0.311	0.419	0.805

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	46	45	46	74	56
N.S.	1	1.00	1.00	0.94	0.94	0.92	0.94	1.51	1.14
time (sec)	N/A	0.024	0.011	0.036	0.272	2.331	0.233	0.404	0.700

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	38	32	39	51	46
N.S.	1	1.00	1.00	0.88	0.93	0.78	0.95	1.24	1.12
time (sec)	N/A	0.015	0.003	0.046	0.557	2.945	0.211	0.414	0.721

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	36	37	37	64	49
N.S.	1	1.00	1.00	0.97	0.92	0.95	0.95	1.64	1.26
time (sec)	N/A	0.018	0.007	0.033	0.274	3.555	0.166	0.399	0.688

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	28	23	31	36	39
N.S.	1	1.00	1.00	0.90	0.90	0.74	1.00	1.16	1.26
time (sec)	N/A	0.008	0.003	0.046	0.475	2.557	0.126	0.404	0.757

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	24	24	24	45	22
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.00	1.88	0.92
time (sec)	N/A	0.004	0.003	0.032	0.274	4.131	0.095	0.401	0.136

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	63	56	0	0	38	-1
N.S.	1	1.00	1.00	1.70	1.51	0.00	0.00	1.03	-0.03
time (sec)	N/A	0.017	0.003	0.033	0.517	0.000	0.000	0.431	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	30	31	24	32	28
N.S.	1	1.00	1.00	1.13	1.00	1.03	0.80	1.07	0.93
time (sec)	N/A	0.015	0.003	0.039	0.283	3.124	0.116	0.398	0.235

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	32	23	24	24	40	44
N.S.	1	1.00	1.16	1.03	0.74	0.77	0.77	1.29	1.42
time (sec)	N/A	0.010	0.003	0.058	0.498	3.566	0.210	0.413	0.701

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	44	42	43	39	44	58
N.S.	1	1.00	0.96	0.96	0.91	0.93	0.85	0.96	1.26
time (sec)	N/A	0.021	0.009	0.047	0.278	2.594	0.253	0.412	0.850

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	40	37	33	32	51	47
N.S.	1	1.00	0.88	0.98	0.90	0.80	0.78	1.24	1.15
time (sec)	N/A	0.013	0.003	0.080	0.491	2.284	0.286	0.409	0.704

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	98	95	78	104	0	85
N.S.	1	1.00	0.76	0.94	0.91	0.75	1.00	0.00	0.82
time (sec)	N/A	0.151	0.018	0.111	0.489	3.097	0.398	0.000	0.809

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	95	208	0	0	0	0	-1
N.S.	1	1.00	0.70	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.380	0.158	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	78	77	60	78	0	66
N.S.	1	1.00	0.76	0.98	0.96	0.75	0.98	0.00	0.82
time (sec)	N/A	0.104	0.015	0.096	0.484	2.924	0.280	0.000	0.204

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	76	188	0	0	0	0	-1
N.S.	1	1.00	0.68	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.211	0.142	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	57	57	40	54	0	44
N.S.	1	1.00	0.79	1.08	1.08	0.75	1.02	0.00	0.83
time (sec)	N/A	0.052	0.011	0.087	0.487	2.227	0.154	0.000	0.137

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	130	0	0	0	0	55
N.S.	1	1.00	0.84	1.94	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.052	0.059	0.195	0.000	0.000	0.000	0.000	0.591

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	959	0	0	0	0	-1
N.S.	1	1.00	1.14	8.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.044	2.072	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	225	0	0	0	0	-1
N.S.	1	1.00	0.97	3.41	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.033	0.320	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	64	56	57	53	60	50
N.S.	1	1.00	0.95	1.08	0.95	0.97	0.90	1.02	0.85
time (sec)	N/A	0.063	0.013	0.114	0.485	2.129	0.220	0.397	0.660

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	96	254	0	0	0	0	-1
N.S.	1	1.00	0.85	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.170	0.276	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	88	95	78	80	91	73
N.S.	1	1.00	0.91	0.99	1.07	0.88	0.90	1.02	0.82
time (sec)	N/A	0.111	0.014	0.138	0.486	2.129	0.296	0.419	0.699

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	125	2455	0	0	0	0	-1
N.S.	1	1.00	0.64	12.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.455	0.498	5.207	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	184	2627	0	0	0	0	-1
N.S.	1	1.00	0.90	12.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.435	8.086	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	96	1017	0	0	0	0	-1
N.S.	1	1.00	0.65	6.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.263	2.133	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	1665	0	0	0	0	-1
N.S.	1	1.00	0.95	10.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	0.254	2.960	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	76	4393	0	0	0	0	-1
N.S.	1	1.00	0.74	42.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.069	0.534	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	90	187	0	0	0	0	-1
N.S.	1	1.00	0.94	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.085	0.484	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	180	1050	0	0	0	0	-1
N.S.	1	1.00	1.01	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.058	0.908	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	1444	0	0	0	0	-1
N.S.	1	1.00	0.89	15.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.054	1.008	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	5367	0	0	0	29	-1
N.S.	1	1.00	0.86	51.11	0.00	0.00	0.00	0.28	-0.01
time (sec)	N/A	0.145	0.119	0.816	0.000	0.000	0.000	0.408	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	151	4745	0	0	0	0	-1
N.S.	1	1.00	0.90	28.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.181	6.392	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	126	855	0	0	0	0	-1
N.S.	1	1.00	0.83	5.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.198	2.892	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.009	0.643	0.245	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.009	0.629	0.151	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.017	0.161	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	32	38	35	31	34	0	32
N.S.	1	1.00	0.80	0.95	0.88	0.78	0.85	0.00	0.80
time (sec)	N/A	0.073	0.022	0.161	0.490	1.675	0.161	0.000	0.734

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	241	128	0	0	0	0	-1
N.S.	1	1.00	3.60	1.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.050	0.125	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	24	19	19	0	19
N.S.	1	1.00	1.00	1.13	1.04	0.83	0.83	0.00	0.83
time (sec)	N/A	0.036	0.012	0.152	0.482	1.893	0.136	0.000	0.636

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	221	112	0	0	0	0	-1
N.S.	1	1.00	4.60	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.035	0.109	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	8	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	1.00	0.75
time (sec)	N/A	0.009	0.004	0.110	0.257	1.865	0.528	0.422	0.617

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	251	163	0	0	0	0	-1
N.S.	1	1.00	5.12	3.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.046	0.115	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	29	29	22	26	26
N.S.	1	1.00	1.00	1.10	0.97	0.97	0.73	0.87	0.87
time (sec)	N/A	0.042	0.012	0.163	0.479	4.085	0.182	0.409	0.092

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	280	180	0	0	0	0	-1
N.S.	1	1.00	3.89	2.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.043	0.145	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	55	47	42	39	35
N.S.	1	1.00	1.00	0.91	1.17	1.00	0.89	0.83	0.74
time (sec)	N/A	0.077	0.015	0.148	0.474	2.135	0.303	0.404	0.097

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	626	280	189	0	0	0	-1
N.S.	1	1.00	3.04	1.36	0.92	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	1.122	0.227	0.483	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	343	317	0	0	0	0	-1
N.S.	1	1.00	1.82	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.062	0.172	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	319	335	187	0	0	0	-1
N.S.	1	1.00	1.74	1.83	1.02	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.059	0.261	0.487	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	379	393	0	0	0	0	-1
N.S.	1	1.00	1.70	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.072	0.121	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	348	312	183	0	0	0	-1
N.S.	1	1.00	1.64	1.47	0.86	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	0.066	0.116	0.488	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	8	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.60	1.00
time (sec)	N/A	0.014	0.021	0.112	0.280	6.539	0.096	0.397	0.589

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.15	1.00
time (sec)	N/A	0.016	0.009	0.178	0.280	2.807	0.921	0.414	0.606

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	212	254	226	237	367	347	234
N.S.	1	1.00	0.87	1.04	0.93	0.97	1.50	1.42	0.96
time (sec)	N/A	0.125	0.065	0.142	0.273	3.015	0.733	0.424	0.217

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	149	175	159	167	243	252	190
N.S.	1	1.00	0.89	1.04	0.95	0.99	1.45	1.50	1.13
time (sec)	N/A	0.087	0.045	0.131	0.267	4.926	0.501	0.421	1.016

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	97	112	103	108	151	171	116
N.S.	1	1.00	0.89	1.03	0.94	0.99	1.39	1.57	1.06
time (sec)	N/A	0.094	0.029	0.122	0.267	2.608	0.330	0.406	0.844

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	62	53	57	73	99	62
N.S.	1	1.00	1.16	1.07	0.91	0.98	1.26	1.71	1.07
time (sec)	N/A	0.046	0.012	0.046	0.271	3.051	0.188	0.413	0.786

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	523	977	528	0	0	0	-1
N.S.	1	1.00	1.30	2.42	1.31	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.692	0.219	0.328	0.547	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	801	801	806	2275	628	0	0	0	-1
N.S.	1	1.00	1.01	2.84	0.78	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.929	5.277	0.420	0.548	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	3.833	0.201	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	2.910	0.190	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	169	0	0	349	0	59	-1
N.S.	1	1.00	2.56	0.00	0.00	5.29	0.00	0.89	-0.02
time (sec)	N/A	0.066	0.184	0.181	0.000	2.450	0.000	0.421	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	262	0	0	712	0	126	-1
N.S.	1	1.00	1.96	0.00	0.00	5.31	0.00	0.94	-0.01
time (sec)	N/A	0.246	0.496	0.182	0.000	1.762	0.000	0.437	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	345	0	0	1278	0	208	-1
N.S.	1	1.00	1.66	0.00	0.00	6.14	0.00	1.00	-0.00
time (sec)	N/A	0.671	0.755	0.181	0.000	1.424	0.000	0.423	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	450	0	0	1986	0	340	-1
N.S.	1	1.00	1.54	0.00	0.00	6.78	0.00	1.16	-0.00
time (sec)	N/A	0.846	1.047	0.183	0.000	2.809	0.000	0.443	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	136	117	0	0	0	0	-1
N.S.	1	1.00	0.70	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.886	0.273	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	89	99	0	0	0	0	-1
N.S.	1	1.00	0.57	0.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.079	0.197	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	21	68	31	29	0	33	-1
N.S.	1	1.00	0.60	1.94	0.89	0.83	0.00	0.94	-0.03
time (sec)	N/A	0.015	0.019	0.161	0.470	1.013	0.000	0.427	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	37	165	63	52	0	55	-1
N.S.	1	1.00	0.47	2.09	0.80	0.66	0.00	0.70	-0.01
time (sec)	N/A	0.030	0.024	0.195	0.483	4.324	0.000	0.431	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	258	93	70	0	83	-1
N.S.	1	1.00	0.40	2.19	0.79	0.59	0.00	0.70	-0.01
time (sec)	N/A	0.046	0.030	0.227	0.490	1.927	0.000	0.442	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	25	27	26	21	31	32	22
N.S.	1	1.00	0.78	0.84	0.81	0.66	0.97	1.00	0.69
time (sec)	N/A	0.018	0.016	0.133	0.479	2.366	0.217	0.430	0.073

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	39	39	88	40	27
N.S.	1	1.00	0.82	0.84	0.89	0.89	2.00	0.91	0.61
time (sec)	N/A	0.019	0.019	0.137	0.464	1.846	0.297	0.424	0.601

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	35	38	26	0	0	22
N.S.	1	1.00	0.82	1.03	1.12	0.76	0.00	0.00	0.65
time (sec)	N/A	0.010	0.012	0.144	0.482	0.958	0.000	0.000	0.614

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	65	75	40	0	0	51
N.S.	1	1.00	0.82	1.16	1.34	0.71	0.00	0.00	0.91
time (sec)	N/A	0.031	0.020	0.273	0.481	1.868	0.000	0.000	0.062

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	38	39	39	40	35
N.S.	1	1.00	1.00	0.98	0.93	0.95	0.95	0.98	0.85
time (sec)	N/A	0.020	0.012	0.066	0.266	0.985	0.416	0.398	0.645

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	34	27	36	38	31
N.S.	1	1.00	1.00	0.97	0.92	0.73	0.97	1.03	0.84
time (sec)	N/A	0.015	0.007	0.081	0.474	2.086	0.268	0.408	0.612

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	28	28	31	47	27
N.S.	1	1.00	1.00	0.97	0.90	0.90	1.00	1.52	0.87
time (sec)	N/A	0.007	0.009	0.043	0.258	4.415	0.190	0.423	0.596

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	57	68	0	0	0	-1
N.S.	1	1.00	1.00	1.54	1.84	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.009	0.079	0.492	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	32	37	29	32	31
N.S.	1	1.00	1.00	0.91	0.94	1.09	0.85	0.94	0.91
time (sec)	N/A	0.013	0.007	0.049	0.276	3.711	0.278	0.414	0.660

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	31	27	28	29	29	32
N.S.	1	1.00	1.09	0.89	0.77	0.80	0.83	0.83	0.91
time (sec)	N/A	0.013	0.007	0.069	0.474	9.080	0.376	0.413	0.649

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	136	112	137	239	136	156	54
N.S.	1	1.00	0.89	0.74	0.90	1.57	0.89	1.03	0.36
time (sec)	N/A	0.072	0.032	0.077	0.459	2.539	14.786	0.442	0.443

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	133	109	135	228	126	153	52
N.S.	1	1.00	0.89	0.73	0.90	1.52	0.84	1.02	0.35
time (sec)	N/A	0.071	0.025	0.081	0.467	2.961	7.088	0.429	0.700

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	102	97	120	189	615	144	42
N.S.	1	1.00	0.77	0.73	0.91	1.43	4.66	1.09	0.32
time (sec)	N/A	0.056	0.023	0.041	0.469	3.047	4.365	0.412	0.129

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	105	98	123	227	110	135	44
N.S.	1	1.00	0.78	0.73	0.91	1.68	0.81	1.00	0.33
time (sec)	N/A	0.058	0.031	0.046	0.465	2.923	8.025	0.412	0.683

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	146	106	133	269	459	149	52
N.S.	1	1.00	0.97	0.71	0.89	1.79	3.06	0.99	0.35
time (sec)	N/A	0.070	0.037	0.069	0.465	3.481	16.953	0.425	0.715

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	32	31	27	39	33	31
N.S.	1	1.00	0.78	0.63	0.61	0.53	0.76	0.65	0.61
time (sec)	N/A	0.009	0.012	0.008	0.481	1.987	1.308	0.426	0.650

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	27	26	20	32	28	26
N.S.	1	1.00	0.79	0.64	0.62	0.48	0.76	0.67	0.62
time (sec)	N/A	0.007	0.009	0.004	0.463	2.977	0.839	0.408	0.659

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	12	19	14	16
N.S.	1	1.00	1.00	0.77	0.73	0.55	0.86	0.64	0.73
time (sec)	N/A	0.004	0.014	0.006	0.489	3.128	0.762	0.401	0.631

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	61	35	0	0	19	-1
N.S.	1	1.00	1.00	1.97	1.13	0.00	0.00	0.61	-0.03
time (sec)	N/A	0.022	0.006	0.017	0.492	0.000	0.000	0.421	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	29	18	17	19	92	19	17
N.S.	1	1.00	1.26	0.78	0.74	0.83	4.00	0.83	0.74
time (sec)	N/A	0.009	0.009	0.007	0.493	3.310	0.585	0.405	0.642

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	26	27	160	26	24
N.S.	1	1.00	0.81	0.64	0.62	0.64	3.81	0.62	0.57
time (sec)	N/A	0.008	0.009	0.010	0.458	1.910	1.214	0.405	0.646

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	25	24	24	85	39	24
N.S.	1	1.00	0.81	0.69	0.67	0.67	2.36	1.08	0.67
time (sec)	N/A	0.010	0.013	0.007	0.256	2.370	1.037	0.410	0.646

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	19	24	30	-1
N.S.	1	1.00	0.86	0.69	0.66	0.66	0.83	1.03	-0.03
time (sec)	N/A	0.009	0.009	0.007	0.252	2.570	0.866	0.403	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	17	18	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.94	1.00	0.78
time (sec)	N/A	0.005	0.008	0.010	0.265	2.362	0.143	0.395	0.770

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	25	20	16	20
N.S.	1	1.00	1.00	0.86	0.82	1.14	0.91	0.73	0.91
time (sec)	N/A	0.006	0.010	0.008	0.258	3.150	0.398	0.404	0.665

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	26	25	33	143	23	27
N.S.	1	1.00	0.78	0.70	0.68	0.89	3.86	0.62	0.73
time (sec)	N/A	0.009	0.014	0.010	0.257	2.705	1.295	0.427	0.650

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	20	15	15	14	13	15
N.S.	1	1.00	1.00	1.18	0.88	0.88	0.82	0.76	0.88
time (sec)	N/A	0.004	0.002	0.062	0.254	3.338	0.071	0.406	0.056

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	83	0	63	0	0	-1
N.S.	1	1.00	0.85	1.77	0.00	1.34	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.013	0.151	0.000	2.553	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	57	68	0	0	0	-1
N.S.	1	1.00	1.00	1.54	1.84	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.009	0.060	0.516	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	157	104	92	155	617	133
N.S.	1	1.00	0.90	1.48	0.98	0.87	1.46	5.82	1.25
time (sec)	N/A	0.078	0.049	0.079	0.478	2.556	0.752	0.788	0.775

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	114	114	85	73	117	423	101
N.S.	1	1.00	1.42	1.42	1.06	0.91	1.46	5.29	1.26
time (sec)	N/A	0.062	0.031	0.068	0.472	1.452	0.534	0.700	1.335

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	55	78	210	61
N.S.	1	1.00	1.50	1.05	1.13	0.92	1.30	3.50	1.02
time (sec)	N/A	0.042	0.024	0.062	0.458	2.186	0.334	0.482	0.973

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	44	30	29	43	46	111	42
N.S.	1	1.00	1.33	0.91	0.88	1.30	1.39	3.36	1.27
time (sec)	N/A	0.009	0.011	0.052	0.263	6.181	0.205	0.445	1.197

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	251	106	133	0	0	0	-1
N.S.	1	1.00	2.09	0.88	1.11	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.019	0.060	0.528	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	66	61	77	64	167	498	62
N.S.	1	1.00	1.06	0.98	1.24	1.03	2.69	8.03	1.00
time (sec)	N/A	0.031	0.044	0.076	0.478	3.320	0.795	0.540	1.222

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	83	112	99	381	1309	230
N.S.	1	1.00	0.97	0.87	1.18	1.04	4.01	13.78	2.42
time (sec)	N/A	0.067	0.072	0.105	0.471	1.830	1.094	0.685	1.386

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	126	115	165	142	760	3449	285
N.S.	1	1.00	0.98	0.89	1.28	1.10	5.89	26.74	2.21
time (sec)	N/A	0.101	0.103	0.113	0.468	3.645	1.573	1.529	1.240

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	563	2074	8519	0	0	0	-1
N.S.	1	1.00	0.88	3.23	13.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.643	0.401	0.704	5.308	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	345	187	283	0	0	0	-1
N.S.	1	1.00	2.27	1.23	1.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.030	0.141	0.549	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	338	422	602	322	280	0	0	0	-1
N.S.	1	1.25	1.78	0.95	0.83	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	7.261	0.105	0.543	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	735	818	5117	13390	8518	0	0	0	-1
N.S.	1	1.11	6.96	18.22	11.59	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.215	30.051	1.212	1.197	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	693	618	364	0	0	0	0	-1
N.S.	1	1.00	0.89	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.516	0.542	0.088	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	830	830	809	388	0	0	0	0	-1
N.S.	1	1.00	0.97	0.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.764	0.501	0.073	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	0	4611	0	0	0	0	-1
N.S.	1	1.00	0.00	12.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	180.020	1.003	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	127	118	0	0	0	0	-1
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.114	0.316	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	138	156	0	0	0	0	-1
N.S.	1	1.00	0.64	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.115	0.068	0.372	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	177	0	0	0	0	0	-1
N.S.	1	0.00	7.70	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	0.289	0.207	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	180	0	0	0	0	0	-1
N.S.	1	0.00	7.20	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	0.126	0.189	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	202	160	0	0	0	0	-1
N.S.	1	1.00	1.08	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.046	0.694	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	207	202	0	0	0	0	-1
N.S.	1	1.00	0.74	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.674	0.806	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	198	0	0	0	0	0	-1
N.S.	1	0.00	6.60	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.641	0.627	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	200	0	0	0	0	0	-1
N.S.	1	0.00	6.25	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.121	0.213	0.642	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	42	93	81	99	203	85
N.S.	1	1.00	0.81	0.81	1.79	1.56	1.90	3.90	1.63
time (sec)	N/A	0.026	0.014	0.131	0.454	3.391	6.069	0.491	0.153

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	141	36	52	33	56	100	49
N.S.	1	1.00	3.62	0.92	1.33	0.85	1.44	2.56	1.26
time (sec)	N/A	0.015	0.051	0.071	0.472	2.240	2.587	0.431	1.510

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	87	112	0	0	100	-1
N.S.	1	1.00	0.84	1.93	2.49	0.00	0.00	2.22	-0.02
time (sec)	N/A	0.032	0.009	0.118	0.509	0.000	0.000	0.653	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	41	53	59	139	238	57
N.S.	1	1.00	0.85	0.87	1.13	1.26	2.96	5.06	1.21
time (sec)	N/A	0.022	0.019	0.143	0.265	2.112	5.251	0.445	0.774

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	68	64	0	0	26	-1
N.S.	1	1.00	1.00	1.94	1.83	0.00	0.00	0.74	-0.03
time (sec)	N/A	0.026	0.005	0.112	0.497	0.000	0.000	0.404	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	38	97	122	0	0	103	-1
N.S.	1	1.00	0.84	2.16	2.71	0.00	0.00	2.29	-0.02
time (sec)	N/A	0.034	0.006	0.108	0.510	0.000	0.000	0.678	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	6.419	0.602	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	157	727	339	325	654	2265	783
N.S.	1	1.00	0.67	3.12	1.45	1.39	2.81	9.72	3.36
time (sec)	N/A	0.282	0.197	0.568	0.466	3.680	12.360	1.649	1.194

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	118	430	216	208	376	1161	409
N.S.	1	1.00	0.77	2.79	1.40	1.35	2.44	7.54	2.66
time (sec)	N/A	0.153	0.110	0.276	0.473	2.445	4.583	1.154	0.956

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	163	150	115	114	177	451	136
N.S.	1	1.00	1.68	1.55	1.19	1.18	1.82	4.65	1.40
time (sec)	N/A	0.089	0.067	0.119	0.483	7.949	1.776	0.520	1.453

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	49	42	34	52	51	116	48
N.S.	1	1.00	1.29	1.11	0.89	1.37	1.34	3.05	1.26
time (sec)	N/A	0.014	0.010	0.068	0.257	6.711	0.153	0.451	1.285

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	336	238	0	0	0	0	-1
N.S.	1	1.00	2.07	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.250	0.171	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	118	235	186	229	0	1264	128
N.S.	1	1.00	0.77	1.54	1.22	1.50	0.00	8.26	0.84
time (sec)	N/A	0.091	0.150	0.210	0.474	3.604	0.000	0.767	2.084

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	180	467	437	727	0	6173	399
N.S.	1	1.00	0.79	2.05	1.92	3.19	0.00	27.07	1.75
time (sec)	N/A	0.234	0.447	0.405	0.492	4.884	0.000	1.985	7.329

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	665	2040	0	0	0	0	-1
N.S.	1	1.00	1.74	5.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	3.485	0.514	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	286	721	0	0	0	0	-1
N.S.	1	1.00	1.30	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.361	0.267	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	118	196	0	0	0	0	123
N.S.	1	1.00	1.16	1.92	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.086	0.104	0.268	0.000	0.000	0.000	0.000	0.966

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	2091	0	0	0	0	-1
N.S.	1	1.00	0.00	8.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	36.353	4.712	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	454	1254	0	0	0	0	-1
N.S.	1	1.00	0.80	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.045	6.157	2.062	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	565	565	2336	12407	0	0	0	0	-1
N.S.	1	1.00	4.13	21.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	9.737	52.640	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	630	18833	0	0	0	0	-1
N.S.	1	1.00	1.87	55.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.783	6.664	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	228	430	0	0	0	0	-1
N.S.	1	1.00	1.59	3.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.226	1.105	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	4291	0	0	0	0	-1
N.S.	1	1.00	0.00	11.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	37.891	11.312	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	6740	0	0	0	0	-1
N.S.	1	1.00	0.00	5.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.733	44.108	21.649	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.294	0.168	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	3.996	0.172	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.194	0.175	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	37	35	51	60	127	230
N.S.	1	1.00	0.88	0.88	0.83	1.21	1.43	3.02	5.48
time (sec)	N/A	0.031	0.016	0.131	0.258	2.852	0.770	0.469	0.763

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	149	38	56	0	60	58
N.S.	1	1.00	0.89	3.31	0.84	1.24	0.00	1.33	1.29
time (sec)	N/A	0.034	0.026	0.117	0.259	1.942	0.000	0.426	1.377

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.068	0.268	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	0	1605	0	0	0	0	-1
N.S.	1	1.00	0.00	3.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.214	0.582	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	823	0	0	0	0	-1
N.S.	1	1.00	0.00	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.297	0.391	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	259	0	0	0	0	-1
N.S.	1	1.00	0.74	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.055	0.258	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.065	0.289	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.554	0.286	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	20	17	15	48	30	21
N.S.	1	1.00	1.12	1.25	1.06	0.94	3.00	1.88	1.31
time (sec)	N/A	0.005	0.007	0.148	0.268	2.232	0.081	0.395	0.077

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	363	8070	0	1965	0	0	-1
N.S.	1	1.00	0.90	20.02	0.00	4.88	0.00	0.00	-0.00
time (sec)	N/A	0.386	0.629	28.517	0.000	3.963	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	272	7714	0	1545	0	0	-1
N.S.	1	1.00	0.89	25.29	0.00	5.07	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.464	3.467	0.000	4.347	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	555	1138	433	1101	0	0	-1
N.S.	1	1.00	2.80	5.75	2.19	5.56	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.277	2.135	0.557	3.735	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.103	0.255	0.125	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1526	310	321	0	0	-1
N.S.	1	1.00	0.88	9.91	2.01	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.168	1.990	0.290	2.877	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1491	218	270	0	0	-1
N.S.	1	1.00	0.89	12.12	1.77	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.070	2.021	0.285	3.565	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	599	455	199	0	0	-1
N.S.	1	1.00	0.88	7.05	5.35	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.466	0.653	0.500	6.283	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.082	0.222	0.155	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	141	1527	312	321	0	0	-1
N.S.	1	1.00	0.91	9.85	2.01	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.174	2.010	0.295	7.421	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1492	220	270	0	0	-1
N.S.	1	1.00	0.90	12.03	1.77	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.073	1.690	0.276	2.788	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	650	450	201	0	0	-1
N.S.	1	1.00	0.88	7.56	5.23	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.601	0.677	0.524	7.268	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.147	0.573	0.141	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	45	10	10	19	10	16
N.S.	1	1.00	1.12	2.81	0.62	0.62	1.19	0.62	1.00
time (sec)	N/A	0.006	0.007	0.177	0.254	3.878	0.082	0.406	0.637

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	359	7900	0	1589	0	0	-1
N.S.	1	1.00	0.90	19.80	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.619	33.047	0.000	2.920	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	270	7532	0	1289	0	0	-1
N.S.	1	1.00	0.89	24.86	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	0.298	0.475	3.359	0.000	2.820	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	1649	1146	532	965	0	0	-1
N.S.	1	1.00	8.33	5.79	2.69	4.87	0.00	0.00	-0.01
time (sec)	N/A	0.177	12.610	2.142	0.549	4.544	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.102	0.272	0.201	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1526	0	174	0	0	-1
N.S.	1	1.00	0.91	9.91	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.153	2.009	0.000	2.210	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1491	0	152	0	0	-1
N.S.	1	1.00	0.89	12.12	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.075	1.875	0.000	4.997	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	1498	0	116	0	0	-1
N.S.	1	1.00	0.88	17.62	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.436	0.853	0.000	2.021	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	0.251	0.021	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	136	1527	0	174	0	0	-1
N.S.	1	1.00	0.88	9.85	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.167	2.039	0.000	2.668	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1492	0	152	0	0	-1
N.S.	1	1.00	0.89	12.03	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.082	1.983	0.000	3.287	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	75	1756	0	116	0	0	-1
N.S.	1	1.00	0.87	20.42	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.504	0.936	0.000	2.829	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.093	0.285	0.024	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	2700	0	0	-1
N.S.	1	1.00	2.01	24.33	0.00	9.03	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.222	18.629	0.000	3.167	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	1511	0	0	-1
N.S.	1	1.00	1.64	23.69	0.00	6.60	0.00	0.00	-0.00
time (sec)	N/A	0.116	0.109	13.659	0.000	2.895	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2688	0	717	0	0	-1
N.S.	1	1.00	1.75	16.91	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.164	1.129	0.000	2.371	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	132	184	0	334	0	0	-1
N.S.	1	1.00	1.81	2.52	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.022	0.645	0.000	2.423	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.031	5.542	0.188	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	305	6939	0	1289	0	0	-1
N.S.	1	1.00	0.86	19.55	0.00	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.335	4.442	26.008	0.000	1.169	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	229	6589	0	1067	0	0	-1
N.S.	1	1.00	0.86	24.68	0.00	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	2.706	3.160	0.000	1.522	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	363	365	0	825	0	0	-1
N.S.	1	1.00	2.09	2.10	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.839	0.941	0.000	1.701	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.101	3.837	0.161	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1473	129	292	0	0	-1
N.S.	1	1.00	0.90	10.37	0.91	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.151	1.867	1.212	5.105	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1437	107	246	0	0	-1
N.S.	1	1.00	0.90	12.72	0.95	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.063	1.721	1.220	1.716	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	597	80	186	0	0	-1
N.S.	1	1.00	0.90	7.56	1.01	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.424	0.592	1.199	2.564	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.097	2.375	0.173	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1486	129	292	0	0	-1
N.S.	1	1.00	0.88	10.25	0.89	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.149	1.854	1.218	1.218	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1450	106	246	0	0	-1
N.S.	1	1.00	0.88	12.50	0.91	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.081	1.614	1.199	3.266	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	554	80	186	0	0	-1
N.S.	1	1.00	0.87	6.76	0.98	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.400	0.626	1.215	2.153	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	2.398	0.202	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	2700	0	0	-1
N.S.	1	1.00	2.01	24.33	0.00	9.03	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.214	19.672	0.000	1.371	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	1511	0	0	-1
N.S.	1	1.00	1.64	23.69	0.00	6.60	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.129	14.640	0.000	1.984	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2688	0	717	0	0	-1
N.S.	1	1.00	1.75	16.91	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.181	1.599	0.000	3.829	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	132	184	0	334	0	0	-1
N.S.	1	1.00	1.78	2.49	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.024	0.684	0.000	2.324	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.025	0.577	0.190	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	299	6888	0	1269	0	0	-1
N.S.	1	1.00	0.85	19.62	0.00	3.62	0.00	0.00	-0.00
time (sec)	N/A	0.341	3.400	26.911	0.000	2.260	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	225	6508	0	1051	0	0	-1
N.S.	1	1.00	0.85	24.56	0.00	3.97	0.00	0.00	-0.00
time (sec)	N/A	0.298	2.700	3.314	0.000	3.579	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	363	365	0	813	0	0	-1
N.S.	1	1.00	2.09	2.10	0.00	4.67	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.782	0.957	0.000	3.833	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.102	3.841	0.181	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1472	129	292	0	0	-1
N.S.	1	1.00	0.90	10.37	0.91	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.153	1.898	1.287	2.721	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1436	107	246	0	0	-1
N.S.	1	1.00	0.90	12.71	0.95	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.073	1.608	1.212	2.071	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	597	80	186	0	0	-1
N.S.	1	1.00	0.90	7.56	1.01	2.35	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.407	0.698	1.268	2.733	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.094	2.413	0.220	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1487	129	292	0	0	-1
N.S.	1	1.00	0.88	10.26	0.89	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.155	1.919	1.227	2.507	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1451	106	246	0	0	-1
N.S.	1	1.00	0.88	12.51	0.91	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.096	1.807	1.247	3.674	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	554	80	186	0	0	-1
N.S.	1	1.00	0.87	6.76	0.98	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.404	0.678	1.220	3.116	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	2.390	0.220	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	132	1058	0	264	0	0	-1
N.S.	1	1.00	0.71	5.66	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.461	0.236	0.214	0.000	3.111	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	59	34	40	0	0	-1
N.S.	1	1.00	1.69	1.69	0.97	1.14	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.056	0.107	0.525	2.993	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	58	50	0	65	0	0	-1
N.S.	1	1.00	0.82	0.70	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.009	0.094	0.000	8.987	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	76	0	87	0	0	-1
N.S.	1	1.00	1.00	0.74	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.008	0.091	0.000	7.057	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	83	95	63	103	0	0	-1
N.S.	1	1.00	1.63	1.86	1.24	2.02	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.119	0.102	0.516	5.379	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	83	355	0	151	0	0	-1
N.S.	1	1.00	0.81	3.45	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.012	0.277	0.000	2.519	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	413	0	187	0	0	-1
N.S.	1	1.00	1.00	2.74	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.008	0.275	0.000	4.368	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	167	162	189	212	0	0	-1
N.S.	1	1.00	0.85	0.83	0.96	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.130	0.349	0.539	2.786	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	261	678	0	304	0	0	-1
N.S.	1	1.00	1.04	2.71	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	2.050	0.484	0.158	0.000	3.276	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	345	764	0	378	0	0	-1
N.S.	1	1.00	1.10	2.44	0.00	1.21	0.00	0.00	-0.00
time (sec)	N/A	1.965	0.379	0.158	0.000	2.026	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	19	28	19	21	22
N.S.	1	1.00	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.016	0.018	0.063	0.283	1.402	1.441	0.536	0.093

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	17	15	12	18	15
N.S.	1	1.00	1.00	1.12	1.00	0.88	0.71	1.06	0.88
time (sec)	N/A	0.027	0.024	0.137	0.267	3.990	0.226	0.399	0.127

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	61	1281	47	131	0	66	65
N.S.	1	1.00	1.30	27.26	1.00	2.79	0.00	1.40	1.38
time (sec)	N/A	0.059	0.057	0.186	0.490	1.367	0.000	0.563	0.730

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1358	131	276	0	154	133
N.S.	1	1.00	1.42	13.18	1.27	2.68	0.00	1.50	1.29
time (sec)	N/A	0.124	0.080	0.266	0.483	2.332	0.000	0.454	0.855

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1323	167	431	0	0	164
N.S.	1	1.00	0.49	7.35	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.134	0.060	0.448	0.493	2.279	0.000	0.000	2.564

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1323	167	431	0	0	164
N.S.	1	1.00	0.49	7.35	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.136	0.057	0.395	0.481	2.032	0.000	0.000	2.372

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	859	169	221	0	154	135
N.S.	1	1.00	1.41	8.34	1.64	2.15	0.00	1.50	1.31
time (sec)	N/A	0.119	0.082	0.224	0.517	1.852	0.000	0.431	0.835

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	903	48	75	0	65	68
N.S.	1	1.00	1.23	18.81	1.00	1.56	0.00	1.35	1.42
time (sec)	N/A	0.060	0.049	0.174	0.504	1.736	0.000	0.437	0.706

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [119] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	2	2	1.00	4	0.500
7	A	3	2	1.00	8	0.250
8	A	5	5	1.00	8	0.625
9	A	3	3	1.00	8	0.375
10	A	4	3	1.00	8	0.375
11	A	4	3	1.00	8	0.375
12	A	15	7	1.00	10	0.700
13	A	14	9	1.00	10	0.900
14	A	10	7	1.00	10	0.700
15	A	9	8	1.00	10	0.800
16	A	5	5	1.00	8	0.625
17	A	5	5	1.00	6	0.833
18	A	6	5	1.00	10	0.500
19	A	4	4	1.00	10	0.400
20	A	8	7	1.00	10	0.700
21	A	8	7	1.00	10	0.700
22	A	13	8	1.00	10	0.800
23	A	33	11	1.00	10	1.100
24	A	22	11	1.00	10	1.100
25	A	18	10	1.00	10	1.000

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	11	9	1.00	10	0.900
27	A	8	8	1.00	8	1.000
28	A	5	6	1.00	6	1.000
29	A	8	6	1.00	10	0.600
30	A	5	6	1.00	10	0.600
31	A	7	6	1.00	10	0.600
32	A	14	11	1.00	10	1.100
33	A	16	8	1.00	10	0.800
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	2	2	1.00	8	0.250
37	A	9	7	1.00	13	0.538
38	A	8	8	1.00	13	0.615
39	A	4	4	1.00	13	0.308
40	A	4	4	1.00	11	0.364
41	A	1	1	1.00	10	0.100
42	A	3	3	1.00	13	0.231
43	A	7	7	1.00	13	0.538
44	A	7	7	1.00	13	0.538
45	A	12	8	1.00	13	0.615
46	A	28	15	1.00	15	1.000
47	A	10	5	1.00	13	0.385
48	A	25	13	1.00	12	1.083
49	A	15	7	1.00	15	0.467
50	A	31	19	1.00	15	1.267
51	A	1	1	1.00	12	0.083
52	A	1	1	1.00	12	0.083
53	A	4	4	1.00	14	0.286
54	A	4	4	1.00	14	0.286
55	A	5	5	1.00	14	0.357
56	A	5	4	1.00	12	0.333
57	A	27	13	1.00	14	0.929
58	A	24	12	1.00	14	0.857
59	A	0	0	0.00	0	0.000
60	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	6	1.00	16	0.375
62	A	7	9	1.00	16	0.562
63	A	8	9	1.00	16	0.562
64	A	8	9	1.00	16	0.562
65	A	3	3	1.00	14	0.214
66	A	2	2	1.00	14	0.143
67	A	1	1	1.00	14	0.071
68	A	2	2	1.00	14	0.143
69	A	3	2	1.00	14	0.143
70	A	3	3	1.00	11	0.273
71	A	4	3	1.00	11	0.273
72	A	2	2	1.00	10	0.200
73	A	4	4	1.00	12	0.333
74	A	4	3	1.00	10	0.300
75	A	4	4	1.00	10	0.400
76	A	2	2	1.00	8	0.250
77	A	4	3	1.00	10	0.300
78	A	5	5	1.00	10	0.500
79	A	4	4	1.00	10	0.400
80	A	11	8	1.00	10	0.800
81	A	11	8	1.00	10	0.800
82	A	10	7	1.00	6	1.167
83	A	10	7	1.00	10	0.700
84	A	11	8	1.00	10	0.800
85	A	6	4	1.00	10	0.400
86	A	5	4	1.00	8	0.500
87	A	4	4	1.00	6	0.667
88	A	4	3	1.00	10	0.300
89	A	4	4	1.00	10	0.400
90	A	5	4	1.00	10	0.400
91	A	3	2	1.00	12	0.167
92	A	3	2	1.00	12	0.167
93	A	2	2	1.00	12	0.167
94	A	4	4	1.00	12	0.333
95	A	3	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	4	0.750
97	A	4	3	1.00	10	0.300
98	A	4	3	1.00	10	0.300
99	A	7	6	1.00	10	0.600
100	A	7	6	1.00	10	0.600
101	A	7	6	1.00	8	0.750
102	A	3	3	1.00	6	0.500
103	A	5	5	1.00	10	0.500
104	A	7	7	1.00	10	0.700
105	A	8	7	1.00	10	0.700
106	A	8	7	1.00	10	0.700
107	A	15	5	1.00	16	0.312
108	A	5	5	1.00	14	0.357
109	A	37	10	1.25	16	0.625
110	A	57	11	1.11	16	0.688
111	A	55	16	1.00	18	0.889
112	A	65	19	1.00	18	1.056
113	A	12	8	1.00	19	0.421
114	A	2	2	1.00	28	0.071
115	A	3	3	1.00	33	0.091
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	4	4	1.00	35	0.114
119	A	5	5	1.00	40	0.125
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	5	4	1.00	14	0.286
123	A	4	4	1.00	12	0.333
124	A	4	3	1.00	14	0.214
125	A	6	6	1.00	14	0.429
126	A	5	4	1.00	12	0.333
127	A	5	4	1.00	19	0.210
128	A	0	0	0.00	0	0.000
129	A	7	6	1.00	18	0.333
130	A	7	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	7	6	1.00	16	0.375
132	A	4	3	1.00	10	0.300
133	A	5	5	1.00	18	0.278
134	A	8	8	1.00	18	0.444
135	A	9	8	1.00	18	0.444
136	A	16	13	1.00	20	0.650
137	A	13	10	1.00	18	0.556
138	A	6	6	1.00	12	0.500
139	A	2	2	1.00	20	0.100
140	A	25	25	1.00	20	1.250
141	A	21	14	1.00	20	0.700
142	A	15	11	1.00	18	0.611
143	A	6	7	1.00	12	0.583
144	A	2	2	1.00	20	0.100
145	A	35	22	1.00	20	1.100
146	A	6	4	1.00	18	0.222
147	A	0	0	0.00	0	0.000
148	A	0	0	0.00	0	0.000
149	A	4	4	1.00	12	0.333
150	A	4	4	1.00	14	0.286
151	A	0	0	0.00	0	0.000
152	A	9	7	1.00	40	0.175
153	A	7	6	1.00	40	0.150
154	A	4	4	1.00	38	0.105
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000
157	A	2	2	1.00	7	0.286
158	A	11	6	1.00	15	0.400
159	A	9	5	1.00	13	0.385
160	A	7	4	1.00	11	0.364
161	A	0	0	0.00	0	0.000
162	A	7	7	1.00	21	0.333
163	A	6	6	1.00	19	0.316
164	A	5	5	1.00	17	0.294
165	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	7	7	1.00	22	0.318
167	A	6	6	1.00	20	0.300
168	A	5	5	1.00	18	0.278
169	A	0	0	0.00	0	0.000
170	A	2	2	1.00	7	0.286
171	A	11	6	1.00	15	0.400
172	A	9	5	1.00	13	0.385
173	A	7	4	1.00	11	0.364
174	A	0	0	0.00	0	0.000
175	A	7	7	1.00	21	0.333
176	A	6	6	1.00	19	0.316
177	A	5	5	1.00	17	0.294
178	A	0	0	0.00	0	0.000
179	A	7	7	1.00	22	0.318
180	A	6	6	1.00	20	0.300
181	A	5	5	1.00	18	0.278
182	A	0	0	0.00	0	0.000
183	A	12	6	1.00	15	0.400
184	A	10	6	1.00	15	0.400
185	A	8	5	1.00	13	0.385
186	A	6	4	1.00	7	0.571
187	A	0	0	0.00	0	0.000
188	A	11	6	1.00	15	0.400
189	A	9	5	1.00	13	0.385
190	A	7	4	1.00	11	0.364
191	A	0	0	0.00	0	0.000
192	A	7	7	1.00	19	0.368
193	A	6	6	1.00	17	0.353
194	A	5	5	1.00	15	0.333
195	A	0	0	0.00	0	0.000
196	A	7	7	1.00	22	0.318
197	A	6	6	1.00	20	0.300
198	A	5	5	1.00	18	0.278
199	A	0	0	0.00	0	0.000
200	A	12	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	10	6	1.00	15	0.400
202	A	8	5	1.00	13	0.385
203	A	6	4	1.00	7	0.571
204	A	0	0	0.00	0	0.000
205	A	11	6	1.00	15	0.400
206	A	9	5	1.00	13	0.385
207	A	7	4	1.00	11	0.364
208	A	0	0	0.00	0	0.000
209	A	7	7	1.00	19	0.368
210	A	6	6	1.00	17	0.353
211	A	5	5	1.00	15	0.333
212	A	0	0	0.00	0	0.000
213	A	7	7	1.00	22	0.318
214	A	6	6	1.00	20	0.300
215	A	5	5	1.00	18	0.278
216	A	0	0	0.00	0	0.000
217	A	13	9	1.00	24	0.375
218	A	4	3	1.00	4	0.750
219	A	7	4	1.00	6	0.667
220	A	9	5	1.00	8	0.625
221	A	4	3	1.00	8	0.375
222	A	7	4	1.00	10	0.400
223	A	9	5	1.00	12	0.417
224	A	6	6	1.00	12	0.500
225	A	25	8	1.00	14	0.571
226	A	29	9	1.00	16	0.562
227	A	5	6	1.00	10	0.600
228	A	1	1	1.00	19	0.053
229	A	5	5	1.00	20	0.250
230	A	8	7	1.00	20	0.350
231	A	13	10	1.00	20	0.500
232	A	13	10	1.00	20	0.500
233	A	8	7	1.00	20	0.350
234	A	5	5	1.00	20	0.250

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=51

$$\frac{x}{6a^5}-\frac{x^3}{18a^3}+\frac{x^5}{30a}+\frac{1}{6}x^6\cot^{-1}(ax)-\frac{\mathrm{ArcTan}(ax)}{6a^6}$$

```
[Out] 1/6*x/a^5-1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccot(a*x)-1/6*arctan(a*x)/a^6
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4947, 308, 209}

$$-\frac{\mathrm{ArcTan}(ax)}{6a^6}+\frac{x}{6a^5}-\frac{x^3}{18a^3}+\frac{1}{6}x^6\cot^{-1}(ax)+\frac{x^5}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCot[a*x],x]
```

```
[Out] x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/(
6*a^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cot^{-1}(ax)\,dx &= \frac{1}{6}x^6\cot^{-1}(ax) + \frac{1}{6}a\int \frac{x^6}{1+a^2x^2}\,dx \\
&= \frac{1}{6}x^6\cot^{-1}(ax) + \frac{1}{6}a\int \left(\frac{1}{a^6} - \frac{x^2}{a^4} + \frac{x^4}{a^2} - \frac{1}{a^6\left(1+a^2x^2\right)}\right)dx \\
&= \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\cot^{-1}(ax) - \frac{\int \frac{1}{1+a^2x^2}\,dx}{6a^5} \\
&= \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\cot^{-1}(ax) - \frac{\tan^{-1}(ax)}{6a^6}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 51, normalized size = 1.00

$$
\frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\cot^{-1}(ax) - \frac{\mathrm{ArcTan}(ax)}{6a^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCot[a*x],x]
```

```
[Out] x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/(
6*a^6)
```

Maple [A]
time = 0.06, size = 44, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{a^6x^6\,\mathrm{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
default	$\frac{\frac{a^6x^6\,\mathrm{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
risch	$\frac{ix^6\ln(iax+1)}{12}-\frac{ix^6\ln(-iax+1)}{12}+\frac{x^6\pi}{12}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\arctan(ax)}{6a^6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arccot(a*x)+1/30*a^5*x^5-1/18*a^3*x^3+1/6*a*x-1/6*arctan
(a*x))
```

Maxima [A]
time = 0.49, size = 47, normalized size = 0.92

$$
\frac{1}{6}\,x^6\,\mathrm{arccot}\,(ax) + \frac{1}{90}\,a\left(\frac{3\,a^4x^5 - 5\,a^2x^3 + 15\,x}{a^6} - \frac{15\,\arctan\,(ax)}{a^7}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccot(a*x) + 1/90*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arcta
n(a*x)/a^7)
```

Fricas [A]
time = 3.34, size = 41, normalized size = 0.80

$$\frac{3\,a^5x^5-5\,a^3x^3+15\,ax+15\,(a^6x^6+1)\,\mathrm{arccot}\,(ax)}{90\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/90*(3*a^5*x^5 - 5*a^3*x^3 + 15*a*x + 15*(a^6*x^6 + 1)*arccot(a*x))/a^6
```

Sympy [A]
time = 0.31, size = 48, normalized size = 0.94

$$\begin{cases} \frac{x^6\,\mathrm{acot}\,(ax)}{6}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}+\frac{\mathrm{acot}\,(ax)}{6a^6} & \text{for}\ a\neq 0 \\ \frac{\pi x^6}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acot(a*x),x)
```

```
[Out] Piecewise((x**6*acot(a*x)/6 + x**5/(30*a) - x**3/(18*a**3) + x/(6*a**5) + a
cot(a*x)/(6*a**6), Ne(a, 0)), (pi*x**6/12, True))
```

Giac [A]
time = 0.42, size = 59, normalized size = 1.16

$$\frac{1}{90}\left(\frac{15\,x^6\arctan\left(\frac{1}{ax}\right)}{a}-\frac{x^5\left(\frac{5}{a^2x^2}-\frac{15}{a^4x^4}-3\right)}{a^2}+\frac{15\,\arctan\left(\frac{1}{ax}\right)}{a^7}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/90*(15*x^6*arctan(1/(a*x))/a - x^5*(5/(a^2*x^2) - 15/(a^4*x^4) - 3)/a^2 +
 15*arctan(1/(a*x))/a^7)*a
```

Mupad [B]
time = 0.80, size = 55, normalized size = 1.08

$$\begin{cases} \frac{\pi\,x^6}{12} & \text{if}\ \ a=0 \\ \frac{x^6\,\mathrm{acot}(a\,x)}{6}-\frac{\frac{\mathrm{atan}(a\,x)}{6}-\frac{a\,x}{6}+\frac{a^3\,x^3}{18}-\frac{a^5\,x^5}{30}}{a^6} & \text{if}\ \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^6*pi)/12, a ~= 0, - (atan(a*x)/6 - (a*x)/6 + (a^3*x^3)
/18 - (a^5*x^5)/30)/a^6 + (x^6*acot(a*x))/6)
```

3.2 $\int x^4 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=49

$$-\frac{x^2}{10a^3}+\frac{x^4}{20a}+\frac{1}{5}x^5\cot^{-1}(ax)+\frac{\log(1+a^2x^2)}{10a^5}$$

```
[Out] -1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccot(a*x)+1/10*ln(a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4947, 272, 45}

$$-\frac{x^2}{10a^3}+\frac{\log(a^2x^2+1)}{10a^5}+\frac{1}{5}x^5\cot^{-1}(ax)+\frac{x^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCot[a*x],x]
```

```
[Out] -1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^5
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \cot^{-1}(ax)\,dx &= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{5}a\int \frac{x^5}{1+a^2x^2}\,dx \\
&= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{10}a\text{Subst}\left(\int \frac{x^2}{1+a^2x}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\cot^{-1}(ax) + \frac{1}{10}a\text{Subst}\left(\int \left(-\frac{1}{a^4}+\frac{x}{a^2}+\frac{1}{a^4(1+a^2x)}\right)\,dx, x, x^2\right) \\
&= -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{10a^5}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{10a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCot[a*x],x]
```

```
[Out] -1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^5
)
```

Maple [A]
time = 0.04, size = 46, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
default	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
risch	$\frac{ix^5\ln(iax+1)}{10}-\frac{ix^5\ln(-iax+1)}{10}+\frac{x^5\pi}{10}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\ln\left(-a^2x^2-1\right)}{10a^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccot(a*x)+1/20*a^4*x^4-1/10*a^2*x^2+1/10*ln(a^2*x^2+1)
)
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.94

$$\frac{1}{5}\,x^5\,\mathrm{arccot}\,(ax) + \frac{1}{20}\,a\left(\frac{a^2x^4-2\,x^2}{a^4} + \frac{2\,\log\left(a^2x^2+1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccot(a*x) + 1/20*a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a^
6)
```

Fricas [A]
time = 2.33, size = 45, normalized size = 0.92

$$\frac{4\,a^5x^5\,\mathrm{arccot}\,(ax)+a^4x^4-2\,a^2x^2+2\,\log\left(a^2x^2+1\right)}{20\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/20*(4*a^5*x^5*arccot(a*x) + a^4*x^4 - 2*a^2*x^2 + 2*log(a^2*x^2 + 1))/a^5
```

Sympy [A]
time = 0.23, size = 46, normalized size = 0.94

$$\begin{cases} \frac{x^5\,\mathrm{acot}\,(ax)}{5}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\log\left(a^2x^2+1\right)}{10a^5} & \text{for}\ a\neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acot(a*x),x)
```

```
[Out] Piecewise((x**5*acot(a*x)/5 + x**4/(20*a) - x**2/(10*a**3) + log(a**2*x**2
+ 1)/(10*a**5), Ne(a, 0)), (pi*x**5/10, True))
```

Giac [A]
time = 0.40, size = 74, normalized size = 1.51

$$\frac{1}{20}\left(\frac{4\,x^5\arctan\left(\frac{1}{ax}\right)}{a}-\frac{x^4\left(\frac{2}{a^2x^2}-\frac{3}{a^4x^4}-1\right)}{a^2}+\frac{2\,\log\left(\frac{1}{a^2x^2}+1\right)}{a^6}-\frac{2\,\log\left(\frac{1}{a^2x^2}\right)}{a^6}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/20*(4*x^5*arctan(1/(a*x))/a - x^4*(2/(a^2*x^2) - 3/(a^4*x^4) - 1)/a^2 + 2
*log(1/(a^2*x^2) + 1)/a^6 - 2*log(1/(a^2*x^2))/a^6)*a
```

Mupad [B]
time = 0.70, size = 56, normalized size = 1.14

$$\begin{cases} \frac{\pi\,x^5}{10} & \text{if}\ \ a=0 \\ \frac{2\,\ln\left(a^2\,x^2+1\right)-2\,a^2\,x^2+a^4\,x^4}{20\,a^5}+\frac{x^5\,\mathrm{acot}\left(a\,x\right)}{5} & \text{if}\ \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^5*pi)/10, a ~= 0, (2*log(a^2*x^2 + 1) - 2*a^2*x^2 + a^
4*x^4)/(20*a^5) + (x^5*acot(a*x))/5)
```

3.3 $\int x^3 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=41

$$-\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\mathrm{ArcTan}(ax)}{4a^4}$$

```
[Out] -1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccot(a*x)+1/4*arctan(a*x)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4947, 308, 209}

$$\frac{\mathrm{ArcTan}(ax)}{4a^4}-\frac{x}{4a^3}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{x^3}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a*x],x]
```

```
[Out] -1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\cot^{-1}(ax)\,dx &= \frac{1}{4}x^4\cot^{-1}(ax)+\frac{1}{4}a\int\frac{x^4}{1+a^2x^2}\,dx\\ &= \frac{1}{4}x^4\cot^{-1}(ax)+\frac{1}{4}a\int\left(-\frac{1}{a^4}+\frac{x^2}{a^2}+\frac{1}{a^4\left(1+a^2x^2\right)}\right)dx\\ &= -\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\int\frac{1}{1+a^2x^2}\,dx}{4a^3}\\ &= -\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\tan^{-1}(ax)}{4a^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.00

$$-\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\cot^{-1}(ax)+\frac{\mathrm{ArcTan}(ax)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a*x],x]
```

```
[Out] -1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
default	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
risch	$\frac{ix^4\ln(iax+1)}{8}-\frac{ix^4\ln(-iax+1)}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12a}-\frac{x}{4a^3}+\frac{\arctan(ax)}{4a^4}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccot(a*x)+1/12*a^3*x^3-1/4*a*x+1/4*arctan(a*x))
```

Maxima [A]
time = 0.56, size = 38, normalized size = 0.93

$$\frac{1}{4}\,x^4\operatorname{arccot}(ax)+\frac{1}{12}\,a\left(\frac{a^2x^3-3\,x}{a^4}+\frac{3\,\arctan(ax)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(a*x) + 1/12*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)
```

Fricas [A]
time = 2.94, size = 32, normalized size = 0.78

$$\frac{a^3x^3 - 3\,ax + 3\,(a^4x^4 - 1)\,\mathrm{arccot}\,(ax)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*x^3 - 3*a*x + 3*(a^4*x^4 - 1)*arccot(a*x))/a^4
```

Sympy [A]
time = 0.21, size = 39, normalized size = 0.95

$$\begin{cases} \frac{x^4\,\mathrm{acot}\,(ax)}{4} + \frac{x^3}{12a} - \frac{x}{4a^3} - \frac{\mathrm{acot}\,(ax)}{4a^4} & \text{for}\ a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(a*x),x)
```

```
[Out] Piecewise((x**4*acot(a*x)/4 + x**3/(12*a) - x/(4*a**3) - acot(a*x)/(4*a**4)
, Ne(a, 0)), (pi*x**4/8, True))
```

Giac [A]
time = 0.41, size = 51, normalized size = 1.24

$$\frac{1}{12}\left(\frac{3\,x^4\,\arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^3\left(\frac{3}{a^2x^2} - 1\right)}{a^2} - \frac{3\,\arctan\left(\frac{1}{ax}\right)}{a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/12*(3*x^4*arctan(1/(a*x))/a - x^3*(3/(a^2*x^2) - 1)/a^2 - 3*arctan(1/(a*x
))/a^5)*a
```

Mupad [B]
time = 0.72, size = 46, normalized size = 1.12

$$\begin{cases} \frac{\pi\,x^4}{8} & \text{if}\ \ a = 0 \\ \frac{3\,\mathrm{atan}(a\,x) - 3\,a\,x + a^3\,x^3}{12\,a^4} + \frac{x^4\,\mathrm{acot}(a\,x)}{4} & \text{if}\ \ a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a*x),x)
```

```
[Out] piecewise(a == 0, (x^4*pi)/8, a ~= 0, (3*atan(a*x) - 3*a*x + a^3*x^3)/(12*a
^4) + (x^4*acot(a*x))/4)
```

3.4 $\int x^2 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{6a}+\frac{1}{3}x^3\cot^{-1}(ax)-\frac{\log(1+a^2x^2)}{6a^3}$$

```
[Out] 1/6*x^2/a+1/3*x^3*arccot(a*x)-1/6*ln(a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 272, 45}

$$-\frac{\log(a^2x^2+1)}{6a^3}+\frac{1}{3}x^3\cot^{-1}(ax)+\frac{x^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a*x],x]
```

```
[Out] x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{3}a\int \frac{x^3}{1+a^2x^2}\,dx$$
$$= \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{6}a\text{Subst}\left(\int \frac{x}{1+a^2x}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^3\cot^{-1}(ax) + \frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{a^2} - \frac{1}{a^2(1+a^2x)}\right)\,dx, x, x^2\right)$$
$$= \frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{x^2}{6a} + \frac{1}{3}x^3\cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a*x],x]
[Out] x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{a^3x^3\,\mathrm{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln(a^2x^2+1)}{6}}{a^3}$	38
default	$\frac{\frac{a^3x^3\,\mathrm{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln(a^2x^2+1)}{6}}{a^3}$	38
risch	$\frac{ix^3\ln(iax+1)}{6}-\frac{ix^3\ln(-iax+1)}{6}+\frac{\pi x^3}{6}+\frac{x^2}{6a}-\frac{\ln(-a^2x^2-1)}{6a^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/3*a^3*x^3*arccot(a*x)+1/6*a^2*x^2-1/6*ln(a^2*x^2+1))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.92

$$\frac{1}{3}\,x^3\,\mathrm{arccot}\,(ax) + \frac{1}{6}\,a\left(\frac{x^2}{a^2} - \frac{\log(a^2x^2+1)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x),x, algorithm="maxima")
[Out] 1/3*x^3*arccot(a*x) + 1/6*a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4)
```

Fricas [A]
time = 3.56, size = 37, normalized size = 0.95

$$\frac{2\,a^3x^3\,\mathrm{arccot}\,(ax)+a^2x^2-\log\left(a^2x^2+1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x),x, algorithm="fricas")
[Out] 1/6*(2*a^3*x^3*arccot(a*x) + a^2*x^2 - log(a^2*x^2 + 1))/a^3
```

Sympy [A]
time = 0.17, size = 37, normalized size = 0.95

$$\begin{cases} \frac{x^3\,\mathrm{acot}\,(ax)}{3}+\frac{x^2}{6a}-\frac{\log\left(a^2x^2+1\right)}{6a^3} & \text{for } a\neq 0\\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(a*x),x)
[Out] Piecewise((x**3*acot(a*x)/3 + x**2/(6*a) - log(a**2*x**2 + 1)/(6*a**3), Ne(
a, 0)), (pi*x**3/6, True))
```

Giac [A]
time = 0.40, size = 64, normalized size = 1.64

$$\frac{1}{6}\left(\frac{2\,x^3\arctan\left(\frac{1}{ax}\right)}{a}-\frac{x^2\left(\frac{1}{a^2x^2}-1\right)}{a^2}-\frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^4}+\frac{\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x),x, algorithm="giac")
[Out] 1/6*(2*x^3*arctan(1/(a*x))/a - x^2*(1/(a^2*x^2) - 1)/a^2 - log(1/(a^2*x^2)
+ 1)/a^4 + log(1/(a^2*x^2))/a^4)*a
```

Mupad [B]
time = 0.69, size = 49, normalized size = 1.26

$$\begin{cases} \frac{\pi\,x^3}{6} & \text{if } \ a=0\\ \frac{\frac{x^2}{2}-\frac{\ln\left(a^2\,x^2+1\right)}{2\,a^2}}{3\,a}+\frac{x^3\,\mathrm{acot}(a\,x)}{3} & \text{if } \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a*x),x)
[Out] piecewise(a == 0, (x^3*pi)/6, a ~= 0, (x^2/2 - log(a^2*x^2 + 1)/(2*a^2))/(3
*a) + (x^3*acot(a*x))/3)
```

3.5 $\int x \cot^{-1}(ax)\, dx$

Optimal. Leaf size=31

$$\frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\text{ArcTan}(ax)}{2a^2}$$

```
[Out] 1/2*x/a+1/2*x^2*arccot(a*x)-1/2*arctan(a*x)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4947, 327, 209}

$$-\frac{\text{ArcTan}(ax)}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax) + \frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}(ax)\,dx &= \frac{1}{2}x^2\cot^{-1}(ax)+\frac{1}{2}a\int\frac{x^2}{1+a^2x^2}\,dx\\ &= \frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\int\frac{1}{1+a^2x^2}\,dx}{2a}\\ &= \frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\tan^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$\frac{x}{2a}+\frac{1}{2}x^2\cot^{-1}(ax)-\frac{\text{ArcTan}(ax)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

Maple [A]
time = 0.05, size = 28, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
default	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
risch	$\frac{ix^2\ln(iax+1)}{4}-\frac{ix^2\ln(-iax+1)}{4}+\frac{\pi x^2}{4}+\frac{x}{2a}-\frac{\arctan(ax)}{2a^2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arccot(a*x)*a^2*x^2+1/2*a*x-1/2*arctan(a*x))
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.90

$$\frac{1}{2}\,x^2\operatorname{arccot}\left(ax\right)+\frac{1}{2}\,a\left(\frac{x}{a^2}-\frac{\arctan\left(ax\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x),x, algorithm="maxima")
```

[Out] 1/2*x^2*arccot(a*x) + 1/2*a*(x/a^2 - arctan(a*x)/a^3)

Fricas [A]
time = 2.56, size = 23, normalized size = 0.74

$$\frac{ax + (a^2x^2 + 1)\operatorname{arccot}(ax)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccot(a*x),x, algorithm="fricas")

[Out] 1/2*(a*x + (a^2*x^2 + 1)*arccot(a*x))/a^2

Sympy [A]
time = 0.13, size = 31, normalized size = 1.00

$$\begin{cases} \frac{x^2 \operatorname{acot}(ax)}{2} + \frac{x}{2a} + \frac{\operatorname{acot}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*acot(a*x),x)

[Out] Piecewise((x**2*acot(a*x)/2 + x/(2*a) + acot(a*x)/(2*a**2), Ne(a, 0)), (pi*x**2/4, True))

Giac [A]
time = 0.40, size = 36, normalized size = 1.16

$$\frac{1}{2}\left(\frac{x^2 \arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arccot(a*x),x, algorithm="giac")

[Out] 1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a

Mupad [B]
time = 0.76, size = 39, normalized size = 1.26

$$\begin{cases} \frac{\pi\,x^2}{4} & \text{if } a = 0 \\ \frac{x - \frac{\operatorname{atan}(a\,x)}{a}}{2\,a} + \frac{x^2 \operatorname{acot}(a\,x)}{2} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*acot(a*x),x)

[Out] piecewise(a == 0, (x^2*pi)/4, a ~= 0, (x - atan(a*x)/a)/(2*a) + (x^2*acot(a*x))/2)

3.6 $\int \cot^{-1}(ax)\,dx$

Optimal. Leaf size=24

$$x\cot^{-1}(ax)+\frac{\log(1+a^2x^2)}{2a}$$

```
[Out] x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{4931, 266\}$

$$\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x],x]
```

```
[Out] x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(ax)\,dx &= x\cot^{-1}(ax)+a\int \frac{x}{1+a^2x^2}\,dx\\ &= x\cot^{-1}(ax)+\frac{\log(1+a^2x^2)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$x\cot^{-1}(ax)+\frac{\log(1+a^2x^2)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x],x]
[Out] x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Maple [A]
time = 0.03, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{ax\,\mathrm{arccot}(ax)+\frac{\ln\left(a^{2}x^{2}+1\right)}{2}}{a}$	25
default	$\frac{ax\,\mathrm{arccot}(ax)+\frac{\ln\left(a^{2}x^{2}+1\right)}{2}}{a}$	25
risch	$\frac{ix\ln(iax+1)}{2}-\frac{ix\ln(-iax+1)}{2}+\frac{\pi x}{2}+\frac{\ln(-a^{2}x^{2}-1)}{2a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x*arccot(a*x)+1/2*ln(a^2*x^2+1))
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.00

$$\frac{2\,ax\,\mathrm{arccot}\,(ax)+\log\,(a^{2}x^{2}+1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x),x, algorithm="maxima")
[Out] 1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Fricas [A]
time = 4.13, size = 24, normalized size = 1.00

$$\frac{2\,ax\,\mathrm{arccot}\,(ax)+\log\,(a^{2}x^{2}+1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x),x, algorithm="fricas")
[Out] 1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Sympy [A]
time = 0.09, size = 24, normalized size = 1.00

$$\begin{cases} x\,\mathrm{acot}\,(ax)+\frac{\log\,(a^{2}x^{2}+1)}{2a} & \text{for } a\neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x),x)
```

```
[Out] Piecewise((x*acot(a*x) + log(a**2*x**2 + 1)/(2*a), Ne(a, 0)), (pi*x/2, True
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.
time = 0.40, size = 45, normalized size = 1.88

$$\frac{1}{2}\,a\left(\frac{2\,x\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^2}-\frac{\log\left(\frac{1}{a^2x^2}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/2*a*(2*x*arctan(1/(a*x))/a + log(1/(a^2*x^2) + 1)/a^2 - log(1/(a^2*x^2))/
a^2)
```

Mupad [B]
time = 0.14, size = 22, normalized size = 0.92

$$x\,\mathrm{acot}(a\,x)+\frac{\ln\left(a^2\,x^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x),x)
```

```
[Out] x*acot(a*x) + log(a^2*x^2 + 1)/(2*a)
```

3.7 $\int \frac{\cot^{-1}(ax)}{x} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

```
[Out] -1/2*I*polylog(2,-I/a/x)+1/2*I*polylog(2,I/a/x)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4941, 2438}

$$\frac{1}{2}i\text{Li}_2\left(\frac{i}{ax}\right)-\frac{1}{2}i\text{Li}_2\left(-\frac{i}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/x,x]
```

```
[Out] (-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{x} \, dx &= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax}\right)}{x} \, dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{ax}\right)}{x} \, dx\\ &= -\frac{1}{2}i\text{Li}_2\left(-\frac{i}{ax}\right)+\frac{1}{2}i\text{Li}_2\left(\frac{i}{ax}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/x,x]
[Out] (-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.03, size = 63, normalized size = 1.70

method	result	siz
risch	$\frac{\pi \ln(-iax)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}$	33
derivativedivides	$\ln\left(ax\right)\mathrm{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}$	63
default	$\ln\left(ax\right)\mathrm{arccot}(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/x,x,method=_RETURNVERBOSE)
[Out] ln(a*x)*arccot(a*x)-1/2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2
*I*dilog(1+I*a*x)+1/2*I*dilog(1-I*a*x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(23) = 46$.
time = 0.52, size = 56, normalized size = 1.51

$$\frac{1}{4}\pi\log\left(a^2x^2+1\right)-\arctan\left(ax\right)\log\left(ax\right)+\mathrm{arccot}\left(ax\right)\log\left(x\right)+\arctan\left(ax\right)\log\left(x\right)+\frac{1}{2}i\,\mathrm{Li}_2(i\,ax+1)-\frac{1}{2}i\,\mathrm{Li}_2(-i\,ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x,x, algorithm="maxima")
[Out] 1/4*pi*log(a^2*x^2 + 1) - arctan(a*x)*log(a*x) + arccot(a*x)*log(x) + arcta
n(a*x)*log(x) + 1/2*I*dilog(I*a*x + 1) - 1/2*I*dilog(-I*a*x + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x,x, algorithm="fricas")
[Out] integral(arccot(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{acot}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/x,x)
```

```
[Out] Integral(acot(a*x)/x, x)
```

Giac [A]
time = 0.43, size = 38, normalized size = 1.03

$$-\frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a}+\frac{x}{a^2}+\frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acot}(a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/x,x)
```

```
[Out] int(acot(a*x)/x, x)
```

3.8 $\int \frac{\cot^{-1}(ax)}{x^2} \, dx$

Optimal. Leaf size=30

$$-\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1 + a^2x^2\right)$$

```
[Out] -arccot(a*x)/x-a*ln(x)+1/2*a*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4947, 272, 36, 29, 31}

$$\frac{1}{2}a\log\left(a^2x^2 + 1\right) - a\log(x) - \frac{\cot^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/x^2,x]
```

```
[Out] -(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)}{x^2}\,dx &= -\frac{\cot^{-1}(ax)}{x} - a\int \frac{1}{x(1+a^2x^2)}\,dx \\
&= -\frac{\cot^{-1}(ax)}{x} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1+a^2x)}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(ax)}{x} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/x^2,x]
```

```
[Out] -(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

Maple [A]
time = 0.04, size = 34, normalized size = 1.13

method	result	size
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)}{ax} - \ln(ax) + \frac{\ln(a^2x^2+1)}{2}\right)$	34
default	$a\left(-\frac{\operatorname{arccot}(ax)}{ax} - \ln(ax) + \frac{\ln(a^2x^2+1)}{2}\right)$	34
risch	$-\frac{i\ln(iax+1)}{2x} - \frac{2\ln(x)ax - a\ln(a^2x^2+1)x - i\ln(-iax+1)+\pi}{2x}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arccot(a*x)/a/x-ln(a*x)+1/2*ln(a^2*x^2+1))
```

Maxima [A]
time = 0.28, size = 30, normalized size = 1.00

$$\frac{1}{2}\,a\left(\log\left(a^2x^2+1\right)-\log\left(x^2\right)\right)-\frac{\operatorname{arccot}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="maxima")
[Out] 1/2*a*(log(a^2*x^2 + 1) - log(x^2)) - arccot(a*x)/x
```

Fricas [A]
time = 3.12, size = 31, normalized size = 1.03

$$\frac{ax\log\left(a^2x^2+1\right)-2\,ax\log\left(x\right)-2\,\operatorname{arccot}\left(ax\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="fricas")
[Out] 1/2*(a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) - 2*arccot(a*x))/x
```

Sympy [A]
time = 0.12, size = 24, normalized size = 0.80

$$-a\log\left(x\right)+\frac{a\log\left(a^2x^2+1\right)}{2}-\frac{\operatorname{acot}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/x**2,x)
[Out] -a*log(x) + a*log(a**2*x**2 + 1)/2 - acot(a*x)/x
```

Giac [A]
time = 0.40, size = 32, normalized size = 1.07

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax}\right)}{ax}-\log\left(\frac{1}{a^2x^2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^2,x, algorithm="giac")
[Out] -1/2*a*(2*arctan(1/(a*x))/(a*x) - log(1/(a^2*x^2) + 1))
```

Mupad [B]
time = 0.23, size = 28, normalized size = 0.93

$$\frac{a\,\left(\ln\left(a^2\,x^2+1\right)-2\,\ln\left(x\right)\right)}{2}-\frac{\mathrm{acot}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/x^2,x)
[Out] (a*(log(a^2*x^2 + 1) - 2*log(x)))/2 - acot(a*x)/x
```

3.9 $\int \frac{\cot^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=31

$$\frac{a}{2x}-\frac{\cot^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\mathrm{ArcTan}(ax)$$

```
[Out] 1/2*a/x-1/2*arccot(a*x)/x^2+1/2*a^2*arctan(a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4947, 331, 209}

$$\frac{1}{2}a^2\mathrm{ArcTan}(ax)-\frac{\cot^{-1}(ax)}{2x^2}+\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/x^3,x]
```

```
[Out] a/(2*x) - ArcCot[a*x]/(2*x^2) + (a^2*ArcTan[a*x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{x^3}\,dx &= -\frac{\cot^{-1}(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2(1+a^2x^2)}\,dx \\ &= \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^3\int \frac{1}{1+a^2x^2}\,dx \\ &= \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tan^{-1}(ax)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 36, normalized size = 1.16

$$-\frac{\cot^{-1}(ax)}{2x^2} + \frac{a\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-a^2x^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/x^3,x]
```

```
[Out] -1/2*ArcCot[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*x
)
```

Maple [A]
time = 0.06, size = 32, normalized size = 1.03

method	result	size
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
default	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
risch	$-\frac{i\ln(iax+1)}{4x^2}-\frac{ia^2\ln(-ax+i)x^2-ia^2\ln(-ax-i)x^2-i\ln(-iax+1)-2ax+\pi}{4x^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arccot(a*x)/a^2/x^2+1/2*arctan(a*x)+1/2/a/x)
```

Maxima [A]
time = 0.50, size = 23, normalized size = 0.74

$$\frac{1}{2}\left(a\arctan(ax) + \frac{1}{x}\right)a - \frac{\operatorname{arccot}(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a*arctan(a*x) + 1/x)*a - 1/2*arccot(a*x)/x^2
```

Fricas [A]
time = 3.57, size = 24, normalized size = 0.77

$$\frac{ax - (a^2x^2 + 1)\operatorname{arccot}(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*x - (a^2*x^2 + 1)*arccot(a*x))/x^2
```

Sympy [A]
time = 0.21, size = 24, normalized size = 0.77

$$-\frac{a^2\operatorname{acot}(ax)}{2} + \frac{a}{2x} - \frac{\operatorname{acot}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/x**3,x)
```

```
[Out] -a**2*acot(a*x)/2 + a/(2*x) - acot(a*x)/(2*x**2)
```

Giac [A]
time = 0.41, size = 40, normalized size = 1.29

$$\frac{1}{2}\left(a\left(\frac{1}{ax} - \arctan\left(\frac{1}{ax}\right)\right) - \frac{\arctan\left(\frac{1}{ax}\right)}{ax^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*(1/(a*x) - arctan(1/(a*x))) - arctan(1/(a*x))/(a*x^2))*a
```

Mupad [B]
time = 0.70, size = 44, normalized size = 1.42

$$\begin{cases} -\frac{\pi}{4\,x^2} & \text{if } a = 0 \\ \frac{a^3\operatorname{atan}(a\,x)+\frac{a^2}{x}}{2\,a} - \frac{\operatorname{acot}(a\,x)}{2\,x^2} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/x^3,x)
```

```
[Out] piecewise(a == 0, -pi/(4*x^2), a ~= 0, (a^3*atan(a*x) + a^2/x)/(2*a) - acot
(a*x)/(2*x^2))
```

3.10 $\int \frac{\cot^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=46

$$\frac{a}{6x^2}-\frac{\cot^{-1}(ax)}{3x^3}+\frac{1}{3}a^3\log(x)-\frac{1}{6}a^3\log\left(1+a^2x^2\right)$$

```
[Out] 1/6*a/x^2-1/3*arccot(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {4947, 272, 46}

$$\frac{1}{3}a^3\log(x)-\frac{1}{6}a^3\log\left(a^2x^2+1\right)-\frac{\cot^{-1}(ax)}{3x^3}+\frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/x^4,x]
```

```
[Out] a/(6*x^2) - ArcCot[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 + a^2*x^2])/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)}{x^4}\,dx &= -\frac{\cot^{-1}(ax)}{3x^3}-\frac{1}{3}a\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx\\
&= -\frac{\cot^{-1}(ax)}{3x^3}-\frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2\left(1+a^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{\cot^{-1}(ax)}{3x^3}-\frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{a^2}{x}+\frac{a^4}{1+a^2x}\right)\,dx,x,x^2\right)\\
&= \frac{a}{6x^2}-\frac{\cot^{-1}(ax)}{3x^3}+\frac{1}{3}a^3\log(x)-\frac{1}{6}a^3\log\left(1+a^2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.96

$$
-\frac{\cot^{-1}(ax)}{3x^3}-\frac{1}{6}a\left(-\frac{1}{x^2}-2a^2\log(x)+a^2\log\left(1+a^2x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/x^4,x]
```

```
[Out] -1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - 2*a^2*Log[x] + a^2*Log[1 + a^2*x^2]))/
6
```

Maple [A]
time = 0.05, size = 44, normalized size = 0.96

method	result	size
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln\left(a^2x^2+1\right)}{6}\right)$	44
default	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}-\frac{\ln\left(a^2x^2+1\right)}{6}\right)$	44
risch	$-\frac{i\ln(iax+1)}{6x^3}-\frac{-2a^3\ln(x)x^3+a^3\ln\left(-a^2x^2-1\right)x^3-i\ln(-iax+1)-ax+\pi}{6x^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*arccot(a*x)/a^3/x^3+1/6/a^2/x^2+1/3*ln(a*x)-1/6*ln(a^2*x^2+1))
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.91

$$
-\frac{1}{6}\left(a^2\log\left(a^2x^2+1\right)-a^2\log\left(x^2\right)-\frac{1}{x^2}\right)a-\frac{\operatorname{arccot}\left(ax\right)}{3\,x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*log(a^2*x^2 + 1) - a^2*log(x^2) - 1/x^2)*a - 1/3*arccot(a*x)/x^3
```

Fricas [A]
time = 2.59, size = 43, normalized size = 0.93

$$-\frac{a^3x^3\log\left(a^2x^2+1\right)-2\,a^3x^3\log\left(x\right)-ax+2\,\operatorname{arccot}\left(ax\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log(a^2*x^2 + 1) - 2*a^3*x^3*log(x) - a*x + 2*arccot(a*x))/x^
3
```

Sympy [A]
time = 0.25, size = 39, normalized size = 0.85

$$\frac{a^3\log\left(x\right)}{3}-\frac{a^3\log\left(a^2x^2+1\right)}{6}+\frac{a}{6x^2}-\frac{\operatorname{acot}\left(ax\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/x**4,x)
```

```
[Out] a**3*log(x)/3 - a**3*log(a**2*x**2 + 1)/6 + a/(6*x**2) - acot(a*x)/(3*x**3)
```

Giac [A]
time = 0.41, size = 44, normalized size = 0.96

$$\frac{1}{6}\left(a^2\left(\frac{1}{a^2x^2}-\log\left(\frac{1}{a^2x^2}+1\right)\right)-\frac{2\,\arctan\left(\frac{1}{ax}\right)}{ax^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*(1/(a^2*x^2) - log(1/(a^2*x^2) + 1)) - 2*arctan(1/(a*x))/(a*x^3))*
a
```

Mupad [B]
time = 0.85, size = 58, normalized size = 1.26

$$\begin{cases} -\frac{\pi}{6\,x^3} & \text{if}\ \ a=0\\ \frac{a^4\,\ln(x)-\frac{a^4\,\ln\left(a^2\,x^2+1\right)}{2}+\frac{a^2}{2\,x^2}}{3\,a}-\frac{\mathrm{acot}(a\,x)}{3\,x^3} & \text{if}\ \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/x^4,x)
```

```
[Out] piecewise(a == 0, -pi/(6*x^3), a ~= 0, (a^4*log(x) - (a^4*log(a^2*x^2 + 1))
/2 + a^2/(2*x^2))/(3*a) - acot(a*x)/(3*x^3))
```

3.11 $\int \frac{\cot^{-1}(ax)}{x^5} \, dx$

Optimal. Leaf size=41

$$\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^4 \text{ArcTan}(ax)$$

[Out] 1/12*a/x^3-1/4*a^3/x-1/4*arccot(a*x)/x^4-1/4*a^4*arctan(a*x)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4947, 331, 209}

$$-\frac{1}{4}a^4 \text{ArcTan}(ax) - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} + \frac{a}{12x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCot[a*x]/x^5,x]

[Out] a/(12*x^3) - a^3/(4*x) - ArcCot[a*x]/(4*x^4) - (a^4*ArcTan[a*x])/4

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(ax)}{x^5}\,dx &= -\frac{\cot^{-1}(ax)}{4x^4}-\frac{1}{4}a\int \frac{1}{x^4\left(1+a^2x^2\right)}\,dx\\ &= \frac{a}{12x^3}-\frac{\cot^{-1}(ax)}{4x^4}+\frac{1}{4}a^3\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx\\ &= \frac{a}{12x^3}-\frac{a^3}{4x}-\frac{\cot^{-1}(ax)}{4x^4}-\frac{1}{4}a^5\int \frac{1}{1+a^2x^2}\,dx\\ &= \frac{a}{12x^3}-\frac{a^3}{4x}-\frac{\cot^{-1}(ax)}{4x^4}-\frac{1}{4}a^4\tan^{-1}(ax)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 36, normalized size = 0.88

$$-\frac{\cot^{-1}(ax)}{4x^4}+\frac{a\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-a^2x^2\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/x^5,x]
```

```
[Out] -1/4*ArcCot[a*x]/x^4 + (a*Hypergeometric2F1[-3/2, 1, -1/2, -(a^2*x^2)])/(12
*x^3)
```

Maple [A]
time = 0.08, size = 40, normalized size = 0.98

method	result	size
derivativedivides	$a^4\left(-\frac{\mathrm{arccot}(ax)}{4a^4x^4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}-\frac{\arctan(ax)}{4}\right)$	40
default	$a^4\left(-\frac{\mathrm{arccot}(ax)}{4a^4x^4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}-\frac{\arctan(ax)}{4}\right)$	40
risch	$-\frac{i\ln(iax+1)}{8x^4}-\frac{3ia^4\ln(-ax-i)x^4-3ia^4\ln(-ax+i)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi}{24x^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arccot(a*x)/a^4/x^4+1/12/a^3/x^3-1/4/a/x-1/4*arctan(a*x))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.90

$$-\frac{1}{12}\left(3\,a^3\arctan\left(ax\right)+\frac{3\,a^2x^2-1}{x^3}\right)a-\frac{\mathrm{arccot}\left(ax\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="maxima")
[Out] -1/12*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a - 1/4*arccot(a*x)/x^4
```

Fricas [A]
time = 2.28, size = 33, normalized size = 0.80

$$-\frac{3\,a^3x^3 - ax - 3\,(a^4x^4 - 1)\operatorname{arccot}(ax)}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="fricas")
[Out] -1/12*(3*a^3*x^3 - a*x - 3*(a^4*x^4 - 1)*arccot(a*x))/x^4
```

Sympy [A]
time = 0.29, size = 32, normalized size = 0.78

$$\frac{a^4\operatorname{acot}(ax)}{4} - \frac{a^3}{4x} + \frac{a}{12x^3} - \frac{\operatorname{acot}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/x**5,x)
[Out] a**4*acot(a*x)/4 - a**3/(4*x) + a/(12*x**3) - acot(a*x)/(4*x**4)
```

Giac [A]
time = 0.41, size = 51, normalized size = 1.24

$$-\frac{1}{12}\left(a^3\left(\frac{3}{ax} - \frac{1}{a^3x^3} - 3\arctan\left(\frac{1}{ax}\right)\right) + \frac{3\arctan\left(\frac{1}{ax}\right)}{ax^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/x^5,x, algorithm="giac")
[Out] -1/12*(a^3*(3/(a*x) - 1/(a^3*x^3) - 3*arctan(1/(a*x))) + 3*arctan(1/(a*x))/
(a*x^4))*a
```

Mupad [B]
time = 0.70, size = 47, normalized size = 1.15

$$\begin{cases} -\frac{\pi}{8\,x^4} & \text{if } a=0 \\ -\frac{a^4\operatorname{atan}(a\,x)}{4} - \frac{\frac{\operatorname{acot}(a\,x)}{4} - \frac{a\,x}{12} + \frac{a^3\,x^3}{4}}{x^4} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/x^5,x)
[Out] piecewise(a == 0, -pi/(8*x^4), a ~= 0, - (a^4*atan(a*x))/4 - (acot(a*x)/4 -
 (a*x)/12 + (a^3*x^3)/4)/x^4)
```

3.12 $\int x^5 \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=104

$$-\frac{4x^2}{45a^4}+\frac{x^4}{60a^2}+\frac{x\cot^{-1}(ax)}{3a^5}-\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^5\cot^{-1}(ax)}{15a}+\frac{\cot^{-1}(ax)^2}{6a^6}+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{23\log(1+a^2x^2)}{90a^6}$$

```
[Out] -4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccot(a*x)/a^5-1/9*x^3*arccot(a*x)/a^3+1/
15*x^5*arccot(a*x)/a+1/6*arccot(a*x)^2/a^6+1/6*x^6*arccot(a*x)^2+23/90*ln(a
^2*x^2+1)/a^6
```

Rubi [A]
time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4947, 5037, 272, 45, 4931, 266, 5005}

$$\frac{\cot^{-1}(ax)^2}{6a^6}+\frac{x\cot^{-1}(ax)}{3a^5}-\frac{4x^2}{45a^4}-\frac{x^3\cot^{-1}(ax)}{9a^3}+\frac{x^4}{60a^2}+\frac{23\log(a^2x^2+1)}{90a^6}+\frac{1}{6}x^6\cot^{-1}(ax)^2+\frac{x^5\cot^{-1}(ax)}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCot[a*x]^2,x]
```

```
[Out] (-4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*ArcCot[a*x])/(3*a^5) - (x^3*ArcCot[a*
x])/(9*a^3) + (x^5*ArcCot[a*x])/(15*a) + ArcCot[a*x]^2/(6*a^6) + (x^6*ArcCo
t[a*x]^2)/6 + (23*Log[1 + a^2*x^2])/(90*a^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cot^{-1}(ax)^2\,dx &= \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{1}{3}a\int \frac{x^6\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{\int x^4\cot^{-1}(ax)\,dx}{3a} - \frac{\int \frac{x^4\cot^{-1}(ax)}{1+a^2x^2}\,dx}{3a} \\
&= \frac{x^5\cot^{-1}(ax)}{15a} + \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{1}{15}\int \frac{x^5}{1+a^2x^2}\,dx - \frac{\int x^2\cot^{-1}(ax)\,dx}{3a^3} + \frac{\int \frac{x^2\cot^{-1}(}{1+a^2x}}{3a^3} \\
&= -\frac{x^3\cot^{-1}(ax)}{9a^3} + \frac{x^5\cot^{-1}(ax)}{15a} + \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{1}{30}\text{Subst}\left(\int \frac{x^2}{1+a^2x}\,dx, x, x^2\right) \\
&= \frac{x\cot^{-1}(ax)}{3a^5} - \frac{x^3\cot^{-1}(ax)}{9a^3} + \frac{x^5\cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\cot^{-1}(ax)^2 + \frac{1}{30}\text{Sub} \\
&= -\frac{x^2}{30a^4} + \frac{x^4}{60a^2} + \frac{x\cot^{-1}(ax)}{3a^5} - \frac{x^3\cot^{-1}(ax)}{9a^3} + \frac{x^5\cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\text{co} \\
&= -\frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x\cot^{-1}(ax)}{3a^5} - \frac{x^3\cot^{-1}(ax)}{9a^3} + \frac{x^5\cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\text{co}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 0.76

$$\frac{-16a^2x^2+3a^4x^4+4ax(15-5a^2x^2+3a^4x^4)\cot^{-1}(ax)+30(1+a^6x^6)\cot^{-1}(ax)^2+46\log\left(1+a^2x^2\right)}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCot[a*x]^2,x]
```

```
[Out] (-16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 5*a^2*x^2 + 3*a^4*x^4)*ArcCot[a*x] +
 30*(1 + a^6*x^6)*ArcCot[a*x]^2 + 46*Log[1 + a^2*x^2])/(180*a^6)
```

Maple [A]
time = 0.11, size = 98, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5\operatorname{arccot}(ax)}{15}-\frac{a^3x^3\operatorname{arccot}(ax)}{9}+\frac{ax\operatorname{arccot}(ax)}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln\left(a^2x^2+1\right)}{90}-\text{arc}}{a^6}$
default	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5\operatorname{arccot}(ax)}{15}-\frac{a^3x^3\operatorname{arccot}(ax)}{9}+\frac{ax\operatorname{arccot}(ax)}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln\left(a^2x^2+1\right)}{90}-\text{arc}}{a^6}$
risch	$-\frac{(x^6a^6+1)\ln(iax+1)^2}{24a^6}+\frac{(15i\pi a^6x^6+15x^6\ln(-iax+1)a^6+6ia^5x^5-10ia^3x^3+30iax+15\ln(-iax+1))\ln(iax+1)}{180a^6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arccot(a*x)^2+1/15*a^5*x^5*arccot(a*x)-1/9*a^3*x^3*arcco
t(a*x)+1/3*a*x*arccot(a*x)-1/3*arccot(a*x)*arctan(a*x)+1/60*a^4*x^4-4/45*a^
2*x^2+23/90*ln(a^2*x^2+1)-1/6*arctan(a*x)^2)
```

Maxima [A]
time = 0.49, size = 95, normalized size = 0.91

$$\frac{1}{6}x^6\operatorname{arccot}(ax)^2+\frac{1}{45}a\left(\frac{3a^4x^5-5a^2x^3+15x}{a^6}-\frac{15\arctan(ax)}{a^7}\right)\operatorname{arccot}(ax)+\frac{3a^4x^4-16a^2x^2-30\arctan(ax)^2+46\log\left(a^2x^2+1\right)}{180a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccot(a*x)^2 + 1/45*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arc
tan(a*x)/a^7)*arccot(a*x) + 1/180*(3*a^4*x^4 - 16*a^2*x^2 - 30*arctan(a*x)^
2 + 46*log(a^2*x^2 + 1))/a^6
```

Fricas [A]
time = 3.10, size = 78, normalized size = 0.75

$$\frac{3a^4x^4-16a^2x^2+30\left(a^6x^6+1\right)\operatorname{arccot}(ax)^2+4\left(3a^5x^5-5a^3x^3+15ax\right)\operatorname{arccot}(ax)+46\log\left(a^2x^2+1\right)}{180a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/180*(3*a^4*x^4 - 16*a^2*x^2 + 30*(a^6*x^6 + 1)*arccot(a*x)^2 + 4*(3*a^5*x
^5 - 5*a^3*x^3 + 15*a*x)*arccot(a*x) + 46*log(a^2*x^2 + 1))/a^6
```

Sympy [A]
time = 0.40, size = 104, normalized size = 1.00

$$\begin{cases} \frac{x^{6}\operatorname{acot}^{2}(ax)}{6} + \frac{x^{5}\operatorname{acot}(ax)}{15a} + \frac{x^{4}}{60a^{2}} - \frac{x^{3}\operatorname{acot}(ax)}{9a^{3}} - \frac{4x^{2}}{45a^{4}} + \frac{x\operatorname{acot}(ax)}{3a^{5}} + \frac{23\log(a^{2}x^{2}+1)}{90a^{6}} + \frac{\operatorname{acot}^{2}(ax)}{6a^{6}} & \text{for}\ a \neq 0 \\ \frac{\pi^{2}x^{6}}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acot(a*x)**2,x)
```

```
[Out] Piecewise((x**6*acot(a*x)**2/6 + x**5*acot(a*x)/(15*a) + x**4/(60*a**2) - x
**3*acot(a*x)/(9*a**3) - 4*x**2/(45*a**4) + x*acot(a*x)/(3*a**5) + 23*log(a
**2*x**2 + 1)/(90*a**6) + acot(a*x)**2/(6*a**6), Ne(a, 0)), (pi**2*x**6/24,
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccot(a*x)^2, x)
```

Mupad [B]
time = 0.81, size = 85, normalized size = 0.82

$$\frac{x^6\,\mathrm{acot}(a\,x)^2}{6}+\frac{\frac{23\,\ln\left(a^2\,x^2+1\right)}{90}-\frac{4\,a^2\,x^2}{45}+\frac{a^4\,x^4}{60}+\frac{\mathrm{acot}(a\,x)^2}{6}-\frac{a^3\,x^3\,\mathrm{acot}(a\,x)}{9}+\frac{a^5\,x^5\,\mathrm{acot}(a\,x)}{15}+\frac{a\,x\,\mathrm{acot}(a\,x)}{3}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acot(a*x)^2,x)
```

```
[Out] (x^6*acot(a*x)^2)/6 + ((23*log(a^2*x^2 + 1))/90 - (4*a^2*x^2)/45 + (a^4*x^4
)/60 + acot(a*x)^2/6 - (a^3*x^3*acot(a*x))/9 + (a^5*x^5*acot(a*x))/15 + (a*
x*acot(a*x))/3)/a^6
```

3.13 $\int x^4 \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=135

$$-\frac{3x}{10a^4}+\frac{x^3}{30a^2}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^4\cot^{-1}(ax)}{10a}+\frac{i\cot^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{3\text{ArcTan}(ax)}{10a^5}-\frac{2\cot^{-1}(ax)\log(}{5a^5}$$

```
[Out] -3/10*x/a^4+1/30*x^3/a^2-1/5*x^2*arccot(a*x)/a^3+1/10*x^4*arccot(a*x)/a+1/5
*I*arccot(a*x)^2/a^5+1/5*x^5*arccot(a*x)^2+3/10*arctan(a*x)/a^5-2/5*arccot(
a*x)*ln(2/(1+I*a*x))/a^5+1/5*I*polylog(2,1-2/(1+I*a*x))/a^5
```

Rubi [A]
time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {4947, 5037, 308, 209, 327, 5041, 4965, 2449, 2352}

$$\frac{3\text{ArcTan}(ax)}{10a^5}+\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{5a^5}+\frac{i\cot^{-1}(ax)^2}{5a^5}-\frac{2\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{5a^5}-\frac{3x}{10a^4}-\frac{x^2\cot^{-1}(ax)}{5a^3}+\frac{x^3}{30a^2}+\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{x^4\cot^{-1}(ax)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCot[a*x]^2,x]
```

```
[Out] (-3*x)/(10*a^4) + x^3/(30*a^2) - (x^2*ArcCot[a*x])/(5*a^3) + (x^4*ArcCot[a*
x])/(10*a) + ((I/5)*ArcCot[a*x]^2)/a^5 + (x^5*ArcCot[a*x]^2)/5 + (3*ArcTan[
a*x])/(10*a^5) - (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/(5*a^5) + ((I/5)*PolyLo
g[2, 1 - 2/(1 + I*a*x)])/a^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{1}{5}(2a) \int \frac{x^5 \cot^{-1}(ax)}{1+a^2x^2} \, dx$$
$$= \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{2 \int x^3 \cot^{-1}(ax) \, dx}{5a} - \frac{2 \int \frac{x^3 \cot^{-1}(ax)}{1+a^2x^2} \, dx}{5a}$$
$$= \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{1}{10} \int \frac{x^4}{1+a^2x^2} \, dx - \frac{2 \int x \cot^{-1}(ax) \, dx}{5a^3} + \frac{2 \int \frac{x \cot^{-1}(}{1+a^2}}{5a^3}$$
$$= -\frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{1}{10} \int \left(-\frac{1}{a^4} + \frac{x^2}{a^2} \right.$$
$$= -\frac{3x}{10a^4} + \frac{x^3}{30a^2} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^2 - \frac{2 \mathrm{co}}{}$$
$$= -\frac{3x}{10a^4} + \frac{x^3}{30a^2} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{3 \mathrm{ta}}{}$$
$$= -\frac{3x}{10a^4} + \frac{x^3}{30a^2} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^2 + \frac{3 \mathrm{ta}}{}$$

Mathematica [A]
time = 0.38, size = 95, normalized size = 0.70

$$\frac{ax(-9+a^2x^2) + 6(i+a^5x^5)\cot^{-1}(ax)^2 + 3\cot^{-1}(ax)\left(-3-2a^2x^2+a^4x^4-4\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right) + 6i\mathrm{PolyLog}\left(2, e^{2i\cot^{-1}(ax)}\right)}{30a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCot[a*x]^2,x]
```

```
[Out] (a*x*(-9 + a^2*x^2) + 6*(I + a^5*x^5)*ArcCot[a*x]^2 + 3*ArcCot[a*x]*(-3 - 2
*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^((2*I)*ArcCot[a*x])]) + (6*I)*PolyLog[2, E
^((2*I)*ArcCot[a*x])])/(30*a^5)
```

Maple [A]
time = 0.16, size = 208, normalized size = 1.54

method	result
derivativedivides	$\frac{a^5x^5\mathrm{arccot}(ax)^2}{5} + \frac{a^4x^4\mathrm{arccot}(ax)}{10} - \frac{\mathrm{arccot}(ax)a^2x^2}{5} + \frac{\mathrm{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{10}$
default	$\frac{a^5x^5\mathrm{arccot}(ax)^2}{5} + \frac{a^4x^4\mathrm{arccot}(ax)}{10} - \frac{\mathrm{arccot}(ax)a^2x^2}{5} + \frac{\mathrm{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{10}$
risch	$\frac{i\ln(iax+1)x^4}{20a} - \frac{i\ln(iax+1)x^2}{10a^3} - \frac{i\ln(-iax+1)x^4}{20a} + \frac{i\ln(-iax+1)x^2}{10a^3} + \frac{i\pi\ln(iax+1)x^5}{10} - \frac{i\pi\ln(-iax+1)x^5}{10} + i$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arccot(a*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/a^5*(1/5*a^5*x^5*arccot(a*x)^2+1/10*a^4*x^4*arccot(a*x)-1/5*arccot(a*x)*a^2*x^2+1/5*arccot(a*x)*ln(a^2*x^2+1)+1/30*a^3*x^3-3/10*a*x+3/10*arctan(a*x)-1/10*I*ln(a*x-I)*ln(a^2*x^2+1)+1/10*I*dilog(-1/2*I*(I+a*x))+1/10*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))+1/20*I*ln(a*x-I)^2+1/10*I*ln(I+a*x)*ln(a^2*x^2+1)-1/10*I*dilog(1/2*I*(a*x-I))-1/10*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/20*I*ln(I+a*x)^2)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccot(a*x)^2,x, algorithm="maxima")

[Out] 1/20*x^5*arctan2(1, a*x)^2 - 1/80*x^5*log(a^2*x^2 + 1)^2 + integrate(1/80*(60*a^2*x^6*arctan2(1, a*x)^2 + 4*a^2*x^6*log(a^2*x^2 + 1) + 8*a*x^5*arctan2(1, a*x) + 60*x^4*arctan2(1, a*x)^2 + 5*(a^2*x^6 + x^4)*log(a^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arccot(a*x)^2,x, algorithm="fricas")

[Out] integral(x^4*arccot(a*x)^2, x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{acot}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*acot(a*x)**2,x)

[Out] Integral(x**4*acot(a*x)**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x)^2,x, algorithm="giac")
[Out] integrate(x^4*arccot(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{acot}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acot(a*x)^2,x)
[Out] int(x^4*acot(a*x)^2, x)
```

3.14 $\int x^3 \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=80

$$\frac{x^2}{12a^2} - \frac{x\cot^{-1}(ax)}{2a^3} + \frac{x^3\cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^2 - \frac{\log(1+a^2x^2)}{3a^4}$$

```
[Out] 1/12*x^2/a^2-1/2*x*arccot(a*x)/a^3+1/6*x^3*arccot(a*x)/a-1/4*arccot(a*x)^2/
a^4+1/4*x^4*arccot(a*x)^2-1/3*ln(a^2*x^2+1)/a^4
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4947, 5037, 272, 45, 4931, 266, 5005}

$$-\frac{\cot^{-1}(ax)^2}{4a^4} - \frac{x\cot^{-1}(ax)}{2a^3} + \frac{x^2}{12a^2} - \frac{\log(a^2x^2+1)}{3a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^2 + \frac{x^3\cot^{-1}(ax)}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a*x]^2,x]
```

```
[Out] x^2/(12*a^2) - (x*ArcCot[a*x])/(2*a^3) + (x^3*ArcCot[a*x])/(6*a) - ArcCot[a
*x]^2/(4*a^4) + (x^4*ArcCot[a*x]^2)/4 - Log[1 + a^2*x^2]/(3*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot^{-1}(ax)^2\,dx &= \frac{1}{4}x^4\cot^{-1}(ax)^2+\frac{1}{2}a\int\frac{x^4\cot^{-1}(ax)}{1+a^2x^2}\,dx\\
&= \frac{1}{4}x^4\cot^{-1}(ax)^2+\frac{\int x^2\cot^{-1}(ax)\,dx}{2a}-\frac{\int\frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a}\\
&= \frac{x^3\cot^{-1}(ax)}{6a}+\frac{1}{4}x^4\cot^{-1}(ax)^2+\frac{1}{6}\int\frac{x^3}{1+a^2x^2}\,dx-\frac{\int\cot^{-1}(ax)\,dx}{2a^3}+\frac{\int\frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{2a^3}\\
&= -\frac{x\cot^{-1}(ax)}{2a^3}+\frac{x^3\cot^{-1}(ax)}{6a}-\frac{\cot^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\cot^{-1}(ax)^2+\frac{1}{12}\mathrm{Subst}\left(\int\frac{x}{1+a^2x}\right.\\
&= -\frac{x\cot^{-1}(ax)}{2a^3}+\frac{x^3\cot^{-1}(ax)}{6a}-\frac{\cot^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\cot^{-1}(ax)^2-\frac{\log\left(1+a^2x^2\right)}{4a^4}+\frac{1}{12}\mathrm{S}\\
&= \frac{x^2}{12a^2}-\frac{x\cot^{-1}(ax)}{2a^3}+\frac{x^3\cot^{-1}(ax)}{6a}-\frac{\cot^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\cot^{-1}(ax)^2-\frac{\log\left(1+a^2x^2\right)}{3a^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.76

$$\frac{a^2x^2+2ax(-3+a^2x^2)\cot^{-1}(ax)+3(-1+a^4x^4)\cot^{-1}(ax)^2-4\log\left(1+a^2x^2\right)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a*x]^2,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(-3 + a^2*x^2)*ArcCot[a*x] + 3*(-1 + a^4*x^4)*ArcCot[a*x]^
2 - 4*Log[1 + a^2*x^2])/(12*a^4)
```

Maple [A]
time = 0.10, size = 78, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{ax\operatorname{arccot}(ax)}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln\left(a^2x^2+1\right)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{ax\operatorname{arccot}(ax)}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln\left(a^2x^2+1\right)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16a^4}+\frac{\left(3i\pi a^4x^4+3x^4\ln(-iax+1)a^4+2ia^3x^3-6iax-3\ln(-iax+1)\right)\ln(iax+1)}{24a^4}-\frac{i\pi x^4\ln(-iax}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccot(a*x)^2+1/6*a^3*x^3*arccot(a*x)-1/2*a*x*arccot(a*x
)+1/2*arccot(a*x)*arctan(a*x)+1/12*a^2*x^2-1/3*ln(a^2*x^2+1)+1/4*arctan(a*x
)^2)
```

Maxima [A]
time = 0.48, size = 77, normalized size = 0.96

$$\frac{1}{4}x^4\operatorname{arccot}(ax)^2+\frac{1}{6}a\left(\frac{a^2x^3-3x}{a^4}+\frac{3\arctan(ax)}{a^5}\right)\operatorname{arccot}(ax)+\frac{a^2x^2+3\arctan(ax)^2-4\log(a^2x^2+1)}{12a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(a*x)^2 + 1/6*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)*arc
cot(a*x) + 1/12*(a^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/a^4
```

Fricas [A]
time = 2.92, size = 60, normalized size = 0.75

$$\frac{a^2x^2+3\left(a^4x^4-1\right)\operatorname{arccot}(ax)^2+2\left(a^3x^3-3ax\right)\operatorname{arccot}(ax)-4\log\left(a^2x^2+1\right)}{12a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(a^2*x^2 + 3*(a^4*x^4 - 1)*arccot(a*x)^2 + 2*(a^3*x^3 - 3*a*x)*arccot(
a*x) - 4*log(a^2*x^2 + 1))/a^4
```

Sympy [A]
time = 0.28, size = 78, normalized size = 0.98

$$\begin{cases} \frac{x^4 \operatorname{acot}^2(ax)}{4} + \frac{x^3 \operatorname{acot}(ax)}{6a} + \frac{x^2}{12a^2} - \frac{x \operatorname{acot}(ax)}{2a^3} - \frac{\log(a^2x^2+1)}{3a^4} - \frac{\operatorname{acot}^2(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acot(a*x)**2/4 + x**3*acot(a*x)/(6*a) + x**2/(12*a**2) - x*
acot(a*x)/(2*a**3) - log(a**2*x**2 + 1)/(3*a**4) - acot(a*x)**2/(4*a**4), N
e(a, 0)), (pi**2*x**4/16, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccot(a*x)^2, x)
```

Mupad [B]
time = 0.20, size = 66, normalized size = 0.82

$$\frac{x^4 \operatorname{acot}(a\,x)^2}{4} - \frac{\frac{\ln(a^2\,x^2+1)}{3} - \frac{a^2\,x^2}{12} + \frac{\operatorname{acot}(a\,x)^2}{4} - \frac{a^3\,x^3 \operatorname{acot}(a\,x)}{6} + \frac{a\,x \operatorname{acot}(a\,x)}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a*x)^2,x)
```

```
[Out] (x^4*acot(a*x)^2)/4 - (log(a^2*x^2 + 1)/3 - (a^2*x^2)/12 + acot(a*x)^2/4 -
(a^3*x^3*acot(a*x))/6 + (a*x*acot(a*x))/2)/a^4
```

3.15 $\int x^2 \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=111

$$\frac{x}{3a^2}+\frac{x^2\cot^{-1}(ax)}{3a}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^2-\frac{\operatorname{ArcTan}(ax)}{3a^3}+\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^3}-\frac{i\mathrm{PolyLog}(2,1-}{3a^3}$$

```
[Out] 1/3*x/a^2+1/3*x^2*arccot(a*x)/a-1/3*I*arccot(a*x)^2/a^3+1/3*x^3*arccot(a*x)
^2-1/3*arctan(a*x)/a^3+2/3*arccot(a*x)*ln(2/(1+I*a*x))/a^3-1/3*I*polylog(2,
1-2/(1+I*a*x))/a^3
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 5037, 327, 209, 5041, 4965, 2449, 2352}

$$-\frac{\operatorname{ArcTan}(ax)}{3a^3}-\frac{i\mathrm{Li}_2\left(1-\frac{2}{iax+1}\right)}{3a^3}-\frac{i\cot^{-1}(ax)^2}{3a^3}+\frac{2\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{3a^3}+\frac{x}{3a^2}+\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{x^2\cot^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a*x]^2,x]
```

```
[Out] x/(3*a^2) + (x^2*ArcCot[a*x])/(3*a) - ((I/3)*ArcCot[a*x]^2)/a^3 + (x^3*ArcC
ot[a*x]^2)/3 - ArcTan[a*x]/(3*a^3) + (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/(3*
a^3) - ((I/3)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{1}{3}x^3 \cot^{-1}(ax)^2 + \frac{1}{3}(2a)\int \frac{x^3 \cot^{-1}(ax)}{1+a^2x^2} \, dx$$
$$= \frac{1}{3}x^3 \cot^{-1}(ax)^2 + \frac{2\int x \cot^{-1}(ax) \, dx}{3a} - \frac{2\int \frac{x \cot^{-1}(ax)}{1+a^2x^2} \, dx}{3a}$$
$$= \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 + \frac{1}{3}\int \frac{x^2}{1+a^2x^2} \, dx + \frac{2\int \frac{\cot^{-1}(ax)}{i-ax} \, dx}{3a^2}$$
$$= \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 + \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{3a^3} - \frac{\int}{}$$
$$= \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 - \frac{\tan^{-1}(ax)}{3a^3} + \frac{2\cot^{-1}(ax)\log}{3a^3}$$
$$= \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 - \frac{\tan^{-1}(ax)}{3a^3} + \frac{2\cot^{-1}(ax)\log}{3a^3}$$

Mathematica [**A**]
time = 0.21, size = 76, normalized size = 0.68

$$\frac{ax + (-i + a^3x^3)\cot^{-1}(ax)^2 + \cot^{-1}(ax)\left(1 + a^2x^2 + 2\log\left(1 - e^{2i\cot^{-1}(ax)}\right)\right) - i\text{PolyLog}\left(2, e^{2i\cot^{-1}(ax)}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a*x]^2,x]
```

```
[Out] (a*x + (-I + a^3*x^3)*ArcCot[a*x]^2 + ArcCot[a*x]*(1 + a^2*x^2 + 2*Log[1 -
E^((2*I)*ArcCot[a*x])]) - I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(3*a^3)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(93) = 186.
time = 0.14, size = 188, normalized size = 1.69

method	result
derivativedivides	$\frac{\frac{a^3x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{6} - \frac{i\,\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)}{6} - \frac{i}{}}{a^3}$
default	$\frac{\frac{a^3x^3 \operatorname{arccot}(ax)^2}{3} + \frac{\operatorname{arccot}(ax)a^2x^2}{3} - \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{3} + \frac{ax}{3} - \frac{\arctan(ax)}{3} + \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{6} - \frac{i\,\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)}{6} - \frac{i}{}}{a^3}$
risch	$-\frac{i\ln(-iax+1)x^2}{6a} + \frac{i\ln(iax+1)x^2}{6a} - \frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{3a^3} - \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{3a^3} - \frac{i\pi\ln(-iax+1)x^3}{6} + \frac{i\pi\ln(i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccot(a*x)^2+1/3*arccot(a*x)*a^2*x^2-1/3*arccot(a*x)*ln
(a^2*x^2+1)+1/3*a*x-1/3*arctan(a*x)+1/6*I*ln(a*x-I)*ln(a^2*x^2+1)-1/6*I*dil
og(-1/2*I*(I+a*x))-1/6*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/12*I*ln(a*x-I)^2-1/
6*I*ln(I+a*x)*ln(a^2*x^2+1)+1/6*I*dilog(1/2*I*(a*x-I))+1/6*I*ln(I+a*x)*ln(1
/2*I*(a*x-I))+1/12*I*ln(I+a*x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*x^3*arctan2(1, a*x)^2 - 1/48*x^3*log(a^2*x^2 + 1)^2 + integrate(1/48*(
36*a^2*x^4*arctan2(1, a*x)^2 + 4*a^2*x^4*log(a^2*x^2 + 1) + 8*a*x^3*arctan2
(1, a*x) + 36*x^2*arctan2(1, a*x)^2 + 3*(a^2*x^4 + x^2)*log(a^2*x^2 + 1)^2)
/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccot(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acot}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(a*x)**2,x)
```

```
[Out] Integral(x**2*acot(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(a*x)^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a*x)^2,x)
```

```
[Out] int(x^2*acot(a*x)^2, x)
```

3.16 $\int x \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=53

$$\frac{x \cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^2 + \frac{\log(1+a^2x^2)}{2a^2}$$

```
[Out] x*arccot(a*x)/a+1/2*arccot(a*x)^2/a^2+1/2*x^2*arccot(a*x)^2+1/2*ln(a^2*x^2+
1)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4947, 5037, 4931, 266, 5005}

$$\frac{\log(a^2x^2+1)}{2a^2} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \cot^{-1}(ax)^2 + \frac{x \cot^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a*x]^2,x]
```

```
[Out] (x*ArcCot[a*x])/a + ArcCot[a*x]^2/(2*a^2) + (x^2*ArcCot[a*x]^2)/2 + Log[1 +
 a^2*x^2]/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\cot^{-1}(ax)^2 + a\int \frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{1}{2}x^2\cot^{-1}(ax)^2 + \frac{\int \cot^{-1}(ax)\,dx}{a} - \frac{\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx}{a} \\
&= \frac{x\cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^2 + \int \frac{x}{1+a^2x^2}\,dx \\
&= \frac{x\cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^2 + \frac{\log\left(1+a^2x^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.79

$$\frac{2ax\cot^{-1}(ax) + \left(1+a^2x^2\right)\cot^{-1}(ax)^2 + \log\left(1+a^2x^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a*x]^2,x]
[Out] (2*a*x*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 + Log[1 + a^2*x^2])/(2*a^2
)
```

Maple [A]
time = 0.09, size = 57, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{a^2x^2\operatorname{arccot}(ax)^2}{2} - \operatorname{arccot}(ax)\arctan(ax) + ax\operatorname{arccot}(ax) + \frac{\ln\left(a^2x^2+1\right)}{2} - \frac{\arctan(ax)^2}{2}}{a^2}$
default	$\frac{\frac{a^2x^2\operatorname{arccot}(ax)^2}{2} - \operatorname{arccot}(ax)\arctan(ax) + ax\operatorname{arccot}(ax) + \frac{\ln\left(a^2x^2+1\right)}{2} - \frac{\arctan(ax)^2}{2}}{a^2}$
risch	$-\frac{\left(a^2x^2+1\right)\ln(iax+1)^2}{8a^2} + \frac{\left(i\pi a^2x^2+x^2\ln(-iax+1)a^2+2iax+\ln(-iax+1)\right)\ln(iax+1)}{4a^2} - \frac{i\pi x^2\ln(-iax+1)}{4} + \frac{\pi^2 a}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccot(a*x)^2-arccot(a*x)*arctan(a*x)+a*x*arccot(a*x)+1/
2*ln(a^2*x^2+1)-1/2*arctan(a*x)^2)
```

Maxima [A]
time = 0.49, size = 57, normalized size = 1.08

$$\frac{1}{2}\,x^2\,\mathrm{arccot}\left(ax\right)^2+a\left(\frac{x}{a^2}-\frac{\arctan\left(ax\right)}{a^3}\right)\mathrm{arccot}\left(ax\right)-\frac{\arctan\left(ax\right)^2-\log\left(a^2x^2+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccot(a*x)^2 + a*(x/a^2 - arctan(a*x)/a^3)*arccot(a*x) - 1/2*(arct
an(a*x)^2 - log(a^2*x^2 + 1))/a^2
```

Fricas [A]
time = 2.23, size = 40, normalized size = 0.75

$$\frac{2\,ax\,\mathrm{arccot}\left(ax\right)+\left(a^2x^2+1\right)\mathrm{arccot}\left(ax\right)^2+\log\left(a^2x^2+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*arccot(a*x) + (a^2*x^2 + 1)*arccot(a*x)^2 + log(a^2*x^2 + 1))/a^
2
```

Sympy [A]
time = 0.15, size = 54, normalized size = 1.02

$$\begin{cases} \frac{x^2\,\mathrm{acot}^2\left(ax\right)}{2}+\frac{x\,\mathrm{acot}\left(ax\right)}{a}+\frac{\log\left(a^2x^2+1\right)}{2a^2}+\frac{\mathrm{acot}^2\left(ax\right)}{2a^2} & \text{for}\ a\neq 0\\ \frac{\pi^2x^2}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acot(a*x)**2/2 + x*acot(a*x)/a + log(a**2*x**2 + 1)/(2*a**2
) + acot(a*x)**2/(2*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccot(a*x)^2, x)
```

Mupad [B]
time = 0.14, size = 44, normalized size = 0.83

$$\frac{x^2\,\mathrm{acot}(a\,x)^2}{2}+\frac{\frac{\mathrm{acot}(a\,x)^2}{2}+a\,x\,\mathrm{acot}(a\,x)+\frac{\ln\left(a^2\,x^2+1\right)}{2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(a*x)^2,x)
```

```
[Out] (x^2*acot(a*x)^2)/2 + (log(a^2*x^2 + 1)/2 + acot(a*x)^2/2 + a*x*acot(a*x))/
a^2
```

3.17 $\int \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=67

$$\frac{i\cot^{-1}(ax)^2}{a} + x\cot^{-1}(ax)^2 - \frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{a}$$

```
[Out] I*arccot(a*x)^2/a+x*arccot(a*x)^2-2*arccot(a*x)*ln(2/(1+I*a*x))/a+I*polylog
(2,1-2/(1+I*a*x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4931, 5041, 4965, 2449, 2352}

$$\frac{i\text{Li}_2\left(1 - \frac{2}{iax+1}\right)}{a} + x\cot^{-1}(ax)^2 + \frac{i\cot^{-1}(ax)^2}{a} - \frac{2\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2,x]
```

```
[Out] (I*ArcCot[a*x]^2)/a + x*ArcCot[a*x]^2 - (2*ArcCot[a*x]*Log[2/(1 + I*a*x)])/
a + (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(ax)^2\,dx &= x\cot^{-1}(ax)^2+(2a)\int \frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx\\ &= \frac{i\cot^{-1}(ax)^2}{a}+x\cot^{-1}(ax)^2-2\int \frac{\cot^{-1}(ax)}{i-ax}\,dx\\ &= \frac{i\cot^{-1}(ax)^2}{a}+x\cot^{-1}(ax)^2-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\\ &= \frac{i\cot^{-1}(ax)^2}{a}+x\cot^{-1}(ax)^2-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{(2i)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+iax}\right)}{a}\\ &= \frac{i\cot^{-1}(ax)^2}{a}+x\cot^{-1}(ax)^2-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\text{Li}_2\left(1-\frac{2}{1+iax}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.84

$$\frac{\cot^{-1}(ax)\left((i+ax)\cot^{-1}(ax)-2\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+i\text{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^2,x]
[Out] (ArcCot[a*x]*((I + a*x)*ArcCot[a*x] - 2*Log[1 - E^((2*I)*ArcCot[a*x])]) + I
*PolyLog[2, E^((2*I)*ArcCot[a*x])])/a
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.
time = 0.20, size = 130, normalized size = 1.94

method	result

derivativedivides	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{pol}}{a}$
default	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{pol}}{a}$
risch	$-\frac{i\ln(-iax+1)^2}{4a}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{a}-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{a}+\frac{i}{a}+\frac{i\pi^2}{4a}+\frac{i\ln(iax+1)^2}{4a}+\frac{\pi\ln(a^2x^2}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)^2*(a*x-I)-2*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-2*
arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)^2+2*I*polylog(2
,(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x*arctan2(1, a*x)^2 + 12*a^2*integrate(1/16*x^2*arctan(1/(a*x))^2/(a^2*
x^2 + 1), x) + a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
+ 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/16*x*log(
a^2*x^2 + 1)^2 + 1/4*arctan(a*x)^3/a + 3/4*arctan(a*x)^2*arctan(1/(a*x))/a
+ 3/4*arctan(a*x)*arctan(1/(a*x))^2/a + 8*a*integrate(1/16*x*arctan(1/(a*x)
)/(a^2*x^2 + 1), x) + integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2,x)
```

```
[Out] Integral(acot(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^2, x)
```

Mupad [B]
time = 0.59, size = 55, normalized size = 0.82

$$\frac{-2\ln\left(1-\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,\mathrm{acot}(a\,x)+\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{acot}(a\,x)\,2\mathrm{i}}\right)\,1\mathrm{i}+\mathrm{acot}(a\,x)^2\,1\mathrm{i}}{a}+x\,\mathrm{acot}(a\,x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2,x)
```

```
[Out] (polylog(2, exp(acot(a*x)*2i))*1i - 2*log(1 - exp(acot(a*x)*2i))*acot(a*x)
+ acot(a*x)^2*1i)/a + x*acot(a*x)^2
```

3.18 $\int \frac{\cot^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=116

$$2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-i\cot^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)+i\cot^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right.$$

```
[Out] 2*arccot(a*x)^2*arccoth(1-2/(1+I*a*x))-I*arccot(a*x)*polylog(2,1-2*I/(I+a*x
))+I*arccot(a*x)*polylog(2,1-2*a*x/(I+a*x))-1/2*polylog(3,1-2*I/(I+a*x))+1/
2*polylog(3,1-2*a*x/(I+a*x))
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4943, 5109, 5005, 5113, 6745}

$$-\frac{1}{2}\text{Li}_3\left(1-\frac{2i}{ax+i}\right)+\frac{1}{2}\text{Li}_3\left(1-\frac{2ax}{ax+i}\right)-i\text{Li}_2\left(1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)+i\text{Li}_2\left(1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax)+2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2/x,x]
```

```
[Out] 2*ArcCot[a*x]^2*ArcCoth[1 - 2/(1 + I*a*x)] - I*ArcCot[a*x]*PolyLog[2, 1 - (
2*I)/(I + a*x)] + I*ArcCot[a*x]*PolyLog[2, 1 - (2*a*x)/(I + a*x)] - PolyLog
[3, 1 - (2*I)/(I + a*x)]/2 + PolyLog[3, 1 - (2*a*x)/(I + a*x)]/2
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^2}{x}\,dx &= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)+(4a)\int \frac{\cot^{-1}(ax)\coth^{-1}\left(1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\\ &= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-(2a)\int \frac{\cot^{-1}(ax)\log\left(\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+(2a)\int \frac{\cot^{-1}(a}{1}\\ &= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-i\cot^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2i}{i+ax}\right)+i\cot^{-1}(ax)\mathrm{Li}_2\Big(1\\ &= 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)-i\cot^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2i}{i+ax}\right)+i\cot^{-1}(ax)\mathrm{Li}_2\Big(1\end{aligned}$$

Mathematica [A]
time = 0.04, size = 132, normalized size = 1.14

$$-\frac{2}{3}i\cot^{-1}(ax)^3-\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)+\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)-i\cot^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)-i\cot^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^2/x,x]
```

```
[Out] ((-2*I)/3)*ArcCot[a*x]^3 - ArcCot[a*x]^2*Log[1 - E^((-2*I)*ArcCot[a*x])] +
ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, E^(
(-2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - P
olyLog[3, E^((-2*I)*ArcCot[a*x])]/2 + PolyLog[3, -E^((2*I)*ArcCot[a*x])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.07, size = 959, normalized size = 8.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccot(a*x)^2-1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2+2*I*arcco
t(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2
*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)
-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arccot(a*x)^2-I*arccot(a*x)*polylog(2,-(I+a*
x)^2/(a^2*x^2+1))-1/2*I*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2+2*I*arccot(a*
x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^
2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a
*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2-1/2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-
1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*arccot(a*x)^2+1/2*I*Pi*csgn(I/((I+a*x)^2/(a
^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1
+(I+a*x)^2/(a^2*x^2+1)))*arccot(a*x)^2+arccot(a*x)^2*ln((I+a*x)^2/(a^2*x^2+
1)-1)-arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*arccot(a*x)^2-2*polylog(3,-
(I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-1/
2*I*Pi*arccot(a*x)^2-2*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(1
/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2+1/2*p
olylog(3,-(I+a*x)^2/(a^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccot(a*x)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{2}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2/x,x)
```

```
[Out] Integral(acot(a*x)**2/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2/x,x)
```

```
[Out] int(acot(a*x)^2/x, x)
```

3.19 $\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=66

$$-ia\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{x}-2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)-ia\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

```
[Out] -I*a*arccot(a*x)^2-arccot(a*x)^2/x-2*a*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a*po
lylog(2,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4947, 5045, 4989, 2497}

$$-ia\text{Li}_2\left(\frac{2}{1-iax}-1\right)-ia\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{x}-2a\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2/x^2,x]
```

```
[Out] (-I)*a*ArcCot[a*x]^2 - ArcCot[a*x]^2/x - 2*a*ArcCot[a*x]*Log[2 - 2/(1 - I*a
*x)] - I*a*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = -\frac{\cot^{-1}(ax)^2}{x} - (2a)\int \frac{\cot^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx$$
$$= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - (2ia)\int \frac{\cot^{-1}(ax)}{x(i+ax)}\,dx$$
$$= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) - (2a^2)\int \frac{\log\left(2-\frac{}{1-}\right.}{1+a^2x}$$
$$= -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) - ia\mathrm{Li}_2\left(-1+\frac{2}{1-iax}\right.$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.97

$$a\left(i\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{ax} - 2\cot^{-1}(ax)\log\left(1+e^{2i\cot^{-1}(ax)}\right) + i\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^2/x^2,x]
[Out] a*(I*ArcCot[a*x]^2 - ArcCot[a*x]^2/(a*x) - 2*ArcCot[a*x]*Log[1 + E^((2*I)*A
rcCot[a*x])] + I*PolyLog[2, -E^((2*I)*ArcCot[a*x])])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(62) = 124.
time = 0.32, size = 225, normalized size = 3.41

method	result
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} - 2\ln(ax)\operatorname{arccot}(ax) + \operatorname{arccot}(ax)\ln\left(a^2x^2+1\right) - \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2} + \right.$
default	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax} - 2\ln(ax)\operatorname{arccot}(ax) + \operatorname{arccot}(ax)\ln\left(a^2x^2+1\right) - \frac{i\ln(ax-i)\ln\left(a^2x^2+1\right)}{2} + \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arccot(a*x)^2/a/x-2*ln(a*x)*arccot(a*x)+arccot(a*x)*ln(a^2*x^2+1)-1/2*I
*ln(a*x-I)*ln(a^2*x^2+1)+1/2*I*dilog(-1/2*I*(I+a*x))+1/2*I*ln(a*x-I)*ln(-1/
2*I*(I+a*x))+1/4*I*ln(a*x-I)^2+1/2*I*ln(I+a*x)*ln(a^2*x^2+1)-1/2*I*dilog(1/
2*I*(a*x-I))-1/2*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/4*I*ln(I+a*x)^2+I*ln(a*x)*
ln(1+I*a*x)-I*ln(a*x)*ln(1-I*a*x)+I*dilog(1+I*a*x)-I*dilog(1-I*a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*a*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a + 3*arctan(a*
x)^2*arctan(1/(a*x))/a)*a^2 + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(
a^2*x^4 + x^2), x) - 16*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4 +
x^2), x) - 32*a*integrate(1/16*x*arctan(1/(a*x))/(a^2*x^4 + x^2), x) + 48*i
ntegrate(1/16*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 4*integrate(1/16*log(
a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x - 4*arctan2(1, a*x)^2 + log(a^2*x^2 +
 1)^2)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{2}(ax)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2/x**2,x)
```

```
[Out] Integral(acot(a*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acot}(a\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2/x^2,x)
```

```
[Out] int(acot(a*x)^2/x^2, x)
```

3.20 $\int \frac{\cot^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=59

$$\frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

```
[Out] a*arccot(a*x)/x-1/2*a^2*arccot(a*x)^2-1/2*arccot(a*x)^2/x^2+a^2*ln(x)-1/2*a
^2*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4947, 5039, 272, 36, 29, 31, 5005}

$$-\frac{1}{2}a^2\log\left(a^2x^2+1\right) + a^2\log(x) - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + \frac{a\cot^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2/x^3,x]
```

```
[Out] (a*ArcCot[a*x])/x - (a^2*ArcCot[a*x]^2)/2 - ArcCot[a*x]^2/(2*x^2) + a^2*Log
[x] - (a^2*Log[1 + a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx &= -\frac{\cot^{-1}(ax)^2}{2x^2} - a\int \frac{\cot^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^2}{2x^2} - a\int \frac{\cot^{-1}(ax)}{x^2}\,dx + a^3\int \frac{\cot^{-1}(ax)}{1+a^2x^2}\,dx \\
&= \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\int \frac{1}{x\left(1+a^2x^2\right)}\,dx \\
&= \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\left(1+a^2x\right)}\,dx, x, x^2\right) \\
&= \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}a^4\text{Subst}\Big(\\
&= \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.95

$$\frac{a\cot^{-1}(ax)}{x} + \frac{\left(-1-a^2x^2\right)\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCot[a*x]^2/x^3,x]

[Out] (a*ArcCot[a*x])/x + ((-1 - a^2*x^2)*ArcCot[a*x]^2)/(2*x^2) + a^2*Log[x] - (a^2*Log[1 + a^2*x^2])/2

Maple [A]
time = 0.11, size = 64, normalized size = 1.08

method	result
derivativedivides	$a^{2}\left(-\frac{\operatorname{arccot}(a x)^{2}}{2 a^{2} x^{2}}+\operatorname{arccot}(a x) \arctan (a x)+\frac{\operatorname{arccot}(a x)}{a x}+\ln (a x)-\frac{\ln \left(a^{2} x^{2}+1\right)}{2}+\frac{\arctan (a x)^{2}}{2}\right)$
default	$a^{2}\left(-\frac{\operatorname{arccot}(a x)^{2}}{2 a^{2} x^{2}}+\operatorname{arccot}(a x) \arctan (a x)+\frac{\operatorname{arccot}(a x)}{a x}+\ln (a x)-\frac{\ln \left(a^{2} x^{2}+1\right)}{2}+\frac{\arctan (a x)^{2}}{2}\right)$
risch	$\frac{\left(a^{2} x^{2}+1\right) \ln (i a x+1)^{2}}{8 x^{2}}-\frac{i\left(-i x^{2} \ln (-i a x+1) a^{2}-2 a x+\pi-i \ln (-i a x+1)\right) \ln (i a x+1)}{4 x^{2}}-\frac{-2 i a^{2} \ln ((-\pi a-6 i a) x-i \pi+6) \pi}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccot(a*x)^2/x^3,x,method=_RETURNVERBOSE)

[Out] a^2*(-1/2*arccot(a*x)^2/a^2/x^2+arccot(a*x)*arctan(a*x)+arccot(a*x)/a/x+ln(a*x)-1/2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2)

Maxima [A]
time = 0.49, size = 56, normalized size = 0.95

$$\frac{1}{2}\left(\arctan\left(ax\right)^{2}-\log\left(a^{2}x^{2}+1\right)+2\,\log\left(x\right)\right)a^{2}+\left(a\arctan\left(ax\right)+\frac{1}{x}\right)a\,\operatorname{arccot}\left(ax\right)-\frac{\operatorname{arccot}\left(ax\right)^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(a*x)^2/x^3,x, algorithm="maxima")

[Out] 1/2*(arctan(a*x)^2 - log(a^2*x^2 + 1) + 2*log(x))*a^2 + (a*arctan(a*x) + 1/x)*a*arccot(a*x) - 1/2*arccot(a*x)^2/x^2

Fricas [A]
time = 2.13, size = 57, normalized size = 0.97

$$-\frac{a^{2}x^{2}\log\left(a^{2}x^{2}+1\right)-2\,a^{2}x^{2}\log\left(x\right)-2\,ax\operatorname{arccot}\left(ax\right)+\left(a^{2}x^{2}+1\right)\operatorname{arccot}\left(ax\right)^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(a*x)^2/x^3,x, algorithm="fricas")

[Out] -1/2*(a^2*x^2*log(a^2*x^2 + 1) - 2*a^2*x^2*log(x) - 2*a*x*arccot(a*x) + (a^2*x^2 + 1)*arccot(a*x)^2)/x^2

Sympy [A]
time = 0.22, size = 53, normalized size = 0.90

$$a^2 \log(x) - \frac{a^2 \log(a^2x^2+1)}{2} - \frac{a^2 \operatorname{acot}^2(ax)}{2} + \frac{a \operatorname{acot}(ax)}{x} - \frac{\operatorname{acot}^2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2/x**3,x)
```

```
[Out] a**2*log(x) - a**2*log(a**2*x**2 + 1)/2 - a**2*acot(a*x)**2/2 + a*acot(a*x)
/x - acot(a*x)**2/(2*x**2)
```

Giac [A]
time = 0.40, size = 60, normalized size = 1.02

$$-\frac{1}{2}\left(\left(\arctan\left(\frac{1}{ax}\right)^2 - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax} + \log\left(\frac{1}{a^2x^2}+1\right)\right)a + \frac{\arctan\left(\frac{1}{ax}\right)^2}{ax^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((arctan(1/(a*x))^2 - 2*arctan(1/(a*x))/(a*x) + log(1/(a^2*x^2) + 1))*
a + arctan(1/(a*x))^2/(a*x^2))*a
```

Mupad [B]
time = 0.66, size = 50, normalized size = 0.85

$$a^2 \ln(x) - \operatorname{acot}(a\,x)^2\left(\frac{a^2}{2} + \frac{1}{2\,x^2}\right) - \frac{a^2 \ln(a^2\,x^2+1)}{2} + \frac{a \operatorname{acot}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2/x^3,x)
```

```
[Out] a^2*log(x) - acot(a*x)^2*(a^2/2 + 1/(2*x^2)) - (a^2*log(a^2*x^2 + 1))/2 + (
a*acot(a*x))/x
```

3.21 $\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=113

$$-\frac{a^2}{3x}+\frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{1}{3}a^3\text{ArcTan}(ax)+\frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)+\frac{1}{3}ia^3$$

```
[Out] -1/3*a^2/x+1/3*a*arccot(a*x)/x^2+1/3*I*a^3*arccot(a*x)^2-1/3*arccot(a*x)^2/
x^3-1/3*a^3*arctan(a*x)+2/3*a^3*arccot(a*x)*ln(2-2/(1-I*a*x))+1/3*I*a^3*pol
ylog(2,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4947, 5039, 331, 209, 5045, 4989, 2497}

$$-\frac{1}{3}a^3\text{ArcTan}(ax)+\frac{1}{3}ia^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{1}{3}ia^3\cot^{-1}(ax)^2+\frac{2}{3}a^3\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)-\frac{a^2}{3x}-\frac{\cot^{-1}(ax)^2}{3x^3}+\frac{a\cot^{-1}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2/x^4,x]
```

```
[Out] -1/3*a^2/x + (a*ArcCot[a*x])/(3*x^2) + (I/3)*a^3*ArcCot[a*x]^2 - ArcCot[a*x
]^2/(3*x^3) - (a^3*ArcTan[a*x])/3 + (2*a^3*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x
)])/3 + (I/3)*a^3*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx &= -\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{1}{3}(2a)\int \frac{\cot^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx\\
&= -\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{1}{3}(2a)\int \frac{\cot^{-1}(ax)}{x^3}\,dx+\frac{1}{3}\left(2a^3\right)\int \frac{\cot^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx\\
&= \frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^2\int \frac{1}{x^2\left(1+a^2x^2\right)}\,dx+\frac{1}{3}\left(2ia^3\right)\int\\
&= -\frac{a^2}{3x}+\frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right.\\
&= -\frac{a^2}{3x}+\frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{1}{3}a^3\tan^{-1}(ax)+\frac{2}{3}a^3\cot^{-1}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 96, normalized size = 0.85

$$\frac{-a^2x^2+(-1-ia^3x^3)\cot^{-1}(ax)^2+ax\cot^{-1}(ax)\left(1+a^2x^2+2a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)-ia^3x^3\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^2/x^4,x]
```

```
[Out] (-(a^2*x^2) + (-1 - I*a^3*x^3)*ArcCot[a*x]^2 + a*x*ArcCot[a*x]*(1 + a^2*x^2
 + 2*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]) - I*a^3*x^3*PolyLog[2, -E^((2*
I)*ArcCot[a*x])])/(3*x^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(95) = 190.
time = 0.28, size = 254, normalized size = 2.25

method	result
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3}+\frac{\operatorname{arccot}(ax)}{3a^2x^2}+\frac{2\ln(ax)\operatorname{arccot}(ax)}{3}-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3}-\frac{i\operatorname{dilog}(iax+1)}{3}+\frac{i\ln(ax)\ln(}{3}\right.$
default	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3}+\frac{\operatorname{arccot}(ax)}{3a^2x^2}+\frac{2\ln(ax)\operatorname{arccot}(ax)}{3}-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3}-\frac{i\operatorname{dilog}(iax+1)}{3}+\frac{i\ln(ax)\ln(}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*arccot(a*x)^2/a^3/x^3+1/3*arccot(a*x)/a^2/x^2+2/3*ln(a*x)*arccot(
a*x)-1/3*arccot(a*x)*ln(a^2*x^2+1)+1/6*I*dilog(1/2*I*(a*x-I))+1/3*I*ln(a*x)
*ln(1-I*a*x)+1/3*I*dilog(1-I*a*x)-1/6*I*ln(I+a*x)*ln(a^2*x^2+1)-1/3*I*ln(a*
x)*ln(1+I*a*x)+1/12*I*ln(I+a*x)^2-1/6*I*dilog(-1/2*I*(I+a*x))+1/6*I*ln(a*x-
I)*ln(a^2*x^2+1)-1/3*arctan(a*x)-1/3/a/x-1/6*I*ln(a*x-I)*ln(-1/2*I*(I+a*x))
+1/6*I*ln(I+a*x)*ln(1/2*I*(a*x-I))-1/12*I*ln(a*x-I)^2-1/3*I*dilog(1+I*a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*(48*x^3*integrate(1/48*(36*a^2*x^2*arctan2(1, a*x)^2 - 4*a^2*x^2*log(a
^2*x^2 + 1) - 8*a*x*arctan2(1, a*x) + 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2 +
36*arctan2(1, a*x)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^2 + log(a^2*x
^2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="fricas")
[Out] integral(arccot(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{2}(ax)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2/x**4,x)
[Out] Integral(acot(a*x)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate(arccot(a*x)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(a\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2/x^4,x)
[Out] int(acot(a*x)^2/x^4, x)
```

3.22 $\int \frac{\cot^{-1}(ax)^2}{x^5} \, dx$

Optimal. Leaf size=89

$$-\frac{a^2}{12x^2}+\frac{a\cot^{-1}(ax)}{6x^3}-\frac{a^3\cot^{-1}(ax)}{2x}+\frac{1}{4}a^4\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{4x^4}-\frac{2}{3}a^4\log(x)+\frac{1}{3}a^4\log\left(1+a^2x^2\right)$$

```
[Out] -1/12*a^2/x^2+1/6*a*arccot(a*x)/x^3-1/2*a^3*arccot(a*x)/x+1/4*a^4*arccot(a*
x)^2-1/4*arccot(a*x)^2/x^4-2/3*a^4*ln(x)+1/3*a^4*ln(a^2*x^2+1)
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 5039, 272, 46, 36, 29, 31, 5005}

$$-\frac{2}{3}a^4\log(x)+\frac{1}{4}a^4\cot^{-1}(ax)^2-\frac{a^3\cot^{-1}(ax)}{2x}-\frac{a^2}{12x^2}+\frac{1}{3}a^4\log\left(a^2x^2+1\right)-\frac{\cot^{-1}(ax)^2}{4x^4}+\frac{a\cot^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 + (a*ArcCot[a*x])/(6*x^3) - (a^3*ArcCot[a*x])/(2*x) + (a^4*Ar
cCot[a*x]^2)/4 - ArcCot[a*x]^2/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^
2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx &= -\frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4}\,dx + \frac{1}{2}a^3\int \frac{\cot^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{\cot^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3\left(1+a^2x^2\right)}\,dx + \frac{1}{2}a^3\int \frac{\cot^{-1}(ax)}{x^2}\,dx - \frac{1}{2}a^5\int \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2\mathrm{Subst}\left(\int \frac{1}{x^2\left(1+\right.}\right. \\
&= \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2\mathrm{Subst}\left(\int\left(\frac{1}{x^2} - \right.\right. \\
&= -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{1}{6}a^4\log(x) + \\
&= -\frac{a^2}{12x^2} + \frac{a\cot^{-1}(ax)}{6x^3} - \frac{a^3\cot^{-1}(ax)}{2x} + \frac{1}{4}a^4\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3}a^4\log(x) +
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 0.91

$$-\frac{a^2}{12x^2}-\frac{a(-1+3a^2x^2)\cot^{-1}(ax)}{6x^3}+\frac{(-1+a^4x^4)\cot^{-1}(ax)^2}{4x^4}-\frac{2}{3}a^4\log(x)+\frac{1}{3}a^4\log\left(1+a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*(-1 + 3*a^2*x^2)*ArcCot[a*x])/(6*x^3) + ((-1 + a^4*x^4)*
ArcCot[a*x]^2)/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^2*x^2])/3
```

Maple [A]
time = 0.14, size = 88, normalized size = 0.99

method	result
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}-\right.$
default	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2ax}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}-\frac{1}{12a^2x^2}-\frac{2\ln(ax)}{3}+\frac{\ln(a^2x^2+1)}{3}-\right.$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16x^4}-\frac{i(3ix^4\ln(-iax+1)a^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi)\ln(iax+1)}{24x^4}-\frac{-6ia^4\ln((-\pi a+8ia$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arccot(a*x)^2/a^4/x^4+1/6*arccot(a*x)/a^3/x^3-1/2*arccot(a*x)/a/x
-1/2*arccot(a*x)*arctan(a*x)-1/12/a^2/x^2-2/3*ln(a*x)+1/3*ln(a^2*x^2+1)-1/4
*arctan(a*x)^2)
```

Maxima [A]
time = 0.49, size = 95, normalized size = 1.07

$$-\frac{1}{6}\left(3a^3\arctan(ax)+\frac{3a^2x^2-1}{x^3}\right)a\operatorname{arccot}(ax)-\frac{(3a^2x^2\arctan(ax)^2-4a^2x^2\log(a^2x^2+1)+8a^2x^2\log(x)+1)a^2}{12x^2}-\frac{\operatorname{arccot}(ax)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a*arccot(a*x) - 1/12*(3*a^2*
x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a^2/
x^2 - 1/4*arccot(a*x)^2/x^4
```

Fricas [A]
time = 2.13, size = 78, normalized size = 0.88

$$\frac{4a^4x^4\log(a^2x^2+1)-8a^4x^4\log(x)-a^2x^2+3(a^4x^4-1)\operatorname{arccot}(ax)^2-2(3a^3x^3-ax)\operatorname{arccot}(ax)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^4*x^4*log(a^2*x^2 + 1) - 8*a^4*x^4*log(x) - a^2*x^2 + 3*(a^4*x^4
- 1)*arccot(a*x)^2 - 2*(3*a^3*x^3 - a*x)*arccot(a*x))/x^4
```

Sympy [A]
time = 0.30, size = 80, normalized size = 0.90

$$-\frac{2a^4\log(x)}{3}+\frac{a^4\log(a^2x^2+1)}{3}+\frac{a^4\operatorname{acot}^2(ax)}{4}-\frac{a^3\operatorname{acot}(ax)}{2x}-\frac{a^2}{12x^2}+\frac{a\operatorname{acot}(ax)}{6x^3}-\frac{\operatorname{acot}^2(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**2/x**5,x)
```

```
[Out] -2*a**4*log(x)/3 + a**4*log(a**2*x**2 + 1)/3 + a**4*acot(a*x)**2/4 - a**3*a
cot(a*x)/(2*x) - a**2/(12*x**2) + a*acot(a*x)/(6*x**3) - acot(a*x)**2/(4*x*
*4)
```

Giac [A]
time = 0.42, size = 91, normalized size = 1.02

$$\frac{1}{12}\left(\left(3\arctan\left(\frac{1}{ax}\right)^2-\frac{6\arctan\left(\frac{1}{ax}\right)}{ax}-\frac{1}{a^2x^2}+\frac{2\arctan\left(\frac{1}{ax}\right)}{a^3x^3}+4\log\left(\frac{1}{a^2x^2}+1\right)\right)a^3-\frac{3\arctan\left(\frac{1}{ax}\right)^2}{ax^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/12*((3*arctan(1/(a*x))^2 - 6*arctan(1/(a*x))/(a*x) - 1/(a^2*x^2) + 2*arct
an(1/(a*x))/(a^3*x^3) + 4*log(1/(a^2*x^2) + 1))*a^3 - 3*arctan(1/(a*x))^2/(
a*x^4))*a
```

Mupad [B]
time = 0.70, size = 73, normalized size = 0.82

$$\operatorname{acot}(a\,x)^2\left(\frac{a^4}{4}-\frac{1}{4\,x^4}\right)-\frac{2\,a^4\,\ln(x)}{3}+\frac{a^4\,\ln(a^2\,x^2+1)}{3}-\frac{a^2}{12\,x^2}-\frac{a^2\operatorname{acot}(a\,x)\,\left(\frac{a\,x^2}{2}-\frac{1}{6\,a}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^2/x^5,x)
```

```
[Out] acot(a*x)^2*(a^4/4 - 1/(4*x^4)) - (2*a^4*log(x))/3 + (a^4*log(a^2*x^2 + 1))
/3 - a^2/(12*x^2) - (a^2*acot(a*x)*((a*x^2)/2 - 1/(6*a)))/x^3
```

3.23 $\int x^5 \cot^{-1}(ax)^3 \, dx$

Optimal. Leaf size=194

$$-\frac{19x}{60a^5}+\frac{x^3}{60a^3}-\frac{4x^2\cot^{-1}(ax)}{15a^4}+\frac{x^4\cot^{-1}(ax)}{20a^2}+\frac{23i\cot^{-1}(ax)^2}{30a^6}+\frac{x\cot^{-1}(ax)^2}{2a^5}-\frac{x^3\cot^{-1}(ax)^2}{6a^3}+\frac{x^5\cot^{-1}(ax)^2}{10a}+$$

```
[Out] -19/60*x/a^5+1/60*x^3/a^3-4/15*x^2*arccot(a*x)/a^4+1/20*x^4*arccot(a*x)/a^2
+23/30*I*arccot(a*x)^2/a^6+1/2*x*arccot(a*x)^2/a^5-1/6*x^3*arccot(a*x)^2/a^
3+1/10*x^5*arccot(a*x)^2/a+1/6*arccot(a*x)^3/a^6+1/6*x^6*arccot(a*x)^3+19/6
0*arctan(a*x)/a^6-23/15*arccot(a*x)*ln(2/(1+I*a*x))/a^6+23/30*I*polylog(2,1
-2/(1+I*a*x))/a^6
```

Rubi [A]
time = 0.45, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {4947, 5037, 308, 209, 327, 5041, 4965, 2449, 2352, 4931, 5005}

$$\frac{19\text{ArcTan}(ax)}{60a^6}+\frac{23i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{30a^6}+\frac{\cot^{-1}(ax)^3}{6a^6}+\frac{23i\cot^{-1}(ax)^2}{30a^6}-\frac{23\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{15a^6}-\frac{19x}{60a^5}+\frac{x\cot^{-1}(ax)^2}{2a^5}-\frac{4x^2\cot^{-1}(ax)}{15a^4}+\frac{x^3}{60a^3}-\frac{x^3\cot^{-1}(ax)^2}{6a^3}+\frac{x^4\cot^{-1}(ax)}{20a^2}+\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{x^5\cot^{-1}(ax)^2}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCot[a*x]^3,x]
```

```
[Out] (-19*x)/(60*a^5) + x^3/(60*a^3) - (4*x^2*ArcCot[a*x])/(15*a^4) + (x^4*ArcCo
t[a*x])/(20*a^2) + (((23*I)/30)*ArcCot[a*x]^2)/a^6 + (x*ArcCot[a*x]^2)/(2*a
^5) - (x^3*ArcCot[a*x]^2)/(6*a^3) + (x^5*ArcCot[a*x]^2)/(10*a) + ArcCot[a*x
]^3/(6*a^6) + (x^6*ArcCot[a*x]^3)/6 + (19*ArcTan[a*x])/(60*a^6) - (23*ArcCo
t[a*x]*Log[2/(1 + I*a*x)])/(15*a^6) + (((23*I)/30)*PolyLog[2, 1 - 2/(1 + I*
a*x)])/a^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cot^{-1}(ax)^3\,dx &= \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{1}{2}a\int \frac{x^6\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{\int x^4\cot^{-1}(ax)^2\,dx}{2a} - \frac{\int \frac{x^4\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a} \\
&= \frac{x^5\cot^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{1}{5}\int \frac{x^5\cot^{-1}(ax)}{1+a^2x^2}\,dx - \frac{\int x^2\cot^{-1}(ax)^2\,dx}{2a^3} + \\
&= -\frac{x^3\cot^{-1}(ax)^2}{6a^3} + \frac{x^5\cot^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\cot^{-1}(ax)^3 + \frac{\int \cot^{-1}(ax)^2\,dx}{2a^5} - \\
&= \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \frac{x^3\cot^{-1}(ax)^2}{6a^3} + \frac{x^5\cot^{-1}(ax)^2}{10a} + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \\
&= -\frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \frac{x^3\cot^{-1}(ax)^2}{6a^3} + \\
&= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \\
&= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} - \\
&= -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2\cot^{-1}(ax)}{15a^4} + \frac{x^4\cot^{-1}(ax)}{20a^2} + \frac{23i\cot^{-1}(ax)^2}{30a^6} + \frac{x\cot^{-1}(ax)^2}{2a^5} -
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 125, normalized size = 0.64

$$
\frac{ax(-19+a^2x^2)+2(23i+15ax-5a^3x^3+3a^5x^5)\cot^{-1}(ax)^2+10(1+a^6x^6)\cot^{-1}(ax)^3+\cot^{-1}(ax)\left(-19-16a^2x^2+3a^4x^4-92\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+46i\,\mathrm{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{60a^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCot[a*x]^3,x]
```

```
[Out] (a*x*(-19 + a^2*x^2) + 2*(23*I + 15*a*x - 5*a^3*x^3 + 3*a^5*x^5)*ArcCot[a*x
]^2 + 10*(1 + a^6*x^6)*ArcCot[a*x]^3 + ArcCot[a*x]*(-19 - 16*a^2*x^2 + 3*a^
4*x^4 - 92*Log[1 - E^((2*I)*ArcCot[a*x])]) + (46*I)*PolyLog[2, E^((2*I)*Arc
Cot[a*x])]))/(60*a^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.21, size = 2455, normalized size = 12.65

method	result	size
risch	Expression too large to display	1260
derivativedivides	Expression too large to display	2455
default	Expression too large to display	2455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(-1/6*a^3*x^3*arccot(a*x)^2+1/10*a^5*x^5*arccot(a*x)^2+1/2*a*x*arccot
(a*x)^2+1/6*arccot(a*x)^3-1/80*arccot(a*x)*(-4*I*(a^2*x^2+1)^(1/2)*a^2*x^2-
2*(a^2*x^2+1)^(1/2)*a^3*x^3+4*I*a^3*x^3+2*a^4*x^4+2*(a^2*x^2+1)^(1/2)*a*x+2
*I*a*x-a^2*x^2-1)+47/240*(I*(a^2*x^2+1)^(1/2)+(a^2*x^2+1)^(1/2)*a*x+I*a*x+a
^2*x^2)*arccot(a*x)-41/240*arccot(a*x)*(-I*(a^2*x^2+1)^(1/2)+I*a*x-1)+3/40*
(2*I*(a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3+I*(a^2*x^2+1)^(1/2)+2*I*a*x+(a^2
*x^2+1)^(1/2)*a*x+a^2*x^2+1)*arccot(a*x)-47/120*(a^2*x^2+1+(a^2*x^2+1)^(1/2
)*a*x)*arccot(a*x)-23/15*arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-41/240
*arccot(a*x)*(I*(a^2*x^2+1)^(1/2)+I*a*x-1)+1/40*(-2*I*(a^2*x^2+1)^(1/2)*a^2
*x^2+2*I*a^3*x^3+I*(a^2*x^2+1)^(1/2)+3*(a^2*x^2+1)^(1/2)*a*x-3*a^2*x^2-1)*a
rccot(a*x)+1/20*arccot(a*x)*(-2*I*(a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3-2*(
a^2*x^2+1)^(1/2)*a^3*x^3+2*a^4*x^4-I*(a^2*x^2+1)^(1/2)+2*I*a*x-(a^2*x^2+1)^
(1/2)*a*x+2*a^2*x^2)+1/4*Pi*arccot(a*x)^2-47/120*(a^2*x^2+1-(a^2*x^2+1)^(1/
2)*a*x)*arccot(a*x)+3/40*(-2*I*(a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3-I*(a^2
*x^2+1)^(1/2)+2*I*a*x-(a^2*x^2+1)^(1/2)*a*x+a^2*x^2+1)*arccot(a*x)+1/20*arc
cot(a*x)*(-2*I*(a^2*x^2+1)^(1/2)*a^2*x^2-2*I*a^3*x^3+2*(a^2*x^2+1)^(1/2)*a^
3*x^3+2*a^4*x^4-I*(a^2*x^2+1)^(1/2)-2*I*a*x+(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2
)+1/20*arccot(a*x)*(2*I*(a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3+2*(a^2*x^2+1)
^(1/2)*a^3*x^3+2*a^4*x^4+I*(a^2*x^2+1)^(1/2)+2*I*a*x+(a^2*x^2+1)^(1/2)*a*x+
2*a^2*x^2)-23/15*I*dilog((I+a*x)/(a^2*x^2+1)^(1/2))+23/15*I*dilog(1+(I+a*x)
/(a^2*x^2+1)^(1/2))+23/30*I*arccot(a*x)^2-3/40*(-2*I*(a^2*x^2+1)^(1/2)*a^2*
x^2+2*I*a^3*x^3-I*(a^2*x^2+1)^(1/2)+2*I*a*x+(a^2*x^2+1)^(1/2)*a*x-a^2*x^2-1
)*arccot(a*x)+47/240*(-I*(a^2*x^2+1)^(1/2)+I*a*x-(a^2*x^2+1)^(1/2)*a*x+a^2*
x^2)*arccot(a*x)-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/
(a^2*x^2+1))^2*arccot(a*x)^2-1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I
+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2+1/8*Pi*csgn(
I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2-
1/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a
*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-3/40*(a^2*x^2+1)*arccot(a*x)*(-2*(a^2
```

```
*x^2+1)^(1/2)*a*x+2*a^2*x^2+1)-3/40*(a^2*x^2+1)*arccot(a*x)*(2*(a^2*x^2+1)^
(1/2)*a*x+2*a^2*x^2+1)+1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*
x^2+1)-1))^3*arccot(a*x)^2+1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*arcco
t(a*x)^2-1/80*arccot(a*x)*(4*I*(a^2*x^2+1)^(1/2)*a^2*x^2+4*I*a^3*x^3+2*(a^2
*x^2+1)^(1/2)*a^3*x^3+2*a^4*x^4+2*I*a*x-2*(a^2*x^2+1)^(1/2)*a*x-a^2*x^2-1)+
1/40*(2*I*(a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3-I*(a^2*x^2+1)^(1/2)-3*(a^2*
x^2+1)^(1/2)*a*x-3*a^2*x^2-1)*arccot(a*x)-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2
+1)-1))^2*arccot(a*x)^2+1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^
2+1/15*I*(a^2*x^2+1)^(3/2)/(24*I*(a^2*x^2+1)^(1/2)*a*x+24*I*a^2*x^2+16*(a^2
*x^2+1)^(1/2)*a^2*x^2+16*a^3*x^3+8*I-8*(a^2*x^2+1)^(1/2))-1/5*I*(a^2*x^2+1)
/(-16*I*(a^2*x^2+1)^(1/2)+16*I*a*x-16*(a^2*x^2+1)^(1/2)*a*x+16*a^2*x^2)-41/
15*I*(a^2*x^2+1)^(1/2)/(8*a*x+8*I-8*(a^2*x^2+1)^(1/2))-1/15*I*(a^2*x^2+1)^(
3/2)/(-24*I*(a^2*x^2+1)^(1/2)*a*x+24*I*a^2*x^2-16*(a^2*x^2+1)^(1/2)*a^2*x^2
+16*a^3*x^3+8*I+8*(a^2*x^2+1)^(1/2))+41/15*I*(a^2*x^2+1)^(1/2)/(8*(a^2*x^2+
1)^(1/2)+8*a*x+8*I)+1/2*I*arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))+1/6*a
^6*x^6*arccot(a*x)^3-1/5*I*(a^2*x^2+1)/(16*I*(a^2*x^2+1)^(1/2)+16*I*a*x+16*
(a^2*x^2+1)^(1/2)*a*x+16*a^2*x^2)+1/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*
csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*
x^2+1)-1))*arccot(a*x)^2+1/20*arccot(a*x)*(2*I*(a^2*x^2+1)^(1/2)*a^2*x^2-2*
I*a^3*x^3-2*(a^2*x^2+1)^(1/2)*a^3*x^3+2*a^4*x^4+I*(a^2*x^2+1)^(1/2)-2*I*a*x
-(a^2*x^2+1)^(1/2)*a*x+2*a^2*x^2)-1/2*arccot(a*x)^2*arctan(a*x)-3/40*(2*I*(
a^2*x^2+1)^(1/2)*a^2*x^2+2*I*a^3*x^3+I*(a^2*x^2+1)^(1/2)+2*I*a*x-(a^2*x^2+1
)^(1/2)*a*x-a^2*x^2-1)*arccot(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/480*(40*a^6*x^6*arctan2(1, a*x)^3 + 12*a^5*x^5*arctan2(1, a*x)^2 - 20*a^3
*x^3*arctan2(1, a*x)^2 + 20*(5760*a^7*integrate(1/480*x^7*arctan(1/(a*x))^3
/(a^7*x^2 + a^5), x) + 1440*a^6*integrate(1/480*x^6*arctan(1/(a*x))^2/(a^7*
x^2 + a^5), x) + 360*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^7*x^2 +
a^5), x) + 288*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x)
 + 5760*a^5*integrate(1/480*x^5*arctan(1/(a*x))^3/(a^7*x^2 + a^5), x) + 576
*a^5*integrate(1/480*x^5*arctan(1/(a*x))/(a^7*x^2 + a^5), x) - 480*a^4*inte
grate(1/480*x^4*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) - 960*a^3*integrate(1/
480*x^3*arctan(1/(a*x))/(a^7*x^2 + a^5), x) + 1440*a^2*integrate(1/480*x^2*
log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) + 2880*a*integrate(1/480*x*arctan(1/(a
*x))/(a^7*x^2 + a^5), x) + arctan(a*x)^3/a^6 + 3*arctan(a*x)^2*arctan(1/(a*
x))/a^6 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^6 + 360*integrate(1/480*log(a^2
*x^2 + 1)^2/(a^7*x^2 + a^5), x))*a^6 + 60*a*x*arctan2(1, a*x)^2 + 40*arctan
2(1, a*x)^3 - (3*a^5*x^5 - 5*a^3*x^3 + 15*a*x)*log(a^2*x^2 + 1)^2)/a^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^5*arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \operatorname{acot}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acot(a*x)**3,x)
```

```
[Out] Integral(x**5*acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \operatorname{acot}(a\,x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acot(a*x)^3,x)
```

```
[Out] int(x^5*acot(a*x)^3, x)
```

3.24 $\int x^4 \cot^{-1}(ax)^3 \, dx$

Optimal. Leaf size=205

$$\frac{x^2}{20a^3}-\frac{9x\cot^{-1}(ax)}{10a^4}+\frac{x^3\cot^{-1}(ax)}{10a^2}-\frac{9\cot^{-1}(ax)^2}{20a^5}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}+\frac{3x^4\cot^{-1}(ax)^2}{20a}+\frac{i\cot^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\cot^{-}$$

```
[Out] 1/20*x^2/a^3-9/10*x*arccot(a*x)/a^4+1/10*x^3*arccot(a*x)/a^2-9/20*arccot(a*
x)^2/a^5-3/10*x^2*arccot(a*x)^2/a^3+3/20*x^4*arccot(a*x)^2/a+1/5*I*arccot(a
*x)^3/a^5+1/5*x^5*arccot(a*x)^3-3/5*arccot(a*x)^2*ln(2/(1+I*a*x))/a^5-1/2*l
n(a^2*x^2+1)/a^5+3/5*I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^5-3/10*polylo
g(3,1-2/(1+I*a*x))/a^5
```

Rubi [A]
time = 0.35, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {4947, 5037, 272, 45, 4931, 266, 5005, 5041, 4965, 5115, 6745}

$$-\frac{3\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{10a^5}+\frac{3i\text{Li}_2\left(1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{5a^5}+\frac{i\cot^{-1}(ax)^3}{5a^5}-\frac{9\cot^{-1}(ax)^2}{20a^5}-\frac{3\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{5a^5}-\frac{9x\cot^{-1}(ax)}{10a^4}+\frac{x^2}{20a^3}-\frac{3x^2\cot^{-1}(ax)^2}{10a^3}+\frac{x^3\cot^{-1}(ax)}{10a^2}-\frac{\log\left(a^2x^2+1\right)}{2a^5}+\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3x^4\cot^{-1}(ax)^2}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCot[a*x]^3,x]
```

```
[Out] x^2/(20*a^3) - (9*x*ArcCot[a*x])/(10*a^4) + (x^3*ArcCot[a*x])/(10*a^2) - (9
*ArcCot[a*x]^2)/(20*a^5) - (3*x^2*ArcCot[a*x]^2)/(10*a^3) + (3*x^4*ArcCot[a
*x]^2)/(20*a) + ((I/5)*ArcCot[a*x]^3)/a^5 + (x^5*ArcCot[a*x]^3)/5 - (3*ArcC
ot[a*x]^2*Log[2/(1 + I*a*x)])/(5*a^5) - Log[1 + a^2*x^2]/(2*a^5) + (((3*I)/
5)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^5 - (3*PolyLog[3, 1 - 2/(1
+ I*a*x)])/(10*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \cot^{-1}(ax)^3 \, dx &= \frac{1}{5}x^5 \cot^{-1}(ax)^3 + \frac{1}{5}(3a) \int \frac{x^5 \cot^{-1}(ax)^2}{1+a^2x^2} \, dx \\
&= \frac{1}{5}x^5 \cot^{-1}(ax)^3 + \frac{3 \int x^3 \cot^{-1}(ax)^2 \, dx}{5a} - \frac{3 \int \frac{x^3 \cot^{-1}(ax)^2}{1+a^2x^2} \, dx}{5a} \\
&= \frac{3x^4 \cot^{-1}(ax)^2}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax)^3 + \frac{3}{10} \int \frac{x^4 \cot^{-1}(ax)}{1+a^2x^2} \, dx - \frac{3 \int x \cot^{-1}(ax)^2 \, dx}{5a^3} + \frac{3 \int
\\
&= -\frac{3x^2 \cot^{-1}(ax)^2}{10a^3} + \frac{3x^4 \cot^{-1}(ax)^2}{20a} + \frac{i \cot^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^3 - \frac{3 \int \frac{\cot^{-1}(ax)^2}{i-ax} \, dx
\\
&= \frac{x^3 \cot^{-1}(ax)}{10a^2} - \frac{3x^2 \cot^{-1}(ax)^2}{10a^3} + \frac{3x^4 \cot^{-1}(ax)^2}{20a} + \frac{i \cot^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \cot^{-1}(ax)^3 -
\\
&= -\frac{9x \cot^{-1}(ax)}{10a^4} + \frac{x^3 \cot^{-1}(ax)}{10a^2} - \frac{9 \cot^{-1}(ax)^2}{20a^5} - \frac{3x^2 \cot^{-1}(ax)^2}{10a^3} + \frac{3x^4 \cot^{-1}(ax)^2}{20a} +
\\
&= -\frac{9x \cot^{-1}(ax)}{10a^4} + \frac{x^3 \cot^{-1}(ax)}{10a^2} - \frac{9 \cot^{-1}(ax)^2}{20a^5} - \frac{3x^2 \cot^{-1}(ax)^2}{10a^3} + \frac{3x^4 \cot^{-1}(ax)^2}{20a} +
\\
&= \frac{x^2}{20a^3} - \frac{9x \cot^{-1}(ax)}{10a^4} + \frac{x^3 \cot^{-1}(ax)}{10a^2} - \frac{9 \cot^{-1}(ax)^2}{20a^5} - \frac{3x^2 \cot^{-1}(ax)^2}{10a^3} + \frac{3x^4 \cot^{-1}(ax
\end{aligned}
$$

Mathematica [A]
time = 0.44, size = 184, normalized size = 0.90

$$
\frac{2 + i\pi^3 + 2a^2x^2 - 36ax\cot^{-1}(ax) + 4a^3x^3\cot^{-1}(ax) - 18\cot^{-1}(ax)^2 - 12a^2x^2\cot^{-1}(ax)^2 + 6a^4x^4\cot^{-1}(ax)^2 - 8i\cot^{-1}(ax)^3 + 8a^5x^5\cot^{-1}(ax)^3 - 24\cot^{-1}(ax)^2\log\left(1 - e^{-2i\cot^{-1}(ax)}\right) + 40\log\left(\frac{1}{a\sqrt{1+\frac{1}{a^2x^2}}\,x}\right) - 24i\cot^{-1}(ax)\operatorname{PolyLog}\left(2, e^{-2i\cot^{-1}(ax)}\right) - 12\operatorname{PolyLog}\left(3, e^{-2i\cot^{-1}(ax)}\right)}{40a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCot[a*x]^3,x]
```

```
[Out] (2 + I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcCot[a*x] + 4*a^3*x^3*ArcCot[a*x] - 18*A
rcCot[a*x]^2 - 12*a^2*x^2*ArcCot[a*x]^2 + 6*a^4*x^4*ArcCot[a*x]^2 - (8*I)*A
rcCot[a*x]^3 + 8*a^5*x^5*ArcCot[a*x]^3 - 24*ArcCot[a*x]^2*Log[1 - E^((-2*I)
*ArcCot[a*x])] + 40*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] - (24*I)*ArcCot[a*x]
*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 12*PolyLog[3, E^((-2*I)*ArcCot[a*x])]
)/(40*a^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.09, size = 2627, normalized size = 12.81

method	result	size
derivativedivides	Expression too large to display	2627
default	Expression too large to display	2627

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccot(a*x)^3+3/20*a^4*x^4*arccot(a*x)^2-3/10*a^2*x^2*ar
ccot(a*x)^2+3/10*arccot(a*x)^2*ln(a^2*x^2+1)-3/5*arccot(a*x)^2*ln((I+a*x)/(
a^2*x^2+1)^(1/2))+3/5*arccot(a*x)^2*ln((I+a*x)^2/(a^2*x^2+1)-1)-3/5*arccot(
a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+6/5*I*arccot(a*x)*polylog(2,-(I+a*x)
/(a^2*x^2+1)^(1/2))-6/5*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/5*arccot(a*
x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+6/5*I*arccot(a*x)*polylog(2,(I+a*x)/(a
^2*x^2+1)^(1/2))-6/5*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))+1/160*I*(18*Pi*ar
ccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2
+2*I*(I+a*x)^2/(a^2*x^2+1)-I)^2*a^2*x^2+32*arccot(a*x)^3-160*arccot(a*x)-8*
I*a^2*x^2-8*I+72*I*arccot(a*x)^2-48*Pi*arccot(a*x)^2-48*Pi*csgn(I*(I+a*x)/(
a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2+24*Pi*csgn(
I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+
144*I*arccot(a*x)*a*x-16*I*arccot(a*x)*a^3*x^3+24*Pi*csgn(I*(I+a*x)^2/(a^2*
x^2+1))^3*arccot(a*x)^2+96*I*arccot(a*x)^2*ln(2)-24*Pi*arccot(a*x)^2*csgn(I
*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3-3*Pi*arccot(a*x)^2*cs
gn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3+21*Pi*arccot(a*x)^2*csgn(-I*(I+a*x)^4/(
a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)^3+48*Pi*arccot(a*x)^2*csgn(I*(I+a
*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+24*Pi*arccot(a*x)^2*csgn(I
*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x
^2+1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)-24*Pi*arccot(a*x)^2*csgn(I*(I+a*
x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1
))-24*Pi*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-
1)^2)^2*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)-3*Pi*arccot(a*x)^2*csgn(I*((I+a
*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)+6*Pi*arccot(a*x
)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2
+21*Pi*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a
^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)+42*Pi*arccot(a*x)^2*csgn(I*(I+a*x)
^2/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)
```

```
-I)^2+9*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*a^2*x^2+9*Pi
*arccot(a*x)^2*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)
^3*a^2*x^2-9*I*Pi*arccot(a*x)^2*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)
^2/(a^2*x^2+1)-I)^3*a*x-9*I*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-
1)^2)^3*a*x+3*I*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*a^3*
x^3+3*I*Pi*arccot(a*x)^2*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2
*x^2+1)-I)^3*a^3*x^3+9*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2
*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*a^2*x^2-18*Pi*arccot(a*x)^2*csgn(I*((I
+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*a^2*x^2+9*Pi*
arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a^2*x^2+
1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*a^2*x^2-9*I*Pi*arccot(a*x)^2*csgn(I*((I+a
*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*a*x+18*I*Pi*arc
cot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-
1)^2)^2*a*x+3*I*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I
*((I+a*x)^2/(a^2*x^2+1)-1)^2)*a^3*x^3-6*I*Pi*arccot(a*x)^2*csgn(I*((I+a*x)^
2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*a^3*x^3+3*I*Pi*arcc
ot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2
+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*a^3*x^3+6*I*Pi*arccot(a*x)^2*csgn(I*(I+a*x)^2
/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I
)^2*a^3*x^3-9*I*Pi*arccot(a*x)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*
(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*a*x-18*I*Pi*arccot(a*x
)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+
a*x)^2/(a^2*x^2+1)-I)^2*a*x)+ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)+ln(1+(I+a*x)/(
a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/40*x^5*arctan2(1, a*x)^3 - 3/160*x^5*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
 integrate(1/160*(140*a^2*x^6*arctan2(1, a*x)^3 + 12*a^2*x^6*arctan2(1, a*x
)*log(a^2*x^2 + 1) + 12*a*x^5*arctan2(1, a*x)^2 + 140*x^4*arctan2(1, a*x)^3
 + 3*(5*a^2*x^6*arctan2(1, a*x) - a*x^5 + 5*x^4*arctan2(1, a*x))*log(a^2*x^
2 + 1)^2)/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4*arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \operatorname{acot}^{3}{\left(ax \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acot(a*x)**3,x)
[Out] Integral(x**4*acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x)^3,x, algorithm="giac")
[Out] integrate(x^4*arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\mathrm{acot}\left(a\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acot(a*x)^3,x)
[Out] int(x^4*acot(a*x)^3, x)
```

3.25 $\int x^3 \cot^{-1}(ax)^3 \, dx$

Optimal. Leaf size=148

$$\frac{x}{4a^3}+\frac{x^2\cot^{-1}(ax)}{4a^2}-\frac{i\cot^{-1}(ax)^2}{a^4}-\frac{3x\cot^{-1}(ax)^2}{4a^3}+\frac{x^3\cot^{-1}(ax)^2}{4a}-\frac{\cot^{-1}(ax)^3}{4a^4}+\frac{1}{4}x^4\cot^{-1}(ax)^3-\frac{\operatorname{ArcTan}(ax)}{4a^4}$$

```
[Out] 1/4*x/a^3+1/4*x^2*arccot(a*x)/a^2-I*arccot(a*x)^2/a^4-3/4*x*arccot(a*x)^2/a
^3+1/4*x^3*arccot(a*x)^2/a-1/4*arccot(a*x)^3/a^4+1/4*x^4*arccot(a*x)^3-1/4*
arctan(a*x)/a^4+2*arccot(a*x)*ln(2/(1+I*a*x))/a^4-I*polylog(2,1-2/(1+I*a*x)
)/a^4
```

Rubi [A]
time = 0.27, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4947, 5037, 327, 209, 5041, 4965, 2449, 2352, 4931, 5005}

$$-\frac{\operatorname{ArcTan}(ax)}{4a^4}-\frac{i\operatorname{Li}_2\left(1-\frac{2}{iax+1}\right)}{a^4}-\frac{\cot^{-1}(ax)^3}{4a^4}-\frac{i\cot^{-1}(ax)^2}{a^4}+\frac{2\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a^4}+\frac{x}{4a^3}-\frac{3x\cot^{-1}(ax)^2}{4a^3}+\frac{x^2\cot^{-1}(ax)}{4a^2}+\frac{1}{4}x^4\cot^{-1}(ax)^3+\frac{x^3\cot^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a*x]^3,x]
```

```
[Out] x/(4*a^3) + (x^2*ArcCot[a*x])/(4*a^2) - (I*ArcCot[a*x]^2)/a^4 - (3*x*ArcCot
[a*x]^2)/(4*a^3) + (x^3*ArcCot[a*x]^2)/(4*a) - ArcCot[a*x]^3/(4*a^4) + (x^4
*ArcCot[a*x]^3)/4 - ArcTan[a*x]/(4*a^4) + (2*ArcCot[a*x]*Log[2/(1 + I*a*x)]
)/a^4 - (I*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot^{-1}(ax)^3\,dx &= \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{1}{4}(3a)\int \frac{x^4\cot^{-1}(ax)^2}{1+a^2x^2}\,dx\\
&= \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{3\int x^2\cot^{-1}(ax)^2\,dx}{4a} - \frac{3\int \frac{x^2\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{4a}\\
&= \frac{x^3\cot^{-1}(ax)^2}{4a} + \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{1}{2}\int \frac{x^3\cot^{-1}(ax)}{1+a^2x^2}\,dx - \frac{3\int \cot^{-1}(ax)^2\,dx}{4a^3} + \frac{3\int \frac{\cot^{-1}}{1+}}{4}\\
&= -\frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\cot^{-1}(ax)^3 + \frac{\int x\cot^{-1}(ax)\,dx}{2a^2}\\
&= \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\,\mathrm{co}\\
&= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} +\\
&= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} +\\
&= \frac{x}{4a^3} + \frac{x^2\cot^{-1}(ax)}{4a^2} - \frac{i\cot^{-1}(ax)^2}{a^4} - \frac{3x\cot^{-1}(ax)^2}{4a^3} + \frac{x^3\cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} +
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 96, normalized size = 0.65

$$\frac{ax + (-4i - 3ax + a^3x^3)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 + \cot^{-1}(ax)\left(1 + a^2x^2 + 8\log\left(1 - e^{2i\cot^{-1}(ax)}\right)\right) - 4i\,\mathrm{PolyLog}\left(2, e^{2i\cot^{-1}(ax)}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a*x]^3,x]
```

```
[Out] (a*x + (-4*I - 3*a*x + a^3*x^3)*ArcCot[a*x]^2 + (-1 + a^4*x^4)*ArcCot[a*x]^
3 + ArcCot[a*x]*(1 + a^2*x^2 + 8*Log[1 - E^((2*I)*ArcCot[a*x])]) - (4*I)*Po
lyLog[2, E^((2*I)*ArcCot[a*x])])/(4*a^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.13, size = 1017, normalized size = 6.87

method	result

risch	$\frac{x}{4a^3} - \frac{511 \arctan(ax)}{768a^4} + \frac{3ix^4 \ln(-iax+1)}{256} + \left(-\frac{3i(a^4x^4-1)\ln(-iax+1)^2}{32a^4} + \frac{x(3\pi a^3x^3+2a^2x^2-6)\ln(-iax+1)}{16a^3}\right.$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccot(a*x)^3+1/4*a^3*x^3*arccot(a*x)^2-3/4*a*x*arccot(a
*x)^2-1/2*I*(a^2*x^2+1)^(1/2)/(2*(a^2*x^2+1)^(1/2)+2*a*x+2*I)-3/4*I*arccot(
a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/4*arccot(a*x)^3-1/8*(I*(a^2*x^2+1)^(
1/2)+(a^2*x^2+1)^(1/2)*a*x+I*a*x+a^2*x^2)*arccot(a*x)+1/8*arccot(a*x)*(-I*(
a^2*x^2+1)^(1/2)+I*a*x-1)+1/4*(a^2*x^2+1+(a^2*x^2+1)^(1/2)*a*x)*arccot(a*x)
+2*arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+1/8*arccot(a*x)*(I*(a^2*x^2+
1)^(1/2)+I*a*x-1)-3/8*Pi*arccot(a*x)^2+1/4*(a^2*x^2+1-(a^2*x^2+1)^(1/2)*a*x
)*arccot(a*x)-1/8*(-I*(a^2*x^2+1)^(1/2)+I*a*x-(a^2*x^2+1)^(1/2)*a*x+a^2*x^2
)*arccot(a*x)+3/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^
2*x^2+1))^2*arccot(a*x)^2+3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a
*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-3/16*Pi*csgn(I
*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+3
/16*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a
*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2+1/2*I*(a^2*x^2+1)^(1/2)/(2*a*x+2*I-2*
(a^2*x^2+1)^(1/2))+2*I*dilog((I+a*x)/(a^2*x^2+1)^(1/2))-I*arccot(a*x)^2-2*I
*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((
I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)^2-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+
1)-1))^3*arccot(a*x)^2+3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*
x)^2-3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2-3/16*Pi*csgn(I/(
(I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a
^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*arccot(a*x)^2+3/4*arccot(a*x)^2*arctan
(a*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(8*a^4*x^4*arctan2(1, a*x)^3 + 4*a^3*x^3*arctan2(1, a*x)^2 + 4*(512*a^
5*integrate(1/64*x^5*arctan(1/(a*x))^3/(a^5*x^2 + a^3), x) + 192*a^4*integr
ate(1/64*x^4*arctan(1/(a*x))^2/(a^5*x^2 + a^3), x) + 48*a^4*integrate(1/64*
x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + 64*a^4*integrate(1/64*x^4*log(
a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 512*a^3*integrate(1/64*x^3*arctan(1/(a*x
```

```
))^3/(a^5*x^2 + a^3), x) + 128*a^3*integrate(1/64*x^3*arctan(1/(a*x))/(a^5*
x^2 + a^3), x) - 192*a^2*integrate(1/64*x^2*log(a^2*x^2 + 1)/(a^5*x^2 + a^3
), x) - 384*a*integrate(1/64*x*arctan(1/(a*x))/(a^5*x^2 + a^3), x) - arctan
(a*x)^3/a^4 - 3*arctan(a*x)^2*arctan(1/(a*x))/a^4 - 3*arctan(a*x)*arctan(1/
(a*x))^2/a^4 - 48*integrate(1/64*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x))*a^
4 - 12*a*x*arctan2(1, a*x)^2 - 8*arctan2(1, a*x)^3 - (a^3*x^3 - 3*a*x)*log(
a^2*x^2 + 1)^2)/a^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="fricas")
[Out] integral(x^3*arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \operatorname{acot}^{3}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(a*x)**3,x)
[Out] Integral(x**3*acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x)^3,x, algorithm="giac")
[Out] integrate(x^3*arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{acot}(a\,x)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a*x)^3,x)
[Out] int(x^3*acot(a*x)^3, x)
```

3.26 $\int x^2 \cot^{-1}(ax)^3 \, dx$

Optimal. Leaf size=157

$$\frac{x\cot^{-1}(ax)}{a^2}+\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{x^2\cot^{-1}(ax)^2}{2a}-\frac{i\cot^{-1}(ax)^3}{3a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^3+\frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3}+\frac{\log(1+}{2a^3}$$

```
[Out] x*arccot(a*x)/a^2+1/2*arccot(a*x)^2/a^3+1/2*x^2*arccot(a*x)^2/a-1/3*I*arcco
t(a*x)^3/a^3+1/3*x^3*arccot(a*x)^3+arccot(a*x)^2*ln(2/(1+I*a*x))/a^3+1/2*ln
(a^2*x^2+1)/a^3-I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/2*polylog(3,1-
2/(1+I*a*x))/a^3
```

Rubi [A]
time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {4947, 5037, 4931, 266, 5005, 5041, 4965, 5115, 6745}

$$\frac{\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a^3}-\frac{i\text{Li}_2\left(1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{a^3}-\frac{i\cot^{-1}(ax)^3}{3a^3}+\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a^3}+\frac{x\cot^{-1}(ax)}{a^2}+\frac{\log\left(a^2x^2+1\right)}{2a^3}+\frac{1}{3}x^3\cot^{-1}(ax)^3+\frac{x^2\cot^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a*x]^3,x]
```

```
[Out] (x*ArcCot[a*x])/a^2 + ArcCot[a*x]^2/(2*a^3) + (x^2*ArcCot[a*x]^2)/(2*a) - (
(I/3)*ArcCot[a*x]^3)/a^3 + (x^3*ArcCot[a*x]^3)/3 + (ArcCot[a*x]^2*Log[2/(1
+ I*a*x)])/a^3 + Log[1 + a^2*x^2]/(2*a^3) - (I*ArcCot[a*x]*PolyLog[2, 1 - 2
/(1 + I*a*x)])/a^3 + PolyLog[3, 1 - 2/(1 + I*a*x)]/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cot^{-1}(ax)^3\,dx &= \frac{1}{3}x^3\cot^{-1}(ax)^3 + a\int \frac{x^3\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\int x\cot^{-1}(ax)^2\,dx}{a} - \frac{\int \frac{x\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{a} \\
&= \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\int \frac{\cot^{-1}(ax)^2}{i-ax}\,dx}{a^2} + \int \frac{x^2\cot^{-1}(ax)}{1+a^2x^2}\,d \\
&= \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3} + \frac{\int \cot^{-1}(a}{a^2} \\
&= \frac{x\cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}}{} \\
&= \frac{x\cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2\cot^{-1}(ax)^2}{2a} - \frac{i\cot^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3\cot^{-1}(ax)^3 + \frac{\cot^{-1}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 149, normalized size = 0.95

$$
\frac{-i\pi^3 + 24ax\cot^{-1}(ax) + 12\cot^{-1}(ax)^2 + 12a^2x^2\cot^{-1}(ax)^2 + 8i\cot^{-1}(ax)^3 + 8a^3x^3\cot^{-1}(ax)^3 + 24\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right) - 24\log\left(\frac{1}{a\sqrt{1+\frac{1}{a^2x^2}}\,x}\right) + 24i\cot^{-1}(ax)\mathrm{PolyLog}\left(2, e^{-2i\cot^{-1}(ax)}\right) + 12\mathrm{PolyLog}\left(3, e^{-2i\cot^{-1}(ax)}\right)}{24a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a*x]^3,x]
```

```
[Out] ((-I)*Pi^3 + 24*a*x*ArcCot[a*x] + 12*ArcCot[a*x]^2 + 12*a^2*x^2*ArcCot[a*x]
^2 + (8*I)*ArcCot[a*x]^3 + 8*a^3*x^3*ArcCot[a*x]^3 + 24*ArcCot[a*x]^2*Log[1
 - E^((-2*I)*ArcCot[a*x])] - 24*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] + (24*I)
*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] + 12*PolyLog[3, E^((-2*I)*A
rcCot[a*x])])/(24*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.96, size = 1665, normalized size = 10.61

method	result	size
derivativedivides	Expression too large to display	1665
default	Expression too large to display	1665

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccot(a*x)^3+1/2*a^2*x^2*arccot(a*x)^2-1/2*arccot(a*x)^
2*ln(a^2*x^2+1)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-arccot(a*x)^2*l
```

```
n((I+a*x)^2/(a^2*x^2+1)-1)+1/24*I*arccot(a*x)*(24+3*I*csgn(I*((I+a*x)^2/(a^
2*x^2+1)-1)^2)^3*arccot(a*x)*Pi*a*x+3*I*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I
*(I+a*x)^2/(a^2*x^2+1)-I)^3*arccot(a*x)*Pi*a*x-8*arccot(a*x)^2+6*csgn(I*(I+
a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)*Pi+3*csgn(I*(
(I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)*Pi-6*csgn(I*(I+a*x)^2/(a^2*x^2+1)
)^3*arccot(a*x)*Pi-12*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1
)^2)^2*arccot(a*x)*Pi-3*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*
x^2+1)-I)^3*arccot(a*x)*Pi-24*I*arccot(a*x)*ln(2)-24*I*a*x+12*arccot(a*x)*P
i-12*I*arccot(a*x)-3*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a
^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*arccot(a*x)*Pi-6*csgn(I*(I+a*x)^2/
(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)
^2*arccot(a*x)*Pi+6*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^
2)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)*Pi+6*csgn(I*(I+a*x)^2/(a^2*x
^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*ar
ccot(a*x)*Pi+3*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x
^2+1)-1)^2)*arccot(a*x)*Pi-6*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a
*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)*Pi-6*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2)
)^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)*Pi+12*csgn(I*(I+a*x)/(a^2*x^2
+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)*Pi-6*csgn(I*(I+a*x)^
2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1))*cs
gn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(a*x)*Pi+3*I*csgn(I*((I+a*x)^2/(a^2
*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(a*x)*Pi*a*x-6*I*cs
gn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*arcco
t(a*x)*Pi*a*x+3*I*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a^2*
x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*arccot(a*x)*Pi*a*x+6*I*csgn(I*(I+a*x)
^2/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)
-I)^2*arccot(a*x)*Pi*a*x)-ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(I+a*x)/(a^2
*x^2+1)^(1/2))+arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-2*I*arccot(a*x
)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1
/2))+arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-2*I*arccot(a*x)*polylog(
2,(I+a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*x^3*arctan2(1, a*x)^3 - 1/32*x^3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
integrate(1/32*(28*a^2*x^4*arctan2(1, a*x)^3 + 4*a^2*x^4*arctan2(1, a*x)*lo
g(a^2*x^2 + 1) + 4*a*x^3*arctan2(1, a*x)^2 + 28*x^2*arctan2(1, a*x)^3 + (3*
a^2*x^4*arctan2(1, a*x) - a*x^3 + 3*x^2*arctan2(1, a*x))*log(a^2*x^2 + 1)^2
)/(a^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acot}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(a*x)**3,x)
```

```
[Out] Integral(x**2*acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a*x)^3,x)
```

```
[Out] int(x^2*acot(a*x)^3, x)
```

3.27 $\int x\cot^{-1}(ax)^3\,dx$

Optimal. Leaf size=103

$$\frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}+\frac{3i\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}$$

```
[Out] 3/2*I*arccot(a*x)^2/a^2+3/2*x*arccot(a*x)^2/a+1/2*arccot(a*x)^3/a^2+1/2*x^2
*arccot(a*x)^3-3*arccot(a*x)*ln(2/(1+I*a*x))/a^2+3/2*I*polylog(2,1-2/(1+I*a
*x))/a^2
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4947, 5037, 4931, 5041, 4965, 2449, 2352, 5005}

$$\frac{3i\text{Li}_2\left(1-\frac{2}{iax+1}\right)}{2a^2}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{3i\cot^{-1}(ax)^2}{2a^2}-\frac{3\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3x\cot^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a*x]^3,x]
```

```
[Out] (((3*I)/2)*ArcCot[a*x]^2)/a^2 + (3*x*ArcCot[a*x]^2)/(2*a) + ArcCot[a*x]^3/(
2*a^2) + (x^2*ArcCot[a*x]^3)/2 - (3*ArcCot[a*x]*Log[2/(1 + I*a*x)])/a^2 + (
((3*I)/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cot^{-1}(ax)^3\,dx = \frac{1}{2}x^2\cot^{-1}(ax)^3 + \frac{1}{2}(3a)\int \frac{x^2\cot^{-1}(ax)^2}{1+a^2x^2}\,dx$$
$$= \frac{1}{2}x^2\cot^{-1}(ax)^3 + \frac{3\int \cot^{-1}(ax)^2\,dx}{2a} - \frac{3\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx}{2a}$$
$$= \frac{3x\cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^3 + 3\int \frac{x\cot^{-1}(ax)}{1+a^2x^2}\,dx$$
$$= \frac{3i\cot^{-1}(ax)^2}{2a^2} + \frac{3x\cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^3 - \frac{3\int \frac{\cot^{-1}(ax)}{i-ax}\,dx}{a}$$
$$= \frac{3i\cot^{-1}(ax)^2}{2a^2} + \frac{3x\cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right.}{a^2}$$
$$= \frac{3i\cot^{-1}(ax)^2}{2a^2} + \frac{3x\cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right.}{a^2}$$
$$= \frac{3i\cot^{-1}(ax)^2}{2a^2} + \frac{3x\cot^{-1}(ax)^2}{2a} + \frac{\cot^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right.}{a^2}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.74

$$\frac{\cot^{-1}(ax)\left(3(i+ax)\cot^{-1}(ax)+(1+a^2x^2)\cot^{-1}(ax)^2-6\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+3i\text{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a*x]^3,x]
```

```
[Out] (ArcCot[a*x]*(3*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 - 6*Log
[1 - E^((2*I)*ArcCot[a*x])]) + (3*I)*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(2*
a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 4393, normalized size = 42.65

method	result
risch	$-\frac{3i\pi\ln(-iax+1)x}{8a}+\frac{3i\pi\arctan(ax)}{16a^2}+\frac{21\arctan(ax)}{32a^2}+\frac{3\pi^2x}{8a}-\frac{i(a^2x^2+1)\ln(iax+1)^3}{16a^2}-\frac{3i\pi^2\ln(-iax+1)x^2}{16}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccot(a*x)^3+3/2*a*x*arccot(a*x)^2+3/8*I*Pi*csgn(I*(I+a
*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)*ln(1-(I+
a*x)/(a^2*x^2+1)^(1/2))-3/4*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot
(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1))^3*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I*(I+a*x)
^2/(a^2*x^2+1))^3*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*I*Pi*csgn
(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)*ln(1-(I+a
*x)/(a^2*x^2+1)^(1/2))+3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*polylog(2
,(I+a*x)/(a^2*x^2+1)^(1/2))+3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*poly
log(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^
3*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1
))^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2
+1)-1))^2*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))+3/4*Pi*csgn(I/((I+a*x)^2/(a
^2*x^2+1)-1))^2*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I/((I+a*x)^2
/(a^2*x^2+1)-1))^2*dilog((I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x)^2
/(a^2*x^2+1))^3*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x)^
2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*polylog(2,-(I+a*x)/(a^2*x^2+1)^(
1/2))+3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*poly
log(2,(I+a*x)/(a^2*x^2+1)^(1/2))-3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*
x)^2/(a^2*x^2+1)-1))^3*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+
a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*dilog((I+a*x)/(a^2*x^2+1)^(
1/2))+3/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^3*dilog((I+a*x)/(a^2*x^2+1)^
(1/2))+3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*polylog(2,-(I+a*x)/(a^2*x^2+1
)^(1/2))+3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*dilog((I+a*x)/(a^2*x^2+1)^(
1/2))-3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*dilog(1+(I+a*x)/(a^2*x^2+1)^(1
/2))+1/2*arccot(a*x)^3+3/2*I*arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))+3/
4*Pi*dilog((I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*dilog(1+(I+a*x)/(a^2*x^2+1)^(1
/2))+3/4*Pi*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+3/4*Pi*polylog(2,-(I+a*x)/
(a^2*x^2+1)^(1/2))+3/2*I*arccot(a*x)^2-3/2*I*dilog((I+a*x)/(a^2*x^2+1)^(1/2
))+3/2*I*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))+3/2*I*polylog(2,(I+a*x)/(a^2*x^
2+1)^(1/2))+3/2*I*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/2*arccot(a*x)*ln(
1-(I+a*x)/(a^2*x^2+1)^(1/2))-3*arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+
3/4*Pi*arccot(a*x)^2+3/8*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a
*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*
arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+
1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2-3/8*Pi*csgn(I*(I+a*
x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2
*arccot(a*x)^2+3/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/
(a^2*x^2+1))*arccot(a*x)^2-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*
(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2+3/4*I*Pi*a
rccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^
2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*dilog((I
+a*x)/(a^2*x^2+1)^(1/2))-3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x
)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*polylog(2,-(I+a*x)/(a^2*x^2+1)
^(1/2))-3/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/(
```

```
(I+a*x)^2/(a^2*x^2+1)-1))^2*dilog((I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I*
(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*dilog((I+a*x)/(a
^2*x^2+1)^(1/2))+3/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/
(a^2*x^2+1))^2*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x)^2/(
a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*dilog
(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn
(I*(I+a*x)^2/(a^2*x^2+1))^2*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/8*Pi*cs
gn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*dilog(1+(I+
a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(
I+a*x)^2/(a^2*x^2+1))*dilog((I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x
)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*polylog(2,(I+a*x)/(a^2
*x^2+1)^(1/2))+3/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/
(a^2*x^2+1))*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/4*Pi*csgn(I*(I+a*x)/(a
^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*polylog(2,(I+a*x)/(a^2*x^2
+1)^(1/2))-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x
^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/
8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/
(a^2*x^2+1)-1))^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I/((I+a*
x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)
)^2*dilog(1+(I+a*x)/(a^2*x^2+1)^(1/2))-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)
-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*polylog(2,(I
+a*x)/(a^2*x^2+1)^(1/2))+3/8*Pi*csgn(I/((I+a*x)...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(8*a^2*x^2*arctan2(1, a*x)^3 + 12*a*x*arctan2(1, a*x)^2 - 3*a*x*log(a^
2*x^2 + 1)^2 + 4*(128*a^3*integrate(1/32*x^3*arctan(1/(a*x))^3/(a^3*x^2 + a
), x) + 96*a^2*integrate(1/32*x^2*arctan(1/(a*x))^2/(a^3*x^2 + a), x) + 24*
a^2*integrate(1/32*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) + 96*a^2*integr
ate(1/32*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 128*a*integrate(1/32*x*ar
ctan(1/(a*x))^3/(a^3*x^2 + a), x) + 192*a*integrate(1/32*x*arctan(1/(a*x))/
(a^3*x^2 + a), x) + arctan(a*x)^3/a^2 + 3*arctan(a*x)^2*arctan(1/(a*x))/a^2
 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^2 + 24*integrate(1/32*log(a^2*x^2 + 1)
^2/(a^3*x^2 + a), x))*a^2 + 8*arctan2(1, a*x)^3)/a^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acot}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(a*x)**3,x)
```

```
[Out] Integral(x*acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acot}(a\,x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(a*x)^3,x)
```

```
[Out] int(x*acot(a*x)^3, x)
```

3.28 $\int \cot^{-1}(ax)^3\,dx$

Optimal. Leaf size=96

$$\frac{i\cot^{-1}(ax)^3}{a}+x\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}+\frac{3i\cot^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a}-\frac{3\text{PolyLog}\left(3,1-\right.}{2a}$$

[Out] I*arccot(a*x)^3/a+x*arccot(a*x)^3-3*arccot(a*x)^2*ln(2/(1+I*a*x))/a+3*I*arc
cot(a*x)*polylog(2,1-2/(1+I*a*x))/a-3/2*polylog(3,1-2/(1+I*a*x))/a

Rubi [A]
time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4931, 5041, 4965, 5005, 5115, 6745}

$$-\frac{3\text{Li}_3\left(1-\frac{2}{iax+1}\right)}{2a}+\frac{3i\text{Li}_2\left(1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{a}+x\cot^{-1}(ax)^3+\frac{i\cot^{-1}(ax)^3}{a}-\frac{3\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3,x]
```

```
[Out] (I*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[2/(1 + I*a*x)]
)/a + ((3*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - (3*PolyLog[3, 1
 - 2/(1 + I*a*x)])/(2*a)
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}(ax)^3\,dx &= x\cot^{-1}(ax)^3 + (3a)\int \frac{x\cot^{-1}(ax)^2}{1+a^2x^2}\,dx\\
&= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - 3\int \frac{\cot^{-1}(ax)^2}{i-ax}\,dx\\
&= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} - 6\int \frac{\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\\
&= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+iax}\right)}{a} +\\
&= \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+iax}\right)}{a} -
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 90, normalized size = 0.94

$$-\frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3i\cot^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3,x]
```

```
[Out] ((-I)*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[1 - E^((-2*
I)*ArcCot[a*x])])/a - ((3*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])]
)/a - (3*PolyLog[3, E^((-2*I)*ArcCot[a*x])])/(2*a)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(89) = 178$.
time = 0.48, size = 187, normalized size = 1.95

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-}{}$
default	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)^3*(a*x-I)+2*I*arccot(a*x)^3-3*arccot(a*x)^2*ln(1-(I+a*x)/(
a^2*x^2+1)^(1/2))+6*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*po
lylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))-3*arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)
^(1/2))+6*I*arccot(a*x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-
(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x*arctan2(1, a*x)^3 - 3/32*x*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 + 21/16
*arctan(a*x)^2*arctan(1/(a*x))^2/a + 7/8*arctan(a*x)*arctan(1/(a*x))^3/a +
28*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x^2 + 1), x) + 3*a^2*integ
rate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^2
*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 12
*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^2 + 1), x) - 3*a*integrate(1/3
2*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*(a*arctan(a*x)^4 + 4*a*arct
an(a*x)^3*arctan(1/(a*x)))/a^2 + 3*integrate(1/32*arctan(1/(a*x))*log(a^2*x
^2 + 1)^2/(a^2*x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}^{3}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3,x)
```

```
[Out] Integral(acot(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3,x)
```

```
[Out] int(acot(a*x)^3, x)
```

3.29 $\int \frac{\cot^{-1}(ax)^3}{x}\,dx$

Optimal. Leaf size=178

$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right)+\frac{3}{2}i\cot^{-1}(ax)^2\text{PolyLog}\left(2,1-\right.$$

```
[Out] 2*arccot(a*x)^3*arccoth(1-2/(1+I*a*x))-3/2*I*arccot(a*x)^2*polylog(2,1-2*I/
(I+a*x))+3/2*I*arccot(a*x)^2*polylog(2,1-2*a*x/(I+a*x))-3/2*arccot(a*x)*pol
ylog(3,1-2*I/(I+a*x))+3/2*arccot(a*x)*polylog(3,1-2*a*x/(I+a*x))+3/4*I*poly
log(4,1-2*I/(I+a*x))-3/4*I*polylog(4,1-2*a*x/(I+a*x))
```

Rubi [A]
time = 0.23, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4943, 5109, 5005, 5113, 5117, 6745}

$$\frac{3}{4}i\text{Li}_4\left(1-\frac{2i}{ax+i}\right)-\frac{3}{4}i\text{Li}_4\left(1-\frac{2ax}{ax+i}\right)-\frac{3}{2}i\text{Li}_2\left(1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2+\frac{3}{2}i\text{Li}_2\left(1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax)^2-\frac{3}{2}\text{Li}_3\left(1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)+\frac{3}{2}\text{Li}_3\left(1-\frac{2ax}{ax+i}\right)\cot^{-1}(ax)+2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3/x,x]
```

```
[Out] 2*ArcCot[a*x]^3*ArcCoth[1 - 2/(1 + I*a*x)] - ((3*I)/2)*ArcCot[a*x]^2*PolyLo
g[2, 1 - (2*I)/(I + a*x)] + ((3*I)/2)*ArcCot[a*x]^2*PolyLog[2, 1 - (2*a*x)/
(I + a*x)] - (3*ArcCot[a*x]*PolyLog[3, 1 - (2*I)/(I + a*x)])/2 + (3*ArcCot[
a*x]*PolyLog[3, 1 - (2*a*x)/(I + a*x)])/2 + ((3*I)/4)*PolyLog[4, 1 - (2*I)/
(I + a*x)] - ((3*I)/4)*PolyLog[4, 1 - (2*a*x)/(I + a*x)]
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
```

```
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5117

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x}\,dx &= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)+(6a)\int\frac{\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)}{1+a^2x^2}\,dx\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-(3a)\int\frac{\cot^{-1}(ax)^2\log\left(\frac{2i}{i+ax}\right)}{1+a^2x^2}\,dx+(3a)\int\frac{\cot^{-1}(\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2i}{i+ax}\right)+\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{I}\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2i}{i+ax}\right)+\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{I}\\
&= 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2i}{i+ax}\right)+\frac{3}{2}i\cot^{-1}(ax)^2\mathrm{I}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 180, normalized size = 1.01

$$\frac{1}{64}i\left(\pi^4-32\cot^{-1}(ax)^4+64i\cot^{-1}(ax)^3\log\left(1-e^{-2i\cot^{-1}(ax)}\right)-64i\cot^{-1}(ax)^3\log\left(1+e^{2i\cot^{-1}(ax)}\right)-96\cot^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)-96\cot^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)+96i\cot^{-1}(ax)\mathrm{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)-96i\cot^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)+48\mathrm{PolyLog}\left(4,e^{-2i\cot^{-1}(ax)}\right)+48\mathrm{PolyLog}\left(4,-e^{2i\cot^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3/x,x]
```

```
[Out] (I/64)*(Pi^4 - 32*ArcCot[a*x]^4 + (64*I)*ArcCot[a*x]^3*Log[1 - E^((-2*I)*Ar
cCot[a*x])] - (64*I)*ArcCot[a*x]^3*Log[1 + E^((2*I)*ArcCot[a*x])] - 96*ArcC
ot[a*x]^2*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 96*ArcCot[a*x]^2*PolyLog[2,
-E^((2*I)*ArcCot[a*x])] + (96*I)*ArcCot[a*x]*PolyLog[3, E^((-2*I)*ArcCot[a*
x])] - (96*I)*ArcCot[a*x]*PolyLog[3, -E^((2*I)*ArcCot[a*x])] + 48*PolyLog[4
, E^((-2*I)*ArcCot[a*x])] + 48*PolyLog[4, -E^((2*I)*ArcCot[a*x])])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 1050, normalized size = 5.90

method	result	size
derivativedivides	Expression too large to display	1050
default	Expression too large to display	1050

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccot(a*x)^3+arccot(a*x)^3*ln((I+a*x)^2/(a^2*x^2+1)-1)-arccot(a*x)
^3*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*arccot(a*x)^3-6*arccot(a*x)*pol
ylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-(I+a*x)/(a^2*x^2+1)^(1/2)
)-arccot(a*x)^3*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/((I+a*x)^2/
(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*arccot(a*x)^3-6*arccot(a*x)*pol
ylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)
*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^3-1/2*I*Pi*csgn(I*(1+(I+a*x)^2/(a
^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*a
rccot(a*x)^3+3*I*arccot(a*x)^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))+1/2*I*P
i*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*
x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arccot(a*x)^3+3*I*arccot(a*x
)^2*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x
^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*arccot(a*x)^3-3/2*I*arccot(a*x)^2*pol
ylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*c
sgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x
)^2/(a^2*x^2+1)))*arccot(a*x)^3+3/4*I*polylog(4,-(I+a*x)^2/(a^2*x^2+1))-6*I
*polylog(4,(I+a*x)/(a^2*x^2+1)^(1/2))+3/2*arccot(a*x)*polylog(3,-(I+a*x)^2/
(a^2*x^2+1))-1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^
2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot
(a*x)^3-1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x
^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccot(a*x)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{3}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3/x,x)
```

```
[Out] Integral(acot(a*x)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(a\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3/x,x)
```

```
[Out] int(acot(a*x)^3/x, x)
```

3.30 $\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx$

Optimal. Leaf size=93

$$-ia\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{x}-3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right)-3ia\cot^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)-$$

```
[Out] -I*a*arccot(a*x)^3-arccot(a*x)^3/x-3*a*arccot(a*x)^2*ln(2-2/(1-I*a*x))-3*I*
a*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))-3/2*a*polylog(3,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4947, 5045, 4989, 5005, 5113, 6745}

$$-\frac{3}{2}a\text{Li}_3\left(\frac{2}{1-iax}-1\right)-3ia\text{Li}_2\left(\frac{2}{1-iax}-1\right)\cot^{-1}(ax)-ia\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{x}-3a\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3/x^2,x]
```

```
[Out] (-I)*a*ArcCot[a*x]^3 - ArcCot[a*x]^3/x - 3*a*ArcCot[a*x]^2*Log[2 - 2/(1 - I
*a*x)] - (3*I)*a*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] - (3*a*PolyLog[
3, -1 + 2/(1 - I*a*x)])/2
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx &= -\frac{\cot^{-1}(ax)^3}{x} - (3a)\int \frac{\cot^{-1}(ax)^2}{x\left(1+a^2x^2\right)}\,dx\\
&= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - (3ia)\int \frac{\cot^{-1}(ax)^2}{x(i+ax)}\,dx\\
&= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - \left(6a^2\right)\int \frac{\cot^{-1}(ax)}{1}\\
&= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\mathrm{Li}_2\Big(\\
&= -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\mathrm{Li}_2\Big(\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.89

$$\frac{(-1+iax)\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) + 3ia\cot^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) - \frac{3}{2}a\mathrm{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3/x^2,x]
```

```
[Out] ((-1 + I*a*x)*ArcCot[a*x]^3)/x - 3*a*ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[
a*x])] + (3*I)*a*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - (3*a*Poly
Log[3, -E^((2*I)*ArcCot[a*x])])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 1444, normalized size = 15.53

method	result	size
derivativedivides	Expression too large to display	1444
default	Expression too large to display	1444

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arccot(a*x)^3/a/x-3*ln(a*x)*arccot(a*x)^2+3/2*arccot(a*x)^2*ln(a^2*x^2+
1)-3*arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-3/4*(2*I*Pi*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+I*Pi*csgn(I*(I+a*x)^2/(a^2*
x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3-I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*cs
gn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(
a^2*x^2+1)-1)^2)+2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*
x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-2*I*Pi
*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+2*
I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*cs
gn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-2*I*Pi*csgn(I*(1+
(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*
x^2+1)))^2+2*I*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x
^2+1))^2+I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3+I*Pi*csgn(I*(I+a*x)^2/(
a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2+I*P
i*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)
^2/(a^2*x^2+1)-1)^2)^2-I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3+2*I*Pi*csgn(1/(
(I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-2*I*Pi*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+
1)))^2-2*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-I
*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))-2*I*P
i*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*
x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+I*Pi*csgn(I*((I+a*x)^2/(a^
2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)-2*I*Pi*csgn(1/((I+a*x)^2
/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+4*ln(2))*arccot(a*x)^2+3*I*arc
cot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))-3/2*polylog(3,-(I+a*x)^2/(a^2*x^
2+1))+I*arccot(a*x)^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4*arctan2(1, a*x)^3 - 3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 - (28*a*a
rctan(a*x)*arctan(1/(a*x))^3 + 7*(6*arctan(a*x)^2*arctan(1/(a*x))^2/a + (a*
arctan(a*x)^4 + 4*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2)*a^2 + 96*a^2*integr
ate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 384*a
^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x)
- 384*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 96*a*integ
rate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*integrate(1/32*arc
tan(1/(a*x))^3/(a^2*x^4 + x^2), x) + 96*integrate(1/32*arctan(1/(a*x))*log(
a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3/x**2,x)
```

```
[Out] Integral(acot(a*x)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{acot}\left(a\,x\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3/x^2,x)
```

```
[Out] int(acot(a*x)^3/x^2, x)
```

3.31 $\int \frac{\cot^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=105

$$\frac{3}{2}ia^2\cot^{-1}(ax)^2+\frac{3a\cot^{-1}(ax)^2}{2x}-\frac{1}{2}a^2\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{2x^2}+3a^2\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ia^2\text{Poly}$$

```
[Out] 3/2*I*a^2*arccot(a*x)^2+3/2*a*arccot(a*x)^2/x-1/2*a^2*arccot(a*x)^3-1/2*arc
cot(a*x)^3/x^2+3*a^2*arccot(a*x)*ln(2-2/(1-I*a*x))+3/2*I*a^2*polylog(2,-1+2
/(1-I*a*x))
```

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4947, 5039, 5045, 4989, 2497, 5005}

$$\frac{3}{2}ia^2\text{Li}_2\left(\frac{2}{1-iax}-1\right)-\frac{1}{2}a^2\cot^{-1}(ax)^3+\frac{3}{2}ia^2\cot^{-1}(ax)^2+3a^2\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)-\frac{\cot^{-1}(ax)^3}{2x^2}+\frac{3a\cot^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3/x^3,x]
```

```
[Out] ((3*I)/2)*a^2*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/(2*x) - (a^2*ArcCot[a*x]^
3)/2 - ArcCot[a*x]^3/(2*x^2) + 3*a^2*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + (
(3*I)/2)*a^2*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx &= -\frac{\cot^{-1}(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\cot^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx + \frac{1}{2}\left(3a^3\right)\int \frac{\cot^{-1}(ax)^2}{1+a^2x^2}\,dx \\
&= \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + \left(3a^2\right)\int \frac{\cot^{-1}(ax)}{x\left(1+a^2x^2\right)}\,dx \\
&= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + \left(3ia^2\right)\int \frac{\cot^{-1}(ax)}{x(i+ax)} \\
&= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\Big(2 \\
&= \frac{3}{2}ia^2\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{2x} - \frac{1}{2}a^2\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2\cot^{-1}(ax)\log\Big(2
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.86

$$-\frac{\cot^{-1}(ax)\left(3iax(i+ax)\cot^{-1}(ax)+\left(1+a^2x^2\right)\cot^{-1}(ax)^2-6a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)}{2x^2}-\frac{3}{2}ia^2\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3/x^3,x]
```

```
[Out] -1/2*(ArcCot[a*x]*((3*I)*a*x*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a
*x]^2 - 6*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]))/x^2 - ((3*I)/2)*a^2*Poly
Log[2, -E^((2*I)*ArcCot[a*x])]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 5367, normalized size = 51.11

method	result	size
derivativedivides	Expression too large to display	5367
default	Expression too large to display	5367

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*x^2*arctan2(1, a*x)^3 - 12*a*x*arctan2(1, a*x)^2 + 3*a*x*log(a
^2*x^2 + 1)^2 + 4*(3*a^2*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a +
 3*arctan(a*x)^2*arctan(1/(a*x))/a)*a^3 + 24*a^3*integrate(1/32*x^3*log(a^2
*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 96*a^3*integrate(1/32*x^3*log(a^2*x^2 + 1
)/(a^2*x^5 + x^3), x) - 128*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x
^5 + x^3), x) - 192*a^2*integrate(1/32*x^2*arctan(1/(a*x))/(a^2*x^5 + x^3),
 x) + 96*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^5 + x^3), x) + 24*a*in
tegrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 128*integrate(1/32*
arctan(1/(a*x))^3/(a^2*x^5 + x^3), x))*x^2 + 8*arctan2(1, a*x)^3)/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{3}(ax)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3/x**3,x)
```

```
[Out] Integral(acot(a*x)**3/x**3, x)
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.28

$$-\frac{1}{2}\, a \arctan\left(\frac{1}{ax}\right)^{3} - \frac{\arctan\left(\frac{1}{ax}\right)^{3}}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(1/(a*x))^3 - 1/2*arctan(1/(a*x))^3/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(a\, x)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3/x^3,x)
```

```
[Out] int(acot(a*x)^3/x^3, x)
```

3.32 $\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx$

Optimal. Leaf size=167

$$-\frac{a^2 \cot^{-1}(ax)}{x}+\frac{1}{2}a^3 \cot^{-1}(ax)^2+\frac{a \cot^{-1}(ax)^2}{2x^2}+\frac{1}{3}ia^3 \cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{3x^3}-a^3 \log(x)+\frac{1}{2}a^3 \log \left(1+a^2x^2\right)$$

```
[Out] -a^2*arccot(a*x)/x+1/2*a^3*arccot(a*x)^2+1/2*a*arccot(a*x)^2/x^2+1/3*I*a^3*
arccot(a*x)^3-1/3*arccot(a*x)^3/x^3-a^3*ln(x)+1/2*a^3*ln(a^2*x^2+1)+a^3*arc
cot(a*x)^2*ln(2-2/(1-I*a*x))+I*a^3*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))+1/
2*a^3*polylog(3,-1+2/(1-I*a*x))
```

Rubi [A]
time = 0.25, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {4947, 5039, 272, 36, 29, 31, 5005, 5045, 4989, 5113, 6745}

$$\frac{1}{2}a^3\text{Li}_3\left(\frac{2}{1-iax}-1\right)+ia^3\text{Li}_2\left(\frac{2}{1-iax}-1\right)\cot^{-1}(ax)-a^3\log(x)+\frac{1}{3}ia^3\cot^{-1}(ax)^3+\frac{1}{2}a^3\cot^{-1}(ax)^2+a^3\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2-\frac{a^2\cot^{-1}(ax)}{x}+\frac{1}{2}a^3\log\left(a^2x^2+1\right)-\frac{\cot^{-1}(ax)^3}{3x^3}+\frac{a\cot^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcCot[a*x])/x) + (a^3*ArcCot[a*x]^2)/2 + (a*ArcCot[a*x]^2)/(2*x^2)
+ (I/3)*a^3*ArcCot[a*x]^3 - ArcCot[a*x]^3/(3*x^3) - a^3*Log[x] + (a^3*Log[1
 + a^2*x^2])/2 + a^3*ArcCot[a*x]^2*Log[2 - 2/(1 - I*a*x)] + I*a^3*ArcCot[a*
x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (a^3*PolyLog[3, -1 + 2/(1 - I*a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx &= -\frac{\cot^{-1}(ax)^3}{3x^3} - a\int \frac{\cot^{-1}(ax)^2}{x^3\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^3}{3x^3} - a\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx + a^3\int \frac{\cot^{-1}(ax)^2}{x\left(1+a^2x^2\right)}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^2\int \frac{\cot^{-1}(ax)}{x^2\left(1+a^2x^2\right)}\,dx + \left(ia^3\right)\int \frac{\cot^{-1}}{x(i-}\\
&= \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) + a^2 \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3 \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3 \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} + a^3 \\
&= -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} - a^3
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 151, normalized size = 0.90

$$\frac{1}{6}\left(-\frac{6a^2\cot^{-1}(ax)}{x}+3a^3\cot^{-1}(ax)^2+\frac{3a\cot^{-1}(ax)^2}{x^2}-2ia^3\cot^{-1}(ax)^3-\frac{2\cot^{-1}(ax)^3}{x^3}+6a^3\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)-6a^3\log\left(\frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}\right)-6ia^3\cot^{-1}(ax)\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)+3a^3\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3/x^4,x]
```

```
[Out] ((-6*a^2*ArcCot[a*x])/x + 3*a^3*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/x^2 - (
2*I)*a^3*ArcCot[a*x]^3 - (2*ArcCot[a*x]^3)/x^3 + 6*a^3*ArcCot[a*x]^2*Log[1
+ E^((2*I)*ArcCot[a*x])] - 6*a^3*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - (6*I)*a^3*A
rcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] + 3*a^3*PolyLog[3, -E^((2*I)*
ArcCot[a*x])])/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.39, size = 4745, normalized size = 28.41

method	result	size
derivativedivides	Expression too large to display	4745
default	Expression too large to display	4745

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*arccot(a*x)^3/a^3/x^3+1/2*arccot(a*x)^2/a^2/x^2+ln(a*x)*arccot(a*
x)^2-1/2*arccot(a*x)^2*ln(a^2*x^2+1)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(
1/2))-I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(I+a*x
)^2/(a^2*x^2+1))+1/24*arccot(a*x)*(-24+12*a*x*arccot(a*x)+3*I*csgn(I*((I+a*
x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)*Pi*a*x-12*I*Pi*csgn(I*(I+a*x)^2/(a^2*x
^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)*a*x+6*I*Pi*csgn((I+a*x)^2/
(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot
(a*x)*a*x+6*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2
*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arcco
t(a*x)*a*x+6*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x
^2+1)+I)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+I/((I+a*x)^
2/(a^2*x^2+1)-1))*arccot(a*x)*a*x-6*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a
*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I/((I+a*x)^2/(a^
2*x^2+1)-1)^2)*arccot(a*x)*a*x+3*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arcc
ot(a*x)*Pi+3*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)^3
*arccot(a*x)*Pi+3*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4/(a^2*
x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*arccot(a*x)*Pi+6*csgn(I*(I+a*x)^2/(a^
2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)^2*
arccot(a*x)*Pi+3*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2
*x^2+1)-1)^2)*arccot(a*x)*Pi-6*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I
+a*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)*Pi-6*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+
1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)+I)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x
)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))*arccot(a*x)-6*I*Pi*csgn(I*(
I+a*x)^2/(a^2*x^2+1)+I)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)
-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)*a*x-6*I*Pi*csgn(I*(I+a*x)^2/
(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn((I+
a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^2+1)-1))^2
*arccot(a*x)*a*x-3*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)-I)^2*csgn(-I*(I+a*x)^4
/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)*arccot(a*x)*a*x-6*I*Pi*csgn(I*(
I+a*x)^2/(a^2*x^2+1)-I)*csgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*
x^2+1)-I)^2*arccot(a*x)*a*x+6*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a
*x)^2/(a^2*x^2+1)))^2*arccot(a*x)*a*x+6*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/(
(I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)*a*x-6*I*Pi*csgn(I*(I+a*x)^2/(a^2*
x^2+1))^3*arccot(a*x)*a*x+6*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a
^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)*a*x-6*I*Pi*csgn((I+
a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^2+1)-1))^3
```

```
*arccot(a*x)*a*x+6*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^
2+1)))^3*arccot(a*x)-6*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^
2*x^2+1)))^3*arccot(a*x)+6*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2
/(a^2*x^2+1)))^2*arccot(a*x)-6*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(
a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)+6*Pi*csgn((I+a*x)^
2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^2+1)-1))^3*arcc
ot(a*x)-6*Pi*csgn((I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x
)^2/(a^2*x^2+1)-1))^2*arccot(a*x)-6*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x
)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn((I+a*x)^2/(a^2*x^2+1)/
((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^2+1)-1))*arccot(a*x)-6*Pi*csg
n(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^
2/(a^2*x^2+1)))^2*arccot(a*x)-6*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I
/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)-6*Pi*cs
gn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2
/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)+6*Pi*csgn(I/((I+a*
x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1
)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arccot(a*x)+6*Pi*csgn(I/((I+a*x)^2/(a^2*x^2
+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2
/(a^2*x^2+1)-1))^2*arccot(a*x)+6*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)+I)*csgn(I*
(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1)
)^2*arccot(a*x)+24*ln(2)*arccot(a*x)*a*x-8*I*arccot(a*x)^2*a*x+6*Pi*csgn(I*
(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+I/((I+a*x)^2/(a^2*x^2+1)-1)
)*csgn((I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)+1/((I+a*x)^2/(a^2*x^
2+1)-1))^2*arccot(a*x)+6*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+
a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2
+1)))*arccot(a*x)+24*I*a*x+3*I*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*(
(I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(a*x)*Pi*a*x-6*I*csgn(I*((I+a*x)^2/(a^2*x
^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)*Pi*a*x-3*I*Pi*c
sgn(-I*(I+a*x)^4/(a^2*x^2+1)^2+2*I*(I+a*x)^2/(a^2*x^2+1)-I)^3*arccot(a*x)*a
*x-6*I*Pi*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3*arc
cot(a*x)*a*x+6*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(96*x^3*integrate(1/32*(28*a^2*x^2*arctan2(1, a*x)^3 - 4*a^2*x^2*arcta
n2(1, a*x)*log(a^2*x^2 + 1) - 4*a*x*arctan2(1, a*x)^2 + 28*arctan2(1, a*x)^
3 + (3*a^2*x^2*arctan2(1, a*x) + a*x + 3*arctan2(1, a*x))*log(a^2*x^2 + 1)^
2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^3 + 3*arctan2(1, a*x)*log(a^2*x^
2 + 1)^2)/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^{3}(ax)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3/x**4,x)
```

```
[Out] Integral(acot(a*x)**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(a\,x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3/x^4,x)
```

```
[Out] int(acot(a*x)^3/x^4, x)
```

3.33 $\int \frac{\cot^{-1}(ax)^3}{x^5} \, dx$

Optimal. Leaf size=152

$$\frac{a^3}{4x}-\frac{a^2\cot^{-1}(ax)}{4x^2}-ia^4\cot^{-1}(ax)^2+\frac{a\cot^{-1}(ax)^2}{4x^3}-\frac{3a^3\cot^{-1}(ax)^2}{4x}+\frac{1}{4}a^4\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^3}{4x^4}+\frac{1}{4}a^4\text{ArcTa}$$

```
[Out] 1/4*a^3/x-1/4*a^2*arccot(a*x)/x^2-I*a^4*arccot(a*x)^2+1/4*a*arccot(a*x)^2/x
^3-3/4*a^3*arccot(a*x)^2/x+1/4*a^4*arccot(a*x)^3-1/4*arccot(a*x)^3/x^4+1/4*
a^4*arctan(a*x)-2*a^4*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a^4*polylog(2,-1+2/(1
-I*a*x))
```

Rubi [A]
time = 0.29, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 5039, 331, 209, 5045, 4989, 2497, 5005}

$$\frac{1}{4}a^4\text{ArcTan}(ax)-ia^4\text{Li}_2\left(\frac{2}{1-iax}-1\right)+\frac{1}{4}a^4\cot^{-1}(ax)^3-ia^4\cot^{-1}(ax)^2-2a^4\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)+\frac{a^3}{4x}-\frac{3a^3\cot^{-1}(ax)^2}{4x}-\frac{a^2\cot^{-1}(ax)}{4x^2}-\frac{\cot^{-1}(ax)^3}{4x^4}+\frac{a\cot^{-1}(ax)^2}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]^3/x^5,x]
```

```
[Out] a^3/(4*x) - (a^2*ArcCot[a*x])/(4*x^2) - I*a^4*ArcCot[a*x]^2 + (a*ArcCot[a*x
]^2)/(4*x^3) - (3*a^3*ArcCot[a*x]^2)/(4*x) + (a^4*ArcCot[a*x]^3)/4 - ArcCot
[a*x]^3/(4*x^4) + (a^4*ArcTan[a*x])/4 - 2*a^4*ArcCot[a*x]*Log[2 - 2/(1 - I*
a*x)] - I*a^4*PolyLog[2, -1 + 2/(1 - I*a*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx &= -\frac{\cot^{-1}(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\cot^{-1}(ax)^2}{x^4\left(1+a^2x^2\right)}\,dx \\
&= -\frac{\cot^{-1}(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\cot^{-1}(ax)^2}{x^2\left(1+a^2x^2\right)}\,dx \\
&= \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\cot^{-1}(ax)}{x^3\left(1+a^2x^2\right)}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx - \frac{1}{4}(\\
&= \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\cot^{-1}(ax)}{x^3}\,dx \\
&= -\frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 - \\
&= \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3 \\
&= \frac{a^3}{4x} - \frac{a^2\cot^{-1}(ax)}{4x^2} - ia^4\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{4x^3} - \frac{3a^3\cot^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\cot^{-1}(ax)^3
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 126, normalized size = 0.83

$$\frac{a^3x^3+\left(ax-3a^3x^3+4ia^4x^4\right)\cot^{-1}(ax)^2+\left(-1+a^4x^4\right)\cot^{-1}(ax)^3-a^2x^2\cot^{-1}(ax)\left(1+a^2x^2+8a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)+4ia^4x^4\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]^3/x^5,x]
```

```
[Out] (a^3*x^3 + (a*x - 3*a^3*x^3 + (4*I)*a^4*x^4)*ArcCot[a*x]^2 + (-1 + a^4*x^4)
*ArcCot[a*x]^3 - a^2*x^2*ArcCot[a*x]*(1 + a^2*x^2 + 8*a^2*x^2*Log[1 + E^((2
*I)*ArcCot[a*x])]) + (4*I)*a^4*x^4*PolyLog[2, -E^((2*I)*ArcCot[a*x])])/(4*x
^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.89, size = 855, normalized size = 5.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arccot(a*x)^3/a^4/x^4-3/4*arccot(a*x)^2/a/x+3/4*I*arccot(a*x)^2*l
n((I+a*x)/(a^2*x^2+1)^(1/2))+1/4*arccot(a*x)^2/a^3/x^3-2*arccot(a*x)*ln(1-I
*(I+a*x)/(a^2*x^2+1)^(1/2))-2*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))
+1/4*arccot(a*x)^3-1/4*arccot(a*x)*(I+a*x)^2/a^2/x^2+1/4*I/a/x*(a*x-I)+3/8*
Pi*arccot(a*x)^2-3/8*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/
(a^2*x^2+1))^2*arccot(a*x)^2-3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(
I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2+3/16*Pi*csg
```

```
n(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^
2-3/16*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((
I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2+3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+
1)/((I+a*x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)^2+3/8*Pi*csgn(I/((I+a*x)^2/(a^2
*x^2+1)-1))^3*arccot(a*x)^2-3/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arcc
ot(a*x)^2+3/16*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2+3/16*Pi*csg
n(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)
^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*arccot(a*x)^2-3/4*arccot(a*x)^2*a
rctan(a*x)+1/2*arccot(a*x)*(I+a*x)/a/x+2*I*dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1
/2))+2*I*dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)^2-1/2*arccot(a*
x)*(a*x-I)*(I+a*x)/a^2/x^2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)^3/x^5, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}^3(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)**3/x**5,x)
```

```
[Out] Integral(acot(a*x)**3/x**5, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)^3/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a\,x)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)^3/x^5,x)
```

```
[Out] int(acot(a*x)^3/x^5, x)
```

3.34 $\int x^m \cot^{-1}(ax)^3 \, dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(x^m \cot^{-1}(ax)^3, x\right)$$

```
[Out] Unintegrable(x^m*arccot(a*x)^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cot^{-1}(ax)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*ArcCot[a*x]^3,x]
[Out] Defer[Int][x^m*ArcCot[a*x]^3, x]
```

Rubi steps

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \cot^{-1}(ax)^3 \, dx$$

Mathematica [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int x^m \cot^{-1}(ax)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*ArcCot[a*x]^3,x]
[Out] Integrate[x^m*ArcCot[a*x]^3, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccot}(ax)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccot(a*x)^3,x)
```

```
[Out] int(x^m*arccot(a*x)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*x*x^m*arctan2(1, a*x)^3 - 3*x*x^m*arctan2(1, a*x)*log(a^2*x^2 + 1)^
2 + 32*(m + 1)*integrate(1/32*(12*a^2*x^2*x^m*arctan2(1, a*x)*log(a^2*x^2 +
 1) + 3*((a^2*m*arctan2(1, a*x) + a^2*arctan2(1, a*x))*x^2 - a*x + m*arctan
2(1, a*x) + arctan2(1, a*x))*x^m*log(a^2*x^2 + 1)^2 + 4*(3*a*x*arctan2(1, a
*x)^2 + 7*m*arctan2(1, a*x)^3 + 7*(a^2*m*arctan2(1, a*x)^3 + a^2*arctan2(1,
 a*x)^3)*x^2 + 7*arctan2(1, a*x)^3)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(
m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccot(a*x)^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{acot}^{3}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acot(a*x)**3,x)
```

```
[Out] Integral(x**m*acot(a*x)**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccot(a*x)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int x^m \operatorname{acot}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acot(a*x)^3,x)
```

```
[Out] int(x^m*acot(a*x)^3, x)
```

3.35 $\int x^m \cot^{-1}(ax)^2 \, dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(x^m \cot^{-1}(ax)^2, x\right)$$

```
[Out] Unintegrable(x^m*arccot(a*x)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \cot^{-1}(ax)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*ArcCot[a*x]^2,x]
[Out] Defer[Int][x^m*ArcCot[a*x]^2, x]
```

Rubi steps

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \cot^{-1}(ax)^2 \, dx$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int x^m \cot^{-1}(ax)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*ArcCot[a*x]^2,x]
[Out] Integrate[x^m*ArcCot[a*x]^2, x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccot}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccot(a*x)^2,x)
```

```
[Out] int(x^m*arccot(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="maxima")
[Out] 1/16*(4*x*x^m*arctan2(1, a*x)^2 - x*x^m*log(a^2*x^2 + 1)^2 + 16*(m + 1)*int
egrate(1/16*(4*a^2*x^2*x^m*log(a^2*x^2 + 1) + ((a^2*m + a^2)*x^2 + m + 1)*x
^m*log(a^2*x^2 + 1)^2 + 4*(3*(a^2*m*arctan2(1, a*x)^2 + a^2*arctan2(1, a*x)
^2)*x^2 + 2*a*x*arctan2(1, a*x) + 3*m*arctan2(1, a*x)^2 + 3*arctan2(1, a*x)
^2)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="fricas")
[Out] integral(x^m*arccot(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\operatorname{acot}^{2}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acot(a*x)**2,x)
[Out] Integral(x**m*acot(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x)^2,x, algorithm="giac")
[Out] integrate(x^m*arccot(a*x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int x^m \operatorname{acot}(a x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acot(a*x)^2,x)
```

```
[Out] int(x^m*acot(a*x)^2, x)
```

3.36 $\int x^m \cot^{-1}(ax)\,dx$

Optimal. Leaf size=57

$$\frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}$$

```
[Out] x^(1+m)*arccot(a*x)/(1+m)+a*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x
^2)/(m^2+3*m+2)
```

Rubi [A]

time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4947, 371}

$$\frac{ax^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-a^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCot[a*x],x]
```

```
[Out] (x^(1 + m)*ArcCot[a*x])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[1, (2 + m)
/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m\cot^{-1}(ax)\,dx&=\frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{a\int\frac{x^{1+m}}{1+a^2x^2}\,dx}{1+m}\\&=\frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{ax^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.91

$$\frac{x^{1+m}\left((2+m)\cot^{-1}(ax)+ax\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-a^2x^2\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCot[a*x],x]
[Out] (x^(1 + m)*((2 + m)*ArcCot[a*x] + a*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2
, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccot}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccot(a*x),x)
[Out] int(x^m*arccot(a*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x),x, algorithm="maxima")
[Out] (x*x^m*arctan2(1, a*x) + (a*m + a)*integrate(x*x^m/((a^2*m + a^2)*x^2 + m +
 1), x))/(m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x),x, algorithm="fricas")
[Out] integral(x^m*arccot(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acot}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acot(a*x),x)
```

```
[Out] Integral(x**m*acot(a*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccot(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccot(a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \operatorname{acot}(a\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acot(a*x),x)
```

```
[Out] int(x^m*acot(a*x), x)
```

3.37 $\int \frac{x^4 \cot^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=40

$$\frac{x^2}{6} - x\cot^{-1}(x) + \frac{1}{3}x^3\cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 - \frac{2}{3}\log\left(1+x^2\right)$$

```
[Out] 1/6*x^2-x*arccot(x)+1/3*x^3*arccot(x)-1/2*arccot(x)^2-2/3*ln(x^2+1)
```

Rubi [A]
time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5037, 4947, 272, 45, 4931, 266, 5005}

$$\frac{1}{3}x^3\cot^{-1}(x) + \frac{x^2}{6} - \frac{2}{3}\log\left(x^2+1\right) - x\cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x^2/6 - x*ArcCot[x] + (x^3*ArcCot[x])/3 - ArcCot[x]^2/2 - (2*Log[1 + x^2])/
3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
```

```
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4 \cot^{-1}(x)}{1+x^2}\,dx &= \int x^2 \cot^{-1}(x)\,dx - \int \frac{x^2 \cot^{-1}(x)}{1+x^2}\,dx \\
&= \frac{1}{3}x^3 \cot^{-1}(x) + \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx - \int \cot^{-1}(x)\,dx + \int \frac{\cot^{-1}(x)}{1+x^2}\,dx \\
&= -x \cot^{-1}(x) + \frac{1}{3}x^3 \cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 + \frac{1}{6}\mathrm{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right) - \int \frac{x}{1+x^2}\,d \\
&= -x \cot^{-1}(x) + \frac{1}{3}x^3 \cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{2}\log\left(1+x^2\right) + \frac{1}{6}\mathrm{Subst}\left(\int \left(1 + \frac{1}{-1-x}\right.\right. \\
&= \frac{x^2}{6} - x \cot^{-1}(x) + \frac{1}{3}x^3 \cot^{-1}(x) - \frac{1}{2}\cot^{-1}(x)^2 - \frac{2}{3}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.80

$$\frac{1}{6}\left(x^2 + 2x\left(-3+x^2\right)\cot^{-1}(x) - 3\cot^{-1}(x)^2 - 4\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCot[x])/(1 + x^2),x]
```

```
[Out] (x^2 + 2*x*(-3 + x^2)*ArcCot[x] - 3*ArcCot[x]^2 - 4*Log[1 + x^2])/6
```

Maple [A]
time = 0.16, size = 38, normalized size = 0.95

method	result
default	$\frac{x^3 \operatorname{arccot}(x)}{3} - x \operatorname{arccot}(x) + \operatorname{arccot}(x) \arctan(x) + \frac{x^2}{6} - \frac{2 \ln(x^2+1)}{3} + \frac{\arctan(x)^2}{2}$
risch	$\frac{\ln(ix+1)^2}{8} + \left(\frac{ix^3}{6} - \frac{ix}{2} - \frac{\ln(-ix+1)}{4}\right) \ln(ix+1) + \frac{\ln(-ix+1)^2}{8} - \frac{ix^3 \ln(-ix+1)}{6} + \frac{i \ln(-ix+1) x}{2} + \frac{\pi x^3}{6} - \frac{\pi x}{2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccot(x)-x*arccot(x)+arccot(x)*arctan(x)+1/6*x^2-2/3*ln(x^2+1)+1/2
*arctan(x)^2
```

Maxima [A]
time = 0.49, size = 35, normalized size = 0.88

$$\frac{1}{6} x^2 + \frac{1}{3} \left(x^3 - 3x + 3 \arctan(x)\right) \operatorname{arccot}(x) + \frac{1}{2} \arctan(x)^2 - \frac{2}{3} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*x^2 + 1/3*(x^3 - 3*x + 3*arctan(x))*arccot(x) + 1/2*arctan(x)^2 - 2/3*l
og(x^2 + 1)
```

Fricas [A]
time = 1.68, size = 31, normalized size = 0.78

$$\frac{1}{6} x^2 + \frac{1}{3} \left(x^3 - 3x\right) \operatorname{arccot}(x) - \frac{1}{2} \operatorname{arccot}(x)^2 - \frac{2}{3} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*x^2 + 1/3*(x^3 - 3*x)*arccot(x) - 1/2*arccot(x)^2 - 2/3*log(x^2 + 1)
```

Sympy [A]
time = 0.16, size = 34, normalized size = 0.85

$$\frac{x^3 \operatorname{acot}(x)}{3} + \frac{x^2}{6} - x \operatorname{acot}(x) - \frac{2 \log(x^2+1)}{3} - \frac{\operatorname{acot}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acot(x)/(x**2+1),x)
```

```
[Out] x**3*acot(x)/3 + x**2/6 - x*acot(x) - 2*log(x**2 + 1)/3 - acot(x)**2/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccot(x)/(x^2 + 1), x)
```

Mupad [B]
time = 0.73, size = 32, normalized size = 0.80

$$\frac{x^3\,\mathrm{acot}(x)}{3}-\frac{2\,\ln\left(x^2+1\right)}{3}-\frac{\mathrm{acot}(x)^2}{2}-x\,\mathrm{acot}(x)+\frac{x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acot(x))/(x^2 + 1),x)
```

```
[Out] (x^3*acot(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 - x*acot(x) + x^2/6
```

3.38 $\int \frac{x^3 \cot^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=67

$$\frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2-\frac{\text{ArcTan}(x)}{2}+\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*I*arccot(x)^2-1/2*arctan(x)+arccot(x)*ln(2/(1+I
*x))-1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5037, 4947, 327, 209, 5041, 4965, 2449, 2352}

$$-\frac{\text{ArcTan}(x)}{2}-\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{ix+1}\right)+\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}i\cot^{-1}(x)^2+\log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - (I/2)*ArcCot[x]^2 - ArcTan[x]/2 + ArcCot[x]*Log[2
/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\cot^{-1}(x)}{1+x^2}\,dx &= \int x\cot^{-1}(x)\,dx-\int \frac{x\cot^{-1}(x)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2+\frac{1}{2}\int \frac{x^2}{1+x^2}\,dx+\int \frac{\cot^{-1}(x)}{i-x}\,dx\\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2+\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{\log\left(\frac{}{1}\right.}{1+}\\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2-\frac{1}{2}\tan^{-1}(x)+\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-i\text{Subst}\Bigg(\int \frac{\text{lo}}{1}\\
&= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}i\cot^{-1}(x)^2-\frac{1}{2}\tan^{-1}(x)+\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)-\frac{1}{2}i\text{Li}_2\Bigg(1-\frac{}{1}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 241 vs. $2(67) = 134$.
time = 0.05, size = 241, normalized size = 3.60

$$\frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{\text{ArcTan}(x)}{2}+\frac{1}{8}i\log^2(i-x)-\frac{1}{4}i\log(i-x)\log\left(-\frac{i-x}{x}\right)-\frac{1}{4}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right)+\frac{1}{4}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x)-\frac{1}{4}i\log\left(-\frac{i-x}{x}\right)\log(i+x)-\frac{1}{8}i\log^2(i+x)+\frac{1}{4}i\log(i-x)\log\left(\frac{i+x}{x}\right)+\frac{1}{4}i\log(i+x)\log\left(\frac{i+x}{x}\right)-\frac{1}{4}i\text{PolyLog}\left(2,-\frac{1}{2}i(i-x)\right)+\frac{1}{4}i\text{PolyLog}\left(2,-\frac{1}{2}i(i+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCot[x])/(1 + x^2),x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2 + (I/8)*Log[I - x]^2 - (I/4)*Log[I -
x]*Log[-((I - x)/x)] - (I/4)*Log[I - x]*Log[(-1/2*I)*(I + x)] + (I/4)*Log[(
-1/2*I)*(I - x)]*Log[I + x] - (I/4)*Log[-((I - x)/x)]*Log[I + x] - (I/8)*Lo
g[I + x]^2 + (I/4)*Log[I - x]*Log[(I + x)/x] + (I/4)*Log[I + x]*Log[(I + x)
/x] - (I/4)*PolyLog[2, (-1/2*I)*(I - x)] + (I/4)*PolyLog[2, (-1/2*I)*(I + x
)]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.
time = 0.12, size = 128, normalized size = 1.91

method	result
default	$\frac{x^2\operatorname{arccot}(x)}{2}-\frac{\operatorname{arccot}(x)\ln(x^2+1)}{2}+\frac{x}{2}-\frac{\arctan(x)}{2}+\frac{i\ln(x-i)\ln(x^2+1)}{4}-\frac{i\operatorname{dilog}\left(-\frac{i(i+x)}{2}\right)}{4}-\frac{i\ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)}{4}-$
risch	$\frac{\pi x^2}{4}+\frac{\pi}{4}-\frac{\pi\ln(x^2+1)}{4}-\frac{i\ln(-ix+1)x^2}{4}-\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}-\frac{i\ln\left(\frac{1}{2}-\frac{ix}{2}\right)\ln(ix+1)}{4}-\frac{i\ln(ix+1)^2}{8}+\frac{i\ln(-ix+1)^2}{8}+\frac{x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccot(x)-1/2*arccot(x)*ln(x^2+1)+1/2*x-1/2*arctan(x)+1/4*I*ln(x-I)
*ln(x^2+1)-1/4*I*dilog(-1/2*I*(I+x))-1/4*I*ln(x-I)*ln(-1/2*I*(I+x))-1/8*I*l
n(x-I)^2-1/4*I*ln(I+x)*ln(x^2+1)+1/4*I*dilog(1/2*I*(x-I))+1/4*I*ln(I+x)*ln(
1/2*I*(x-I))+1/8*I*ln(I+x)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arccot(x)/(x^2 + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arccot(x)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{acot}(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*acot(x)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccot(x)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \operatorname{acot}(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acot(x))/(x^2 + 1),x)
```

```
[Out] int((x^3*acot(x))/(x^2 + 1), x)
```

3.39 $\int \frac{x^2 \cot^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$x \cot^{-1}(x) + \frac{1}{2} \cot^{-1}(x)^2 + \frac{1}{2} \log \left(1 + x^2\right)$$

[Out] x*arccot(x)+1/2*arccot(x)^2+1/2*ln(x^2+1)

Rubi [A]

time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5037, 4931, 266, 5005}

$$\frac{1}{2} \log \left(x^2 + 1\right) + \frac{1}{2} \cot^{-1}(x)^2 + x \cot^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*ArcCot[x])/(1 + x^2),x]

[Out] x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \cot^{-1}(x)}{1+x^2}\,dx &= \int \cot^{-1}(x)\,dx - \int \frac{\cot^{-1}(x)}{1+x^2}\,dx \\
&= x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \int \frac{x}{1+x^2}\,dx \\
&= x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$x\cot^{-1}(x) + \frac{1}{2}\cot^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCot[x])/(1 + x^2),x]
[Out] x*ArcCot[x] + ArcCot[x]^2/2 + Log[1 + x^2]/2
```

Maple [A]
time = 0.15, size = 26, normalized size = 1.13

method	result	siz
default	$-\operatorname{arccot}(x)\arctan(x) + x\operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2} - \frac{\arctan(x)^2}{2}$	26
risch	$-\frac{\ln(ix+1)^2}{8} + \left(\frac{ix}{2} + \frac{\ln(-ix+1)}{4}\right)\ln(ix+1) - \frac{\ln(-ix+1)^2}{8} - \frac{i\ln(-ix+1)x}{2} + \frac{\pi x}{2} - \frac{\pi\arctan(x)}{2} + \frac{\ln(x^2+1)}{2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arccot(x)*arctan(x)+x*arccot(x)+1/2*ln(x^2+1)-1/2*arctan(x)^2
```

Maxima [A]
time = 0.48, size = 24, normalized size = 1.04

$$(x - \arctan(x))\operatorname{arccot}(x) - \frac{1}{2}\arctan(x)^2 + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] (x - arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 1.89, size = 19, normalized size = 0.83

$$x \operatorname{arccot}(x) + \frac{1}{2} \operatorname{arccot}(x)^2 + \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] x*arccot(x) + 1/2*arccot(x)^2 + 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.14, size = 19, normalized size = 0.83

$$x \operatorname{acot}(x) + \frac{\log(x^2 + 1)}{2} + \frac{\operatorname{acot}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(x)/(x**2+1),x)
[Out] x*acot(x) + log(x**2 + 1)/2 + acot(x)**2/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x^2*arccot(x)/(x^2 + 1), x)
```

Mupad [B]
time = 0.64, size = 19, normalized size = 0.83

$$\frac{\operatorname{acot}(x)^2}{2} + x \operatorname{acot}(x) + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acot(x))/(x^2 + 1),x)
[Out] log(x^2 + 1)/2 + acot(x)^2/2 + x*acot(x)
```

3.40 $\int \frac{x \cot^{-1}(x)}{1+x^2}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}i\cot^{-1}(x)^2-\cot^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)$$

```
[Out] 1/2*I*arccot(x)^2-arccot(x)*ln(2/(1+I*x))+1/2*I*polylog(2,1-2/(1+I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {5041, 4965, 2449, 2352}

$$\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{ix+1}\right)+\frac{1}{2}i\cot^{-1}(x)^2-\log\left(\frac{2}{1+ix}\right)\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCot[x])/(1 + x^2),x]
```

```
[Out] (I/2)*ArcCot[x]^2 - ArcCot[x]*Log[2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1
+ I*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x \cot^{-1}(x)}{1+x^2}\,dx &= \frac{1}{2} i \cot^{-1}(x)^2 - \int \frac{\cot^{-1}(x)}{i-x}\,dx \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x) \log\left(\frac{2}{1+ix}\right) - \int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x) \log\left(\frac{2}{1+ix}\right) + i\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+ix}\right) \\
&= \frac{1}{2} i \cot^{-1}(x)^2 - \cot^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} i \operatorname{Li}_2\left(1-\frac{2}{1+ix}\right)
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 221 vs. $2(48) = 96$.
time = 0.04, size = 221, normalized size = 4.60

$$
-\frac{1}{8} i \log^2(i-x) + \frac{1}{4} i \log(i-x) \log\left(-\frac{i-x}{x}\right) + \frac{1}{4} i \log(i-x) \log\left(-\frac{1}{2} i (i+x)\right) - \frac{1}{4} i \log\left(-\frac{1}{2} i (i-x)\right) \log(i+x) + \frac{1}{4} i \log\left(-\frac{i-x}{x}\right) \log(i+x) + \frac{1}{8} i \log^2(i+x) - \frac{1}{4} i \log(i-x) \log\left(\frac{i+x}{x}\right) - \frac{1}{4} i \log(i+x) \log\left(\frac{i+x}{x}\right) + \frac{1}{4} i \text{PolyLog}\left(2, -\frac{1}{2} i (i-x)\right) - \frac{1}{4} i \text{PolyLog}\left(2, -\frac{1}{2} i (i+x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2),x]
```

```
[Out] (-1/8*I)*Log[I - x]^2 + (I/4)*Log[I - x]*Log[-((I - x)/x)] + (I/4)*Log[I -
x]*Log[(-1/2*I)*(I + x)] - (I/4)*Log[(-1/2*I)*(I - x)]*Log[I + x] + (I/4)*L
og[-((I - x)/x)]*Log[I + x] + (I/8)*Log[I + x]^2 - (I/4)*Log[I - x]*Log[(I
+ x)/x] - (I/4)*Log[I + x]*Log[(I + x)/x] + (I/4)*PolyLog[2, (-1/2*I)*(I -
x)] - (I/4)*PolyLog[2, (-1/2*I)*(I + x)]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(40) = 80$.
time = 0.11, size = 112, normalized size = 2.33

method	result
risch	$\frac{\pi \ln\left(-2+(-ix+1)^2+2ix\right)}{4} - \frac{i \ln(-ix+1)^2}{8} - \frac{i \ln\left(\frac{1}{2}+\frac{ix}{2}\right) \ln(-ix+1)}{4} + \frac{i \operatorname{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} + \frac{i \ln(ix+1)^2}{8} + \frac{i \ln\left(\frac{1}{2}-\frac{ix}{2}\right) \ln(ix+1)}{4}$
default	$\frac{\operatorname{arccot}(x) \ln(x^2+1)}{2} - \frac{i\left(\ln(x-i)\ln(x^2+1) - \frac{\ln(x-i)^2}{2} - \operatorname{dilog}\left(-\frac{i(i+x)}{2}\right) - \ln(x-i)\ln\left(-\frac{i(i+x)}{2}\right)\right)}{4} + \frac{i\left(\ln(i+x)\ln(x^2+1) - \frac{\ln(i+x)^2}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arccot(x)*ln(x^2+1)-1/4*I*(ln(x-I)*ln(x^2+1)-1/2*ln(x-I)^2-dilog(-1/2*I
*(I+x))-ln(x-I)*ln(-1/2*I*(I+x)))+1/4*I*(ln(I+x)*ln(x^2+1)-1/2*ln(I+x)^2-di
log(1/2*I*(x-I))-ln(I+x)*ln(1/2*I*(x-I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="maxima")
[Out] integrate(x*arccot(x)/(x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="fricas")
[Out] integral(x*arccot(x)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{acot}(x)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x)/(x**2+1),x)
[Out] Integral(x*acot(x)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x*arccot(x)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{acot}(x)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acot(x))/(x^2 + 1),x)
[Out] int((x*acot(x))/(x^2 + 1), x)
```

3.41 $\int \frac{\cot^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\cot^{-1}(x)^2$$

[Out] -1/2*arccot(x)^2

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5005}

$$-\frac{1}{2}\cot^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCot[x]/(1 + x^2),x]

[Out] -1/2*ArcCot[x]^2

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^{-1}(x)}{1+x^2} \, dx = -\frac{1}{2}\cot^{-1}(x)^2$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cot^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCot[x]/(1 + x^2),x]

[Out] -1/2*ArcCot[x]^2

Maple [A]
time = 0.11, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^{2}}{2}$	7
default	$-\frac{\operatorname{arccot}(x)^{2}}{2}$	7
risch	$\frac{\ln(ix+1)^{2}}{8}-\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{\ln(-ix+1)^{2}}{8}+\frac{\pi\arctan(x)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccot(x)^2
```

Maxima [A]
time = 0.26, size = 6, normalized size = 0.75

$$-\frac{1}{2}\operatorname{arccot}(x)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*arccot(x)^2
```

Fricas [A]
time = 1.86, size = 6, normalized size = 0.75

$$-\frac{1}{2}\operatorname{arccot}(x)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*arccot(x)^2
```

Sympy [A]
time = 0.53, size = 7, normalized size = 0.88

$$-\frac{\operatorname{acot}^{2}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(x**2+1),x)
```

```
[Out] -acot(x)**2/2
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.00

$$-\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/x)^2
```

Mupad [B]
time = 0.62, size = 6, normalized size = 0.75

$$-\frac{\operatorname{acot}(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x^2 + 1),x)
```

```
[Out] -acot(x)^2/2
```

3.42 $\int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}i\cot^{-1}(x)^2+\cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

```
[Out] 1/2*I*arccot(x)^2+arccot(x)*ln(2-2/(1-I*x))+1/2*I*polylog(2,-1+2/(1-I*x))
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5045, 4989, 2497}

$$\frac{1}{2}i\text{Li}_2\left(\frac{2}{1-ix}-1\right)+\frac{1}{2}i\cot^{-1}(x)^2+\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(x*(1 + x^2)),x]
```

```
[Out] (I/2)*ArcCot[x]^2 + ArcCot[x]*Log[2 - 2/(1 - I*x)] + (I/2)*PolyLog[2, -1 +
2/(1 - I*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(x)}{x\,(1+x^2)}\,dx &= \frac{1}{2}i\cot^{-1}(x)^2 + i\int \frac{\cot^{-1}(x)}{x(i+x)}\,dx \\
&= \frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx \\
&= \frac{1}{2}i\cot^{-1}(x)^2 + \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \frac{1}{2}i\mathrm{Li}_2\left(-1+\frac{2}{1-ix}\right)
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 251 vs. $2(49) = 98$.
time = 0.05, size = 251, normalized size = 5.12

$$\frac{1}{8}i\log^2(i-x) - \frac{1}{4}i\log(i-x)\log\left(-\frac{i-x}{x}\right) - \frac{1}{4}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right) + \frac{1}{4}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x) - \frac{1}{4}i\log\left(-\frac{i-x}{x}\right)\log(i+x) - \frac{1}{8}i\log^2(i+x) + \frac{1}{4}i\log(i-x)\log\left(\frac{i+x}{x}\right) + \frac{1}{4}i\log(i+x)\log\left(\frac{i+x}{x}\right) - \frac{1}{4}i\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i-x)\right) - \frac{1}{2}i\mathrm{PolyLog}\left(2,-\frac{i}{x}\right) + \frac{1}{2}i\mathrm{PolyLog}\left(2,\frac{i}{x}\right) + \frac{1}{4}i\mathrm{PolyLog}\left(2,-\frac{1}{2}i(i+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(x*(1 + x^2)),x]
```

```
[Out] (I/8)*Log[I - x]^2 - (I/4)*Log[I - x]*Log[-((I - x)/x)] - (I/4)*Log[I - x]*
Log[(-1/2*I)*(I + x)] + (I/4)*Log[(-1/2*I)*(I - x)]*Log[I + x] - (I/4)*Log[
-((I - x)/x)]*Log[I + x] - (I/8)*Log[I + x]^2 + (I/4)*Log[I - x]*Log[(I + x
)/x] + (I/4)*Log[I + x]*Log[(I + x)/x] - (I/4)*PolyLog[2, (-1/2*I)*(I - x)]
 - (I/2)*PolyLog[2, (-I)/x] + (I/2)*PolyLog[2, I/x] + (I/4)*PolyLog[2, (-1/
2*I)*(I + x)]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(41) = 82$.
time = 0.12, size = 163, normalized size = 3.33

method	result
risch	$-\frac{\pi\ln(x^2+1)}{4} + \frac{\pi\ln(-ix)}{2} + \frac{i\ln(-ix+1)^2}{8} + \frac{i\,\mathrm{dilog}(-ix+1)}{2} + \frac{i\ln\left(\frac{1}{2}+\frac{ix}{2}\right)\ln(-ix+1)}{4} - \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4} - \frac{i\ln(ix+1)^2}{8} -$
default	$\mathrm{arccot}(x)\ln(x) - \frac{\mathrm{arccot}(x)\ln(x^2+1)}{2} - \frac{i\ln(x)\ln(ix+1)}{2} + \frac{i\ln(x)\ln(-ix+1)}{2} - \frac{i\,\mathrm{dilog}(ix+1)}{2} + \frac{i\,\mathrm{dilog}(-ix+1)}{2} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arccot(x)*ln(x)-1/2*arccot(x)*ln(x^2+1)-1/2*I*ln(x)*ln(1+I*x)+1/2*I*ln(x)*l
n(1-I*x)-1/2*I*dilog(1+I*x)+1/2*I*dilog(1-I*x)+1/4*I*ln(x-I)*ln(x^2+1)-1/4*
I*dilog(-1/2*I*(I+x))-1/4*I*ln(x-I)*ln(-1/2*I*(I+x))-1/8*I*ln(x-I)^2-1/4*I*
ln(I+x)*ln(x^2+1)+1/4*I*dilog(1/2*I*(x-I))+1/4*I*ln(I+x)*ln(1/2*I*(x-I))+1/
8*I*ln(I+x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="maxima")
[Out] integrate(arccot(x)/((x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="fricas")
[Out] integral(arccot(x)/(x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(x)}{x(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/x/(x**2+1),x)
[Out] Integral(acot(x)/(x*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(x)/((x^2 + 1)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acot}(x)}{x\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x*(x^2 + 1)),x)
[Out] int(acot(x)/(x*(x^2 + 1)), x)
```

3.43 $\int \frac{\cot^{-1}(x)}{x^2(1+x^2)} \, dx$

Optimal. Leaf size=30

$$-\frac{\cot^{-1}(x)}{x}+\frac{1}{2}\cot^{-1}(x)^2-\log(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -arccot(x)/x+1/2*arccot(x)^2-ln(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5039, 4947, 272, 36, 29, 31, 5005}

$$\frac{1}{2}\log\left(x^2+1\right)-\log(x)+\frac{1}{2}\cot^{-1}(x)^2-\frac{\cot^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[x]/x) + ArcCot[x]^2/2 - Log[x] + Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(x)}{x^2(1+x^2)}\,dx &= \int \frac{\cot^{-1}(x)}{x^2}\,dx - \int \frac{\cot^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \int \frac{1}{x(1+x^2)}\,dx \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$
-\frac{\cot^{-1}(x)}{x} + \frac{1}{2}\cot^{-1}(x)^2 - \log(x) + \frac{1}{2}\log\left(1+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(x^2*(1 + x^2)),x]
[Out] -(ArcCot[x]/x) + ArcCot[x]^2/2 - Log[x] + Log[1 + x^2]/2
```

Maple [A]
time = 0.16, size = 33, normalized size = 1.10

method	result
default	$-\mathrm{arccot}(x)\arctan(x)-\frac{\mathrm{arccot}(x)}{x}-\ln(x)+\frac{\ln(x^2+1)}{2}-\frac{\arctan(x)^2}{2}$
risch	$-\frac{\ln(ix+1)^2}{8}+\frac{(\ln(-ix+1)x-2i)\ln(ix+1)}{4x}-\frac{-2i\ln((-\pi+6i)x+6+i\pi)\pi x+2i\ln((-\pi-6i)x+6-i\pi)\pi x+\ln(-ix+1)^2x-4i\ln(-ix+1}{8x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arccot(x)*arctan(x)-arccot(x)/x-ln(x)+1/2*ln(x^2+1)-1/2*arctan(x)^2
```

Maxima [A]
time = 0.48, size = 29, normalized size = 0.97

$$-\left(\frac{1}{x}+\arctan(x)\right)\mathrm{arccot}(x)-\frac{1}{2}\arctan(x)^2+\frac{1}{2}\log(x^2+1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -(1/x + arctan(x))*arccot(x) - 1/2*arctan(x)^2 + 1/2*log(x^2 + 1) - log(x)
```

Fricas [A]
time = 4.09, size = 29, normalized size = 0.97

$$\frac{x\,\mathrm{arccot}(x)^2+x\log(x^2+1)-2\,x\log(x)-2\,\mathrm{arccot}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(x*arccot(x)^2 + x*log(x^2 + 1) - 2*x*log(x) - 2*arccot(x))/x
```

Sympy [A]
time = 0.18, size = 22, normalized size = 0.73

$$-\log(x)+\frac{\log(x^2+1)}{2}+\frac{\mathrm{acot}^2(x)}{2}-\frac{\mathrm{acot}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/x**2/(x**2+1),x)
[Out] -log(x) + log(x**2 + 1)/2 + acot(x)**2/2 - acot(x)/x
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.87

$$\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2-\frac{\arctan\left(\frac{1}{x}\right)}{x}+\frac{1}{2}\log\left(\frac{1}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/x)^2 - arctan(1/x)/x + 1/2*log(1/x^2 + 1)
```

Mupad [B]
time = 0.09, size = 26, normalized size = 0.87

$$\frac{\ln\left(x^2+1\right)}{2}-\ln\left(x\right)-\frac{\mathrm{acot}(x)}{x}+\frac{{\mathrm{acot}(x)}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x^2*(x^2 + 1)),x)
```

```
[Out] log(x^2 + 1)/2 - log(x) - acot(x)/x + acot(x)^2/2
```

3.44 $\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx$

Optimal. Leaf size=72

$$\frac{1}{2x}-\frac{\cot^{-1}(x)}{2x^2}-\frac{1}{2}i\cot^{-1}(x)^2+\frac{\text{ArcTan}(x)}{2}-\cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)-\frac{1}{2}i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)$$

```
[Out] 1/2/x-1/2*arccot(x)/x^2-1/2*I*arccot(x)^2+1/2*arctan(x)-arccot(x)*ln(2-2/(1
-I*x))-1/2*I*polylog(2,-1+2/(1-I*x))
```

Rubi [A]
time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {5039, 4947, 331, 209, 5045, 4989, 2497}

$$\frac{\text{ArcTan}(x)}{2}-\frac{1}{2}i\text{Li}_2\left(\frac{2}{1-ix}-1\right)-\frac{\cot^{-1}(x)}{2x^2}+\frac{1}{2x}-\frac{1}{2}i\cot^{-1}(x)^2-\log\left(2-\frac{2}{1-ix}\right)\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(x^3*(1 + x^2)),x]
```

```
[Out] 1/(2*x) - ArcCot[x]/(2*x^2) - (I/2)*ArcCot[x]^2 + ArcTan[x]/2 - ArcCot[x]*L
og[2 - 2/(1 - I*x)] - (I/2)*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4989

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Di
st[b*c*(p/d), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5045

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Dist[
I/d, Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(x)}{x^3(1+x^2)}\,dx &= \int \frac{\cot^{-1}(x)}{x^3}\,dx - \int \frac{\cot^{-1}(x)}{x(1+x^2)}\,dx \\
&= -\frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 - i\int \frac{\cot^{-1}(x)}{x(i+x)}\,dx - \frac{1}{2}\int \frac{1}{x^2(1+x^2)}\,dx \\
&= \frac{1}{2x} - \frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 - \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \int \frac{\log}{} \\
&= \frac{1}{2x} - \frac{\cot^{-1}(x)}{2x^2} - \frac{1}{2}i\cot^{-1}(x)^2 + \frac{1}{2}\tan^{-1}(x) - \cot^{-1}(x)\log\left(2-\frac{2}{1-ix}\right) - \frac{1}{2}i\mathrm{Li}_2\left(-1\right.
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.04, size = 280, normalized size = 3.89

$$-\frac{\cot^{-1}(x)}{2x^2}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x^2\right)}{2x}-\frac{1}{8}i\log^2(i-x)+\frac{1}{4}i\log(i-x)\log\left(-\frac{i-x}{x}\right)+\frac{1}{4}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right)-\frac{1}{4}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x)+\frac{1}{4}i\log\left(-\frac{i-x}{x}\right)\log(i+x)+\frac{1}{8}i\log^2(i+x)-\frac{1}{4}i\log(i-x)\log\left(\frac{i+x}{x}\right)-\frac{1}{4}i\log(i+x)\log\left(\frac{i+x}{x}\right)+\frac{1}{4}i\text{PolyLog}\left(2,-\frac{1}{2}i(i-x)\right)+\frac{1}{2}i\text{PolyLog}\left(2,-\frac{i}{x}\right)-\frac{1}{2}i\text{PolyLog}\left(2,\frac{i}{x}\right)-\frac{1}{4}i\text{PolyLog}\left(2,-\frac{1}{2}i(i+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(x^3*(1 + x^2)),x]
```

```
[Out] -1/2*ArcCot[x]/x^2 + Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x) - (I/8)*Lo
g[I - x]^2 + (I/4)*Log[I - x]*Log[-((I - x)/x)] + (I/4)*Log[I - x]*Log[(-1/
2*I)*(I + x)] - (I/4)*Log[(-1/2*I)*(I - x)]*Log[I + x] + (I/4)*Log[-((I - x
)/x)]*Log[I + x] + (I/8)*Log[I + x]^2 - (I/4)*Log[I - x]*Log[(I + x)/x] - (
I/4)*Log[I + x]*Log[(I + x)/x] + (I/4)*PolyLog[2, (-1/2*I)*(I - x)] + (I/2)
*PolyLog[2, (-I)/x] - (I/2)*PolyLog[2, I/x] - (I/4)*PolyLog[2, (-1/2*I)*(I
+ x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(58) = 116.
time = 0.14, size = 180, normalized size = 2.50

method	result
risch	$\frac{\pi\ln(x^2+1)}{4}-\frac{i\ln(-ix)}{4}-\frac{\pi}{4x^2}-\frac{\pi\ln(-ix)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ix}{2}\right)}{4}+\frac{i\ln(ix+1)^2}{8}-\frac{i\ln(-ix+1)^2}{8}-\frac{i\,\mathrm{dilog}(-ix+1)}{2}-\frac{i\ln\left(\frac{1}{2}+\right.}{}$
default	$-\frac{\mathrm{arccot}(x)}{2x^2}-\mathrm{arccot}(x)\ln(x)+\frac{\mathrm{arccot}(x)\ln(x^2+1)}{2}-\frac{i\ln(x-i)\ln(x^2+1)}{4}+\frac{i\ln(i+x)\ln(x^2+1)}{4}-\frac{i\ln(i+x)^2}{8}+\frac{i\ln(x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/x^3/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccot(x)/x^2-arccot(x)*ln(x)+1/2*arccot(x)*ln(x^2+1)-1/4*I*ln(x-I)*ln
(x^2+1)+1/4*I*ln(I+x)*ln(x^2+1)-1/2*I*dilog(1-I*x)+1/2*I*dilog(1+I*x)-1/8*I
*ln(I+x)^2+1/4*I*ln(x-I)*ln(-1/2*I*(I+x))+1/2*I*ln(x)*ln(1+I*x)+1/4*I*dilog
(-1/2*I*(I+x))+1/2/x+1/2*arctan(x)-1/4*I*ln(I+x)*ln(1/2*I*(x-I))-1/2*I*ln(x
)*ln(1-I*x)+1/8*I*ln(x-I)^2-1/4*I*dilog(1/2*I*(x-I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(x)/(x^5 + x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(x)}{x^3(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/x**3/(x**2+1),x)
```

```
[Out] Integral(acot(x)/(x**3*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^3/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(arccot(x)/((x^2 + 1)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(x)}{x^3\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x^3*(x^2 + 1)),x)
```

```
[Out] int(acot(x)/(x^3*(x^2 + 1)), x)
```

3.45 $\int \frac{\cot^{-1}(x)}{x^4(1+x^2)}\,dx$

Optimal. Leaf size=47

$$\frac{1}{6x^2}-\frac{\cot^{-1}(x)}{3x^3}+\frac{\cot^{-1}(x)}{x}-\frac{1}{2}\cot^{-1}(x)^2+\frac{4\log(x)}{3}-\frac{2}{3}\log\left(1+x^2\right)$$

```
[Out] 1/6/x^2-1/3*arccot(x)/x^3+arccot(x)/x-1/2*arccot(x)^2+4/3*ln(x)-2/3*ln(x^2+
1)
```

Rubi [A]
time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.615$, Rules used = {5039, 4947, 272, 46, 36, 29, 31, 5005}

$$-\frac{\cot^{-1}(x)}{3x^3}+\frac{1}{6x^2}-\frac{2}{3}\log\left(x^2+1\right)+\frac{4\log(x)}{3}-\frac{1}{2}\cot^{-1}(x)^2+\frac{\cot^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(x^4*(1 + x^2)),x]
```

```
[Out] 1/(6*x^2) - ArcCot[x]/(3*x^3) + ArcCot[x]/x - ArcCot[x]^2/2 + (4*Log[x])/3
- (2*Log[1 + x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(x)}{x^4\,(1+x^2)}\,dx &= \int \frac{\cot^{-1}(x)}{x^4}\,dx - \int \frac{\cot^{-1}(x)}{x^2\,(1+x^2)}\,dx \\
&= -\frac{\cot^{-1}(x)}{3x^3} - \frac{1}{3}\int \frac{1}{x^3\,(1+x^2)}\,dx - \int \frac{\cot^{-1}(x)}{x^2}\,dx + \int \frac{\cot^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, x^2\right) + \int \frac{1}{x\,(1+x
\\
&= -\frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 - \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x}\right)dx, x, x^2\right) + \frac{1}{2}
\\
&= \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,d
\\
&= \frac{1}{6x^2} - \frac{\cot^{-1}(x)}{3x^3} + \frac{\cot^{-1}(x)}{x} - \frac{1}{2}\cot^{-1}(x)^2 + \frac{4\log(x)}{3} - \frac{2}{3}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$\frac{1}{6x^2}-\frac{\cot^{-1}(x)}{3x^3}+\frac{\cot^{-1}(x)}{x}-\frac{1}{2}\cot^{-1}(x)^2+\frac{4\log(x)}{3}-\frac{2}{3}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(x^4*(1 + x^2)),x]
```

```
[Out] 1/(6*x^2) - ArcCot[x]/(3*x^3) + ArcCot[x]/x - ArcCot[x]^2/2 + (4*Log[x])/3
- (2*Log[1 + x^2])/3
```

Maple [A]
time = 0.15, size = 43, normalized size = 0.91

method	result
default	$-\frac{\operatorname{arccot}(x)}{3x^3}+\frac{\operatorname{arccot}(x)}{x}+\operatorname{arccot}(x)\arctan\left(x\right)+\frac{1}{6x^2}+\frac{4\ln(x)}{3}-\frac{2\ln\left(x^2+1\right)}{3}+\frac{\arctan(x)^2}{2}$
risch	$\frac{\ln(ix+1)^2}{8}-\frac{\left(3\ln(-ix+1)x^3-6ix^2+2i\right)\ln(ix+1)}{12x^3}+\frac{-6i\ln((-\pi+8i)x+8+i\pi)\pi\,x^3+6i\ln((-\pi-8i)x+8-i\pi)\pi\,x^3+3\ln(-ix+1)^2x^3-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccot(x)/x^3+arccot(x)/x+arccot(x)*arctan(x)+1/6/x^2+4/3*ln(x)-2/3*ln
(x^2+1)+1/2*arctan(x)^2
```

Maxima [A]
time = 0.47, size = 55, normalized size = 1.17

$$\frac{1}{3}\left(\frac{3\,x^2-1}{x^3}+3\,\arctan\left(x\right)\right)\operatorname{arccot}\left(x\right)+\frac{3\,x^2\arctan\left(x\right)^2-4\,x^2\log\left(x^2+1\right)+8\,x^2\log\left(x\right)+1}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*((3*x^2 - 1)/x^3 + 3*arctan(x))*arccot(x) + 1/6*(3*x^2*arctan(x)^2 - 4*
x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2
```

Fricas [A]
time = 2.13, size = 47, normalized size = 1.00

$$-\frac{3\,x^3\operatorname{arccot}\left(x\right)^2+4\,x^3\log\left(x^2+1\right)-8\,x^3\log\left(x\right)-2\left(3\,x^2-1\right)\operatorname{arccot}\left(x\right)-x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/6*(3*x^3*arccot(x)^2 + 4*x^3*log(x^2 + 1) - 8*x^3*log(x) - 2*(3*x^2 - 1)
*arccot(x) - x)/x^3
```

Sympy [A]
time = 0.30, size = 42, normalized size = 0.89

$$\frac{4\log(x)}{3} - \frac{2\log(x^2+1)}{3} - \frac{\operatorname{acot}^2(x)}{2} + \frac{\operatorname{acot}(x)}{x} + \frac{1}{6x^2} - \frac{\operatorname{acot}(x)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/x**4/(x**2+1),x)
```

```
[Out] 4*log(x)/3 - 2*log(x**2 + 1)/3 - acot(x)**2/2 + acot(x)/x + 1/(6*x**2) - ac
ot(x)/(3*x**3)
```

Giac [A]
time = 0.40, size = 39, normalized size = 0.83

$$-\frac{1}{2}\arctan\left(\frac{1}{x}\right)^2 + \frac{\arctan\left(\frac{1}{x}\right)}{x} + \frac{1}{6\,x^2} - \frac{\arctan\left(\frac{1}{x}\right)}{3\,x^3} - \frac{2}{3}\log\left(\frac{1}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/x)^2 + arctan(1/x)/x + 1/6/x^2 - 1/3*arctan(1/x)/x^3 - 2/3*lo
g(1/x^2 + 1)
```

Mupad [B]
time = 0.10, size = 35, normalized size = 0.74

$$\frac{4\ln(x)}{3} - \frac{2\ln(x^2+1)}{3} - \frac{\operatorname{acot}(x)^2}{2} + \frac{1}{6\,x^2} + \frac{\operatorname{acot}(x)\,\left(x^2-\frac{1}{3}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x^4*(x^2 + 1)),x)
```

```
[Out] (4*log(x))/3 - (2*log(x^2 + 1))/3 - acot(x)^2/2 + 1/(6*x^2) + (acot(x)*(x^2
 - 1/3))/x^3
```

3.46 $\int \frac{x^2 \cot^{-1}(cx)}{1+x^2} \, dx$

Optimal. Leaf size=206

$$x \cot^{-1}(cx) - \frac{1}{2} i \text{ArcTan}(x) \log \left(1 - \frac{i}{cx}\right) + \frac{1}{2} i \text{ArcTan}(x) \log \left(1 + \frac{i}{cx}\right) + \frac{1}{2} i \text{ArcTan}(x) \log \left(-\frac{2i(i - cx)}{(1-c)(1-ix)}\right.$$

```
[Out] x*arccot(c*x)-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)+1/2*I
*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*I*(c*x+I)/(
1+c)/(1-I*x))+1/2*ln(c^2*x^2+1)/c+1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I*x)
)-1/4*polylog(2,1+2*I*(c*x+I)/(1+c)/(1-I*x))
```

Rubi [A]
time = 0.44, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 15, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5037, 4931, 266, 5029, 209, 2520, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$-\frac{1}{2} i \text{ArcTan}(x) \log \left(1 - \frac{i}{cx}\right) + \frac{1}{2} i \text{ArcTan}(x) \log \left(1 + \frac{i}{cx}\right) + \frac{1}{2} i \text{ArcTan}(x) \log \left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right) - \frac{1}{2} i \text{ArcTan}(x) \log \left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right) + \frac{\log(c^2x^2+1)}{2c} + \frac{1}{4} \text{Li}_2\left(\frac{2i(i-cx)}{(1-c)(1-ix)} + 1\right) - \frac{1}{4} \text{Li}_2\left(\frac{2i(cx+i)}{(c+1)(1-ix)} + 1\right) + x \cot^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] x*ArcCot[c*x] - (I/2)*ArcTan[x]*Log[1 - I/(c*x)] + (I/2)*ArcTan[x]*Log[1 +
I/(c*x)] + (I/2)*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1 - I*x))] - (I
/2)*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] + Log[1 + c^2*x^2
]/(2*c) + PolyLog[2, 1 + ((2*I)*(I - c*x))/((1 - c)*(1 - I*x))]/4 - PolyLog
[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
```

```
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{x^2 \cot^{-1}(cx)}{1+x^2}\,dx = \int \cot^{-1}(cx)\,dx - \int \frac{\cot^{-1}(cx)}{1+x^2}\,dx$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx + \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx + c\int \frac{x}{1+c^2x^2}\,dx$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c}$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c}$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c}$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c}$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{\log\left(1+c^2x^2\right)}{2c}$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log$$
$$= x\cot^{-1}(cx) - \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right) + \frac{1}{2}i\tan^{-1}(x)\log$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 626 vs. $2(206) = 412$.
time = 1.12, size = 626, normalized size = 3.04

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] (c*x*ArcCot[c*x] - Log[1/(c*Sqrt[1 + 1/(c^2*x^2)]*x)] + (Sqrt[-c^2]*(2*ArcC
os[(1 + c^2)/(-1 + c^2)]*ArcTanh[Sqrt[-c^2]/(c*x)] - 4*ArcCot[c*x]*ArcTanh[
(c*x)/Sqrt[-c^2]] - (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2
]/(c*x)])*Log[(-2*(c^2 + I*Sqrt[-c^2])*(-I + c*x))/((-1 + c^2)*(Sqrt[-c^2]
- c*x))] - (ArcCos[(1 + c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)])
*Log[((2*I)*(I*c^2 + Sqrt[-c^2])*(I + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))
] + (ArcCos[(1 + c^2)/(-1 + c^2)] - (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] + (2*I)
*ArcTanh[(c*x)/Sqrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2])/(Sqrt[-1 + c^2]*E^(I*A
rcCot[c*x])*Sqrt[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + (ArcCos[(1 +
 c^2)/(-1 + c^2)] + (2*I)*ArcTanh[Sqrt[-c^2]/(c*x)] - (2*I)*ArcTanh[(c*x)/S
```

```
qrt[-c^2]])*Log[(Sqrt[2]*Sqrt[-c^2]*E^(I*ArcCot[c*x]))/(Sqrt[-1 + c^2]*Sqrt
[-1 - c^2 + (-1 + c^2)*Cos[2*ArcCot[c*x]]])] + I*(-PolyLog[2, ((1 + c^2 - (
2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt[-c^2] - c*x))] + Pol
yLog[2, ((1 + c^2 + (2*I)*Sqrt[-c^2])*(Sqrt[-c^2] + c*x))/((-1 + c^2)*(Sqrt
[-c^2] - c*x))])))/4)/c
```

Maple [A]
time = 0.23, size = 280, normalized size = 1.36

method	result
risch	$\frac{\pi x}{2}+\frac{i\pi}{2c}-\frac{i\ln(-icx+1)x}{2}+\frac{i\ln(icx+1)x}{2}-\frac{\pi\arctan(x)}{2}+\frac{\ln(c^2x^2+1)}{2c}-\frac{1}{c}+\frac{\ln\left(\frac{-icx-c}{-c-1}\right)\ln(-icx+1)}{4}+\frac{\mathrm{dil}}{}$
derivativedivides	$\frac{-\mathrm{arccot}(cx)\arctan(x)c^3+\mathrm{arccot}(cx)c^3x+c^3\left(\frac{\ln\left(c^2x^2+1\right)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{\left(x^2+1\right)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\mathrm{polylog}\left(2,\frac{(c-1)(}{\left(x^2+1\right.}\right.}{4}\right.}{}$
default	$\frac{-\mathrm{arccot}(cx)\arctan(x)c^3+\mathrm{arccot}(cx)c^3x+c^3\left(\frac{\ln\left(c^2x^2+1\right)}{2c}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{\left(x^2+1\right)(1+c)}\right)}{2}-\frac{\arctan(x)^2}{2}-\frac{\mathrm{polylog}\left(2,\frac{(c-1)(}{\left(x^2+1\right.}\right.}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-arccot(c*x)*arctan(x)*c^3+arccot(c*x)*c^3*x+c^3*(1/2*ln(c^2*x^2+1)/
c-1/2*I*arctan(x)*ln(1-(c-1)*(1+I*x)^2/(x^2+1)/(1+c))-1/2*arctan(x)^2-1/4*p
olylog(2,(c-1)*(1+I*x)^2/(x^2+1)/(1+c))+1/2*I*c/(c-1)*ln(1-(1+c)*(1+I*x)^2/
(x^2+1)/(c-1))*arctan(x)-1/2*I/(c-1)*ln(1-(1+c)*(1+I*x)^2/(x^2+1)/(c-1))*ar
ctan(x)+1/2*c/(c-1)*arctan(x)^2+1/4*c/(c-1)*polylog(2,(1+c)*(1+I*x)^2/(x^2+
1)/(c-1))-1/2/(c-1)*arctan(x)^2-1/4/(c-1)*polylog(2,(1+c)*(1+I*x)^2/(x^2+1)
/(c-1))))
```

Maxima [A]
time = 0.48, size = 189, normalized size = 0.92

$$(x-\arctan(x))\,\mathrm{arccot}(cx)-\frac{4\,c\arctan(cx)\arctan(x)-4\,c\arctan(x)\arctan\left(\frac{cx}{c-1},-\frac{1}{c-1}\right)+c\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2+2\,c+1}\right)-c\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2-2\,c+1}\right)+2\,c\mathrm{Li}_2\left(\frac{icx+c}{c+1}\right)+2\,c\mathrm{Li}_2\left(-\frac{icx-c}{c+1}\right)-2\,c\mathrm{Li}_2\left(\frac{icx+c}{c-1}\right)-2\,c\mathrm{Li}_2\left(-\frac{icx-c}{c-1}\right)-4\log(c^2x^2+1)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - arctan(x))*arccot(c*x) - 1/8*(4*c*arctan(c*x)*arctan(x) - 4*c*arctan(x
)*arctan2(c*x/(c - 1), -1/(c - 1)) + c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2
+ 2*c + 1)) - c*log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*c*dilog
((I*c*x + c)/(c + 1)) + 2*c*dilog(-(I*c*x - c)/(c + 1)) - 2*c*dilog((I*c*x
+ c)/(c - 1)) - 2*c*dilog(-(I*c*x - c)/(c - 1)) - 4*log(c^2*x^2 + 1))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="fricas")
[Out] integral(x^2*arccot(c*x)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{acot}(cx)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c*x)/(x**2+1),x)
[Out] Integral(x**2*acot(c*x)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c*x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x^2*arccot(c*x)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \operatorname{acot}(c\,x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acot(c*x))/(x^2 + 1),x)
[Out] int((x^2*acot(c*x))/(x^2 + 1), x)
```

3.47 $\int \frac{x \cot^{-1}(cx)}{1+x^2} \, dx$

Optimal. Leaf size=188

$$-\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}iL$$

```
[Out] -arccot(c*x)*ln(2/(1-I*c*x))+1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x)
)+1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,1-2/(1-I
*c*x))+1/4*I*polylog(2,1-2*I*c*(I-x)/(1-c)/(1-I*c*x))+1/4*I*polylog(2,1+2*I
*c*(I+x)/(1+c)/(1-I*c*x))
```

Rubi [A]
time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5049, 4967, 2449, 2352, 2497}

$$-\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{1-icx}\right)+\frac{1}{4}i\text{Li}_2\left(1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{4}i\text{Li}_2\left(\frac{2ic(x+i)}{(c+1)(1-icx)}+1\right)+\log\left(\frac{2}{1-icx}\right)\left(-\cot^{-1}(cx)\right)+\frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx)+\frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] -(ArcCot[c*x]*Log[2/(1 - I*c*x)]) + (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1
- c)*(1 - I*c*x))])/2 + (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 - I
*c*x))])/2 - (I/2)*PolyLog[2, 1 - 2/(1 - I*c*x)] + (I/4)*PolyLog[2, 1 - ((2
*I)*c*(I - x))/((1 - c)*(1 - I*c*x))] + (I/4)*PolyLog[2, 1 + ((2*I)*c*(I +
x))/((1 + c)*(1 - I*c*x))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5049

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx &= \int \left(-\frac{\cot^{-1}(cx)}{2(i-x)}+\frac{\cot^{-1}(cx)}{2(i+x)}\right)dx\\
&= -\left(\frac{1}{2}\int \frac{\cot^{-1}(cx)}{i-x}\,dx\right)+\frac{1}{2}\int \frac{\cot^{-1}(cx)}{i+x}\,dx\\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{2}\cot^{-1}(cx)\log\Bigg(-\\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{2}\cot^{-1}(cx)\log\Bigg(-\\
&= -\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)+\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)+\frac{1}{2}\cot^{-1}(cx)\log\Bigg(-
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 343, normalized size = 1.82

$$-\frac{1}{4}i\log(i-x)\log\left(-\frac{i(i-cx)}{1-c}\right)-\frac{1}{4}i\log(i+x)\log\left(-\frac{i(i-cx)}{1+c}\right)+\frac{1}{4}i\log(i-x)\log\left(-\frac{i-cx}{cx}\right)+\frac{1}{4}i\log(i+x)\log\left(-\frac{i-cx}{cx}\right)+\frac{1}{4}i\log(i+x)\log\left(-\frac{i(i+cx)}{1-c}\right)+\frac{1}{4}i\log(i-x)\log\left(-\frac{i(i+cx)}{1+c}\right)-\frac{1}{4}i\log(i-x)\log\left(\frac{i+cx}{cx}\right)-\frac{1}{4}i\log(i+x)\log\left(\frac{i+cx}{cx}\right)-\frac{1}{4}i\text{PolyLog}\left(2,\frac{ic(i-x)}{1-c}\right)+\frac{1}{4}i\text{PolyLog}\left(2,-\frac{ic(i-x)}{1+c}\right)+\frac{1}{4}i\text{PolyLog}\left(2,\frac{ic(i+x)}{1-c}\right)-\frac{1}{4}i\text{PolyLog}\left(2,-\frac{ic(i+x)}{1+c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCot[c*x])/(1 + x^2),x]
```

```
[Out] (-1/4*I)*Log[I - x]*Log[((-I)*(I - c*x))/(1 - c)] - (I/4)*Log[I + x]*Log[((
-I)*(I - c*x))/(1 + c)] + (I/4)*Log[I - x]*Log[-((I - c*x)/(c*x))] + (I/4)*
Log[I + x]*Log[-((I - c*x)/(c*x))] + (I/4)*Log[I + x]*Log[((-I)*(I + c*x))/
(1 - c)] + (I/4)*Log[I - x]*Log[((-I)*(I + c*x))/(1 + c)] - (I/4)*Log[I - x
]*Log[(I + c*x)/(c*x)] - (I/4)*Log[I + x]*Log[(I + c*x)/(c*x)] - (I/4)*Poly
```

```
Log[2, (I*c*(I - x))/(1 - c)] + (I/4)*PolyLog[2, ((-I)*c*(I - x))/(1 + c)]
+ (I/4)*PolyLog[2, (I*c*(I + x))/(1 - c)] - (I/4)*PolyLog[2, ((-I)*c*(I + x
))/(1 + c)]
```

Maple [A]
time = 0.17, size = 317, normalized size = 1.69

method	result
risch	$\frac{\pi \ln\left(-c^2+(-icx+1)^2-1+2icx\right)}{4} - \frac{i \operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} - \frac{i \ln(-icx+1) \ln\left(\frac{-icx-c}{-c-1}\right)}{4} - \frac{i \operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4} - \frac{i \ln(-icx+$
derivativedivides	$\frac{c^2 \ln\left(c^2x^2+c^2\right) \operatorname{arccot}(cx)}{2} + c^2 \Bigg(-\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+c^2\right)-i\left(-i \operatorname{dilog}\left(\frac{i(cx-i)-c-1}{-c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)-c-1}{-c-1}\right)\right)-i\Big(-i \operatorname{dilog}}{2}$
default	$\frac{c^2 \ln\left(c^2x^2+c^2\right) \operatorname{arccot}(cx)}{2} + c^2 \Bigg(-\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+c^2\right)-i\left(-i \operatorname{dilog}\left(\frac{i(cx-i)-c-1}{-c-1}\right)-i\ln(cx-i)\ln\left(\frac{i(cx-i)-c-1}{-c-1}\right)\right)-i\Big(-i \operatorname{dilog}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*ln(c^2*x^2+c^2)*arccot(c*x)+1/2*c^2*(-1/2*I*(ln(c*x-I)*ln(c^
2*x^2+c^2)-I*(-I*dilog((I*(c*x-I)-c-1)/(-c-1))-I*ln(c*x-I)*ln((I*(c*x-I)-c-
1)/(-c-1)))-I*(-I*dilog((I*(c*x-I)+c-1)/(c-1))-I*ln(c*x-I)*ln((I*(c*x-I)+c-
1)/(c-1))))+1/2*I*(ln(I+c*x)*ln(c^2*x^2+c^2)+I*(I*dilog((-I*(I+c*x)-c-1)/(-
c-1))+I*ln(I+c*x)*ln((-I*(I+c*x)-c-1)/(-c-1)))+I*(I*dilog((-I*(I+c*x)+c-1)/
(c-1))+I*ln(I+c*x)*ln((-I*(I+c*x)+c-1)/(c-1))))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x*arccot(c*x)/(x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x*arccot(c*x)/(x^2 + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{acot}(cx)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c*x)/(x**2+1),x)
[Out] Integral(x*acot(c*x)/(x**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c*x)/(x^2+1),x, algorithm="giac")
[Out] integrate(x*arccot(c*x)/(x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \operatorname{acot}(c\,x)}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acot(c*x))/(x^2 + 1),x)
[Out] int((x*acot(c*x))/(x^2 + 1), x)
```

3.48 $\int \frac{\cot^{-1}(cx)}{1+x^2} \, dx$

Optimal. Leaf size=183

$$\frac{1}{2}i\text{ArcTan}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\text{ArcTan}(x)\log\left(1+\frac{i}{cx}\right)-\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-ix)}\right)+\frac{1}{2}i\text{ArcTan}$$

```
[Out] 1/2*I*arctan(x)*ln(1-I/c/x)-1/2*I*arctan(x)*ln(1+I/c/x)-1/2*I*arctan(x)*ln(
-2*I*(I-c*x)/(1-c)/(1-I*x))+1/2*I*arctan(x)*ln(-2*I*(c*x+I)/(1+c)/(1-I*x))-
1/4*polylog(2,1+2*I*(I-c*x)/(1-c)/(1-I*x))+1/4*polylog(2,1+2*I*(c*x+I)/(1+c
)/(1-I*x))
```

Rubi [A]
time = 0.34, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5029, 209, 2520, 266, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\frac{1}{2}i\text{ArcTan}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\text{ArcTan}(x)\log\left(1+\frac{i}{cx}\right)-\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right)+\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right)-\frac{1}{4}\text{Li}_2\left(\frac{2i(i-cx)}{(1-c)(1-ix)}+1\right)+\frac{1}{4}\text{Li}_2\left(\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c*x]/(1 + x^2),x]
```

```
[Out] (I/2)*ArcTan[x]*Log[1 - I/(c*x)] - (I/2)*ArcTan[x]*Log[1 + I/(c*x)] - (I/2)
*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1 - I*x))] + (I/2)*ArcTan[x]*Lo
g[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] - PolyLog[2, 1 + ((2*I)*(I - c*x)
)/((1 - c)*(1 - I*x))]/4 + PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I
*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
```

```
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx &= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)+\frac{\int \frac{\tan^{-1}(x)}{\left(1-\frac{i}{cx}\right)x^2}\,dx}{2c}+\frac{\int \frac{\tan^{-1}(x)}{\left(1+\frac{i}{cx}\right)x^2}\,dx}{2c}\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)+\frac{\int \frac{c\tan^{-1}(x)}{x(-i+cx)}\,dx}{2c}+\frac{\int \frac{c\tan^{-1}(x)}{x(i+cx)}\,dx}{2c}\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x(-i+cx)}\,dx+\frac{1}{2}\int \frac{\tan^{-1}}{x(i}\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)+\frac{1}{2}\int \left(\frac{i\tan^{-1}(x)}{x}-\frac{ic\tan^{-1}(x)}{-i+cx}\right.\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-\frac{1}{2}(ic)\int \frac{\tan^{-1}(x)}{-i+cx}\,dx+\frac{1}{2}(ic)\int\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-}\right.\\
&= \frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-\frac{1}{2}i\tan^{-1}(x)\log\left(-\frac{2i(i-cx)}{(1-c)(1-}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 319, normalized size = 1.74

$$-\frac{1}{4}\log(i-x)\log\left(-\frac{i(i-cx)}{1-c}\right)+\frac{1}{4}\log(i+x)\log\left(-\frac{i(i-cx)}{1+c}\right)+\frac{1}{4}\log(i-x)\log\left(-\frac{i-cx}{cx}\right)-\frac{1}{4}\log(i+x)\log\left(-\frac{i-cx}{cx}\right)-\frac{1}{4}\log(i+x)\log\left(-\frac{i(i+cx)}{1-c}\right)+\frac{1}{4}\log(i-x)\log\left(-\frac{i(i+cx)}{1+c}\right)-\frac{1}{4}\log(i-x)\log\left(\frac{i+cx}{cx}\right)+\frac{1}{4}\log(i+x)\log\left(\frac{i+cx}{cx}\right)-\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{ic(i-x)}{1-c}\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,-\frac{ic(i-x)}{1+c}\right)-\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{ic(i+x)}{1-c}\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,-\frac{ic(i+x)}{1+c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c*x]/(1 + x^2),x]
```

```
[Out] -1/4*(Log[I - x]*Log[((-I)*(I - c*x))/(1 - c)]) + (Log[I + x]*Log[((-I)*(I
- c*x))/(1 + c)])/4 + (Log[I - x]*Log[-((I - c*x)/(c*x))])/4 - (Log[I + x]*
Log[-((I - c*x)/(c*x))])/4 - (Log[I + x]*Log[((-I)*(I + c*x))/(1 - c)])/4 +
 (Log[I - x]*Log[((-I)*(I + c*x))/(1 + c)])/4 - (Log[I - x]*Log[(I + c*x)/(
c*x)])/4 + (Log[I + x]*Log[(I + c*x)/(c*x)])/4 - PolyLog[2, (I*c*(I - x))/(
1 - c)]/4 + PolyLog[2, ((-I)*c*(I - x))/(1 + c)]/4 - PolyLog[2, (I*c*(I + x
))/(1 - c)]/4 + PolyLog[2, ((-I)*c*(I + x))/(1 + c)]/4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(153) = 306$.
time = 0.26, size = 335, normalized size = 1.83

method	result
risch	$\frac{\pi\arctan(x)}{2} - \frac{\ln\left(\frac{-icx-c}{-c-1}\right)\ln(-icx+1)}{4} - \frac{\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{\ln\left(\frac{-icx+c}{c-1}\right)\ln(-icx+1)}{4} + \frac{\operatorname{dilog}\left(\frac{-icx+c}{c-1}\right)}{4} - \cdots$
derivativedivides	$c\arctan(x)\operatorname{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{-\frac{ic\arctan(cx)\ln\left(1-\frac{(1+c)(icx+1)^2}{(c^2x^2+1)(1-c)}\right)}{2} - \frac{c\arctan(cx)^2}{2} - \frac{c\operatorname{polylog}\left(2, \frac{(1+c)(\cdots}{(c^2x^2\cdots}\right)}{4}\cdots}{\cdots}\right.$
default	$c\arctan(x)\operatorname{arccot}(cx)+c^2\left(\frac{\arctan(cx)\arctan(x)}{c} - \frac{-\frac{ic\arctan(cx)\ln\left(1-\frac{(1+c)(icx+1)^2}{(c^2x^2+1)(1-c)}\right)}{2} - \frac{c\arctan(cx)^2}{2} - \frac{c\operatorname{polylog}\left(2, \frac{(1+c)(\cdots}{(c^2x^2\cdots}\right)}{4}\cdots}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*arctan(x)*arccot(c*x)+c^2*(1/c*arctan(c*x)*arctan(x)-1/c^2*(-1/2*I*c
*arctan(c*x)*ln(1-(1+c)*(1+I*c*x)^2/(c^2*x^2+1)/(1-c))-1/2*c*arctan(c*x)^2-
1/4*c*polylog(2,(1+c)*(1+I*c*x)^2/(c^2*x^2+1)/(1-c))+1/2*I*c^2/(1+c)*ln(1-(
c-1)*(1+I*c*x)^2/(c^2*x^2+1)/(-c-1))*arctan(c*x)+1/2*I*c/(1+c)*arctan(c*x)*
ln(1-(c-1)*(1+I*c*x)^2/(c^2*x^2+1)/(-c-1))+1/2*c^2/(1+c)*arctan(c*x)^2+1/4*
c^2/(1+c)*polylog(2,(c-1)*(1+I*c*x)^2/(c^2*x^2+1)/(-c-1))+1/2*c/(1+c)*arcta
n(c*x)^2+1/4*c/(1+c)*polylog(2,(c-1)*(1+I*c*x)^2/(c^2*x^2+1)/(-c-1)))))
```

Maxima [**A**]
time = 0.49, size = 187, normalized size = 1.02

$$-\frac{1}{8}\,c\left(\frac{8\,\arctan\left(cx\right)\arctan\left(x\right)}{c} - \frac{4\,\arctan\left(cx\right)\arctan\left(x\right) - 4\,\arctan\left(x\right)\arctan\left(\frac{cx}{c-1}, -\frac{1}{c-1}\right) + \log\left(x^2+1\right)\log\left(\frac{c^2x^2+1}{c^2+2\,c+1}\right) - \log\left(x^2+1\right)\log\left(\frac{c^2x^2+1}{c^2-2\,c+1}\right) + 2\,\mathrm{Li}_2\left(\frac{i\,cx+c}{c+1}\right) + 2\,\mathrm{Li}_2\left(-\frac{i\,cx-c}{c+1}\right) - 2\,\mathrm{Li}_2\left(\frac{i\,cx+c}{c-1}\right) - 2\,\mathrm{Li}_2\left(-\frac{i\,cx-c}{c-1}\right)}{c}\right) + \operatorname{arccot}\left(cx\right)\arctan\left(x\right) + \arctan\left(cx\right)\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*c*(8*arctan(c*x)*arctan(x)/c - (4*arctan(c*x)*arctan(x) - 4*arctan(x)*
arctan2(c*x/(c - 1), -1/(c - 1)) + log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 + 2*
c + 1)) - log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 - 2*c + 1)) + 2*dilog((I*c*x
+ c)/(c + 1)) + 2*dilog(-(I*c*x - c)/(c + 1)) - 2*dilog((I*c*x + c)/(c - 1)
) - 2*dilog(-(I*c*x - c)/(c - 1)))/c) + arccot(c*x)*arctan(x) + arctan(c*x)
*arctan(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(c*x)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(cx)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c*x)/(x**2+1),x)
```

```
[Out] Integral(acot(c*x)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(arccot(c*x)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(c\,x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c*x)/(x^2 + 1),x)
```

```
[Out] int(acot(c*x)/(x^2 + 1), x)
```

3.49 $\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx$

Optimal. Leaf size=223

$$\cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right)-\frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right)-\frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{2ic(i+x)}{(1+c)(1-icx)}\right)-\frac{1}{2}i\mathrm{P}$$

```
[Out] arccot(c*x)*ln(2/(1-I*c*x))-1/2*arccot(c*x)*ln(2*I*c*(I-x)/(1-c)/(1-I*c*x))
-1/2*arccot(c*x)*ln(-2*I*c*(I+x)/(1+c)/(1-I*c*x))-1/2*I*polylog(2,-I/c/x)+1
/2*I*polylog(2,I/c/x)+1/2*I*polylog(2,1-2/(1-I*c*x))-1/4*I*polylog(2,1-2*I*
c*(I-x)/(1-c)/(1-I*c*x))-1/4*I*polylog(2,1+2*I*c*(I+x)/(1+c)/(1-I*c*x))
```

Rubi [A]
time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5049, 4941, 2438, 4967, 2449, 2352, 2497}

$$-\frac{1}{2}i\mathrm{Li}_2\left(-\frac{i}{cx}\right)+\frac{1}{2}i\mathrm{Li}_2\left(\frac{i}{cx}\right)+\frac{1}{2}i\mathrm{Li}_2\left(1-\frac{2}{1-icx}\right)-\frac{1}{4}i\mathrm{Li}_2\left(1-\frac{2ic(i-x)}{(1-c)(1-icx)}\right)-\frac{1}{4}i\mathrm{Li}_2\left(\frac{2ic(x+i)}{(c+1)(1-icx)}+1\right)+\log\left(\frac{2}{1-icx}\right)\cot^{-1}(cx)-\frac{1}{2}\log\left(\frac{2ic(-x+i)}{(1-c)(1-icx)}\right)\cot^{-1}(cx)-\frac{1}{2}\log\left(-\frac{2ic(x+i)}{(c+1)(1-icx)}\right)\cot^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c*x]/(x*(1 + x^2)),x]
```

```
[Out] ArcCot[c*x]*Log[2/(1 - I*c*x)] - (ArcCot[c*x]*Log[((2*I)*c*(I - x))/((1 - c
)*(1 - I*c*x))])/2 - (ArcCot[c*x]*Log[((-2*I)*c*(I + x))/((1 + c)*(1 - I*c*
x))])/2 - (I/2)*PolyLog[2, (-I)/(c*x)] + (I/2)*PolyLog[2, I/(c*x)] + (I/2)*
PolyLog[2, 1 - 2/(1 - I*c*x)] - (I/4)*PolyLog[2, 1 - ((2*I)*c*(I - x))/((1
- c)*(1 - I*c*x))] - (I/4)*PolyLog[2, 1 + ((2*I)*c*(I + x))/((1 + c)*(1 - I
*c*x))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5049

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCot[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(cx)}{x(1+x^2)}\,dx &= \int \left(\frac{\cot^{-1}(cx)}{x} - \frac{x\cot^{-1}(cx)}{1+x^2}\right) dx \\
&= \int \frac{\cot^{-1}(cx)}{x}\,dx - \int \frac{x\cot^{-1}(cx)}{1+x^2}\,dx \\
&= \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{x}\,dx - \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{x}\,dx - \int \left(-\frac{\cot^{-1}(cx)}{2(i-x)} + \frac{\cot^{-1}(cx)}{2(i+x)}\right) dx \\
&= -\frac{1}{2}i\mathrm{Li}_2\left(-\frac{i}{cx}\right) + \frac{1}{2}i\mathrm{Li}_2\left(\frac{i}{cx}\right) + \frac{1}{2}\int \frac{\cot^{-1}(cx)}{i-x}\,dx - \frac{1}{2}\int \frac{\cot^{-1}(cx)}{i+x}\,dx \\
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{}{(1}\right. \\
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{}{(1}\right. \\
&= \cot^{-1}(cx)\log\left(\frac{2}{1-icx}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(\frac{2ic(i-x)}{(1-c)(1-icx)}\right) - \frac{1}{2}\cot^{-1}(cx)\log\left(-\frac{}{(1}\right.
\end{aligned}
$$

<u>Mathematica</u> [**A**]
time = 0.07, size = 379, normalized size = 1.70

$$
\frac{1}{4}i\log(i-x)\log\left(-\frac{i(i-cx)}{1-c}\right) + \frac{1}{4}i\log(i+x)\log\left(-\frac{i(i-cx)}{1+c}\right) - \frac{1}{4}i\log(i-x)\log\left(-\frac{i-cx}{cx}\right) - \frac{1}{4}i\log(i+x)\log\left(-\frac{i-cx}{cx}\right) - \frac{1}{4}i\log(i+x)\log\left(-\frac{i(i+cx)}{1-c}\right) - \frac{1}{4}i\log(i-x)\log\left(-\frac{i(i+cx)}{1+c}\right) + \frac{1}{4}i\log(i-x)\log\left(\frac{i+cx}{cx}\right) + \frac{1}{4}i\log(i+x)\log\left(\frac{i+cx}{cx}\right) + \frac{1}{4}i\mathrm{PolyLog}\left(2,\frac{ic(i-x)}{1-c}\right) - \frac{1}{4}i\mathrm{PolyLog}\left(2,-\frac{ic(i-x)}{1+c}\right) - \frac{1}{2}i\mathrm{PolyLog}\left(2,-\frac{i}{cx}\right) + \frac{1}{2}i\mathrm{PolyLog}\left(2,\frac{i}{cx}\right) - \frac{1}{4}i\mathrm{PolyLog}\left(2,\frac{ic(i+x)}{1-c}\right) + \frac{1}{4}i\mathrm{PolyLog}\left(2,-\frac{ic(i+x)}{1+c}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c*x]/(x*(1 + x^2)),x]
```

```
[Out] (I/4)*Log[I - x]*Log[((-I)*(I - c*x))/(1 - c)] + (I/4)*Log[I + x]*Log[((-I)
*(I - c*x))/(1 + c)] - (I/4)*Log[I - x]*Log[-((I - c*x)/(c*x))] - (I/4)*Log
[I + x]*Log[-((I - c*x)/(c*x))] - (I/4)*Log[I + x]*Log[((-I)*(I + c*x))/(1
- c)] - (I/4)*Log[I - x]*Log[((-I)*(I + c*x))/(1 + c)] + (I/4)*Log[I - x]*L
og[(I + c*x)/(c*x)] + (I/4)*Log[I + x]*Log[(I + c*x)/(c*x)] + (I/4)*PolyLog
[2, (I*c*(I - x))/(1 - c)] - (I/4)*PolyLog[2, ((-I)*c*(I - x))/(1 + c)] - (
I/2)*PolyLog[2, (-I)/(c*x)] + (I/2)*PolyLog[2, I/(c*x)] - (I/4)*PolyLog[2,
(I*c*(I + x))/(1 - c)] + (I/4)*PolyLog[2, ((-I)*c*(I + x))/(1 + c)]
```

<u>Maple</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(188) = 376.
time = 0.12, size = 393, normalized size = 1.76

method	result
risch	$-\frac{\pi\ln(c^2x^2+c^2)}{4} + \frac{\pi\ln(-icx)}{2} + \frac{i\ln(-icx+1)\ln\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{i\,\mathrm{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4} + \frac{i\ln(-icx+1)\ln\left(\frac{-icx+c}{c-1}\right)}{4} +$

derivativedivides	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2}+\operatorname{arccot}(cx)\ln(cx)+\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2}+\frac{i\ln(cx)\ln(-icx+1)}{c^2}-\frac{i\operatorname{dilog}(icx+1)}{c^2}+\frac{i\,\mathrm{di}}{}\right.}{}$
default	$-\frac{\operatorname{arccot}(cx)\ln(c^2x^2+c^2)}{2}+\operatorname{arccot}(cx)\ln(cx)+\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2}+\frac{i\ln(cx)\ln(-icx+1)}{c^2}-\frac{i\operatorname{dilog}(icx+1)}{c^2}+\frac{i\,\mathrm{di}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c*x)/x/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccot(c*x)*ln(c^2*x^2+c^2)+arccot(c*x)*ln(c*x)+1/2*c^2*(-I/c^2*ln(c*x
)*ln(1+I*c*x)+I/c^2*ln(c*x)*ln(1-I*c*x)-I/c^2*dilog(1+I*c*x)+I/c^2*dilog(1-
I*c*x)+1/2*I/c^2*ln(c^2*x^2+c^2)*ln(c*x-I)-1/2*I/c^2*ln(c*x-I)*ln((I*(c*x-I
)+c-1)/(c-1))-1/2*I/c^2*ln(c*x-I)*ln((I*(c*x-I)-c-1)/(-c-1))-1/2*I/c^2*dilo
g((I*(c*x-I)+c-1)/(c-1))-1/2*I/c^2*dilog((I*(c*x-I)-c-1)/(-c-1))-1/2*I/c^2*
ln(c^2*x^2+c^2)*ln(I+c*x)+1/2*I/c^2*ln(I+c*x)*ln((-I*(I+c*x)+c-1)/(c-1))+1/
2*I/c^2*ln(I+c*x)*ln((-I*(I+c*x)-c-1)/(-c-1))+1/2*I/c^2*dilog((-I*(I+c*x)+c
-1)/(c-1))+1/2*I/c^2*dilog((-I*(I+c*x)-c-1)/(-c-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arccot(c*x)/(x^3 + x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(cx)}{x(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c*x)/x/(x**2+1),x)
```

```
[Out] Integral(acot(c*x)/(x*(x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(arccot(c*x)/((x^2 + 1)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(c\,x)}{x\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c*x)/(x*(x^2 + 1)),x)
```

```
[Out] int(acot(c*x)/(x*(x^2 + 1)), x)
```

3.50 $\int \frac{\cot^{-1}(cx)}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=212

$$-\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\text{ArcTan}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\text{ArcTan}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(}{(1-}\right.$$

```
[Out] -arccot(c*x)/x-1/2*I*arctan(x)*ln(1-I/c/x)+1/2*I*arctan(x)*ln(1+I/c/x)-c*ln
(x)+1/2*I*arctan(x)*ln(-2*I*(I-c*x)/(1-c)/(1-I*x))-1/2*I*arctan(x)*ln(-2*I*
(c*x+I)/(1+c)/(1-I*x))+1/2*c*ln(c^2*x^2+1)+1/4*polylog(2,1+2*I*(I-c*x)/(1-c
)/(1-I*x))-1/4*polylog(2,1+2*I*(c*x+I)/(1+c)/(1-I*x))
```

Rubi [A]
time = 0.37, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 19, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.267$, Rules used = {5039, 4947, 272, 36, 29, 31, 5029, 209, 2520, 266, 6820, 12, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$-\frac{1}{2}i\text{ArcTan}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\text{ArcTan}(x)\log\left(1+\frac{i}{cx}\right)+\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(-cx+i)}{(1-c)(1-ix)}\right)-\frac{1}{2}i\text{ArcTan}(x)\log\left(-\frac{2i(cx+i)}{(c+1)(1-ix)}\right)+\frac{1}{2}c\log\left(c^2x^2+1\right)+\frac{1}{4}\text{Li}_2\left(\frac{2i(i-cx)}{(1-c)(1-ix)}+1\right)-\frac{1}{4}\text{Li}_2\left(\frac{2i(cx+i)}{(c+1)(1-ix)}+1\right)-c\log(x)-\frac{\cot^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[c*x]/x) - (I/2)*ArcTan[x]*Log[1 - I/(c*x)] + (I/2)*ArcTan[x]*Log[1
 + I/(c*x)] - c*Log[x] + (I/2)*ArcTan[x]*Log[((-2*I)*(I - c*x))/((1 - c)*(1
 - I*x))] - (I/2)*ArcTan[x]*Log[((-2*I)*(I + c*x))/((1 + c)*(1 - I*x))] + (
c*Log[1 + c^2*x^2])/2 + PolyLog[2, 1 + ((2*I)*(I - c*x))/((1 - c)*(1 - I*x)
)]/4 - PolyLog[2, 1 + ((2*I)*(I + c*x))/((1 + c)*(1 - I*x))]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5039

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(cx)}{x^2\left(1+x^2\right)}\,dx &= \int \frac{\cot^{-1}(cx)}{x^2}\,dx-\int \frac{\cot^{-1}(cx)}{1+x^2}\,dx\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{cx}\right)}{1+x^2}\,dx+\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{cx}\right)}{1+x^2}\,dx-c\int \frac{1}{x\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-\frac{\int \frac{\tan^{-1}(x)}{\left(1-\frac{i}{cx}\right)x^2}\,dx}{2c}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-\frac{\int \frac{c\tan^{-1}(x)}{x(-i+cx)}\,dx}{2c}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}c\,\mathrm{l}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}c\,\mathrm{l}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}c\,\mathrm{l}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}i\,\mathrm{t}\\
&= -\frac{\cot^{-1}(cx)}{x}-\frac{1}{2}i\tan^{-1}(x)\log\left(1-\frac{i}{cx}\right)+\frac{1}{2}i\tan^{-1}(x)\log\left(1+\frac{i}{cx}\right)-c\log(x)+\frac{1}{2}i\,\mathrm{t}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 348, normalized size = 1.64

$$-\frac{\cot^{-1}(cx)}{x}-c\log(x)+\frac{1}{4}\log(i-x)\log\left(-\frac{i(i-cx)}{1-c}\right)-\frac{1}{4}\log(i+x)\log\left(-\frac{i(i-cx)}{1+c}\right)-\frac{1}{4}\log(i-x)\log\left(-\frac{i-cx}{cx}\right)+\frac{1}{4}\log(i+x)\log\left(-\frac{i-cx}{cx}\right)+\frac{1}{4}\log(i+x)\log\left(-\frac{i(i+cx)}{1-c}\right)-\frac{1}{4}\log(i-x)\log\left(-\frac{i(i+cx)}{1+c}\right)+\frac{1}{4}\log(i-x)\log\left(\frac{i+cx}{cx}\right)-\frac{1}{4}\log(i+x)\log\left(\frac{i+cx}{cx}\right)+\frac{1}{2}c\log\left(1+c^2x^2\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{ic(i-x)}{1-c}\right)-\frac{1}{4}\mathrm{PolyLog}\left(2,-\frac{ic(i-x)}{1+c}\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{ic(i+x)}{1-c}\right)-\frac{1}{4}\mathrm{PolyLog}\left(2,-\frac{ic(i+x)}{1+c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c*x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcCot[c*x]/x) - c*Log[x] + (Log[I - x]*Log[((-I)*(I - c*x))/(1 - c)])/4
- (Log[I + x]*Log[((-I)*(I - c*x))/(1 + c)])/4 - (Log[I - x]*Log[-((I - c*x
)/(c*x))])/4 + (Log[I + x]*Log[-((I - c*x)/(c*x))])/4 + (Log[I + x]*Log[((-
I)*(I + c*x))/(1 - c)])/4 - (Log[I - x]*Log[((-I)*(I + c*x))/(1 + c)])/4 +
(Log[I - x]*Log[(I + c*x)/(c*x)])/4 - (Log[I + x]*Log[(I + c*x)/(c*x)])/4 +
```

```
 (c*Log[1 + c^2*x^2])/2 + PolyLog[2, (I*c*(I - x))/(1 - c)]/4 - PolyLog[2,
((-I)*c*(I - x))/(1 + c)]/4 + PolyLog[2, (I*c*(I + x))/(1 - c)]/4 - PolyLog
[2, ((-I)*c*(I + x))/(1 + c)]/4
```

Maple [A]
time = 0.12, size = 312, normalized size = 1.47

method	result
risch	$-\frac{i\ln(icx+1)}{2x}-\frac{\pi\arctan(x)}{2}+\frac{i\ln(-icx+1)}{2x}-\frac{\pi}{2x}+\frac{\ln\left(\frac{-icx-c}{-c-1}\right)\ln(-icx+1)}{4}+\frac{\operatorname{dilog}\left(\frac{-icx-c}{-c-1}\right)}{4}-\frac{\ln\left(\frac{-icx+c}{c-1}\right)}{}$
derivativedivides	$c\left(-\frac{\operatorname{arccot}(cx)}{cx}-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}+c^3\left(\frac{\ln(c^2x^2+1)}{2c^3}-\frac{\ln(x)}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\operatorname{arct}\right.\right.$
default	$c\left(-\frac{\operatorname{arccot}(cx)}{cx}-\frac{\operatorname{arccot}(cx)\arctan(x)}{c}+c^3\left(\frac{\ln(c^2x^2+1)}{2c^3}-\frac{\ln(x)}{c^3}-\frac{i\arctan(x)\ln\left(1-\frac{(c-1)(ix+1)^2}{(x^2+1)(1+c)}\right)}{2c^4}-\operatorname{arct}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-arccot(c*x)/c/x-arccot(c*x)/c*arctan(x)+c^3*(1/2/c^3*ln(c^2*x^2+1)-1/c^
3*ln(x)-1/2*I/c^4*arctan(x)*ln(1-(c-1)*(1+I*x)^2/(x^2+1)/(1+c))-1/2/c^4*arc
tan(x)^2-1/4/c^4*polylog(2,(c-1)*(1+I*x)^2/(x^2+1)/(1+c))+1/2*I/c^3/(c-1)*l
n(1-(1+c)*(1+I*x)^2/(x^2+1)/(c-1))*arctan(x)-1/2*I/c^4/(c-1)*ln(1-(1+c)*(1+
I*x)^2/(x^2+1)/(c-1))*arctan(x)+1/2/c^3/(c-1)*arctan(x)^2+1/4/c^3/(c-1)*pol
ylog(2,(1+c)*(1+I*x)^2/(x^2+1)/(c-1))-1/2/c^4/(c-1)*arctan(x)^2-1/4/c^4/(c-
1)*polylog(2,(1+c)*(1+I*x)^2/(x^2+1)/(c-1))))
```

Maxima [A]
time = 0.49, size = 183, normalized size = 0.86

$$-\left(\frac{1}{x}+\arctan(x)\right)\operatorname{arccot}(cx)-\frac{1}{2}\arctan(cx)\arctan(x)+\frac{1}{2}\arctan(x)\arctan\left(\frac{cx}{c-1},-\frac{1}{c-1}\right)+\frac{1}{2}c\log(c^2x^2+1)-c\log(x)-\frac{1}{8}\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2+2c+1}\right)+\frac{1}{8}\log(x^2+1)\log\left(\frac{c^2x^2+1}{c^2-2c+1}\right)-\frac{1}{4}\operatorname{Li}_2\left(\frac{icx+c}{c+1}\right)-\frac{1}{4}\operatorname{Li}_2\left(-\frac{icx-c}{c+1}\right)+\frac{1}{4}\operatorname{Li}_2\left(\frac{icx+c}{c-1}\right)+\frac{1}{4}\operatorname{Li}_2\left(-\frac{icx-c}{c-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arccot(c*x) - 1/2*arctan(c*x)*arctan(x) + 1/2*arctan(x)*
arctan2(c*x/(c - 1), -1/(c - 1)) + 1/2*c*log(c^2*x^2 + 1) - c*log(x) - 1/8*
log(x^2 + 1)*log((c^2*x^2 + 1)/(c^2 + 2*c + 1)) + 1/8*log(x^2 + 1)*log((c^2
*x^2 + 1)/(c^2 - 2*c + 1)) - 1/4*dilog((I*c*x + c)/(c + 1)) - 1/4*dilog(-(I
*c*x - c)/(c + 1)) + 1/4*dilog((I*c*x + c)/(c - 1)) + 1/4*dilog(-(I*c*x - c
)/(c - 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] integral(arccot(c*x)/(x^4 + x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(cx\right)}{x^{2}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c*x)/x**2/(x**2+1),x)
[Out] Integral(acot(c*x)/(x**2*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c*x)/x^2/(x^2+1),x, algorithm="giac")
[Out] integrate(arccot(c*x)/((x^2 + 1)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(c\,x)}{x^2\,\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c*x)/(x^2*(x^2 + 1)),x)
[Out] int(acot(c*x)/(x^2*(x^2 + 1)), x)
```

3.51 $\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx$

Optimal. Leaf size=5

$$-\log\left(\cot^{-1}(x)\right)$$

```
[Out] -ln(arccot(x))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5003}

$$-\log\left(\cot^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*ArcCot[x]),x]
```

```
[Out] -Log[ArcCot[x]]
```

Rule 5003

```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{(1+x^2)\cot^{-1}(x)}\,dx = -\log\left(\cot^{-1}(x)\right)$$

Mathematica [A]
time = 0.02, size = 5, normalized size = 1.00

$$-\log\left(\cot^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*ArcCot[x]),x]
```

```
[Out] -Log[ArcCot[x]]
```

Maple [A]
time = 0.11, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$-\ln(\operatorname{arccot}(x))$	6
default	$-\ln(\operatorname{arccot}(x))$	6
risch	$-\ln(\ln(ix+1)+i(i\ln(-ix+1)-\pi))$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/arccot(x),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(arccot(x))
```

Maxima [**A**]
time = 0.28, size = 5, normalized size = 1.00

$$-\log(\operatorname{arccot}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="maxima")
```

```
[Out] -log(arccot(x))
```

Fricas [**A**]
time = 6.54, size = 5, normalized size = 1.00

$$-\log(\operatorname{arccot}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="fricas")
```

```
[Out] -log(arccot(x))
```

Sympy [**A**]
time = 0.10, size = 5, normalized size = 1.00

$$-\log(\operatorname{acot}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/acot(x),x)
```

```
[Out] -log(acot(x))
```

Giac [**A**]
time = 0.40, size = 8, normalized size = 1.60

$$-\log\left(\left|\arctan\left(\frac{1}{x}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/arccot(x),x, algorithm="giac")
```

```
[Out] -log(abs(arctan(1/x)))
```

Mupad [B]
time = 0.59, size = 5, normalized size = 1.00

$$-\ln(\mathrm{acot}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acot(x)*(x^2 + 1)),x)
```

```
[Out] -log(acot(x))
```

3.52 $\int \frac{\cot^{-1}(x)^n}{1+x^2} dx$

Optimal. Leaf size=13

$$-\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

```
[Out] -arccot(x)^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5005}

$$-\frac{\cot^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]^n/(1 + x^2),x]
```

```
[Out] -(ArcCot[x]^(1 + n)/(1 + n))
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^{-1}(x)^n}{1+x^2} dx = -\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\cot^{-1}(x)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]^n/(1 + x^2),x]
```

```
[Out] -(ArcCot[x]^(1 + n)/(1 + n))
```

Maple [A]
time = 0.18, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}(x)^{1+n}}{1+n}$	14
default	$-\frac{\operatorname{arccot}(x)^{1+n}}{1+n}$	14
risch	$-\frac{(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))\left(\frac{1}{2}\right)^n(\pi-i\ln(-i(i+x))+i\ln(-i(i-x)))^n}{2(1+n)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)^n/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(x)^(1+n)/(1+n)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 1.00

$$-\frac{\operatorname{arccot}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="maxima")
```

```
[Out] -arccot(x)^(n + 1)/(n + 1)
```

Fricas [A]
time = 2.81, size = 13, normalized size = 1.00

$$-\frac{\operatorname{arccot}(x)^{n}\operatorname{arccot}(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="fricas")
```

```
[Out] -arccot(x)^n*arccot(x)/(n + 1)
```

Sympy [A]
time = 0.92, size = 17, normalized size = 1.31

$$-\begin{cases}\frac{\operatorname{acot}^{n+1}(x)}{n+1} & \text{for}\ n\neq -1\\ \log(\operatorname{acot}(x)) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)**n/(x**2+1),x)
```

```
[Out] -Piecewise((acot(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acot(x)), True))
```

Giac [A]
time = 0.41, size = 15, normalized size = 1.15

$$-\frac{\arctan\left(\frac{1}{x}\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^n/(x^2+1),x, algorithm="giac")
```

```
[Out] -arctan(1/x)^(n + 1)/(n + 1)
```

Mupad [B]
time = 0.61, size = 13, normalized size = 1.00

$$-\frac{\operatorname{acot}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)^n/(x^2 + 1),x)
```

```
[Out] -acot(x)^(n + 1)/(n + 1)
```

3.53 $\int \left(c+dx^2\right)^4 \cot^{-1}(ax) \, dx$

Optimal. Leaf size=244

$$\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2-180a^2cd+35d^2)\,x^4}{1260a^5}+\frac{(36a^2c-7d)\,d^3x^6}{378a^3}+\frac{d^4x^8}{72a}+$$

```
[Out] 1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7+1/1260*d^2
*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5+1/378*(36*a^2*c-7*d)*d^3*x^6/a^3+
1/72*d^4*x^8/a+c^4*x*arccot(a*x)+4/3*c^3*d*x^3*arccot(a*x)+6/5*c^2*d^2*x^5*
arccot(a*x)+4/7*c*d^3*x^7*arccot(a*x)+1/9*d^4*x^9*arccot(a*x)+1/630*(315*a^
8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1)/a^9
```

Rubi [A]
time = 0.12, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 5033, 1824, 266}

$$\frac{d^3x^6(36a^2c-7d)}{378a^3}+\frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{1260a^5}+\frac{dx^2(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)}{630a^7}+\frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log\left(a^2x^2+1\right)}{630a^9}+c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\frac{1}{9}d^4x^9\cot^{-1}(ax)+\frac{d^4x^8}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCot[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + ((36*a^2*c - 7*
d)*d^3*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcCot[a*x] + (4*c^3*d*x^3
*ArcCot[a*x])/3 + (6*c^2*d^2*x^5*ArcCot[a*x])/5 + (4*c*d^3*x^7*ArcCot[a*x])
/7 + (d^4*x^9*ArcCot[a*x])/9 + ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*
d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(630*a^9)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(c+dx^2\right)^4 \cot^{-1}(ax)\,dx &= c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\\
&= c^4x\cot^{-1}(ax)+\frac{4}{3}c^3dx^3\cot^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cot^{-1}(ax)+\frac{4}{7}cd^3x^7\cot^{-1}(ax)+\\
&= \frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2-180a^2cd+35d^2}{1260a^5}\\
&= \frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2-180a^2cd+35d^2}{1260a^5}
\end{aligned}
$$

Mathematica [A]

time = 0.06, size = 212, normalized size = 0.87

$$
\frac{a^2dx^2(-420d^3+30a^2d^2(72c+7dx^2)-4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240cd^2x^4+35d^3x^6))+24a^9x(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8)\cot^{-1}(ax)+12(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log\left(1+a^2x^2\right)}{7560a^9}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCot[a*x],x]
```

```
[Out] (a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134*c^2 + 27
0*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
 35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c
*d^3*x^6 + 35*d^4*x^8)*ArcCot[a*x] + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*
a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/(7560*a^9)
```

Maple [A]

time = 0.14, size = 254, normalized size = 1.04

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)c^4ax+\frac{4a\operatorname{arccot}(ax)c^3dx^3}{3}+\frac{6a\operatorname{arccot}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccot}(ax)cd^3x^7}{7}+\frac{a\operatorname{arccot}(ax)d^4x^9}{9}+\frac{210c^3a^8dx^2+\frac{189c^2a^8d^2}{2}}{}}{}$
default	$\frac{\operatorname{arccot}(ax)c^4ax+\frac{4a\operatorname{arccot}(ax)c^3dx^3}{3}+\frac{6a\operatorname{arccot}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccot}(ax)cd^3x^7}{7}+\frac{a\operatorname{arccot}(ax)d^4x^9}{9}+\frac{210c^3a^8dx^2+\frac{189c^2a^8d^2}{2}}{}}{}$
risch	$-\frac{2ic^3dx^3\ln(-iax+1)}{3}+\frac{d^4x^8}{72a}+\frac{i\left(35d^4x^9+180cd^3x^7+378c^2d^2x^5+420c^3dx^3+315c^4x\right)\ln(iax+1)}{630}-\frac{2icd^3x^7\ln(}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)*c^4*a*x+4/3*a*arccot(a*x)*c^3*d*x^3+6/5*a*arccot(a*x)*c^2*
d^2*x^5+4/7*a*arccot(a*x)*c*d^3*x^7+1/9*a*arccot(a*x)*d^4*x^9+1/315/a^8*(21
0*c^3*a^8*d*x^2+189/2*c^2*a^8*d^2*x^4-189*c^2*a^6*d^2*x^2+30*c*a^8*d^3*x^6-
45*a^6*c*d^3*x^4+35/8*d^4*a^8*x^8+90*a^4*c*d^3*x^2-35/6*d^4*a^6*x^6+35/4*d^
4*a^4*x^4-35/2*d^4*a^2*x^2+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-1
80*a^2*c*d^3+35*d^4)*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 226, normalized size = 0.93

$$\frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3-7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2-180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d-378\,a^4c^2d^2+180\,a^2cd^3-35\,d^4)x^2}{a^8}+\frac{12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(a^2x^2+1\right)}{a^{10}}\right)+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\operatorname{arccot}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1/315*
(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*
arccot(a*x)
```

Fricas [A]
time = 3.01, size = 237, normalized size = 0.97

$$\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3-7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2-180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d-378\,a^6c^2d^2+180\,a^4cd^3-35\,a^2d^4)x^2+24\,(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x)\operatorname{arccot}\left(ax\right)+12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(a^2x^2+1\right)}{7560\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 + 24*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*x
^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arccot(a*x) +
 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4
)*log(a^2*x^2 + 1))/a^9
```

Sympy [A]
time = 0.73, size = 367, normalized size = 1.50

$$\begin{cases} c^4x\operatorname{acot}\left(ax\right)+\frac{4c^3dx^3\operatorname{acot}\left(ax\right)}{3}+\frac{6c^2d^2x^5\operatorname{acot}\left(ax\right)}{5}+\frac{4cd^3x^7\operatorname{acot}\left(ax\right)}{7}+\frac{d^4x^9\operatorname{acot}\left(ax\right)}{9}+\frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a}+\frac{2c^3dx^2}{3a}+\frac{3c^2d^2x^4}{10a}+\frac{2cd^3x^6}{21a}+\frac{d^4x^8}{72a}-\frac{2c^3d\log\left(x^2+\frac{1}{a^2}\right)}{3a^3}-\frac{3c^2d^2x^2}{5a^3}-\frac{cd^3x^4}{7a^3}-\frac{d^4x^6}{54a^3}+\frac{3c^2d^2\log\left(x^2+\frac{1}{a^2}\right)}{5a^5}+\frac{2cd^3x^2}{7a^5}+\frac{d^4x^4}{36a^5}-\frac{2cd^3\log\left(x^2+\frac{1}{a^2}\right)}{7a^7}-\frac{d^4x^2}{18a^7}+\frac{d^4\log\left(x^2+\frac{1}{a^2}\right)}{18a^9} & \text{for}\: a \neq 0 \\ \frac{\pi\left(c^4x+\frac{4c^3dx^3}{3}+\frac{6c^2d^2x^5}{5}+\frac{4cd^3x^7}{7}+\frac{d^4x^9}{9}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acot(a*x),x)
```

```
[Out] Piecewise((c**4*x*acot(a*x) + 4*c**3*d*x**3*acot(a*x)/3 + 6*c**2*d**2*x**5*
acot(a*x)/5 + 4*c*d**3*x**7*acot(a*x)/7 + d**4*x**9*acot(a*x)/9 + c**4*log(
x**2 + a**(-2))/(2*a) + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**4/(10*a) + 2*c
*d**3*x**6/(21*a) + d**4*x**8/(72*a) - 2*c**3*d*log(x**2 + a**(-2))/(3*a**3
) - 3*c**2*d**2*x**2/(5*a**3) - c*d**3*x**4/(7*a**3) - d**4*x**6/(54*a**3)
+ 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*
x**4/(36*a**5) - 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) - d**4*x**2/(18*a**7
) + d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (pi*(c**4*x + 4*c**3*d*x
**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, True))
```

Giac [A]
time = 0.42, size = 347, normalized size = 1.42

$$\frac{1}{7560}\left(\frac{24\left(35\,d^4+\frac{180\,cd^3}{x^2}+\frac{378\,c^2d^2}{x^4}+\frac{420\,c^3d}{x^6}+\frac{315\,c^4}{x^8}\right)x^9\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(105\,d^4+\frac{720\,cd^3}{x^2}+\frac{2268\,c^2d^2}{x^4}-\frac{140\,d^4}{a^2x^2}+\frac{5040\,c^3d}{x^6}-\frac{1080\,cd^3}{a^2x^4}+\frac{7875\,c^4}{x^8}-\frac{4536\,c^2d^2}{a^2x^6}+\frac{210\,d^4}{a^4x^4}-\frac{10500\,c^3d}{a^2x^8}+\frac{2160\,cd^3}{a^4x^6}+\frac{9450\,c^2d^2}{a^4x^8}-\frac{420\,d^4}{a^6x^6}-\frac{4500\,cd^3}{a^6x^8}+\frac{875\,d^4}{a^8x^8}\right)x^8}{a^2}+\frac{12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(\frac{1}{a^2x^2}+1\right)}{a^{10}}-\frac{12\,(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4)\log\left(\frac{1}{a^2x^2}\right)}{a^{10}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/7560*(24*(35*d^4 + 180*c*d^3/x^2 + 378*c^2*d^2/x^4 + 420*c^3*d/x^6 + 315*
c^4/x^8)*x^9*arctan(1/(a*x))/a + (105*d^4 + 720*c*d^3/x^2 + 2268*c^2*d^2/x^
4 - 140*d^4/(a^2*x^2) + 5040*c^3*d/x^6 - 1080*c*d^3/(a^2*x^4) + 7875*c^4/x^
8 - 4536*c^2*d^2/(a^2*x^6) + 210*d^4/(a^4*x^4) - 10500*c^3*d/(a^2*x^8) + 21
60*c*d^3/(a^4*x^6) + 9450*c^2*d^2/(a^4*x^8) - 420*d^4/(a^6*x^6) - 4500*c*d^
3/(a^6*x^8) + 875*d^4/(a^8*x^8))*x^8/a^2 + 12*(315*a^8*c^4 - 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(1/(a^2*x^2) + 1)/a^10 - 12*
(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*lo
g(1/(a^2*x^2))/a^10)*a
```

Mupad [B]
time = 0.22, size = 234, normalized size = 0.96

$$\mathrm{acot}(a\,x)\,\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)-x^2\,\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)-x^6\,\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)+x^4\,\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)+\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}+\frac{d^4\,x^8}{72\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)*(c + d*x^2)^4,x)
```

```
[Out] acot(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^2
*d^2*x^5)/5) - x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a
))/(2*a^2) - (2*c^3*d)/(3*a)) - x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) + x^4
*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) + (log(a^2*
x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a^4*c^
2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

3.54 $\int (c+dx^2)^3 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=168

$$\frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5}+\frac{(21a^2c-5d)\,d^2x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+$$

```
[Out] 1/70*d*(35*a^4*c^2-21*a^2*c*d+5*d^2)*x^2/a^5+1/140*(21*a^2*c-5*d)*d^2*x^4/a
^3+1/42*d^3*x^6/a+c^3*x*arccot(a*x)+c^2*d*x^3*arccot(a*x)+3/5*c*d^2*x^5*arc
cot(a*x)+1/7*d^3*x^7*arccot(a*x)+1/70*(35*a^6*c^3-35*a^4*c^2*d+21*a^2*c*d^2
-5*d^3)*ln(a^2*x^2+1)/a^7
```

Rubi [A]
time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 5033, 1824, 266}

$$\frac{d^2x^4(21a^2c-5d)}{140a^3}+\frac{dx^2(35a^4c^2-21a^2cd+5d^2)}{70a^5}+\frac{(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log(a^2x^2+1)}{70a^7}+c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)+\frac{d^3x^6}{42a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCot[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 - 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + ((21*a^2*c - 5*d)*d^2*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCot[a*x] + c^2*d*x^3*ArcCot[a*
x] + (3*c*d^2*x^5*ArcCot[a*x])/5 + (d^3*x^7*ArcCot[a*x])/7 + ((35*a^6*c^3 -
 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[1 + a^2*x^2])/(70*a^7)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
```

```
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \cot^{-1}(ax)\,dx = c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)+a\int$$

$$= c^3x\cot^{-1}(ax)+c^2dx^3\cot^{-1}(ax)+\frac{3}{5}cd^2x^5\cot^{-1}(ax)+\frac{1}{7}d^3x^7\cot^{-1}(ax)+a\int$$

$$= \frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5}+\frac{(21a^2c-5d)\,d^2x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\cot^{-1}(ax)+c$$

$$= \frac{d(35a^4c^2-21a^2cd+5d^2)\,x^2}{70a^5}+\frac{(21a^2c-5d)\,d^2x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\cot^{-1}(ax)+c$$

Mathematica [A]

time = 0.04, size = 149, normalized size = 0.89

$$\frac{a^2dx^2(30d^2-3a^2d(42c+5dx^2)+a^4(210c^2+63cdx^2+10d^2x^4))+12a^7x(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6)\cot^{-1}(ax)+6(35a^6c^3-35a^4c^2d+21a^2cd^2-5d^3)\log\left(1+a^2x^2\right)}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCot[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 - 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcCot[a*x] + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a^2*c*d^2 - 5*d^3)*Log[1 +
 a^2*x^2])/(420*a^7)
```

Maple [A]

time = 0.13, size = 175, normalized size = 1.04

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)c^3ax+a\operatorname{arccot}(ax)c^2d\,x^3+\frac{3a\operatorname{arccot}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccot}(ax)d^3x^7}{7}+\frac{\frac{35c^2a^6d\,x^2}{2}+\frac{21c\,a^6d^2x^4}{4}-\frac{21c\,a^4d^2x^2}{2}+\frac{5d^3a^6}{6}}{a}$
default	$\frac{\operatorname{arccot}(ax)c^3ax+a\operatorname{arccot}(ax)c^2d\,x^3+\frac{3a\operatorname{arccot}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccot}(ax)d^3x^7}{7}+\frac{\frac{35c^2a^6d\,x^2}{2}+\frac{21c\,a^6d^2x^4}{4}-\frac{21c\,a^4d^2x^2}{2}+\frac{5d^3a^6}{6}}{a}$
risch	$\frac{i\left(5d^3x^7+21c\,d^2x^5+35c^2d\,x^3+35c^3x\right)\ln(iax+1)}{70}-\frac{ic^3x\ln(-iax+1)}{2}+\frac{\pi\,d^3x^7}{14}-\frac{id^3x^7\ln(-iax+1)}{14}+\frac{3\pi c\,d^2x^5}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)*c^3*a*x+a*arccot(a*x)*c^2*d*x^3+3/5*a*arccot(a*x)*c*d^2*x^
5+1/7*a*arccot(a*x)*d^3*x^7+1/35/a^6*(35/2*c^2*a^6*d*x^2+21/4*c*a^6*d^2*x^4
```

```
-21/2*c*a^4*d^2*x^2+5/6*d^3*a^6*x^6-5/4*d^3*a^4*x^4+5/2*d^3*a^2*x^2+1/2*(35
*a^6*c^3-35*a^4*c^2*d+21*a^2*c*d^2-5*d^3)*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 159, normalized size = 0.95

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\,(21\,a^4cd^2-5\,a^2d^3)x^4+6\,(35\,a^4c^2d-21\,a^2cd^2+5\,d^3)x^2}{a^6}+\frac{6\,(35\,a^6c^3-35\,a^4c^2d+21\,a^2cd^2-5\,d^3)\log\left(a^2x^2+1\right)}{a^8}\right)+\frac{1}{35}\,(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x)\,\mathrm{arccot}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 - 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d - 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 - 35*a^4*c^2*d + 21*a^2
*c*d^2 - 5*d^3)*log(a^2*x^2 + 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35
*c^2*d*x^3 + 35*c^3*x)*arccot(a*x)
```

Fricas [A]
time = 4.93, size = 167, normalized size = 0.99

$$\frac{10\,a^6d^3x^6+3\,(21\,a^6cd^2-5\,a^4d^3)x^4+6\,(35\,a^6c^2d-21\,a^4cd^2+5\,a^2d^3)x^2+12\,(5\,a^7d^3x^7+21\,a^7cd^2x^5+35\,a^7c^2dx^3+35\,a^7c^3x)\,\mathrm{arccot}\left(ax\right)+6\,(35\,a^6c^3-35\,a^4c^2d+21\,a^2cd^2-5\,d^3)\log\left(a^2x^2+1\right)}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 - 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
- 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 12*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35
*a^7*c^2*d*x^3 + 35*a^7*c^3*x)*arccot(a*x) + 6*(35*a^6*c^3 - 35*a^4*c^2*d +
 21*a^2*c*d^2 - 5*d^3)*log(a^2*x^2 + 1))/a^7
```

Sympy [A]
time = 0.50, size = 243, normalized size = 1.45

$$\begin{cases} c^3x\,\mathrm{acot}\left(ax\right)+c^2dx^3\,\mathrm{acot}\left(ax\right)+\frac{3cd^2x^5\,\mathrm{acot}\left(ax\right)}{5}+\frac{d^3x^7\,\mathrm{acot}\left(ax\right)}{7}+\frac{c^3\log\left(x^2+\frac{1}{a^2}\right)}{2a}+\frac{c^2dx^2}{2a}+\frac{3cd^2x^4}{20a}+\frac{d^3x^6}{42a}-\frac{c^2d\log\left(x^2+\frac{1}{a^2}\right)}{2a^3}-\frac{3cd^2x^2}{10a^3}-\frac{d^3x^4}{28a^3}+\frac{3cd^2\log\left(x^2+\frac{1}{a^2}\right)}{10a^5}+\frac{d^3x^2}{14a^5}-\frac{d^3\log\left(x^2+\frac{1}{a^2}\right)}{14a^7} & \text{for}\ a\neq 0\\ \frac{\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acot(a*x),x)
```

```
[Out] Piecewise((c**3*x*acot(a*x) + c**2*d*x**3*acot(a*x) + 3*c*d**2*x**5*acot(a*
x)/5 + d**3*x**7*acot(a*x)/7 + c**3*log(x**2 + a**(-2))/(2*a) + c**2*d*x**2
/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) - c**2*d*log(x**2 + a**(-2
))/(2*a**3) - 3*c*d**2*x**2/(10*a**3) - d**3*x**4/(28*a**3) + 3*c*d**2*log(
x**2 + a**(-2))/(10*a**5) + d**3*x**2/(14*a**5) - d**3*log(x**2 + a**(-2))/
(14*a**7), Ne(a, 0)), (pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x*
*7/7)/2, True))
```

Giac [A]
time = 0.42, size = 252, normalized size = 1.50

$$\frac{1}{420}\left(\frac{12\left(5d^{3}+\frac{21cd^{2}}{x^{2}}+\frac{35c^{2}d}{x^{4}}+\frac{35c^{3}}{x^{6}}\right)x^{7}\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(10d^{3}+\frac{63cd^{2}}{x^{2}}+\frac{210c^{2}d}{x^{4}}-\frac{15d^{3}}{a^{2}x^{2}}+\frac{385c^{3}}{x^{6}}-\frac{126cd^{2}}{a^{2}x^{4}}-\frac{385c^{2}d}{a^{2}x^{6}}+\frac{30d^{3}}{a^{4}x^{4}}+\frac{231cd^{2}}{a^{4}x^{6}}-\frac{55d^{3}}{a^{6}x^{6}}\right)x^{6}}{a^{2}}+\frac{6\left(35a^{6}c^{3}-35a^{4}c^{2}d+21a^{2}cd^{2}-5d^{3}\right)\log\left(\frac{1}{a^{2}x^{2}}+1\right)}{a^{8}}-\frac{6\left(35a^{6}c^{3}-35a^{4}c^{2}d+21a^{2}cd^{2}-5d^{3}\right)\log\left(\frac{1}{a^{2}x^{2}}\right)}{a^{8}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/420*(12*(5*d^3 + 21*c*d^2/x^2 + 35*c^2*d/x^4 + 35*c^3/x^6)*x^7*arctan(1/(
a*x))/a + (10*d^3 + 63*c*d^2/x^2 + 210*c^2*d/x^4 - 15*d^3/(a^2*x^2) + 385*c
^3/x^6 - 126*c*d^2/(a^2*x^4) - 385*c^2*d/(a^2*x^6) + 30*d^3/(a^4*x^4) + 231
*c*d^2/(a^4*x^6) - 55*d^3/(a^6*x^6))*x^6/a^2 + 6*(35*a^6*c^3 - 35*a^4*c^2*d
 + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2) + 1)/a^8 - 6*(35*a^6*c^3 - 35*a^4*
c^2*d + 21*a^2*c*d^2 - 5*d^3)*log(1/(a^2*x^2))/a^8)*a
```

Mupad [B]
time = 1.02, size = 190, normalized size = 1.13

$$c^{3}x\,\mathrm{acot}(a\,x)+\frac{d^{3}x^{7}\,\mathrm{acot}(a\,x)}{7}+\frac{c^{3}\ln\left(a^{2}x^{2}+1\right)}{2\,a}-\frac{d^{3}\ln\left(a^{2}x^{2}+1\right)}{14\,a^{7}}+\frac{d^{3}x^{6}}{42\,a}-\frac{d^{3}x^{4}}{28\,a^{3}}+\frac{d^{3}x^{2}}{14\,a^{5}}-\frac{c^{2}d\ln\left(a^{2}x^{2}+1\right)}{2\,a^{3}}+\frac{3\,c\,d^{2}\ln\left(a^{2}x^{2}+1\right)}{10\,a^{5}}+\frac{c^{2}d\,x^{2}}{2\,a}+\frac{3\,c\,d^{2}x^{4}}{20\,a}-\frac{3\,c\,d^{2}x^{2}}{10\,a^{3}}+c^{2}d\,x^{3}\,\mathrm{acot}(a\,x)+\frac{3\,c\,d^{2}x^{5}\,\mathrm{acot}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)*(c + d*x^2)^3,x)
```

```
[Out] c^3*x*acot(a*x) + (d^3*x^7*acot(a*x))/7 + (c^3*log(a^2*x^2 + 1))/(2*a) - (d
^3*log(a^2*x^2 + 1))/(14*a^7) + (d^3*x^6)/(42*a) - (d^3*x^4)/(28*a^3) + (d^
3*x^2)/(14*a^5) - (c^2*d*log(a^2*x^2 + 1))/(2*a^3) + (3*c*d^2*log(a^2*x^2 +
 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) - (3*c*d^2*x^2)/(1
0*a^3) + c^2*d*x^3*acot(a*x) + (3*c*d^2*x^5*acot(a*x))/5
```

3.55 $\int (c+dx^2)^2 \cot^{-1}(ax)\,dx$

Optimal. Leaf size=109

$$\frac{(10a^2c-3d)\,dx^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{(15a^4c^2-10a^2cd+3d^2)\log(1}{30a^5}$$

```
[Out] 1/30*(10*a^2*c-3*d)*d*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arccot(a*x)+2/3*c*d*x^3*
arccot(a*x)+1/5*d^2*x^5*arccot(a*x)+1/30*(15*a^4*c^2-10*a^2*c*d+3*d^2)*ln(a
^2*x^2+1)/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {200, 5033, 1608, 1261, 712}

$$\frac{dx^2(10a^2c-3d)}{30a^3}+\frac{(15a^4c^2-10a^2cd+3d^2)\log(a^2x^2+1)}{30a^5}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{d^2x^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCot[a*x],x]
```

```
[Out] ((10*a^2*c - 3*d)*d*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcCot[a*x] +
(2*c*d*x^3*ArcCot[a*x])/3 + (d^2*x^5*ArcCot[a*x])/5 + ((15*a^4*c^2 - 10*a^2
*c*d + 3*d^2)*Log[1 + a^2*x^2])/(30*a^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int (c+dx^2)^2 \cot^{-1}(ax)\,dx = c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+a\int \frac{c^2x+\frac{2}{3}cdx^3+\frac{d^2x}{5}}{1+a^2x^2}$$

$$=c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+a\int \frac{x\left(c^2+\frac{2}{3}cdx^2+\frac{d}{}\right.}{1+a^2x^2}$$

$$=c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{1}{2}a\text{Subst}\left(\int \frac{c^2+\frac{2cdx}{3}}{1+a}\right.$$

$$=c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(ax)+\frac{1}{2}a\text{Subst}\left(\int \left(\frac{(10a^2c}{15}\right.\right.$$

$$=\frac{(10a^2c-3d)\,dx^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\cot^{-1}(ax)+\frac{2}{3}cdx^3\cot^{-1}(ax)+\frac{1}{5}d^2x^5\cot^{-1}(a$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 0.89

$$\frac{a^2dx^2(-6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\cot^{-1}(ax)+(30a^4c^2-20a^2cd+6d^2)\log(1+a^2x^2)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCot[a*x],x]
```

```
[Out] (a^2*d*x^2*(-6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3
*d^2*x^4)*ArcCot[a*x] + (30*a^4*c^2 - 20*a^2*c*d + 6*d^2)*Log[1 + a^2*x^2])
/(60*a^5)
```

Maple [A]
time = 0.12, size = 112, normalized size = 1.03

method	result

derivativedivides	$\frac{\operatorname{arccot}(ax)c^2ax+\frac{2a\operatorname{arccot}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{(15a^4c^2-10a^2cd+3d^2)\ln(a^2x^2+1)}{2}}{15a^4}}{a}$
default	$\frac{\operatorname{arccot}(ax)c^2ax+\frac{2a\operatorname{arccot}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccot}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}-\frac{3a^2d^2x^2}{2}+\frac{(15a^4c^2-10a^2cd+3d^2)\ln(a^2x^2+1)}{2}}{15a^4}}{a}$
risch	$\frac{i(3d^2x^5+10cd\,x^3+15c^2x)\ln(iax+1)}{30}-\frac{id^2x^5\ln(-iax+1)}{10}+\frac{\pi\,d^2x^5}{10}-\frac{icd\,x^3\ln(-iax+1)}{3}+\frac{\pi cd\,x^3}{3}-\frac{ic^2x\ln(-ia}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)*c^2*a*x+2/3*a*arccot(a*x)*c*d*x^3+1/5*a*arccot(a*x)*d^2*x^
5+1/15/a^4*(5*c*a^4*d*x^2+3/4*d^2*a^4*x^4-3/2*a^2*d^2*x^2+1/2*(15*a^4*c^2-1
0*a^2*c*d+3*d^2)*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 103, normalized size = 0.94

$$\frac{1}{60}\,a\left(\frac{3\,a^2d^2x^4+2\,(10\,a^2cd-3\,d^2)x^2}{a^4}+\frac{2\,(15\,a^4c^2-10\,a^2cd+3\,d^2)\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\operatorname{arccot}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d - 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 -
10*a^2*c*d + 3*d^2)*log(a^2*x^2 + 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 +
15*c^2*x)*arccot(a*x)
```

Fricas [A]
time = 2.61, size = 108, normalized size = 0.99

$$\frac{3\,a^4d^2x^4+2\,(10\,a^4cd-3\,a^2d^2)x^2+4\,(3\,a^5d^2x^5+10\,a^5cdx^3+15\,a^5c^2x)\operatorname{arccot}\left(ax\right)+2\,(15\,a^4c^2-10\,a^2cd+3\,d^2)\log\left(a^2x^2+1\right)}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(3*a^5*d^2*x^5 + 1
0*a^5*c*d*x^3 + 15*a^5*c^2*x)*arccot(a*x) + 2*(15*a^4*c^2 - 10*a^2*c*d + 3*
d^2)*log(a^2*x^2 + 1))/a^5
```

Sympy [A]
time = 0.33, size = 151, normalized size = 1.39

$$\begin{cases} c^2x\operatorname{acot}\left(ax\right)+\frac{2cdx^3\operatorname{acot}\left(ax\right)}{3}+\frac{d^2x^5\operatorname{acot}\left(ax\right)}{5}+\frac{c^2\log\left(x^2+\frac{1}{a^2}\right)}{2a}+\frac{cdx^2}{3a}+\frac{d^2x^4}{20a}-\frac{cd\log\left(x^2+\frac{1}{a^2}\right)}{3a^3}-\frac{d^2x^2}{10a^3}+\frac{d^2\log\left(x^2+\frac{1}{a^2}\right)}{10a^5} & \text{for}\: a\neq 0 \\ \frac{\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acot(a*x),x)
```

```
[Out] Piecewise((c**2*x*acot(a*x) + 2*c*d*x**3*acot(a*x)/3 + d**2*x**5*acot(a*x)/
5 + c**2*log(x**2 + a**(-2))/(2*a) + c*d*x**2/(3*a) + d**2*x**4/(20*a) - c*
d*log(x**2 + a**(-2))/(3*a**3) - d**2*x**2/(10*a**3) + d**2*log(x**2 + a**(
-2))/(10*a**5), Ne(a, 0)), (pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, Tru
e))
```

Giac [A]
time = 0.41, size = 171, normalized size = 1.57

$$\frac{1}{60}\left(\frac{4\left(3\,d^{2}+\frac{10\,cd}{x^{2}}+\frac{15\,c^{2}}{x^{4}}\right)x^{5}\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(3\,d^{2}+\frac{20\,cd}{x^{2}}+\frac{45\,c^{2}}{x^{4}}-\frac{6\,d^{2}}{a^{2}x^{2}}-\frac{30\,cd}{a^{2}x^{4}}+\frac{9\,d^{2}}{a^{4}x^{4}}\right)x^{4}}{a^{2}}+\frac{2\,(15\,a^{4}c^{2}-10\,a^{2}cd+3\,d^{2})\log\left(\frac{1}{a^{2}x^{2}}+1\right)}{a^{6}}-\frac{2\,(15\,a^{4}c^{2}-10\,a^{2}cd+3\,d^{2})\log\left(\frac{1}{a^{2}x^{2}}\right)}{a^{6}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/60*(4*(3*d^2 + 10*c*d/x^2 + 15*c^2/x^4)*x^5*arctan(1/(a*x))/a + (3*d^2 +
20*c*d/x^2 + 45*c^2/x^4 - 6*d^2/(a^2*x^2) - 30*c*d/(a^2*x^4) + 9*d^2/(a^4*x
^4))*x^4/a^2 + 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2) + 1)/a^6
 - 2*(15*a^4*c^2 - 10*a^2*c*d + 3*d^2)*log(1/(a^2*x^2))/a^6)*a
```

Mupad [B]
time = 0.84, size = 116, normalized size = 1.06

$$\frac{a^{4}\left(\frac{c^{2}\,\ln\left(a^{2}\,x^{2}+1\right)}{2}+\frac{d^{2}\,x^{4}}{20}+\frac{c\,d\,x^{2}}{3}\right)-a^{2}\left(\frac{d^{2}\,x^{2}}{10}+\frac{c\,d\,\ln\left(a^{2}\,x^{2}+1\right)}{3}\right)+\frac{d^{2}\,\ln\left(a^{2}\,x^{2}+1\right)}{10}}{a^{5}}+c^{2}\,x\,\mathrm{acot}(a\,x)+\frac{d^{2}\,x^{5}\,\mathrm{acot}(a\,x)}{5}+\frac{2\,c\,d\,x^{3}\,\mathrm{acot}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)*(c + d*x^2)^2,x)
```

```
[Out] (a^4*((c^2*log(a^2*x^2 + 1))/2 + (d^2*x^4)/20 + (c*d*x^2)/3) - a^2*((d^2*x^
2)/10 + (c*d*log(a^2*x^2 + 1))/3) + (d^2*log(a^2*x^2 + 1))/10)/a^5 + c^2*x*
acot(a*x) + (d^2*x^5*acot(a*x))/5 + (2*c*d*x^3*acot(a*x))/3
```

3.56 $\int (c + dx^2) \cot^{-1}(ax) \, dx$

Optimal. Leaf size=58

$$\frac{dx^2}{6a} + cx\cot^{-1}(ax) + \frac{1}{3}dx^3\cot^{-1}(ax) + \frac{(3a^2c - d)\log(1 + a^2x^2)}{6a^3}$$

```
[Out] 1/6*d*x^2/a+c*x*arccot(a*x)+1/3*d*x^3*arccot(a*x)+1/6*(3*a^2*c-d)*ln(a^2*x^
2+1)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5033, 1607, 455, 45}

$$\frac{(3a^2c - d)\log(a^2x^2 + 1)}{6a^3} + cx\cot^{-1}(ax) + \frac{1}{3}dx^3\cot^{-1}(ax) + \frac{dx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCot[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + ((3*a^2*c - d)*Lo
g[1 + a^2*x^2])/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
```

```
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx^2)\cot^{-1}(ax)\,dx &= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+a\int\frac{cx+\frac{dx^3}{3}}{1+a^2x^2}\,dx \\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1+a^2x^2}\,dx \\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{1}{2}a\mathrm{Subst}\left(\int\frac{c+\frac{dx}{3}}{1+a^2x}\,dx,x,x^2\right) \\
&= cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{1}{2}a\mathrm{Subst}\left(\int\left(\frac{d}{3a^2}+\frac{3a^2c-d}{3a^2(1+a^2x)}\right)\,dx,x,x
\end{aligned}
$$

$$
= \frac{dx^2}{6a}+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{(3a^2c-d)\log(1+a^2x^2)}{6a^3}
$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.16

$$
\frac{dx^2}{6a}+cx\cot^{-1}(ax)+\frac{1}{3}dx^3\cot^{-1}(ax)+\frac{c\log(1+a^2x^2)}{2a}-\frac{d\log(1+a^2x^2)}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCot[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCot[a*x] + (d*x^3*ArcCot[a*x])/3 + (c*Log[1 + a^2*x^
2])/(2*a) - (d*Log[1 + a^2*x^2])/(6*a^3)
```

Maple [A]
time = 0.05, size = 62, normalized size = 1.07

method	result
derivativedivides	$\frac{\mathrm{arccot}(ax)cax+\frac{a\,\mathrm{arccot}(ax)d\,x^3}{3}+\frac{\frac{d\,a^2x^2}{2}+\frac{\left(3a^2c-d\right)\ln\left(a^2x^2+1\right)}{2}}{3a^2}}{a}$
default	$\frac{\mathrm{arccot}(ax)cax+\frac{a\,\mathrm{arccot}(ax)d\,x^3}{3}+\frac{\frac{d\,a^2x^2}{2}+\frac{\left(3a^2c-d\right)\ln\left(a^2x^2+1\right)}{2}}{3a^2}}{a}$
risch	$\frac{i(d\,x^3+3cx)\ln(iax+1)}{6}-\frac{id\,x^3\ln(-iax+1)}{6}+\frac{\pi d\,x^3}{6}-\frac{icx\ln(-iax+1)}{2}+\frac{\pi cx}{2}+\frac{d\,x^2}{6a}+\frac{\ln(-a^2x^2-1)c}{2a}-\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccot(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccot(a*x)*c*a*x+1/3*a*arccot(a*x)*d*x^3+1/3/a^2*(1/2*d*a^2*x^2+1/2*(
3*a^2*c-d)*ln(a^2*x^2+1)))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.91

$$\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{(3\,a^2c-d)\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\operatorname{arccot}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c - d)*log(a^2*x^2 + 1)/a^4) + 1/3*(d*x^3 + 3*c*x
)*arccot(a*x)
```

Fricas [A]
time = 3.05, size = 57, normalized size = 0.98

$$\frac{a^2dx^2+2\left(a^3dx^3+3\,a^3cx\right)\operatorname{arccot}\left(ax\right)+(3\,a^2c-d)\log\left(a^2x^2+1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + 2*(a^3*d*x^3 + 3*a^3*c*x)*arccot(a*x) + (3*a^2*c - d)*log(
a^2*x^2 + 1))/a^3
```

Sympy [A]
time = 0.19, size = 73, normalized size = 1.26

$$\begin{cases} cx\operatorname{acot}\left(ax\right)+\frac{dx^3\operatorname{acot}\left(ax\right)}{3}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{2a}+\frac{dx^2}{6a}-\frac{d\log\left(x^2+\frac{1}{a^2}\right)}{6a^3} & \text{for } a\neq 0\\ \frac{\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acot(a*x),x)
```

```
[Out] Piecewise((c*x*acot(a*x) + d*x**3*acot(a*x)/3 + c*log(x**2 + a**(-2))/(2*a)
 + d*x**2/(6*a) - d*log(x**2 + a**(-2))/(6*a**3), Ne(a, 0)), (pi*(c*x + d*x
**3/3)/2, True))
```

Giac [A]
time = 0.41, size = 99, normalized size = 1.71

$$\frac{1}{6}\left(\frac{2\left(d+\frac{3\,c}{x^2}\right)x^3\arctan\left(\frac{1}{ax}\right)}{a}+\frac{\left(d+\frac{3\,c}{x^2}-\frac{d}{a^2x^2}\right)x^2}{a^2}+\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}+1\right)}{a^4}-\frac{(3\,a^2c-d)\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccot(a*x),x, algorithm="giac")
```

```
[Out] 1/6*(2*(d + 3*c/x^2)*x^3*arctan(1/(a*x))/a + (d + 3*c/x^2 - d/(a^2*x^2))*x^
2/a^2 + (3*a^2*c - d)*log(1/(a^2*x^2) + 1)/a^4 - (3*a^2*c - d)*log(1/(a^2*x
^2))/a^4)*a
```

Mupad [B]
time = 0.79, size = 62, normalized size = 1.07

$$\frac{d\,x^3\,\mathrm{acot}(a\,x)}{3}-\frac{\frac{d\,\ln\left(a^2\,x^2+1\right)}{6}-a^2\,\left(\frac{c\,\ln\left(a^2\,x^2+1\right)}{2}+\frac{d\,x^2}{6}\right)}{a^3}+c\,x\,\mathrm{acot}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)*(c + d*x^2),x)
```

```
[Out] (d*x^3*acot(a*x))/3 - ((d*log(a^2*x^2 + 1))/6 - a^2*((c*log(a^2*x^2 + 1))/2
 + (d*x^2)/6))/a^3 + c*x*acot(a*x)
```

3.57 $\int \frac{\cot^{-1}(ax)}{c+dx^2} \, dx$

Optimal. Leaf size=403

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\,\sqrt{d}\,(i-ax}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\right.}\right.}{2\sqrt{c}\,\sqrt{d}}$$

```
[Out] 1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(1-I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x*
d^(1/2)/c^(1/2))*ln(1+I/a/x)/c^(1/2)/d^(1/2)-1/2*I*arctan(x*d^(1/2)/c^(1/2)
)*ln(2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2)))
/c^(1/2)/d^(1/2)+1/2*I*arctan(x*d^(1/2)/c^(1/2))*ln(-2*I*(I+a*x)*c^(1/2)*d^
(1/2)/(a*c^(1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylo
g(2,1-2*I*(I-a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2))
)/c^(1/2)/d^(1/2)+1/4*polylog(2,1+2*I*(I+a*x)*c^(1/2)*d^(1/2)/(a*c^(1/2)+d^
(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5029, 211, 2520, 12, 266, 6820, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\frac{i\log\left(1-\frac{i}{ax}\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{i\log\left(1+\frac{i}{ax}\right)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{i\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{2i\sqrt{c}\,\sqrt{d}\,(-ax+i)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{i\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{2i\sqrt{c}\,\sqrt{d}\,(ax+i)}{\left(a\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{\text{Li}_2\left(1-\frac{2i\sqrt{c}\,\sqrt{d}\,(i-ax)}{\left(a\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{2i\sqrt{c}\,\sqrt{d}\,(ax+i)}{\left(\sqrt{c}\,a+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}+1\right)}{4\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - I/(a*x)])/(Sqrt[c]*Sqrt[d]) - ((
I/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + I/(a*x)])/(Sqrt[c]*Sqrt[d]) - ((I/
2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[((2*I)*Sqrt[c]*Sqrt[d]*(I - a*x))/((a*Sq
rt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d]) + ((I/2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]]*Log[((-2*I)*Sqrt[c]*Sqrt[d]*(I + a*x))/((a*Sqrt[c]
+ Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))])/(Sqrt[c]*Sqrt[d]) - PolyLog[2, 1 - ((
2*I)*Sqrt[c]*Sqrt[d]*(I - a*x))/((a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]
*x))]/(4*Sqrt[c]*Sqrt[d]) + PolyLog[2, 1 + ((2*I)*Sqrt[c]*Sqrt[d]*(I + a*x)
)/((a*Sqrt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]/(4*Sqrt[c]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5029

```
Int[ArcCot[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I/(c*x)]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I/(c*x)]/(d + e
*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{c+dx^2}\,dx = \frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax}\right)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log\left(1+\frac{i}{ax}\right)}{c+dx^2}\,dx$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}\,\left(1-\frac{i}{ax}\right)x^2}\,dx}{2a} +$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\left(1-\frac{i}{ax}\right)x^2}\,dx}{2a\sqrt{c}\,\sqrt{d}} + \int$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\int \frac{a\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x(-i+ax)}\,dx}{2a\sqrt{c}\,\sqrt{d}} +$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x(-i+ax)}\,dx}{2\sqrt{c}\,\sqrt{d}} + \int$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\int \left(\frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x} - \frac{ia}{}\right.}{2\sqrt{c}\,\sqrt{}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{(ia)\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{-i+ax}\,dx}{2\sqrt{c}\,\sqrt{d}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\right.}{2}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{i}{ax}\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\right.}{2}$$

Mathematica [A]
time = 0.22, size = 523, normalized size = 1.30

$$\frac{i\left(\log\left(1-\frac{i}{ax}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(1+\frac{i}{ax}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(\frac{\sqrt{d}\,(-i+ax)}{a\sqrt{-c}-i\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)+\log\left(\frac{\sqrt{d}\,(i+ax)}{a\sqrt{-c}+i\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(1-\frac{i}{ax}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)+\log\left(1+\frac{i}{ax}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)+\log\left(\frac{\sqrt{d}\,(i-ax)}{a\sqrt{-c}+i\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)-\log\left(-\frac{\sqrt{d}\,(i+ax)}{a\sqrt{-c}-i\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)-\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}-i\sqrt{d}}\right)+\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{-c}-\sqrt{d}\,x\right)}{a\sqrt{-c}+i\sqrt{d}}\right)-\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}-i\sqrt{d}}\right)+\mathrm{PolyLog}\left(2,\frac{a\left(\sqrt{-c}+\sqrt{d}\,x\right)}{a\sqrt{-c}+i\sqrt{d}}\right)\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCot[a*x]/(c + d*x^2),x]

[Out] ((I/4)*(Log[1 - I/(a*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - Log[1 + I/(a*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - Log[(Sqrt[d]*(-I + a*x))/(a*Sqrt[-c] - I*Sqrt[d])]*Log[Sqrt[-c] - Sqrt[d]*x] + Log[(Sqrt[d]*(I + a*x))/(a*Sqrt[-c] + I*Sqrt[d])]*Log[Sqrt[-c] - Sqrt[d]*x] - Log[1 - I/(a*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + Log[1 + I/(a*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + Log[(Sqrt[d]*(I - a*x))/(a*Sqrt[-c] + I*Sqrt[d])]*Log[Sqrt[-c] + Sqrt[d]*x] - Log[-((Sqrt[d]*(I + a*x))/(a*Sqrt[-c] - I*Sqrt[d]))]*Log[Sqrt[-c] + Sqrt[d]*x] - PolyLog[2, (a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - I*Sqrt[d])] + PolyLog[2, (a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] + I*Sqrt[d])] - PolyLog[2, (a*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] - I*Sqrt[d])] + PolyLog[2, (a*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] + I*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d])

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(285) = 570$.
time $= 0.33$, size $= 977$, normalized size $= 2.42$ Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccot(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] 1/a*(-1/2*I*(a^2*c*d)^(1/2)/c/d*arccot(a*x)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2*c*d)^(1/2)+d))-1/2*(a^2*c*d)^(1/2)/c/d*arccot(a*x)^2-1/4*(a^2*c*d)^(1/2)/c/d*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2*c*d)^(1/2)+d))+1/2*I*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*a^2*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))*arccot(a*x)/d/(a^4*c^2-2*a^2*c*d+d^2)+1/2*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*a^2*arccot(a*x)^2/d/(a^4*c^2-2*a^2*c*d+d^2)-I*(a^2*c+2*(a^2*c*d)^(1/2)+d)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))*arccot(a*x)*a^2/(a^4*c^2-2*a^2*c*d+d^2)+1/2*I*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))*arccot(a*x)/c/(a^4*c^2-2*a^2*c*d+d^2)+1/4*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*a^2*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))/d/(a^4*c^2-2*a^2*c*d+d^2)-(a^2*c+2*(a^2*c*d)^(1/2)+d)*a^2*arccot(a*x)^2/(a^4*c^2-2*a^2*c*d+d^2)+1/2*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*arccot(a*x)^2/c/(a^4*c^2-2*a^2*c*d+d^2)-1/2*(a^2*c+2*(a^2*c*d)^(1/2)+d)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))*a^2/(a^4*c^2-2*a^2*c*d+d^2)+1/4*((a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(a^2*c*d)^(1/2)*d)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))/c/(a^4*c^2-2*a^2*c*d+d^2))

Maxima [**A**]
time $= 0.55$, size $= 528$, normalized size $= 1.31$

$$-\frac{a\left(\frac{8\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{a}-\frac{4\arctan(ax)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)+4\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\arctan\left(-\frac{a\sqrt{d}x}{a\sqrt{c}-\sqrt{d}},-\frac{\sqrt{d}}{a\sqrt{c}-\sqrt{d}}\right)+\log(dx^2+c)\log\left(\frac{a^2cd+(a^4cd+a^2d^2)x^2+2(a^3dx^2+ad)\sqrt{c}\sqrt{d}+d^2}{a^4c^2+6a^2cd+4(a^3c+ad)\sqrt{c}\sqrt{d}+d^2}\right)-\log(dx^2+c)\log\left(\frac{a^2cd+(a^4cd+a^2d^2)x^2-2(a^3dx^2+ad)\sqrt{c}\sqrt{d}+d^2}{a^4c^2+6a^2cd-4(a^3c+ad)\sqrt{c}\sqrt{d}+d^2}\right)+2\,\mathrm{Li}_2\left(\frac{a^2c+iadx+(ia^2x+a)\sqrt{c}\sqrt{d}}{a^2c+2a\sqrt{c}\sqrt{d}+d}\right)+2\,\mathrm{Li}_2\left(\frac{a^2c-iadx-(ia^2x-a)\sqrt{c}\sqrt{d}}{a^2c+2a\sqrt{c}\sqrt{d}+d}\right)-2\,\mathrm{Li}_2\left(\frac{a^2c+iadx-(ia^2x+a)\sqrt{c}\sqrt{d}}{a^2c-2a\sqrt{c}\sqrt{d}+d}\right)-2\,\mathrm{Li}_2\left(\frac{a^2c-iadx+(ia^2x-a)\sqrt{c}\sqrt{d}}{a^2c-2a\sqrt{c}\sqrt{d}+d}\right)}{a}\right)}{8\sqrt{cd}}+\frac{\operatorname{arccot}(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{\arctan(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/8*a*(8*arctan(a*x)*arctan(d*x/sqrt(c*d))/a - (4*arctan(a*x)*arctan(sqrt(
d)*x/sqrt(c)) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2(-a*sqrt(d)*x/(a*sqrt(c)
 - sqrt(d)), -sqrt(d)/(a*sqrt(c) - sqrt(d))) + log(d*x^2 + c)*log((a^2*c*d
+ (a^4*c*d + a^2*d^2)*x^2 + 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4
*c^2 + 6*a^2*c*d + 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) - log(d*x^2 + c)
*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(
d) + d^2)/(a^4*c^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) +
2*dilog((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c + 2*a*sqrt
(c)*sqrt(d) + d)) + 2*dilog((a^2*c - I*a*d*x - (I*a^2*x - a)*sqrt(c)*sqrt(d
))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*dilog((a^2*c + I*a*d*x - (I*a^2*x
 + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)) - 2*dilog((a^2*c
- I*a*d*x + (I*a^2*x - a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d
)))/a)/sqrt(c*d) + arccot(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + arctan(a*x
)*arctan(d*x/sqrt(c*d))/sqrt(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)/(d*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acot}(a\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2),x)
[Out] int(acot(a*x)/(c + d*x^2), x)
```

3.58 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=801

$$\frac{x\cot^{-1}(ax)}{2c\,(c+dx^2)}+\frac{\cot^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}-\frac{ia\log\left(\frac{\sqrt{d}\,\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\,\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{ia\log\left(-\frac{\sqrt{d}\,(1}{i\sqrt{-a^2}}\right.}{8}$$

```
[Out] 1/2*x*arccot(a*x)/c/(d*x^2+c)+1/4*a*ln(a^2*x^2+1)/c/(a^2*c-d)-1/4*a*ln(d*x^
2+c)/c/(a^2*c-d)+1/2*arccot(a*x)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+
1/8*I*a*ln(-(1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)-d^(1/2)))*ln
(1-I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*ln((1-x*(-a^2)
^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))
/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*ln(-(1-x*(-a^2)^(1/2))*d^(1/2)/(I*(-a
^2)^(1/2)*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/
d^(1/2)-1/8*I*a*ln((1+x*(-a^2)^(1/2))*d^(1/2)/(I*(-a^2)^(1/2)*c^(1/2)+d^(1/
2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*polylog
(2,(-a^2)^(1/2)*(c^(1/2)-I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)-I*d^(1/2)))/c^(
3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2)-I*x*d^(1/
2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)-1/8*I*a*
polylog(2,(-a^2)^(1/2)*(c^(1/2)+I*x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)-I*d^(1/2
)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)+1/8*I*a*polylog(2,(-a^2)^(1/2)*(c^(1/2)+I*
x*d^(1/2))/((-a^2)^(1/2)*c^(1/2)+I*d^(1/2)))/c^(3/2)/(-a^2)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 801, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {205, 211, 5033, 6857, 455, 36, 31, 5028, 2456, 2441, 2440, 2438}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\cot^{-1}(ax)}{2c^{3/2}\sqrt{d}}+\frac{x\cot^{-1}(ax)}{2c\,(dx^2+c)}-\frac{ia\log\left(\frac{\sqrt{d}\,\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\,\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{ia\log\left(-\frac{\sqrt{d}\,\left(\sqrt{-a^2}\,x+1\right)}{i\sqrt{-a^2}\,\sqrt{c}-\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{ia\log\left(-\frac{\sqrt{d}\,\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\,\sqrt{c}-\sqrt{d}}\right)\log\left(\frac{i\sqrt{d}\,x}{\sqrt{c}}+1\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}-\frac{ia\log\left(\frac{\sqrt{d}\,\left(\sqrt{-a^2}\,x+1\right)}{i\sqrt{-a^2}\,\sqrt{c}+\sqrt{d}}\right)\log\left(\frac{i\sqrt{d}\,x}{\sqrt{c}}+1\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{a\log\left(a^2x^2+1\right)}{4c\left(a^2c-d\right)}-\frac{a\log\left(dx^2+c\right)}{4c\left(a^2c-d\right)}-\frac{ia\text{Li}_2\left(\frac{\sqrt{-a^2}\,\left(\sqrt{c}-i\sqrt{d}\,x\right)}{\sqrt{-a^2}\,\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{ia\text{Li}_2\left(\frac{\sqrt{-a^2}\,\left(\sqrt{c}-i\sqrt{d}\,x\right)}{\sqrt{-a^2}\,\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}-\frac{ia\text{Li}_2\left(\frac{\sqrt{-a^2}\,\left(i\sqrt{d}\,x+\sqrt{c}\right)}{\sqrt{-a^2}\,\sqrt{c}-i\sqrt{d}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}+\frac{ia\text{Li}_2\left(\frac{\sqrt{-a^2}\,\left(i\sqrt{d}\,x+\sqrt{c}\right)}{\sqrt{-a^2}\,\sqrt{c}+i\sqrt{d}}\right)}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcCot[a*x])/(2*c*(c + d*x^2)) + (ArcCot[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*Sqrt[d]) - ((I/8)*a*Log[(Sqrt[d]*(1 - Sqrt[-a^2]*x))/(I*Sqrt[
-a^2]*Sqrt[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/
2)*Sqrt[d]) + ((I/8)*a*Log[-((Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqr
t[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[
d]) + ((I/8)*a*Log[-((Sqrt[d]*(1 - Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt[c] - S
qrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) - ((
I/8)*a*Log[(Sqrt[d]*(1 + Sqrt[-a^2]*x))/(I*Sqrt[-a^2]*Sqrt[c] + Sqrt[d])]*L
og[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + (a*Log[1 + a^
```

```
2*x^2])/(4*c*(a^2*c - d)) - (a*Log[c + d*x^2])/(4*c*(a^2*c - d)) - ((I/8)*a
*PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] - I*Sq
rt[d])])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/8)*a*PolyLog[2, (Sqrt[-a^2]*(Sq
rt[c] - I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])])/(Sqrt[-a^2]*c^(3/2
)*Sqrt[d]) - ((I/8)*a*PolyLog[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt
[-a^2]*Sqrt[c] - I*Sqrt[d])])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d]) + ((I/8)*a*PolyL
og[2, (Sqrt[-a^2]*(Sqrt[c] + I*Sqrt[d]*x))/(Sqrt[-a^2]*Sqrt[c] + I*Sqrt[d])
])/(Sqrt[-a^2]*c^(3/2)*Sqrt[d])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + a\int \frac{\frac{x}{2c(c+dx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1+a^2x^2}\,dx$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + a\int \left(\frac{x}{2c\left(1+a^2x^2\right)\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\cdots\right)}{2c^{3/2}\sqrt{d}\,(1\cdots}\right.$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(1+a^2x^2)(c+dx^2)}\,dx}{2c} + \frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{1+a^2x^2}\,dx}{2c^{3/2}\sqrt{d}}$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \frac{1}{(1+a^2x)(c+dx)}\,dx, x, x^2\right)}{4c} + \frac{(ia)\int \cdots}{4c\cdots}$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^2\right)}{4c\left(a^2c-d\right)} + \frac{(ia)\int \left(\frac{\log\left(1-\cdots\right)}{2\left(1-\sqrt{\cdots}\right.}\cdots\right.}{\cdots}$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\log\left(1+a^2x^2\right)}{4c\left(a^2c-d\right)} - \frac{a\log\left(c+dx^2\right)}{4c\left(a^2c-d\right)} + \frac{(ia)\int \cdots}{8\cdots}$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\cdots}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}\cdots$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\cdots}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}\cdots$$

$$= \frac{x\cot^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\cot^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{ia\log\left(\frac{\sqrt{d}\left(1-\sqrt{-a^2}\,x\right)}{i\sqrt{-a^2}\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\cdots}{8\sqrt{-a^2}\,c^{3/2}\sqrt{d}}\cdots$$

Mathematica [A]
time = 5.28, size = 806, normalized size = 1.01

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -1/8*(a*((2*Log[(a^2*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])/(a^2*c + d)
])/(a^2*c - d) + (2*ArcCos[(a^2*c + d)/(a^2*c - d)]*ArcTanh[(a*c)/(Sqrt[-(a
^2*c*d)]*x)] + 4*ArcCot[a*x]*ArcTanh[(a*d*x)/Sqrt[-(a^2*c*d)]] + (ArcCos[(a
^2*c + d)/(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)])*Log[((2
*I)*d*(I*a^2*c + Sqrt[-(a^2*c*d)])*(I + a*x))/((a^2*c - d)*(Sqrt[-(a^2*c*d)
] - a*d*x))] + (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2*I)*ArcTanh[(a*c)/(Sqrt
[-(a^2*c*d)]*x)])*Log[(2*d*(a^2*c + I*Sqrt[-(a^2*c*d)])*(-I + a*x))/((a^2*c
 - d)*(-Sqrt[-(a^2*c*d)] + a*d*x))] - (ArcCos[(a^2*c + d)/(a^2*c - d)] + (2
*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] + (2*I)*ArcTanh[(a*d*x)/Sqrt[-(a^2*
c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)])/(Sqrt[a^2*c - d]*E^(I*ArcCot[a*x])*S
qrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] - (ArcCos[(a^2*c + d)/
(a^2*c - d)] - (2*I)*ArcTanh[(a*c)/(Sqrt[-(a^2*c*d)]*x)] - (2*I)*ArcTanh[(a
*d*x)/Sqrt[-(a^2*c*d)]])*Log[(Sqrt[2]*Sqrt[-(a^2*c*d)]*E^(I*ArcCot[a*x]))/(
Sqrt[a^2*c - d]*Sqrt[-(a^2*c) - d + (a^2*c - d)*Cos[2*ArcCot[a*x]]])] + I*(
PolyLog[2, ((a^2*c + d - (2*I)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x)
)/((a^2*c - d)*(Sqrt[-(a^2*c*d)] - a*d*x))] - PolyLog[2, ((a^2*c + d + (2*I
)*Sqrt[-(a^2*c*d)])*(Sqrt[-(a^2*c*d)] + a*d*x))/((a^2*c - d)*(Sqrt[-(a^2*c*
d)] - a*d*x))]))/Sqrt[-(a^2*c*d)] - (4*ArcCot[a*x]*Sin[2*ArcCot[a*x]])/(a^2
*c + d + (-(a^2*c) + d)*Cos[2*ArcCot[a*x]])))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2274 vs. 2(593) = 1186.
time = 0.42, size = 2275, normalized size = 2.84

method	result	size
risch	Expression too large to display	2141
derivativedivides	Expression too large to display	2275
default	Expression too large to display	2275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/4*(c*d)^(1/2)/c^2*d*a*arctanh(1/4*(2*(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)
-2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2*c-d)^2-1/2*I*a^2*arccot(a*x)*(I*c*a^2+a*d
*x)*(a*x-I)/c/(a^2*c-d)/(a^2*d*x^2+a^2*c)-1/4*I*(-(a^2*c*d)^(1/2)*a^2*c+2*a
^2*c*d-(a^2*c*d)^(1/2)*d)*d*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(
```

```
a^2*c*d)^(1/2)+d))*arccot(a*x)/c^2/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)+1/4*I*
(a^2*c*d)^(1/2)/c*a^2/d/(a^2*c-d)*arccot(a*x)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2
*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))+1/4*I*(-(a^2*c*d)^(1/2)*a^2*c+2*a^2*c*
d-(a^2*c*d)^(1/2)*d)*a^4*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2
*c*d)^(1/2)+d))*arccot(a*x)/d/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)+1/4*(a^2*c-
2*(a^2*c*d)^(1/2)+d)/(a^2*c-d)/c/(a^4*c^2-2*a^2*c*d+d^2)*a^2*d*polylog(2,(a
^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2*c*d)^(1/2)+d))-1/2*I*(a^2*c-2*(
a^2*c*d)^(1/2)+d)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2*c*d)^(
1/2)+d))*arccot(a*x)*a^4/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)+1/2*(a^2*c-2*(a^
2*c*d)^(1/2)+d)/(a^2*c-d)/c/(a^4*c^2-2*a^2*c*d+d^2)*a^2*d*arccot(a*x)^2-1/2
*(a^2*c-2*(a^2*c*d)^(1/2)+d)/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)*a^4*arccot(a
*x)^2+a^4/(a^2*c-d)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/4*a^4/(a^2*c-d)^2*ln(
a^2*c*(I+a*x)^4/(a^2*x^2+1)^2-2*c*a^2*(I+a*x)^2/(a^2*x^2+1)-d*(I+a*x)^4/(a^
2*x^2+1)^2+a^2*c-2*d*(I+a*x)^2/(a^2*x^2+1)-d)+1/4*a^2/c/(a^2*c-d)^2*d*ln(a^
2*c*(I+a*x)^4/(a^2*x^2+1)^2-2*c*a^2*(I+a*x)^2/(a^2*x^2+1)-d*(I+a*x)^4/(a^2*
x^2+1)^2+a^2*c-2*d*(I+a*x)^2/(a^2*x^2+1)-d)-1/4*(a^2*c-2*(a^2*c*d)^(1/2)+d)
/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)*a^4*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x
^2+1)/(a^2*c+2*(a^2*c*d)^(1/2)+d))-a^2/c/(a^2*c-d)^2*d*ln((I+a*x)/(a^2*x^2+
1)^(1/2))+1/4*(c*d)^(1/2)/c^2*a*arctanh(1/4*(2*(a^2*c-d)*(I+a*x)^2/(a^2*x^2
+1)-2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2*c-d)-1/4*(c*d)^(1/2)/d*a^5*arctanh(1/4
*(2*(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)-2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2*c-d)^2
+1/2*I*(a^2*c-2*(a^2*c*d)^(1/2)+d)*ln(1-(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^
2*c+2*(a^2*c*d)^(1/2)+d))*arccot(a*x)*a^2*d/(a^2*c-d)/c/(a^4*c^2-2*a^2*c*d+
d^2)+1/8*(-(a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d-(a^2*c*d)^(1/2)*d)*a^4/d/(a^2*c-
d)/(a^4*c^2-2*a^2*c*d+d^2)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c
+2*(a^2*c*d)^(1/2)+d))-1/4*(-(a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d-(a^2*c*d)^(1/2
)*d)/c^2*d/(a^2*c-d)/(a^4*c^2-2*a^2*c*d+d^2)*arccot(a*x)^2+1/4*(-(a^2*c*d)^
(1/2)*a^2*c+2*a^2*c*d-(a^2*c*d)^(1/2)*d)*a^4/d/(a^2*c-d)/(a^4*c^2-2*a^2*c*d
+d^2)*arccot(a*x)^2+1/4*(a^2*c*d)^(1/2)/c*a^2/d/(a^2*c-d)*arccot(a*x)^2+1/8
*(a^2*c*d)^(1/2)/c*a^2/d/(a^2*c-d)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1
)/(a^2*c-2*(a^2*c*d)^(1/2)+d))+1/4*(c*d)^(1/2)/c/d*a^3*arctanh(1/4*(2*(a^2*
c-d)*(I+a*x)^2/(a^2*x^2+1)-2*a^2*c-2*d)/a/(c*d)^(1/2))/(a^2*c-d)-1/8*(-(a^2
*c*d)^(1/2)*a^2*c+2*a^2*c*d-(a^2*c*d)^(1/2)*d)/c^2*d/(a^2*c-d)/(a^4*c^2-2*a
^2*c*d+d^2)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c+2*(a^2*c*d)^(1
/2)+d))-1/4*I*(a^2*c*d)^(1/2)/c^2/(a^2*c-d)*arccot(a*x)*ln(1-(a^2*c-d)*(I+a
*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d))-1/8*(a^2*c*d)^(1/2)/c^2/(a^2
*c-d)*polylog(2,(a^2*c-d)*(I+a*x)^2/(a^2*x^2+1)/(a^2*c-2*(a^2*c*d)^(1/2)+d)
)-1/4*(a^2*c*d)^(1/2)/c^2/(a^2*c-d)*arccot(a*x)^2)
```

Maxima [A]
time = 0.55, size = 628, normalized size = 0.78

$$\frac{1}{2}\left(\frac{x}{cdx^2+c^2}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}\right)\operatorname{arccot}(ax)+\frac{\left(4\,acd\log\left(a^2x^2+1\right)-4\,acd\log\left(dx^2+c\right)+\left(4\left(a^2c-d\right)\arctan\left(ax\right)\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)+4\left(a^2c-d\right)\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\arctan\left(-\frac{a\sqrt{d}\,x}{a\sqrt{c}-\sqrt{d}},-\frac{\sqrt{d}}{a\sqrt{c}-\sqrt{d}}\right)+\left(a^2c-d\right)\log\left(dx^2+c\right)\log\left(\frac{a^2cd+\left(a^4cd+a^2d^2\right)x^2+2\left(a^3dx^2+ad\right)\sqrt{c}\sqrt{d}+d^2}{a^4c^2+6\,a^2cd+4\left(a^3c+ad\right)\sqrt{c}\sqrt{d}+d^2}\right)-\left(a^2c-d\right)\log\left(dx^2+c\right)\log\left(\frac{a^2cd+\left(a^4cd+a^2d^2\right)x^2-2\left(a^3dx^2+ad\right)\sqrt{c}\sqrt{d}+d^2}{a^4c^2+6\,a^2cd-4\left(a^3c+ad\right)\sqrt{c}\sqrt{d}+d^2}\right)+2\left(a^2c-d\right)\mathrm{Li}_2\left(\frac{a^2c+i\,adx+\left(i\,a^2x+a\right)\sqrt{c}\sqrt{d}}{a^2c+2\,a\sqrt{c}\sqrt{d}+d}\right)+2\left(a^2c-d\right)\mathrm{Li}_2\left(\frac{a^2c-i\,adx-\left(i\,a^2x-a\right)\sqrt{c}\sqrt{d}}{a^2c+2\,a\sqrt{c}\sqrt{d}+d}\right)-2\left(a^2c-d\right)\mathrm{Li}_2\left(\frac{a^2c+i\,adx-\left(i\,a^2x+a\right)\sqrt{c}\sqrt{d}}{a^2c-2\,a\sqrt{c}\sqrt{d}+d}\right)-2\left(a^2c-d\right)\mathrm{Li}_2\left(\frac{a^2c-i\,adx+\left(i\,a^2x-a\right)\sqrt{c}\sqrt{d}}{a^2c-2\,a\sqrt{c}\sqrt{d}+d}\right)\right)\sqrt{c}\sqrt{d}\right)a}{16\left(a^3c^3d-ac^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccot(a*x) +
 1/16*(4*a*c*d*log(a^2*x^2 + 1) - 4*a*c*d*log(d*x^2 + c) + (4*(a^2*c - d)*a
rctan(a*x)*arctan(sqrt(d)*x/sqrt(c)) + 4*(a^2*c - d)*arctan(sqrt(d)*x/sqrt(
c))*arctan2(-a*sqrt(d)*x/(a*sqrt(c) - sqrt(d)), -sqrt(d)/(a*sqrt(c) - sqrt(
d))) + (a^2*c - d)*log(d*x^2 + c)*log((a^2*c*d + (a^4*c*d + a^2*d^2)*x^2 +
2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4*c^2 + 6*a^2*c*d + 4*(a^3*c
+ a*d)*sqrt(c)*sqrt(d) + d^2)) - (a^2*c - d)*log(d*x^2 + c)*log((a^2*c*d +
(a^4*c*d + a^2*d^2)*x^2 - 2*(a^3*d*x^2 + a*d)*sqrt(c)*sqrt(d) + d^2)/(a^4*c
^2 + 6*a^2*c*d - 4*(a^3*c + a*d)*sqrt(c)*sqrt(d) + d^2)) + 2*(a^2*c - d)*di
log((a^2*c + I*a*d*x + (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*
sqrt(d) + d)) + 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x - (I*a^2*x - a)*sqrt(c
)*sqrt(d))/(a^2*c + 2*a*sqrt(c)*sqrt(d) + d)) - 2*(a^2*c - d)*dilog((a^2*c
+ I*a*d*x - (I*a^2*x + a)*sqrt(c)*sqrt(d))/(a^2*c - 2*a*sqrt(c)*sqrt(d) + d
)) - 2*(a^2*c - d)*dilog((a^2*c - I*a*d*x + (I*a^2*x - a)*sqrt(c)*sqrt(d))/
(a^2*c - 2*a*sqrt(c)*sqrt(d) + d)))*sqrt(c)*sqrt(d))*a/(a^3*c^3*d - a*c^2*d
^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a\,x)}{{\left(d\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^2,x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^2, x)
```

3.59 $\int \sqrt{c+dx^2} \cot^{-1}(ax)\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\sqrt{c+dx^2}\cot^{-1}(ax),x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arccot(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx^2} \cot^{-1}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCot[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCot[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2} \cot^{-1}(ax)\,dx = \int \sqrt{c+dx^2} \cot^{-1}(ax)\,dx$$

Mathematica [A]
time = 3.83, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2} \cot^{-1}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCot[a*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]*ArcCot[a*x], x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c}\,\operatorname{arccot}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccot(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arccot(a*x),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*arccot(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2}\,\operatorname{acot}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acot(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*acot(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccot(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccot(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{acot}(a\,x)\,\sqrt{d\,x^2+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)*(c + d*x^2)^(1/2),x)
[Out] int(acot(a*x)*(c + d*x^2)^(1/2), x)
```

3.60 $\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcCot[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [A]
time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcCot[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(ax)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{\sqrt{c+dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(acot(a*x)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{acot}(a\,x)}{\sqrt{d\,x^2+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^(1/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(1/2), x)
```

3.61 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

```
[Out] -arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c/(a^2*c-d)^(1/2)+x*arccot(a*x)
/c/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {197, 5033, 12, 455, 65, 214}

$$\frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCot[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*
c - d]]/(c*Sqrt[a^2*c - d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx &= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}+a\int \frac{x}{c\,(1+a^2x^2)\sqrt{c+dx^2}}\,dx\\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}+\frac{a\int \frac{x}{(1+a^2x^2)\sqrt{c+dx^2}}\,dx}{c}\\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}+\frac{a\mathrm{Subst}\left(\int \frac{1}{(1+a^2x)\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}\\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}+\frac{a\mathrm{Subst}\left(\int \frac{1}{1-\frac{a^2c}{d}+\frac{a^2x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{cd}\\
&= \frac{x\cot^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{c\sqrt{a^2c-d}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 169, normalized size = 2.56

$$
\frac{\frac{2x\cot^{-1}(ax)}{\sqrt{c+dx^2}}+\frac{-\log\left(\frac{4ac\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}\,(i+ax)}\right)-\log\left(\frac{4ac\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{\sqrt{a^2c-d}\,(-i+ax)}\right)}{\sqrt{a^2c-d}}}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcCot[a*x])/Sqrt[c + d*x^2] + (-Log[(4*a*c*(a*c - I*d*x + Sqrt[a^2*c
 - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(I + a*x))] - Log[(4*a*c*(a*c + I*
d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/(Sqrt[a^2*c - d]*(-I + a*x))])/Sqrt
[a^2*c - d])/(2*c)
```

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(58) = 116$.
time = 2.45, size = 349, normalized size = 5.29

$$\left[\frac{4\,(a^2c-d)\sqrt{dx^2+c}\,x\operatorname{arccot}(ax)+\sqrt{a^2c-d}\,(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2-8\,a^2cd+2\,(4\,a^4cd-3\,a^2d^2)x^2-4\,(a^3dx^2+2\,a^3c-ad)\sqrt{a^2c-d}\,\sqrt{dx^2+c}+d^2}{a^4x^4+2\,a^2x^2+1}\right)}{4\,(a^2c^3-c^2d+(a^2c^2d-cd^2)x^2)},\ \frac{2\,(a^2c-d)\sqrt{dx^2+c}\,x\operatorname{arccot}(ax)-\sqrt{-a^2c+d}\,(dx^2+c)\arctan\left(-\frac{(a^2dx^2+2\,a^2c-d)\sqrt{-a^2c+d}\,\sqrt{dx^2+c}}{2\,(a^3c^2-acd+(a^3cd-ad^2)x^2)}\right)}{2\,(a^2c^3-c^2d+(a^2c^2d-cd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) + sqrt(a^2*c - d)*(d*x^2
+ c)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x
^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(
a^4*x^4 + 2*a^2*x^2 + 1)))/(a^2*c^3 - c^2*d + (a^2*c^2*d - c*d^2)*x^2), 1/2
```

```
*(2*(a^2*c - d)*sqrt(d*x^2 + c)*x*arccot(a*x) - sqrt(-a^2*c + d)*(d*x^2 + c
)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a
^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)))/(a^2*c^3 - c^2*d + (a^2*c^2*d - c
*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 0.42, size = 59, normalized size = 0.89

$$\frac{x\arctan\left(\frac{1}{ax}\right)}{\sqrt{dx^2+c}\,c}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c+d}}\right)}{\sqrt{-a^2c+d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*arctan(1/(a*x))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*
c + d))/(sqrt(-a^2*c + d)*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acot}(a\,x)}{{(d\,x^2+c)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(3/2), x)
```

3.62 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}} dx$

Optimal. Leaf size=134

$$\frac{a}{3c\left(a^2c-d\right)\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}-\frac{\left(3a^2c-2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2\left(a^2c-d\right)^{3/2}}$$

```
[Out] 1/3*x*arccot(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c-2*d)*arctanh(a*(d*x^2+c)^(
1/2)/(a^2*c-d)^(1/2))/c^2/(a^2*c-d)^(3/2)+1/3*a/c/(a^2*c-d)/(d*x^2+c)^(1/2)
+2/3*x*arccot(a*x)/c^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$,
Rules used = {198, 197, 5033, 6820, 12, 585, 79, 65, 214}

$$-\frac{\left(3a^2c-2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{3c^2\left(a^2c-d\right)^{3/2}}+\frac{a}{3c\left(a^2c-d\right)\sqrt{c+dx^2}}+\frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c - d)*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(3*c*(c + d*x^2)^(3/2
)) + (2*x*ArcCot[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c - 2*d)*ArcTanh[(
a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]])/(3*c^2*(a^2*c - d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + a\int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1+a^2x^2}\,dx$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + a\int \frac{x(3c+2dx^2)}{3c^2\,(1+a^2x^2)\,(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{a\int \frac{x(3c+2dx^2)}{(1+a^2x^2)(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{a\mathrm{Subst}\left(\int \frac{3c+2dx}{(1+a^2x)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{(a(3a^2c-2d))\,\mathrm{Subst}\Big(\int}{6c^2\,(a}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} + \frac{(a(3a^2c-2d))\,\mathrm{Subst}\Big(\int}{3c^2\,(a}$$

$$= \frac{a}{3c\,(a^2c-d)\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cot^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c-2d)\tanh^{-1}\Big(\frac{a\sqrt{c}}{\sqrt{a}}}{3c^2\,(a^2c-d)^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.50, size = 262, normalized size = 1.96

$$-\frac{-\frac{2ac}{(a^2c-d)\sqrt{c+dx^2}} - \frac{2x(3c+2dx^2)\cot^{-1}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c-2d)\log\left(\frac{12ac^2\sqrt{a^2c-d}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(3a^2c-2d)(i+ax)}\right)}{(a^2c-d)^{3/2}} + \frac{(3a^2c-2d)\log\left(\frac{12ac^2\sqrt{a^2c-d}\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(3a^2c-2d)(-i+ax)}\right)}{(a^2c-d)^{3/2}}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/6*((-2*a*c)/((a^2*c - d)*Sqrt[c + d*x^2]) - (2*x*(3*c + 2*d*x^2)*ArcCot[
a*x])/(c + d*x^2)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*(a
*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(I + a*x))]
)/(a^2*c - d)^(3/2) + ((3*a^2*c - 2*d)*Log[(12*a*c^2*Sqrt[a^2*c - d]*(a*c +
 I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((3*a^2*c - 2*d)*(-I + a*x))])/(
a^2*c - d)^(3/2))/c^2
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(114) = 228.
time = 1.76, size = 712, normalized size = 5.31

$$\left[\frac{\left(3\,a^{2}c^{3}+\left(3\,a^{2}cd^{2}-2\,d^{3}\right)x^{4}-2\,c^{2}d+2\left(3\,a^{2}c^{2}d-2\,cd^{2}\right)x^{2}\right)\sqrt{a^{2}c-d}\,\log\left(\frac{a^{4}d^{2}x^{4}+8a^{4}c^{2}-8a^{2}cd+2\left(4a^{4}cd-3a^{2}d^{2}\right)x^{2}-4\left(a^{3}dx^{2}+2a^{3}c-ad\right)\sqrt{a^{2}c-d}\,\sqrt{dx^{2}+c}+d^{2}}{a^{4}x^{4}+2a^{2}x^{2}+1}\right)+4\left(a^{3}c^{3}-ac^{2}d+\left(a^{3}c^{2}d-acd^{2}\right)x^{2}+\left(2\left(a^{4}c^{2}d-2\,a^{2}cd^{2}+d^{3}\right)x^{3}+3\left(a^{4}c^{3}-2\,a^{2}c^{2}d+cd^{2}\right)x\right)\operatorname{arccot}\left(ax\right)\right)\sqrt{dx^{2}+c}}{12\left(a^{4}c^{6}-2\,a^{2}c^{5}d+c^{4}d^{2}+\left(a^{4}c^{4}d^{2}-2\,a^{2}c^{3}d^{3}+c^{2}d^{4}\right)x^{4}+2\left(a^{4}c^{5}d-2\,a^{2}c^{4}d^{2}+c^{3}d^{3}\right)x^{2}\right)},\ -\frac{\left(3\,a^{2}c^{3}+\left(3\,a^{2}cd^{2}-2\,d^{3}\right)x^{4}-2\,c^{2}d+2\left(3\,a^{2}c^{2}d-2\,cd^{2}\right)x^{2}\right)\sqrt{-a^{2}c+d}\,\arctan\left(-\frac{\left(a^{2}dx^{2}+2a^{2}c-d\right)\sqrt{-a^{2}c+d}\,\sqrt{dx^{2}+c}}{2\left(a^{3}c^{2}-acd+\left(a^{3}cd-ad^{2}\right)x^{2}\right)}\right)-2\left(a^{3}c^{3}-ac^{2}d+\left(a^{3}c^{2}d-acd^{2}\right)x^{2}+\left(2\left(a^{4}c^{2}d-2\,a^{2}cd^{2}+d^{3}\right)x^{3}+3\left(a^{4}c^{3}-2\,a^{2}c^{2}d+cd^{2}\right)x\right)\operatorname{arccot}\left(ax\right)\right)\sqrt{dx^{2}+c}}{6\left(a^{4}c^{6}-2\,a^{2}c^{5}d+c^{4}d^{2}+\left(a^{4}c^{4}d^{2}-2\,a^{2}c^{3}d^{3}+c^{2}d^{4}\right)x^{4}+2\left(a^{4}c^{5}d-2\,a^{2}c^{4}d^{2}+c^{3}d^{3}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 - 2*d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d -
2*c*d^2)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*
(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*(a^3*c^3 - a*c^2*d +
 (a^3*c^2*d - a*c*d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^
4*c^3 - 2*a^2*c^2*d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^6 - 2*
a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*
c^5*d - 2*a^2*c^4*d^2 + c^3*d^3)*x^2), -1/6*((3*a^2*c^3 + (3*a^2*c*d^2 - 2*
d^3)*x^4 - 2*c^2*d + 2*(3*a^2*c^2*d - 2*c*d^2)*x^2)*sqrt(-a^2*c + d)*arctan
(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 -
 a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(a^3*c^3 - a*c^2*d + (a^3*c^2*d - a*c*
d^2)*x^2 + (2*(a^4*c^2*d - 2*a^2*c*d^2 + d^3)*x^3 + 3*(a^4*c^3 - 2*a^2*c^2*
d + c*d^2)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^4*c^6 - 2*a^2*c^5*d + c^4*d^
2 + (a^4*c^4*d^2 - 2*a^2*c^3*d^3 + c^2*d^4)*x^4 + 2*(a^4*c^5*d - 2*a^2*c^4*
d^2 + c^3*d^3)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(ax\right)}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [A]
time = 0.44, size = 126, normalized size = 0.94

$$\frac{1}{3}\,a\left(\frac{(3\,a^2c-2\,d)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c+d}}\right)}{(a^2c^3-c^2d)\sqrt{-a^2c+d}\,a}+\frac{1}{(a^2c^2-cd)\sqrt{dx^2+c}}\right)+\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\arctan\left(\frac{1}{ax}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c - 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^2*c^3
 - c^2*d)*sqrt(-a^2*c + d)*a) + 1/((a^2*c^2 - c*d)*sqrt(d*x^2 + c))) + 1/3*
x*(2*d*x^2/c^2 + 3/c)*arctan(1/(a*x))/(d*x^2 + c)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a\,x)}{{(d\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(5/2), x)
```

3.63 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=208

$$\frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}}+\frac{a(7a^2c-4d)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-$$

```
[Out] 1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)+1/5*x*arccot(a*x)/c/(d*x^2+c)^(5/2)+4/15
*x*arccot(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2-20*a^2*c*d+8*d^2)*arcta
nh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(5/2)+1/15*a*(7*a^2*c-4
*d)/c^2/(a^2*c-d)^2/(d*x^2+c)^(1/2)+8/15*x*arccot(a*x)/c^3/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 5033, 6820, 12, 6847, 911, 1275, 214}

$$\frac{a(7a^2c-4d)}{15c^2\left(a^2c-d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c-d\right)\left(c+dx^2\right)^{3/2}}-\frac{\left(15a^4c^2-20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3\left(a^2c-d\right)^{5/2}}+\frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cot^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\cot^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c - d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c - 4*d))/(15*c^2*(a^2*c
 - d)^2*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*A
rcCot[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCot[a*x])/(15*c^3*Sqrt[c +
 d*x^2]) - ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/S
qrt[a^2*c - d]])/(15*c^3*(a^2*c - d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}}}{1+a^2x^2}$$

$$= \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3\,(1+a^2x^2)\,(c+dx^2)^{5/}}$$

$$= \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1+a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1+a^2x)(c+dx)^{5/2}}\,dx, x\right.}{30c^3}$$

$$= \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{-a^2c+d}{d}+\frac{a^2x^2}{d}\right)}\,dx, x\right.}{15c^3d}$$

$$= \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cot^{-1}(ax)}{15c^3\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int \left(\frac{3c^2d}{(-a^2c+d)x^4} - \frac{c(7a^2c-}{(-a^2c+c}\right.\right.}{}$$

$$= \frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(}{15c^2\,(c+dx}$$

$$= \frac{a}{15c\,(a^2c-d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c-4d)}{15c^2\,(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\cot^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cot^{-1}(}{15c^2\,(c+dx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.76, size = 345, normalized size = 1.66

$$-\frac{-\frac{2ac(-d(5c+4dx^2)+a^2c(8c+7dx^2))}{(-a^2c+d)^2(c+dx^2)^{3/2}} - \frac{2x(15c^2+20cdx^2+8d^2x^4)\cot^{-1}(ax)}{(c+dx^2)^{5/2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\log\left(\frac{60ac^3(a^2c-d)^{3/2}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(15a^4c^2-20a^2cd+8d^2)(i+ax)}\right)}{(a^2c-d)^{5/2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\log\left(\frac{60ac^3(a^2c-d)^{3/2}\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(15a^4c^2-20a^2cd+8d^2)(-i+ax)}\right)}{(a^2c-d)^{5/2}}}{30c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/30*((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) +
 d)^2*(c + d*x^2)^(3/2)) - (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCot[a*
x])/(c + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(60*a*c^3*(a
^2*c - d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c
^2 - 20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20
*a^2*c*d + 8*d^2)*Log[(60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2*c
 - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])/(a
^2*c - d)^(5/2))/c^3
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^{2}+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(180) = 360$.
time = 1.42, size = 1278, normalized size = 6.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x
^2 + 1)) + 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*
a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3
)*x^2 + (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 + 20*(a^6*
c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 - 3*a^4*c^
4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arccot(a*x))*sqrt(d*x^2 + c))/(a^6*c^9 -
3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5*d^4 + 3*a^
2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 -
```

```
 c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 - c^5*d^4)*x^2
), -1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d
^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3
*(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*c + d)*arctan(-
1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a
*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2
+ (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3
*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4
 - d^5)*x^5 + 20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 +
15*(a^6*c^5 - 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arccot(a*x))*sqrt(d
*x^2 + c))/(a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3
- 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6
*d^3 + 3*a^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*
c^6*d^3 - c^5*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(7/2), x)
```

Giac [A]
time = 0.42, size = 208, normalized size = 1.00

$$\frac{1}{15}\,a\left(\frac{(15\,a^4c^2-20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c+d}}\right)}{(a^4c^5-2\,a^2c^4d+c^3d^2)\sqrt{-a^2c+d}\,a}+\frac{7\,(dx^2+c)a^2c+a^2c^2-4\,(dx^2+c)d-cd}{(a^4c^4-2\,a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{15\,(dx^2+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c + d))/((a^4*c^5 - 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c + d)*a) + (7*(d*x^
2 + c)*a^2*c + a^2*c^2 - 4*(d*x^2 + c)*d - c*d)/((a^4*c^4 - 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/15*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*arctan(1/(a*x))/(d*x^2 + c)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acot}(a\,x)}{{\left(d\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(7/2), x)
```

3.64 $\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=293

$$\frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}}+\frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}}+\frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\,(a^2c-d)^3\,\sqrt{c+dx^2}}+\frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x\,\text{c}}{35c^2\,(}$$

```
[Out] 1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2/(
d*x^2+c)^(3/2)+1/7*x*arccot(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccot(a*x)/c^2/(
d*x^2+c)^(5/2)+8/35*x*arccot(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3-70*a
^4*c^2*d+56*a^2*c*d^2-16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^
4/(a^2*c-d)^(7/2)+1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^3/(d*x
^2+c)^(1/2)+16/35*x*arccot(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 5033, 6820, 12, 6847, 1633, 65, 214}

$$\frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}}+\frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}}+\frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\,(a^2c-d)^3\,\sqrt{c+dx^2}}-\frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4\,(a^2c-d)^{7/2}}+\frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}}+\frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c - d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c - 6*d))/(105*c^2*(a^2
*c - d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c - d)^3*Sqrt[c + d*x^2]) + (x*ArcCot[a*x])/(7*c*(c + d*x^2)^(7/2))
+ (6*x*ArcCot[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCot[a*x])/(35*c^3*
(c + d*x^2)^(3/2)) + (16*x*ArcCot[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35*a^6
*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2])/Sq
rt[a^2*c - d]])/(35*c^4*(a^2*c - d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 5033

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCot[c*x], u, x]
+ Dist[b*c, Int[u/(1 + c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (I
ntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + a\int \frac{\frac{x}{7c(c+dx}}{}$$

$$= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + a\int \frac{x(35c^3}{3}$$

$$= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{a\int \frac{x(35c^3+7}{(1}}{}$$

$$= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int\right.}{}$$

$$= \frac{x\cot^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cot^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cot^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cot^{-1}(ax)}{35c^4\sqrt{c+dx^2}} + \frac{a\text{Subst}\left(\int\right.}{}$$

$$= \frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\,(a^2c-d)^3\,\sqrt{c+dx^2}}$$

$$= \frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\,(a^2c-d)^3\,\sqrt{c+dx^2}}$$

$$= \frac{a}{35c\,(a^2c-d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c-6d)}{105c^2\,(a^2c-d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3\,(a^2c-d)^3\,\sqrt{c+dx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 450, normalized size = 1.54

$$\frac{\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}} + \frac{6x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\cot^{-1}(ax)}{(c+dx^2)^{7/2}} - \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\log\left(\frac{140ac^4(a^2c-d)^{5/2}\left(ac-idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(i+ax)}\right)}{(a^2c-d)^{7/2}} - \frac{3(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\log\left(\frac{140ac^4(a^2c-d)^{5/2}\left(ac+idx+\sqrt{a^2c-d}\sqrt{c+dx^2}\right)}{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)(-i+ax)}\right)}{(a^2c-d)^{7/2}}}{210c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] ((2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x^2
) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(c +
 d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*A
rcCot[a*x])/(c + d*x^2)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^
2 - 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2*c - d]
*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(I
+ a*x))])/(a^2*c - d)^(7/2) - (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2
```

```
- 16*d^3)*Log[(140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a^2*c - d]*S
qrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*(-I +
 a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(ax)}{(d\,x^2+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccot(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(257) = 514$.
time = 2.81, size = 1986, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a^2*x^2
+ 1)) + 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*
(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a
```

```
^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a
^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 + 3*(16*(a
^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a
^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 7
0*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3
 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*
arccot(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4
*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a
^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5
- 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^
8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a
^4*c^9*d^3 - 4*a^2*c^8*d^4 + c^7*d^5)*x^2), -1/210*(3*(35*a^6*c^7 - 70*a^4*
c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 - 70*a^4*c^2*d^5 + 56*a^2*c*d^6 -
16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*
d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 - 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 -
 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c
^3*d^4)*x^2)*sqrt(-a^2*c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a
^2*c + d)*sqrt(d*x^2 + c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(7
1*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*d^3 + 3*(19*a^7*c^4*
d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6 + (182*a^7*c^5*d^2 -
 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^4 + (196*a^7*c^6*d - 4
34*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2 + 3*(16*(a^8*c^4*d^3 -
 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)*x^7 + 56*(a^8*c^5*d^2 -
 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^8*c^6*d
 - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^8*c
^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3 + c^3*d^4)*x)*arccot(a*x))
*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*a^4*c^10*d^2 - 4*a^2*c^9*d^3
 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*a^4*c^6*d^6 - 4*a^2*c^5*d^7 +
 c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*
d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^
2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d - 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 -
 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(acot(a*x)/(c + d*x**2)**(9/2), x)
```

Giac [**A**]

time = 0.44, size = 340, normalized size = 1.16

$$\frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3-70\,a^4c^2d+56\,a^2cd^2-16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c+d}}\right)}{(a^6c^7-3\,a^4c^6d+3\,a^2c^5d^2-c^4d^3)\sqrt{-a^2c+d}\,a}+\frac{57\,(dx^2+c)^2a^4c^2+11\,(dx^2+c)a^4c^3+3\,a^4c^4-66\,(dx^2+c)^2a^2cd-17\,(dx^2+c)a^2c^2d-6\,a^2c^3d+24\,(dx^2+c)^2d^2+6\,(dx^2+c)cd^2+3\,c^2d^2}{(a^6c^6-3\,a^4c^5d+3\,a^2c^4d^2-c^3d^3)(dx^2+c)^{\frac{5}{2}}}\right)+\frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\arctan\left(\frac{1}{ax}\right)}{35\,(dx^2+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^6*c^7 - 3*a^4*c^6*d + 3*a^2*c^5*d^2 - c^
4*d^3)*sqrt(-a^2*c + d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 - 66*(d*x^2 + c)^2*a^2*c*d - 17*(d*x^2 + c)*a^2*c^2*d - 6*
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 - 3*a^4*c^5*d + 3*a^2*c^4*d^2 - c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/35*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*arctan(1/(a*x))
/(d*x^2 + c)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acot}(a\,x)}{{\left(d\,x^2+c\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(acot(a*x)/(c + d*x^2)^(9/2), x)
```

3.65 $\int \sqrt{a+ax^2} \cot^{-1}(x)\,dx$

Optimal. Leaf size=195

$$\frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)-\frac{ia\sqrt{1+x^2}\cot^{-1}(x)\text{ArcTan}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}-\frac{ia\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\right.}{2\sqrt{a+ax^2}}$$

```
[Out] -I*a*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^
(1/2)-1/2*I*a*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^
2+a)^(1/2)+1/2*I*a*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(
a*x^2+a)^(1/2)+1/2*(a*x^2+a)^(1/2)+1/2*x*arccot(x)*(a*x^2+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4999, 5011, 5007}

$$-\frac{ia\sqrt{x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)}{\sqrt{ax^2+a}}-\frac{ia\sqrt{x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{2\sqrt{ax^2+a}}+\frac{ia\sqrt{x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{2\sqrt{ax^2+a}}+\frac{1}{2}\sqrt{ax^2+a}+\frac{1}{2}x\sqrt{ax^2+a}\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x^2]*ArcCot[x],x]
```

```
[Out] Sqrt[a + a*x^2]/2 + (x*Sqrt[a + a*x^2]*ArcCot[x])/2 - (I*a*Sqrt[1 + x^2]*Ar
cCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]])/Sqrt[a + a*x^2] - ((I/2)*a*Sqr
t[1 + x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*x])/Sqrt[1 - I*x]])/Sqrt[a + a*x^2]
+ ((I/2)*a*Sqrt[1 + x^2]*PolyLog[2, (I*Sqrt[1 + I*x])/Sqrt[1 - I*x]])/Sqrt[
a + a*x^2]
```

Rule 4999

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1)
), Int[(d + e*x^2)^(q - 1)*(a + b*ArcCot[c*x]), x], x] + Simp[x*(d + e*x^2)
^q*((a + b*ArcCot[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ
[e, c^2*d] && GtQ[q, 0]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ax^2}\cot^{-1}(x)\,dx &= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)+\frac{1}{2}a\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx\\ &= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)+\frac{\left(a\sqrt{1+x^2}\right)\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx}{2\sqrt{a+ax^2}}\\ &= \frac{1}{2}\sqrt{a+ax^2}+\frac{1}{2}x\sqrt{a+ax^2}\cot^{-1}(x)-\frac{ia\sqrt{1+x^2}\cot^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right.}{\sqrt{a+ax^2}}\end{aligned}$$

Mathematica [A]
time = 0.89, size = 136, normalized size = 0.70

$$-\frac{(a(1+x^2))^{3/2}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(x)\right)-\cot^{-1}(x)\csc^2\left(\frac{1}{2}\cot^{-1}(x)\right)+4\cot^{-1}(x)\log\left(1-e^{i\cot^{-1}(x)}\right)-4\cot^{-1}(x)\log\left(1+e^{i\cot^{-1}(x)}\right)+4i\text{PolyLog}\left(2,-e^{i\cot^{-1}(x)}\right)-4i\text{PolyLog}\left(2,e^{i\cot^{-1}(x)}\right)+\cot^{-1}(x)\sec^2\left(\frac{1}{2}\cot^{-1}(x)\right)-2\tan\left(\frac{1}{2}\cot^{-1}(x)\right)\right)}{8a\left(1+\frac{1}{x^2}\right)^{3/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x^2]*ArcCot[x],x]
```

```
[Out] -1/8*((a*(1 + x^2))^(3/2)*(-2*Cot[ArcCot[x]/2] - ArcCot[x]*Csc[ArcCot[x]/2]
^2 + 4*ArcCot[x]*Log[1 - E^(I*ArcCot[x])] - 4*ArcCot[x]*Log[1 + E^(I*ArcCot
[x])] + (4*I)*PolyLog[2, -E^(I*ArcCot[x])] - (4*I)*PolyLog[2, E^(I*ArcCot[x
])] + ArcCot[x]*Sec[ArcCot[x]/2]^2 - 2*Tan[ArcCot[x]/2]))/(a*(1 + x^(-2))^(
3/2)*x^3)
```

Maple [A]
time = 0.27, size = 117, normalized size = 0.60

method	result
default	$\frac{\sqrt{a(i+x)(x-i)}\,(x\,\mathrm{arccot}(x)+1)}{2}-\frac{i\sqrt{a(i+x)(x-i)}\left(i\mathrm{arccot}(x)\ln\left(\frac{i+x}{\sqrt{x^2+1}}+1\right)-i\mathrm{arccot}(x)\ln\left(1-\right.\right.}{2\sqrt{x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+a)^(1/2)*arccot(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*(I+x)*(x-I))^(1/2)*(x*arccot(x)+1)-1/2*I*(a*(I+x)*(x-I))^(1/2)*(I*ar
ccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(1/2))+pol
ylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))/(x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="fricas")
[Out] integral(sqrt(a*x^2 + a)*arccot(x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(x^2+1\right)}\,\operatorname{acot}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+a)**(1/2)*acot(x),x)
[Out] Integral(sqrt(a*(x**2 + 1))*acot(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+a)^(1/2)*arccot(x),x, algorithm="giac")
[Out] integrate(sqrt(a*x^2 + a)*arccot(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(x)\,\sqrt{a\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)*(a + a*x^2)^(1/2),x)
```

```
[Out] int(acot(x)*(a + a*x^2)^(1/2), x)
```

3.66 $\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}} \, dx$

Optimal. Leaf size=155

$$-\frac{2i\sqrt{1+x^2}\cot^{-1}(x)\text{ArcTan}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}-\frac{i\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}+\frac{i\sqrt{1+x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}}$$

```
[Out] -2*I*arccot(x)*arctan((1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^
(1/2)-I*polylog(2,-I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^(
1/2)+I*polylog(2,I*(1+I*x)^(1/2)/(1-I*x)^(1/2))*(x^2+1)^(1/2)/(a*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5011, 5007}

$$-\frac{2i\sqrt{x^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)\cot^{-1}(x)}{\sqrt{ax^2+a}}-\frac{i\sqrt{x^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{\sqrt{ax^2+a}}+\frac{i\sqrt{x^2+1}\,\text{Li}_2\left(\frac{i\sqrt{ix+1}}{\sqrt{1-ix}}\right)}{\sqrt{ax^2+a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/Sqrt[a + a*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + x^2]*ArcCot[x]*ArcTan[Sqrt[1 + I*x]/Sqrt[1 - I*x]])/Sqrt[a
 + a*x^2] - (I*Sqrt[1 + x^2]*PolyLog[2, ((-I)*Sqrt[1 + I*x])/Sqrt[1 - I*x]]
)/Sqrt[a + a*x^2] + (I*Sqrt[1 + x^2]*PolyLog[2, (I*Sqrt[1 + I*x])/Sqrt[1 -
I*x]])/Sqrt[a + a*x^2]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(x)}{\sqrt{a+ax^2}}\,dx = \frac{\sqrt{1+x^2}\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx}{\sqrt{a+ax^2}}$$
$$= -\frac{2i\sqrt{1+x^2}\cot^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} - \frac{i\sqrt{1+x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1+ix}}{\sqrt{1-ix}}\right)}{\sqrt{a+ax^2}} + \frac{i\sqrt{1+x^2}}{\sqrt{}}$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 0.57

$$-\frac{\sqrt{a(1+x^2)}\left(\cot^{-1}(x)\left(\log\left(1-e^{i\cot^{-1}(x)}\right)-\log\left(1+e^{i\cot^{-1}(x)}\right)\right)+i\mathrm{PolyLog}\left(2,-e^{i\cot^{-1}(x)}\right)-i\mathrm{PolyLog}\left(2,e^{i\cot^{-1}(x)}\right)\right)}{a\sqrt{1+\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/Sqrt[a + a*x^2],x]
```

```
[Out] -((Sqrt[a*(1 + x^2)]*(ArcCot[x]*(Log[1 - E^(I*ArcCot[x])] - Log[1 + E^(I*Ar
cCot[x])]) + I*PolyLog[2, -E^(I*ArcCot[x])] - I*PolyLog[2, E^(I*ArcCot[x])]
))/(a*Sqrt[1 + x^(-2)]*x))
```

Maple [A]
time = 0.20, size = 99, normalized size = 0.64

method	result
default	$-\frac{i\left(i\mathrm{arccot}(x)\ln\left(\frac{i+x}{\sqrt{x^2+1}}+1\right)-i\mathrm{arccot}(x)\ln\left(1-\frac{i+x}{\sqrt{x^2+1}}\right)+\mathrm{polylog}\left(2,-\frac{i+x}{\sqrt{x^2+1}}\right)-\mathrm{polylog}\left(2,\frac{i+x}{\sqrt{x^2+1}}\right)\right)}{\sqrt{x^2+1}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(I*arccot(x)*ln((I+x)/(x^2+1)^(1/2)+1)-I*arccot(x)*ln(1-(I+x)/(x^2+1)^(1
/2))+polylog(2,-(I+x)/(x^2+1)^(1/2))-polylog(2,(I+x)/(x^2+1)^(1/2)))*(a*(I+
x)*(x-I))^(1/2)/(x^2+1)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccot(x)/sqrt(a*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccot(x)/sqrt(a*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(x)}{\sqrt{a(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(a*x**2+a)**(1/2),x)
```

```
[Out] Integral(acot(x)/sqrt(a*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccot(x)/sqrt(a*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(x)}{\sqrt{a\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(a + a*x^2)^(1/2),x)
```

```
[Out] int(acot(x)/(a + a*x^2)^(1/2), x)
```

3.67 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{a\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{a\sqrt{a+ax^2}}$$

[Out] -1/a/(a*x^2+a)^(1/2)+x*arccot(x)/a/(a*x^2+a)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5015}

$$\frac{x\cot^{-1}(x)}{a\sqrt{ax^2+a}}-\frac{1}{a\sqrt{ax^2+a}}$$

Antiderivative was successfully verified.

[In] Int[ArcCot[x]/(a + a*x^2)^(3/2),x]

[Out] -(1/(a*Sqrt[a + a*x^2])) + (x*ArcCot[x])/(a*Sqrt[a + a*x^2])

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{a\sqrt{a+ax^2}}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 0.60

$$\frac{-1+x\cot^{-1}(x)}{a\sqrt{a\left(1+x^2\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCot[x]/(a + a*x^2)^(3/2),x]

[Out] (-1 + x*ArcCot[x])/(a*Sqrt[a*(1 + x^2)])

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 68, normalized size = 1.94

method	result	size
risch	$\frac{ix\ln(ix+1)}{2a\sqrt{a\left(x^2+1\right)}}+\frac{-i\ln(-ix+1)x+\pi x-2}{2a\sqrt{a\left(x^2+1\right)}}$	55
default	$\frac{(\operatorname{arccot}(x)+i)(i+x)\sqrt{a\left(i+x\right)\left(x-i\right)}}{2(x^2+1)a^2}+\frac{\sqrt{a\left(i+x\right)\left(x-i\right)}\,(x-i)(\operatorname{arccot}(x)-i)}{2(x^2+1)a^2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arccot(x)+I)*(I+x)*(a*(I+x)*(x-I))^(1/2)/(x^2+1)/a^2+1/2*(a*(I+x)*(x-I
))^(1/2)*(x-I)*(arccot(x)-I)/(x^2+1)/a^2
```

Maxima [A]
time = 0.47, size = 31, normalized size = 0.89

$$\frac{x\operatorname{arccot}\left(x\right)}{\sqrt{ax^2+a}\,a}-\frac{1}{\sqrt{ax^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arccot(x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

Fricas [A]
time = 1.01, size = 29, normalized size = 0.83

$$\frac{\sqrt{ax^2+a}\left(x\operatorname{arccot}\left(x\right)-1\right)}{a^2x^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*x^2 + a)*(x*arccot(x) - 1)/(a^2*x^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acot}\left(x\right)}{\left(a\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(a*x**2+a)**(3/2),x)
```

```
[Out] Integral(acot(x)/(a*(x**2 + 1))**(3/2), x)
```

Giac [A]
time = 0.43, size = 33, normalized size = 0.94

$$\frac{x \arctan\left(\frac{1}{x}\right)}{\sqrt{ax^2+a}\, a} - \frac{1}{\sqrt{ax^2+a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(3/2),x, algorithm="giac")
[Out] x*arctan(1/x)/(sqrt(a*x^2 + a)*a) - 1/(sqrt(a*x^2 + a)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acot}(x)}{{\left(a\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(a + a*x^2)^(3/2),x)
[Out] int(acot(x)/(a + a*x^2)^(3/2), x)
```

3.68 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}} dx$

Optimal. Leaf size=79

$$-\frac{1}{9a(a+ax^2)^{3/2}}-\frac{2}{3a^2\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{3a(a+ax^2)^{3/2}}+\frac{2x\cot^{-1}(x)}{3a^2\sqrt{a+ax^2}}$$

```
[Out] -1/9/a/(a*x^2+a)^(3/2)+1/3*x*arccot(x)/a/(a*x^2+a)^(3/2)-2/3/a^2/(a*x^2+a)^
(1/2)+2/3*x*arccot(x)/a^2/(a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5017, 5015}

$$-\frac{2}{3a^2\sqrt{ax^2+a}}+\frac{2x\cot^{-1}(x)}{3a^2\sqrt{ax^2+a}}-\frac{1}{9a(ax^2+a)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(ax^2+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(a + a*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*(a + a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a + a*x^2]) + (x*ArcCot[x])/(3
*a*(a + a*x^2)^(3/2)) + (2*x*ArcCot[x])/(3*a^2*Sqrt[a + a*x^2])
```

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5017

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3
)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x] - Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a
, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}} dx &= -\frac{1}{9a(a+ax^2)^{3/2}}+\frac{x\cot^{-1}(x)}{3a(a+ax^2)^{3/2}}+\frac{2\int \frac{\cot^{-1}(x)}{(a+ax^2)^{3/2}} dx}{3a}\\
&= -\frac{1}{9a(a+ax^2)^{3/2}}-\frac{2}{3a^2\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{3a(a+ax^2)^{3/2}}+\frac{2x\cot^{-1}(x)}{3a^2\sqrt{a+ax^2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.47

$$\frac{-7-6x^2+(9x+6x^3)\cot^{-1}(x)}{9a\left(a\left(1+x^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(a + a*x^2)^(5/2),x]
```

```
[Out] (-7 - 6*x^2 + (9*x + 6*x^3)*ArcCot[x])/(9*a*(a*(1 + x^2))^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 165, normalized size = 2.09

method	result
risch	$\frac{ix(2x^2+3)\ln(ix+1)}{6a^2(x^2+1)\sqrt{a\left(x^2+1\right)}}+\frac{-6ix^3\ln(-ix+1)+6\pi x^3-9i\ln(-ix+1)x+9\pi x-12x^2-14}{18a^2(x^2+1)\sqrt{a\left(x^2+1\right)}}$
default	$-\frac{(3\,\mathrm{arccot}(x)+i)(x^3+3ix^2-3x-i)\sqrt{a\left(i+x\right)\left(x-i\right)}}{72(x^2+1)^2a^3}+\frac{3(\mathrm{arccot}(x)+i)(i+x)\sqrt{a\left(i+x\right)\left(x-i\right)}}{8a^3(x^2+1)}+\frac{3\sqrt{a\left(i+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(3*arccot(x)+I)*(3*I*x^2+x^3-I-3*x)*(a*(I+x)*(x-I))^(1/2)/(x^2+1)^2/a
^3+3/8*(arccot(x)+I)*(I+x)*(a*(I+x)*(x-I))^(1/2)/a^3/(x^2+1)+3/8*(a*(I+x)*(
x-I))^(1/2)*(x-I)*(arccot(x)-I)/a^3/(x^2+1)-1/72*(-I+3*arccot(x))*(a*(I+x)*
(x-I))^(1/2)*(-3*x-3*I*x^2+x^3+I)/(x^4+2*x^2+1)/a^3
```

Maxima [A]
time = 0.48, size = 63, normalized size = 0.80

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{ax^2+a}\,a^2}+\frac{x}{\left(ax^2+a\right)^{\frac{3}{2}}a}\right)\mathrm{arccot}\left(x\right)-\frac{2}{3\,\sqrt{ax^2+a}\,a^2}-\frac{1}{9\,\left(ax^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(a*x^2 + a)*a^2) + x/((a*x^2 + a)^(3/2)*a))*arccot(x) - 2/3/(
sqrt(a*x^2 + a)*a^2) - 1/9/((a*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 4.32, size = 52, normalized size = 0.66

$$-\frac{\sqrt{ax^2+a}\,\left(6\,x^2-3\,\left(2\,x^3+3\,x\right)\mathrm{arccot}\left(x\right)+7\right)}{9\,\left(a^3x^4+2\,a^3x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(a*x^2 + a)*(6*x^2 - 3*(2*x^3 + 3*x)*arccot(x) + 7)/(a^3*x^4 + 2*a
^3*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(x)}{(a(x^2+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(a*x**2+a)**(5/2),x)
```

```
[Out] Integral(acot(x)/(a*(x**2 + 1))**(5/2), x)
```

Giac [A]
time = 0.43, size = 55, normalized size = 0.70

$$\frac{x\left(\frac{2x^2}{a}+\frac{3}{a}\right)\arctan\left(\frac{1}{x}\right)}{3(ax^2+a)^{\frac{3}{2}}}-\frac{6ax^2+7a}{9(ax^2+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*x^2/a + 3/a)*arctan(1/x)/(a*x^2 + a)^(3/2) - 1/9*(6*a*x^2 + 7*a)/(
(a*x^2 + a)^(3/2)*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(x)}{(a\,x^2+a)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(a + a*x^2)^(5/2),x)
```

```
[Out] int(acot(x)/(a + a*x^2)^(5/2), x)
```

3.69 $\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{1}{25a\left(a+ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a+ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a+ax^2}}+\frac{x\cot^{-1}(x)}{5a\left(a+ax^2\right)^{5/2}}+\frac{4x\cot^{-1}(x)}{15a^2\left(a+ax^2\right)^{3/2}}+\frac{8x\cot^{-1}(x)}{15a^3\sqrt{a+ax^2}}$$

```
[Out] -1/25/a/(a*x^2+a)^(5/2)-4/45/a^2/(a*x^2+a)^(3/2)+1/5*x*arccot(x)/a/(a*x^2+a
)^(5/2)+4/15*x*arccot(x)/a^2/(a*x^2+a)^(3/2)-8/15/a^3/(a*x^2+a)^(1/2)+8/15*
x*arccot(x)/a^3/(a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5017, 5015}

$$-\frac{8}{15a^3\sqrt{ax^2+a}}+\frac{8x\cot^{-1}(x)}{15a^3\sqrt{ax^2+a}}-\frac{4}{45a^2\left(ax^2+a\right)^{3/2}}+\frac{4x\cot^{-1}(x)}{15a^2\left(ax^2+a\right)^{3/2}}-\frac{1}{25a\left(ax^2+a\right)^{5/2}}+\frac{x\cot^{-1}(x)}{5a\left(ax^2+a\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(a + a*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*(a + a*x^2)^(5/2)) - 4/(45*a^2*(a + a*x^2)^(3/2)) - 8/(15*a^3*Sq
rt[a + a*x^2]) + (x*ArcCot[x])/(5*a*(a + a*x^2)^(5/2)) + (4*x*ArcCot[x])/(1
5*a^2*(a + a*x^2)^(3/2)) + (8*x*ArcCot[x])/(15*a^3*Sqrt[a + a*x^2])
```

Rule 5015

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCot[c*x])/(d*Sq
rt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5017

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q + 3
)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCot[c*x]), x], x] - Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])/(2*d*(q + 1))), x]) /; FreeQ[{a
, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\cot^{-1}(x)}{(a+ax^2)^{7/2}}\,dx = -\frac{1}{25a\,(a+ax^2)^{5/2}} + \frac{x\cot^{-1}(x)}{5a\,(a+ax^2)^{5/2}} + \frac{4\int \frac{\cot^{-1}(x)}{(a+ax^2)^{5/2}}\,dx}{5a}$$

$$= -\frac{1}{25a\,(a+ax^2)^{5/2}} - \frac{4}{45a^2\,(a+ax^2)^{3/2}} + \frac{x\cot^{-1}(x)}{5a\,(a+ax^2)^{5/2}} + \frac{4x\cot^{-1}(x)}{15a^2\,(a+ax^2)^{3/2}} + \frac{8\int \frac{\mathrm{co}}{(a+}}{1}$$

$$= -\frac{1}{25a\,(a+ax^2)^{5/2}} - \frac{4}{45a^2\,(a+ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a+ax^2}} + \frac{x\cot^{-1}(x)}{5a\,(a+ax^2)^{5/2}} + \frac{4x\,\mathrm{co}}{15a^2\,(a\,}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.40

$$\frac{-149-260x^2-120x^4+15x(15+20x^2+8x^4)\cot^{-1}(x)}{225a\,(a\,(1+x^2))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(a + a*x^2)^(7/2),x]
```

```
[Out] (-149 - 260*x^2 - 120*x^4 + 15*x*(15 + 20*x^2 + 8*x^4)*ArcCot[x])/(225*a*(a
*(1 + x^2))^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 258, normalized size = 2.19

method	result
risch	$\frac{ix(8x^4+20x^2+15)\ln(ix+1)}{30a^3(x^2+1)^2\sqrt{a\,(x^2+1)}} + \frac{-120ix^5\ln(-ix+1)+120x^5\pi-300ix^3\ln(-ix+1)+300\pi x^3-240x^4-225i\ln(-ix+1)x+225\pi x-520}{450a^3(x^2+1)^2\sqrt{a\,(x^2+1)}}$
default	$\frac{(i+5\,\mathrm{arccot}(x))(x^5+5ix^4-10x^3-10ix^2+5x+i)\sqrt{a\,(i+x)\,(x-i)}}{800(x^2+1)^3a^4} + \frac{5(\mathrm{arccot}(x)+i)(i+x)\sqrt{a\,(i+x)\,(x-i)}}{16(x^2+1)a^4} + \frac{5\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/800*(I+5*arccot(x))*(5*I*x^4+x^5-10*I*x^2-10*x^3+I+5*x)*(a*(I+x)*(x-I))^(
1/2)/(x^2+1)^3/a^4+5/16*(arccot(x)+I)*(I+x)*(a*(I+x)*(x-I))^(1/2)/(x^2+1)/a
^4+5/16*(a*(I+x)*(x-I))^(1/2)*(x-I)*(arccot(x)-I)/(x^2+1)/a^4-5/288*(-I+3*a
rccot(x))*(a*(I+x)*(x-I))^(1/2)*(-3*x-3*I*x^2+x^3+I)/(x^4+2*x^2+1)/a^4-1/36
00*(a*(I+x)*(x-I))^(1/2)*(x-I)*(67*I+165*arccot(x))*cos(4*arccot(x))/(x^2+1
)/a^4-1/1800*(a*(I+x)*(x-I))^(1/2)*(1+I*x)*(29*I+105*arccot(x))*sin(4*arcco
t(x))/(x^2+1)/a^4
```

Maxima [A]
time = 0.49, size = 93, normalized size = 0.79

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{ax^2+a}\,a^3}+\frac{4\,x}{(ax^2+a)^{\frac{3}{2}}a^2}+\frac{3\,x}{(ax^2+a)^{\frac{5}{2}}a}\right)\operatorname{arccot}(x)-\frac{8}{15\,\sqrt{ax^2+a}\,a^3}-\frac{4}{45\,(ax^2+a)^{\frac{3}{2}}a^2}-\frac{1}{25\,(ax^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(a*x^2 + a)*a^3) + 4*x/((a*x^2 + a)^(3/2)*a^2) + 3*x/((a*x^2
 + a)^(5/2)*a))*arccot(x) - 8/15/(sqrt(a*x^2 + a)*a^3) - 4/45/((a*x^2 + a)^
(3/2)*a^2) - 1/25/((a*x^2 + a)^(5/2)*a)
```

Fricas [A]
time = 1.93, size = 70, normalized size = 0.59

$$-\frac{\left(120\,x^4+260\,x^2-15\,(8\,x^5+20\,x^3+15\,x)\operatorname{arccot}(x)+149\right)\sqrt{ax^2+a}}{225\,(a^4x^6+3\,a^4x^4+3\,a^4x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/225*(120*x^4 + 260*x^2 - 15*(8*x^5 + 20*x^3 + 15*x)*arccot(x) + 149)*sqr
t(a*x^2 + a)/(a^4*x^6 + 3*a^4*x^4 + 3*a^4*x^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acot}(x)}{(a\,(x^2+1))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(a*x**2+a)**(7/2),x)
```

```
[Out] Integral(acot(x)/(a*(x**2 + 1))**(7/2), x)
```

Giac [A]
time = 0.44, size = 83, normalized size = 0.70

$$\frac{\left(4\,x^2\left(\frac{2\,x^2}{a}+\frac{5}{a}\right)+\frac{15}{a}\right)x\arctan\left(\frac{1}{x}\right)}{15\,(ax^2+a)^{\frac{5}{2}}}-\frac{120\,(ax^2+a)^2+20\,(ax^2+a)a+9\,a^2}{225\,(ax^2+a)^{\frac{5}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*x^2/a + 5/a) + 15/a)*x*arctan(1/x)/(a*x^2 + a)^(5/2) - 1/225
*(120*(a*x^2 + a)^2 + 20*(a*x^2 + a)*a + 9*a^2)/((a*x^2 + a)^(5/2)*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(x)}{\left(a\,x^2+a\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(a + a*x^2)^(7/2),x)
[Out] int(acot(x)/(a + a*x^2)^(7/2), x)
```

3.70 $\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{x}{4(1+x^2)}-\frac{\cot^{-1}(x)}{2(1+x^2)}-\frac{\text{ArcTan}(x)}{4}$$

```
[Out] -1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)-1/4*arctan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5051, 205, 209}

$$-\frac{\text{ArcTan}(x)}{4}-\frac{x}{4(x^2+1)}-\frac{\cot^{-1}(x)}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCot[x])/(1 + x^2)^2,x]
```

```
[Out] -1/4*x/(1 + x^2) - ArcCot[x]/(2*(1 + x^2)) - ArcTan[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\cot^{-1}(x)}{(1+x^2)^2}\,dx = -\frac{\cot^{-1}(x)}{2\,(1+x^2)} - \frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx$$
$$= -\frac{x}{4\,(1+x^2)} - \frac{\cot^{-1}(x)}{2\,(1+x^2)} - \frac{1}{4}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{4\,(1+x^2)} - \frac{\cot^{-1}(x)}{2\,(1+x^2)} - \frac{1}{4}\tan^{-1}(x)$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.78

$$-\frac{x+2\cot^{-1}(x)+\mathrm{ArcTan}(x)+x^2\mathrm{ArcTan}(x)}{4+4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2)^2,x]
[Out] -((x + 2*ArcCot[x] + ArcTan[x] + x^2*ArcTan[x])/(4 + 4*x^2))
```

Maple [A]
time = 0.13, size = 27, normalized size = 0.84

method	result	size
default	$-\frac{x}{4(x^2+1)}-\frac{\mathrm{arccot}(x)}{2(x^2+1)}-\frac{\arctan(x)}{4}$	27
risch	$-\frac{i\ln(ix+1)}{4(x^2+1)}-\frac{-2i\ln(-ix+1)-i\ln(x-i)x^2-i\ln(x-i)+i\ln(i+x)x^2+i\ln(i+x)+2\pi+2x}{8(i+x)(x-i)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)-1/4*arctan(x)
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.81

$$-\frac{x}{4\,(x^2+1)}-\frac{\mathrm{arccot}\,(x)}{2\,(x^2+1)}-\frac{1}{4}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/4*x/(x^2 + 1) - 1/2*arccot(x)/(x^2 + 1) - 1/4*arctan(x)
```

Fricas [A]
time = 2.37, size = 21, normalized size = 0.66

$$\frac{(x^2-1)\operatorname{arccot}(x)-x}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 - 1)*arccot(x) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.22, size = 31, normalized size = 0.97

$$\frac{x^2\operatorname{acot}(x)}{4x^2+4}-\frac{x}{4x^2+4}-\frac{\operatorname{acot}(x)}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x)/(x**2+1)**2,x)
[Out] x**2*acot(x)/(4*x**2 + 4) - x/(4*x**2 + 4) - acot(x)/(4*x**2 + 4)
```

Giac [A]
time = 0.43, size = 32, normalized size = 1.00

$$-\frac{\arctan\left(\frac{1}{x}\right)}{2(x^2+1)}-\frac{1}{4x\left(\frac{1}{x^2}+1\right)}+\frac{1}{4}\arctan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*arctan(1/x)/(x^2 + 1) - 1/4/(x*(1/x^2 + 1)) + 1/4*arctan(1/x)
```

Mupad [B]
time = 0.07, size = 22, normalized size = 0.69

$$\frac{\operatorname{acot}(x)}{4}-\frac{\frac{x}{4}+\frac{\operatorname{acot}(x)}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acot(x))/(x^2 + 1)^2,x)
[Out] acot(x)/4 - (x/4 + acot(x)/2)/(x^2 + 1)
```

3.71 $\int \frac{x \cot^{-1}(x)}{(1+x^2)^3} \, dx$

Optimal. Leaf size=44

$$-\frac{x}{16(1+x^2)^2}-\frac{3x}{32(1+x^2)}-\frac{\cot^{-1}(x)}{4(1+x^2)^2}-\frac{3\text{ArcTan}(x)}{32}$$

```
[Out] -1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5051, 205, 209}

$$-\frac{3\text{ArcTan}(x)}{32}-\frac{3x}{32(x^2+1)}-\frac{x}{16(x^2+1)^2}-\frac{\cot^{-1}(x)}{4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCot[x])/(1 + x^2)^3,x]
```

```
[Out] -1/16*x/(1 + x^2)^2 - (3*x)/(32*(1 + x^2)) - ArcCot[x]/(4*(1 + x^2)^2) - (3
*ArcTan[x])/32
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cot^{-1}(x)}{(1+x^2)^3}\,dx &= -\frac{\cot^{-1}(x)}{4(1+x^2)^2}-\frac{1}{4}\int \frac{1}{(1+x^2)^3}\,dx\\
&= -\frac{x}{16(1+x^2)^2}-\frac{\cot^{-1}(x)}{4(1+x^2)^2}-\frac{3}{16}\int \frac{1}{(1+x^2)^2}\,dx\\
&= -\frac{x}{16(1+x^2)^2}-\frac{3x}{32(1+x^2)}-\frac{\cot^{-1}(x)}{4(1+x^2)^2}-\frac{3}{32}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{x}{16(1+x^2)^2}-\frac{3x}{32(1+x^2)}-\frac{\cot^{-1}(x)}{4(1+x^2)^2}-\frac{3}{32}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.82

$$
-\frac{x(5+3x^2)+8\cot^{-1}(x)+3(1+x^2)^2\,\mathrm{ArcTan}(x)}{32(1+x^2)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCot[x])/(1 + x^2)^3,x]
[Out] -1/32*(x*(5 + 3*x^2) + 8*ArcCot[x] + 3*(1 + x^2)^2*ArcTan[x])/(1 + x^2)^2
```

Maple [A]
time = 0.14, size = 37, normalized size = 0.84

method	result
default	$-\frac{x}{16(x^2+1)^2}-\frac{3x}{32(x^2+1)}-\frac{\mathrm{arccot}(x)}{4(x^2+1)^2}-\frac{3\arctan(x)}{32}$
risch	$-\frac{i\ln(ix+1)}{8(x^2+1)^2}-\frac{-8i\ln(-ix+1)-3i\ln(x-i)x^4-6i\ln(x-i)x^2-3i\ln(x-i)+3i\ln(i+x)x^4+6i\ln(i+x)x^2+3i\ln(i+x)+6x^3+8\pi+10x}{64(i+x)(x^2+1)(x-i)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] -1/16*x/(x^2+1)^2-3/32*x/(x^2+1)-1/4*arccot(x)/(x^2+1)^2-3/32*arctan(x)
```

Maxima [A]
time = 0.46, size = 39, normalized size = 0.89

$$
-\frac{3\,x^3+5\,x}{32\,(x^4+2\,x^2+1)}-\frac{\mathrm{arccot}\,(x)}{4\,(x^2+1)^2}-\frac{3}{32}\,\arctan\,(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arccot(x)/(x^2 + 1)^2 - 3/32*ar
ctan(x)
```

Fricas [A]
time = 1.85, size = 39, normalized size = 0.89

$$-\frac{3\,x^3-(3\,x^4+6\,x^2-5)\operatorname{arccot}(x)+5\,x}{32\,(x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(3*x^3 - (3*x^4 + 6*x^2 - 5)*arccot(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(39) = 78$.
time = 0.30, size = 88, normalized size = 2.00

$$\frac{3x^4\operatorname{acot}(x)}{32x^4+64x^2+32}-\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{acot}(x)}{32x^4+64x^2+32}-\frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{acot}(x)}{32x^4+64x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x)/(x**2+1)**3,x)
```

```
[Out] 3*x**4*acot(x)/(32*x**4 + 64*x**2 + 32) - 3*x**3/(32*x**4 + 64*x**2 + 32) +
 6*x**2*acot(x)/(32*x**4 + 64*x**2 + 32) - 5*x/(32*x**4 + 64*x**2 + 32) - 5
*acot(x)/(32*x**4 + 64*x**2 + 32)
```

Giac [A]
time = 0.42, size = 40, normalized size = 0.91

$$-\frac{\frac{3}{x}+\frac{5}{x^3}}{32\left(\frac{1}{x^2}+1\right)^2}-\frac{\arctan\left(\frac{1}{x}\right)}{4\,(x^2+1)^2}+\frac{3}{32}\arctan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/32*(3/x + 5/x^3)/(1/x^2 + 1)^2 - 1/4*arctan(1/x)/(x^2 + 1)^2 + 3/32*arct
an(1/x)
```

Mupad [B]
time = 0.60, size = 27, normalized size = 0.61

$$-\frac{3\operatorname{atan}(x)}{32}-\frac{\frac{5\,x}{32}+\frac{\operatorname{acot}(x)}{4}+\frac{3\,x^3}{32}}{(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acot(x))/(x^2 + 1)^3,x)
```

```
[Out] - (3*atan(x))/32 - ((5*x)/32 + acot(x)/4 + (3*x^3)/32)/(x^2 + 1)^2
```

3.72 $\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{4\,(1+x^2)}+\frac{x\cot^{-1}(x)}{2\,(1+x^2)}-\frac{1}{4}\cot^{-1}(x)^2$$

```
[Out] -1/4/(x^2+1)+1/2*x*arccot(x)/(x^2+1)-1/4*arccot(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5013, 267}

$$-\frac{1}{4\,(x^2+1)}+\frac{x\cot^{-1}(x)}{2\,(x^2+1)}-\frac{1}{4}\cot^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]/(1 + x^2)^2,x]
```

```
[Out] -1/4*1/(1 + x^2) + (x*ArcCot[x])/(2*(1 + x^2)) - ArcCot[x]^2/4
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5013

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (Dist[b*c*(p
/2), Int[x*((a + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x] - Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}(x)}{(1+x^2)^2}\,dx &= \frac{x\cot^{-1}(x)}{2\,(1+x^2)}-\frac{1}{4}\cot^{-1}(x)^2+\frac{1}{2}\int \frac{x}{(1+x^2)^2}\,dx\\ &= -\frac{1}{4\,(1+x^2)}+\frac{x\cot^{-1}(x)}{2\,(1+x^2)}-\frac{1}{4}\cot^{-1}(x)^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$-\frac{1-2x\cot^{-1}(x)+(1+x^2)\cot^{-1}(x)^2}{4(1+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]/(1 + x^2)^2,x]
```

```
[Out] -1/4*(1 - 2*x*ArcCot[x] + (1 + x^2)*ArcCot[x]^2)/(1 + x^2)
```

Maple [A]
time = 0.14, size = 35, normalized size = 1.03

method	result
default	$\frac{x\,\mathrm{arccot}(x)}{2x^2+2}+\frac{\mathrm{arccot}(x)\arctan(x)}{2}-\frac{1}{4(x^2+1)}+\frac{\arctan(x)^2}{4}$
risch	$\frac{\ln(ix+1)^2}{16}-\frac{(x^2\ln(-ix+1)-2ix+\ln(-ix+1))\ln(ix+1)}{8(x^2+1)}+\frac{x^2\ln(-ix+1)^2+\ln(-ix+1)^2+2i\pi\ln(i+x)x^2+2i\ln(i+x)\pi-2i\pi\ln(x-i)}{16(i+x)(x-i)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*arccot(x)/(x^2+1)+1/2*arccot(x)*arctan(x)-1/4/(x^2+1)+1/4*arctan(x)^2
```

Maxima [A]
time = 0.48, size = 38, normalized size = 1.12

$$\frac{1}{2}\left(\frac{x}{x^2+1}+\arctan(x)\right)\mathrm{arccot}(x)+\frac{(x^2+1)\arctan(x)^2-1}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(x^2 + 1) + arctan(x))*arccot(x) + 1/4*((x^2 + 1)*arctan(x)^2 - 1)/(
x^2 + 1)
```

Fricas [A]
time = 0.96, size = 26, normalized size = 0.76

$$-\frac{(x^2+1)\,\mathrm{arccot}(x)^2-2x\,\mathrm{arccot}(x)+1}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((x^2 + 1)*arccot(x)^2 - 2*x*arccot(x) + 1)/(x^2 + 1)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)/(x**2+1)**2,x)
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)/(x^2+1)^2,x, algorithm="giac")
[Out] integrate(arccot(x)/(x^2 + 1)^2, x)
```

Mupad [**B**]
time = 0.61, size = 22, normalized size = 0.65

$$\frac{\frac{x\,\mathrm{acot}(x)}{2}-\frac{1}{4}}{x^2+1}-\frac{\mathrm{acot}(x)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)/(x^2 + 1)^2,x)
[Out] ((x*acot(x))/2 - 1/4)/(x^2 + 1) - acot(x)^2/4
```

3.73 $\int \frac{\cot^{-1}(x)^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=56

$$-\frac{x}{4(1+x^2)}-\frac{\cot^{-1}(x)}{2(1+x^2)}+\frac{x\cot^{-1}(x)^2}{2(1+x^2)}-\frac{1}{6}\cot^{-1}(x)^3-\frac{\text{ArcTan}(x)}{4}$$

```
[Out] -1/4*x/(x^2+1)-1/2*arccot(x)/(x^2+1)+1/2*x*arccot(x)^2/(x^2+1)-1/6*arccot(x
)^3-1/4*arctan(x)
```

Rubi [A]

time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5013, 5051, 205, 209}

$$-\frac{\text{ArcTan}(x)}{4}-\frac{x}{4(x^2+1)}+\frac{x\cot^{-1}(x)^2}{2(x^2+1)}-\frac{\cot^{-1}(x)}{2(x^2+1)}-\frac{1}{6}\cot^{-1}(x)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x]^2/(1 + x^2)^2,x]
```

```
[Out] -1/4*x/(1 + x^2) - ArcCot[x]/(2*(1 + x^2)) + (x*ArcCot[x]^2)/(2*(1 + x^2))
- ArcCot[x]^3/6 - ArcTan[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5013

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcCot[c*x])^p/(2*d*(d + e*x^2))), x] + (Dist[b*c*(p
/2), Int[x*((a + b*ArcCot[c*x])^(p - 1)/(d + e*x^2)^2), x], x] - Simp[(a +
b*ArcCot[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5051

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCot[c*x])^p/(2*e*(q +
1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCot[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(x)^2}{\left(1+x^2\right)^2}\,dx &= \frac{x\cot^{-1}(x)^2}{2\left(1+x^2\right)}-\frac{1}{6}\cot^{-1}(x)^3+\int \frac{x\cot^{-1}(x)}{\left(1+x^2\right)^2}\,dx \\
&= -\frac{\cot^{-1}(x)}{2\left(1+x^2\right)}+\frac{x\cot^{-1}(x)^2}{2\left(1+x^2\right)}-\frac{1}{6}\cot^{-1}(x)^3-\frac{1}{2}\int \frac{1}{\left(1+x^2\right)^2}\,dx \\
&= -\frac{x}{4\left(1+x^2\right)}-\frac{\cot^{-1}(x)}{2\left(1+x^2\right)}+\frac{x\cot^{-1}(x)^2}{2\left(1+x^2\right)}-\frac{1}{6}\cot^{-1}(x)^3-\frac{1}{4}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{x}{4\left(1+x^2\right)}-\frac{\cot^{-1}(x)}{2\left(1+x^2\right)}+\frac{x\cot^{-1}(x)^2}{2\left(1+x^2\right)}-\frac{1}{6}\cot^{-1}(x)^3-\frac{1}{4}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.82

$$
-\frac{6\cot^{-1}(x)-6x\cot^{-1}(x)^2+2(1+x^2)\cot^{-1}(x)^3+3(x+(1+x^2)\,\mathrm{ArcTan}(x))}{12\left(1+x^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x]^2/(1 + x^2)^2,x]
```

```
[Out] -1/12*(6*ArcCot[x] - 6*x*ArcCot[x]^2 + 2*(1 + x^2)*ArcCot[x]^3 + 3*(x + (1
+ x^2)*ArcTan[x]))/(1 + x^2)
```

Maple [A]
time = 0.27, size = 65, normalized size = 1.16

method	result
default	$\frac{x\mathrm{arccot}(x)^2}{2x^2+2}+\frac{\mathrm{arccot}(x)^2\arctan(x)}{2}-\frac{\mathrm{arccot}(x)^2\pi}{4}+\frac{\mathrm{arccot}(x)^3}{3}+\frac{x^2\mathrm{arccot}(x)}{2x^2+2}-\frac{x}{4(x^2+1)}-\frac{\mathrm{arccot}(x)}{4}$
risch	$\frac{i\ln(ix+1)^3}{48}+\frac{\left(-i\ln(-ix+1)x^2+\pi x^2-i\ln(-ix+1)+\pi-2x\right)\ln(ix+1)^2}{16x^2+16}-\frac{\left(-ix^2\ln(-ix+1)^2-i\ln(-ix+1)^2-4\ln(-ix+1)x+2\pi\,\mathrm{l}\right.}{16(i+x)(x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*arccot(x)^2/(x^2+1)+1/2*arccot(x)^2*arctan(x)-1/4*arccot(x)^2*Pi+1/3*
arccot(x)^3+1/2*x^2*arccot(x)/(x^2+1)-1/4*x/(x^2+1)-1/4*arccot(x)
```

Maxima [A]
time = 0.48, size = 75, normalized size = 1.34

$$\frac{1}{2}\left(\frac{x}{x^2+1}+\arctan\left(x\right)\right)\mathrm{arccot}\left(x\right)^2+\frac{\left(\left(x^2+1\right)\arctan\left(x\right)^2-1\right)\mathrm{arccot}\left(x\right)}{2\left(x^2+1\right)}+\frac{2\left(x^2+1\right)\arctan\left(x\right)^3-3\left(x^2+1\right)\arctan\left(x\right)-3\,x}{12\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(x^2 + 1) + arctan(x))*arccot(x)^2 + 1/2*((x^2 + 1)*arctan(x)^2 - 1)
*arccot(x)/(x^2 + 1) + 1/12*(2*(x^2 + 1)*arctan(x)^3 - 3*(x^2 + 1)*arctan(x
) - 3*x)/(x^2 + 1)
```

Fricas [A]
time = 1.87, size = 40, normalized size = 0.71

$$-\frac{2\left(x^2+1\right)\mathrm{arccot}\left(x\right)^3-6\,x\,\mathrm{arccot}\left(x\right)^2-3\left(x^2-1\right)\mathrm{arccot}\left(x\right)+3\,x}{12\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(x^2 + 1)*arccot(x)^3 - 6*x*arccot(x)^2 - 3*(x^2 - 1)*arccot(x) +
3*x)/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acot}^2\left(x\right)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x)**2/(x**2+1)**2,x)
```

```
[Out] Integral(acot(x)**2/(x**2 + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x)^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arccot(x)^2/(x^2 + 1)^2, x)
```

Mupad [B]
time = 0.06, size = 51, normalized size = 0.91

$$\frac{x\,\mathrm{acot}(x)^2}{2\,(x^2+1)} - \frac{\mathrm{acot}(x)^3}{6} - \frac{x}{4\,(x^2+1)} - \frac{\mathrm{acot}(x)}{2\,(x^2+1)} - \frac{\mathrm{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x)^2/(x^2 + 1)^2,x)
```

```
[Out] (x*acot(x)^2)/(2*(x^2 + 1)) - acot(x)^3/6 - x/(4*(x^2 + 1)) - acot(x)/(2*(x
^2 + 1)) - atan(x)/4
```

3.74 $\int x^5 \cot^{-1}(ax^2)\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{12a}+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)-\frac{\log\left(1+a^2x^4\right)}{12a^3}$$

```
[Out] 1/12*x^4/a+1/6*x^6*arccot(a*x^2)-1/12*ln(a^2*x^4+1)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4947, 272, 45}

$$-\frac{\log\left(a^2x^4+1\right)}{12a^3}+\frac{x^4}{12a}+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCot[a*x^2],x]
```

```
[Out] x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cot^{-1}\left(ax^2\right) dx &= \frac{1}{6}x^6 \cot^{-1}\left(ax^2\right) + \frac{1}{3}a \int \frac{x^7}{1+a^2x^4} dx \\
&= \frac{1}{6}x^6 \cot^{-1}\left(ax^2\right) + \frac{1}{12}a\mathrm{Subst}\left(\int \frac{x}{1+a^2x} dx, x, x^4\right) \\
&= \frac{1}{6}x^6 \cot^{-1}\left(ax^2\right) + \frac{1}{12}a\mathrm{Subst}\left(\int \left(\frac{1}{a^2} - \frac{1}{a^2(1+a^2x)}\right) dx, x, x^4\right) \\
&= \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}\left(ax^2\right) - \frac{\log\left(1+a^2x^4\right)}{12a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$
\frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}\left(ax^2\right) - \frac{\log\left(1+a^2x^4\right)}{12a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCot[a*x^2],x]
```

```
[Out] x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

Maple [A]
time = 0.07, size = 40, normalized size = 0.98

method	result	size
default	$\frac{x^6 \operatorname{arccot}(a x^2)}{6}+\frac{a\left(\frac{x^4}{4a^2}-\frac{\ln(a^2x^4+1)}{4a^4}\right)}{3}$	40
risch	$\frac{i x^6 \ln(i a x^2+1)}{12}-\frac{i x^6 \ln(-i a x^2+1)}{12}+\frac{x^6 \pi}{12}+\frac{x^4}{12a}-\frac{\ln(-a^2x^4-1)}{12a^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*arccot(a*x^2)+1/3*a*(1/4*x^4/a^2-1/4/a^4*ln(a^2*x^4+1))
```

Maxima [A]
time = 0.27, size = 38, normalized size = 0.93

$$
\frac{1}{6}\,x^6 \operatorname{arccot}\left(ax^2\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccot(a*x^2) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Fricas [A]
time = 0.98, size = 39, normalized size = 0.95

$$\frac{2\,a^3x^6\,\mathrm{arccot}\,(ax^2)+a^2x^4-\log\,(a^2x^4+1)}{12\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^3*x^6*arccot(a*x^2) + a^2*x^4 - log(a^2*x^4 + 1))/a^3
```

Sympy [A]
time = 0.42, size = 39, normalized size = 0.95

$$\begin{cases}\frac{x^6\,\mathrm{acot}\,(ax^2)}{6}+\frac{x^4}{12a}-\frac{\log\,(a^2x^4+1)}{12a^3} & \text{for } a\neq 0\\ \frac{\pi x^6}{12} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acot(a*x**2),x)
```

```
[Out] Piecewise((x**6*acot(a*x**2)/6 + x**4/(12*a) - log(a**2*x**4 + 1)/(12*a**3)
, Ne(a, 0)), (pi*x**6/12, True))
```

Giac [A]
time = 0.40, size = 40, normalized size = 0.98

$$\frac{1}{6}\,x^6\,\arctan\left(\frac{1}{ax^2}\right)+\frac{1}{12}\left(\frac{x^4}{a^2}-\frac{\log\,(a^2x^4+1)}{a^4}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*x^6*arctan(1/(a*x^2)) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Mupad [B]
time = 0.65, size = 35, normalized size = 0.85

$$\frac{x^6\,\mathrm{acot}(a\,x^2)}{6}-\frac{\ln\,(a^2\,x^4+1)}{12\,a^3}+\frac{x^4}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acot(a*x^2),x)
```

```
[Out] (x^6*acot(a*x^2))/6 - log(a^2*x^4 + 1)/(12*a^3) + x^4/(12*a)
```

3.75 $\int x^3 \cot^{-1}(ax^2)\, dx$

Optimal. Leaf size=37

$$\frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\text{ArcTan}(ax^2)}{4a^2}$$

```
[Out] 1/4*x^2/a+1/4*x^4*arccot(a*x^2)-1/4*arctan(a*x^2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4947, 281, 327, 209}

$$-\frac{\text{ArcTan}(ax^2)}{4a^2} + \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a*x^2],x]
```

```
[Out] x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3 \cot^{-1}\left(ax^2\right) dx &= \frac{1}{4}x^4 \cot^{-1}\left(ax^2\right) + \frac{1}{2}a \int \frac{x^5}{1+a^2x^4} dx \\ &= \frac{1}{4}x^4 \cot^{-1}\left(ax^2\right) + \frac{1}{4}a\text{Subst}\left(\int \frac{x^2}{1+a^2x^2} dx, x, x^2\right) \\ &= \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}\left(ax^2\right) - \frac{\text{Subst}\left(\int \frac{1}{1+a^2x^2} dx, x, x^2\right)}{4a} \\ &= \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}\left(ax^2\right) - \frac{\tan^{-1}\left(ax^2\right)}{4a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}\left(ax^2\right) - \frac{\text{ArcTan}(ax^2)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a*x^2],x]
[Out] x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.97

method	result	size
default	$\frac{x^4 \operatorname{arccot}(a x^2)}{4} + \frac{a\left(\frac{x^2}{2a^2} - \frac{\arctan(a x^2)}{2a^3}\right)}{2}$	36
risch	$\frac{i x^4 \ln(i a x^2+1)}{8} - \frac{i x^4 \ln(-i a x^2+1)}{8} + \frac{\pi x^4}{8} + \frac{x^2}{4a} - \frac{\arctan(a x^2)}{4a^2} + \frac{1}{8\pi a^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arccot(a*x^2)+1/2*a*(1/2*x^2/a^2-1/2*arctan(a*x^2)/a^3)
```

Maxima [A]
time = 0.47, size = 34, normalized size = 0.92

$$\frac{1}{4}\,x^4 \operatorname{arccot}\left(ax^2\right) + \frac{1}{4}\,a\left(\frac{x^2}{a^2} - \frac{\arctan\left(ax^2\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(a*x^2) + 1/4*a*(x^2/a^2 - arctan(a*x^2)/a^3)
```

Fricas [A]
time = 2.09, size = 27, normalized size = 0.73

$$\frac{ax^2 + (a^2x^4 + 1)\operatorname{arccot}(ax^2)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(a*x^2 + (a^2*x^4 + 1)*arccot(a*x^2))/a^2
```

Sympy [A]
time = 0.27, size = 36, normalized size = 0.97

$$\begin{cases} \frac{x^4 \operatorname{acot}(ax^2)}{4} + \frac{x^2}{4a} + \frac{\operatorname{acot}(ax^2)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(a*x**2),x)
```

```
[Out] Piecewise((x**4*acot(a*x**2)/4 + x**2/(4*a) + acot(a*x**2)/(4*a**2), Ne(a,
0)), (pi*x**4/8, True))
```

Giac [A]
time = 0.41, size = 38, normalized size = 1.03

$$\frac{1}{4}\left(\frac{x^4 \arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{x^2}{a^2} + \frac{\arctan\left(\frac{1}{ax^2}\right)}{a^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*(x^4*arctan(1/(a*x^2))/a + x^2/a^2 + arctan(1/(a*x^2))/a^3)*a
```

Mupad [B]
time = 0.61, size = 31, normalized size = 0.84

$$\frac{x^4 \operatorname{acot}(a\,x^2)}{4} - \frac{\operatorname{atan}(a\,x^2)}{4\,a^2} + \frac{x^2}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a*x^2),x)
```

```
[Out] (x^4*acot(a*x^2))/4 - atan(a*x^2)/(4*a^2) + x^2/(4*a)
```

3.76 $\int x \cot^{-1}(ax^2)\, dx$

Optimal. Leaf size=31

$$\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+\frac{\log\left(1+a^2x^4\right)}{4a}$$

```
[Out] 1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4947, 266}

$$\frac{\log\left(a^2x^4+1\right)}{4a}+\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a*x^2],x]
```

```
[Out] (x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}\left(ax^2\right)\,dx&=\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+a\int\frac{x^3}{1+a^2x^4}\,dx\\&=\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+\frac{\log\left(1+a^2x^4\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}x^2\cot^{-1}\left(ax^2\right)+\frac{\log\left(1+a^2x^4\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a*x^2],x]
[Out] (x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\operatorname{arccot}\left(a\,x^{2}\right)a\,x^{2}+\frac{\ln\left(a^{2}x^{4}+1\right)}{2}}{2a}$	30
default	$\frac{\operatorname{arccot}\left(a\,x^{2}\right)a\,x^{2}+\frac{\ln\left(a^{2}x^{4}+1\right)}{2}}{2a}$	30
risch	$\frac{i x^{2} \ln\left(i a\,x^{2}+1\right)}{4}-\frac{i x^{2} \ln\left(-i a\,x^{2}+1\right)}{4}+\frac{\pi\,x^{2}}{4}+\frac{\ln\left(-a^{2}x^{4}-1\right)}{4a}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(arccot(a*x^2)*a*x^2+1/2*ln(a^2*x^4+1))
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.90

$$\frac{2\,ax^{2}\operatorname{arccot}\left(ax^{2}\right)+\log\left(a^{2}x^{4}+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x^2),x, algorithm="maxima")
[Out] 1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Fricas [A]
time = 4.42, size = 28, normalized size = 0.90

$$\frac{2\,ax^{2}\operatorname{arccot}\left(ax^{2}\right)+\log\left(a^{2}x^{4}+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x^2),x, algorithm="fricas")
[Out] 1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Sympy [A]
time = 0.19, size = 31, normalized size = 1.00

$$\begin{cases} \frac{x^{2}\operatorname{acot}\left(ax^{2}\right)}{2}+\frac{\log\left(a^{2}x^{4}+1\right)}{4a} & \text{for}\: a \neq 0 \\ \frac{\pi x^{2}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(a*x**2),x)
```

```
[Out] Piecewise((x**2*acot(a*x**2)/2 + log(a**2*x**4 + 1)/(4*a), Ne(a, 0)), (pi*x
**2/4, True))
```

Giac [A]
time = 0.42, size = 47, normalized size = 1.52

$$\frac{1}{4}\left(\frac{2\,x^2\arctan\left(\frac{1}{ax^2}\right)}{a}+\frac{\log\left(\frac{1}{a^2x^4}+1\right)}{a^2}-\frac{\log\left(\frac{1}{a^2x^4}\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x^2*arctan(1/(a*x^2))/a + log(1/(a^2*x^4) + 1)/a^2 - log(1/(a^2*x^4)
)/a^2)*a
```

Mupad [B]
time = 0.60, size = 27, normalized size = 0.87

$$\frac{x^2\,\mathrm{acot}(a\,x^2)}{2}+\frac{\ln\left(a^2\,x^4+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(a*x^2),x)
```

```
[Out] (x^2*acot(a*x^2))/2 + log(a^2*x^4 + 1)/(4*a)
```

3.77 $\int \frac{\cot^{-1}(ax^2)}{x}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{4}i\text{PolyLog}\left(2,-\frac{i}{ax^2}\right)+\frac{1}{4}i\text{PolyLog}\left(2,\frac{i}{ax^2}\right)$$

```
[Out] -1/4*I*polylog(2,-I/a/x^2)+1/4*I*polylog(2,I/a/x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4945, 4941, 2438}

$$\frac{1}{4}i\text{Li}_2\left(\frac{i}{ax^2}\right)-\frac{1}{4}i\text{Li}_2\left(-\frac{i}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2]/x,x]
```

```
[Out] (-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{4}i\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^2\right) - \frac{1}{4}i\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^2\right)$$
$$= -\frac{1}{4}i\mathrm{Li}_2\left(-\frac{i}{ax^2}\right) + \frac{1}{4}i\mathrm{Li}_2\left(\frac{i}{ax^2}\right)$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{1}{4}i\mathrm{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4}i\mathrm{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2]/x,x]
[Out] (-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 57, normalized size = 1.54

method	result
default	$\ln(x)\,\mathrm{arccot}(a\,x^2) + \frac{\sum_{\_R1=\mathrm{RootOf}\left(a^2\_Z^4+1\right)} \frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}}{2a}$
risch	$\frac{\pi\ln(x)}{2} + \frac{i\ln\left(1+ix\sqrt{-ia}\right)\ln(x)}{2} + \frac{i\ln\left(1-ix\sqrt{-ia}\right)\ln(x)}{2} + \frac{i\,\mathrm{dilog}\left(1-ix\sqrt{-ia}\right)}{2} + \frac{i\,\mathrm{dilog}\left(1+ix\sqrt{-ia}\right)}{2} - $

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arccot(a*x^2)+1/2/a*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^4*a^2+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(23) = 46.
time = 0.49, size = 68, normalized size = 1.84

$$\frac{1}{8}\pi\log\left(a^2x^4+1\right) - \frac{1}{2}\arctan\left(ax^2\right)\log\left(ax^2\right) + \mathrm{arccot}\left(ax^2\right)\log(x) + \arctan\left(ax^2\right)\log(x) + \frac{1}{4}i\,\mathrm{Li}_2\left(i\,ax^2+1\right) - \frac{1}{4}i\,\mathrm{Li}_2\left(-i\,ax^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*pi*log(a^2*x^4 + 1) - 1/2*arctan(a*x^2)*log(a*x^2) + arccot(a*x^2)*log(
x) + arctan(a*x^2)*log(x) + 1/4*I*dilog(I*a*x^2 + 1) - 1/4*I*dilog(-I*a*x^2
 + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(ax^{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2)/x,x)
```

```
[Out] Integral(acot(a*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x^2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acot}(a\,x^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2)/x,x)
```

```
[Out] int(acot(a*x^2)/x, x)
```

3.78 $\int \frac{\cot^{-1}(ax^2)}{x^3} \, dx$

Optimal. Leaf size=34

$$-\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1 + a^2x^4\right)$$

```
[Out] -1/2*arccot(a*x^2)/x^2-a*ln(x)+1/4*a*ln(a^2*x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4947, 272, 36, 29, 31}

$$\frac{1}{4}a\log\left(a^2x^4 + 1\right) - \frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2]/x^3,x]
```

```
[Out] -1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx &= -\frac{\cot^{-1}(ax^2)}{2x^2} - a\int \frac{1}{x(1+a^2x^4)}\,dx \\
&= -\frac{\cot^{-1}(ax^2)}{2x^2} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x(1+a^2x)}\,dx, x, x^4\right) \\
&= -\frac{\cot^{-1}(ax^2)}{2x^2} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) + \frac{1}{4}a^3\text{Subst}\left(\int \frac{1}{1+a^2x}\,dx, x, x^4\right) \\
&= -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2]/x^3,x]
[Out] -1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

Maple [A]
time = 0.05, size = 31, normalized size = 0.91

method	result	size
default	$-\frac{\operatorname{arccot}(a\,x^2)}{2x^2} - a\left(\ln(x) - \frac{\ln(a^2x^4+1)}{4}\right)$	31
risch	$-\frac{i\ln(ia\,x^2+1)}{4x^2} - \frac{4\ln(x)a\,x^2 - a\ln(a^2x^4+1)x^2 - i\ln(-ia\,x^2+1)+\pi}{4x^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arccot(a*x^2)/x^2-a*(ln(x)-1/4*ln(a^2*x^4+1))
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.94

$$\frac{1}{4}\,a\left(\log\left(a^2x^4+1\right) - \log\left(x^4\right)\right) - \frac{\operatorname{arccot}(ax^2)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="maxima")
[Out] 1/4*a*(log(a^2*x^4 + 1) - log(x^4)) - 1/2*arccot(a*x^2)/x^2
```

Fricas [A]
time = 3.71, size = 37, normalized size = 1.09

$$\frac{ax^2\log\left(a^2x^4+1\right)-4\,ax^2\log\left(x\right)-2\,\operatorname{arccot}\left(ax^2\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="fricas")
[Out] 1/4*(a*x^2*log(a^2*x^4 + 1) - 4*a*x^2*log(x) - 2*arccot(a*x^2))/x^2
```

Sympy [A]
time = 0.28, size = 29, normalized size = 0.85

$$-a\log\left(x\right)+\frac{a\log\left(a^2x^4+1\right)}{4}-\frac{\operatorname{acot}\left(ax^2\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2)/x**3,x)
[Out] -a*log(x) + a*log(a**2*x**4 + 1)/4 - acot(a*x**2)/(2*x**2)
```

Giac [A]
time = 0.41, size = 32, normalized size = 0.94

$$-\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax^2}\right)}{ax^2}-\log\left(\frac{1}{a^2x^4}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^3,x, algorithm="giac")
[Out] -1/4*a*(2*arctan(1/(a*x^2))/(a*x^2) - log(1/(a^2*x^4) + 1))
```

Mupad [B]
time = 0.66, size = 31, normalized size = 0.91

$$\frac{a\,\ln\left(-a^2\,x^4-1\right)}{4}-\frac{\operatorname{acot}(a\,x^2)}{2\,x^2}-a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2)/x^3,x)
[Out] (a*log(- a^2*x^4 - 1))/4 - acot(a*x^2)/(2*x^2) - a*log(x)
```

3.79 $\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx$

Optimal. Leaf size=35

$$\frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2\text{ArcTan}(ax^2)$$

```
[Out] 1/4*a/x^2-1/4*arccot(a*x^2)/x^4+1/4*a^2*arctan(a*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4947, 281, 331, 209}

$$\frac{1}{4}a^2\text{ArcTan}(ax^2) + \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2]/x^5,x]
```

```
[Out] a/(4*x^2) - ArcCot[a*x^2]/(4*x^4) + (a^2*ArcTan[a*x^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx &= -\frac{\cot^{-1}(ax^2)}{4x^4} - \frac{1}{2}a\int \frac{1}{x^3(1+a^2x^4)}\,dx \\
&= -\frac{\cot^{-1}(ax^2)}{4x^4} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x^2(1+a^2x^2)}\,dx, x, x^2\right) \\
&= \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^3\text{Subst}\left(\int \frac{1}{1+a^2x^2}\,dx, x, x^2\right) \\
&= \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2\tan^{-1}(ax^2)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 38, normalized size = 1.09

$$-\frac{\cot^{-1}(ax^2)}{4x^4} + \frac{a\,{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -a^2x^4\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2]/x^5,x]
[Out] -1/4*ArcCot[a*x^2]/x^4 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^4)])/(4
*x^2)
```

Maple [A]
time = 0.07, size = 31, normalized size = 0.89

method	result	size
default	$-\frac{\operatorname{arccot}(a\,x^2)}{4x^4} - \frac{a\left(-\frac{1}{2x^2} - \frac{a\arctan(a\,x^2)}{2}\right)}{2}$	31
risch	$-\frac{i\ln(ia\,x^2+1)}{8x^4} - \frac{ia^2\ln(-a\,x^2+i)x^4 - ia^2\ln(-a\,x^2-i)x^4 - 2a\,x^2 - i\ln(-ia\,x^2+1) + \pi}{8x^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*arccot(a*x^2)/x^4-1/2*a*(-1/2/x^2-1/2*a*arctan(a*x^2))
```

Maxima [A]
time = 0.47, size = 27, normalized size = 0.77

$$\frac{1}{4}\left(a\arctan\left(ax^2\right)+\frac{1}{x^2}\right)a-\frac{\operatorname{arccot}\left(ax^2\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="maxima")
[Out] 1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arccot(a*x^2)/x^4
```

Fricas [A]
time = 9.08, size = 28, normalized size = 0.80

$$\frac{ax^2-\left(a^2x^4+1\right)\operatorname{arccot}\left(ax^2\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="fricas")
[Out] 1/4*(a*x^2 - (a^2*x^4 + 1)*arccot(a*x^2))/x^4
```

Sympy [A]
time = 0.38, size = 29, normalized size = 0.83

$$-\frac{a^2\operatorname{acot}\left(ax^2\right)}{4}+\frac{a}{4x^2}-\frac{\operatorname{acot}\left(ax^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2)/x**5,x)
[Out] -a**2*acot(a*x**2)/4 + a/(4*x**2) - acot(a*x**2)/(4*x**4)
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.83

$$\frac{1}{4}\left(a\arctan\left(ax^2\right)+\frac{1}{x^2}\right)a-\frac{\arctan\left(\frac{1}{ax^2}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^5,x, algorithm="giac")
[Out] 1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arctan(1/(a*x^2))/x^4
```

Mupad [B]
time = 0.65, size = 32, normalized size = 0.91

$$\frac{a\,x^2-\operatorname{acot}(a\,x^2)+a^2\,x^4\,\operatorname{atan}(a\,x^2)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2)/x^5,x)
[Out] (a*x^2 - acot(a*x^2) + a^2*x^4*atan(a*x^2))/(4*x^4)
```

3.80 $\int x^4 \cot^{-1}\left(ax^2\right)\,dx$

Optimal. Leaf size=152

$$\frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)+\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{5\sqrt{2}\,a^{5/2}}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{5\sqrt{2}\,a^{5/2}}-\frac{\log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{10\sqrt{2}\,a^{5/2}}+\frac{\log}{}$$

```
[Out] 2/15*x^3/a+1/5*x^5*arccot(a*x^2)-1/10*arctan(-1+x*2^(1/2)*a^(1/2))/a^(5/2)*
2^(1/2)-1/10*arctan(1+x*2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)-1/20*ln(1+a*x^2-x*
2^(1/2)*a^(1/2))/a^(5/2)*2^(1/2)+1/20*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(5/2)
*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4947, 327, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{5\sqrt{2}\,a^{5/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{a}\,x+1\right)}{5\sqrt{2}\,a^{5/2}}-\frac{\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{10\sqrt{2}\,a^{5/2}}+\frac{\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{10\sqrt{2}\,a^{5/2}}+\frac{2x^3}{15a}+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCot[a*x^2],x]
```

```
[Out] (2*x^3)/(15*a) + (x^5*ArcCot[a*x^2])/5 + ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(5*S
qrt[2]*a^(5/2)) - ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(5*Sqrt[2]*a^(5/2)) - Log[1
 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(10*Sqrt[2]*a^(5/2)) + Log[1 + Sqrt[2]*Sqrt[a
]*x + a*x^2]/(10*Sqrt[2]*a^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \cot^{-1}(ax^2)\,dx = \frac{1}{5}x^5\cot^{-1}(ax^2) + \frac{1}{5}(2a)\int \frac{x^6}{1+a^2x^4}\,dx$$
$$= \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) - \frac{2\int \frac{x^2}{1+a^2x^4}\,dx}{5a}$$
$$= \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) + \frac{\int \frac{1-ax^2}{1+a^2x^4}\,dx}{5a^2} - \frac{\int \frac{1+ax^2}{1+a^2x^4}\,dx}{5a^2}$$
$$= \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) - \frac{\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{10a^3} - \frac{\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{10a^3} - \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{10\sqrt{2}\,a^{5/2}} -$$
$$= \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) - \frac{\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right)}{10\sqrt{2}\,a^{5/2}} + \frac{\log\left(1+\sqrt{2}\sqrt{a}\,x+ax^2\right)}{10\sqrt{2}\,a^{5/2}} -$$
$$= \frac{2x^3}{15a} + \frac{1}{5}x^5\cot^{-1}(ax^2) + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{a}\,x\right)}{5\sqrt{2}\,a^{5/2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{a}\,x\right)}{5\sqrt{2}\,a^{5/2}} - \log(1$$

Mathematica [A]
time = 0.03, size = 136, normalized size = 0.89

$$\frac{8a^{3/2}x^3 + 12a^{5/2}x^5\cot^{-1}(ax^2) + 6\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{a}\,x\right) - 6\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{a}\,x\right) - 3\sqrt{2}\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right) + 3\sqrt{2}\log\left(1+\sqrt{2}\sqrt{a}\,x+ax^2\right)}{60a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCot[a*x^2],x]
```

```
[Out] (8*a^(3/2)*x^3 + 12*a^(5/2)*x^5*ArcCot[a*x^2] + 6*Sqrt[2]*ArcTan[1 - Sqrt[2
]*Sqrt[a]*x] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] - 3*Sqrt[2]*Log[1 -
Sqrt[2]*Sqrt[a]*x + a*x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/
(60*a^(5/2))
```

Maple [A]
time = 0.08, size = 112, normalized size = 0.74

method	result	size

default	$\frac{x^5 \operatorname{arccot}(a\,x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*arccot(a*x^2)+2/5*a*(1/3*x^3/a^2-1/8/a^4/(1/a^2)^(1/4)*2^(1/2)*(ln(
(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1
/a^2)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^
(1/4)*x-1)))
```

Maxima [A]
time = 0.46, size = 137, normalized size = 0.90

$$\frac{1}{5}\,x^5\operatorname{arccot}\left(ax^2\right)+\frac{1}{60}\,a\left(\frac{8\,x^3}{a^2}-\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\,\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\,\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\,\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\,\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccot(a*x^2) + 1/60*a*(8*x^3/a^2 - 3*(2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sq
rt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2))/
a^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(107) = 214$.
time = 2.54, size = 239, normalized size = 1.57

$$\frac{12\,ax^5\arctan\left(\frac{1}{ax^2}\right)+8\,x^3+12\,\sqrt{2}\,a\frac{1}{a^{10}}^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a^3\frac{1}{a^{10}}^{\frac{1}{4}}x+\sqrt{2}\,\sqrt{\sqrt{2}\,a^7\frac{1}{a^{10}}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^{10}}}+x^2}\,a^3\frac{1}{a^{10}}^{\frac{1}{4}}-1\right)+12\,\sqrt{2}\,a\frac{1}{a^{10}}^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a^3\frac{1}{a^{10}}^{\frac{1}{4}}x+\sqrt{2}\,\sqrt{-\sqrt{2}\,a^7\frac{1}{a^{10}}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^{10}}}+x^2}\,a^3\frac{1}{a^{10}}^{\frac{1}{4}}+1\right)+3\,\sqrt{2}\,a\frac{1}{a^{10}}^{\frac{1}{4}}\log\left(\sqrt{2}\,a^7\frac{1}{a^{10}}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^{10}}}+x^2\right)-3\,\sqrt{2}\,a\frac{1}{a^{10}}^{\frac{1}{4}}\log\left(-\sqrt{2}\,a^7\frac{1}{a^{10}}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^{10}}}+x^2\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*x^5*arctan(1/(a*x^2)) + 8*x^3 + 12*sqrt(2)*a*(a^(-10))^(1/4)*arc
tan(-sqrt(2)*a^3*(a^(-10))^(1/4)*x + sqrt(2)*sqrt(sqrt(2)*a^7*(a^(-10))^(3/
4)*x + a^4*sqrt(a^(-10)) + x^2)*a^3*(a^(-10))^(1/4) - 1) + 12*sqrt(2)*a*(a^
(-10))^(1/4)*arctan(-sqrt(2)*a^3*(a^(-10))^(1/4)*x + sqrt(2)*sqrt(-sqrt(2)*
a^7*(a^(-10))^(3/4)*x + a^4*sqrt(a^(-10)) + x^2)*a^3*(a^(-10))^(1/4) + 1) +
 3*sqrt(2)*a*(a^(-10))^(1/4)*log(sqrt(2)*a^7*(a^(-10))^(3/4)*x + a^4*sqrt(a
^(-10)) + x^2) - 3*sqrt(2)*a*(a^(-10))^(1/4)*log(-sqrt(2)*a^7*(a^(-10))^(3/
4)*x + a^4*sqrt(a^(-10)) + x^2))/a
```

Sympy [A]
time = 14.79, size = 136, normalized size = 0.89

$$\begin{cases} \frac{x^5 \operatorname{acot}(ax^2)}{5} + \frac{2x^3}{15a} - \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} + \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{10a^3\sqrt[4]{-\frac{1}{a^2}}} - \frac{\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} - \frac{\operatorname{acot}(ax^2)}{5a^6\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acot(a*x**2),x)
```

```
[Out] Piecewise((x**5*acot(a*x**2)/5 + 2*x**3/(15*a) - log(x - (-1/a**2)**(1/4))/
(5*a**3*(-1/a**2)**(1/4)) + log(x**2 + sqrt(-1/a**2))/(10*a**3*(-1/a**2)**(
1/4)) - atan(x/(-1/a**2)**(1/4))/(5*a**3*(-1/a**2)**(1/4)) - acot(a*x**2)/(
5*a**6*(-1/a**2)**(7/4)), Ne(a, 0)), (pi*x**5/10, True))
```

Giac [A]
time = 0.44, size = 156, normalized size = 1.03

$$\frac{1}{5}x^5\arctan\left(\frac{1}{ax^2}\right) + \frac{1}{60}a\left(\frac{8x^3}{a^2} - \frac{6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2|a|^{\frac{3}{2}}} - \frac{6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2|a|^{\frac{3}{2}}} + \frac{3\sqrt{2}\sqrt{|a|}\log\left(x^2 + \frac{\sqrt{2}x}{\sqrt{|a|}} + \frac{1}{|a|}\right)}{a^4} - \frac{3\sqrt{2}\log\left(x^2 - \frac{\sqrt{2}x}{\sqrt{|a|}} + \frac{1}{|a|}\right)}{a^2|a|^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arctan(1/(a*x^2)) + 1/60*a*(8*x^3/a^2 - 6*sqrt(2)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^(3/2)) - 6*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^
(3/2)) + 3*sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a)
)/a^4 - 3*sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*abs(a)^
(3/2)))
```

Mupad [B]
time = 0.44, size = 54, normalized size = 0.36

$$\frac{x^5\operatorname{acot}(a\,x^2)}{5} + \frac{2\,x^3}{15\,a} - \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{a}\;x\right)}{5\,a^{5/2}} - \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{a}\;x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acot(a*x^2),x)
```

```
[Out] (x^5*acot(a*x^2))/5 + (2*x^3)/(15*a) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*
x))/(5*a^(5/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/(5*a^(5/2))
```

3.81 $\int x^2 \cot^{-1}(ax^2)\,dx$

Optimal. Leaf size=150

$$\frac{2x}{3a}+\frac{1}{3}x^3\cot^{-1}(ax^2)+\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}\,a^{3/2}}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}\,a^{3/2}}+\frac{\log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{6\sqrt{2}\,a^{3/2}}-\frac{\log}{}$$

```
[Out] 2/3*x/a+1/3*x^3*arccot(a*x^2)-1/6*arctan(-1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1
/2)-1/6*arctan(1+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1/2)+1/12*ln(1+a*x^2-x*2^(1/
2)*a^(1/2))/a^(3/2)*2^(1/2)-1/12*ln(1+a*x^2+x*2^(1/2)*a^(1/2))/a^(3/2)*2^(1
/2)
```

Rubi [A]

time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 327, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}\,a^{3/2}}-\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{a}\,x+1\right)}{3\sqrt{2}\,a^{3/2}}+\frac{\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{6\sqrt{2}\,a^{3/2}}-\frac{\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{6\sqrt{2}\,a^{3/2}}+\frac{1}{3}x^3\cot^{-1}(ax^2)+\frac{2x}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a*x^2],x]
```

```
[Out] (2*x)/(3*a) + (x^3*ArcCot[a*x^2])/3 + ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(3*Sqrt
[2]*a^(3/2)) - ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(3*Sqrt[2]*a^(3/2)) + Log[1 -
Sqrt[2]*Sqrt[a]*x + a*x^2]/(6*Sqrt[2]*a^(3/2)) - Log[1 + Sqrt[2]*Sqrt[a]*x
+ a*x^2]/(6*Sqrt[2]*a^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{1}{3}(2a)\int \frac{x^4}{1+a^2x^4}\,dx$$
$$= \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) - \frac{2\int \frac{1}{1+a^2x^4}\,dx}{3a}$$
$$= \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) - \frac{\int \frac{1-ax^2}{1+a^2x^4}\,dx}{3a} - \frac{\int \frac{1+ax^2}{1+a^2x^4}\,dx}{3a}$$
$$= \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) - \frac{\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{6a^2} - \frac{\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx}{6a^2} + \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}-x^2}\,dx}{6\sqrt{2}\,a^{3/2}} +$$
$$= \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right)}{6\sqrt{2}\,a^{3/2}} - \frac{\log\left(1+\sqrt{2}\sqrt{a}\,x+ax^2\right)}{6\sqrt{2}\,a^{3/2}} -$$
$$= \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{a}\,x\right)}{3\sqrt{2}\,a^{3/2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{a}\,x\right)}{3\sqrt{2}\,a^{3/2}} + \frac{\log\left(1-\right.}{}$$

Mathematica [A]
time = 0.02, size = 133, normalized size = 0.89

$$\frac{8\sqrt{a}\,x+4a^{3/2}x^3\cot^{-1}(ax^2)+2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{a}\,x\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{a}\,x\right)+\sqrt{2}\,\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\sqrt{a}\,x+ax^2\right)}{12a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a*x^2],x]
```

```
[Out] (8*Sqrt[a]*x + 4*a^(3/2)*x^3*ArcCot[a*x^2] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*S
qrt[a]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Log[1 - Sqrt[
2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/(12*a^(
3/2))
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.73

method	result	siz

default	$\frac{x^3 \operatorname{arccot}(a\,x^2)}{3}+\frac{2a\left(\frac{x}{a^2}-\frac{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}\right)}{3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccot(a*x^2)+2/3*a*(x/a^2-1/8/a^2*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(
1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^
(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*
x-1)))
```

Maxima [A]
time = 0.47, size = 135, normalized size = 0.90

$$\frac{1}{3}\,x^3\operatorname{arccot}\left(ax^2\right)+\frac{1}{12}\,a\left(\frac{8\,x}{a^2}-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}\,x+1\right)}{\sqrt{a}}-\frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{a}\,x+1\right)}{\sqrt{a}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(a*x^2) + 1/12*a*(8*x/a^2 - (2*sqrt(2)*arctan(1/2*sqrt(2)*(2*
a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a
*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a
)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a))/a^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(105) = 210.
time = 2.96, size = 228, normalized size = 1.52

$$\frac{4\,ax^3\arctan\left(\frac{1}{ax^2}\right)+4\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a^5\left(\frac{1}{a^6}\right)^{\frac{3}{4}}x+\sqrt{2}\sqrt{\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}x+a^2\sqrt{\frac{1}{a^6}}+x^2}\,a^5\left(\frac{1}{a^6}\right)^{\frac{3}{4}}-1\right)+4\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a^5\left(\frac{1}{a^6}\right)^{\frac{3}{4}}x+\sqrt{2}\sqrt{-\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}x+a^2\sqrt{\frac{1}{a^6}}+x^2}\,a^5\left(\frac{1}{a^6}\right)^{\frac{3}{4}}+1\right)-\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}x+a^2\sqrt{\frac{1}{a^6}}+x^2\right)+\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,a\left(\frac{1}{a^6}\right)^{\frac{1}{4}}x+a^2\sqrt{\frac{1}{a^6}}+x^2\right)+8\,x}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*x^3*arctan(1/(a*x^2)) + 4*sqrt(2)*a*(a^(-6))^(1/4)*arctan(-sqrt(2
)*a^5*(a^(-6))^(3/4)*x + sqrt(2)*sqrt(sqrt(2)*a*(a^(-6))^(1/4)*x + a^2*sqrt
(a^(-6)) + x^2)*a^5*(a^(-6))^(3/4) - 1) + 4*sqrt(2)*a*(a^(-6))^(1/4)*arctan
```

```
(-sqrt(2)*a^5*(a^(-6))^(3/4)*x + sqrt(2)*sqrt(-sqrt(2)*a*(a^(-6))^(1/4)*x +
 a^2*sqrt(a^(-6)) + x^2)*a^5*(a^(-6))^(3/4) + 1) - sqrt(2)*a*(a^(-6))^(1/4)
*log(sqrt(2)*a*(a^(-6))^(1/4)*x + a^2*sqrt(a^(-6)) + x^2) + sqrt(2)*a*(a^(-
6))^(1/4)*log(-sqrt(2)*a*(a^(-6))^(1/4)*x + a^2*sqrt(a^(-6)) + x^2) + 8*x)/
a
```

Sympy [A]
time = 7.09, size = 126, normalized size = 0.84

$$\begin{cases} \frac{x^3\operatorname{acot}(ax^2)}{3} + \frac{\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{acot}(ax^2)}{3} + \frac{2x}{3a} + \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{3a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{3a} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(a*x**2),x)
```

```
[Out] Piecewise((x**3*acot(a*x**2)/3 + (-1/a**2)**(3/4)*acot(a*x**2)/3 + 2*x/(3*a
) + (-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4))/(3*a) - (-1/a**2)**(1/4)*log
(x**2 + sqrt(-1/a**2))/(6*a) - (-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4))/(3
*a), Ne(a, 0)), (pi*x**3/6, True))
```

Giac [A]
time = 0.43, size = 153, normalized size = 1.02

$$\frac{1}{3}x^3\arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}a\left(\frac{8x}{a^2} - \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{a^2\sqrt{|a|}} + \frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{a^2\sqrt{|a|}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(1/(a*x^2)) + 1/12*a*(8*x/a^2 - 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(a))) - 2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(
a))) - sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(abs(a
))) + sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(abs(a)
)))
```

Mupad [B]
time = 0.70, size = 52, normalized size = 0.35

$$\frac{x^3\operatorname{acot}(a\,x^2)}{3} + \frac{2\,x}{3\,a} + \frac{{(-1)}^{1/4}\operatorname{atan}\left({(-1)}^{1/4}\,\sqrt{a}\,x\right)\,1\mathrm{i}}{3\,a^{3/2}} + \frac{{(-1)}^{1/4}\operatorname{atan}\left({(-1)}^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)}{3\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a*x^2),x)
```

```
[Out] (x^3*acot(a*x^2))/3 + (2*x)/(3*a) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x)*
1i)/(3*a^(3/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i))/(3*a^(3/2))
```

3.82 $\int \cot^{-1}\left(ax^2\right)\,dx$

Optimal. Leaf size=132

$$x\cot^{-1}\left(ax^2\right)-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}\,\sqrt{a}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}\,\sqrt{a}}+\frac{\log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}\,\sqrt{a}}-\frac{\log\left(1+\right.}{2}$$

```
[Out] x*arccot(a*x^2)+1/2*arctan(-1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/2*arctan
(1+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*2^(
1/2)/a^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {4931, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}\,\sqrt{a}}+\frac{\text{ArcTan}\left(\sqrt{2}\,\sqrt{a}\,x+1\right)}{\sqrt{2}\,\sqrt{a}}+\frac{\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{2\sqrt{2}\,\sqrt{a}}-\frac{\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{2\sqrt{2}\,\sqrt{a}}+x\cot^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2],x]
```

```
[Out] x*ArcCot[a*x^2] - ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]) + ArcTan[
1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]) + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^
2]/(2*Sqrt[2]*Sqrt[a]) - Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt
[a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \cot^{-1}(ax^2)\,dx &= x\cot^{-1}(ax^2) + (2a)\int \frac{x^2}{1+a^2x^4}\,dx \\
&= x\cot^{-1}(ax^2) - \int \frac{1-ax^2}{1+a^2x^4}\,dx + \int \frac{1+ax^2}{1+a^2x^4}\,dx \\
&= x\cot^{-1}(ax^2) + \frac{\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}\,x}{\sqrt{a}}+x^2}\,dx}{2a} + \frac{\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}\,x}{\sqrt{a}}+x^2}\,dx}{2a} + \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}\,x}{\sqrt{a}}-x^2}\,dx}{2\sqrt{2}\,\sqrt{a}} + \frac{\int \frac{\frac{\sqrt{2}}{\sqrt{a}}}{-\frac{1}{a}+\frac{\sqrt{}}{\sqrt{}}}}{2\sqrt{2}} \\
&= x\cot^{-1}(ax^2) + \frac{\log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}\,\sqrt{a}} - \frac{\log\left(1+\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}\,\sqrt{a}} + \frac{\mathrm{Subst}\left(\int\right.}{} \\
&= x\cot^{-1}(ax^2) - \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}\,\sqrt{a}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}\,\sqrt{a}} + \frac{\log\left(1-\sqrt{2}\,\sqrt{a}\right.}{2\sqrt{2}\,\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 102, normalized size = 0.77

$$x\cot^{-1}(ax^2) + \frac{-2\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right) + 2\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right) + \log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right) - \log\left(1+\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}\,\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2],x]
```

```
[Out] x*ArcCot[a*x^2] + (-2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] + 2*ArcTan[1 + Sqrt[2]*
Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2]*Sqrt[a]*x
 + a*x^2])/(2*Sqrt[2]*Sqrt[a])
```

Maple [A]
time = 0.04, size = 97, normalized size = 0.73

method	result	size
default	$x\,\mathrm{arccot}(a\,x^2) + \frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccot(a*x^2)+1/4/a/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^(1/4)*x*2^(1/2
)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^(1
/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Maxima [A]
time = 0.47, size = 120, normalized size = 0.91

$$\frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,ax+\sqrt{2}\,\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,ax-\sqrt{2}\,\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\,\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\,\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}\right)+x\,\mathrm{arccot}\left(ax^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3
/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/
2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2
 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + x*arccot(a*x^2)
```

Fricas [A]
time = 3.05, size = 189, normalized size = 1.43

$$x\arctan\left(\frac{1}{ax^2}\right)-\sqrt{2}\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{3}{4}}x+x^2+\sqrt{\frac{1}{a^2}}}\,a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}-1\right)-\sqrt{2}\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{-\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{3}{4}}x+x^2+\sqrt{\frac{1}{a^2}}}\,a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}+1\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{3}{4}}x+x^2+\sqrt{\frac{1}{a^2}}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,a\left(\frac{1}{a^2}\right)^{\frac{3}{4}}x+x^2+\sqrt{\frac{1}{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2),x, algorithm="fricas")
```

```
[Out] x*arctan(1/(a*x^2)) - sqrt(2)*(a^(-2))^(1/4)*arctan(-sqrt(2)*a*(a^(-2))^(1/
4)*x + sqrt(2)*sqrt(sqrt(2)*a*(a^(-2))^(3/4)*x + x^2 + sqrt(a^(-2)))*a*(a^(
-2))^(1/4) - 1) - sqrt(2)*(a^(-2))^(1/4)*arctan(-sqrt(2)*a*(a^(-2))^(1/4)*x
 + sqrt(2)*sqrt(-sqrt(2)*a*(a^(-2))^(3/4)*x + x^2 + sqrt(a^(-2)))*a*(a^(-2)
)^(1/4) + 1) - 1/4*sqrt(2)*(a^(-2))^(1/4)*log(sqrt(2)*a*(a^(-2))^(3/4)*x +
x^2 + sqrt(a^(-2))) + 1/4*sqrt(2)*(a^(-2))^(1/4)*log(-sqrt(2)*a*(a^(-2))^(3
/4)*x + x^2 + sqrt(a^(-2)))
```

Sympy [C] Result contains complex when optimal does not.
time = 4.37, size = 615, normalized size = 4.66

$$\begin{cases} \infty ix & \text{for}\: a = -\frac{i}{x^2} \\ -\infty ix & \text{for}\: a = \frac{i}{x^2} \\ \frac{\pi x}{2} & \text{for}\: a = 0 \\ \frac{2a^5x^5\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}\operatorname{acot}\left(ax^2\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2a^4x^4\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}-\frac{a^4x^4\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2a^4x^4\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2a^3x\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}\operatorname{acot}\left(ax^2\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2a^2\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}-\frac{a^2\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2a^2\left(-\frac{1}{a^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2ax^4\operatorname{acot}\left(ax^2\right)}{2a^5x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^3\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}}+\frac{2\operatorname{acot}\left(ax^2\right)}{2a^6x^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}+2a^4\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2),x)
```

```
[Out] Piecewise((oo*I*x, Eq(a, -I/x**2)), (-oo*I*x, Eq(a, I/x**2)), (pi*x/2, Eq(a
, 0)), (2*a**5*x**5*(-1/a**2)**(7/4)*acot(a*x**2)/(2*a**5*x**4*(-1/a**2)**(
```

```
7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a**4*x**4*(-1/a**2)**(3/2)*log(x - (-1/
a**2)**(1/4))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) - a*
*4*x**4*(-1/a**2)**(3/2)*log(x**2 + sqrt(-1/a**2))/(2*a**5*x**4*(-1/a**2)**
(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a**4*x**4*(-1/a**2)**(3/2)*atan(x/(-1/
a**2)**(1/4))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*
a**3*x*(-1/a**2)**(7/4)*acot(a*x**2)/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3
*(-1/a**2)**(7/4)) + 2*a**2*(-1/a**2)**(3/2)*log(x - (-1/a**2)**(1/4))/(2*a
**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) - a**2*(-1/a**2)**(3/2
)*log(x**2 + sqrt(-1/a**2))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2
)**(7/4)) + 2*a**2*(-1/a**2)**(3/2)*atan(x/(-1/a**2)**(1/4))/(2*a**5*x**4*(
-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a*x**4*acot(a*x**2)/(2*a**5*
x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*acot(a*x**2)/(2*a**6*x
**4*(-1/a**2)**(7/4) + 2*a**4*(-1/a**2)**(7/4)), True))
```

Giac [A]
time = 0.41, size = 144, normalized size = 1.09

$$\frac{1}{4}a\left(\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}+\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}-\frac{\sqrt{2}\sqrt{|a|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{a^2}+\frac{\sqrt{2}\sqrt{|a|}\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{a^2}\right)+x\arctan\left(\frac{1}{ax^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*a*(2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)
))*sqrt(abs(a)))/a^2 + 2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)/sqrt(abs(a)))*sqrt(abs(a)))/a^2 - sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(
2)*x/sqrt(abs(a)) + 1/abs(a))/a^2 + sqrt(2)*sqrt(abs(a))*log(x^2 - sqrt(2)*
x/sqrt(abs(a)) + 1/abs(a))/a^2) + x*arctan(1/(a*x^2))
```

Mupad [B]
time = 0.13, size = 42, normalized size = 0.32

$$x\,\mathrm{acot}\left(a\,x^2\right)+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2),x)
```

```
[Out] x*acot(a*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x))/a^(1/2) - ((-1)^(1/
4)*atanh((-1)^(1/4)*a^(1/2)*x))/a^(1/2)
```

3.83 $\int \frac{\cot^{-1}(ax^2)}{x^2} dx$

Optimal. Leaf size=135

$$-\frac{\cot^{-1}(ax^2)}{x}+\frac{\sqrt{a}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{a}\,x\right)}{\sqrt{2}}-\frac{\sqrt{a}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{a}\,x\right)}{\sqrt{2}}+\frac{\sqrt{a}\,\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}}$$

```
[Out] -arccot(a*x^2)/x-1/2*arctan(-1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)-1/2*arcta
n(1+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)+1/4*ln(1+a*x^2-x*2^(1/2)*a^(1/2))*a^
(1/2)*2^(1/2)-1/4*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*a^(1/2)*2^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4947, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{a}\,x\right)}{\sqrt{2}}-\frac{\sqrt{a}\,\text{ArcTan}\left(\sqrt{2}\sqrt{a}\,x+1\right)}{\sqrt{2}}+\frac{\sqrt{a}\,\log\left(ax^2-\sqrt{2}\sqrt{a}\,x+1\right)}{2\sqrt{2}}-\frac{\sqrt{a}\,\log\left(ax^2+\sqrt{2}\sqrt{a}\,x+1\right)}{2\sqrt{2}}-\frac{\cot^{-1}(ax^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2]/x^2,x]
```

```
[Out] -(ArcCot[a*x^2]/x) + (Sqrt[a]*ArcTan[1 - Sqrt[2]*Sqrt[a]*x])/Sqrt[2] - (Sqr
t[a]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x])/Sqrt[2] + (Sqrt[a]*Log[1 - Sqrt[2]*Sqrt
[a]*x + a*x^2])/(2*Sqrt[2]) - (Sqrt[a]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/
(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{\cot^{-1}(ax^2)}{x} - (2a)\int \frac{1}{1+a^2x^4}\,dx$$
$$= -\frac{\cot^{-1}(ax^2)}{x} - a\int \frac{1-ax^2}{1+a^2x^4}\,dx - a\int \frac{1+ax^2}{1+a^2x^4}\,dx$$
$$= -\frac{\cot^{-1}(ax^2)}{x} - \frac{1}{2}\int \frac{1}{\frac{1}{a} - \frac{\sqrt{2}\,x}{\sqrt{a}} + x^2}\,dx - \frac{1}{2}\int \frac{1}{\frac{1}{a} + \frac{\sqrt{2}\,x}{\sqrt{a}} + x^2}\,dx + \frac{\sqrt{a}\int \frac{\frac{\sqrt{2}}{\sqrt{a}}+2x}{-\frac{1}{a}-\frac{\sqrt{2}\,x}{\sqrt{a}}-x^2}}{2\sqrt{2}}$$
$$= -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\,\log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{a}\,\log\left(1+\sqrt{2}\,\sqrt{a}\,x+ax^2\right)}{2\sqrt{2}} - \frac{\sqrt{}}{}$$
$$= -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\,\tan^{-1}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}} - \frac{\sqrt{a}\,\tan^{-1}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{\sqrt{2}} + \frac{\sqrt{a}\,\log\left(\right.}{}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 0.78

$$-\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\left(2\mathrm{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right) - 2\mathrm{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right) + \log\left(1-\sqrt{2}\,\sqrt{a}\,x+ax^2\right) - \log\left(1+\sqrt{2}\,\sqrt{a}\,x+ax^2\right)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2]/x^2,x]
```

```
[Out] -(ArcCot[a*x^2]/x) + (Sqrt[a]*(2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2
]*Sqrt[a]*x + a*x^2]))/(2*Sqrt[2])
```

Maple [A]
time = 0.05, size = 98, normalized size = 0.73

method	result	size
default	$-\frac{\mathrm{arccot}(a\,x^2)}{x} - \frac{a\left(\frac{1}{a^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(a*x^2)/x-1/4*a*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(1/a^2)^(1/4)*x*2^(1/
2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^(
1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Maxima [A]
time = 0.46, size = 123, normalized size = 0.91

$$-\frac{1}{4}a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}-\frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}\right)-\frac{\operatorname{arccot}\left(ax^2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqr
t(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt
(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^
2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)) - arccot(a*x^2)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(99) = 198$.
time = 2.92, size = 227, normalized size = 1.68

$$\frac{4\sqrt{2}\left(a^2\right)^{\frac{1}{4}}x\arctan\left(-\frac{\sqrt{2}\left(a^2\right)^{\frac{3}{4}}ax+a^2-\sqrt{2}\sqrt{a^2x^2+\sqrt{2}\left(a^2\right)^{\frac{1}{4}}ax+\sqrt{a^2}}\left(a^2\right)^{\frac{3}{4}}}{a^2}\right)+4\sqrt{2}\left(a^2\right)^{\frac{1}{4}}x\arctan\left(-\frac{\sqrt{2}\left(a^2\right)^{\frac{3}{4}}ax-a^2-\sqrt{2}\sqrt{a^2x^2-\sqrt{2}\left(a^2\right)^{\frac{1}{4}}ax+\sqrt{a^2}}\left(a^2\right)^{\frac{3}{4}}}{a^2}\right)-\sqrt{2}\left(a^2\right)^{\frac{1}{4}}x\log\left(a^2x^2+\sqrt{2}\left(a^2\right)^{\frac{1}{4}}ax+\sqrt{a^2}\right)+\sqrt{2}\left(a^2\right)^{\frac{1}{4}}x\log\left(a^2x^2-\sqrt{2}\left(a^2\right)^{\frac{1}{4}}ax+\sqrt{a^2}\right)-4\arctan\left(\frac{1}{ax^2}\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(a^2)^(1/4)*x*arctan(-(sqrt(2)*(a^2)^(3/4)*a*x + a^2 - sqrt(
2)*sqrt(a^2*x^2 + sqrt(2)*(a^2)^(1/4)*a*x + sqrt(a^2))*(a^2)^(3/4))/a^2) +
4*sqrt(2)*(a^2)^(1/4)*x*arctan(-(sqrt(2)*(a^2)^(3/4)*a*x - a^2 - sqrt(2)*sq
rt(a^2*x^2 - sqrt(2)*(a^2)^(1/4)*a*x + sqrt(a^2))*(a^2)^(3/4))/a^2) - sqrt(
2)*(a^2)^(1/4)*x*log(a^2*x^2 + sqrt(2)*(a^2)^(1/4)*a*x + sqrt(a^2)) + sqrt(
2)*(a^2)^(1/4)*x*log(a^2*x^2 - sqrt(2)*(a^2)^(1/4)*a*x + sqrt(a^2)) - 4*arc
tan(1/(a*x^2)))/x
```

Sympy [A]
time = 8.03, size = 110, normalized size = 0.81

$$\begin{cases} a\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)-\frac{a\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2}-a\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)-\frac{\operatorname{acot}\left(ax^2\right)}{\sqrt[4]{-\frac{1}{a^2}}}-\frac{\operatorname{acot}\left(ax^2\right)}{x} & \text{for}\ a\neq 0 \\ -\frac{\pi}{2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2)/x**2,x)
```

```
[Out] Piecewise((a*(-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4)) - a*(-1/a**2)**(1/4
)*log(x**2 + sqrt(-1/a**2))/2 - a*(-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4))
 - acot(a*x**2)/(-1/a**2)**(1/4) - acot(a*x**2)/x, Ne(a, 0)), (-pi/(2*x), T
rue))
```

Giac [A]
time = 0.41, size = 135, normalized size = 1.00

$$-\frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{\sqrt{2}\,\log\left(x^2+\frac{\sqrt{2}\,x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{\sqrt{|a|}}-\frac{\sqrt{2}\,\log\left(x^2-\frac{\sqrt{2}\,x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{\sqrt{|a|}}\right)-\frac{\arctan\left(\frac{1}{ax^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(
a)))/sqrt(abs(a)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)
))*sqrt(abs(a)))/sqrt(abs(a)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) +
1/abs(a))/sqrt(abs(a)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a
))/sqrt(abs(a))) - arctan(1/(a*x^2))/x
```

Mupad [B]
time = 0.68, size = 44, normalized size = 0.33

$$-\frac{\mathrm{acot}\left(a\,x^2\right)}{x}+{\left(-1\right)}^{1/4}\,\sqrt{a}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\right)\,1{}\mathrm{i}+{\left(-1\right)}^{1/4}\,\sqrt{a}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2)/x^2,x)
```

```
[Out] (-1)^(1/4)*a^(1/2)*atan((-1)^(1/4)*a^(1/2)*x)*1i - acot(a*x^2)/x + (-1)^(1/
4)*a^(1/2)*atanh((-1)^(1/4)*a^(1/2)*x)*1i
```

3.84 $\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$

Optimal. Leaf size=150

$$\frac{2a}{3x}-\frac{\cot^{-1}(ax^2)}{3x^3}-\frac{a^{3/2}\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}}+\frac{a^{3/2}\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}}+\frac{a^{3/2}\log\left(1-\sqrt{2}\,\sqrt{a}\,x+a\right.}{6\sqrt{2}}$$

```
[Out] 2/3*a/x-1/3*arccot(a*x^2)/x^3+1/6*a^(3/2)*arctan(-1+x*2^(1/2)*a^(1/2))*2^(1
/2)+1/6*a^(3/2)*arctan(1+x*2^(1/2)*a^(1/2))*2^(1/2)+1/12*a^(3/2)*ln(1+a*x^2
-x*2^(1/2)*a^(1/2))*2^(1/2)-1/12*a^(3/2)*ln(1+a*x^2+x*2^(1/2)*a^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4947, 331, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a^{3/2}\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{a}\,x\right)}{3\sqrt{2}}+\frac{a^{3/2}\text{ArcTan}\left(\sqrt{2}\,\sqrt{a}\,x+1\right)}{3\sqrt{2}}+\frac{a^{3/2}\log\left(ax^2-\sqrt{2}\,\sqrt{a}\,x+1\right)}{6\sqrt{2}}-\frac{a^{3/2}\log\left(ax^2+\sqrt{2}\,\sqrt{a}\,x+1\right)}{6\sqrt{2}}-\frac{\cot^{-1}(ax^2)}{3x^3}+\frac{2a}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^2]/x^4,x]
```

```
[Out] (2*a)/(3*x) - ArcCot[a*x^2]/(3*x^3) - (a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[a]*x
])/(3*Sqrt[2]) + (a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[a]*x])/(3*Sqrt[2]) + (a^(
3/2)*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2])/(6*Sqrt[2]) - (a^(3/2)*Log[1 + Sqr
t[2]*Sqrt[a]*x + a*x^2])/(6*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx &= -\frac{\cot^{-1}(ax^2)}{3x^3} - \frac{1}{3}(2a)\int \frac{1}{x^2(1+a^2x^4)}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{1}{3}(2a^3)\int \frac{x^2}{1+a^2x^4}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{1}{3}a^2\int \frac{1-ax^2}{1+a^2x^4}\,dx + \frac{1}{3}a^2\int \frac{1+ax^2}{1+a^2x^4}\,dx \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{1}{6}a\int \frac{1}{\frac{1}{a}-\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx + \frac{1}{6}a\int \frac{1}{\frac{1}{a}+\frac{\sqrt{2}x}{\sqrt{a}}+x^2}\,dx + \frac{a^{3/2}\int \frac{\sqrt{\ }}{\sqrt{\ }}}{6\sqrt{\ }} \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} + \frac{a^{3/2}\log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} - \frac{a^{3/2}\log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{6\sqrt{2}} \\
&= \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{a^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\tan^{-1}\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\,\mathrm{l}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 146, normalized size = 0.97

$$\frac{-4\cot^{-1}(ax^2)+ax^2\left(8-2\sqrt{2}\sqrt{a}\,x\text{ArcTan}\left(1-\sqrt{2}\sqrt{a}\,x\right)+2\sqrt{2}\sqrt{a}\,x\text{ArcTan}\left(1+\sqrt{2}\sqrt{a}\,x\right)+\sqrt{2}\sqrt{a}\,x\log\left(1-\sqrt{2}\sqrt{a}\,x+ax^2\right)-\sqrt{2}\sqrt{a}\,x\log\left(1+\sqrt{2}\sqrt{a}\,x+ax^2\right)\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^2]/x^4,x]
```

```
[Out] (-4*ArcCot[a*x^2] + a*x^2*(8 - 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 - Sqrt[2]*Sqrt[
a]*x] + 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Sqrt[a]
*x*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Sqrt[a]*x*Log[1 + Sqrt[2]*S
qrt[a]*x + a*x^2]))/(12*x^3)
```

Maple [A]
time = 0.07, size = 106, normalized size = 0.71

method	result	size

default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{3x^{3}}-\frac{2a\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccot(a*x^2)/x^3-2/3*a*(-1/8/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^(
1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+
2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))-1/
x)
```

Maxima [A]
time = 0.46, size = 133, normalized size = 0.89

$$\frac{1}{12}\left(a^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(ax^{2}+\sqrt{2}\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(ax^{2}-\sqrt{2}\sqrt{a}\,x+1\right)}{a^{\frac{3}{2}}}\right)+\frac{8}{x}\right)a-\frac{\operatorname{arccot}\left(ax^{2}\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/
a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a
^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a
*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + 8/x)*a - 1/3*arccot(a*x^2)/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(105) = 210$.
time = 3.48, size = 269, normalized size = 1.79

$$-\frac{4\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}x^{3}\arctan\left(-\frac{\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}a^{5}x+a^{6}-\sqrt{2}\sqrt{a^{10}x^{2}+\sqrt{2}\left(a^{6}\right)^{\frac{3}{4}}a^{5}x+\sqrt{a^{6}}\,a^{6}}\left(a^{6}\right)^{\frac{1}{4}}}{a^{6}}\right)+4\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}x^{3}\arctan\left(-\frac{\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}a^{5}x-a^{6}-\sqrt{2}\sqrt{a^{10}x^{2}-\sqrt{2}\left(a^{6}\right)^{\frac{3}{4}}a^{5}x+\sqrt{a^{6}}\,a^{6}}\left(a^{6}\right)^{\frac{1}{4}}}{a^{6}}\right)+\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}x^{3}\log\left(a^{10}x^{2}+\sqrt{2}\left(a^{6}\right)^{\frac{3}{4}}a^{5}x+\sqrt{a^{6}}\,a^{6}\right)-\sqrt{2}\left(a^{6}\right)^{\frac{1}{4}}x^{3}\log\left(a^{10}x^{2}-\sqrt{2}\left(a^{6}\right)^{\frac{3}{4}}a^{5}x+\sqrt{a^{6}}\,a^{6}\right)-8\,ax^{2}+4\arctan\left(\frac{1}{ax^{2}}\right)}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(2)*(a^6)^(1/4)*x^3*arctan(-(sqrt(2)*(a^6)^(1/4)*a^5*x + a^6 -
 sqrt(2)*sqrt(a^10*x^2 + sqrt(2)*(a^6)^(3/4)*a^5*x + sqrt(a^6)*a^6)*(a^6)^(
1/4))/a^6) + 4*sqrt(2)*(a^6)^(1/4)*x^3*arctan(-(sqrt(2)*(a^6)^(1/4)*a^5*x -
 a^6 - sqrt(2)*sqrt(a^10*x^2 - sqrt(2)*(a^6)^(3/4)*a^5*x + sqrt(a^6)*a^6)*(
```

```
a^6)^(1/4))/a^6) + sqrt(2)*(a^6)^(1/4)*x^3*log(a^10*x^2 + sqrt(2)*(a^6)^(3/
4)*a^5*x + sqrt(a^6)*a^6) - sqrt(2)*(a^6)^(1/4)*x^3*log(a^10*x^2 - sqrt(2)*
(a^6)^(3/4)*a^5*x + sqrt(a^6)*a^6) - 8*a*x^2 + 4*arctan(1/(a*x^2)))/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 16.95, size = 459, normalized size = 3.06

$$\begin{cases} -\frac{\pi}{6x^3} & \text{for } a = 0 \\ -\frac{\infty i}{x^3} & \text{for } a = -\frac{i}{x^2} \\ \frac{\infty i}{x^3} & \text{for } a = \frac{i}{x^2} \\ -\frac{2a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} + \frac{a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} - \frac{2a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{6x^7+\frac{6x^3}{a^2}} + \frac{2a^2x^7\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}(ax^2)}{6x^7+\frac{6x^3}{a^2}} + \frac{4ax^6}{6x^7+\frac{6x^3}{a^2}} - \frac{2ax^3\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} + \frac{ax^3\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} - \frac{2ax^3\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{6x^7+\frac{6x^3}{a^2}} - \frac{2x^4\operatorname{acot}(ax^2)}{6x^7+\frac{6x^3}{a^2}} + \frac{2x^3\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}(ax^2)}{6x^7+\frac{6x^3}{a^2}} + \frac{4x^2}{6ax^7+\frac{6x^3}{a}} - \frac{2\operatorname{acot}(ax^2)}{6a^2x^7+6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**2)/x**4,x)
```

```
[Out] Piecewise((-pi/(6*x**3), Eq(a, 0)), (-oo*I/x**3, Eq(a, -I/x**2)), (oo*I/x**
3, Eq(a, I/x**2)), (-2*a**3*x**7*(-1/a**2)**(3/4)*log(x - (-1/a**2)**(1/4))
/(6*x**7 + 6*x**3/a**2) + a**3*x**7*(-1/a**2)**(3/4)*log(x**2 + sqrt(-1/a**
2))/(6*x**7 + 6*x**3/a**2) - 2*a**3*x**7*(-1/a**2)**(3/4)*atan(x/(-1/a**2)*
*(1/4))/(6*x**7 + 6*x**3/a**2) + 2*a**2*x**7*(-1/a**2)**(1/4)*acot(a*x**2)/
(6*x**7 + 6*x**3/a**2) + 4*a*x**6/(6*x**7 + 6*x**3/a**2) - 2*a*x**3*(-1/a**
2)**(3/4)*log(x - (-1/a**2)**(1/4))/(6*x**7 + 6*x**3/a**2) + a*x**3*(-1/a**
2)**(3/4)*log(x**2 + sqrt(-1/a**2))/(6*x**7 + 6*x**3/a**2) - 2*a*x**3*(-1/a
**2)**(3/4)*atan(x/(-1/a**2)**(1/4))/(6*x**7 + 6*x**3/a**2) - 2*x**4*acot(a
*x**2)/(6*x**7 + 6*x**3/a**2) + 2*x**3*(-1/a**2)**(1/4)*acot(a*x**2)/(6*x**
7 + 6*x**3/a**2) + 4*x**2/(6*a*x**7 + 6*x**3/a) - 2*acot(a*x**2)/(6*a**2*x*
*7 + 6*x**3), True))
```

Giac [A]
time = 0.42, size = 149, normalized size = 0.99

$$\frac{1}{12}\left(\frac{2\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} + \frac{2\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} - \sqrt{2}\sqrt{|a|}\log\left(x^2+\frac{\sqrt{2}\,x}{\sqrt{|a|}}+\frac{1}{|a|}\right) + \frac{\sqrt{2}\,a^2\log\left(x^2-\frac{\sqrt{2}\,x}{\sqrt{|a|}}+\frac{1}{|a|}\right)}{|a|^{\frac{3}{2}}} + \frac{8}{x}\right)a - \frac{\arctan\left(\frac{1}{ax^2}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(ab
s(a)))/abs(a)^(3/2) + 2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(
abs(a)))*sqrt(abs(a)))/abs(a)^(3/2) - sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2
)*x/sqrt(abs(a)) + 1/abs(a)) + sqrt(2)*a^2*log(x^2 - sqrt(2)*x/sqrt(abs(a))
 + 1/abs(a))/abs(a)^(3/2) + 8/x)*a - 1/3*arctan(1/(a*x^2))/x^3
```

Mupad [B]
time = 0.71, size = 52, normalized size = 0.35

$$\frac{2\,a}{3\,x} - \frac{\mathrm{acot}\left(a\,x^2\right)}{3\,x^3} + \frac{{\left(-1\right)}^{1/4}\,a^{3/2}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\right)}{3} + \frac{{\left(-1\right)}^{1/4}\,a^{3/2}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{a}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^2)/x^4,x)
```

```
[Out] (2*a)/(3*x) - acot(a*x^2)/(3*x^3) + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(
1/2)*x))/3 + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/3
```

3.85 $\int x^2 \cot^{-1}\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{3}$$

```
[Out] -1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*x
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 52, 65, 209}

$$-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{3}+\frac{x^{5/2}}{15}-\frac{x^{3/2}}{9}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/3 - x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCot[Sqrt[x]])/3 - ArcTan[Sqrt[
x]]/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int \frac{x^{5/2}}{1+x}\,dx \\
&= \frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int \frac{x^{3/2}}{1+x}\,dx \\
&= -\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int \frac{\sqrt{x}}{1+x}\,dx \\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx \\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right) \\
&= \frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.78

$$\frac{1}{45}\left(\sqrt{x}\left(15-5x+3x^2\right)+15x^3\cot^{-1}\left(\sqrt{x}\right)-15\text{ArcTan}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[Sqrt[x]],x]
[Out] (Sqrt[x]*(15 - 5*x + 3*x^2) + 15*x^3*ArcCot[Sqrt[x]] - 15*ArcTan[Sqrt[x]])/
45
```

Maple [A]
time = 0.01, size = 32, normalized size = 0.63

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^3\,\text{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32

default	$-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^(1/2))+1/3*x
^(1/2)
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.61

$$\frac{1}{3}\,x^{3}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{15}\,x^{\frac{5}{2}}-\frac{1}{9}\,x^{\frac{3}{2}}+\frac{1}{3}\,\sqrt{x}-\frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(sqrt(x)) + 1/15*x^(5/2) - 1/9*x^(3/2) + 1/3*sqrt(x) - 1/3*ar
ctan(sqrt(x))
```

Fricas [A]
time = 1.99, size = 27, normalized size = 0.53

$$\frac{1}{3}\,\left(x^{3}+1\right)\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{45}\,\left(3\,x^{2}-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arccot(sqrt(x)) + 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A]
time = 1.31, size = 39, normalized size = 0.76

$$\frac{x^{\frac{5}{2}}}{15}-\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{x^{3}\operatorname{acot}\left(\sqrt{x}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(x**(1/2)),x)
```

```
[Out] x**(5/2)/15 - x**(3/2)/9 + sqrt(x)/3 + x**3*acot(sqrt(x))/3 - atan(sqrt(x))
/3
```

Giac [A]
time = 0.43, size = 33, normalized size = 0.65

$$\frac{1}{3}\,x^{3}\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{45}\,x^{\frac{5}{2}}\left(\frac{5}{x}-\frac{15}{x^{2}}-3\right)+\frac{1}{3}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(1/sqrt(x)) - 1/45*x^(5/2)*(5/x - 15/x^2 - 3) + 1/3*arctan(1/
sqrt(x))
```

Mupad [B]
time = 0.65, size = 31, normalized size = 0.61

$$\frac{x^3\,\mathrm{acot}\left(\sqrt{x}\right)}{3}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}-\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(x^(1/2)),x)
```

```
[Out] (x^3*acot(x^(1/2)))/3 - atan(x^(1/2))/3 + x^(1/2)/3 - x^(3/2)/9 + x^(5/2)/1
5
```

3.86 $\int x \cot^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}$$

```
[Out] 1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4947, 52, 65, 209}

$$\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[Sqrt[x]],x]
```

```
[Out] -1/2*Sqrt[x] + x^(3/2)/6 + (x^2*ArcCot[Sqrt[x]])/2 + ArcTan[Sqrt[x]]/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x^{3/2}}{1+x}\,dx\\
&= \frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.79

$$\frac{1}{6}\left((-3+x)\sqrt{x}+3x^2\cot^{-1}\left(\sqrt{x}\right)+3\mathrm{ArcTan}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[Sqrt[x]],x]
[Out] ((-3 + x)*Sqrt[x] + 3*x^2*ArcCot[Sqrt[x]] + 3*ArcTan[Sqrt[x]])/6
```

Maple [A]
time = 0.00, size = 27, normalized size = 0.64

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^2\,\mathrm{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27
default	$\frac{x^{\frac{3}{2}}}{6}+\frac{x^2\,\mathrm{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))-1/2*x^(1/2)
```

Maxima [A]
time = 0.46, size = 26, normalized size = 0.62

$$\frac{1}{2}\,x^2\,\mathrm{arccot}\left(\sqrt{x}\right)+\frac{1}{6}\,x^{\frac{3}{2}}-\frac{1}{2}\,\sqrt{x}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(sqrt(x)) + 1/6*x^(3/2) - 1/2*sqrt(x) + 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 2.98, size = 20, normalized size = 0.48

$$\frac{1}{2}\left(x^2-1\right)\mathrm{arccot}\left(\sqrt{x}\right)+\frac{1}{6}\,(x-3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*arccot(sqrt(x)) + 1/6*(x - 3)*sqrt(x)
```

Sympy [A]
time = 0.84, size = 32, normalized size = 0.76

$$\frac{x^{\frac{3}{2}}}{6}-\frac{\sqrt{x}}{2}+\frac{x^2\,\mathrm{acot}\left(\sqrt{x}\right)}{2}+\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x**(1/2)),x)
[Out] x**(3/2)/6 - sqrt(x)/2 + x**2*acot(sqrt(x))/2 + atan(sqrt(x))/2
```

Giac [A]
time = 0.41, size = 28, normalized size = 0.67

$$\frac{1}{2}\,x^2\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{3}{x}-1\right)-\frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*arctan(1/sqrt(x)) - 1/6*x^(3/2)*(3/x - 1) - 1/2*arctan(1/sqrt(x))
```

Mupad [B]
time = 0.66, size = 26, normalized size = 0.62

$$\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{x^2\,\mathrm{acot}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(x^(1/2)),x)
[Out] atan(x^(1/2))/2 + (x^2*acot(x^(1/2)))/2 - x^(1/2)/2 + x^(3/2)/6
```

3.87 $\int \cot^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\text{ArcTan}\left(\sqrt{x}\right)$$

[Out] x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.667$, Rules used = {4931, 52, 65, 209}

$$-\text{ArcTan}\left(\sqrt{x}\right)+\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCot[Sqrt[x]],x]

[Out] Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}\left(\sqrt{x}\right)\,dx &= x\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{x}+x\cot^{-1}\left(\sqrt{x}\right)-\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]],x]
[Out] Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$x\,\text{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17
default	$x\,\text{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)+\sqrt{x}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccot(x^(1/2))-arctan(x^(1/2))+x^(1/2)
```

Maxima [A]
time = 0.49, size = 16, normalized size = 0.73

$$x\,\text{arccot}\left(\sqrt{x}\right)+\sqrt{x}-\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arccot(sqrt(x)) + sqrt(x) - arctan(sqrt(x))
```

Fricas [A]
time = 3.13, size = 12, normalized size = 0.55

$$(x+1)\operatorname{arccot}\left(\sqrt{x}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arccot(sqrt(x)) + sqrt(x)
```

Sympy [A]
time = 0.76, size = 19, normalized size = 0.86

$$\sqrt{x}+x\operatorname{acot}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2)),x)
```

```
[Out] sqrt(x) + x*acot(sqrt(x)) - atan(sqrt(x))
```

Giac [A]
time = 0.40, size = 14, normalized size = 0.64

$$x\arctan\left(\frac{1}{\sqrt{x}}\right)+\sqrt{x}+\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(1/sqrt(x)) + sqrt(x) + arctan(1/sqrt(x))
```

Mupad [B]
time = 0.63, size = 16, normalized size = 0.73

$$x\operatorname{acot}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2)),x)
```

```
[Out] x*acot(x^(1/2)) - atan(x^(1/2)) + x^(1/2)
```

3.88 $\int \frac{\cot^{-1}(\sqrt{x})}{x} dx$

Optimal. Leaf size=31

$$-i\text{PolyLog}\left(2,-\frac{i}{\sqrt{x}}\right)+i\text{PolyLog}\left(2,\frac{i}{\sqrt{x}}\right)$$

```
[Out] -I*polylog(2,-I/x^(1/2))+I*polylog(2,I/x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4945, 4941, 2438}

$$i\text{Li}_2\left(\frac{i}{\sqrt{x}}\right)-i\text{Li}_2\left(-\frac{i}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Sqrt[x]]/x,x]
```

```
[Out] (-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(\sqrt{x})}{x}\,dx = 2\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)$$
$$= i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= -i\text{Li}_2\left(-\frac{i}{\sqrt{x}}\right) + i\text{Li}_2\left(\frac{i}{\sqrt{x}}\right)$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-i\text{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i\text{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/x,x]
[Out] (-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.
time = 0.02, size = 61, normalized size = 1.97

method	result
derivativedivides	$\ln(x)\,\mathrm{arccot}\left(\sqrt{x}\right) - \frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2} + \frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2} - i\,\mathrm{dilog}\left(1+i\sqrt{x}\right) + i\,\mathrm{dilog}$
default	$\ln(x)\,\mathrm{arccot}\left(\sqrt{x}\right) - \frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2} + \frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2} - i\,\mathrm{dilog}\left(1+i\sqrt{x}\right) + i\,\mathrm{dilog}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arccot(x^(1/2))-1/2*I*ln(x)*ln(1+I*x^(1/2))+1/2*I*ln(x)*ln(1-I*x^(1/2
))-I*dilog(1+I*x^(1/2))+I*dilog(1-I*x^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.
time = 0.49, size = 35, normalized size = 1.13

$$\frac{1}{2}\pi\log(x+1) + \mathrm{arccot}\left(\sqrt{x}\right)\log(x) + i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right) - i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] 1/2*pi*log(x + 1) + arccot(sqrt(x))*log(x) + I*dilog(I*sqrt(x) + 1) - I*dil
og(-I*sqrt(x) + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x,x)
```

```
[Out] Integral(acot(sqrt(x))/x, x)
```

Giac [A]
time = 0.42, size = 19, normalized size = 0.61

$$-x\arctan\left(\frac{1}{\sqrt{x}}\right)-\sqrt{x}-\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] -x*arctan(1/sqrt(x)) - sqrt(x) - arctan(1/sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x,x)
```

```
[Out] int(acot(x^(1/2))/x, x)
```

3.89 $\int \frac{\cot^{-1}(\sqrt{x})}{x^2} dx$

Optimal. Leaf size=23

$$\frac{1}{\sqrt{x}}-\frac{\cot^{-1}(\sqrt{x})}{x}+\operatorname{ArcTan}(\sqrt{x})$$

```
[Out] -arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 53, 65, 209}

$$\operatorname{ArcTan}(\sqrt{x})+\frac{1}{\sqrt{x}}-\frac{\cot^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Sqrt[x]]/x^2,x]
```

```
[Out] 1/Sqrt[x] - ArcCot[Sqrt[x]]/x + ArcTan[Sqrt[x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}-\frac{1}{2}\int \frac{1}{x^{3/2}(1+x)}\,dx\\ &= \frac{1}{\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= \frac{1}{\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= \frac{1}{\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}+\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 29, normalized size = 1.26

$$-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}+\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/x^2,x]
[Out] -(ArcCot[Sqrt[x]]/x) + Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18
default	$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{\sqrt{x}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(x^(1/2))/x+arctan(x^(1/2))+1/x^(1/2)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.74

$$-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\frac{1}{\sqrt{x}}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arccot(sqrt(x))/x + 1/sqrt(x) + arctan(sqrt(x))
```

Fricas [A]
time = 3.31, size = 19, normalized size = 0.83

$$-\frac{(x+1)\operatorname{arccot}\left(\sqrt{x}\right)-\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -((x + 1)*arccot(sqrt(x)) - sqrt(x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(20) = 40$.
time = 0.58, size = 92, normalized size = 4.00

$$-\frac{x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}+\frac{x^{2}}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}+\frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x**2,x)
```

```
[Out] -x**(5/2)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*acot(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) + x**2/(x
**(5/2) + x**(3/2)) + x/(x**(5/2) + x**(3/2))
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.83

$$-\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{x}+\frac{1}{\sqrt{x}}-\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -arctan(1/sqrt(x))/x + 1/sqrt(x) - arctan(1/sqrt(x))
```

Mupad [B]
time = 0.64, size = 17, normalized size = 0.74

$$\mathrm{atan}\left(\sqrt{x}\right)-\frac{\mathrm{acot}\left(\sqrt{x}\right)}{x}+\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x^2,x)
```

```
[Out] atan(x^(1/2)) - acot(x^(1/2))/x + 1/x^(1/2)
```

3.90 $\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=42

$$\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{\mathrm{ArcTan}\left(\sqrt{x}\right)}{2}$$

```
[Out] 1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4947, 53, 65, 209}

$$-\frac{\mathrm{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{1}{6x^{3/2}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Sqrt[x]]/x^3,x]
```

```
[Out] 1/(6*x^(3/2)) - 1/(2*Sqrt[x]) - ArcCot[Sqrt[x]]/(2*x^2) - ArcTan[Sqrt[x]]/2
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= \frac{1}{6x^{3/2}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= \frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 34, normalized size = 0.81

$$-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x\right)}{6x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/x^3,x]
[Out] -1/2*ArcCot[Sqrt[x]]/x^2 + Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.64

method	result	size
derivativedivides	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^2}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27

default	$\frac{1}{6x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] 1/6/x^(3/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))-1/2/x^(1/2)
```

Maxima [A]
time = 0.46, size = 26, normalized size = 0.62

$$-\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2\,x^{2}}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="maxima")
[Out] -1/6*(3*x - 1)/x^(3/2) - 1/2*arccot(sqrt(x))/x^2 - 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 1.91, size = 27, normalized size = 0.64

$$\frac{3\,(x^{2}-1)\operatorname{arccot}\left(\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*(3*(x^2 - 1)*arccot(sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(36)=72$.
time = 1.21, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{3}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{2x^{2}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x**3,x)
[Out] 3*x**(7/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*acot(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**3/(6*x**(
7/2) + 6*x**(5/2)) - 2*x**2/(6*x**(7/2) + 6*x**(5/2)) + x/(6*x**(7/2) + 6*x
**(5/2))
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.62

$$-\frac{1}{2\sqrt{x}}-\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2}+\frac{1}{6\,x^{\frac{3}{2}}}+\frac{1}{2}\arctan\left(\frac{1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2/sqrt(x) - 1/2*arctan(1/sqrt(x))/x^2 + 1/6/x^(3/2) + 1/2*arctan(1/sqrt(
x))
```

Mupad [B]
time = 0.65, size = 24, normalized size = 0.57

$$-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}-\frac{x-\frac{1}{3}}{2\,x^{3/2}}-\frac{\operatorname{acot}\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x^3,x)
```

```
[Out] - atan(x^(1/2))/2 - (x - 1/3)/(2*x^(3/2)) - acot(x^(1/2))/(2*x^2)
```

3.91 $\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=36

$$-\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1+x)$$

```
[Out] -1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4947, 45}

$$\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{x^2}{10}-\frac{x}{5}+\frac{1}{5}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcCot[Sqrt[x]],x]
```

```
[Out] -1/5*x + x^2/10 + (2*x^(5/2)*ArcCot[Sqrt[x]])/5 + Log[1 + x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)dx &= \frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\ &= \frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)dx\\ &= -\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.81

$$\frac{1}{10}\left((-2+x)x+4x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+2\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcCot[Sqrt[x]],x]
[Out] ((-2 + x)*x + 4*x^(5/2)*ArcCot[Sqrt[x]] + 2*Log[1 + x])/10
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{x}{5}+\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln(1+x)}{5}$	25
default	$-\frac{x}{5}+\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln(1+x)}{5}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Fricas [A]
time = 2.37, size = 24, normalized size = 0.67

$$\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.
time = 1.04, size = 85, normalized size = 2.36

$$\frac{4x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{x^3}{10x+10}-\frac{x^2}{10x+10}+\frac{2x\log\left(x+1\right)}{10x+10}+\frac{2\log\left(x+1\right)}{10x+10}+\frac{2}{10x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*acot(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*acot(sqrt(x))/(10*x + 10) + 4*x**(5/2)*acot(sqrt(x))/(10*x + 10)
 + x**3/(10*x + 10) - x**2/(10*x + 10) + 2*x*log(x + 1)/(10*x + 10) + 2*log
(x + 1)/(10*x + 10) + 2/(10*x + 10)
```

Giac [A]
time = 0.41, size = 39, normalized size = 1.08

$$\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{10}\,x^2\left(\frac{2}{x}-\frac{3}{x^2}-1\right)+\frac{1}{5}\,\log\left(x\right)+\frac{1}{5}\,\log\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*arctan(1/sqrt(x)) - 1/10*x^2*(2/x - 3/x^2 - 1) + 1/5*log(x) + 1
/5*log(1/x + 1)
```

Mupad [B]
time = 0.65, size = 24, normalized size = 0.67

$$\frac{\ln\left(x+1\right)}{5}-\frac{x}{5}+\frac{2\,x^{5/2}\,\operatorname{acot}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*acot(x^(1/2)),x)
```

```
[Out] log(x + 1)/5 - x/5 + (2*x^(5/2)*acot(x^(1/2)))/5 + x^2/10
```

3.92 $\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=29

$$\frac{x}{3}+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\log(1+x)$$

```
[Out] 1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4947, 45}

$$\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{x}{3}-\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcCot[Sqrt[x]])/3 - Log[1 + x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right) dx &= \frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int \frac{x}{1+x}\,dx\\
&= \frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int \left(1+\frac{1}{-1-x}\right) dx\\
&= \frac{x}{3}+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{3}\left(x+2x^{3/2}\cot^{-1}\left(\sqrt{x}\right)-\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcCot[Sqrt[x]],x]
[Out] (x + 2*x^(3/2)*ArcCot[Sqrt[x]] - Log[1 + x])/3
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln(1+x)}{3}$	20
default	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln(1+x)}{3}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Maxima [A]
time = 0.25, size = 19, normalized size = 0.66

$$\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{3}\,x-\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Fricas [A]
time = 2.57, size = 19, normalized size = 0.66

$$\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{3}\,x-\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="fricas")
[Out] 2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Sympy [A]
time = 0.87, size = 24, normalized size = 0.83

$$\frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3}+\frac{x}{3}-\frac{\log\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*acot(x**(1/2)),x)
```

```
[Out] 2*x**(3/2)*acot(sqrt(x))/3 + x/3 - log(x + 1)/3
```

Giac [A]
time = 0.40, size = 30, normalized size = 1.03

$$\frac{2}{3}x^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{3}x\left(\frac{1}{x}-1\right)-\frac{1}{3}\log\left(x\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(1/sqrt(x)) - 1/3*x*(1/x - 1) - 1/3*log(x) - 1/3*log(1/x
+ 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{x}\,\operatorname{acot}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*acot(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*acot(x^(1/2)), x)
```

3.93 $\int \frac{\cot^{-1}(\sqrt{x})}{\sqrt{x}} dx$

Optimal. Leaf size=18

$$2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\log(1+x)$$

```
[Out] ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4947, 31}

$$\log(x+1)+2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.83

method	result	size
derivativedivides	$\ln(1+x)+2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)$	15
default	$\ln(1+x)+2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+x)+2*x^(1/2)*arccot(x^(1/2))
```

Maxima [A]
time = 0.26, size = 14, normalized size = 0.78

$$2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Fricas [A]
time = 2.36, size = 14, normalized size = 0.78

$$2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Sympy [A]
time = 0.14, size = 17, normalized size = 0.94

$$2\sqrt{x}\,\operatorname{acot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*acot(sqrt(x)) + log(x + 1)
```

Giac [**A**]
time = 0.39, size = 18, normalized size = 1.00

$$2\sqrt{x}\arctan\left(\frac{1}{\sqrt{x}}\right)+\log\left(x\right)+\log\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arctan(1/sqrt(x)) + log(x) + log(1/x + 1)
```

Mupad [**B**]
time = 0.77, size = 14, normalized size = 0.78

$$\ln\left(x+1\right)+2\sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x^(1/2),x)
```

```
[Out] log(x + 1) + 2*x^(1/2)*acot(x^(1/2))
```

3.94 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}} dx$

Optimal. Leaf size=22

$$-\frac{2\cot^{-1}(\sqrt{x})}{\sqrt{x}} - \log(x) + \log(1+x)$$

```
[Out] -ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4947, 36, 29, 31}

$$-\log(x) + \log(x+1) - \frac{2\cot^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m + 1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \int \frac{1}{x(1+x)}\,dx \\
&= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \int \frac{1}{x}\,dx + \int \frac{1}{1+x}\,dx \\
&= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$-\ln(x)+\ln(1+x)-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}$	19
default	$-\ln(x)+\ln(1+x)-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(1+x)-2*arccot(x^(1/2))/x^(1/2)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.82

$$-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}} + \log(x+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arccot(sqrt(x))/sqrt(x) + log(x + 1) - log(x)
```

Fricas [A]
time = 3.15, size = 25, normalized size = 1.14

$$\frac{x\log\left(x+1\right)-x\log\left(x\right)-2\sqrt{x}\ \mathrm{arccot}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] (x*log(x + 1) - x*log(x) - 2*sqrt(x)*arccot(sqrt(x)))/x
```

Sympy [A]
time = 0.40, size = 20, normalized size = 0.91

$$-\log\left(x\right)+\log\left(x+1\right)-\frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x**(3/2),x)
[Out] -log(x) + log(x + 1) - 2*acot(sqrt(x))/sqrt(x)
```

Giac [A]
time = 0.40, size = 16, normalized size = 0.73

$$-\frac{2\ \arctan\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}}+\log\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -2*arctan(1/sqrt(x))/sqrt(x) + log(1/x + 1)
```

Mupad [B]
time = 0.67, size = 20, normalized size = 0.91

$$\ln\left(x+1\right)-2\ \ln\left(\sqrt{x}\right)-\frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x^(3/2),x)
[Out] log(x + 1) - 2*log(x^(1/2)) - (2*acot(x^(1/2)))/x^(1/2)
```

3.95 $\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3x}-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{\log(x)}{3}-\frac{1}{3}\log(1+x)$$

[Out] 1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{4947, 46\}$

$$-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3x}+\frac{\log(x)}{3}-\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[ArcCot[Sqrt[x]]/x^(5/2),x]

[Out] 1/(3*x) - (2*ArcCot[Sqrt[x]])/(3*x^(3/2)) + Log[x]/3 - Log[1 + x]/3

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx &= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3}\int \frac{1}{x^2(1+x)}\,dx\\
&= -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3}\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx\\
&= \frac{1}{3x}-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{\log(x)}{3}-\frac{1}{3}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.78

$$\frac{1}{3}\left(\frac{1}{x}-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\log(x)-\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Sqrt[x]]/x^(5/2),x]
[Out] (x^(-1) - (2*ArcCot[Sqrt[x]])/x^(3/2) + Log[x] - Log[1 + x])/3
```

Maple [A]
time = 0.01, size = 26, normalized size = 0.70

method	result	size
derivativedivides	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln(x)}{3}-\frac{\ln(1+x)}{3}$	26
default	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln(x)}{3}-\frac{\ln(1+x)}{3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.68

$$-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}}+\frac{1}{3\,x}-\frac{1}{3}\,\log\left(x+1\right)+\frac{1}{3}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] -2/3*arccot(sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(x + 1) + 1/3*log(x)
```

Fricas [A]
time = 2.70, size = 33, normalized size = 0.89

$$-\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)+2\,\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right)-x}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^2*log(x + 1) - x^2*log(x) + 2*sqrt(x)*arccot(sqrt(x)) - x)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.
time = 1.30, size = 143, normalized size = 3.86

$$-\frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3x^3+3x^2}-\frac{2\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{3x^3+3x^2}+\frac{x^3\log\left(x\right)}{3x^3+3x^2}-\frac{x^3\log\left(x+1\right)}{3x^3+3x^2}+\frac{x^2\log\left(x\right)}{3x^3+3x^2}-\frac{x^2\log\left(x+1\right)}{3x^3+3x^2}+\frac{x^2}{3x^3+3x^2}+\frac{x}{3x^3+3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x**(1/2))/x**(5/2),x)
```

```
[Out] -2*x**(3/2)*acot(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*acot(sqrt(x))/(3*x*
*3 + 3*x**2) + x**3*log(x)/(3*x**3 + 3*x**2) - x**3*log(x + 1)/(3*x**3 + 3*
x**2) + x**2*log(x)/(3*x**3 + 3*x**2) - x**2*log(x + 1)/(3*x**3 + 3*x**2) +
 x**2/(3*x**3 + 3*x**2) + x/(3*x**3 + 3*x**2)
```

Giac [A]
time = 0.43, size = 23, normalized size = 0.62

$$-\frac{2\arctan\left(\frac{1}{\sqrt{x}}\right)}{3\,x^{\frac{3}{2}}}+\frac{1}{3\,x}-\frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(1/sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(1/x + 1)
```

Mupad [B]
time = 0.65, size = 27, normalized size = 0.73

$$\frac{2\ln\left(\sqrt{x}\right)}{3}-\frac{\ln\left(x+1\right)}{3}-\frac{2\operatorname{acot}\left(\sqrt{x}\right)}{3\,x^{3/2}}+\frac{1}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x^(1/2))/x^(5/2),x)
```

```
[Out] (2*log(x^(1/2)))/3 - log(x + 1)/3 - (2*acot(x^(1/2)))/(3*x^(3/2)) + 1/(3*x)
```

3.96 $\int \cot^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=17

$$x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(1+x^2\right)$$

[Out] x*arccot(1/x)-1/2*ln(x^2+1)

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4931, 269, 266}

$$x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCot[x^(-1)],x]

[Out] x*ArcCot[x^(-1)] - Log[1 + x^2]/2

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}\left(\frac{1}{x}\right) dx &= x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{1}{\left(1+\frac{1}{x^2}\right)x}\,dx\\
&= x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{x}{1+x^2}\,dx\\
&= x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x^(-1)],x]
[Out] x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

Maple [A]
time = 0.06, size = 20, normalized size = 1.18

method	result
derivativedivides	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^2}+1\right)}{2}+\ln\left(\frac{1}{x}\right)$
default	$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^2}+1\right)}{2}+\ln\left(\frac{1}{x}\right)$
risch	$\frac{ix\ln(i+x)}{2}-\frac{i\ln(x-i)x}{2}-\frac{\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(i\left(x-\mathrm{RootOf}\left(\_Z^2+1,index=1\right)\right)\right)\mathrm{csgn}\left(\frac{i\left(x-\mathrm{RootOf}\left(\_Z^2+1,index=1\right.\right.}{x}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(1/x),x,method=_RETURNVERBOSE)
[Out] x*arccot(1/x)-1/2*ln(1/x^2+1)+ln(1/x)
```

Maxima [A]
time = 0.25, size = 15, normalized size = 0.88

$$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1/x),x, algorithm="maxima")
[Out] x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Fricas [A]
time = 3.34, size = 15, normalized size = 0.88

$$x\,\mathrm{arccot}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1/x),x, algorithm="fricas")
```

```
[Out] x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.07, size = 14, normalized size = 0.82

$$x \operatorname{acot}\left(\frac{1}{x}\right) - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(1/x),x)
```

```
[Out] x*acot(1/x) - log(x**2 + 1)/2
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.76

$$x \arctan\left(x\right) - \frac{1}{2} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1/x),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.88

$$x \operatorname{acot}\left(\frac{1}{x}\right) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(1/x),x)
```

```
[Out] x*acot(1/x) - log(x^2 + 1)/2
```

3.97 $\int \frac{\cot^{-1}(ax^n)}{x}\,dx$

Optimal. Leaf size=47

$$-\frac{i\text{PolyLog}\left(2,-\frac{ix^{-n}}{a}\right)}{2n}+\frac{i\text{PolyLog}\left(2,\frac{ix^{-n}}{a}\right)}{2n}$$

```
[Out] -1/2*I*polylog(2,-I/a/(x^n))/n+1/2*I*polylog(2,I/a/(x^n))/n
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4945, 4941, 2438}

$$\frac{i\text{Li}_2\left(\frac{ix^{-n}}{a}\right)}{2n}-\frac{i\text{Li}_2\left(-\frac{ix^{-n}}{a}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^n]/x,x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a*x^n)])/n + ((I/2)*PolyLog[2, I/(a*x^n)])/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^n\right)}{2n} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^n\right)}{2n}$$
$$= -\frac{i\text{Li}_2\left(-\frac{ix^{-n}}{a}\right)}{2n} + \frac{i\text{Li}_2\left(\frac{ix^{-n}}{a}\right)}{2n}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.85

$$-\frac{i\left(\text{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right) - \text{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^n]/x,x]
```

```
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a*x^n)] - PolyLog[2, I/(a*x^n)]))/n
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.
time = 0.15, size = 83, normalized size = 1.77

method	result
derivativedivides	$\frac{\ln(a\,x^n)\operatorname{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ix^n a)}{2}+\frac{i\ln(a\,x^n)\ln(1-ix^n a)}{2}-\frac{i\,\mathrm{dilog}(1+ix^n a)}{2}+\frac{i\,\mathrm{dilog}(1-ix^n a)}{2}}{n}$
default	$\frac{\ln(a\,x^n)\operatorname{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ix^n a)}{2}+\frac{i\ln(a\,x^n)\ln(1-ix^n a)}{2}-\frac{i\,\mathrm{dilog}(1+ix^n a)}{2}+\frac{i\,\mathrm{dilog}(1-ix^n a)}{2}}{n}$
risch	$\frac{i\ln(x)\ln(1+ix^n a)}{2}+\frac{\pi\ln(x)}{2}+\frac{i\,\mathrm{dilog}(1-ix^n a)}{2n}-\frac{i\ln(-i(-a\,x^n+i))\ln(x)}{2}+\frac{i\ln(-i(-a\,x^n+i))\ln(-ix^n a)}{2n}+ i\,\mathrm{dil}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(a*x^n)*arccot(a*x^n)-1/2*I*ln(a*x^n)*ln(1+I*x^n*a)+1/2*I*ln(a*x^n)*
ln(1-I*x^n*a)-1/2*I*dilog(1+I*x^n*a)+1/2*I*dilog(1-I*x^n*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(1/(a*x^n))*log(x)
```

Fricas [A]
time = 2.55, size = 63, normalized size = 1.34

$$\frac{2\,n\,\mathrm{arccot}\left(ax^n\right)\log\left(x\right)-i\,n\log\left(i\,ax^n+1\right)\log\left(x\right)+i\,n\log\left(-i\,ax^n+1\right)\log\left(x\right)+i\,\mathrm{Li}_2(i\,ax^n)-i\,\mathrm{Li}_2(-i\,ax^n)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*n*arccot(a*x^n)*log(x) - I*n*log(I*a*x^n + 1)*log(x) + I*n*log(-I*a*
x^n + 1)*log(x) + I*dilog(I*a*x^n) - I*dilog(-I*a*x^n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{acot}\left(ax^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**n)/x,x)
```

```
[Out] Integral(acot(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acot}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^n)/x,x)
```

```
[Out] int(acot(a*x^n)/x, x)
```

3.98 $\int \frac{\cot^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=37

$$-\frac{1}{10}i\text{PolyLog}\left(2,-\frac{i}{ax^5}\right)+\frac{1}{10}i\text{PolyLog}\left(2,\frac{i}{ax^5}\right)$$

```
[Out] -1/10*I*polylog(2,-I/a/x^5)+1/10*I*polylog(2,I/a/x^5)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4945, 4941, 2438}

$$\frac{1}{10}i\text{Li}_2\left(\frac{i}{ax^5}\right)-\frac{1}{10}i\text{Li}_2\left(-\frac{i}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a*x^5]/x,x]
```

```
[Out] (-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \frac{1}{5}\text{Subst}\left(\int \frac{\cot^{-1}(ax)}{x}\,dx, x, x^5\right)$$
$$= \frac{1}{10}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{ax}\right)}{x}\,dx, x, x^5\right) - \frac{1}{10}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{ax}\right)}{x}\,dx, x, x^5\right)$$
$$= -\frac{1}{10}i\text{Li}_2\left(-\frac{i}{ax^5}\right) + \frac{1}{10}i\text{Li}_2\left(\frac{i}{ax^5}\right)$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{1}{10}i\text{PolyLog}\left(2, -\frac{i}{ax^5}\right) + \frac{1}{10}i\text{PolyLog}\left(2, \frac{i}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a*x^5]/x,x]
```

```
[Out] (-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 57, normalized size = 1.54

method	result
default	$\ln(x)\,\text{arccot}(a\,x^5) + \frac{\sum_{\_R1=\text{RootOf}(a^2\_Z^{10}+1)} \frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
risch	$\frac{\pi\ln(x)}{2} + \frac{i\left(\sum_{\_R1=\text{RootOf}(a\_Z^5+\text{RootOf}(\_Z^2+1,index=1))}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2} - \frac{i\ln(x)\ln(-ia\,x^5+1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arccot(a*x^5)+1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(23) = 46.
time = 0.52, size = 68, normalized size = 1.84

$$\frac{1}{20}\pi\log\left(a^2x^{10}+1\right) - \frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right) + \text{arccot}\left(ax^5\right)\log\left(x\right) + \arctan\left(ax^5\right)\log\left(x\right) + \frac{1}{10}i\,\text{Li}_2\left(i\,ax^5+1\right) - \frac{1}{10}i\,\text{Li}_2\left(-i\,ax^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] 1/20*pi*log(a^2*x^10 + 1) - 1/5*arctan(a*x^5)*log(a*x^5) + arccot(a*x^5)*lo
g(x) + arctan(a*x^5)*log(x) + 1/10*I*dilog(I*a*x^5 + 1) - 1/10*I*dilog(-I*a
*x^5 + 1)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(a*x^5)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(ax^{5}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a*x**5)/x,x)
```

```
[Out] Integral(acot(a*x**5)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(a*x^5)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acot}(a\,x^5)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a*x^5)/x,x)
```

```
[Out] int(acot(a*x^5)/x, x)
```

3.99 $\int x^3 \cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$-\frac{(1-6a^2)\,x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{(1-6a^2+a^4)\,\mathrm{ArcTan}(a+bx)}{4b^4}+\frac{a(1-a^2)\log(1$$

```
[Out] -1/4*(-6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arccot
(b*x+a)+1/4*(a^4-6*a^2+1)*arctan(b*x+a)/b^4+1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)/
b^4
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\frac{a(1-a^2)\log((a+bx)^2+1)}{2b^4}-\frac{(1-6a^2)\,x}{4b^3}+\frac{(a^4-6a^2+1)\,\mathrm{ArcTan}(a+bx)}{4b^4}+\frac{(a+bx)^3}{12b^4}-\frac{a(a+bx)^2}{2b^4}+\frac{1}{4}x^4\cot^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a + b*x],x]
```

```
[Out] -1/4*((1 - 6*a^2)*x)/b^3 - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) +
 (x^4*ArcCot[a + b*x])/4 + ((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (a
*(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cot^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx, x, a+bx\right)\\
&= \frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{1-6a^2}{b^4}-\frac{4ax}{b^4}+\frac{x^2}{b^4}+\frac{1-6a^2+a^4+4a(1-}{b^4\left(1+x^2\right)}\right.\right.\\
&= -\frac{\left(1-6a^2\right)x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{\text{Subst}\left(\int \frac{1-6a^2+a^4+4}{1+x}\right.}{}\\
&= -\frac{\left(1-6a^2\right)x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{\left(a\left(1-a^2\right)\right)\text{Subst}\left(\right.}{b}\\
&= -\frac{\left(1-6a^2\right)x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\cot^{-1}(a+bx)+\frac{\left(1-6a^2+a^4\right)\tan^{-}}{4b^4}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 95, normalized size = 0.90

$$\frac{6(-1+6a^2)\,bx-12a(a+bx)^2+2(a+bx)^3+6b^4x^4\cot^{-1}(a+bx)-3i(-i+a)^4\log(i-a-bx)+3i(i+a)^4\log(i+a+bx)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a + b*x],x]
```

```
[Out] (6*(-1 + 6*a^2)*b*x - 12*a*(a + b*x)^2 + 2*(a + b*x)^3 + 6*b^4*x^4*ArcCot[a
 + b*x] - (3*I)*(-I + a)^4*Log[I - a - b*x] + (3*I)*(I + a)^4*Log[I + a + b
*x])/(24*b^4)
```

Maple [A]
time = 0.08, size = 157, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(b}{2}}{b^4}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)a^4}{4}-\operatorname{arccot}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccot}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccot}(bx+a)a(bx+a)^3+\frac{\operatorname{arccot}(bx+a)(bx+a)^4}{4}+\frac{3a^2(b}{2}}{b^4}$
risch	$\frac{ix^4\ln(1+i(bx+a))}{8}-\frac{ix^4\ln(1-i(bx+a))}{8}+\frac{\pi x^4}{8}+\frac{x^3}{12b}+\frac{a^4\arctan(bx+a)}{4b^4}-\frac{a x^2}{4b^2}-\frac{a^3\ln(b^2x^2+2abx+a^2+1)}{2b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arccot(b*x+a)*a^4-arccot(b*x+a)*a^3*(b*x+a)+3/2*arccot(b*x+a)*a^
2*(b*x+a)^2-arccot(b*x+a)*a*(b*x+a)^3+1/4*arccot(b*x+a)*(b*x+a)^4+3/2*a^2*(
b*x+a)-1/2*(b*x+a)^2*a+1/12*(b*x+a)^3-1/4*b*x-1/4*a+1/8*(-4*a^3+4*a)*ln(1+(
b*x+a)^2)+1/4*(a^4-6*a^2+1)*arctan(b*x+a))
```

Maxima [A]
time = 0.48, size = 104, normalized size = 0.98

$$\frac{1}{4}x^4\operatorname{arccot}(bx+a)+\frac{1}{12}b\left(\frac{b^2x^3-3abx^2+3(3a^2-1)x}{b^4}+\frac{3(a^4-6a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5}-\frac{6(a^3-a)\log(b^2x^2+2abx+a^2+1)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccot(b*x + a) + 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Fricas [A]
time = 2.56, size = 92, normalized size = 0.87

$$\frac{3b^4x^4\operatorname{arccot}(bx+a)+b^3x^3-3ab^2x^2+3(3a^2-1)bx+3(a^4-6a^2+1)\arctan(bx+a)-6(a^3-a)\log(b^2x^2+2abx+a^2+1)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4*arccot(b*x + a) + b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x
 + 3*(a^4 - 6*a^2 + 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x
+ a^2 + 1))/b^4
```

Sympy [A]
time = 0.75, size = 155, normalized size = 1.46

$$\begin{cases} -\frac{a^4 \operatorname{acot}(a+bx)}{4b^4} - \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{acot}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} + \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{acot}(a+bx)}{4} + \frac{x^3}{12b} - \frac{x}{4b^3} - \frac{\operatorname{acot}(a+bx)}{4b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acot}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acot(a + b*x)/(4*b**4) - a**3*log(a**2 + 2*a*b*x + b**2*x**
*2 + 1)/(2*b**4) + 3*a**2*x/(4*b**3) + 3*a**2*acot(a + b*x)/(2*b**4) - a*x*
*2/(4*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*acot(a
+ b*x)/4 + x**3/(12*b) - x/(4*b**3) - acot(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*acot(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(92) = 184.
time = 0.79, size = 617, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/192*(96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 72*a^2*a
rctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a*arctan(1/(b*x + a)
)*tan(1/2*arctan(1/(b*x + a)))^7 + 3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(
b*x + a)))^8 + 96*a^3*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan
(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/
(b*x + a)))^4 - 96*a^3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 +
 144*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 144*a^2*tan(1
/2*arctan(1/(b*x + a)))^5 - 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x
+ a)))^5 - 24*a*tan(1/2*arctan(1/(b*x + a)))^6 - 12*arctan(1/(b*x + a))*tan
(1/2*arctan(1/(b*x + a)))^6 - 2*tan(1/2*arctan(1/(b*x + a)))^7 - 96*a*log(1
6*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/
2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 72*a^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 144*a^2*tan(1/2*arctan(1/(
b*x + a)))^3 + 72*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 48
*a*tan(1/2*arctan(1/(b*x + a)))^4 - 30*arctan(1/(b*x + a))*tan(1/2*arctan(1
/(b*x + a)))^4 + 30*tan(1/2*arctan(1/(b*x + a)))^5 - 24*a*arctan(1/(b*x + a
))*tan(1/2*arctan(1/(b*x + a))) - 24*a*tan(1/2*arctan(1/(b*x + a)))^2 - 12*
arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 30*tan(1/2*arctan(1/(b
*x + a)))^3 + 3*arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^4*
tan(1/2*arctan(1/(b*x + a)))^4)
```

Mupad [B]
time = 0.77, size = 133, normalized size = 1.25

$$\frac{\operatorname{atan}(a+b\,x)}{4\,b^4}+\frac{x^4\,\operatorname{acot}(a+b\,x)}{4}-\frac{x}{4\,b^3}+\frac{x^3}{12\,b}-\frac{a^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}-\frac{3\,a^2\,\operatorname{atan}(a+b\,x)}{2\,b^4}+\frac{a^4\,\operatorname{atan}(a+b\,x)}{4\,b^4}-\frac{a\,x^2}{4\,b^2}+\frac{3\,a^2\,x}{4\,b^3}+\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a + b*x),x)
```

```
[Out] atan(a + b*x)/(4*b^4) + (x^4*acot(a + b*x))/4 - x/(4*b^3) + x^3/(12*b) - (a
^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) - (3*a^2*atan(a + b*x))/(2*b^4
) + (a^4*atan(a + b*x))/(4*b^4) - (a*x^2)/(4*b^2) + (3*a^2*x)/(4*b^3) + (a*
log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.100 $\int x^2 \cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{a(3-a^2)\,\mathrm{ArcTan}(a+bx)}{3b^3}-\frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

```
[Out] -a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccot(b*x+a)+1/3*a*(-a^2+3)*arctan(b*x+
a)/b^3-1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\frac{a(3-a^2)\,\mathrm{ArcTan}(a+bx)}{3b^3}-\frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3}+\frac{(a+bx)^2}{6b^3}-\frac{ax}{b^2}+\frac{1}{3}x^3\cot^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a + b*x],x]
```

```
[Out] -((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*ArcCot[a + b*x])/3 + (a*(3 - a^2)
*ArcTan[a + b*x])/(3*b^3) - ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\cot^{-1}(a+bx)\,dx &= \frac{\mathrm{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cot^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{1}{3}\mathrm{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx,x,a+bx\right)\\
&= \frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{1}{3}\mathrm{Subst}\left(\int\left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a(3-a^2)-(1-3a^2)\,x}{b^3\,(1+x^2)}\right)\,dx,x,a\right.\\
&= -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{\mathrm{Subst}\left(\int\frac{a(3-a^2)-(1-3a^2)x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}\\
&= -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)-\frac{(1-3a^2)\,\mathrm{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}+\\
&= -\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\cot^{-1}(a+bx)+\frac{a(3-a^2)\tan^{-1}(a+bx)}{3b^3}-\frac{(1-3a^2)\log}{6}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 114, normalized size = 1.42

$$\frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\cot^{-1}(a+bx)+\frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcCot[a + b*x])/3 + (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

Maple [A]
time = 0.07, size = 114, normalized size = 1.42

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+1)}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)a^3}{3}+\operatorname{arccot}(bx+a)a^2(bx+a)-\operatorname{arccot}(bx+a)a(bx+a)^2+\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(-3a^2+1)}{}}{b^3}$
risch	$\frac{ix^3\ln(1+i(bx+a))}{6}-\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{\pi x^3}{6}-\frac{a^3\arctan(bx+a)}{3b^3}+\frac{x^2}{6b}+\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}-\frac{2ax}{3b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arccot(b*x+a)*a^3+arccot(b*x+a)*a^2*(b*x+a)-arccot(b*x+a)*a*(b*
x+a)^2+1/3*arccot(b*x+a)*(b*x+a)^3-(b*x+a)*a+1/6*(b*x+a)^2-1/6*(-3*a^2+1)*l
n(1+(b*x+a)^2)-1/3*(a^3-3*a)*arctan(b*x+a))
```

Maxima [A]
time = 0.47, size = 85, normalized size = 1.06

$$\frac{1}{3}x^3\operatorname{arccot}(bx+a)+\frac{1}{6}b\left(\frac{bx^2-4ax}{b^3}-\frac{2(a^3-3a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4}+\frac{(3a^2-1)\log(b^2x^2+2abx+a^2+1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Fricas [A]
time = 1.45, size = 73, normalized size = 0.91

$$\frac{2b^3x^3\operatorname{arccot}(bx+a)+b^2x^2-4abx-2(a^3-3a)\arctan(bx+a)+(3a^2-1)\log(b^2x^2+2abx+a^2+1)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arccot(b*x + a) + b^2*x^2 - 4*a*b*x - 2*(a^3 - 3*a)*arctan(b
*x + a) + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A]
time = 0.53, size = 117, normalized size = 1.46

$$\begin{cases}\frac{a^3\operatorname{acot}(a+bx)}{3b^3}+\frac{a^2\log(a^2+2abx+b^2x^2+1)}{2b^3}-\frac{2ax}{3b^2}-\frac{a\operatorname{acot}(a+bx)}{b^3}+\frac{x^3\operatorname{acot}(a+bx)}{3}+\frac{x^2}{6b}-\frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b\neq 0\\ \frac{x^3\operatorname{acot}(a)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(b*x+a),x)
```

```
[Out] Piecewise((a**3*acot(a + b*x)/(3*b**3) + a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) - 2*a*x/(3*b**2) - a*acot(a + b*x)/b**3 + x**3*acot(a + b*x
)/3 + x**2/(6*b) - log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*acot(a)/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(70) = 140$.
time = 0.70, size = 423, normalized size = 5.29

$$-\frac{12\,a^2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+6\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^5+\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^6+12\,a^2\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3-12\,a^2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+12\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3-12\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4-3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4-\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^5-4\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+6\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)+12\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3-\arctan\left(\frac{1}{bx+a}\right)-\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{24\,b^3\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*(12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 6*a*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + arctan(1/(b*x + a))*tan(1/
2*arctan(1/(b*x + a)))^6 + 12*a^2*log(16*tan(1/2*arctan(1/(b*x + a)))^2/(ta
n(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1
/2*arctan(1/(b*x + a)))^3 - 12*a^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*
x + a)))^2 + 12*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 12*a
*tan(1/2*arctan(1/(b*x + a)))^4 - 3*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b
*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*log(16*tan(1/2*arctan(1/(b
*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a))
)^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 6*a*arctan(1/(b*x + a))*tan(1/2*
arctan(1/(b*x + a))) + 12*a*tan(1/2*arctan(1/(b*x + a)))^2 + 3*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 -
 arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b^3*tan(1/2*arctan(1/
(b*x + a)))^3)
```

Mupad [B]
time = 1.34, size = 101, normalized size = 1.26

$$\frac{x^3\,\mathrm{acot}(a+b\,x)}{3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3}+\frac{x^2}{6\,b}+\frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3}-\frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3}+\frac{a\,\mathrm{atan}(a+b\,x)}{b^3}-\frac{2\,a\,x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a + b*x),x)
```

```
[Out] (x^3*acot(a + b*x))/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + x^2/(6*b
) + (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) - (a^3*atan(a + b*x))/(3
*b^3) + (a*atan(a + b*x))/b^3 - (2*a*x)/(3*b^2)
```

3.101 $\int x \cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{(1-a^2)\,\text{ArcTan}(a+bx)}{2b^2}-\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*x/b+1/2*x^2*arccot(b*x+a)-1/2*(-a^2+1)*arctan(b*x+a)/b^2-1/2*a*ln(1+(b*
x+a)^2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5156, 4973, 716, 649, 209, 266}

$$-\frac{(1-a^2)\,\text{ArcTan}(a+bx)}{2b^2}-\frac{a\log\left((a+bx)^2+1\right)}{2b^2}+\frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a + b*x],x]
```

```
[Out] x/(2*b) + (x^2*ArcCot[a + b*x])/2 - ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) - (
a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\cot^{-1}(x)\,dx, x, a+bx\right)}{b}\\
&= \frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx, x, a+bx\right)\\
&= \frac{1}{2}x^2\cot^{-1}(a+bx)+\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{1-a^2+2ax}{b^2\left(1+x^2\right)}\right)\,dx, x, a+bx\right)\\
&= \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{\operatorname{Subst}\left(\int \frac{1-a^2+2ax}{1+x^2}\,dx, x, a+bx\right)}{2b^2}\\
&= \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{a\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b^2}-\frac{\left(1-a^2\right)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{2b^2}\\
&= \frac{x}{2b}+\frac{1}{2}x^2\cot^{-1}(a+bx)-\frac{\left(1-a^2\right)\tan^{-1}(a+bx)}{2b^2}-\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 90, normalized size = 1.50

$$
\frac{2bx+2b^2x^2\cot^{-1}(a+bx)-i(-i+a)^2\log(i-a-bx)-i\log(i+a+bx)-2a\log(i+a+bx)+ia^2\log(i+a+bx)}{4b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a + b*x],x]
```

```
[Out] (2*b*x + 2*b^2*x^2*ArcCot[a + b*x] - I*(-I + a)^2*Log[I - a - b*x] - I*Log[
I + a + b*x] - 2*a*Log[I + a + b*x] + I*a^2*Log[I + a + b*x])/(4*b^2)
```

Maple [A]
time = 0.06, size = 63, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)(bx+a)a+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}-\operatorname{arccot}(bx+a)(bx+a)a+\frac{bx}{2}+\frac{a}{2}-\frac{a\ln\left(1+(bx+a)^2\right)}{2}-\frac{\arctan(bx+a)}{2}}{b^2}$
risch	$\frac{ix^2\ln(1+i(bx+a))}{4}-\frac{ix^2\ln(1-i(bx+a))}{4}+\frac{\pi x^2}{4}+\frac{a^2\arctan(bx+a)}{2b^2}-\frac{a\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{x}{2b}-\frac{\arctan(b}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*(b*x+a)^2*arccot(b*x+a)-arccot(b*x+a)*(b*x+a)*a+1/2*b*x+1/2*a-1/
2*a*ln(1+(b*x+a)^2)-1/2*arctan(b*x+a))
```

Maxima [A]
time = 0.46, size = 68, normalized size = 1.13

$$\frac{1}{2}\,x^2\operatorname{arccot}(bx+a)+\frac{1}{2}\,b\left(\frac{x}{b^2}+\frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}-\frac{a\log(b^2x^2+2\,abx+a^2+1)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccot(b*x + a) + 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Fricas [A]
time = 2.19, size = 55, normalized size = 0.92

$$\frac{b^2x^2\operatorname{arccot}(bx+a)+bx+(a^2-1)\arctan(bx+a)-a\log(b^2x^2+2\,abx+a^2+1)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2*arccot(b*x + a) + b*x + (a^2 - 1)*arctan(b*x + a) - a*log(b^2*
x^2 + 2*a*b*x + a^2 + 1))/b^2
```

Sympy [A]
time = 0.33, size = 78, normalized size = 1.30

$$\begin{cases} -\frac{a^2\operatorname{acot}(a+bx)}{2b^2}-\frac{a\log(a^2+2abx+b^2x^2+1)}{2b^2}+\frac{x^2\operatorname{acot}(a+bx)}{2}+\frac{x}{2b}+\frac{\operatorname{acot}(a+bx)}{2b^2} & \text{for}\ b\neq 0\\ \frac{x^2\operatorname{acot}(a)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acot(a + b*x)/(2*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*acot(a + b*x)/2 + x/(2*b) + acot(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*acot(a)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(50) = 100$.
time = 0.48, size = 210, normalized size = 3.50

$$\frac{4\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+4\,a\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-4\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)+2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3+\arctan\left(\frac{1}{bx+a}\right)+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{8\,b^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a*log(16*tan(1/2*arctan(1/(b*x +
a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 +
 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 4*a*arctan(1/(b*x + a))*tan(1/2*arcta
n(1/(b*x + a))) + 2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*
tan(1/2*arctan(1/(b*x + a)))^3 + arctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(
b*x + a))))/(b^2*tan(1/2*arctan(1/(b*x + a)))^2)
```

Mupad [B]
time = 0.97, size = 61, normalized size = 1.02

$$\frac{x^2\,\mathrm{acot}(a+b\,x)}{2}+\frac{\frac{\mathrm{acot}(a+b\,x)}{2}+\frac{b\,x}{2}-\frac{a^2\,\mathrm{acot}(a+b\,x)}{2}-\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(a + b*x),x)
```

```
[Out] (x^2*acot(a + b*x))/2 + (acot(a + b*x)/2 + (b*x)/2 - (a^2*acot(a + b*x))/2
- (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.102 $\int \cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{(a+bx)\cot^{-1}(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] (b*x+a)*arccot(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5148, 4931, 266}

$$\frac{\log\left((a+bx)^2+1\right)}{2b}+\frac{(a+bx)\cot^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcCot[a + b*x])/b + Log[1 + (a + b*x)^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cot^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cot^{-1}(a+bx)}{b} + \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.33

$$x\cot^{-1}(a+bx) + \frac{-2a\text{ArcTan}(a+bx) + \log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x],x]
```

```
[Out] x*ArcCot[a + b*x] + (-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2
])/(2*b)
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\operatorname{arccot}(bx+a)+\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
risch	$\frac{ix\ln(1+i(bx+a))}{2} - \frac{ix\ln(1-i(bx+a))}{2} + \frac{\pi x}{2} - \frac{a\arctan(bx+a)}{b} + \frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arccot(b*x+a)+1/2*ln(1+(b*x+a)^2))
```

Maxima [A]
time = 0.26, size = 29, normalized size = 0.88

$$\frac{2\,(bx+a)\operatorname{arccot}(bx+a) + \log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arccot(b*x + a) + log((b*x + a)^2 + 1))/b
```

Fricas [A]
time = 6.18, size = 43, normalized size = 1.30

$$\frac{2\,bx\,\mathrm{arccot}\left(bx+a\right)-2\,a\arctan\left(bx+a\right)+\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arccot(b*x + a) - 2*a*arctan(b*x + a) + log(b^2*x^2 + 2*a*b*x +
a^2 + 1))/b
```

Sympy [A]
time = 0.21, size = 46, normalized size = 1.39

$$\begin{cases} \frac{a\,\mathrm{acot}\left(a+bx\right)}{b}+x\,\mathrm{acot}\left(a+bx\right)+\frac{\log\left(a^2+2abx+b^2x^2+1\right)}{2b} & \text{for}\; b\neq 0 \\ x\,\mathrm{acot}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a),x)
```

```
[Out] Piecewise((a*acot(a + b*x)/b + x*acot(a + b*x) + log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*acot(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.
time = 0.44, size = 111, normalized size = 3.36

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)-\arctan\left(\frac{1}{bx+a}\right)}{2\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + log(16*tan(1/2*a
rctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - arctan(1/(b*x + a)))/(b*
tan(1/2*arctan(1/(b*x + a))))
```

Mupad [B]
time = 1.20, size = 42, normalized size = 1.27

$$\frac{\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}+a\,\mathrm{acot}\left(a+b\,x\right)}{b}+x\,\mathrm{acot}\left(a+b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x),x)
[Out] (log(a^2 + b^2*x^2 + 2*a*b*x + 1)/2 + a*acot(a + b*x))/b + x*acot(a + b*x)
```

3.103 $\int \frac{\cot^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=120

$$-\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a$$

```
[Out] -arccot(b*x+a)*ln(2/(1-I*(b*x+a)))+arccot(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+a
)))-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))+1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I*(
b*x+a)))
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5156, 4967, 2449, 2352, 2497}

$$-\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)+\frac{1}{2}i\text{Li}_2\left(1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\log\left(\frac{2}{1-i(a+bx)}\right)\left(-\cot^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)\cot^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/x,x]
```

```
[Out] -(ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcCot[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] + (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
```

```
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

$$= -\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\cot^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 251 vs. $2(120) = 240$.
time = 0.02, size = 251, normalized size = 2.09

$$-\frac{1}{2}i\log\left(\frac{-i+a+bx}{\left(-\frac{i}{b}+\frac{a}{b}\right)b}\right)\log\left(-\frac{a}{b}+\frac{a+bx}{b}\right)+\frac{1}{2}i\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(-\frac{a}{b}+\frac{a+bx}{b}\right)+\frac{1}{2}i\log\left(\frac{i+a+bx}{\left(\frac{i}{b}+\frac{a}{b}\right)b}\right)\log\left(-\frac{a}{b}+\frac{a+bx}{b}\right)-\frac{1}{2}i\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{a}{b}+\frac{a+bx}{b}\right)-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-i+a}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-\frac{b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{i+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*Log[(-I + a + b*x)/(((-I)/b + a/b)*b)]*Log[-(a/b) + (a + b*x)/b] +
 (I/2)*Log[(-I + a + b*x)/(a + b*x)]*Log[-(a/b) + (a + b*x)/b] + (I/2)*Log[
(I + a + b*x)/((I/b + a/b)*b)]*Log[-(a/b) + (a + b*x)/b] - (I/2)*Log[(I + a
 + b*x)/(a + b*x)]*Log[-(a/b) + (a + b*x)/b] - (I/2)*PolyLog[2, -((b*(-(a/b
) + (a + b*x)/b))/(-I + a))] + (I/2)*PolyLog[2, -((b*(-(a/b) + (a + b*x)/b)
)/(I + a))]
```

Maple [A]
time = 0.06, size = 106, normalized size = 0.88

method	result
risch	$\frac{\pi \ln(-ibx)}{2} - \frac{i \ln\left(-\frac{ixb}{ia-1}\right) \ln(-ibx-ia+1)}{2} - \frac{i\,\mathrm{dilog}\left(-\frac{ixb}{ia-1}\right)}{2} + \frac{i \ln\left(\frac{ixb}{-ia-1}\right) \ln(ibx+ia+1)}{2} + \frac{i\,\mathrm{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}$
derivativedivides	$\ln(-bx)\,\mathrm{arccot}(bx+a) + \frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2} + \frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i\,\mathrm{dilog}(}{}$
default	$\ln(-bx)\,\mathrm{arccot}(bx+a) + \frac{i \ln(-bx) \ln\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i \ln(-bx) \ln\left(\frac{-bx-a+i}{i-a}\right)}{2} + \frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2} - \frac{i\,\mathrm{dilog}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-b*x)*arccot(b*x+a)+1/2*I*ln(-b*x)*ln((I+a+b*x)/(I+a))-1/2*I*ln(-b*x)*ln
((I-a-b*x)/(I-a))+1/2*I*dilog((I+a+b*x)/(I+a))-1/2*I*dilog((I-a-b*x)/(I-a))
```

Maxima [A]
time = 0.53, size = 133, normalized size = 1.11

$$\frac{1}{2}\arctan\left(\frac{bx}{a^2+1}, -\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right) - \frac{1}{2}\arctan\left(bx+a\right)\log\left(\frac{b^2x^2}{a^2+1}\right) + \mathrm{arccot}\left(bx+a\right)\log\left(x\right) + \arctan\left(\frac{b^2x+ab}{b}\right)\log\left(x\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right) - \frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
1) - 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arccot(b*x + a)*log(x) +
arctan((b^2*x + a*b)/b)*log(x) + 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1)) -
 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(b*x + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/x,x)
```

```
[Out] int(acot(a + b*x)/x, x)
```

3.104 $\int \frac{\cot^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=62

$$-\frac{\cot^{-1}(a+bx)}{x}+\frac{ab\text{ArcTan}(a+bx)}{1+a^2}-\frac{b\log(x)}{1+a^2}+\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}$$

```
[Out] -arccot(b*x+a)/x+a*b*arctan(b*x+a)/(a^2+1)-b*ln(x)/(a^2+1)+1/2*b*ln(1+(b*x+
a)^2)/(a^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5154, 378, 720, 31, 649, 209, 266}

$$\frac{ab\text{ArcTan}(a+bx)}{a^2+1}-\frac{b\log(x)}{a^2+1}+\frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)}-\frac{\cot^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/x^2,x]
```

```
[Out] -(ArcCot[a + b*x]/x) + (a*b*ArcTan[a + b*x])/(1 + a^2) - (b*Log[x])/(1 + a^
2) + (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{x^2}\,dx &= -\frac{\cot^{-1}(a+bx)}{x} - b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx \\
&= -\frac{\cot^{-1}(a+bx)}{x} - b\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx, x, a+bx\right) \\
&= -\frac{\cot^{-1}(a+bx)}{x} - \frac{b\text{Subst}\left(\int \frac{1}{-a+x}\,dx, x, a+bx\right)}{1+a^2} - \frac{b\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2} \\
&= -\frac{\cot^{-1}(a+bx)}{x} - \frac{b\log(x)}{1+a^2} + \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2} + \frac{(ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{1+a^2} \\
&= -\frac{\cot^{-1}(a+bx)}{x} + \frac{ab\tan^{-1}(a+bx)}{1+a^2} - \frac{b\log(x)}{1+a^2} + \frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 66, normalized size = 1.06

$$
-\frac{\cot^{-1}(a+bx)}{x} + \frac{b(-2\log(x) + (1-ia)\log(i-a-bx) + (1+ia)\log(i+a+bx))}{2\left(1+a^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/x^2,x]
```

```
[Out] -(ArcCot[a + b*x]/x) + (b*(-2*Log[x] + (1 - I*a)*Log[I - a - b*x] + (1 + I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Maple [A]
time = 0.08, size = 61, normalized size = 0.98

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+\arctan(bx+a)a}{a^2+1}\right)$
default	$b\left(-\frac{\operatorname{arccot}(bx+a)}{bx}-\frac{\ln(-bx)}{a^2+1}+\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+\arctan(bx+a)a}{a^2+1}\right)$
risch	$-\frac{i\ln(1+i(bx+a))}{2x}-\frac{-ia^2\ln(1-i(bx+a))-i\ln(1-i(bx+a))+\pi a^2+\pi+2\ln(x)xb-xb\ln\left((iab-3b)x+ia^2+3i-2a\right)+ixb\ln}{2x(a-i)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arccot(b*x+a)/b/x-1/(a^2+1)*ln(-b*x)+1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+arc
tan(b*x+a)*a))
```

Maxima [A]
time = 0.48, size = 77, normalized size = 1.24

$$\frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(x\right)}{a^2+1}\right)-\frac{\operatorname{arccot}\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
+ 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arccot(b*x + a)/x
```

Fricas [A]
time = 3.32, size = 64, normalized size = 1.03

$$\frac{2\,abx\arctan\left(bx+a\right)+bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)-2\,\left(a^2+1\right)\operatorname{arccot}\left(bx+a\right)}{2\,\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x*arctan(b*x + a) + b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x
*log(x) - 2*(a^2 + 1)*arccot(b*x + a))/((a^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.80, size = 167, normalized size = 2.69

$$\begin{cases} -\frac{ib\,\mathrm{acot}\,(bx-i)}{2} - \frac{\mathrm{acot}\,(bx-i)}{x} + \frac{i}{2x} & \text{for}\ a = -i \\ \frac{ib\,\mathrm{acot}\,(bx+i)}{2} - \frac{\mathrm{acot}\,(bx+i)}{x} - \frac{i}{2x} & \text{for}\ a = i \\ -\frac{2a^2\,\mathrm{acot}\,(a+bx)}{2a^2x+2x} - \frac{2abx\,\mathrm{acot}\,(a+bx)}{2a^2x+2x} - \frac{2bx\log{(x)}}{2a^2x+2x} + \frac{bx\log{(a^2+2abx+b^2x^2+1)}}{2a^2x+2x} - \frac{2\,\mathrm{acot}\,(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b*acot(b*x - I)/2 - acot(b*x - I)/x + I/(2*x), Eq(a, -I)), (I
*b*acot(b*x + I)/2 - acot(b*x + I)/x - I/(2*x), Eq(a, I)), (-2*a**2*acot(a
+ b*x)/(2*a**2*x + 2*x) - 2*a*b*x*acot(a + b*x)/(2*a**2*x + 2*x) - 2*b*x*lo
g(x)/(2*a**2*x + 2*x) + b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2*x +
 2*x) - 2*acot(a + b*x)/(2*a**2*x + 2*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(60) = 120.
time = 0.54, size = 498, normalized size = 8.03

$$-\frac{\left(2\,a\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2} + 2\,a\log\left(\frac{4\left(4\,a^{2}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}+\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}-4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right) + \log\left(\frac{4\left(4\,a^{2}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}+\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}-4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2} - 2\,a\arctan\left(\frac{1}{bx+a}\right) - 4\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right) - \log\left(\frac{4\left(4\,a^{2}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}+\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}-4\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1}\right)\right)b}{2\left(2\,a^{3}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right) + a^{2}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2} - a^{2} + 2\,a\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right) + \tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 + 2*a*log(4*(4
*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 +
tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*
arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) + log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2
*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*a*arctan(1/(b*x +
a)) - 4*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - log(4*(4*a^2*tan
(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*
arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arcta
n(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1
/(b*x + a)))^2 + 1)))*b/(2*a^3*tan(1/2*arctan(1/(b*x + a))) + a^2*tan(1/2*a
rctan(1/(b*x + a)))^2 - a^2 + 2*a*tan(1/2*arctan(1/(b*x + a))) + tan(1/2*ar
ctan(1/(b*x + a)))^2 - 1)
```

Mupad [B]
time = 1.22, size = 62, normalized size = 1.00

$$-\frac{\mathrm{acot}(a+b\,x)}{x}-\frac{b\,x\,\ln\left(x\right)-\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}+a\,b\,x\,\mathrm{acot}(a+b\,x)}{x\,\left(a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/x^2,x)
```

```
[Out] - acot(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2
+ a*b*x*acot(a + b*x))/(x*(a^2 + 1))
```

3.105 $\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=95

$$\frac{b}{2(1+a^2)x}-\frac{\cot^{-1}(a+bx)}{2x^2}+\frac{(1-a^2)\,b^2\mathrm{ArcTan}(a+bx)}{2(1+a^2)^2}+\frac{ab^2\log(x)}{(1+a^2)^2}-\frac{ab^2\log\left(1+(a+bx)^2\right)}{2(1+a^2)^2}$$

```
[Out] 1/2*b/(a^2+1)/x-1/2*arccot(b*x+a)/x^2+1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1
)^2+a*b^2*ln(x)/(a^2+1)^2-1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5154, 378, 724, 815, 649, 209, 266}

$$\frac{(1-a^2)\,b^2\mathrm{ArcTan}(a+bx)}{2(a^2+1)^2}+\frac{ab^2\log(x)}{(a^2+1)^2}-\frac{ab^2\log\left((a+bx)^2+1\right)}{2(a^2+1)^2}+\frac{b}{2(a^2+1)x}-\frac{\cot^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/x^3,x]
```

```
[Out] b/(2*(1 + a^2)*x) - ArcCot[a + b*x]/(2*x^2) + ((1 - a^2)*b^2*ArcTan[a + b*x
])/(2*(1 + a^2)^2) + (a*b^2*Log[x])/(1 + a^2)^2 - (a*b^2*Log[1 + (a + b*x)^
2])/(2*(1 + a^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{x^3}\,dx &= -\frac{\cot^{-1}(a+bx)}{2x^2}-\frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\cot^{-1}(a+bx)}{2x^2}-\frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= \frac{b}{2\left(1+a^2\right)x}-\frac{\cot^{-1}(a+bx)}{2x^2}-\frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)(1+x^2)}\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= \frac{b}{2\left(1+a^2\right)x}-\frac{\cot^{-1}(a+bx)}{2x^2}-\frac{b^2\text{Subst}\left(\int \left(\frac{2a}{(1+a^2)(a-x)}+\frac{-1+a^2+2ax}{(1+a^2)(1+x^2)}\right)\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= \frac{b}{2\left(1+a^2\right)x}-\frac{\cot^{-1}(a+bx)}{2x^2}+\frac{ab^2\log(x)}{\left(1+a^2\right)^2}-\frac{b^2\text{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2\left(1+a^2\right)^2}\\
&= \frac{b}{2\left(1+a^2\right)x}-\frac{\cot^{-1}(a+bx)}{2x^2}+\frac{ab^2\log(x)}{\left(1+a^2\right)^2}-\frac{\left(ab^2\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{\left(1+a^2\right)^2}+\frac{((1
\end{aligned}
$$

$$
= \frac{b}{2\left(1+a^2\right)x}-\frac{\cot^{-1}(a+bx)}{2x^2}+\frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(1+a^2\right)^2}+\frac{ab^2\log(x)}{\left(1+a^2\right)^2}-\frac{ab^2\log(1+}{2(1+}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 92, normalized size = 0.97

$$\frac{-2\cot^{-1}(a+bx)+\frac{bx\left(4abx\log(x)+i(i+a)^2bx\log(i-a-bx)+(-i+a)(2(i+a)+(-1-ia)bx\log(i+a+bx))\right)}{(1+a^2)^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcCot[a + b*x] + (b*x*(4*a*b*x*Log[x] + I*(I + a)^2*b*x*Log[I - a - b*
x] + (-I + a)*(2*(I + a) + (-1 - I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)/
(4*x^2)
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.87

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}\right)$
default	$b^2\left(-\frac{\operatorname{arccot}(bx+a)}{2b^2x^2}+\frac{1}{2(a^2+1)bx}+\frac{a\ln(-bx)}{(a^2+1)^2}-\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}\right)$
risch	$-\frac{i\ln(1+i(bx+a))}{4x^2}+\frac{ia^4\ln(1-i(bx+a))+2ia^2\ln(1-i(bx+a))+i\ln(1-i(bx+a))+4b^2a\ln(-x)x^2-\pi a^4+2a^2bx-2\pi a^2 \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arccot(b*x+a)/b^2/x^2+1/2/(a^2+1)/b/x+1/(a^2+1)^2*a*ln(-b*x)-1/2/
(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*arctan(b*x+a)))
```

Maxima [A]
time = 0.47, size = 112, normalized size = 1.18

$$-\frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1}+\frac{ab\log\left(b^2x^2+2abx+a^2+1\right)}{a^4+2a^2+1}-\frac{2ab\log(x)}{a^4+2a^2+1}-\frac{1}{(a^2+1)x}\right)b-\frac{\operatorname{arccot}(bx+a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x
^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1)
- 1/((a^2 + 1)*x))*b - 1/2*arccot(b*x + a)/x^2
```

Fricas [A]
time = 1.83, size = 99, normalized size = 1.04

$$-\frac{(a^2-1)b^2x^2\arctan(bx+a)+ab^2x^2\log(b^2x^2+2\,abx+a^2+1)-2\,ab^2x^2\log(x)-(a^2+1)bx+(a^4+2\,a^2+1)\operatorname{arccot}(bx+a)}{2\,(a^4+2\,a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 - 1)*b^2*x^2*arctan(b*x + a) + a*b^2*x^2*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2 + 1)*b*x + (a^4 + 2*a^2 + 1)*arccot(b
*x + a))/((a^4 + 2*a^2 + 1)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.09, size = 381, normalized size = 4.01

$$\begin{cases} -\frac{b^2\operatorname{acot}(bx-i)}{8}+\frac{b}{8x}-\frac{\operatorname{acot}(bx-i)}{2x^2}+\frac{i}{8x^2} & \text{for}\ a=-i \\ -\frac{b^2\operatorname{acot}(bx+i)}{8}+\frac{b}{8x}-\frac{\operatorname{acot}(bx+i)}{2x^2}-\frac{i}{8x^2} & \text{for}\ a=i \\ -\frac{a^4\operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2}-\frac{2a^2\operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{2ab^2x^2\log(x)}{2a^4x^2+4a^2x^2+2x^2}-\frac{ab^2x^2\log(a^2+2abx+b^2x^2+1)}{2a^4x^2+4a^2x^2+2x^2}-\frac{b^2x^2\operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{bx}{2a^4x^2+4a^2x^2+2x^2}-\frac{\operatorname{acot}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-b**2*acot(b*x - I)/8 + b/(8*x) - acot(b*x - I)/(2*x**2) + I/(8*
x**2), Eq(a, -I)), (-b**2*acot(b*x + I)/8 + b/(8*x) - acot(b*x + I)/(2*x**2
) - I/(8*x**2), Eq(a, I)), (-a**4*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2
+ 2*x**2) + a**2*b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**
2) + a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*acot(a + b*x)/(
2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) - a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a
**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*acot(a + b*x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) + b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - acot(a +
b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. 2(85) = 170.
time = 0.69, size = 1309, normalized size = 13.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^3*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a^2*b*arc
tan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 4*a^3*b*log(4*(4*a^2*tan(
1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*a
rctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan
```

```
(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a^2*b*log(4*(4*a^2*t
an(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/
2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan
(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*log(4*(4*a^2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2
*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arct
an(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(
1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^3*b*arctan(1/(b*
x + a))*tan(1/2*arctan(1/(b*x + a))) - 14*a^2*b*arctan(1/(b*x + a))*tan(1/2
*arctan(1/(b*x + a)))^2 + 2*a^2*b*tan(1/2*arctan(1/(b*x + a)))^3 - 4*a*b*ar
ctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + a*b*tan(1/2*arctan(1/(b*
x + a)))^4 - b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 - 4*a^2*b
*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x +
a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a)))
- 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2
*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))) - 2*a*b*
log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a
)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) -
 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 + a^2*b
*arctan(1/(b*x + a)) - 2*a^2*b*tan(1/2*arctan(1/(b*x + a))) + 4*a*b*arctan(
1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) - 6*a*b*tan(1/2*arctan(1/(b*x + a
)))^2 - 2*b*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*b*tan(1/
2*arctan(1/(b*x + a)))^3 + a*b*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2
+ 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a
*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(
1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)) + a*b -
 b*arctan(1/(b*x + a)) + 2*b*tan(1/2*arctan(1/(b*x + a))))*b/(4*a^6*tan(1/2
*arctan(1/(b*x + a)))^2 + 4*a^5*tan(1/2*arctan(1/(b*x + a)))^3 + a^4*tan(1/
2*arctan(1/(b*x + a)))^4 - 4*a^5*tan(1/2*arctan(1/(b*x + a))) + 6*a^4*tan(1
/2*arctan(1/(b*x + a)))^2 + 8*a^3*tan(1/2*arctan(1/(b*x + a)))^3 + 2*a^2*ta
n(1/2*arctan(1/(b*x + a)))^4 + a^4 - 8*a^3*tan(1/2*arctan(1/(b*x + a))) + 4
*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 + 2*a^2
- 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)
```

Mupad [B]
time = 1.39, size = 230, normalized size = 2.42

$$\frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)}-\frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,{\left(a^2+1\right)}^2}-\frac{\mathrm{acot}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)-\frac{b\,x}{2}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{2}-\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^4\,x^4}{{\left(a^2+1\right)}^2}+a\,b\,x\,\mathrm{acot}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2}+\frac{a\,b^2\,\ln\left(x\right)}{{\left(a^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/x^3,x)
```

```
[Out] (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3
))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2)) - (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x +
 1))/(2*(a^2 + 1)^2) - (acot(a + b*x)*(a^2/2 + 1/2) - (b*x)/2 + (b^2*x^2*ac
ot(a + b*x))/2 - (x^3*(b^3 - 3*a^2*b^3))/(2*(2*a^2 + a^4 + 1)) + (a*b^4*x^4
)/(a^2 + 1)^2 + a*b*x*acot(a + b*x))/(x^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3)
+ (a*b^2*log(x))/(a^2 + 1)^2
```

3.106 $\int \frac{\cot^{-1}(a+bx)}{x^4} dx$

Optimal. Leaf size=129

$$\frac{b}{6\left(1+a^2\right)x^2}-\frac{2ab^2}{3\left(1+a^2\right)^2x}-\frac{\cot^{-1}(a+bx)}{3x^3}-\frac{a(3-a^2)\,b^3\text{ArcTan}(a+bx)}{3\left(1+a^2\right)^3}+\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}-\frac{\left(1-3a^2\right)b$$

```
[Out] 1/6*b/(a^2+1)/x^2-2/3*a*b^2/(a^2+1)^2/x-1/3*arccot(b*x+a)/x^3-1/3*a*(-a^2+3
)*b^3*arctan(b*x+a)/(a^2+1)^3+1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3-1/6*(-3*a^
2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5154, 378, 724, 815, 649, 209, 266}

$$-\frac{a(3-a^2)\,b^3\text{ArcTan}(a+bx)}{3\left(a^2+1\right)^3}+\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(a^2+1\right)^3}-\frac{\left(1-3a^2\right)b^3\log\left((a+bx)^2+1\right)}{6\left(a^2+1\right)^3}-\frac{2ab^2}{3\left(a^2+1\right)^2x}+\frac{b}{6\left(a^2+1\right)x^2}-\frac{\cot^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/x^4,x]
```

```
[Out] b/(6*(1 + a^2)*x^2) - (2*a*b^2)/(3*(1 + a^2)^2*x) - ArcCot[a + b*x]/(3*x^3)
 - (a*(3 - a^2)*b^3*ArcTan[a + b*x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log
[x])/(3*(1 + a^2)^3) - ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^2)^
3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{x^4}\,dx = -\frac{\cot^{-1}(a+bx)}{3x^3} - \frac{1}{3}b\int \frac{1}{x^3\left(1+(a+bx)^2\right)}\,dx$$

$$= -\frac{\cot^{-1}(a+bx)}{3x^3} - \frac{1}{3}b^3\text{Subst}\left(\int \frac{1}{(-a+x)^3\,(1+x^2)}\,dx, x, a+bx\right)$$

$$= \frac{b}{6\,(1+a^2)\,x^2} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{b^3\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2(1+x^2)}\,dx, x, a+bx\right)}{3\,(1+a^2)}$$

$$= \frac{b}{6\,(1+a^2)\,x^2} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{b^3\text{Subst}\left(\int \left(-\frac{2a}{(1+a^2)(a-x)^2} + \frac{1-3a^2}{(1+a^2)^2(a-x)} + \frac{a(3-a^2)+(}{(1+a^2)^2(}\right.\right.}{3\,(1+a^2)}$$

$$= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} + \frac{(1-3a^2)\,b^3\log(x)}{3\,(1+a^2)^3} - \frac{b^3\text{Subst}\left(\int \frac{a(3}{}\right.}{}$$

$$= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} + \frac{(1-3a^2)\,b^3\log(x)}{3\,(1+a^2)^3} - \frac{((1-3a^2)\,b^3)}{}$$

$$= \frac{b}{6\,(1+a^2)\,x^2} - \frac{2ab^2}{3\,(1+a^2)^2\,x} - \frac{\cot^{-1}(a+bx)}{3x^3} - \frac{a(3-a^2)\,b^3\tan^{-1}(a+bx)}{3\,(1+a^2)^3} + \frac{(1-3}{3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 126, normalized size = 0.98

$$\frac{-2(1+a^2)^3\cot^{-1}(a+bx) + 2(1-3a^2)\,b^3x^3\log(x) + (-1+ia)^3b^3x^3\log(i-a-bx) + (-i+a)bx\left((i+a)\,(1+a^2-4abx) + i(-i+a)^2b^2x^2\log(i+a+bx)\right)}{6\,(1+a^2)^3\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/x^4,x]
```

```
[Out] (-2*(1 + a^2)^3*ArcCot[a + b*x] + 2*(1 - 3*a^2)*b^3*x^3*Log[x] + (-1 + I*a)
^3*b^3*x^3*Log[I - a - b*x] + (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Maple [A]
time = 0.11, size = 115, normalized size = 0.89

method	result
derivativedivides	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3} + \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} + \frac{1}{6(a^2+1)b^2x^2} - \frac{2a}{3(a^2+1)^2bx} + \frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2} + (a^3-3a)\,a}{3(a^2+1)^3}\right.$
default	$b^3\left(-\frac{\operatorname{arccot}(bx+a)}{3b^3x^3} + \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} + \frac{1}{6(a^2+1)b^2x^2} - \frac{2a}{3(a^2+1)^2bx} + \frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2} + (a^3-3a)\,a}{3(a^2+1)^3}\right.$

risch	$-\frac{i\ln(1+i(bx+a))}{6x^3} - \frac{ix^3\ln\left(\left(-a^7b-5ia^6b-27a^5b+41ia^4b+29a^3b-15ia^2b-9ab+3ib\right)x-a^8-32a^6-4ia^7+70a^4+68ia^5-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*arccot(b*x+a)/b^3/x^3+1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)+1/6/(a^2+
1)/b^2/x^2-2/3/(a^2+1)^2*a/b/x+1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^2)
+(a^3-3*a)*arctan(b*x+a)))
```

Maxima [A]
time = 0.47, size = 165, normalized size = 1.28

$$\frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{4\,abx-a^2-1}{(a^4+2\,a^2+1)x^2}\right)b-\frac{\operatorname{arccot}\left(bx+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
(3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
- 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)/
((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arccot(b*x + a)/x^3
```

Fricas [A]
time = 3.65, size = 142, normalized size = 1.10

$$\frac{2\,(a^3-3\,a)b^3x^3\arctan\left(bx+a\right)+(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4\,(a^3+a)b^2x^2+(a^4+2\,a^2+1)bx-2\,(a^6+3\,a^4+3\,a^2+1)\operatorname{arccot}\left(bx+a\right)}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^3 - 3*a)*b^3*x^3*arctan(b*x + a) + (3*a^2 - 1)*b^3*x^3*log(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2
 + (a^4 + 2*a^2 + 1)*b*x - 2*(a^6 + 3*a^4 + 3*a^2 + 1)*arccot(b*x + a))/((a
^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.57, size = 760, normalized size = 5.89

$$\begin{cases} \frac{ib^3\operatorname{acot}(bx-i)}{24}-\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}(bx-i)}{3x^3}+\frac{i}{18x^3} & \text{for}\ a=-i \\ -\frac{ib^3\operatorname{acot}(bx+i)}{24}+\frac{ib^2}{24x}+\frac{b}{24x^2}-\frac{\operatorname{acot}(bx+i)}{3x^3}-\frac{i}{18x^3} & \text{for}\ a=i \\ -\frac{2a^6\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^4\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^3b^3x^3\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{4a^3b^2x^2}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^2b^3x^3\log(x)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{3a^2b^3x^3\log(a^2+2abx+b^2x^2+1)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{2a^2bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^2\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{6ab^3x^3\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{4ab^2x^2}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{2b^3x^3\log(x)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{b^3x^3\log(a^2+2abx+b^2x^2+1)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2\operatorname{acot}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/x**4,x)
```

```
[Out] Piecewise((I*b**3*acot(b*x - I)/24 - I*b**2/(24*x) + b/(24*x**2) - acot(b*x
 - I)/(3*x**3) + I/(18*x**3), Eq(a, -I)), (-I*b**3*acot(b*x + I)/24 + I*b**
2/(24*x) + b/(24*x**2) - acot(b*x + I)/(3*x**3) - I/(18*x**3), Eq(a, I)), (
-2*a**6*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
+ a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*ac
ot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b
**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 - 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) -
6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) + 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3
 + 18*a**2*x**3 + 6*x**3) - 6*a**2*acot(a + b*x)/(6*a**6*x**3 + 18*a**4*x**
3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*acot(a + b*x)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x
**3 + 18*a**2*x**3 + 6*x**3) + 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) - b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) + b*x/(6*a**6*x**3 +
18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*acot(a + b*x)/(6*a**6*x**3 + 18*a
**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3449 vs. $2(115) = 230$.
time = 1.53, size = 3449, normalized size = 26.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(24*a^5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 12*a^
4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 2*a^3*b^2*arctan
(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 + 24*a^5*b^2*log(4*(4*a^2*tan(
1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*a
rctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan
(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/
(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 36*a^4*b^2*log(4*(4*a^
2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan
(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*
arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arc
tan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a^3*b^2*log(4
*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3
 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*ta
n(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1
/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^5 + 3*a^2*b^2*
log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a
```

```
)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) -
 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*
tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^6 - 24*a^
5*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 120*a^4*b^2*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 + 24*a^4*b^2*tan(1/2*arctan(
1/(b*x + a)))^4 - 78*a^3*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)
))^4 + 22*a^3*b^2*tan(1/2*arctan(1/(b*x + a)))^5 - 36*a^2*b^2*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^5 + 5*a^2*b^2*tan(1/2*arctan(1/(b*x + a
)))^6 - 6*a*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 36*a^4
*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*
x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a
))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4
 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 -
44*a^3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan
(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b
*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x +
a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a))
)^3 - 21*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/2*
arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arcta
n(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(
b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x
 + a)))^4 - 6*a*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1
/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*ar
ctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(
1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(
b*x + a)))^5 - b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x + a)))^2 + 4*a*tan(1/
2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))^4 - 4*a*tan(1/2*arc
tan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1
/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b
*x + a)))^6 + 12*a^4*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) -
 24*a^4*b^2*tan(1/2*arctan(1/(b*x + a)))^2 + 78*a^3*b^2*arctan(1/(b*x + a))
*tan(1/2*arctan(1/(b*x + a)))^2 - 100*a^3*b^2*tan(1/2*arctan(1/(b*x + a)))^
3 + 24*a^2*b^2*arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^3 - 67*a^2*
b^2*tan(1/2*arctan(1/(b*x + a)))^4 + 18*a*b^2*arctan(1/(b*x + a))*tan(1/2*a
rctan(1/(b*x + a)))^4 - 14*a*b^2*tan(1/2*arctan(1/(b*x + a)))^5 - b^2*tan(1
/2*arctan(1/(b*x + a)))^6 + 18*a^3*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b*x +
 a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x + a)))
^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^2 +
1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1))
*tan(1/2*arctan(1/(b*x + a))) + 21*a^2*b^2*log(4*(4*a^2*tan(1/2*arctan(1/(b
*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*x +
a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a)))^
2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 +
 1))*tan(1/2*arctan(1/(b*x + a)))^2 + 12*a*b^2*log(4*(4*a^2*tan(1/2*arctan(
1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b*
```

```
x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x + a
)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))
^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*b^2*log(4*(4*a^2*tan(1/2*arctan
(1/(b*x + a)))^2 + 4*a*tan(1/2*arctan(1/(b*x + a)))^3 + tan(1/2*arctan(1/(b
*x + a)))^4 - 4*a*tan(1/2*arctan(1/(b*x + a))) - 2*tan(1/2*arctan(1/(b*x +
a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2...
```

Mupad [B]
time = 1.24, size = 285, normalized size = 2.21

$$\frac{\ln(x)\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{\mathrm{acot}(a+b\,x)\,\left(\frac{a^2}{3}+\frac{1}{3}\right)-\frac{b\,x}{6}+\frac{b^2\,x^2\,\mathrm{acot}(a+b\,x)}{3}-\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}+\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}+\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\mathrm{acot}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3}+\frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)}+\frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/x^4,x)

[Out] (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) - (acot(a + b*x)*(a^2/
3 + 1/3) - (b*x)/6 + (b^2*x^2*acot(a + b*x))/3 - (x^3*(b^3 - 7*a^2*b^3))/(6
*(2*a^2 + a^4 + 1)) + (a*b^2*x^2)/(3*(a^2 + 1)) + (2*a*b^4*x^4)/(3*(a^2 + 1
)^2) + (2*a*b*x*acot(a + b*x))/3)/(x^3 + a^2*x^3 + b^2*x^5 + 2*a*b*x^4) + (
b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 + a^6 +
 1)) + (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(a^2
- 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.107 $\int \frac{\cot^{-1}(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=642

$$-\frac{\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(-\frac{b\left(i\sqrt{c}-\sqrt{d}\,x\right)}{\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{\log\left(-\frac{i-a-bx}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}\,x\right)}{\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}-\frac{\log\left(-\frac{i-a-b}{a+bx}\right.}{}$$

```
[Out] -1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)-x*d^(1/2))/(b*x+a)/(b*c^(1/2)+(
1-I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((-I+a+b*x)/(b*x+a))*ln(I*b*(c^(1/2)
+I*x*d^(1/2))/(b*x+a)/(b*c^(1/2)-(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*ln((
-I+a+b*x)/(b*x+a))*ln(b*(I*c^(1/2)+x*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a)*d^
(1/2)))/c^(1/2)/d^(1/2)+1/4*ln((I+a+b*x)/(b*x+a))*ln(-b*(I*c^(1/2)+x*d^(1/2
))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2,(I+a+
b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+(1-I*a)*d^(1/2)))/c^(1/2)/d
^(1/2)+1/4*polylog(2,-(I-a-b*x)*(b*c^(1/2)-I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)-
(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*polylog(2,-(I-a-b*x)*(b*c^(1/2)+I*a*d
^(1/2))/(b*x+a)/(b*c^(1/2)+(1+I*a)*d^(1/2)))/c^(1/2)/d^(1/2)+1/4*polylog(2,
(I+a+b*x)*(b*c^(1/2)+I*a*d^(1/2))/(b*x+a)/(b*c^(1/2)+I*(I+a)*d^(1/2)))/c^(1
/2)/d^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5160, 2576, 2404, 2354, 2438}

$$\frac{\text{Li}_2\left(-\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(-a-bx+i)}{\left(b\sqrt{c}-(ia+1)\sqrt{d}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\left(i\sqrt{d}\,a+b\sqrt{c}\right)(-a-bx+i)}{\left(\sqrt{d}\,(ia+1)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}-\frac{\text{Li}_2\left(\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)(a+bx+i)}{\left(\sqrt{d}\,(1-ia)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\left(i\sqrt{d}\,a+b\sqrt{c}\right)(a+bx+i)}{\left(i\sqrt{d}\,(a+i)+b\sqrt{c}\right)(a+bx)}\right)}{4\sqrt{c}\,\sqrt{d}}-\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(-\frac{b\left(-\sqrt{d}\,x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+(1-ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(\frac{ib\left(\sqrt{c}+i\sqrt{d}\,x\right)}{(a+bx)\left(b\sqrt{c}-(1+ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\,\sqrt{d}}-\frac{\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(\frac{b\left(\sqrt{d}\,x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+(1+ia)\sqrt{d}\right)}\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(-\frac{b\left(\sqrt{d}\,x+i\sqrt{c}\right)}{(a+bx)\left(b\sqrt{c}+i(a+i)\sqrt{d}\right)}\right)}{4\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/4*(Log[(I + a + b*x)/(a + b*x)]*Log[-((b*(I*Sqrt[c] - Sqrt[d]*x))/((b*Sq
rt[c] + (1 - I*a)*Sqrt[d])*(a + b*x)))])/(Sqrt[c]*Sqrt[d]) + (Log[-((I - a
- b*x)/(a + b*x))]*Log[(I*b*(Sqrt[c] + I*Sqrt[d]*x))/((b*Sqrt[c] - (1 + I*a
)*Sqrt[d])*(a + b*x))]])/(4*Sqrt[c]*Sqrt[d]) - (Log[-((I - a - b*x)/(a + b*x
))]*Log[(b*(I*Sqrt[c] + Sqrt[d]*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a + b
*x))]])/(4*Sqrt[c]*Sqrt[d]) + (Log[(I + a + b*x)/(a + b*x)]*Log[-((b*(I*Sqrt
[c] + Sqrt[d]*x))/((b*Sqrt[c] + I*(I + a)*Sqrt[d])*(a + b*x)))])/(4*Sqrt[c]
*Sqrt[d]) + PolyLog[2, -(((b*Sqrt[c] - I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt
[c] - (1 + I*a)*Sqrt[d])*(a + b*x)))]/(4*Sqrt[c]*Sqrt[d]) - PolyLog[2, -(((
b*Sqrt[c] + I*a*Sqrt[d])*(I - a - b*x))/((b*Sqrt[c] + (1 + I*a)*Sqrt[d])*(a
 + b*x)))]/(4*Sqrt[c]*Sqrt[d]) - PolyLog[2, ((b*Sqrt[c] - I*a*Sqrt[d])*(I +
 a + b*x))/((b*Sqrt[c] + (1 - I*a)*Sqrt[d])*(a + b*x))]/(4*Sqrt[c]*Sqrt[d])
```

```
 + PolyLog[2, ((b*Sqrt[c] + I*a*Sqrt[d])*(I + a + b*x))/((b*Sqrt[c] + I*(I
+ a)*Sqrt[d])*(a + b*x))]/(4*Sqrt[c]*Sqrt[d])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{c+dx^2}\,dx = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+dx^2}\,dx$$

$$= \frac{1}{2}i\int \frac{\log(-i+a+bx)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log(i+a+bx)}{c+dx^2}\,dx - \frac{1}{2}\Bigg(i\Bigg(-\log(a+bx)+\log(-i$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.}{}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} - \frac{i\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.}{}$$

<u>Mathematica</u> [**A**]
time = 0.40, size = 563, normalized size = 0.88

$$-\frac{i\left(\log\left(\frac{\sqrt{d}(-i+a+bx)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)+\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right)-\log\left(-\frac{\sqrt{d}(-i+a+bx)}{b\sqrt{-c}-(-i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)+\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)+\log\left(-\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)-\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right)+\text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)-\text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)-\text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(-i+a)\sqrt{d}}\right)+\text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*(Log[(Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])]*Lo
g[Sqrt[-c] - Sqrt[d]*x] - Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt
[d]*x] - Log[(Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]*Log[Sq
rt[-c] - Sqrt[d]*x] + Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x
] - Log[-((Sqrt[d]*(-I + a + b*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d]))]*Log[Sq
rt[-c] + Sqrt[d]*x] + Log[(-I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*
```

```
x] + Log[-((Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))]*Log[Sqr
t[-c] + Sqrt[d]*x] - Log[(I + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x]
 + PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] -
 PolyLog[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Po
lyLog[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Poly
Log[2, (b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])]))/(Sqrt[-
c]*Sqrt[d])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2073 vs. $2(495) = 990$.
time = 0.70, size = 2074, normalized size = 3.23

method	result
risch	$-\frac{\operatorname{dilog}\left(\frac{iad-b\sqrt{cd}+(-ibx-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)}{4\sqrt{cd}}+\frac{\operatorname{dilog}\left(\frac{iad+b\sqrt{cd}+(-ibx-ia+1)d-d}{iad+b\sqrt{cd}-d}\right)}{4\sqrt{cd}}-\frac{\operatorname{dilog}\left(\frac{iad+b\sqrt{cd}-(ibx+i}{iad+b\sqrt{cd}+}\right)}{4\sqrt{cd}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*ln(1-(-2*I*
a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)
+d))*arccot(b*x+a)+1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d
)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c-2*(b
^2*c*d)^(1/2)+d))*a^2*arccot(b*x+a)+1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c-2*
(b^2*c*d)^(1/2)+d)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/
(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))*arccot(b*x+a)-I*b^2/(a^2*d+b^2*c+2*(b^2*
c*d)^(1/2)+d)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*
d+b^2*c+2*(b^2*c*d)^(1/2)+d))*arccot(b*x+a)-1/2*b^2/d*(b^2*c*d)^(1/2)/(a^2*
d+b^2*c+2*(b^2*c*d)^(1/2)+d)*arccot(b*x+a)^2-b^2/(a^2*d+b^2*c+2*(b^2*c*d)^(
1/2)+d)*arccot(b*x+a)^2-1/2*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2
)+d)*arccot(b*x+a)^2-1/2*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d
)*a^2*arccot(b*x+a)^2-1/4*b^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1
/2)+d)*polylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+
b^2*c+2*(b^2*c*d)^(1/2)+d))-1/2*b^2/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*polyl
og(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*(b^2
*c*d)^(1/2)+d))-1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*pol
ylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*(b
^2*c*d)^(1/2)+d))-1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*p
olylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*
(b^2*c*d)^(1/2)+d))*a^2-I*b^2/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*ln(1-(-2*I*
a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)
```

```
+d))*arccot(b*x+a)-1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d
)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*(b
^2*c*d)^(1/2)+d))*a^2*arccot(b*x+a)+1/2*I*b^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*
c-2*(b^2*c*d)^(1/2)+d)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x+a)
^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))*arccot(b*x+a)-1/2*I*b^2/d*(b^2*c*d)^
(1/2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*ln(1-(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+
b*x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d))*arccot(b*x+a)+1/2*b
^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arccot(b*x+a)^2-b^2/
(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arccot(b*x+a)^2+1/2*(b^2*c*d)^(1/2)/c/(a^
2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arccot(b*x+a)^2+1/2*(b^2*c*d)^(1/2)/c/(a^2*d
+b^2*c-2*(b^2*c*d)^(1/2)+d)*a^2*arccot(b*x+a)^2+1/4*b^2/d*(b^2*c*d)^(1/2)/(
a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*
x)^2/(1+(b*x+a)^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))-1/2*b^2/(a^2*d+b^2*c-
2*(b^2*c*d)^(1/2)+d)*polylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b*x
+a)^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))+1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*
c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+(b
*x+a)^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))+1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^
2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(-2*I*a*d+a^2*d+b^2*c-d)*(I+a+b*x)^2/(1+
(b*x+a)^2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d))*a^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8519 vs. $2(456) = 912$.
time = 5.31, size = 8519, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(
d)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a
^2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c
^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*b*d
)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b^2*d
*x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^
2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (
a^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2((2*a*
b^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*sqrt(c)*s
qrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)), ((a^2 +
 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(a^2 + 1)*b*
d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3
)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(
d))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b
^20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 +
```

260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 +
570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10
 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 2
2*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 +
 121771*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a
^10 + 9438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(
5*a^18 + 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980
*a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465
*a^16 + 2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^
4 + 1102*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^1
6 + 2730*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211
*a^2 + 21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 7
92*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^
12 + (b^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b
^20*c^9*d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^
8 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^
8 + 13650*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 +
 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c
^5*d^7 + 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 +
4199*a^2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^
10 + 30030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55
*(a^18 + 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 +
 3060*a^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 8
40*a^14 + 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 +
 21)*b^4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^1
2 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 + 2*(
11*(a^2 + 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6
+ 205*a^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 6
46*a^2 + 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 +
 6137*a^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 30
8880*a^6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 4
83*a^12 + 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)
*b^9*c^4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6
890*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 +
 2044*a^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 29
83*a^2 + 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*
a^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 1
1*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 3
30*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2
 + 7)*b^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6
+ 85*a^4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 +
 3876*a^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 2187
90*a^4 + 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*
a^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 +
105*a^12 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*

```
c^3*d^8 + 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 1092
0*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a
^14 + 252*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c
*d^10 + 11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210
*a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d +
 110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 19...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="fricas")
[Out] integral(arccot(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(d*x**2+c),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d*x^2),x)
[Out] int(acot(a + b*x)/(c + d*x^2), x)
```

3.108 $\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=152

$$-\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}+\frac{i\text{Po}}{}$$

```
[Out] -arccot(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arccot(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d-
a*d)/(1-I*(b*x+a)))/d-1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d+1/2*I*polylog(2,
1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

Rubi [A]
time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = $\{5156, 4967, 2449, 2352, 2497\}$

$$\frac{i\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)}{2d}-\frac{\log\left(\frac{2}{1-i(a+bx)}\right)\cot^{-1}(a+bx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcCot[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcCot[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d - ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d + ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \frac{i\text{Li}_2\Big(1-}{}$$

$$= -\frac{\cot^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\cot^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - \frac{i\text{Li}_2\Big(1-}{2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 345 vs. $2(152) = 304$.
time $= 0.03$, size $= 345$, normalized size $= 2.27$

$$-\frac{i\log\left(\frac{d(-i+a+bx)}{b\left(-\frac{id}{b}-\frac{bc-ad}{b}\right)}\right)\log\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{2d} + \frac{i\log\left(\frac{-i+a+bx}{a+bx}\right)\log\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{2d} + \frac{i\log\left(\frac{d(i+a+bx)}{b\left(\frac{id}{b}-\frac{bc-ad}{b}\right)}\right)\log\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{2d} - \frac{i\log\left(\frac{i+a+bx}{a+bx}\right)\log\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{2d} + \frac{i\text{PolyLog}\left(2, \frac{b\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{bc-id-ad}\right)}{2d} - \frac{i\text{PolyLog}\left(2, \frac{b\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{bc+id-ad}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(c + d*x),x]
```

```
[Out] ((-1/2*I)*Log[(d*(-I + a + b*x))/(b*(((-I)*d)/b - (b*c - a*d)/b))]*Log[(b*c
 - a*d)/b + (d*(a + b*x))/b])/d + ((I/2)*Log[(-I + a + b*x)/(a + b*x)]*Log[
(b*c - a*d)/b + (d*(a + b*x))/b])/d + ((I/2)*Log[(d*(I + a + b*x))/(b*((I*d
```

```
)/b - (b*c - a*d)/b))]*Log[(b*c - a*d)/b + (d*(a + b*x))/b])/d - ((I/2)*Log
[(I + a + b*x)/(a + b*x)]*Log[(b*c - a*d)/b + (d*(a + b*x))/b])/d + ((I/2)*
PolyLog[2, (b*((b*c - a*d)/b + (d*(a + b*x))/b))/(b*c - I*d - a*d)])/d - ((
I/2)*PolyLog[2, (b*((b*c - a*d)/b + (d*(a + b*x))/b))/(b*c + I*d - a*d)])/d
```

Maple [A]
time = 0.14, size = 187, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}\right.}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\operatorname{arccot}(bx+a)}{d}-b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\right.}{2d}\right.}{b}$
risch	$-\frac{i\operatorname{dilog}\left(\frac{iad-icb+(-ibx-ia+1)d-d}{iad-icb-d}\right)}{2d}-\frac{i\ln(-ibx-ia+1)\ln\left(\frac{iad-icb+(-ibx-ia+1)d-d}{iad-icb-d}\right)}{2d}+\frac{\pi\ln(iad-icb+(-ibx-ia+}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arccot(b*x+a)-b*(-1/2*I*ln(a*d-b*c-d*(b*x+a)
)*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d-1
/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*d-a
*d)))/d))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(130) = 260.
time = 0.55, size = 283, normalized size = 1.86

$$\frac{\operatorname{arccot}(bx+a)\log(dx+c)}{d}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d}+\frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2},\frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log(b^2x^2+2\,abx+a^2+1)-\arctan(bx+a)\log\left(\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)+i\operatorname{Li}_2\left(\frac{i\,bdx+(i\,a+1)d}{-i\,bc+(i\,a+1)d}\right)-i\operatorname{Li}_2\left(\frac{i\,bdx+(i\,a-1)d}{-i\,bc+(i\,a-1)d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arccot(b*x + a)*log(d*x + c)/d + arctan((b^2*x + a*b)/b)*log(d*x + c)/d + 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/(d*x + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arccot(b*x + a)/(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d*x),x)
```

```
[Out] int(acot(a + b*x)/(c + d*x), x)
```

3.109 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=338

$$\frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}+\frac{\log(i+a+bx)}{2bc}-\frac{i(a+bx)\log\left(\frac{i+a+bx}{a+bx}\right)}{2bc}+\frac{id\log\left(\frac{c(i-a-bx)}{ic-ac+bd}\right)\log(d}{2c^2}$$

```
[Out] 1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*x
)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c+1/2*I*d*ln(c*(I-a-b*x)/(I*c-a
*c+b*d))*ln(c*x+d)/c^2-1/2*I*d*ln((-I+a+b*x)/(b*x+a))*ln(c*x+d)/c^2-1/2*I*d
*ln(c*(I+a+b*x)/((I+a)*c-b*d))*ln(c*x+d)/c^2+1/2*I*d*ln((I+a+b*x)/(b*x+a))*
ln(c*x+d)/c^2-1/2*I*d*polylog(2,-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*polyl
og(2,b*(c*x+d)/(I*c-a*c+b*d))/c^2
```

Rubi [A]

time = 0.38, antiderivative size = 422, normalized size of antiderivative = 1.25, number of steps used = 37, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5160, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 45}

$$-\frac{id\,\text{Li}_2\left(\frac{c(-a-bx+i)}{(i-a)c+bd}\right)}{2c^2}+\frac{id\,\text{Li}_2\left(\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{2c^2}-\frac{id\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)\log(cx+d)}{2c^2}+\frac{id\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)\log(cx+d)}{2c^2}+\frac{id\log(a+bx+i)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{2c^2}-\frac{id\log(a+bx-i)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}+\frac{ix\left(\log\left(-\frac{-a-bx+i}{a+bx}\right)+\log(a+bx)-\log(a+bx-i)\right)}{2c}-\frac{i(-a-bx+i)\log(a+bx-i)}{2bc}-\frac{i(a+bx+i)\log(a+bx+i)}{2bc}-\frac{ix\left(\log(a+bx)-\log(a+bx+i)+\log\left(\frac{a+bx+i}{a+bx}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d/x),x]
```

```
[Out] ((I/2)*x*(Log[-((I - a - b*x)/(a + b*x))] + Log[a + b*x] - Log[-I + a + b*x
]))/c - ((I/2)*(I - a - b*x)*Log[-I + a + b*x])/(b*c) - ((I/2)*(I + a + b*x
)*Log[I + a + b*x])/(b*c) - ((I/2)*x*(Log[a + b*x] - Log[I + a + b*x] + Log
[(I + a + b*x)/(a + b*x)]))/c - ((I/2)*d*(Log[-((I - a - b*x)/(a + b*x))] +
 Log[a + b*x] - Log[-I + a + b*x])*Log[d + c*x])/c^2 + ((I/2)*d*(Log[a + b*
x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a + b*x)])*Log[d + c*x])/c^2 + (
(I/2)*d*Log[I + a + b*x]*Log[-((b*(d + c*x))/((I + a)*c - b*d))])/c^2 - ((I
/2)*d*Log[-I + a + b*x]*Log[(b*(d + c*x))/((I - a)*c + b*d)])/c^2 - ((I/2)*
d*PolyLog[2, (c*(I - a - b*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2
, (c*(I + a + b*x))/((I + a)*c - b*d)])/c^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx$$

$$= \frac{1}{2}i\int \frac{\log(-i+a+bx)}{c+\frac{d}{x}}\,dx - \frac{1}{2}i\int \frac{\log(i+a+bx)}{c+\frac{d}{x}}\,dx - \frac{1}{2}\bigg(i\bigg(-\log(a+bx)+\log(-$$

$$= \frac{1}{2}i\int \left(\frac{\log(-i+a+bx)}{c} - \frac{d\log(-i+a+bx)}{c(d+cx)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(i+a+bx)}{c} - \frac{d\,\mathrm{lo}$$

$$= \frac{i\int \log(-i+a+bx)\,dx}{2c} - \frac{i\int \log(i+a+bx)\,dx}{2c} - \frac{(id)\int \frac{\log(-i+a+bx)}{d+cx}\,dx}{2c} + \frac{(id)\int \frac{\log(}{}}{2}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{ix\left(\log(a+bx)-\log(i+a+\right.}{2c}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i(i-a-bx)\log(-i+a+b}{2bc}$$

$$= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i(i-a-bx)\log(-i+a+b}{2bc}$$

Mathematica [A]
time = 7.26, size = 602, normalized size = 1.78

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[a + b*x]/(c + d/x),x]
```

```
[Out] -1/2*((1 + (a + b*x)^2)*(I*b*c*d*Pi*ArcCot[a + b*x] - 2*c^2*(a + b*x)*ArcCo
t[a + b*x] + I*b*c*d*ArcCot[a + b*x]^2 - a*b*c*d*ArcCot[a + b*x]^2 + b^2*d^
2*ArcCot[a + b*x]^2 + (a*b*c*d*Sqrt[((1 + a^2)*c^2 - 2*a*b*c*d + b^2*d^2)/(
a*c - b*d)^2]*ArcCot[a + b*x]^2)/E^(I*ArcTan[c/(a*c - b*d)]) - (b^2*d^2*Sqr
t[((1 + a^2)*c^2 - 2*a*b*c*d + b^2*d^2)/(a*c - b*d)^2]*ArcCot[a + b*x]^2)/E
^(I*ArcTan[c/(a*c - b*d)]) + (2*I)*b*c*d*ArcCot[a + b*x]*ArcTan[c/(a*c - b*
```

```
d)] + b*c*d*Pi*Log[1 + E^((-2*I)*ArcCot[a + b*x])] - 2*b*c*d*ArcCot[a + b*x
]*Log[1 - E^((2*I)*ArcCot[a + b*x])] + 2*b*c*d*ArcCot[a + b*x]*Log[1 - E^((
2*I)*(ArcCot[a + b*x] - ArcTan[c/(a*c - b*d)]))] - 2*b*c*d*ArcTan[c/(a*c -
b*d)]*Log[1 - E^((2*I)*(ArcCot[a + b*x] - ArcTan[c/(a*c - b*d)]))] - b*c*d*
Pi*Log[1/Sqrt[1 + (a + b*x)^(-2)]] + 2*c^2*Log[1/((a + b*x)*Sqrt[1 + (a + b
*x)^(-2)])] + 2*b*c*d*ArcTan[c/(a*c - b*d)]*Log[Sin[ArcCot[a + b*x] - ArcTa
n[c/(a*c - b*d)]]] + I*b*c*d*PolyLog[2, E^((2*I)*ArcCot[a + b*x])] - I*b*c*
d*PolyLog[2, E^((2*I)*(ArcCot[a + b*x] - ArcTan[c/(a*c - b*d)]))]))/(b*c^3*
(a + b*x)^2*Sqrt[(1 + a^2 + 2*a*b*x + b^2*x^2)/(a + b*x)^2]*Sqrt[1 + (a + b
*x)^(-2)])
```

Maple [A]
time = 0.10, size = 322, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\mathrm{arccot}(bx+a)(bx+a)}{c}-\frac{\mathrm{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$
default	$\frac{\frac{\mathrm{arccot}(bx+a)(bx+a)}{c}-\frac{\mathrm{arccot}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$
risch	$-\frac{id\,\mathrm{dilog}\left(\frac{-iac+ibd+(ibx+ia+1)c-c}{-iac+ibd-c}\right)}{2c^2}+\frac{i\pi}{2bc}-\frac{a\arctan(bx+a)}{bc}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2bc}-\frac{i\ln(-ibx-ia+1)x}{2c}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccot(b*x+a)/c*(b*x+a)-arccot(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))-1/
c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c-b
*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)+1/2*I*b*d/c*ln(a*c-b*d-c*(b*x+a))*
ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-1/2*I*b*d/c*ln(a*c-b*d-c*(b*x+a))*ln((I*c
-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I*b*d/c*dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))
-1/2*I*b*d/c*dilog((I*c-c*(b*x+a))/(I*c-a*c+b*d))))
```

Maxima [A]
time = 0.54, size = 280, normalized size = 0.83

$$\frac{2\,bcx\arctan\left(1,bx+a\right)-bd\arctan\left(1,bx+a\right)\log\left(-\frac{b^2c^2x^2+2\,b^2cdx+b^2d^2}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)-2\,ac\arctan\left(bx+a\right)+i\,bd\mathrm{Li}_2\left(\frac{bcx+(a+i)c}{(a+i)c-bd}\right)-i\,bd\mathrm{Li}_2\left(\frac{bcx+(a-i)c}{(a-i)c-bd}\right)-\left(bd\arctan\left(-\frac{bc^2x+bcd}{2\,abcd-b^2d^2-(a^2+1)c^2},\frac{abcd-b^2d^2+(abc^2-b^2cd)x}{2\,abcd-b^2d^2-(a^2+1)c^2}\right)-c\right)\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*c*x*arctan2(1, b*x + a) - b*d*arctan2(1, b*x + a)*log(-(b^2*c^2*x^
2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - 2*a*c*a
rctan(b*x + a) + I*b*d*dilog((b*c*x + (a + I)*c)/((a + I)*c - b*d)) - I*b*d
*dilog((b*c*x + (a - I)*c)/((a - I)*c - b*d)) - (b*d*arctan2(-(b*c^2*x + b*
```

```
c*d)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 -
 b^2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b*c^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="fricas")
[Out] integral(x*arccot(b*x + a)/(c*x + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(c+d/x),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x),x, algorithm="giac")
[Out] integrate(arccot(b*x + a)/(c + d/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d/x),x)
[Out] int(acot(a + b*x)/(c + d/x), x)
```

3.110 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=735

$$\frac{\log(i-a-bx)}{2bc}+\frac{i(a+bx)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2bc}-\frac{i\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c^{3/2}}+\frac{\log(i+a+bx)}{2bc}-\frac{i(a+bx)\,\ldots}{2\ldots}$$

```
[Out] 1/2*ln(I-a-b*x)/b/c+1/2*I*(b*x+a)*ln((-I+a+b*x)/(b*x+a))/b/c+1/2*ln(I+a+b*x
)/b/c-1/2*I*(b*x+a)*ln((I+a+b*x)/(b*x+a))/b/c-1/2*I*arctan(x*c^(1/2)/d^(1/2
))*ln((-I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/2*I*arctan(x*c^(1/2)/d^(1/2))*l
n((I+a+b*x)/(b*x+a))*d^(1/2)/c^(3/2)+1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((I-a-
b*x)*c^(1/2)/((I-a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*ln(1-I*x*c^(1
/2)/d^(1/2))*ln((I+a+b*x)*c^(1/2)/((I+a)*c^(1/2)-I*b*d^(1/2)))*d^(1/2)/c^(3
/2)-1/4*ln(1-I*x*c^(1/2)/d^(1/2))*ln((I-a-b*x)*c^(1/2)/((I-a)*c^(1/2)+I*b*d
^(1/2)))*d^(1/2)/c^(3/2)-1/4*ln(1+I*x*c^(1/2)/d^(1/2))*ln((I+a+b*x)*c^(1/2)
/((I+a)*c^(1/2)+I*b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4*polylog(2,-b*(I*x*c^(1/2)
+d^(1/2))/((1+I*a)*c^(1/2)-b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*polylog(2,b*(I*x
*c^(1/2)+d^(1/2))/((1-I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^(3/2)-1/4*polylog(
2,b*(-I*x*c^(1/2)+d^(1/2))/((1+I*a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^(3/2)+1/4
*polylog(2,b*(-I*x*c^(1/2)+d^(1/2))/(I*(I+a)*c^(1/2)+b*d^(1/2)))*d^(1/2)/c^
(3/2)
```

Rubi [A]
time = 1.21, antiderivative size = 818, normalized size of antiderivative = 1.11, number of steps used = 57, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5160, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 327, 211}

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((I/2)*x*(Log[-((I - a - b*x)/(a + b*x))] + Log[a + b*x] - Log[-I + a + b*x
]))/c - ((I/2)*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[-((I - a - b*x)/(a
+ b*x))] + Log[a + b*x] - Log[-I + a + b*x]))/c^(3/2) - ((I/2)*(I - a - b*x
)*Log[-I + a + b*x])/(b*c) - ((I/2)*(I + a + b*x)*Log[I + a + b*x])/(b*c) -
 ((I/2)*x*(Log[a + b*x] - Log[I + a + b*x] + Log[(I + a + b*x)/(a + b*x)]))
/c + ((I/2)*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[I + a +
 b*x] + Log[(I + a + b*x)/(a + b*x)]))/c^(3/2) - ((I/4)*Sqrt[d]*Log[-I + a
+ b*x]*Log[-((b*(Sqrt[d] - Sqrt[-c]*x))/((I - a)*Sqrt[-c] - b*Sqrt[d]))])/(
-c)^(3/2) + ((I/4)*Sqrt[d]*Log[I + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/
((I + a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[I + a + b*
```

```
x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d]))])/(-c)^
(3/2) + ((I/4)*Sqrt[d]*Log[-I + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((I
 - a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[
-c]*(I - a - b*x))/((I - a)*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqr
t[d]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))/((I - a)*Sqrt[-c] + b*Sqrt[d])])/(
-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/((I + a)*Sqr
t[-c] - b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I +
a + b*x))/((I + a)*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx &= \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx \\
&= \frac{1}{2}i\int \frac{\log(-i+a+bx)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}i\int \frac{\log(i+a+bx)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}\Big(i\Big(-\log(a+bx)+\log(- \cdots \\
&= \frac{1}{2}i\int \left(\frac{\log(-i+a+bx)}{c} - \frac{d\log(-i+a+bx)}{c\,(d+cx^2)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(i+a+bx)}{c} - \frac{d\log \cdots \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{ix\left(\log(a+bx)-\log(i+a+ \cdots\right.}{2c} \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\left(\log\left(- \cdots\right.\right.}{\cdots} \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\left(\log\left(- \cdots\right.\right.}{\cdots} \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\left(\log\left(- \cdots\right.\right.}{\cdots} \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\left(\log\left(- \cdots\right.\right.}{\cdots} \\
&= \frac{ix\left(\log\left(-\frac{i-a-bx}{a+bx}\right)+\log(a+bx)-\log(-i+a+bx)\right)}{2c} - \frac{i\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\left(\log\left(- \cdots\right.\right.}{\cdots}
\end{aligned}
$$

__Mathematica__ [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5117 vs. 2(735) = 1470.
time = 30.05, size = 5117, normalized size = 6.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[a + b*x]/(c + d/x^2),x]
```

[Out] Result too large to show

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 13390, normalized size = 18.22

method	result
risch	$\frac{i\pi}{2bc}+\frac{\pi a}{2bc}-\frac{i\ln(-ibx-ia+1)x}{2c}+\frac{\ln(b^2x^2+2abx+a^2+1)}{2bc}+\frac{i\ln(ibx+ia+1)x}{2c}+\frac{ib\pi d\arctan\left(\frac{2iac+2(-ibx-ia+1)c-}{2\sqrt{-b^2cd}}\right.}{2c\sqrt{-b^2cd}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccot(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)

[Out] result too large to display

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. 2(502) = 1004.
time = 1.20, size = 8518, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="maxima")

[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccot(b*x + a) - 1/8*(8*a*c *arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d + (a *b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3* b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*s qrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + 4*b*a rctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*(b^3*d + (a ^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^ 3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4 *(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d)*log(((a^2 + 1)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 1 71*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 22 61)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a

^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10
 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 +
 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*
a^4 + 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 158
76*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261
)*b^6*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 101
64*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 +
11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^
10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*
a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495
*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b
^22*c^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 +
 1615*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 9
69)*b^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a
^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6
+ 109395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*
a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(
5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*
a^4 + 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*
a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10
d^2 + 11(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2
730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 +
 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 +
55*a^4 + 11*a^2 + 1)*b^2*c^12)*x^2 + 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4
 + 8*a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3
d^8 + 264(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(2
1*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*
(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2
+ 88179)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 2
0735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^
14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323
)*b^7*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10
 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^
20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^
6 + 261*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a
^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2
+ 1)*b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 1
90*a^2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*
d^7 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*
(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c
^6*d^5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a
^2 + 2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 35
75*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 +
1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*
b^7*c^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8

```
 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^1
6 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 +
1)*b^3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="fricas")
[Out] integral(x^2*arccot(b*x + a)/(c*x^2 + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(c+d/x**2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x^2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d/x^2),x)
[Out] int(acot(a + b*x)/(c + d/x^2), x)
```

3.111 $\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=693

$$-\frac{2i\sqrt{i+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d}+\frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d}-\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+\right.}{d^2}$$

```
[Out] -I*c*ln((-I+a+b*x)/(b*x+a))*ln(c+d*x^(1/2))/d^2+I*c*ln((I+a+b*x)/(b*x+a))*l
n(c+d*x^(1/2))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/
(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/
2)*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a
)^(1/2)+b^(1/2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2
))*ln(-d*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*
polylog(2,b^(1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylo
g(2,b^(1/2)*(c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(
1/2)*(c+d*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(1/2)*(c
+d*x^(1/2))/(d*(I-a)^(1/2)+c*b^(1/2)))/d^2+2*I*arctanh(b^(1/2)*x^(1/2)/(I-a
)^(1/2))*(I-a)^(1/2)/d/b^(1/2)-2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a
)^(1/2)/d/b^(1/2)+I*ln((-I+a+b*x)/(b*x+a))*x^(1/2)/d-I*ln((I+a+b*x)/(b*x+a)
)*x^(1/2)/d
```

Rubi [A]
time = 1.52, antiderivative size = 693, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5160, 196, 45, 2608, 2603, 12, 492, 211, 214, 2604, 2465, 266, 2463, 2441, 2440, 2438}

$$-\frac{2i\sqrt{a+i}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}\,d}-\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-i}\,d}\right)}{d^2}-\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-a-i}\,d}\right)}{d^2}+\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right)}{d^2}+\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)}{d^2}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\,\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}\,c+\sqrt{-a-i}\,d}\right)}{d^2}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\,\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}\,c+\sqrt{-a+i}\,d}\right)}{d^2}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\,\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}\,c-\sqrt{-a-i}\,d}\right)}{d^2}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\,\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}\,c-\sqrt{-a+i}\,d}\right)}{d^2}-\frac{ic\log\left(-\frac{-a-bx+i}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{ic\log\left(\frac{a+bx+i}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{i\sqrt{x}\,\log\left(-\frac{-a-bx+i}{a+bx}\right)}{d}-\frac{i\sqrt{x}\,\log\left(\frac{a+bx+i}{a+bx}\right)}{d}+\frac{2i\sqrt{-a+i}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) + ((
2*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) - (I*c
*Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log
[c + d*Sqrt[x]])/d^2 + (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 - (I*c*Log[-((d*(Sqrt[-I - a]
 + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
 + (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*
d))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[-((I - a - b*x)/(a + b*x))])/
d - (I*c*Log[c + d*Sqrt[x]]*Log[-((I - a - b*x)/(a + b*x))])/d^2 - (I*Sqrt[
x]*Log[(I + a + b*x)/(a + b*x)])/d + (I*c*Log[c + d*Sqrt[x]]*Log[(I + a + b
```

```
*x)/(a + b*x)])/d^2 - (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c
- Sqrt[-I - a]*d)])/d^2 - (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b
]*c + Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sq
rt[b]*c - Sqrt[I - a]*d)])/d^2 + (I*c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/
(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx$$

$$= i\text{Subst}\left(\int \frac{x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{x\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)$$

$$= i\text{Subst}\left(\int \left(\frac{\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d} - \frac{c\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \left(\frac{\log}{} \right.\right.$$

$$= \frac{i\text{Subst}\left(\int \log\left(\frac{-i+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} - \frac{i\text{Subst}\left(\int \log\left(\frac{i+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} - \frac{(ic)}{}$$

$$= \frac{i\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\,\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log}{}$$

$$= \frac{i\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\,\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log}{}$$

$$= \frac{i\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{d^2} - \frac{i\sqrt{x}\,\log\left(\frac{i+a+bx}{a+bx}\right)}{d} + \frac{ic\log}{}$$

$$= -\frac{2i\sqrt{i+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} + \frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{i\sqrt{x}\,\log\left(-\frac{i-a-b}{a+ba}\right.}{d}$$

$$= -\frac{2i\sqrt{i+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} + \frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{i\sqrt{x}\,\log\left(-\frac{i-a-b}{a+ba}\right.}{d}$$

$$= -\frac{2i\sqrt{i+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} + \frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} - \frac{ic\log\left(\frac{d\left(\sqrt{-i-}\right.}{\sqrt{b}\,c+}\right.}{}$$

$$= -\frac{2i\sqrt{i+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} + \frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} - \frac{ic\log\left(\frac{d\left(\sqrt{-i-}\right.}{\sqrt{b}\,c+}\right.}{}$$

$$= -\frac{2i\sqrt{i+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} + \frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} - \frac{ic\log\left(\frac{d\left(\sqrt{-i-}\right.}{\sqrt{b}\,c+}\right.}{}$$

Mathematica [A]
time = 0.54, size = 618, normalized size = 0.89

$$-\frac{i\left(\frac{2\sqrt{i+a}\,d\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}}-\frac{2\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}}+c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\log\left(c+d\sqrt{x}\right)+c\log\left(\frac{d\left(\sqrt{-i-a}+\sqrt{b}\,\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{i-a}+\sqrt{b}\,\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-d\sqrt{x}\log\left(\frac{-i+a+bx}{a+bx}\right)+c\log\left(c+d\sqrt{x}\right)\log\left(\frac{-i+a+bx}{a+bx}\right)+d\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)-c\log\left(c+d\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)+c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-i-a}\,d}\right)+c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)-c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right)-c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((-I)*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2
*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(
Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sq
rt[x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]
*d)]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqr
t[b]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqr
t[b]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[
x]*Log[(-I + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-I + a + b*x)/
(a + b*x)] + d*Sqrt[x]*Log[(I + a + b*x)/(a + b*x)] - c*Log[c + d*Sqrt[x]]*
Log[(I + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt
[b]*c - Sqrt[-I - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*
c + Sqrt[-I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c -
Sqrt[I - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[
I - a]*d)]))/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 364, normalized size = 0.53

method	result
derivativedivides	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2-\right.}\right.\right.}{}$
default	$\frac{2\,\mathrm{arccot}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccot}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}+\frac{4b\left(d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2ab\,d^2+6b^2c^2\right)\_Z^2-\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arccot(b*x+a)/d*x^(1/2)-2*arccot(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*(1/
4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^
2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^
2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*
x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^
2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a
*b*c^2*d^2+b^2*c^4+d^4)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arccot(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arccot(b*x + a) - c*arccot(b*x + a))/(d^2*x - c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acot}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d*x^(1/2)),x)
```

```
[Out] int(acot(a + b*x)/(c + d*x^(1/2)), x)
```

3.112 $\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=830

$$\frac{2i\sqrt{i+a}\,d\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2}-\frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2}+\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log(d}{c^3}$$

```
[Out] 1/2*(1+I*a)*ln(I-a-b*x)/b/c-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)
^(1/2)-d*b^(1/2)))/c^3+1/2*(1-I*a)*ln(I+a+b*x)/b/c+I*d^2*polylog(2,-b^(1/2)
*(d+c*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)))/c^3-I*d*ln((-I+a+b*x)/(b*x+a))*x
^(1/2)/c^2-I*d^2*ln((I+a+b*x)/(b*x+a))*ln(d+c*x^(1/2))/c^3-I*d^2*ln(d+c*x^(
1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3+I*d
*ln((I+a+b*x)/(b*x+a))*x^(1/2)/c^2+I*d^2*ln((-I+a+b*x)/(b*x+a))*ln(d+c*x^(1
/2))/c^3+2*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)-
I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(I-a)^(1/2)+d*b
^(1/2)))/c^3-2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a)^(1/2)/c^2/b^(
1/2)+1/2*I*x*ln((-I+a+b*x)/(b*x+a))/c-I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))
/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3+I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-b^
(1/2)*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3-1/2*I*x*ln((I+a+b*x)/(b*x+a)
)/c+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3+I
*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)-d*
b^(1/2)))/c^3
```

Rubi [A]
time = 1.76, antiderivative size = 830, normalized size of antiderivative = 1.00, number of steps used = 65, number of rules used = 19, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {5160, 196, 46, 2608, 2603, 12, 492, 211, 214, 2605, 457, 78, 2604, 2465, 266, 2463, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2) -
 ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2)
 + (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b
]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x
]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(S
qrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqr
t[x]])/c^3 - (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
```

```
- Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 + ((1 + I*a)*Log[I - a - b*x])/(2*b*c
) - (I*d*Sqrt[x]*Log[-((I - a - b*x)/(a + b*x))])/c^2 + ((I/2)*x*Log[-((I -
 a - b*x)/(a + b*x))])/c + (I*d^2*Log[d + c*Sqrt[x]]*Log[-((I - a - b*x)/(a
 + b*x))])/c^3 + ((1 - I*a)*Log[I + a + b*x])/(2*b*c) + (I*d*Sqrt[x]*Log[(I
 + a + b*x)/(a + b*x)])/c^2 - ((I/2)*x*Log[(I + a + b*x)/(a + b*x)])/c - (I
*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)])/c^3 + (I*d^2*PolyLog[
2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2
*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3
+ (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]
)/c^3 - (I*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b
]*d)])/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5160

```
Int[ArcCot[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[(-I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] - Dist[I/2, Int[
Log[(I + a + b*x)/(a + b*x)]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x]
&& RationalQ[n]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{1}{2}i\int \frac{\log\left(\frac{-i+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2}i\int \frac{\log\left(\frac{i+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= i\text{Subst}\left(\int \frac{x^2\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{x^2\log\left(\frac{i+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right)$$

$$= i\text{Subst}\left(\int \left(-\frac{d\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right) dx, x, \sqrt{x}$$

$$= \frac{i\text{Subst}\left(\int x\log\left(\frac{-i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{i\text{Subst}\left(\int x\log\left(\frac{i+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{(i}{}$$

$$= -\frac{id\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{}}{}$$

$$= -\frac{id\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{}}{}$$

$$= -\frac{id\sqrt{x}\,\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^2} + \frac{ix\log\left(-\frac{i-a-bx}{a+bx}\right)}{2c} + \frac{id^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{i-a-bx}{a+bx}\right)}{c^3} + \frac{id\sqrt{}}{}$$

$$= \frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} - \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} - \frac{id\sqrt{x}\,\log\left(-\frac{i-a-}{a+b}\right.}{c^2}$$

$$= \frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} - \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} + \frac{(1+ia)\log(i-a}{2bc}$$

$$= \frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} - \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} + \frac{id^2\log\left(\frac{c\left(\sqrt{-i-}\right.}{\sqrt{-i}}\right.}{}$$

$$= \frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} - \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} + \frac{id^2\log\left(\frac{c\left(\sqrt{-i-}\right.}{\sqrt{-i}}\right.}{}$$

$$= \frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2} - \frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2} + \frac{id^2\log\left(\frac{c\left(\sqrt{-i-}\right.}{\sqrt{-i}}\right.}{}$$

Mathematica [A]
time = 0.50, size = 809, normalized size = 0.97

$$\frac{4i\sqrt{i+a}\sqrt{b}\,cd\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)-4i\sqrt{i-a}\sqrt{b}\,cd\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)+2ibd^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)-2ibd^2\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)+2ibd^2\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c-\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)-2ibd^2\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}\,c-\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)+c^2\log(i-a-bx)+iac^2\log(i-a-bx)-2ibcd\sqrt{x}\log\left(\frac{-i+a+bx}{a+bx}\right)+ibc^2x\log\left(\frac{-i+a+bx}{a+bx}\right)+2ibd^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{-i+a+bx}{a+bx}\right)+c^2\log(i+a+bx)-iac^2\log(i+a+bx)+2ibcd\sqrt{x}\log\left(\frac{i+a+bx}{a+bx}\right)-ibc^2x\log\left(\frac{i+a+bx}{a+bx}\right)-2ibd^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{i+a+bx}{a+bx}\right)+2ibd^2\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)+2ibd^2\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)-2ibd^2\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{i-a}\,c+\sqrt{b}\,d}\right)-2ibd^2\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((4*I)*Sqrt[I + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]] - (4*I
)*Sqrt[I - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]] + (2*I)*b*
d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)]*
Log[d + c*Sqrt[x]] - (2*I)*b*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(S
qrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + (2*I)*b*d^2*Log[(c*(Sqrt[-I
 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]]
- (2*I)*b*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt
[b]*d)]*Log[d + c*Sqrt[x]] + c^2*Log[I - a - b*x] + I*a*c^2*Log[I - a - b*x
] - (2*I)*b*c*d*Sqrt[x]*Log[(-I + a + b*x)/(a + b*x)] + I*b*c^2*x*Log[(-I +
 a + b*x)/(a + b*x)] + (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(-I + a + b*x)/(a
 + b*x)] + c^2*Log[I + a + b*x] - I*a*c^2*Log[I + a + b*x] + (2*I)*b*c*d*Sq
rt[x]*Log[(I + a + b*x)/(a + b*x)] - I*b*c^2*x*Log[(I + a + b*x)/(a + b*x)]
 - (2*I)*b*d^2*Log[d + c*Sqrt[x]]*Log[(I + a + b*x)/(a + b*x)] + (2*I)*b*d^
2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + Sqrt[b]*d)] + (
2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d
)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I - a]*c) + S
qrt[b]*d)] - (2*I)*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*
c + Sqrt[b]*d)])/(2*b*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 388, normalized size = 0.47

method	result
derivativedivides	$\frac{\mathrm{arccot}(bx+a)x}{c}-\frac{2\,\mathrm{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\,\mathrm{arccot}(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3}+\frac{4b\left(-\frac{c\left(\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\_\right.\right.}{}\right.}{}$

default	$\frac{\operatorname{arccot}(bx+a)x}{c}-\frac{2\operatorname{arccot}(bx+a)d\sqrt{x}}{c^2}+\frac{2\operatorname{arccot}(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3}+\frac{4b\left(-\frac{c\left(\_R=\operatorname{RootOf}\left(b^2\_Z^4-4b^2d\_Z\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arccot(b*x+a)/c*x-2*arccot(b*x+a)/c^2*d*x^(1/2)+2*arccot(b*x+a)*d^2/c^3*ln(
d+c*x^(1/2))+4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*b-
3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=RootOf(
b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_
Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*d+
a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+
_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-
4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arccot(b*x + a)/(c + d/sqrt(x)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arccot(b*x + a) - d*sqrt(x)*arccot(b*x + a))/(c^2*x - d^2), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c + d/x^(1/2)),x)
```

```
[Out] int(acot(a + b*x)/(c + d/x^(1/2)), x)
```

3.113 $\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=367

$$\frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}}-\frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-}\right.}{\sqrt{b^2-4ac}}$$

```
[Out] arccot(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+
e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arccot(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-
4*a*c+b^2)^(1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(
1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(
1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(1-I*
(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {632, 212, 6860, 5156, 4967, 2449, 2352, 2497}

$$\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}-\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcCot[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - (ArcCot
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] + ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
- ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b
^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\,\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\cot^{-1}(d+ex)}{\sqrt{b^2-4ac}\,\left(b+\sqrt{b^2-4ac}+2cx\right)}\right) d$$

$$= \frac{(2c)\int \frac{\cot^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\cot^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2c)\text{Subst}\left(\int \frac{\cot^{-1}}{\frac{-2cd+\left(b+\sqrt{b^2-\right.}}{e}}\right.}{\sqrt{b^2-}}$$

$$= \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{}{(2c(i-d}\right.}{\sqrt{b}}$$

$$= \frac{\cot^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\cot^{-1}(d+ex)\log\left(\frac{}{(2c(i-d}\right.}{\sqrt{b}}$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] $Aborted
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4610 vs. 2(329) = 658.
time = 1.00, size = 4611, normalized size = 12.56

method	result
risch	$-\frac{e\,\mathrm{dilog}\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}}+\frac{e\,\mathrm{dilog}\left(\frac{ibe-2icd-2(-iex-id+1)c-\sqrt{4ac\,e^2-b}{ibe-2icd+2c-\sqrt{4ac\,e^2-b^2e^2}}\right.}{2\sqrt{4ac\,e^2-b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(I*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c
-b^2))^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+
d)^2+1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c))*d^2*arccot(e*x+d)-I*
(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^
(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)
/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*d^2*arccot(e*x+d)-1/4*I*e^2
*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))
^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1
)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*b^2*arccot(e*x+d)+1/4*I*e^
2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2)
)^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+
1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c))*b^2*arccot(e*x+d)-I*e*(e^
2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)
+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e
^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c))*b*d*arccot(e*x+d)+I*e*(e^2*(4*a*
c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(
1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*
d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*b*d*arccot(e*x+d)+1/2*(e^2*(4*a*c-b^2))
^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(
2,(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*
d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c))*d^2-1/2*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*
c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*c
*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2+(e^2*(
4*a*c-b^2))^(1/2)+c))*d^2+(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*
d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*d^2*arccot(e*x+d)^2-(e^2*(4*a*c-b^2))^(1
/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*d^2*arccot(
e*x+d)^2-e^2/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arccot(e*x+d)^2-
1/2*e^2/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*
c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2+(e^2*
(4*a*c-b^2))^(1/2)+c))-e^2/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ar
ccot(e*x+d)^2-1/2*e^2/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog
(2,(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e
*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c))+1/4*I*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a
```

```
*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d
+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1
/2)+c))*arccot(e*x+d)-1/8*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-
b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*c*d+a*e^2-b*e*d
+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1
/2)+c))*b^2-e*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(
4*a*c-b^2))^(1/2)+c)*b*d*arccot(e*x+d)^2-1/2*e*(e^2*(4*a*c-b^2))^(1/2)/(4*a
*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*
c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2-(e^2*
(4*a*c-b^2))^(1/2)+c))*b*d-1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a
*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*b^2*arccot(e*x+d)^2-I*(e^2*(4*a
*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*
ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b
*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*arccot(e*x+d)+1/2*e*(e^2*(4*a*c-b^2)
)^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2
,(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d
+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*b*d+I*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b
^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2
-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^
2))^(1/2)+c))*arccot(e*x+d)+1/8*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(
a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*c*d+a*e^2
-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^
2))^(1/2)+c))*b^2+1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*
d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*b^2*arccot(e*x+d)^2+e*(e^2*(4*a*c-b^2))^
(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*b*d*arccot(
e*x+d)^2-1/4*I*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c
-b^2))^(1/2)+c)*ln(1-(I*b*e-2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+
d)^2+1)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c))*arccot(e*x+d)+1/4*e^
2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*a
rccot(e*x+d)^2-1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2+(e^2*(4
*a*c-b^2))^(1/2)+c)*arccot(e*x+d)^2-1/2*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^
2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(I*b*e-2*I*c*d+a
*e^2-b*e*d+c*d^2-c)*(e*x+d+I)^2/((e*x+d)^2+1)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*
c-b^2))^(1/2)+c))+(e^2*(4*a*c-b^2))^(1/2)*c/(4*...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccot(x*e + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(d + e*x)/(a + b*x + c*x^2),x)
```

```
[Out] int(acot(d + e*x)/(a + b*x + c*x^2), x)
```

3.114 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{2i\cot^{-1}(a+bx)\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] -2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*poly
log(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*polylog(2,I*(1+I*(b*x
+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5164, 5007}

$$-\frac{2i\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b - (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
+ (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5164

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{2i\cot^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} - \frac{i\text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Mathematica [A]
time = 0.11, size = 127, normalized size = 0.96

$$-\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\text{PolyLog}\left(2,-e^{i\cot^{-1}(a+bx)}\right)-i\text{PolyLog}\left(2,e^{i\cot^{-1}(a+bx)}\right)\right)}{b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -((Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[
a + b*x])] - Log[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a
+ b*x])] - I*PolyLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[1 + (a +
 b*x)^(-2)]))
```

Maple [A]
time = 0.32, size = 118, normalized size = 0.89

method	result
default	$-\frac{\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)+i\,\text{dilog}\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-arccot(b*x+a)*ln((I
+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+I*dilog((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-I*d
ilog(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(a + bx)}{\sqrt{a^2 + 2abx + b^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acot}(a + b\,x)}{\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.115 $\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2i\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}-\frac{i\sqrt{1+(a+bx)^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a}}{\sqrt{1-i(a}}\right.}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] -2*I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+
a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I
*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,I*
(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)
^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5164, 5011, 5007}

$$-\frac{2i\sqrt{(a+bx)^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5164

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} - \frac{i\sqrt{1}}{}$$

Mathematica [A]
time = 0.07, size = 138, normalized size = 0.64

$$-\frac{(1+(a+bx)^2)\left(\cot^{-1}(a+bx)\left(\log\left(1-e^{i\cot^{-1}(a+bx)}\right)-\log\left(1+e^{i\cot^{-1}(a+bx)}\right)\right)+i\text{PolyLog}\left(2,-e^{i\cot^{-1}(a+bx)}\right)-i\text{PolyLog}\left(2,e^{i\cot^{-1}(a+bx)}\right)\right)}{b(a+bx)\sqrt{c(1+a^2+2abx+b^2x^2)}\sqrt{1+\frac{1}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] -(((1 + (a + b*x)^2)*(ArcCot[a + b*x]*(Log[1 - E^(I*ArcCot[a + b*x])] - Log
[1 + E^(I*ArcCot[a + b*x])]) + I*PolyLog[2, -E^(I*ArcCot[a + b*x])] - I*Pol
yLog[2, E^(I*ArcCot[a + b*x])]))/(b*(a + b*x)*Sqrt[c*(1 + a^2 + 2*a*b*x + b
^2*x^2)]*Sqrt[1 + (a + b*x)^(-2)]))
```

Maple [A]
time = 0.37, size = 156, normalized size = 0.72

method	result

default	$\frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arccot}(bx+a)\ln\left(\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}+1\right)-\operatorname{polylog}\left(2,-\frac{bx+a+i}{\sqrt{1+(bx+}}\right.\right.}{\sqrt{b^2x^2+2abx+a^2+1}\, bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*ln((
I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))+p
olylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}(a+bx)}{\sqrt{c(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

```
[Out] Integral(acot(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"giac")
[Out] integrate(arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
[Out] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.116 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcCot[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(23) = 46.
time = 0.29, size = 177, normalized size = 7.70

$$\frac{6\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)-5\sqrt[3]{2}\sqrt{\pi}\,\text{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+a^2+2abx+b^2x^2}\right)}{20b\left(1+a^2+2abx+b^2x^2\right)^{4/3}\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a + b*x
)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/3,
11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*
HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x^2)
^(-1)])/(20*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(4/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(bx + a)}{(b^2x^2 + 2abx + a^2 + 1)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
```

```
[Out] integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}{\left(a + bx \right)}}{\sqrt[3]{a^2 + 2abx + b^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral(acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acot}(a+b\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

```
[Out] int(acot(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.117 $\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcCot[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(25) = 50.
time = 0.13, size = 180, normalized size = 7.20

$$\frac{c\left(6\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(5(1+a^2+2abx+b^2x^2)\left(-3+2(a+bx)\cot^{-1}(a+bx)\right)+4(a+bx)\cot^{-1}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)-5\sqrt[3]{2}\,\sqrt{\pi}\,\text{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)}{20b\left(c\left(1+a^2+2abx+b^2x^2\right)\right)^{4/3}\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (c*(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a +
b*x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/
3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/
3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x
```

```
^2)^(-1)]))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(4/3)*Gamma[11/6]*Gamma
[7/3])
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(bx+a)}{\left(\left(a^{2}+1\right)c+2abcx+b^{2}c\,x^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"maxima")
```

```
[Out] integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"fricas")
```

```
[Out] integral(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(a+bx\right)}{\sqrt[3]{c\left(a^{2}+2abx+b^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
```

```
[Out] Integral(acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"giac")
```

```
[Out] integrate(arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acot}(a+b\,x)}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

```
[Out] int(acot(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.118 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=187

$$\frac{\sqrt{1+(a+bx)^2}}{2b}+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)}{2b}+\frac{i\cot^{-1}(a+bx)\mathrm{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\ i$$

```
[Out] I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*polylog(2,I*(1+I
*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a
)*arccot(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5166, 5073, 267, 5007}

$$\frac{i\mathrm{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b}+\frac{i\mathrm{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{i\mathrm{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{\sqrt{(a+bx)^2+1}}{2b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\cot^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] Sqrt[1 + (a + b*x)^2]/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b
*x])/(2*b) + (I*ArcCot[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
+ b*x)]])/b + ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
+ b*x)]])/b - ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b
*x)]])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5073

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
+ b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2*
m)), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right.}{2b}$$

$$= \frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)}{2b} + \frac{i\cot^{-1}(a+b}{}$$

Mathematica [A]
time = 1.05, size = 202, normalized size = 1.08

$$-\frac{\sqrt{(a+bx)^2\left(1+\frac{1}{(a+bx)^2}\right)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}(a+bx)\log\left(1-e^{i\cot^{-1}(a+bx)}\right)+4\cot^{-1}(a+bx)\log\left(1+e^{i\cot^{-1}(a+bx)}\right)-4i\text{PolyLog}\left(2,-e^{i\cot^{-1}(a+bx)}\right)+4i\text{PolyLog}\left(2,e^{i\cot^{-1}(a+bx)}\right)+\cot^{-1}(a+bx)\sec^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-2\tan\left(\frac{1}{2}\cot^{-1}(a+bx)\right)\right)}{8b(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] -1/8*(Sqrt[(a + b*x)^2*(1 + (a + b*x)^(-2))]*(-2*Cot[ArcCot[a + b*x]/2] - A
rcCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*Ar
cCot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I)*
PolyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*x])
```

```
] + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]))/(
b*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Maple [A]
time = 0.69, size = 160, normalized size = 0.86

method	result
default	$\frac{(\operatorname{arccot}(bx+a)bx+\operatorname{arccot}(bx+a)a+1)\sqrt{b^2x^2+2abx+a^2+1}}{2b}-\frac{i\left(i\operatorname{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right)-i\operatorname{arcc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2*(arccot(b*x+a)*b*x+arccot(b*x+a)*a+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b-1
/2*I*(I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))-I*arccot(b*x+a)*l
n((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)-polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(1/2)
)+polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*acot(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.119 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\cot^{-1}(a+bx)}{2bc}+\frac{i\sqrt{1+(a+bx)^2}\,\cot^{-1}(a+bx)\text{ArcTan}\Big(\frac{\sqrt{1}}{\sqrt{1}}}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] I*arccot(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^
2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog
(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b
*x+a)^2)^(1/2)+1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arccot(b*x+a)*(c+c
*(b*x+a)^2)^(1/2)/b/c
```

Rubi [A]
time = 0.24, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5166, 5073, 267, 5011, 5007}

$$\frac{i\sqrt{(a+bx)^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\cot^{-1}(a+bx)}{b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{\sqrt{c(a+bx)^2+c}}{2bc}+\frac{(a+bx)\sqrt{c(a+bx)^2+c}\,\cot^{-1}(a+bx)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```

```
[Out] Sqrt[c + c*(a + b*x)^2]/(2*b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*ArcCot
[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcCot[a + b*x]*ArcTan[Sqrt[1
 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2
)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*
(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((I/2)*Sqrt[1 + (a + b*x)^2]*Pol
yLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a
+ b*x)^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5007

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcCot[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
```

```
(c*Sqrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*
c*x])]/(c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 -
I*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 GtQ[d, 0]
```

Rule 5011

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCot[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5073

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcCot[c*x])^p/(c^2*d*m)), x] + (Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
+ b*ArcCot[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2*
m)), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\cot^{-1}(a+bx)}{2bc} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx,\right.}{2b}$$

$$= \frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\cot^{-1}(a+bx)}{2bc} - \frac{\sqrt{1}}{}$$

$$= \frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\cot^{-1}(a+bx)}{2bc} + \frac{i\sqrt{}}{}$$

Mathematica [A]
time = 0.67, size = 207, normalized size = 0.74

$$-\frac{\sqrt{c(1+a^2+2abx+b^2x^2)}\left(-2\cot\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-\cot^{-1}(a+bx)\csc^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-4\cot^{-1}(a+bx)\log\left(1-e^{i\cot^{-1}(a+bx)}\right)+4\cot^{-1}(a+bx)\log\left(1+e^{i\cot^{-1}(a+bx)}\right)-4i\text{PolyLog}\left(2,-e^{i\cot^{-1}(a+bx)}\right)+4i\text{PolyLog}\left(2,e^{i\cot^{-1}(a+bx)}\right)+\cot^{-1}(a+bx)\sec^2\left(\frac{1}{2}\cot^{-1}(a+bx)\right)-2\tan\left(\frac{1}{2}\cot^{-1}(a+bx)\right)\right)}{8bc(a+bx)\sqrt{1+\frac{1}{(a+bx)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] -1/8*(Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)]*(-2*Cot[ArcCot[a + b*x]/2] - Ar
cCot[a + b*x]*Csc[ArcCot[a + b*x]/2]^2 - 4*ArcCot[a + b*x]*Log[1 - E^(I*Arc
Cot[a + b*x])] + 4*ArcCot[a + b*x]*Log[1 + E^(I*ArcCot[a + b*x])] - (4*I)*P
olyLog[2, -E^(I*ArcCot[a + b*x])] + (4*I)*PolyLog[2, E^(I*ArcCot[a + b*x])]
 + ArcCot[a + b*x]*Sec[ArcCot[a + b*x]/2]^2 - 2*Tan[ArcCot[a + b*x]/2]))/(b
*c*(a + b*x)*Sqrt[1 + (a + b*x)^(-2)])
```

Maple [A]
time = 0.81, size = 202, normalized size = 0.72

method	result
default	$\frac{(\text{arccot}(bx+a)bx+\text{arccot}(bx+a)a+1)\sqrt{c\,(bx+a-i)\,(bx+a+i)}}{2bc} + \frac{i\left(-i\text{arccot}(bx+a)\ln\left(1-\frac{bx+a+i}{\sqrt{1+(bx+a)^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/2*(arccot(b*x+a)*b*x+arccot(b*x+a)*a+1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c+1/2*I*(-I*arccot(b*x+a)*ln(1-(I+a+b*x)/(1+(b*x+a)^2)^(1/2))+I*arccot(b*x+
a)*ln((I+a+b*x)/(1+(b*x+a)^2)^(1/2)+1)+polylog(2,-(I+a+b*x)/(1+(b*x+a)^2)^(
1/2))-polylog(2,(I+a+b*x)/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(I+a+b*x))^(1
/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt{c\left(a^2+2abx+b^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),
x)
```

```
[Out] Integral((a + b*x)**2*acot(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
,x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
, x)
```

3.120 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$

Optimal. Leaf size=30

$$\mathrm{Int}\left(\frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+(a+bx)^2}}, x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcCot[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}} \, dx = \frac{\mathrm{Subst}\left(\int \frac{x^2 \cot^{-1}(x)}{\sqrt[3]{1+x^2}} \, dx, x, a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(30) =$ 60.
time = 0.64, size = 198, normalized size = 6.60

$$\frac{3\left(\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)(3(7+(a+bx)^2)+4(a+bx)(-2+(a+bx)^2)\cot^{-1}(a+bx))-24(a+bx)\cot^{-1}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)+5\sqrt[3]{2}\sqrt{\pi}\,\mathrm{Gamma}\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)}{140b\sqrt[3]{1+a^2+2abx+b^2x^2}\,(1+(a+bx)^2)\,\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
```

```
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/
3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral((a + b*x)**2*acot(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acot}(a+b\,x)\,{(a+b\,x)}^2}{{(a^2+2\,a\,b\,x+b^2\,x^2+1)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.121 $\int \frac{(a+bx)^2 \cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=32

$$\operatorname{Int}\left(\frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/
3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcCot[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/
b
```

Rubi steps

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\frac{\operatorname{Subst}\left(\int \frac{x^2\cot^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(32) = 64.
time = 0.21, size = 200, normalized size = 6.25

$$\frac{3\left(\operatorname{Gamma}\left(\frac{11}{6}\right)\operatorname{Gamma}\left(\frac{7}{3}\right)\left(5(1+(a+bx)^2)\left(3(7+(a+bx)^2)+4(a+bx)\left(-2+(a+bx)^2\right)\cot^{-1}(a+bx)\right)-24(a+bx)\cot^{-1}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)+5\sqrt[3]{2}\sqrt{\pi}\operatorname{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+a^2+2abx+b^2x^2}\right)\right)}{140b\sqrt[3]{c\left(1+a^2+2abx+b^2x^2\right)}\left(1+(a+bx)^2\right)\operatorname{Gamma}\left(\frac{11}{6}\right)\operatorname{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
```

```
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))
^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \operatorname{arccot}(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{acot}(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),
x)
```

```
[Out] Integral((a + b*x)**2*acot(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(
1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*arccot(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acot}(a+b\,x)\,\left(a+b\,x\right)^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
,x)
```

```
[Out] int((acot(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
, x)
```

3.122 $\int (a+bx)^2 \cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=52

$$\frac{(a+bx)^2}{6b}+\frac{(a+bx)^3\cot^{-1}(a+bx)}{3b}-\frac{\log\left(1+(a+bx)^2\right)}{6b}$$

```
[Out] 1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccot(b*x+a)/b-1/6*ln(1+(b*x+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5152, 4947, 272, 45}

$$\frac{(a+bx)^2}{6b}-\frac{\log\left((a+bx)^2+1\right)}{6b}+\frac{(a+bx)^3\cot^{-1}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*ArcCot[a + b*x],x]
```

```
[Out] (a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCot[a + b*x])/(3*b) - Log[1 + (a + b*x)
^2]/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
```

```
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)^2 \cot^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2 \cot^{-1}(x)\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{x^3}{1+x^2}\,dx, x, a+bx\right)}{3b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, (a+bx)^2\right)}{6b} \\
&= \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} + \frac{\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, (a+bx)^2\right)}{6b} \\
&= \frac{(a+bx)^2}{6b} + \frac{(a+bx)^3 \cot^{-1}(a+bx)}{3b} - \frac{\log\left(1+(a+bx)^2\right)}{6b}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.81

$$
\frac{(a+bx)^2 + 2(a+bx)^3 \cot^{-1}(a+bx) - \log\left(1+(a+bx)^2\right)}{6b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*ArcCot[a + b*x],x]
[Out] ((a + b*x)^2 + 2*(a + b*x)^3*ArcCot[a + b*x] - Log[1 + (a + b*x)^2])/(6*b)
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
default	$\frac{\frac{\operatorname{arccot}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}-\frac{\ln\left(1+(bx+a)^2\right)}{6}}{b}$
risch	$\frac{i(bx+a)^3\ln(1+i(bx+a))}{6b}-\frac{ib^2x^3\ln(1-i(bx+a))}{6}+\frac{\pi b^2x^3}{6}-\frac{iba\,x^2\ln(1-i(bx+a))}{2}+\frac{b\pi a\,x^2}{2}-\frac{ia^2x\ln(1-i(bx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*arccot(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2-1/6*ln(1+(b*x+a)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(46) = 92$.
time = 0.45, size = 93, normalized size = 1.79

$$-\frac{1}{6}\left(\frac{2\,a^{3}\arctan\left(\frac{b^{2}x+ab}{b}\right)}{b^{2}}-\frac{bx^{2}+2\,ax}{b}+\frac{\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)}{b^{2}}\right)b+\frac{1}{3}\left(b^{2}x^{3}+3\,abx^{2}+3\,a^{2}x\right)\operatorname{arccot}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*arctan((b^2*x + a*b)/b)/b^2 - (b*x^2 + 2*a*x)/b + log(b^2*x^2 +
 2*a*b*x + a^2 + 1)/b^2)*b + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccot(b*x
 + a)
```

Fricas [A]
time = 3.39, size = 81, normalized size = 1.56

$$\frac{b^{2}x^{2}-2\,a^{3}\arctan\left(bx+a\right)+2\,abx+2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx\right)\operatorname{arccot}\left(bx+a\right)-\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*x^2 - 2*a^3*arctan(b*x + a) + 2*a*b*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x)*arccot(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 6.07, size = 99, normalized size = 1.90

$$\begin{cases} \frac{a^{3}\operatorname{acot}\left(a+bx\right)}{3b}+a^{2}x\operatorname{acot}\left(a+bx\right)+abx^{2}\operatorname{acot}\left(a+bx\right)+\frac{ax}{3}+\frac{b^{2}x^{3}\operatorname{acot}\left(a+bx\right)}{3}+\frac{bx^{2}}{6}-\frac{\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{3b}-\frac{i\operatorname{acot}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ a^{2}x\operatorname{acot}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a),x)
```

```
[Out] Piecewise((a**3*acot(a + b*x)/(3*b) + a**2*x*acot(a + b*x) + a*b*x**2*acot(
a + b*x) + a*x/3 + b**2*x**3*acot(a + b*x)/3 + b*x**2/6 - log(a/b + x - I/b
)/(3*b) - I*acot(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acot(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(46) = 92$.
time = 0.49, size = 203, normalized size = 3.90

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{6}-3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}-\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{5}-4\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4}+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}+3\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}-\arctan\left(\frac{1}{bx+a}\right)-\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{24\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] -1/24*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^6 - 3*arctan(1/(b*x
 + a))*tan(1/2*arctan(1/(b*x + a)))^4 - tan(1/2*arctan(1/(b*x + a)))^5 - 4*
log(16*tan(1/2*arctan(1/(b*x + a)))^2/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*t
an(1/2*arctan(1/(b*x + a)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^3 + 3*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x +
a)))^3 - arctan(1/(b*x + a)) - tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arc
tan(1/(b*x + a)))^3)
```

Mupad [B]
time = 0.15, size = 85, normalized size = 1.63

$$\frac{a\,x}{3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b}+\frac{b\,x^2}{6}-\frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b}+\frac{b^2\,x^3\,\mathrm{acot}(a+b\,x)}{3}+a^2\,x\,\mathrm{acot}(a+b\,x)+a\,b\,x^2\,\mathrm{acot}(a+b\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a*x)/3 - log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b) + (b*x^2)/6 - (a^3*atan(a
+ b*x))/(3*b) + (b^2*x^3*acot(a + b*x))/3 + a^2*x*acot(a + b*x) + a*b*x^2*a
cot(a + b*x)
```

3.123 $\int (a+bx)\cot^{-1}(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\text{ArcTan}(a+bx)}{2b}$$

```
[Out] 1/2*x+1/2*(b*x+a)^2*arccot(b*x+a)/b-1/2*arctan(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5152, 4947, 327, 209}

$$-\frac{\text{ArcTan}(a+bx)}{2b}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*ArcCot[a + b*x],x]
```

```
[Out] x/2 + ((a + b*x)^2*ArcCot[a + b*x])/(2*b) - ArcTan[a + b*x]/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
```

```
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)\cot^{-1}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x\cot^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,a+bx\right)}{2b}\\
&= \frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,a+bx\right)}{2b}\\
&= \frac{x}{2}+\frac{(a+bx)^2\cot^{-1}(a+bx)}{2b}-\frac{\tan^{-1}(a+bx)}{2b}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 141, normalized size = 3.62

$$ax\cot^{-1}(a+bx)+\frac{1}{2}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^2\cot^{-1}(a+bx)+\frac{1}{2}b\left(\frac{x}{b}-\frac{i(i-a)^2\log(i-a-bx)}{2b^2}+\frac{i(i+a)^2\log(i+a+bx)}{2b^2}\right)+\frac{a(-2a\mathrm{ArcTan}(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*ArcCot[a + b*x],x]
```

```
[Out] a*x*ArcCot[a + b*x] + (b*(-(a/b) + (a + b*x)/b)^2*ArcCot[a + b*x])/2 + (b*(
x/b - ((I/2)*(I - a)^2*Log[I - a - b*x])/b^2 + ((I/2)*(I + a)^2*Log[I + a +
 b*x])/b^2))/2 + (a*(-2*a*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2
]))/(2*b)
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccot}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}-\frac{\arctan(bx+a)}{2}}{b}$
risch	$\frac{i\left(x^2b+2ax\right)\ln(1+i(bx+a))}{4}-\frac{ib\,x^2\ln(1-i(bx+a))}{4}+\frac{\pi b\,x^2}{4}-\frac{iax\ln(1-i(bx+a))}{2}+\frac{\pi ax}{2}-\frac{a^2\arctan(bx+a)}{2b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*arccot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*(b*x+a)^2*arccot(b*x+a)+1/2*b*x+1/2*a-1/2*arctan(b*x+a))
```

Maxima [A]
time = 0.47, size = 52, normalized size = 1.33

$$\frac{1}{2}\, b\left(\frac{x}{b} - \frac{(a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2}\right) + \frac{1}{2}\left(bx^2 + 2\,ax\right)\operatorname{arccot}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*(x/b - (a^2 + 1)*arctan((b^2*x + a*b)/b)/b^2) + 1/2*(b*x^2 + 2*a*x)*a
rccot(b*x + a)
```

Fricas [A]
time = 2.24, size = 33, normalized size = 0.85

$$\frac{bx + \left(b^2x^2 + 2\,abx + a^2 + 1\right)\operatorname{arccot}\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)*arccot(b*x + a))/b
```

Sympy [A]
time = 2.59, size = 56, normalized size = 1.44

$$\begin{cases} \frac{a^2 \operatorname{acot}(a+bx)}{2b} + ax \operatorname{acot}(a+bx) + \frac{bx^2 \operatorname{acot}(a+bx)}{2} + \frac{x}{2} + \frac{\operatorname{acot}(a+bx)}{2b} & \text{for}\ b \neq 0 \\ ax \operatorname{acot}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*acot(b*x+a),x)
```

```
[Out] Piecewise((a**2*acot(a + b*x)/(2*b) + a*x*acot(a + b*x) + b*x**2*acot(a + b
*x)/2 + x/2 + acot(a + b*x)/(2*b), Ne(b, 0)), (a*x*acot(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(33) = 66.
time = 0.43, size = 100, normalized size = 2.56

$$\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4 + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2 - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^3 + \arctan\left(\frac{1}{bx+a}\right) + 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x +
 a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + ar
ctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*tan(1/2*arctan(1/(b*
x + a)))^2)
```

Mupad [B]
time = 1.51, size = 49, normalized size = 1.26

$$\frac{x}{2}+\frac{\frac{\mathrm{acot}(a+b\,x)}{2}+\frac{a^2\,\mathrm{acot}(a+b\,x)}{2}}{b}+a\,x\,\mathrm{acot}(a+b\,x)+\frac{b\,x^2\,\mathrm{acot}(a+b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)*(a + b*x),x)
```

```
[Out] x/2 + (acot(a + b*x)/2 + (a^2*acot(a + b*x))/2)/b + a*x*acot(a + b*x) + (b*
x^2*acot(a + b*x))/2
```

3.124 $\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx$

Optimal. Leaf size=45

$$-\frac{i\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2b}+\frac{i\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2b}$$

```
[Out] -1/2*I*polylog(2,-I/(b*x+a))/b+1/2*I*polylog(2,I/(b*x+a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5152, 4941, 2438}

$$\frac{i\text{Li}_2\left(\frac{i}{a+bx}\right)}{2b}-\frac{i\text{Li}_2\left(-\frac{i}{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(a + b*x),x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a + b*x)])/b + ((I/2)*PolyLog[2, I/(a + b*x)])/b
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^{-1}(a+bx)}{a+bx}\,dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, a+bx\right)}{b}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2b}$$
$$= -\frac{i\text{Li}_2\left(-\frac{i}{a+bx}\right)}{2b} + \frac{i\text{Li}_2\left(\frac{i}{a+bx}\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.84

$$-\frac{i\left(\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)-\text{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(a + b*x),x]
```

```
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.
time = 0.12, size = 87, normalized size = 1.93

method	result	siz
risch	$\frac{i\,\text{dilog}(-ibx-ia+1)}{2b}+\frac{\pi\ln(-ibx-ia)}{2b}-\frac{i\,\text{dilog}(ibx+ia+1)}{2b}$	55
derivativedivides	$\frac{\ln(bx+a)\text{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\text{dilog}(1+i(bx+a))}{2}+\frac{i\,\text{dilog}(1-i(bx+a))}{2}}{b}$	87
default	$\frac{\ln(bx+a)\text{arccot}(bx+a)-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\text{dilog}(1+i(bx+a))}{2}+\frac{i\,\text{dilog}(1-i(bx+a))}{2}}{b}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(b*x+a)*arccot(b*x+a)-1/2*I*ln(b*x+a)*ln(1+I*(b*x+a))+1/2*I*ln(b*x+a
)*ln(1-I*(b*x+a))-1/2*I*dilog(1+I*(b*x+a))+1/2*I*dilog(1-I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(31) = 62$.
time = 0.51, size = 112, normalized size = 2.49

$$\frac{\text{arccot}\,(bx+a)\log(bx+a)}{b}+\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(bx+a)}{b}+\frac{\arctan(bx+a,0)\log(b^2x^2+2\,abx+a^2+1)-2\,\arctan(bx+a)\log(|bx+a|)+i\,\text{Li}_2(i\,bx+i\,a+1)-i\,\text{Li}_2(-i\,bx-i\,a+1)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] arccot(b*x + a)*log(b*x + a)/b + arctan((b^2*x + a*b)/b)*log(b*x + a)/b + 1
/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan(b*x + a
)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I*a + 1))
/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccot(b*x + a)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}{\left(a + bx \right)}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(b*x+a),x)
```

```
[Out] Integral(acot(a + b*x)/(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(31) = 62.
time = 0.65, size = 100, normalized size = 2.22

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{4} + 2\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2} - 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{3} + \arctan\left(\frac{1}{bx+a}\right) + 2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)}{8\,b^{2}\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + a
rctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b^2*tan(1/2*arctan(1/
(b*x + a)))^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acot}(a+b\,x)}{a+b\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(a + b*x),x)
[Out] int(acot(a + b*x)/(a + b*x), x)
```

3.125 $\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\log(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] -arccot(b*x+a)/b/(b*x+a)-ln(b*x+a)/b+1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5152, 4947, 272, 36, 29, 31}

$$-\frac{\log(a+bx)}{b}+\frac{\log\left((a+bx)^2+1\right)}{2b}-\frac{\cot^{-1}(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(ArcCot[a + b*x]/(b*(a + b*x))) - Log[a + b*x]/b + Log[1 + (a + b*x)^2]/(2
*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{(a+bx)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\cot^{-1}(x)}{x^2}\,dx,x,a+bx\right)}{b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx,x,a+bx\right)}{b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,(a+bx)^2\right)}{2b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,(a+bx)^2\right)}{2b}+\frac{\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,(a+bx)^2\right)}{2b}\\
&= -\frac{\cot^{-1}(a+bx)}{b(a+bx)}-\frac{\log(a+bx)}{b}+\frac{\log\left(1+(a+bx)^2\right)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.85

$$
-\frac{\frac{\cot^{-1}(a+bx)}{a+bx}+\log(a+bx)-\frac{1}{2}\log\left(1+(a+bx)^2\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -((ArcCot[a + b*x]/(a + b*x) + Log[a + b*x] - Log[1 + (a + b*x)^2]/2)/b)
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}+\frac{\ln\left(1+(bx+a)^2\right)}{2}-\ln(bx+a)}{b}$
default	$\frac{-\frac{\operatorname{arccot}(bx+a)}{bx+a}+\frac{\ln\left(1+(bx+a)^2\right)}{2}-\ln(bx+a)}{b}$
risch	$-\frac{i\ln(1+i(bx+a))}{2b(bx+a)}-\frac{2\ln(-bx-a)bx-\ln(b^2x^2+2abx+a^2+1)bx-i\ln(1-i(bx+a))+2\ln(-bx-a)a-\ln(b^2x^2+2abx+a^2+}{2b(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-arccot(b*x+a)/(b*x+a)+1/2*ln(1+(b*x+a)^2)-ln(b*x+a))
```

Maxima [A]
time = 0.26, size = 53, normalized size = 1.13

$$\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}-\frac{\log\left(bx+a\right)}{b}-\frac{\operatorname{arccot}\left(bx+a\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b - log(b*x + a)/b - arccot(b*x + a)/(
(b*x + a)*b)
```

Fricas [A]
time = 2.11, size = 59, normalized size = 1.26

$$\frac{(bx+a)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(bx+a)\log\left(bx+a\right)-2\,\operatorname{arccot}\left(bx+a\right)}{2\,(b^2x+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(b*x + a)*log(b*x + a)
- 2*arccot(b*x + a))/(b^2*x + a*b)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.25, size = 139, normalized size = 2.96

$$\begin{cases}-\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x}+\frac{a\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x}+\frac{ia\operatorname{acot}(a+bx)}{ab+b^2x}-\frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x}+\frac{bx\log\left(\frac{a}{b}+x-\frac{i}{b}\right)}{ab+b^2x}+\frac{ibx\operatorname{acot}(a+bx)}{ab+b^2x}-\frac{\operatorname{acot}(a+bx)}{ab+b^2x} & \text{for}\ b\neq 0\\ \frac{x\operatorname{acot}(a)}{a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(b*x+a)**2,x)
```

```
[Out] Piecewise((-a*log(a/b + x)/(a*b + b**2*x) + a*log(a/b + x - I/b)/(a*b + b**
2*x) + I*a*acot(a + b*x)/(a*b + b**2*x) - b*x*log(a/b + x)/(a*b + b**2*x) +
 b*x*log(a/b + x - I/b)/(a*b + b**2*x) + I*b*x*acot(a + b*x)/(a*b + b**2*x)
 - acot(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acot(a)/a**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(45) = 90.
time = 0.44, size = 238, normalized size = 5.06

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)^2-\frac{\arctan\left(\frac{1}{bx+a}\right)^2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-\log\left(\frac{4\left(\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-\arctan\left(\frac{1}{bx+a}\right)^2+4\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)+\log\left(\frac{4\left(\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4-2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2+1}\right)}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx+a}\right)\right)^2-1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(1/(b*x + a))^2 - (arctan(1/(b*x + a))^2*tan(1/2*arctan(1/(b*x
+ a)))^2 - log(4*(tan(1/2*arctan(1/(b*x + a)))^4 - 2*tan(1/2*arctan(1/(b*x
+ a)))^2 + 1)/(tan(1/2*arctan(1/(b*x + a)))^4 + 2*tan(1/2*arctan(1/(b*x + a
)))^2 + 1))*tan(1/2*arctan(1/(b*x + a)))^2 - arctan(1/(b*x + a))^2 + 4*arct
an(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a))) + log(4*(tan(1/2*arctan(1/(b*x
 + a)))^4 - 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)/(tan(1/2*arctan(1/(b*x +
a)))^4 + 2*tan(1/2*arctan(1/(b*x + a)))^2 + 1)))/(tan(1/2*arctan(1/(b*x + a
)))^2 - 1))/b
```

Mupad [B]
time = 0.77, size = 57, normalized size = 1.21

$$\frac{\ln\left(-a^2-2\,a\,b\,x-b^2\,x^2-1\right)}{2\,b}-\frac{\ln\left(a+b\,x\right)}{b}-\frac{\mathrm{acot}(a+b\,x)}{x\,b^2+a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(a + b*x)^2,x)
```

```
[Out] log(- a^2 - b^2*x^2 - 2*a*b*x - 1)/(2*b) - log(a + b*x)/b - acot(a + b*x)/(
a*b + b^2*x)
```

3.126 $\int \frac{\cot^{-1}(1+x)}{2+2x} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{4} i \text{PolyLog}\left(2,-\frac{i}{1+x}\right)+\frac{1}{4} i \text{PolyLog}\left(2,\frac{i}{1+x}\right)$$

```
[Out] -1/4*I*polylog(2,-I/(1+x))+1/4*I*polylog(2,I/(1+x))
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5152, 12, 4941, 2438}

$$\frac{1}{4} i \text{Li}_2\left(\frac{i}{x+1}\right)-\frac{1}{4} i \text{Li}_2\left(-\frac{i}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[1 + x]/(2 + 2*x),x]
```

```
[Out] (-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(1+x)}{2+2x}\,dx &= \text{Subst}\left(\int \frac{\cot^{-1}(x)}{2x}\,dx, x, 1+x\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, 1+x\right)\\
&= \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, 1+x\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, 1+x\right)\\
&= -\frac{1}{4}i\text{Li}_2\left(-\frac{i}{1+x}\right) + \frac{1}{4}i\text{Li}_2\left(\frac{i}{1+x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$
-\frac{1}{4}i\text{PolyLog}\left(2, -\frac{i}{1+x}\right) + \frac{1}{4}i\text{PolyLog}\left(2, \frac{i}{1+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[1 + x]/(2 + 2*x),x]
```

```
[Out] (-1/4*I)*PolyLog[2, (-I)/(1 + x)] + (I/4)*PolyLog[2, I/(1 + x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(27) = 54.
time = 0.11, size = 68, normalized size = 1.94

method	result
risch	$\frac{\pi \ln(-ix-i)}{4} + \frac{i\,\text{dilog}(-ix-i+1)}{4} - \frac{i\,\text{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\text{arccot}(1+x)}{2} - \frac{i\ln(1+x)\ln(1+i(1+x))}{4} + \frac{i\ln(1+x)\ln(1-i(1+x))}{4} - \frac{i\,\text{dilog}(1+i(1+x))}{4} + \frac{i\,\text{dilog}(1-i(1+}{4}$
default	$\frac{\ln(1+x)\text{arccot}(1+x)}{2} - \frac{i\ln(1+x)\ln(1+i(1+x))}{4} + \frac{i\ln(1+x)\ln(1-i(1+x))}{4} - \frac{i\,\text{dilog}(1+i(1+x))}{4} + \frac{i\,\text{dilog}(1-i(1+}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1+x)*arccot(1+x)-1/4*I*ln(1+x)*ln(1+I*(1+x))+1/4*I*ln(1+x)*ln(1-I*(1
+x))-1/4*I*dilog(1+I*(1+x))+1/4*I*dilog(1-I*(1+x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(21) = 42.
time = 0.50, size = 64, normalized size = 1.83

$$
\frac{1}{4}\arctan(x+1,0)\log\left(x^2+2x+2\right) + \frac{1}{2}\,\text{arccot}(x+1)\log(x+1) + \frac{1}{2}\arctan(x+1)\log(x+1) - \frac{1}{2}\arctan(x+1)\log(|x+1|) + \frac{1}{4}i\,\text{Li}_2(ix+i+1) - \frac{1}{4}i\,\text{Li}_2(-ix-i+1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arccot(x + 1)*log(x + 1) + 1
/2*arctan(x + 1)*log(x + 1) - 1/2*arctan(x + 1)*log(abs(x + 1)) + 1/4*I*dil
og(I*x + I + 1) - 1/4*I*dilog(-I*x - I + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arccot(x + 1)/(x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acot}(x+1)}{x+1}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(1+x)/(2+2*x),x)
```

```
[Out] Integral(acot(x + 1)/(x + 1), x)/2
```

Giac [A]
time = 0.40, size = 26, normalized size = 0.74

$$-\frac{1}{4}\,(x+1)^2\arctan\left(\frac{1}{x+1}\right)-\frac{1}{4}\,x-\frac{1}{4}\,\arctan\left(\frac{1}{x+1}\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] -1/4*(x + 1)^2*arctan(1/(x + 1)) - 1/4*x - 1/4*arctan(1/(x + 1)) - 1/4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acot}(x+1)}{2\,x+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x + 1)/(2*x + 2),x)
```

```
[Out] int(acot(x + 1)/(2*x + 2), x)
```

3.127 $\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=45

$$-\frac{i\text{PolyLog}\left(2,-\frac{i}{a+bx}\right)}{2d}+\frac{i\text{PolyLog}\left(2,\frac{i}{a+bx}\right)}{2d}$$

```
[Out] -1/2*I*polylog(2,-I/(b*x+a))/d+1/2*I*polylog(2,I/(b*x+a))/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5152, 12, 4941, 2438}

$$\frac{i\text{Li}_2\left(\frac{i}{a+bx}\right)}{2d}-\frac{i\text{Li}_2\left(-\frac{i}{a+bx}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)/(a + b*x)])/d + ((I/2)*PolyLog[2, I/(a + b*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5152

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCot[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{b\cot^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b} \\
&= \frac{\operatorname{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, a+bx\right)}{d} \\
&= \frac{i\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2d} - \frac{i\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, a+bx\right)}{2d} \\
&= -\frac{i\operatorname{Li}_2\left(-\frac{i}{a+bx}\right)}{2d} + \frac{i\operatorname{Li}_2\left(\frac{i}{a+bx}\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.84

$$
-\frac{i\left(\operatorname{PolyLog}\left(2,-\frac{i}{a+bx}\right)-\operatorname{PolyLog}\left(2,\frac{i}{a+bx}\right)\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((-1/2*I)*(PolyLog[2, (-I)/(a + b*x)] - PolyLog[2, I/(a + b*x)]))/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(37) = 74.
time = 0.11, size = 97, normalized size = 2.16

method	result	si
risch	$\frac{i\,\mathrm{dilog}(-ibx-ia+1)}{2d}+\frac{\pi\ln(-ibx-ia)}{2d}-\frac{i\,\mathrm{dilog}(ibx+ia+1)}{2d}$	5
derivativedivides	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9
default	$\frac{\frac{b\ln(bx+a)\,\mathrm{arccot}(bx+a)}{d}+\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b/d*ln(b*x+a)*arccot(b*x+a)+b/d*(-1/2*I*ln(b*x+a)*ln(1+I*(b*x+a))+1/2*
I*ln(b*x+a)*ln(1-I*(b*x+a))-1/2*I*dilog(1+I*(b*x+a))+1/2*I*dilog(1-I*(b*x+a
))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(31) = 62$.
time = 0.51, size = 122, normalized size = 2.71

$$\frac{\operatorname{arccot}\left(bx+a\right)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan\left(\frac{b^{2}x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}+\frac{\arctan\left(bx+a,0\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)-2\,\arctan\left(bx+a\right)\log\left(|bx+a|\right)+i\,\mathrm{Li}_{2}(i\,bx+i\,a+1)-i\,\mathrm{Li}_{2}(-i\,bx-i\,a+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] arccot(b*x + a)*log(d*x + a*d/b)/d + arctan((b^2*x + a*b)/b)*log(d*x + a*d/
b)/d + 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan
(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I
*a + 1))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccot(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{acot}\left(a+bx\right)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acot(a + b*x)/(a + b*x), x)/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(31) = 62$.
time = 0.68, size = 103, normalized size = 2.29

$$-\frac{\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^{4}+2\,\arctan\left(\frac{1}{bx+a}\right)\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^{2}-2\,\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^{3}+\arctan\left(\frac{1}{bx+a}\right)+2\,\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)}{8\,bd\tan\left(\frac{1}{2}\,\arctan\left(\frac{1}{bx+a}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] -1/8*(arctan(1/(b*x + a))*tan(1/2*arctan(1/(b*x + a)))^4 + 2*arctan(1/(b*x
+ a))*tan(1/2*arctan(1/(b*x + a)))^2 - 2*tan(1/2*arctan(1/(b*x + a)))^3 + a
```

```
rctan(1/(b*x + a)) + 2*tan(1/2*arctan(1/(b*x + a))))/(b*d*tan(1/2*arctan(1/
(b*x + a)))^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acot}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)/(d*x + (a*d)/b),x)
[Out] int(acot(a + b*x)/(d*x + (a*d)/b), x)
```

3.128 $\int (a+bx)^2 \sqrt{\cot^{-1}(a+bx)}\, dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left((a+bx)^2\sqrt{\cot^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

```
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

Rubi steps

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\, dx = \int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\, dx$$

Mathematica [A]
time = 6.42, size = 0, normalized size = 0.00

$$\int (a+bx)^2\sqrt{\cot^{-1}(a+bx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]],x]
```

```
[Out] Integrate[(a + b*x)^2*Sqrt[ArcCot[a + b*x]], x]
```

Maple [A]
time = 0.60, size = 0, normalized size = 0.00

$$\int (bx+a)^2\sqrt{\operatorname{arccot}(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

```
[Out] int((b*x+a)^2*arccot(b*x+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2 \sqrt{\operatorname{acot}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*acot(b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(acot(a + b*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccot(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*sqrt(arccot(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\operatorname{acot}(a+b\,x)}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*x)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(acot(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.129 $\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal. Leaf size=233

$$\frac{bf\left(6d^2e^2-12cdef-(1-6c^2)\,f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}+\frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f}$$

```
[Out] 1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d
*x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccot(d*x+c))/f+1/
4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4
-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(1+c)
*f)*ln(1+(d*x+c)^2)/d^4
```

Rubi [A]

time = 0.28, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\frac{(e+fx)^4\left(a+b\cot^{-1}(c+dx)\right)}{4f}+\frac{b\text{ArcTan}(c+dx)\left(-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+(c^4-6c^2+1)\,f^4-4cd^3e^3f+d^4e^4\right)}{4d^4f}+\frac{bfx\left(-(1-6c^2)\,f^2-12cdef+6d^2e^2\right)}{4d^3}+\frac{bf^2(c+dx)^2(de-cf)}{2d^4}+\frac{b(de-cf)(-cf+de+f)(de-(c+1)f)\log\left((c+dx)^2+1\right)}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcCot[c + d*x]))/(4*f) + (b*(d^4*e^4 - 4*c*d^3*e^3*f - 6*(1 - c^2)*d^2
*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2 + c^4)*f^4)*ArcTan[c + d*x])/
(4*d^4*f) + (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c + d
*x)^2])/(2*d^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\cot^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4 \left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2} dx, x, c+ \right.}{4f}$$

$$= \frac{(e+fx)^4 \left(a+b\cot^{-1}(c+dx)\right)}{4f} + \frac{b\text{Subst}\left(\int \left(\frac{f^2\left(6d^2e^2-12cdef-(1-6c^2)\right.}{d^4}\right.\right.}{}$$

$$= \frac{bf\left(6d^2e^2-12cdef-(1-6c^2)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{b}{}$$

$$= \frac{bf\left(6d^2e^2-12cdef-(1-6c^2)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{b}{}$$

$$= \frac{bf\left(6d^2e^2-12cdef-(1-6c^2)f^2\right)x}{4d^3} + \frac{bf^2(de-cf)(c+dx)^2}{2d^4} + \frac{b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 157, normalized size = 0.67

$$\frac{(e+fx)^4 \left(a+b\cot^{-1}(c+dx)\right) + \frac{b\left(6df^2\left(6d^2e^2-12cdef+(-1+6c^2)f^2\right)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4\log(i-c-dx)+3i(de-(i+c)f)^4\log(i+c+dx)\right)}{6d^4}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(221) = 442$.
time = 0.57, size = 727, normalized size = 3.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b*ln(1+(d*x+c)^2)*e^3-b/d^3*f^3*arccot(d*x+c)*c^3*(d*x+c)-3*b/d^2*
f^2*c*e*(d*x+c)+3*b/d^2*f^2*arctan(d*x+c)*c*e+3/2*b/d^2*f^2*ln(1+(d*x+c)^2)
*c^2*e-3/2*b/d*f*ln(1+(d*x+c)^2)*c*e^2-3*b/d*f*arccot(d*x+c)*c*e^2*(d*x+c)-
3*b/d^2*f^2*arccot(d*x+c)*c*e*(d*x+c)^2+3*b/d^2*f^2*arccot(d*x+c)*c^2*e*(d*
x+c)-b/d^2*f^2*arctan(d*x+c)*c^3*e+3/2*b/d*f*arctan(d*x+c)*c^2*e^2+3/2*b/d*
f*arccot(d*x+c)*c^2*e^2+3/2*b/d*f*arccot(d*x+c)*e^2*(d*x+c)^2+b/d^2*f^2*arc
cot(d*x+c)*e*(d*x+c)^3-b/d^2*f^2*arccot(d*x+c)*c^3*e-b/d^3*f^3*arccot(d*x+c
)*c*(d*x+c)^3+3/2*b/d^3*f^3*arccot(d*x+c)*c^2*(d*x+c)^2-b*arctan(d*x+c)*c*e
^3-b*arccot(d*x+c)*c*e^3+b*arccot(d*x+c)*e^3*(d*x+c)+1/4*b*d/f*arccot(d*x+c
)*e^4+1/4*b/d^3*f^3*arccot(d*x+c)*c^4+1/4*b/d^3*f^3*arccot(d*x+c)*(d*x+c)^4
+1/4*b/d^3*f^3*arctan(d*x+c)*c^4+1/4*b*d/f*arctan(d*x+c)*e^4-1/2*b/d^3*f^3*
c*(d*x+c)^2+3/2*b/d*f*e^2*(d*x+c)+1/2*b/d^2*f^2*e*(d*x+c)^2-1/2*b/d^2*f^2*l
n(1+(d*x+c)^2)*e-3/2*b/d*f*arctan(d*x+c)*e^2-1/2*b/d^3*f^3*ln(1+(d*x+c)^2)*
c^3+1/2*b/d^3*f^3*ln(1+(d*x+c)^2)*c-3/2*b/d^3*f^3*arctan(d*x+c)*c^2+3/2*b/d
^3*f^3*c^2*(d*x+c)+1/4*(c*f-d*e-f*(d*x+c))^4*a/d^3/f+1/12*b/d^3*f^3*(d*x+c)
^3-1/4*b/d^3*f^3*(d*x+c)+1/4*b/d^3*f^3*arctan(d*x+c))
```

Maxima [A]
time = 0.47, size = 339, normalized size = 1.45

$$\frac{1}{4}af^3x^4 + af^2x^3e + \frac{1}{12}\left(3x^4\operatorname{arccot}(dx+c) + d\left(\frac{d^2x^3 - 3cdx^2 + 3(3c^2-1)x}{d^4} + \frac{3(c^4-6c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5} - \frac{6(c^3-c)\log(d^2x^2+2cdx+c^2+1)}{d^5}\right)\right)bf^3 + \frac{3}{2}afx^2e^2 + \frac{1}{2}\left(2x^3\operatorname{arccot}(dx+c) + d\left(\frac{dx^2-4cx}{d^3} - \frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3c^2-1)\log(d^2x^2+2cdx+c^2+1)}{d^4}\right)\right)bf^2e + \frac{3}{2}\left(x^2\operatorname{arccot}(dx+c) + d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bfe^2 + axe^3 + \frac{(2(dx+c)\operatorname{arccot}(dx+c) + \log((dx+c)^2+1))be^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*f^2*x^3*e + 1/12*(3*x^4*arccot(d*x + c) + d*((d^2*x^3 - 3
*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x + c*d)/
d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + 3/2*a*f
*x^2*e^2 + 1/2*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3
*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^4))*b*f^2*e + 3/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan(
```

```
(d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f*e^2 + a
*x*e^3 + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e^3/d
```

Fricas [A]
time = 3.68, size = 325, normalized size = 1.39

$$\frac{3\,ad^4f^3x^4 + bd^3f^3x^3 - 3\,bcd^2f^3x^2 + 12\,ad^4xe^3 + 3\,(3\,bc^2 - b)df^3x + 3\,(bd^4f^3x^4 + 4\,bd^4f^2x^3e + 6\,bd^4fx^2e^2 + 4\,bd^4xe^3)\operatorname{arccot}(dx+c) - 3\,(4\,bcd^3e^3 - 6\,(bc^2 - b)d^2fe^2 + 4\,(bc^3 - 3\,bc)df^2e - (bc^4 - 6\,bc^2 + b)f^3)\arctan(dx+c) + 18\,(ad^4fx^2 + bd^3fx)e^2 + 6\,(2\,ad^4f^2x^3 + bd^3f^2x^2 - 4\,bcd^2f^2x)e - 6\,(3\,bcd^2fe^2 - bd^3e^3 - (3\,bc^2 - b)df^2e + (bc^3 - bc)f^3)\log(d^2x^2 + 2\,cdx + c^2 + 1)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 + b*d^3*f^3*x^3 - 3*b*c*d^2*f^3*x^2 + 12*a*d^4*x*e^3
+ 3*(3*b*c^2 - b)*d*f^3*x + 3*(b*d^4*f^3*x^4 + 4*b*d^4*f^2*x^3*e + 6*b*d^4*
f*x^2*e^2 + 4*b*d^4*x*e^3)*arccot(d*x + c) - 3*(4*b*c*d^3*e^3 - 6*(b*c^2 -
b)*d^2*f*e^2 + 4*(b*c^3 - 3*b*c)*d*f^2*e - (b*c^4 - 6*b*c^2 + b)*f^3)*arcta
n(d*x + c) + 18*(a*d^4*f*x^2 + b*d^3*f*x)*e^2 + 6*(2*a*d^4*f^2*x^3 + b*d^3*
f^2*x^2 - 4*b*c*d^2*f^2*x)*e - 6*(3*b*c*d^2*f*e^2 - b*d^3*e^3 - (3*b*c^2 -
b)*d*f^2*e + (b*c^3 - b*c)*f^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 12.36, size = 654, normalized size = 2.81

$$\begin{cases} ae^3x + \frac{3ae^2fx^2}{2} + aef^2x^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3\operatorname{acot}(c+dx)}{4d^4} + \frac{bc^3ef^2\operatorname{acot}(c+dx)}{d^3} - \frac{bc^3f^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^4} - \frac{ibc^3f^3\operatorname{acot}(c+dx)}{d^4} - \frac{3bc^2e^2f\operatorname{acot}(c+dx)}{2d^2} + \frac{3bc^2ef^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{3ibc^2ef^2\operatorname{acot}(c+dx)}{d^3} + \frac{3bc^2f^3x}{4d^3} + \frac{3bc^2f^3\operatorname{acot}(c+dx)}{2d^4} + \frac{bce^3\operatorname{acot}(c+dx)}{d} - \frac{3bce^2f\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{3ibce^2f\operatorname{acot}(c+dx)}{d^2} - \frac{2bcef^2x}{d^2} - \frac{bcf^3x^2}{4d^2} - \frac{3bcef^2\operatorname{acot}(c+dx)}{d^3} + \frac{bcf^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^4} + \frac{ibcf^3\operatorname{acot}(c+dx)}{d^4} + be^3x\operatorname{acot}(c+dx) + \frac{3be^2fx^2\operatorname{acot}(c+dx)}{2} + bef^2x^3\operatorname{acot}(c+dx) + \frac{bf^3x^4\operatorname{acot}(c+dx)}{4} + \frac{be^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe^3\operatorname{acot}(c+dx)}{d} + \frac{3be^2fx}{2d} + \frac{bef^2x^2}{2d} + \frac{bf^3x^3}{12d} + \frac{3be^2f\operatorname{acot}(c+dx)}{2d^2} - \frac{bef^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} - \frac{ibef^2\operatorname{acot}(c+dx)}{d^3} - \frac{bf^3x}{4d^3} - \frac{bf^3\operatorname{acot}(c+dx)}{4d^4} & \text{for}\: d \neq 0 \\ (a + b\operatorname{acot}(c))\left(e^3x + \frac{3e^2fx^2}{2} + ef^2x^3 + \frac{f^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*acot(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*acot(c + d*x)/(4*d**4) + b*c**3*e*f**2*acot(c + d*x)/d**3 - b*c*
*3*f**3*log(c/d + x - I/d)/d**4 - I*b*c**3*f**3*acot(c + d*x)/d**4 - 3*b*c*
*2*e**2*f*acot(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x - I/d)/d**3
+ 3*I*b*c**2*e*f**2*acot(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) + 3*b*c**
2*f**3*acot(c + d*x)/(2*d**4) + b*c*e**3*acot(c + d*x)/d - 3*b*c*e**2*f*log
(c/d + x - I/d)/d**2 - 3*I*b*c*e**2*f*acot(c + d*x)/d**2 - 2*b*c*e*f**2*x/d
**2 - b*c*f**3*x**2/(4*d**2) - 3*b*c*e*f**2*acot(c + d*x)/d**3 + b*c*f**3*l
og(c/d + x - I/d)/d**4 + I*b*c*f**3*acot(c + d*x)/d**4 + b*e**3*x*acot(c +
d*x) + 3*b*e**2*f*x**2*acot(c + d*x)/2 + b*e*f**2*x**3*acot(c + d*x) + b*f*
*3*x**4*acot(c + d*x)/4 + b*e**3*log(c/d + x - I/d)/d + I*b*e**3*acot(c + d
*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/(12*d) + 3*b
*e**2*f*acot(c + d*x)/(2*d**2) - b*e*f**2*log(c/d + x - I/d)/d**3 - I*b*e*f
**2*acot(c + d*x)/d**3 - b*f**3*x/(4*d**3) - b*f**3*acot(c + d*x)/(4*d**4),
 Ne(d, 0)), ((a + b*acot(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3
*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2265 vs. $2(216) = 432$.

time = 1.65, size = 2265, normalized size = 9.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(96*b*d^3*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 2
88*b*c*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 288*b
*c^2*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 96*b*c^3*
f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 72*b*d^2*e^2*f*arc
tan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 144*b*c*d*e*f^2*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*c^2*f^3*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^6 + 24*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2
*arctan(1/(d*x + c)))^7 - 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^7 - 3*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^8 +
 96*b*d^3*e^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x
+ c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c
)))^4 - 288*b*c*d^2*e^2*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*ar
ctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arcta
n(1/(d*x + c)))^4 + 288*b*c^2*d*e*f^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2
/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*t
an(1/2*arctan(1/(d*x + c)))^4 - 96*b*c^3*f^3*log(16*tan(1/2*arctan(1/(d*x +
 c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2
+ 1))*tan(1/2*arctan(1/(d*x + c)))^4 + 96*a*d^3*e^3*tan(1/2*arctan(1/(d*x +
 c)))^5 - 288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^5 + 288*a*c^2*d*e*
f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 96*a*c^3*f^3*tan(1/2*arctan(1/(d*x + c
)))^5 - 72*a*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^6 + 144*a*c*d*e*f^2*tan
(1/2*arctan(1/(d*x + c)))^6 - 72*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^6 +
 24*a*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^7 - 24*a*c*f^3*tan(1/2*arctan(1/
(d*x + c)))^7 - 3*a*f^3*tan(1/2*arctan(1/(d*x + c)))^8 - 96*b*d^3*e^3*arcta
n(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 288*b*c*d^2*e^2*f*arctan(1/
(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 288*b*c^2*d*e*f^2*arctan(1/(d*x
 + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 96*b*c^3*f^3*arctan(1/(d*x + c))*ta
n(1/2*arctan(1/(d*x + c)))^3 - 144*b*d^2*e^2*f*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c)))^4 + 288*b*c*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan
(1/(d*x + c)))^4 - 144*b*c^2*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
+ c)))^4 + 144*b*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^5 - 288*b*c*d*e*f^2
*tan(1/2*arctan(1/(d*x + c)))^5 + 144*b*c^2*f^3*tan(1/2*arctan(1/(d*x + c))
)^5 - 72*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 72*
b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 - 24*b*d*e*f^2*t
an(1/2*arctan(1/(d*x + c)))^6 + 24*b*c*f^3*tan(1/2*arctan(1/(d*x + c)))^6 +
 12*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^6 + 2*b*f^3*tan(
1/2*arctan(1/(d*x + c)))^7 - 96*a*d^3*e^3*tan(1/2*arctan(1/(d*x + c)))^3 +
288*a*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^3 - 288*a*c^2*d*e*f^2*tan(1/
```

```
2*arctan(1/(d*x + c)))^3 + 96*a*c^3*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 14
4*a*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^4 + 288*a*c*d*e*f^2*tan(1/2*arct
an(1/(d*x + c)))^4 - 144*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^4 - 96*b*d*
e*f^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4
 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 +
96*b*c*f^3*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c
)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))
^4 - 72*a*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^5 + 72*a*c*f^3*tan(1/2*arcta
n(1/(d*x + c)))^5 + 12*a*f^3*tan(1/2*arctan(1/(d*x + c)))^6 - 72*b*d^2*e^2*
f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 144*b*c*d*e*f^2*arct
an(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 72*b*c^2*f^3*arctan(1/(d*x
 + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 144*b*d^2*e^2*f*tan(1/2*arctan(1/(d
*x + c)))^3 + 288*b*c*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 144*b*c^2*f^
3*tan(1/2*arctan(1/(d*x + c)))^3 + 72*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2
*arctan(1/(d*x + c)))^3 - 72*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^3 - 48*b*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 48*b*c*f^3*tan
(1/2*arctan(1/(d*x + c)))^4 + 30*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1
/(d*x + c)))^4 - 30*b*f^3*tan(1/2*arctan(1/(d*x + c)))^5 - 72*a*d^2*e^2*f*t
an(1/2*arctan(1/(d*x + c)))^2 + 144*a*c*d*e*f^2*tan(1/2*arctan(1/(d*x + c))
)^2 - 72*a*c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + 72*a*d*e*f^2*tan(1/2*ar
ctan(1/(d*x + c)))^3 - 72*a*c*f^3*tan(1/2*arctan(1/(d*x + c)))^3 + 30*a*f^3
*tan(1/2*arctan(1/(d*x + c)))^4 - 24*b*d*e*f^2*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c))) + 24*b*c*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
 + c))) - 24*b*d*e*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + 24*b*c*f^3*tan(1/2*
arctan(1/(d*x + c)))^2 + 12*b*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x
 + c)))^2 + 30*b*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 24*a*d*e*f^2*tan(1/2*
arctan(1/(d*x + c))) + 24*a*c*f^3*tan(1/2*arctan(1/(d*x + c))) + 12*a*f^3*t
an(1/2*arctan(1/(d*x + c)))^2 - 3*b*f^3*arctan(1/(d*x + c)) - 2*b*f^3*tan(1
/2*arctan(1/(d*x + c))) - 3*a*f^3)/(d^4*tan(1/2...
```

Mupad [B]
time = 1.19, size = 783, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*acot(c + d*x)),x)
```

```
[Out] acot(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3) +
 x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 + 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/
d))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f
^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*
d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(b*f +
8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 + 4
```

```
*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c^2 + 4))
/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(12*d) - (2*a*c*f^3)/(3*d
)) + (a*f^3*x^4)/4 + (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*e^3 - 64*b
*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2*f + 192*b*
c^2*d^5*e*f^2))/(128*d^8) + (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3 + c^4*f^3 -
 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2
))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*
c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^3 - 6*c^2*f^3 +
 c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c
^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2
*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.130 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal. Leaf size=154

$$\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)f^2\right)\text{Arc}}{3d^3f}$$

```
[Out] b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccot(d*x+
c))/f+1/3*b*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f
+1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Rubi [A]
time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b\text{ArcTan}(c+dx)(de-cf)\left(-(3-c^2)f^2-2cdef+d^2e^2\right)}{3d^3f}+\frac{b\left(-(1-3c^2)f^2-6cdef+3d^2e^2\right)\log\left((c+dx)^2+1\right)}{6d^3}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{bfx(de-cf)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcCot[c + d*x]))/(3*f) + (b*(d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*
f^2)*ArcTan[c + d*x])/(3*d^3*f) + (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f
^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2\left(a+b\cot^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b\text{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2}\,dx,x,c+d\right.}{3f}$$

$$=\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b\text{Subst}\left(\int\left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-\right.\right.}{}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b}{}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{(}{}$$

$$=\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)}{3f}+\frac{b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 118, normalized size = 0.77

$$\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)+\frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcCot[c + d*x]) + (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.
time = 0.28, size = 430, normalized size = 2.79

method	result
risch	$-\frac{ifbe\,x^{2}\ln(1-i(dx+c))}{2}+\frac{x^{3}f^{2}a}{3}+\frac{f^{2}b\,x^{2}}{6d}-\frac{ib\,e^{3}\ln\left(d^{2}x^{2}+2cdx+c^{2}+1\right)}{12f}+\frac{i(fx+e)^{3}b\ln(1+i(dx+c))}{6f}-\frac{f^{2}b\,\mathrm{l}}{}$
derivativedivides	$-\frac{(cf-de-f(dx+c))^{3}a}{3d^{2}f}-\frac{b\,f^{2}\operatorname{arccot}(dx+c)c^{3}}{3d^{2}}+\frac{bf\,\operatorname{arccot}(dx+c)c^{2}e}{d}+\frac{b\,f^{2}\operatorname{arccot}(dx+c)c^{2}(dx+c)}{d^{2}}-b\operatorname{arccot}(dx+c)c\,e^{2}-\frac{2bf\,\operatorname{arccot}}{}$
default	$-\frac{(cf-de-f(dx+c))^{3}a}{3d^{2}f}-\frac{b\,f^{2}\operatorname{arccot}(dx+c)c^{3}}{3d^{2}}+\frac{bf\,\operatorname{arccot}(dx+c)c^{2}e}{d}+\frac{b\,f^{2}\operatorname{arccot}(dx+c)c^{2}(dx+c)}{d^{2}}-b\operatorname{arccot}(dx+c)c\,e^{2}-\frac{2bf\,\operatorname{arccot}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*(c*f-d*e-f*(d*x+c))^3*a/d^2/f-1/3*b/d^2*f^2*arccot(d*x+c)*c^3+b/d
*f*arccot(d*x+c)*c^2*e+b/d^2*f^2*arccot(d*x+c)*c^2*(d*x+c)-b*arccot(d*x+c)*
c*e^2-2*b/d*f*arccot(d*x+c)*c*e*(d*x+c)-b/d^2*f^2*arccot(d*x+c)*c*(d*x+c)^2
+1/3*b*d/f*arccot(d*x+c)*e^3+b*arccot(d*x+c)*e^2*(d*x+c)+b/d*f*arccot(d*x+c
)*e*(d*x+c)^2+1/3*b/d^2*f^2*arccot(d*x+c)*(d*x+c)^3-b/d^2*f^2*c*(d*x+c)+b/d
*f*e*(d*x+c)+1/6*b/d^2*f^2*(d*x+c)^2+1/2*b/d^2*f^2*ln(1+(d*x+c)^2)*c^2-b/d*
f*ln(1+(d*x+c)^2)*c*e+1/2*e^2*b*ln(1+(d*x+c)^2)-1/6*b/d^2*f^2*ln(1+(d*x+c)^
2)-1/3*b/d^2*f^2*arctan(d*x+c)*c^3+b/d*f*arctan(d*x+c)*c^2*e-b*arctan(d*x+c
)*c*e^2+1/3*b*d/f*arctan(d*x+c)*e^3+b/d^2*f^2*arctan(d*x+c)*c-b/d*f*arctan(
d*x+c)*e)
```

Maxima [A]
time = 0.47, size = 216, normalized size = 1.40

$$\frac{1}{3}af^{2}x^{3}+afx^{2}e+\frac{1}{6}\left(2x^{3}\operatorname{arccot}(dx+c)+d\left(\frac{dx^{2}-4cx}{d^{3}}-\frac{2(c^{3}-3c)\arctan\left(\frac{d^{2}x+cd}{d}\right)}{d^{4}}+\frac{(3c^{2}-1)\log(d^{2}x^{2}+2cdx+c^{2}+1)}{d^{4}}\right)\right)bf^{2}+\left(x^{2}\operatorname{arccot}(dx+c)+d\left(\frac{x}{d^{2}}+\frac{(c^{2}-1)\arctan\left(\frac{d^{2}x+cd}{d}\right)}{d^{3}}-\frac{c\log(d^{2}x^{2}+2cdx+c^{2}+1)}{d^{3}}\right)\right)bfe+axe^{2}+\frac{(2(dx+c)\operatorname{arccot}(dx+c)+\log((dx+c)^{2}+1))be^{2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + 1/6*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)
/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/d^4))*b*f^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 -
 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*
b*f*e + a*x*e^2 + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*
b*e^2/d
```

Fricas [**A**]
time = 2.45, size = 208, normalized size = 1.35

$$\frac{2\,ad^{3}f^{2}x^{3} + bd^{2}f^{2}x^{2} - 4\,bcdf^{2}x + 6\,ad^{3}xe^{2} + 2\,(bd^{3}f^{2}x^{3} + 3\,bd^{3}fx^{2}e + 3\,bd^{3}xe^{2})\operatorname{arccot}(dx + c) - 2\,(3\,bcd^{2}e^{2} - 3\,(bc^{2} - b)dfe + (bc^{3} - 3\,bc)f^{2})\arctan(dx + c) + 6\,(ad^{3}fx^{2} + bd^{2}fx)e - (6\,bcdfe - 3\,bd^{2}e^{2} - (3\,bc^{2} - b)f^{2})\log(d^{2}x^{2} + 2\,cdx + c^{2} + 1)}{6\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + b*d^2*f^2*x^2 - 4*b*c*d*f^2*x + 6*a*d^3*x*e^2 + 2*(b
*d^3*f^2*x^3 + 3*b*d^3*f*x^2*e + 3*b*d^3*x*e^2)*arccot(d*x + c) - 2*(3*b*c*
d^2*e^2 - 3*(b*c^2 - b)*d*f*e + (b*c^3 - 3*b*c)*f^2)*arctan(d*x + c) + 6*(a
*d^3*f*x^2 + b*d^2*f*x)*e - (6*b*c*d*f*e - 3*b*d^2*e^2 - (3*b*c^2 - b)*f^2)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^3
```

Sympy [**C**] Result contains complex when optimal does not.
time = 4.58, size = 376, normalized size = 2.44

$$\begin{cases} ae^{2}x + aefx^{2} + \frac{af^{2}x^{3}}{3} + \frac{bc^{3}f^{2}\operatorname{acot}(c+dx)}{3d^{3}} - \frac{bc^{2}ef\operatorname{acot}(c+dx)}{d^{2}} + \frac{bc^{2}f^{2}\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^{3}} + \frac{ibc^{2}f^{2}\operatorname{acot}(c+dx)}{d^{3}} + \frac{bce^{2}\operatorname{acot}(c+dx)}{d} - \frac{2bcef\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^{2}} - \frac{2ibcef\operatorname{acot}(c+dx)}{d^{2}} - \frac{2bcf^{2}x}{3d^{2}} - \frac{bcf^{2}\operatorname{acot}(c+dx)}{d^{3}} + be^{2}x\operatorname{acot}(c+dx) + befx^{2}\operatorname{acot}(c+dx) + \frac{bf^{2}x^{3}\operatorname{acot}(c+dx)}{3} + \frac{be^{2}\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe^{2}\operatorname{acot}(c+dx)}{d} + \frac{befx}{d} + \frac{bf^{2}x^{2}}{6d} + \frac{bef\operatorname{acot}(c+dx)}{d^{2}} - \frac{bf^{2}\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{3d^{3}} - \frac{ibf^{2}\operatorname{acot}(c+dx)}{3d^{3}} & \text{for}\: d \neq 0 \\ (a + b\operatorname{acot}(c))\left(e^{2}x + efx^{2} + \frac{f^{2}x^{3}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acot(c + d*x
)/(3*d**3) - b*c**2*e*f*acot(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x - I/d)
/d**3 + I*b*c**2*f**2*acot(c + d*x)/d**3 + b*c*e**2*acot(c + d*x)/d - 2*b*c
*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*acot(c + d*x)/d**2 - 2*b*c*f**2*
x/(3*d**2) - b*c*f**2*acot(c + d*x)/d**3 + b*e**2*x*acot(c + d*x) + b*e*f*x
**2*acot(c + d*x) + b*f**2*x**3*acot(c + d*x)/3 + b*e**2*log(c/d + x - I/d)
/d + I*b*e**2*acot(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) + b*e*f*acot(
c + d*x)/d**2 - b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*acot(c + d*x)
/(3*d**3), Ne(d, 0)), ((a + b*acot(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), T
rue))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(142) = 284$.
time = 1.15, size = 1161, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(12*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 24
*b*c*d*e*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 12*b*c^2*f^
2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(1/(
```

```
d*x + c))*tan(1/2*arctan(1/(d*x + c)))^5 + 6*b*c*f^2*arctan(1/(d*x + c))*ta
n(1/2*arctan(1/(d*x + c)))^5 + b*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(
d*x + c)))^6 + 12*b*d^2*e^2*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*
arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arc
tan(1/(d*x + c)))^3 - 24*b*c*d*e*f*log(16*tan(1/2*arctan(1/(d*x + c)))^2/(t
an(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(
1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2*f^2*log(16*tan(1/2*arctan(1/(d*x + c)
))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1
))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*a*d^2*e^2*tan(1/2*arctan(1/(d*x + c)
))^4 - 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 + 12*a*c^2*f^2*tan(1/2*a
rctan(1/(d*x + c)))^4 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x + c)))^5 + 6*a*c*f^
2*tan(1/2*arctan(1/(d*x + c)))^5 + a*f^2*tan(1/2*arctan(1/(d*x + c)))^6 - 1
2*b*d^2*e^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 24*b*c*d*e
*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*c^2*f^2*arctan
(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 12*b*d*e*f*arctan(1/(d*x + c
))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c*f^2*arctan(1/(d*x + c))*tan(1/2*
arctan(1/(d*x + c)))^3 + 12*b*d*e*f*tan(1/2*arctan(1/(d*x + c)))^4 - 12*b*c
*f^2*tan(1/2*arctan(1/(d*x + c)))^4 - 3*b*f^2*arctan(1/(d*x + c))*tan(1/2*a
rctan(1/(d*x + c)))^4 - b*f^2*tan(1/2*arctan(1/(d*x + c)))^5 - 12*a*d^2*e^2
*tan(1/2*arctan(1/(d*x + c)))^2 + 24*a*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))
^2 - 12*a*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 12*a*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^3 + 12*a*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*f^2*log
(16*tan(1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(
1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 - 3*a*f^2*t
an(1/2*arctan(1/(d*x + c)))^4 - 6*b*d*e*f*arctan(1/(d*x + c))*tan(1/2*arcta
n(1/(d*x + c))) + 6*b*c*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))
) - 12*b*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 12*b*c*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 + 3*b*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2
 - 2*b*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 6*a*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) + 6*a*c*f^2*tan(1/2*arctan(1/(d*x + c))) + 3*a*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - b*f^2*arctan(1/(d*x + c)) - b*f^2*tan(1/2*arctan(1/(d*x +
c))) - a*f^2)/(d^3*tan(1/2*arctan(1/(d*x + c)))^3)
```

Mupad [B]
time = 0.96, size = 409, normalized size = 2.66

$$x^2\left(\frac{f\,\left(b\,f+6\,a\,c\,f+6\,a\,d\,e\right)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-x\,\left(\frac{2\,c\,\left(\frac{f\,\left(b\,f+6\,a\,c\,f+6\,a\,d\,e\right)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,\left(3\,c^2+3\right)}{3\,d^2}\right)+\mathrm{acot}(c+d\,x)\,\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3}+\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6}-\frac{b\,\mathrm{atan}\left(\frac{3\,d^2\,\left(\frac{c\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^2}+\frac{x\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\,\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x)),x)
```

```
[Out] x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(b*f
+ 6*a*c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2 +
3*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3*d^
2)) + acot(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)/3 +
```

```
 (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b*c^2*
d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) - (b*atan((3*d^2*((c*(c^3*f^2 - 3*c*f^2
 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 + 3*c*d^2*
e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f -
3*c^2*d*e*f))/(3*d^3)
```

3.131 $\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)(de-(1+c)f)\text{ArcTan}(c+dx)}{2d^2f}+\frac{b(de-cf)\log\left(1+\right.}{2d^2}$$

[Out] 1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))/f+1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5156, 4973, 716, 649, 209, 266}

$$\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\text{ArcTan}(c+dx)(-cf+de+f)(de-(c+1)f)}{2d^2f}+\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}+\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcCot[c + d*x]))/(2*f) + (b*(d*e + f -
 c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d^2*f) + (b*(d*e - c*f)*Log[1 +
 (c + d*x)^2])/(2*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)\,dx=\frac{\operatorname{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\operatorname{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}\,dx,x,c+dx\right.}{2f}$$

$$=\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\operatorname{Subst}\left(\int\left(\frac{f^2}{d^2}+\frac{(de-f-cf)(de+f-cf)+}{d^2(1+x^2)}\right.\right.}{2f}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b\operatorname{Subst}\left(\int\frac{(de-f-cf)(de+f-cf)}{1+x^2}\right.}{2d^2f}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{(b(de-cf))\operatorname{Subst}\left(\int\frac{x}{1+x^2}\,d\right.}{d^2}$$

$$=\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)(de-(1+c)}{2d^2f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 163, normalized size = 1.68

$$aex+\frac{1}{2}afx^2+bex\cot^{-1}(c+dx)+\frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\cot^{-1}(c+dx)+\frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d}+\frac{be\left(-2c\operatorname{ArcTan}(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcCot[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcCot[c + d*x])/2 + (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
```

```
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d + (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Maple [A]

time = 0.12, size = 150, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arccot}(dx+c)fc(dx+c)}{d}+b\,\mathrm{arccot}(dx+c)e(dx+c)+\frac{b\,\mathrm{arccot}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx+c}{2d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arccot}(dx+c)fc(dx+c)}{d}+b\,\mathrm{arccot}(dx+c)e(dx+c)+\frac{b\,\mathrm{arccot}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx+c}{2d}}{d}$
risch	$\frac{ib(f\,x^2+2ex)\ln(1+i(dx+c))}{4}-\frac{ibf\,x^2\ln(1-i(dx+c))}{4}+\frac{\pi bf\,x^2}{4}-\frac{ibex\ln(1-i(dx+c))}{2}+\frac{\pi bex}{2}+\frac{af\,x^2}{2}+\frac{b\,c^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccot(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)-b/d*arccot(d*x+c)*f*c*(
d*x+c)+b*arccot(d*x+c)*e*(d*x+c)+1/2*b/d*arccot(d*x+c)*f*(d*x+c)^2+1/2*b/d*
f*(d*x+c)-1/2*b/d*ln(1+(d*x+c)^2)*f*c+1/2*b*ln(1+(d*x+c)^2)*e-1/2*b/d*f*arc
tan(d*x+c))
```

Maxima [A]

time = 0.48, size = 115, normalized size = 1.19

$$\frac{1}{2}afx^2+\frac{1}{2}\left(x^2\,\mathrm{arccot}\,(dx+c)+d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2\,cdx+c^2+1)}{d^3}\right)\right)bf+axe+\frac{(2\,(dx+c)\,\mathrm{arccot}\,(dx+c)+\log\left((dx+c)^2+1\right))be}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*x*e + 1/2
*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b*e/d
```

Fricas [A]

time = 7.95, size = 114, normalized size = 1.18

$$\frac{ad^2fx^2+2\,ad^2xe+bdfx+(bd^2fx^2+2\,bd^2xe)\,\mathrm{arccot}\,(dx+c)-(2\,bcde-(bc^2-b)f)\arctan(dx+c)-(bcf-bde)\log(d^2x^2+2\,cdx+c^2+1)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + 2*a*d^2*x*e + b*d*f*x + (b*d^2*f*x^2 + 2*b*d^2*x*e)*arcc
ot(d*x + c) - (2*b*c*d*e - (b*c^2 - b)*f)*arctan(d*x + c) - (b*c*f - b*d*e)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.78, size = 177, normalized size = 1.82

$$\begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2 f \operatorname{acot}(c+dx)}{2d^2} + \frac{bce \operatorname{acot}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{ibcf \operatorname{acot}(c+dx)}{d^2} + bex \operatorname{acot}(c+dx) + \frac{bfx^2 \operatorname{acot}(c+dx)}{2} + \frac{be \log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe \operatorname{acot}(c+dx)}{d} + \frac{bfx}{2d} + \frac{bf \operatorname{acot}(c+dx)}{2d^2} & \text{for } d \neq 0 \\ (a + b \operatorname{acot}(c)) \left(ex + \frac{fx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acot(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acot(c + d*x)/(2*d**2) + b*c*e*aco
t(c + d*x)/d - b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*acot(c + d*x)/d**2 +
 b*e*x*acot(c + d*x) + b*f*x**2*acot(c + d*x)/2 + b*e*log(c/d + x - I/d)/d
+ I*b*e*acot(c + d*x)/d + b*f*x/(2*d) + b*f*acot(c + d*x)/(2*d**2), Ne(d, 0
)), ((a + b*acot(c))*(e*x + f*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(89) = 178.
time = 0.52, size = 451, normalized size = 4.65

$$-\frac{4 \, b d e \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{3} - 4 \, b c f \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{3} - b f \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{4} + 4 \, b d e \log\left(\frac{16 \, \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2}}{\tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{4} + 2 \, \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} + 1}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} - 4 \, b c f \log\left(\frac{16 \, \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2}}{\tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{4} + 2 \, \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} + 1}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} + 4 \, a d e \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{3} - 4 \, a c f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{3} - a f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{4} - 4 \, b d e \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right) + 4 \, b c f \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right) - 2 \, b f \arctan\left(\frac{1}{d x + c}\right) \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} + 2 \, b f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{3} - 4 \, a d e \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right) + 4 \, a c f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right) - 2 \, a f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2} - b f \arctan\left(\frac{1}{d x + c}\right) - 2 \, b f \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right) - a f}{8 \, d^{2} \tan\left(\frac{1}{2} \arctan\left(\frac{1}{d x + c}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8*(4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*f*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d*e*log(16*tan(1/2*arctan(1/(d*x + c
)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 +
1))*tan(1/2*arctan(1/(d*x + c)))^2 - 4*b*c*f*log(16*tan(1/2*arctan(1/(d*x +
 c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2
+ 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*a*d*e*tan(1/2*arctan(1/(d*x + c)))
^3 - 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))^3 - a*f*tan(1/2*arctan(1/(d*x + c
)))^4 - 4*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) + 4*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*b*f*arctan(1/(d*x + c)
)*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*f*tan(1/2*arctan(1/(d*x + c)))^3 - 4
*a*d*e*tan(1/2*arctan(1/(d*x + c))) + 4*a*c*f*tan(1/2*arctan(1/(d*x + c)))
- 2*a*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*arctan(1/(d*x + c)) - 2*b*f*ta
n(1/2*arctan(1/(d*x + c))) - a*f)/(d^2*tan(1/2*arctan(1/(d*x + c)))^2)
```

Mupad [B]
time = 1.45, size = 136, normalized size = 1.40

$$a\,e\,x+\frac{a\,f\,x^2}{2}+\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,f\,\mathrm{acot}\left(c+d\,x\right)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{acot}\left(c+d\,x\right)}{2}+\frac{b\,f\,x}{2\,d}+b\,e\,x\,\mathrm{acot}\left(c+d\,x\right)-\frac{b\,c^2\,f\,\mathrm{acot}\left(c+d\,x\right)}{2\,d^2}-\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2}+\frac{b\,c\,e\,\mathrm{acot}\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acot(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*a
cot(c + d*x))/(2*d^2) + (b*f*x^2*acot(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x*a
cot(c + d*x) - (b*c^2*f*acot(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x^2 +
 2*c*d*x + 1))/(2*d^2) + (b*c*e*acot(c + d*x))/d
```

3.132 $\int (a + b\cot^{-1}(c + dx))\,dx$

Optimal. Leaf size=38

$$ax + \frac{b(c+dx)\cot^{-1}(c+dx)}{d} + \frac{b\log\left(1+(c+dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arccot(d*x+c)/d+1/2*b*ln(1+(d*x+c)^2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5148, 4931, 266}

$$ax + \frac{b\log\left((c+dx)^2+1\right)}{2d} + \frac{b(c+dx)\cot^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCot[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcCot[c + d*x])/d + (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cot^{-1}(c+dx)\right)\,dx &= ax+b\int \cot^{-1}(c+dx)\,dx\\
&= ax+\frac{b\text{Subst}\left(\int \cot^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}+\frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\cot^{-1}(c+dx)}{d}+\frac{b\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.29

$$
ax+bx\cot^{-1}(c+dx)+\frac{b(-2c\text{ArcTan}(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right))}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCot[c + d*x],x]
```

```
[Out] a*x + b*x*ArcCot[c + d*x] + (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

Maple [A]
time = 0.07, size = 42, normalized size = 1.11

method	result	size
derivativedivides	$\frac{(dx+c)a+b\,\mathrm{arccot}(dx+c)(dx+c)+\frac{b\ln\left(1+(dx+c)^2\right)}{2}}{d}$	39
default	$ax+b\,\mathrm{arccot}(dx+c)\,x+\frac{b\,\mathrm{arccot}(dx+c)c}{d}+\frac{b\ln\left(1+(dx+c)^2\right)}{2d}$	42
risch	$ax+\frac{ibx\ln(1+i(dx+c))}{2}-\frac{ibx\ln(1-i(dx+c))}{2}+\frac{\pi bx}{2}-\frac{bc\arctan(dx+c)}{d}+\frac{b\ln(d^2x^2+2cdx+c^2+1)}{2d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccot(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*arccot(d*x+c)*x+b/d*arccot(d*x+c)*c+1/2*b*ln(1+(d*x+c)^2)/d
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.89

$$
ax+\frac{\left(2\,(dx+c)\,\mathrm{arccot}\,(dx+c)+\log\left((dx+c)^2+1\right)\right)b}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*b/d
```

Fricas [A]
time = 6.71, size = 52, normalized size = 1.37

$$\frac{2\,bdx\,\mathrm{arccot}\,(dx+c)+2\,adx-2\,bc\arctan\,(dx+c)+b\log\,(d^2x^2+2\,cdx+c^2+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*arccot(d*x + c) + 2*a*d*x - 2*b*c*arctan(d*x + c) + b*log(d^2*
x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [A]
time = 0.15, size = 51, normalized size = 1.34

$$ax+b\left(\begin{cases}\frac{c\,\mathrm{acot}\,(c+dx)}{d}+x\,\mathrm{acot}\,(c+dx)+\frac{\log\,(c^2+2cdx+d^2x^2+1)}{2d} & \text{for } d\neq 0\\ x\,\mathrm{acot}\,(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acot(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acot(c + d*x)/d + x*acot(c + d*x) + log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*acot(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.
time = 0.45, size = 116, normalized size = 3.05

$$ax-\frac{\left(\arctan\,\left(\frac{1}{dx+c}\right)\tan\,\left(\frac{1}{2}\arctan\,\left(\frac{1}{dx+c}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{dx+c}\right)\right)^2+1}\right)\tan\,\left(\frac{1}{2}\arctan\,\left(\frac{1}{dx+c}\right)\right)-\arctan\,\left(\frac{1}{dx+c}\right)\right)b}{2\,d\tan\,\left(\frac{1}{2}\arctan\,\left(\frac{1}{dx+c}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccot(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - 1/2*(arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + log(16*tan(
1/2*arctan(1/(d*x + c)))^2/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arct
an(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) - arctan(1/(d*x + c))
)*b/(d*tan(1/2*arctan(1/(d*x + c))))
```

Mupad [B]
time = 1.29, size = 48, normalized size = 1.26

$$a\,x+\frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2}+b\,c\,\mathrm{acot}\left(c+d\,x\right)}{d}+b\,x\,\mathrm{acot}\left(c+d\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acot(c + d*x),x)
```

```
[Out] a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/2 + b*c*acot(c + d*x))/d + b*x*
acot(c + d*x)
```

3.133 $\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=162

$$-\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}-\frac{ib\text{PolyLog}\left(2,1-\frac{\ }{1\ }\right.}{2f}$$

[Out] -(a+b*arccot(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f+1/2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f

Rubi [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5156, 4967, 2449, 2352, 2497}

$$\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}+\frac{ib\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{ib\text{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[c
+ d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - ((I
/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + ((I/2)*b*PolyLog[2, 1 - (2*d
*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{a+b\cot^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+\ldots}{(de+if-cf)(1\ldots}\right.}{f}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+\ldots}{(de+if-cf)(1\ldots}\right.}{f}$$

$$= -\frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))\log\left(\frac{2d(e+\ldots}{(de+if-cf)(1\ldots}\right.}{f}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 336 vs. 2(162) = 324.
time = 0.25, size = 336, normalized size = 2.07

$$\frac{a\log(e+fx)+b\left((\cot^{-1}(c+dx)+\text{ArcTan}(c+dx))\log(e+fx)+\text{ArcTan}(c+dx)\left(\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-\log\left(\sin\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\right)\right)+\frac{1}{2}\left(\frac{1}{4}i(\pi-2\text{ArcTan}(c+dx))^2+i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)^2-(\pi-2\text{ArcTan}(c+dx))\log\left(1+e^{-2i\text{ArcTan}(c+dx)}\right)-2\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)+(\pi-2\text{ArcTan}(c+dx))\log\left(\frac{2}{\sqrt{1+(c+dx)^2}}\right)+2\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\log\left(2\sin\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\right)+i\text{PolyLog}\left(2,-e^{-2i\text{ArcTan}(c+dx)}\right)+i\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] + b*((ArcCot[c + d*x] + ArcTan[c + d*x])*Log[e + f*x] + Arc
Tan[c + d*x]*(Log[1/Sqrt[1 + (c + d*x)^2]] - Log[Sin[ArcTan[(d*e - c*f)/f]
+ ArcTan[c + d*x]]]) + ((I/4)*(Pi - 2*ArcTan[c + d*x])^2 + I*(ArcTan[(d*e -
```

```
c*f)/f] + ArcTan[c + d*x])^2 - (Pi - 2*ArcTan[c + d*x])*Log[1 + E^((-2*I)*
ArcTan[c + d*x])] - 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^(
(2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))] + (Pi - 2*ArcTan[c + d*x]
)*Log[2/Sqrt[1 + (c + d*x)^2]] + 2*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]
)*Log[2*Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, -E^((-
2*I)*ArcTan[c + d*x])] + I*PolyLog[2, E^((2*I)*(ArcTan[(d*e - c*f)/f] + Arc
Tan[c + d*x]))])/2))/f
```

Maple [A]
time = 0.17, size = 238, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}+\frac{ibd\ln(cf-de-f(dx+c))\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)}{2f}-\frac{ibd\ln(cf-de-f(dx+c))\,\mathrm{l}}{2f}}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\operatorname{arccot}(dx+c)}{f}+\frac{ibd\ln(cf-de-f(dx+c))\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)}{2f}-\frac{ibd\ln(cf-de-f(dx+c))\,\mathrm{l}}{2f}}{d}$
risch	$-\frac{ib\,\mathrm{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}-\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{\ln(icf-ide+(-idx-ic+1}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d*ln(c*f-d*e-f*(d*x+c))/f+b*d*ln(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)+
1/2*I*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))-1/2*I*b
*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))+1/2*I*b*d/f*di
log((I*f+f*(d*x+c))/(c*f-d*e+I*f))-1/2*I*b*d/f*dilog((I*f-f*(d*x+c))/(d*e+I
*f-c*f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan2(1, d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))/(f*x+e),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arccot(d*x + c) + a)/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acot}(c+d\,x)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))/(e + f*x),x)
[Out] int((a + b*acot(c + d*x))/(e + f*x), x)
```

3.134 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd(de-cf)\text{ArcTan}(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}-\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)\,f^2}+\frac{bd\log\left(1+c^2+2cdx+\right.}{2\left(d^2e^2-2cdef+(1+\right.}$$

```
[Out] (-a-b*arccot(d*x+c))/f/(f*x+e)-b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*
d*e*f+(c^2+1)*f^2)-b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+1/2*b*d*ln
(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5154, 2007, 719, 31, 648, 632, 210, 642}

$$-\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)}-\frac{bd\text{ArcTan}(c+dx)(de-cf)}{f\left((c^2+1)\,f^2-2cdef+d^2e^2\right)}+\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left((c^2+1)\,f^2-2cdef+d^2e^2\right)}-\frac{bd\log(e+fx)}{(c^2+1)\,f^2-2cdef+d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCot[c + d*x])/(f*(e + f*x))) - (b*d*(d*e - c*f)*ArcTan[c + d*x]
)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (b*d*Log[e + f*x])/(d^2*e^2 -
 2*c*d*e*f + (1 + c^2)*f^2) + (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{(bd)\int \frac{1}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{(bd)\int \frac{1}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{(bd)\int \frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(bdf)\int \frac{1}{e+fx}\,dx}{d^2e^2-2cdef+(1+c}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{(bd)\int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}\,d}{2\left(d^2e^2-2cdef+(1+c^2\right.}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{bd\log\left(1+c^2+2cdx+d\right.}{2\left(d^2e^2-2cdef+(1+c^2\right.}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{f(e+fx)} - \frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 118, normalized size = 0.77

$$\frac{-\frac{a+b\cot^{-1}(c+dx)}{e+fx}+\frac{bd((ide+f-icf)\log(i-c-dx)+(-ide+f+icf)\log(i+c+dx)-2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (-((a + b*ArcCot[c + d*x])/(e + f*x)) + (b*d*((I*d*e + f - I*c*f)*Log[I - c
 - d*x] + ((-I)*d*e + f + I*c*f)*Log[I + c + d*x] - 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Maple [A]
time = 0.21, size = 235, normalized size = 1.54

method	result
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{b\,d^2\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{b\,d^2\arctan(dx+c)c}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{b\,c}{f\left(c^2f\right.}}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\operatorname{arccot}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{b\,d^2\ln\left(1+(dx+c)^2\right)}{2c^2f^2-4cdef+2d^2e^2+2f^2}+\frac{b\,d^2\arctan(dx+c)c}{c^2f^2-2cdef+d^2e^2+f^2}-\frac{b\,c}{f\left(c^2f\right.}}{d}$
risch	$-\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}-\frac{2\ln(fx+e)bd\,f^2x+2\ln(fx+e)bdef+i\ln\left(\left(-cdf+d^2e+3idf\right)x+2icf+ide-c^2f+cde-3f\right)b\,d^2e^2-1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d^2/(c*f-d*e-f*(d*x+c))/f+b*d^2/(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)-
b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))+1/2*b*d^2/(c^2*
f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)+b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2
+f^2)*arctan(d*x+c)*c-b*d^3/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)
*e)
```

Maxima [A]
time = 0.47, size = 186, normalized size = 1.22

$$-\frac{1}{2}\left(d\left(\frac{2\,(cdf-d^2e)\arctan\left(\frac{d^2x+cd}{d}\right)}{(2\,cdf^2e-(c^2+1)f^3-d^2fe^2)d}+\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,cdfe-(c^2+1)f^2-d^2e^2}-\frac{2\,\log\left(fx+e\right)}{2\,cdfe-(c^2+1)f^2-d^2e^2}\right)+\frac{2\,\mathrm{arccot}\left(dx+c\right)}{f^2x+fe}\right)b-\frac{a}{f^2x+fe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1
)*f^3 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2
+ 1)*f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2))
 + 2*arccot(d*x + c)/(f^2*x + f*e))*b - a/(f^2*x + f*e)
```

Fricas [A]
time = 3.60, size = 229, normalized size = 1.50

$$\frac{4\,acdfe-2\,ad^2e^2-2\,(ac^2+a)f^2+2\,(2\,bcdfe-bd^2e^2-(bc^2+b)f^2)\,\mathrm{arccot}\left(dx+c\right)+2\,(bcdf^2x-bd^2e^2-(bd^2fx-bcdf)e)\arctan\left(dx+c\right)+(bdf^2x+bdfe)\log\left(d^2x^2+2\,cdx+c^2+1\right)-2\,(bdf^2x+bdfe)\log\left(fx+e\right)}{2\,((c^2+1)f^4x+d^2fe^3+(d^2f^2x-2\,cdf^2)e^2-(2\,cdf^3x-(c^2+1)f^3)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*c*d*f*e - 2*a*d^2*e^2 - 2*(a*c^2 + a)*f^2 + 2*(2*b*c*d*f*e - b*d^2
*e^2 - (b*c^2 + b)*f^2)*arccot(d*x + c) + 2*(b*c*d*f^2*x - b*d^2*e^2 - (b*d
^2*f*x - b*c*d*f)*e)*arctan(d*x + c) + (b*d*f^2*x + b*d*f*e)*log(d^2*x^2 +
2*c*d*x + c^2 + 1) - 2*(b*d*f^2*x + b*d*f*e)*log(f*x + e))/((c^2 + 1)*f^4*x
 + d^2*f*e^3 + (d^2*f^2*x - 2*c*d*f^2)*e^2 - (2*c*d*f^3*x - (c^2 + 1)*f^3)*
e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(151) = 302$.

time = 0.77, size = 1264, normalized size = 8.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*e*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 - 2*b*c*f*
arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^2 + 2*b*d*e*log(4*(4*d^2*e
^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^
2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*
x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/
(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arct
an(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arc
tan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan
(1/(d*x + c))) - 2*b*c*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 -
8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x
+ c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(
1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arct
an(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arc
tan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arct
an(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c))) + 2*a*d*e*tan(1/2*arc
tan(1/(d*x + c)))^2 - 2*a*c*f*tan(1/2*arctan(1/(d*x + c)))^2 - b*f*log(4*(4
*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x +
 c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan
(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arc
tan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/
2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1
/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*
arctan(1/(d*x + c)))^2 - 2*b*d*e*arctan(1/(d*x + c)) + 2*b*c*f*arctan(1/(d*
x + c)) + 4*b*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c))) - 2*a*d*e
+ 2*a*c*f + b*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f
*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2
- 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x +
c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*
x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x
 + c)))^2 + 1)) + 4*a*f*tan(1/2*arctan(1/(d*x + c))))*d/(2*d^3*e^3*tan(1/2*
arctan(1/(d*x + c))) - 6*c*d^2*e^2*f*tan(1/2*arctan(1/(d*x + c))) + 6*c^2*d
*e*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*c^3*f^3*tan(1/2*arctan(1/(d*x + c))
) - d^2*e^2*f*tan(1/2*arctan(1/(d*x + c)))^2 + 2*c*d*e*f^2*tan(1/2*arctan(1
/(d*x + c)))^2 - c^2*f^3*tan(1/2*arctan(1/(d*x + c)))^2 + d^2*e^2*f - 2*c*d
```

```
*e*f^2 + c^2*f^3 + 2*d*e*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*c*f^3*tan(1/2
*arctan(1/(d*x + c))) - f^3*tan(1/2*arctan(1/(d*x + c)))^2 + f^3)
```

Mupad [B]
time = 2.08, size = 128, normalized size = 0.84

$$-\frac{a}{x\,f^2+e\,f}-\frac{b\,\mathrm{acot}(c+d\,x)}{f\,\left(e+f\,x\right)}-\frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2}+\frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}+\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))/(e + f*x)^2,x)
```

```
[Out] (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i - c*f + d*e)) - (b*acot(c + d*x))/(f*
(e + f*x)) - (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - a/(
e*f + f^2*x) + (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.135 $\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=228

$$\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2(de+f-cf)(de-(1+c)f)\text{ArcTan}(c+d}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}$$

```
[Out] 1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*(-a-b*arccot(d*x+c))/f/
(f*x+e)^2-1/2*b*d^2*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)
*f^2)^2+1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(
c^2+1)*f^2)^2
```

Rubi [A]

time = 0.23, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5154, 2007, 723, 814, 648, 632, 210, 642}

$$-\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\text{ArcTan}(c+dx)(-cf+de+f)(de-(c+1)f)}{2f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}+\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}+\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}-\frac{bd^2(de-cf)\log(e+fx)}{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (b*d)/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot[c
 + d*x])/(2*f*(e + f*x)^2) - (b*d^2*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTa
n[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2) - (b*d^2*(d*e - c
*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 + (b*d^2*(d*e - c
*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{(bd)\int \frac{1}{(e+fx)^2(1+(c+dx)^2)}\,dx}{2f}$$

$$= -\frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{(bd)\int \frac{1}{(e+fx)^2(1+c^2+2cdx+d^2x^2)}\,dx}{2f}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{(bd)\int \frac{d(de\cdots}{(e+fx)(1\cdots}}{2f\left(d^2e^2-2cd\cdots\right.}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{(bd)\int \left(\frac{\cdots}{(d^2e^2-2\cdots}\right.}{\cdots}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\cdots}{(d^2e^2-2cdef\cdots}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\cdots}{(d^2e^2-2cdef\cdots}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-cf)\cdots}{(d^2e^2-2cdef\cdots}$$

$$= \frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2(de-f-c\cdots}{2f(d^2e^2\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 180, normalized size = 0.79

$$\frac{\frac{bdf}{(d^2e^2-2cdef+(1+c^2)f^2)(e+fx)} - \frac{a+b\cot^{-1}(c+dx)}{(e+fx)^2} + \frac{ibd^2\log(i-c-dx)}{2(de-(-i+c)f)^2} - \frac{ibd^2\log(i+c+dx)}{2(de-(i+c)f)^2} - \frac{2bd^2f(de-cf)\log(d(e+fx))}{(d^2e^2-2cdef+(1+c^2)f^2)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[c + d*x])/(e + f*x)^3,x]
```

```
[Out] ((b*d*f)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (a + b*ArcCot[
c + d*x])/(e + f*x)^2 + ((I/2)*b*d^2*Log[I - c - d*x])/(d*e - (-I + c)*f)^2
 - ((I/2)*b*d^2*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (2*b*d^2*f*(d*e - c
*f)*Log[d*(e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2)/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(223) = 446.
time = 0.40, size = 467, normalized size = 2.05

method	result

derivativedivides	$-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3 \operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3 f \arctan(dx+c)c^2}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}+\frac{b\,d^4 \arctan(dx+c)ce}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}-\frac{b\,d^5 \arctan(dx}{2f\left(c^2f^2-2cdef+d\right.}$
default	$-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3 \operatorname{arccot}(dx+c)}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3 f \arctan(dx+c)c^2}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}+\frac{b\,d^4 \arctan(dx+c)ce}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}-\frac{b\,d^5 \arctan(dx}{2f\left(c^2f^2-2cdef+d\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a*d^3/(c*f-d*e-f*(d*x+c))^2/f-1/2*b*d^3/(c*f-d*e-f*(d*x+c))^2/f*a
rccot(d*x+c)-1/2*b*d^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c^
2+b*d^4/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c*e-1/2*b*d^5/f/(c^
2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*e^2-1/2*b*d^3*f/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)^2*ln(1+(d*x+c)^2)*c+1/2*b*d^4/(c^2*f^2-2*c*d*e*f+d^2*e^2+
f^2)^2*ln(1+(d*x+c)^2)*e+1/2*b*d^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arct
an(d*x+c)-1/2*b*d^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(c*f-d*e-f*(d*x+c))+b*d
^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(c*f-d*e-f*(d*x+c))*c-b*d^4/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(c*f-d*e-f*(d*x+c))*e)
```

<u>Maxima</u> [A]
time = 0.49, size = 437, normalized size = 1.92

$$\frac{1}{2}\left(d\left(\frac{(cdf-d^2e)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{4\,cd^3fe^3-2\,(3\,c^2e^2+e^2)d^2f^2+4\,(c^3e+ce)df^3-(c^4+2\,c^2+1)f^4-d^4e^4}-\frac{2\,(cdf-d^2e)\log\left(fx+e\right)}{4\,cd^3fe^3-2\,(3\,c^2e^2+e^2)d^2f^2+4\,(c^3e+ce)df^3-(c^4+2\,c^2+1)f^4-d^4e^4}-\frac{(2\,cd^3fe-(c^2-1)d^2f^2-d^4e^2)\arctan\left(\frac{d^2x+cd}{d}\right)}{(4\,cd^3f^2e^3-2\,(3\,c^2e^2+e^2)d^2f^3+4\,(c^3e+ce)df^4-(c^4+2\,c^2+1)f^5-d^4fe^4)d}-\frac{1}{2\,cdfe^2-(c^2e+e)f^2-d^2e^3+(2\,cdf^2e-(c^2+1)f^3-d^2fe^2)x}\right)-\frac{\operatorname{arccot}\left(dx+c\right)}{f^3x^2+2\,f^2xe+fe^2}\right)b-\frac{a}{2\,(f^3x^2+2\,f^2xe+fe^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(d*((c*d*f - d^2*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(4*c*d^3*f*e^3 - 2
*(3*c^2*e^2 + e^2)*d^2*f^2 + 4*(c^3*e + c*e)*d*f^3 - (c^4 + 2*c^2 + 1)*f^4
- d^4*e^4) - 2*(c*d*f - d^2*e)*log(f*x + e)/(4*c*d^3*f*e^3 - 2*(3*c^2*e^2 +
 e^2)*d^2*f^2 + 4*(c^3*e + c*e)*d*f^3 - (c^4 + 2*c^2 + 1)*f^4 - d^4*e^4) -
(2*c*d^3*f*e - (c^2 - 1)*d^2*f^2 - d^4*e^2)*arctan((d^2*x + c*d)/d)/((4*c*d
^3*f^2*e^3 - 2*(3*c^2*e^2 + e^2)*d^2*f^3 + 4*(c^3*e + c*e)*d*f^4 - (c^4 + 2
*c^2 + 1)*f^5 - d^4*f*e^4)*d) - 1/(2*c*d*f*e^2 - (c^2*e + e)*f^2 - d^2*e^3
+ (2*c*d*f^2*e - (c^2 + 1)*f^3 - d^2*f*e^2)*x)) - arccot(d*x + c)/(f^3*x^2
+ 2*f^2*x*e + f*e^2))*b - 1/2*a/(f^3*x^2 + 2*f^2*x*e + f*e^2)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(236) = 472$.
time = 4.88, size = 727, normalized size = 3.19

$$\frac{(bc^2+b)df^4x-ad^4e^4+(4\,ac+b)d^3fe^3-(ac^4+2\,ac^2+a)f^4+(4\,bcd^3fe^3-bd^4e^4-2\,(3\,bc^2+b)d^2f^2e^2+4\,(bc^3+bc)df^3e-(bc^4+2\,bc^2+b)f^4)\operatorname{arccot}\left(dx+c\right)-((bc^2-b)d^2f^4x^2+bd^4e^4+2\,(bd^4fx-bcd^3f)e^3+(bd^4f^2x^2-4\,bcd^3f^2x+(bc^2-b)d^2f^2)e^2-2\,(bcd^3f^3x^2-(bc^2-b)d^2f^3x)e)\arctan\left(dx+c\right)+(bd^3f^2x-2\,(3\,ac^2+bc+a)d^2f^2)e^2-(2\,bcd^2f^3x-(4\,ac^3+bc^2+4\,ac+b)df^3)e-(bcd^2f^4x^2-bd^3fe^3-(2\,bd^3f^2x-bcd^2f^2)e^2-(bd^3f^3x^2-2\,bcd^2f^3x)e)\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\,(bcd^2f^4x^2-bd^3fe^3-(2\,bd^3f^2x-bcd^2f^2)e^2-(bd^3f^3x^2-2\,bcd^2f^3x)e)\log\left(fx+e\right)}{2\,((c^4+2\,c^2+1)f^7x^2+d^4fe^6+2\,(d^4f^2x-2\,cd^3f^2)e^5+(d^4f^3x^2-8\,cd^3f^3x+2\,(3\,c^2+1)d^2f^3)e^4-4\,(cd^3f^4x^2-(3\,c^2+1)d^2f^4x+(c^3+c)df^4)e^3+(2\,(3\,c^2+1)d^2f^5x^2-8\,(c^3+c)df^5x+(c^4+2\,c^2+1)f^5)e^2-2\,(2\,(c^3+c)df^6x^2-(c^4+2\,c^2+1)f^6x)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="fricas")

[Out] 1/2*((b*c^2 + b)*d*f^4*x - a*d^4*e^4 + (4*a*c + b)*d^3*f*e^3 - (a*c^4 + 2*a *c^2 + a)*f^4 + (4*b*c*d^3*f*e^3 - b*d^4*e^4 - 2*(3*b*c^2 + b)*d^2*f^2*e^2 + 4*(b*c^3 + b*c)*d*f^3*e - (b*c^4 + 2*b*c^2 + b)*f^4)*arccot(d*x + c) - ((b*c^2 - b)*d^2*f^4*x^2 + b*d^4*e^4 + 2*(b*d^4*f*x - b*c*d^3*f)*e^3 + (b*d^4 *f^2*x^2 - 4*b*c*d^3*f^2*x + (b*c^2 - b)*d^2*f^2)*e^2 - 2*(b*c*d^3*f^3*x^2 - (b*c^2 - b)*d^2*f^3*x)*e)*arctan(d*x + c) + (b*d^3*f^2*x - 2*(3*a*c^2 + b *c + a)*d^2*f^2)*e^2 - (2*b*c*d^2*f^3*x - (4*a*c^3 + b*c^2 + 4*a*c + b)*d*f ^3)*e - (b*c*d^2*f^4*x^2 - b*d^3*f*e^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*e^2 - (b*d^3*f^3*x^2 - 2*b*c*d^2*f^3*x)*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2 *(b*c*d^2*f^4*x^2 - b*d^3*f*e^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*e^2 - (b*d^ 3*f^3*x^2 - 2*b*c*d^2*f^3*x)*e)*log(f*x + e))/((c^4 + 2*c^2 + 1)*f^7*x^2 + d^4*f*e^6 + 2*(d^4*f^2*x - 2*c*d^3*f^2)*e^5 + (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 + 1)*d^2*f^3)*e^4 - 4*(c*d^3*f^4*x^2 - (3*c^2 + 1)*d^2*f^4*x + (c^3 + c)*d*f^4)*e^3 + (2*(3*c^2 + 1)*d^2*f^5*x^2 - 8*(c^3 + c)*d*f^5*x + (c ^4 + 2*c^2 + 1)*f^5)*e^2 - 2*(2*(c^3 + c)*d*f^6*x^2 - (c^4 + 2*c^2 + 1)*f^6 *x)*e)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acot(d*x+c))/(f*x+e)**3,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6173 vs. $2(220) = 440$.
time = 1.98, size = 6173, normalized size = 27.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccot(d*x+c))/(f*x+e)^3,x, algorithm="giac")

[Out] -1/2*(4*b*d^4*e^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 12*b *c*d^3*e^2*f*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 + 12*b*c^2* d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^3*d*f^ 3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^3 - b*d^3*e^2*f*arctan(1 /(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 2*b*c*d^2*e*f^2*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c^2*d*f^3*arctan(1/(d*x + c))*tan(1/2*arctan(1/(d*x + c)))^4 + 4*b*d^4*e^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/ (d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/

```
2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2
*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*
e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2
*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4
+ 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 - 1
2*b*c*d^3*e^2*f*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f
*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2
- 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x +
c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x
 + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*
x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x
 + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 12*b*c^2*d^2*e*f^2*log(4*(
4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x
+ c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*ar
ctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1
/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(
1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2
*arctan(1/(d*x + c)))^2 - 4*b*c^3*d*f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(
d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2
*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*
tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e
*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*
f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 +
 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^2 + 4*
a*d^4*e^3*tan(1/2*arctan(1/(d*x + c)))^3 - 12*a*c*d^3*e^2*f*tan(1/2*arctan(
1/(d*x + c)))^3 + 12*a*c^2*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^3 - 4*a*c
^3*d*f^3*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*d^3*e^2*f*log(4*(4*d^2*e^2*ta
n(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4
*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c
)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x
+ c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/
(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1
/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d
*x + c)))^3 + 8*b*c*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))
^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/
(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*ar
ctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2
*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/
2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2
*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^3 - 4*b*c^2*d*f^
3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arcta
n(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*tan(
1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*arctan(1/(d*x + c)))^3 + f^2*t
an(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*tan(1/2*arctan(1/(d*x + c))) - 4*c*
```

```
f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 + f
^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*tan(1/2*arctan(1/(d*x + c)))^2 + 1)
)*tan(1/2*arctan(1/(d*x + c)))^3 - a*d^3*e^2*f*tan(1/2*arctan(1/(d*x + c)))
^4 + 2*a*c*d^2*e*f^2*tan(1/2*arctan(1/(d*x + c)))^4 - a*c^2*d*f^3*tan(1/2*a
rctan(1/(d*x + c)))^4 + b*d^2*e*f^2*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x
+ c)))^2 - 8*c*d*e*f*tan(1/2*arctan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arc
tan(1/(d*x + c)))^2 - 4*d*e*f*tan(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(
1/2*arctan(1/(d*x + c)))^3 + f^2*tan(1/2*arctan(1/(d*x + c)))^4 + 4*d*e*f*t
an(1/2*arctan(1/(d*x + c))) - 4*c*f^2*tan(1/2*arctan(1/(d*x + c))) - 2*f^2*
tan(1/2*arctan(1/(d*x + c)))^2 + f^2)/(tan(1/2*arctan(1/(d*x + c)))^4 + 2*t
an(1/2*arctan(1/(d*x + c)))^2 + 1))*tan(1/2*arctan(1/(d*x + c)))^4 - b*c*d*
f^3*log(4*(4*d^2*e^2*tan(1/2*arctan(1/(d*x + c)))^2 - 8*c*d*e*f*tan(1/2*arc
tan(1/(d*x + c)))^2 + 4*c^2*f^2*tan(1/2*arctan(1/(d*x + c)))^2 - 4*d*e*f*ta
n(1/2*arctan(1/(d*x + c)))^3 + 4*c*f^2*tan(1/2*...
```

Mupad [B]

time = 7.33, size = 399, normalized size = 1.75

$$\frac{b\,d\,e}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{a\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,\mathrm{acot}\left(c+d\,x\right)}{2\,f\,{\left(e+f\,x\right)}^2}-\frac{a\,c^2\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}+\frac{b\,c\,d^2\,f\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}+\frac{a\,c\,d\,e}{{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}+\frac{b\,d\,f\,x}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{a\,d^2\,e^2}{2\,f\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}+\frac{b\,d^2\,\ln\left(c+d\,x-\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(d\,e-c\,f+f\,1{}\mathrm{i}\right)}^2}-\frac{b\,d^2\,\ln\left(c+d\,x+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(c\,f-d\,e+f\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))/(e + f*x)^3,x)

[Out] (b*d*e)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (a*f)/(2*(e
 + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*acot(c + d*x))/(2*f*(
e + f*x)^2) + (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i - c*f + d*e)^2) - (b*
d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*c^2*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^3*e*log(e + f*x))/(f^2 +
 c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 + (b*c*d^2*f*log(e + f*x))/(f^2 + c^2*f^2
 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) + (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*
e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f))
```

3.136 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=382

$$\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3}+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3}+\frac{i(3d^2e$$

```
[Out] 1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcco
t(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))/d^3+1/3*I*(3*d^2*e^2-6
*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-
2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^2/d^3/f+1/3*(f*x+e)^3*(a+b*arcc
ot(d*x+c))^2/f-1/3*b^2*f^2*arctan(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+d*e)*l
n(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(2
,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A]
time = 0.43, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5156, 4975, 4931, 266, 4947, 327, 209, 5105, 5005, 5041, 4965, 2449, 2352}

$$\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{3d^3f}-\frac{2b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{3d^3}+\frac{bf^2(c+dx)^2\left(a+b\cot^{-1}(c+dx)\right)}{3d^3}+\frac{2abfx(de-cf)}{d^2}+\frac{(e+fx)^3\left(a+b\cot^{-1}(c+dx)\right)^2}{3f}-\frac{b^2f^2\text{ArcTan}(c+dx)}{3d^3}+\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{3d^3}+\frac{b^2f(de-cf)\log\left((c+dx)^2+1\right)}{d^3}+\frac{2b^2f(c+dx)(de-cf)\cot^{-1}(c+dx)}{d^3}+\frac{b^2f^2x}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 + (2*b^2*f*(d*e - c*f)*(c
 + d*x)*ArcCot[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcCot[c + d*x]))/
(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c
+ d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*
ArcCot[c + d*x])^2)/(3*d^3*f) + ((e + f*x)^3*(a + b*ArcCot[c + d*x])^2)/(3*
f) - (b^2*f^2*ArcTan[c + d*x])/(3*d^3) - (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\cot^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f} + \frac{(2b)\text{Subst}\left(\int \left(\frac{3f^2(de-cf)(a+b\cot^{-1}}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^2}{3f} + \frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-}{}\right.}{}$$

$$= \frac{2abf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2 \left(a+b\cot^{-1}(c+dx)\right)}{3d^3} + \frac{(e+fx)^3 (}{}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} +$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} +$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} +$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} +$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\cot^{-1}(c+dx)}{d^3} +$$

<u>Mathematica</u> [A]
time = 3.49, size = 665, normalized size = 1.74

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(d*f*x*(6*d*e - 4*c*f + d*
f*x) + 2*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c + d*x] - 2*(3*c*d^2*e^2
 + 3*d*e*f - 3*c^2*d*e*f - 3*c*f^2 + c^3*f^2)*ArcTan[c + d*x] + (3*d^2*e^2
- 6*c*d*e*f + (-1 + 3*c^2)*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(3*d^3)
+ (b^2*e^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Log[1 - E^((
2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]))/d + (b^
2*e*f*((1 - (2*I)*c - c^2 + d^2*x^2)*ArcCot[c + d*x]^2 + 2*ArcCot[c + d*x]*
```

```
(c + d*x + 2*c*Log[1 - E^((2*I)*ArcCot[c + d*x])]) - 2*Log[1/((c + d*x)*Sqr
t[1 + (c + d*x)^(-2)])] - (2*I)*c*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]))/d
^2 + (b^2*f^2*((c + d*x)*(1 + (c + d*x)^2)*(1 - 6*c*ArcCot[c + d*x] + 3*(1
+ c^2)*ArcCot[c + d*x]^2) - (c + d*x)*Sqrt[1 + (c + d*x)^(-2)]*(1 + (c + d*
x)^2)*(1 - 6*c*ArcCot[c + d*x] + (-1 + 3*c^2)*ArcCot[c + d*x]^2)*Cos[3*ArcC
ot[c + d*x]] + 2*(1 + (c + d*x)^2)*((-I)*ArcCot[c + d*x]^2*(1 - (6*I)*c - 3
*c^2 + (-1 + 3*c^2)*Cos[2*ArcCot[c + d*x]]) + 2*ArcCot[c + d*x]*(1 + (1 - 3
*c^2)*Log[1 - E^((2*I)*ArcCot[c + d*x])] + (-1 + 3*c^2)*Cos[2*ArcCot[c + d*
x]]*Log[1 - E^((2*I)*ArcCot[c + d*x])]) - 6*c*(-1 + Cos[2*ArcCot[c + d*x]])
*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + (4*I)*(-1 + 3*c^2)*PolyLog[
2, E^((2*I)*ArcCot[c + d*x])]))/(12*d^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(362) = 724$.
time = 0.51, size = 2040, normalized size = 5.34

method	result	size
derivativedivides	Expression too large to display	2040
default	Expression too large to display	2040
risch	Expression too large to display	3402

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*I*b^2/d^2*f^2*ln(d*x+c+I)^2*c^2-I*b^2/d*f*ln(d*x+c-I)*ln(-1/2*I*(
d*x+c+I))*c*e+I*b^2/d*f*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c*e+I*b^2/d*f*ln(d*x+c+
I)*ln(1/2*I*(d*x+c-I))*c*e-I*b^2/d*f*ln(d*x+c+I)*ln(1+(d*x+c)^2)*c*e-4*a*b/
d*f*arccot(d*x+c)*c*e*(d*x+c)-1/3*(c*f-d*e-f*(d*x+c))^3*a^2/d^2/f+a*b*ln(1+
(d*x+c)^2)*e^2+b^2*arccot(d*x+c)*ln(1+(d*x+c)^2)*e^2-1/2*I*b^2*dilog(1/2*I*
(d*x+c-I))*e^2-1/4*I*b^2*ln(d*x+c+I)^2*e^2+1/2*I*b^2*dilog(-1/2*I*(d*x+c+I)
)*e^2+1/4*I*b^2*ln(d*x+c-I)^2*e^2+1/3*b^2/d^2*f^2*(d*x+c)-1/3*b^2/d^2*f^2*a
rctan(d*x+c)+b^2*arccot(d*x+c)^2*e^2*(d*x+c)-b^2*arccot(d*x+c)^2*c*e^2-b^2*
arctan(d*x+c)^2*c*e^2+2*b^2/d*f*arccot(d*x+c)*e*(d*x+c)+b^2/d*f*arctan(d*x+
c)^2*c^2*e-2*b^2/d*f*arccot(d*x+c)*arctan(d*x+c)*e+2/3*b^2*d/f*arccot(d*x+c
)*arctan(d*x+c)*e^3+2*b^2/d^2*f^2*arccot(d*x+c)*arctan(d*x+c)*c+b^2/d^2*f^2
*arccot(d*x+c)*ln(1+(d*x+c)^2)*c^2-2/3*b^2/d^2*f^2*arccot(d*x+c)*arctan(d*x
+c)*c^3-2*b^2/d^2*f^2*arccot(d*x+c)*c*(d*x+c)-1/6*I*b^2/d^2*f^2*ln(d*x+c-I)
*ln(-1/2*I*(d*x+c+I))-1/6*I*b^2/d^2*f^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)+1/6*I*b
^2/d^2*f^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+1/6*I*b^2/d^2*f^2*ln(d*x+c-I)*ln
(1+(d*x+c)^2)+1/4*I*b^2/d^2*f^2*ln(d*x+c-I)^2*c^2-1/2*I*b^2/d^2*f^2*dilog(1
/2*I*(d*x+c-I))*c^2+1/2*I*b^2/d*f*ln(d*x+c+I)^2*c*e-2*b^2/d*f*arccot(d*x+c)
^2*c*e*(d*x+c)+I*b^2/d*f*dilog(1/2*I*(d*x+c-I))*c*e-I*b^2/d*f*dilog(-1/2*I*
(d*x+c+I))*c*e-1/2*I*b^2/d*f*ln(d*x+c-I)^2*c*e+1/2*I*b^2/d^2*f^2*ln(d*x+c-I
)*ln(-1/2*I*(d*x+c+I))*c^2+2*a*b/d*f*arccot(d*x+c)*c^2*e+2*a*b/d^2*f^2*arcc
ot(d*x+c)*c^2*(d*x+c)-2*a*b/d^2*f^2*arccot(d*x+c)*c*(d*x+c)^2+2*a*b/d*f*arc
```

```
cot(d*x+c)*e*(d*x+c)^2-2*a*b/d*f*ln(1+(d*x+c)^2)*c*e+2*a*b/d*f*arctan(d*x+c
)*c^2*e-1/2*I*b^2/d^2*f^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c^2+b^2/d^2*f^2*arcta
n(d*x+c)^2*c-1/3*b^2/d^2*f^2*arctan(d*x+c)^2*c^3+1/2*I*b^2*ln(d*x+c-I)*ln(-
1/2*I*(d*x+c+I))*e^2-1/12*I*b^2/d^2*f^2*ln(d*x+c-I)^2+1/12*I*b^2/d^2*f^2*ln
(d*x+c+I)^2-1/6*I*b^2/d^2*f^2*dilog(-1/2*I*(d*x+c+I))+1/6*I*b^2/d^2*f^2*dil
og(1/2*I*(d*x+c-I))-b^2/d*f*arctan(d*x+c)^2*e+1/3*b^2*d/f*arctan(d*x+c)^2*e
^3+1/3*b^2*d/f*arccot(d*x+c)^2*e^3-1/3*b^2/d^2*f^2*arccot(d*x+c)^2*c^3+1/3*
b^2/d^2*f^2*arccot(d*x+c)^2*(d*x+c)^3+1/3*b^2/d^2*f^2*arccot(d*x+c)*(d*x+c)
^2-1/3*b^2/d^2*f^2*arccot(d*x+c)*ln(1+(d*x+c)^2)+1/2*I*b^2*ln(d*x+c+I)*ln(1
+(d*x+c)^2)*e^2-1/2*I*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*e^2-1/2*I*b^2*ln(
d*x+c-I)*ln(1+(d*x+c)^2)*e^2-2*b^2*arccot(d*x+c)*arctan(d*x+c)*c*e^2-b^2/d^
2*f^2*ln(1+(d*x+c)^2)*c+2*a*b*arccot(d*x+c)*e^2*(d*x+c)-1/3*a*b/d^2*f^2*ln(
1+(d*x+c)^2)-2*a*b*arctan(d*x+c)*c*e^2+1/3*a*b/d^2*f^2*(d*x+c)^2-2*a*b*arcc
ot(d*x+c)*c*e^2+b^2/d*f*ln(1+(d*x+c)^2)*e+1/2*I*b^2/d^2*f^2*dilog(-1/2*I*(d
*x+c+I))*c^2-2*a*b/d^2*f^2*c*(d*x+c)+2*a*b/d*f*e*(d*x+c)-2/3*a*b/d^2*f^2*ar
ccot(d*x+c)*c^3+2/3*a*b*d/f*arccot(d*x+c)*e^3+2/3*a*b/d^2*f^2*arccot(d*x+c)
*(d*x+c)^3+a*b/d^2*f^2*ln(1+(d*x+c)^2)*c^2-2/3*a*b/d^2*f^2*arctan(d*x+c)*c^
3+2/3*a*b*d/f*arctan(d*x+c)*e^3+2*a*b/d^2*f^2*arctan(d*x+c)*c-2*a*b/d*f*arc
tan(d*x+c)*e+b^2/d^2*f^2*arccot(d*x+c)^2*c^2*(d*x+c)-b^2/d^2*f^2*arccot(d*x
+c)^2*c*(d*x+c)^2+b^2/d*f*arccot(d*x+c)^2*e*(d*x+c)^2+b^2/d*f*arccot(d*x+c)
^2*c^2*e+1/2*I*b^2/d^2*f^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*c^2-1/2*I*b^2/d^2*f^
2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*c^2+2*b^2/d*f*arccot(d*x+c)*arctan(d*x+c)
*c^2*e-2*b^2/d*f*arccot(d*x+c)*ln(1+(d*x+c)^2)*c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*f^2*x^3*arctan2(1, d*x + c)^2 + 1/4*b^2*f*x^2*arctan2(1, d*x + c)^
2*e + 1/3*a^2*f^2*x^3 + 1/4*b^2*x*arctan2(1, d*x + c)^2*e^2 + a^2*f*x^2*e +
 1/3*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
a*b*f^2 + 2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d
)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f*e + a^2*x*e^2 + (
2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e^2/d - 1/48*(b^2*f
^2*x^3 + 3*b^2*f*x^2*e + 3*b^2*x*e^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 +
integrate(1/48*(36*b^2*d^2*f^2*x^4*arctan2(1, d*x + c)^2 + 36*b^2*c^2*arcta
n2(1, d*x + c)^2*e^2 + 36*b^2*arctan2(1, d*x + c)^2*e^2 + 8*(9*b^2*d^2*f*ar
ctan2(1, d*x + c)^2*e + (9*b^2*c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x
 + c))*d*f^2)*x^3 + 12*(3*b^2*d^2*arctan2(1, d*x + c)^2*e^2 + 2*(6*b^2*c*ar
ctan2(1, d*x + c)^2*e + b^2*arctan2(1, d*x + c)*e)*d*f + 3*(b^2*c^2*arctan2
```

```
(1, d*x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f^2)*x^2 + 3*(b^2*d^2*f^2*x^4 +
 b^2*c^2*e^2 + 2*(b^2*c*d*f^2 + b^2*d^2*f*e)*x^3 + (4*b^2*c*d*f*e + b^2*d^2
*e^2 + (b^2*c^2 + b^2)*f^2)*x^2 + b^2*e^2 + 2*(b^2*c*d*e^2 + (b^2*c^2*e + b
^2*e)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*((3*b^2*c*arctan2(1, d*
x + c)^2*e^2 + b^2*arctan2(1, d*x + c)*e^2)*d + 3*(b^2*c^2*arctan2(1, d*x +
 c)^2*e + b^2*arctan2(1, d*x + c)^2*e)*f)*x + 4*(b^2*d^2*f^2*x^4 + 3*b^2*c*
d*x*e^2 + (b^2*c*d*f^2 + 3*b^2*d^2*f*e)*x^3 + 3*(b^2*c*d*f*e + b^2*d^2*e^2)
*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
[Out] integral(a^2*f^2*x^2 + 2*a^2*f*x*e + (b^2*f^2*x^2 + 2*b^2*f*x*e + b^2*e^2)*
arccot(d*x + c)^2 + a^2*e^2 + 2*(a*b*f^2*x^2 + 2*a*b*f*x*e + a*b*e^2)*arcco
t(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acot}(c + dx))^2 (e + fx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c))**2,x)
[Out] Integral((a + b*acot(c + d*x))**2*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2\,(a + b\,\mathrm{acot}(c + d\,x))^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x))^2,x)
[Out] int((e + f*x)^2*(a + b*acot(c + d*x))^2, x)
```

3.137 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=220

$$\frac{abfx}{d}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}-\frac{(de+f-cf)(de-(1+c)f)\left(a+b\mathrm{co}\right.}{2d^2f}$$

```
[Out] a*b*f*x/d+b^2*f*(d*x+c)*arccot(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^
2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arccot(d*x+c))^2/d^2/f+1/2*(f*x+e
)^2*(a+b*arccot(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccot(d*x+c))*ln(2/(1+I*(d
*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-2/(1
+I*(d*x+c)))/d^2
```

Rubi [A]

time = 0.28, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5156, 4975, 4931, 266, 5105, 5005, 5041, 4965, 2449, 2352}

$$\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2f}-\frac{2b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f}+\frac{abfx}{d}+\frac{ib^2(de-cf)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{d^2}+\frac{b^2f\log\left((c+dx)^2+1\right)}{2d^2}+\frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcCot[c + d*x])/d^2 + (I*(d*e - c*f)*(a + b
*ArcCot[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot
[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcCot[c + d*x])^2)/(2*f) - (
2*b*(d*e - c*f)*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (b^
2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \left(\frac{f^2\left(a+b\cot^{-1}(x)\right)}{d^2}+\frac{((de-\cdots}{}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2f(d\cdots}{1+x^2}\right.}{d^2\cdots}$$

$$= \frac{abfx}{d} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2f} + \frac{b\text{Subst}\left(\int \left(\frac{(de+f-cf)(de-(\cdots}{}\right.\right.}{}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)\cdots}{2f}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx\cdots\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx\cdots\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx\cdots\right.}{d^2}$$

$$= \frac{abfx}{d} + \frac{b^2f(c+dx)\cot^{-1}(c+dx)}{d^2} + \frac{i(de-cf)\left(a+b\cot^{-1}(c+dx\cdots\right.}{d^2}$$

Mathematica [A]
time = 0.36, size = 286, normalized size = 1.30

$$\frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(i+c+dx)(-((i+c)f)+d(2e+fx))\cot^{-1}(c+dx)^2-2abf\,\text{ArcTan}(c+dx)+2b\cot^{-1}(c+dx)\left(-((c+dx)(-bf+acf-ad(2e+fx)))-2b(de-cf)\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)-4abde\log\left(\frac{1}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)-2b^2f\log\left(\frac{1}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)+4abcf\log\left(\frac{1}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)+2ib^2(de-cf)\text{PolyLog}\left(2,e^{2i\cot^{-1}(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(I + c + d*x)*(-((I + c)*f) + d*(2*e + f*x))*ArcCot[c + d*x]^
2 - 2*a*b*f*ArcTan[c + d*x] + 2*b*ArcCot[c + d*x]*(-((c + d*x)*(-(b*f) + a*
```

```
c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^((2*I)*ArcCot[c + d*x])
]) - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] - 2*b^2*f*Log[1/
((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sqrt[1 +
 (c + d*x)^(-2)])] + (2*I)*b^2*(d*e - c*f)*PolyLog[2, E^((2*I)*ArcCot[c + d
*x])])/(2*d^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(210) = 420$.
time = 0.27, size = 721, normalized size = 3.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)+1/4*I*b^2/d*ln(d*x+c+
I)^2*c*f-1/2*I*b^2/d*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*c*f-1/2*I*b^2/d*ln(d*
x+c+I)*ln(1+(d*x+c)^2)*c*f+1/2*I*b^2/d*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c*f-2*a*
b/d*arccot(d*x+c)*f*c*(d*x+c)+1/2*I*b^2/d*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*c
*f+a*b/d*f*(d*x+c)+2*a*b*arccot(d*x+c)*e*(d*x+c)-a*b/d*f*arctan(d*x+c)+b^2/
d*arccot(d*x+c)*f*(d*x+c)+1/2*I*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*e-1/2*I*b^2
*ln(d*x+c-I)*ln(1+(d*x+c)^2)*e-1/2*I*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*e+
1/2*I*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*e+1/2*b^2/d*arccot(d*x+c)^2*f*(d
*x+c)^2-b^2/d*arctan(d*x+c)*arccot(d*x+c)*f-b^2/d*arccot(d*x+c)^2*f*c*(d*x+
c)+1/2*I*b^2/d*dilog(1/2*I*(d*x+c-I))*c*f-1/4*I*b^2/d*ln(d*x+c-I)^2*c*f+a*b
/d*arccot(d*x+c)*f*(d*x+c)^2-a*b/d*ln(1+(d*x+c)^2)*c*f-1/2*I*b^2/d*dilog(-1
/2*I*(d*x+c+I))*c*f-b^2/d*ln(1+(d*x+c)^2)*arccot(d*x+c)*c*f-1/2*I*b^2*dilog
(1/2*I*(d*x+c-I))*e+1/2*I*b^2*dilog(-1/2*I*(d*x+c+I))*e-1/4*I*b^2*ln(d*x+c+
I)^2*e+1/4*I*b^2*ln(d*x+c-I)^2*e+a*b*ln(1+(d*x+c)^2)*e+b^2*ln(1+(d*x+c)^2)*
arccot(d*x+c)*e+b^2*arccot(d*x+c)^2*e*(d*x+c)+1/2*b^2/d*f*ln(1+(d*x+c)^2)-1
/2*b^2/d*arctan(d*x+c)^2*f)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*f*x^2*arctan2(1, d*x + c)^2 + 1/4*b^2*x*arctan2(1, d*x + c)^2*e + 1
/2*a^2*f*x^2 + (x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f + a^2*x*e + (2
*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a*b*e/d - 1/32*(b^2*f*x^
2 + 2*b^2*x*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/16*(12*b^2*
d^2*f*x^3*arctan2(1, d*x + c)^2 + 12*b^2*c^2*arctan2(1, d*x + c)^2*e + 12*b
^2*arctan2(1, d*x + c)^2*e + 4*(3*b^2*d^2*arctan2(1, d*x + c)^2*e + (6*b^2*
c*arctan2(1, d*x + c)^2 + b^2*arctan2(1, d*x + c))*d*f)*x^2 + (b^2*d^2*f*x^
```

```
3 + b^2*c^2*e + (2*b^2*c*d*f + b^2*d^2*e)*x^2 + b^2*e + (2*b^2*c*d*e + (b^2
*c^2 + b^2)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(2*(3*b^2*c*arctan
2(1, d*x + c)^2*e + b^2*arctan2(1, d*x + c)*e)*d + 3*(b^2*c^2*arctan2(1, d*
x + c)^2 + b^2*arctan2(1, d*x + c)^2)*f)*x + 2*(b^2*d^2*f*x^3 + 2*b^2*c*d*x
*e + (b^2*c*d*f + 2*b^2*d^2*e)*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
[Out] integral(a^2*f*x + (b^2*f*x + b^2*e)*arccot(d*x + c)^2 + a^2*e + 2*(a*b*f*x
 + a*b*e)*arccot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acot}\,(c + dx))^2\,(e + fx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acot(d*x+c))**2,x)
[Out] Integral((a + b*acot(c + d*x))**2*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*(b*arccot(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\;(a + b\,\mathrm{acot}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acot(c + d*x))^2,x)
[Out] int((e + f*x)*(a + b*acot(c + d*x))^2, x)
```

3.138 $\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=102

$$\frac{i(a+b\cot^{-1}(c+dx))^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b(a+b\cot^{-1}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\text{PolyL}}{}$$

```
[Out] I*(a+b*arccot(d*x+c))^2/d+(d*x+c)*(a+b*arccot(d*x+c))^2/d-2*b*(a+b*arccot(d
*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A]
time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5148, 4931, 5041, 4965, 2449, 2352}

$$\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{i(a+b\cot^{-1}(c+dx))^2}{d}-\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{d}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcCot[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCot[c + d*x])^2)/d -
 (2*b*(a + b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
```

```
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cot^{-1}(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}\\
&= \frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(2b)\text{Subst}\left(\int \frac{x(a+b\cot^{-1}(x))}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{(2b)\text{Subst}\left(\int \frac{a+b}{}\right.}{}\\
&= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b(a+b\cot^{-1}(c}{}\\
&= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b(a+b\cot^{-1}(c}{}\\
&= \frac{i\left(a+b\cot^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d}-\frac{2b(a+b\cot^{-1}(c}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 118, normalized size = 1.16

$$
\frac{b^2(i+c+dx)\cot^{-1}(c+dx)^2+2b\cot^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+a\left(ac+adx-2b\log\left(\frac{1}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)\right)+ib^2\text{PolyLog}\left(2,e^{2i\cot^{-1}(c+dx)}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] (b^2*(I + c + d*x)*ArcCot[c + d*x]^2 + 2*b*ArcCot[c + d*x]*(a*c + a*d*x - b
*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d*x
)*Sqrt[1 + (c + d*x)^(-2)])]) + I*b^2*PolyLog[2, E^((2*I)*ArcCot[c + d*x])]
)/d
```

Maple [A]
time = 0.27, size = 196, normalized size = 1.92

method	result
derivativedivides	$\frac{(dx+c)a^2+i\operatorname{arccot}(dx+c)^2b^2+\operatorname{arccot}(dx+c)^2b^2(dx+c)+2i\operatorname{polylog}\left(2,-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)b^2+2i\operatorname{polylog}\left(2,\frac{}{\sqrt{}}\right.}{}$
default	$\frac{(dx+c)a^2+i\operatorname{arccot}(dx+c)^2b^2+\operatorname{arccot}(dx+c)^2b^2(dx+c)+2i\operatorname{polylog}\left(2,-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)b^2+2i\operatorname{polylog}\left(2,\frac{}{\sqrt{}}\right.}{}$
risch	$\frac{\pi abc}{d}+\frac{\pi^2b^2c}{4d}+\frac{\pi b^2\ln(d^2x^2+2cdx+c^2+1)}{2d}+\frac{ab\ln(d^2x^2+2cdx+c^2+1)}{d}+\frac{b^2c\ln(d^2x^2+2cdx+c^2+1)}{4d}-\frac{b^2\pi c\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2+I*arccot(d*x+c)^2*b^2+arccot(d*x+c)^2*b^2*(d*x+c)+2*I*poly
log(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*b^2+2*I*polylog(2,(d*x+c+I)/(1+(d*x+c
)^2)^(1/2))*b^2-2*arccot(d*x+c)*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*b^2-2*a
rccot(d*x+c)*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*b^2+2*a*b*arccot(d*x+c)*(d
*x+c)+a*b*ln(1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*arctan2(1, d*x + c)^2 - x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 16
*integrate(1/16*(12*d^2*x^2*arctan2(1, d*x + c)^2 + 12*c^2*arctan2(1, d*x +
 c)^2 + 8*(3*c*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c))*d*x + (d^2*x^2
+ 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 12*arctan2(1, d*x
 + c)^2 + 4*(d^2*x^2 + c*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 +
2*c*d*x + c^2 + 1), x))*b^2 + a^2*x + (2*(d*x + c)*arccot(d*x + c) + log((d
*x + c)^2 + 1))*a*b/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acot}\,(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acot(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^2, x)
```

Mupad [B]
time = 0.97, size = 123, normalized size = 1.21

$$a^2\,x+\frac{a\,b\,\left(\ln\left({\left(c+d\,x\right)}^2+1\right)+2\,\mathrm{acot}(c+d\,x)\,\left(c+d\,x\right)\right)}{d}-\frac{2\,b^2\,\ln\left(1-{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,\mathrm{acot}(c+d\,x)}{d}+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,\left(c+d\,x\right)}{d}+\frac{b^2\,\mathrm{polylog}\left(2,{\mathrm{e}}^{\mathrm{acot}(c+d\,x)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{d}+\frac{b^2\,{\mathrm{acot}(c+d\,x)}^2\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^2,x)
```

```
[Out] a^2*x + (b^2*polylog(2, exp(acot(c + d*x)*2i))*1i)/d + (b^2*acot(c + d*x)^2
*1i)/d + (a*b*(log((c + d*x)^2 + 1) + 2*acot(c + d*x)*(c + d*x)))/d - (2*b^
2*log(1 - exp(acot(c + d*x)*2i))*acot(c + d*x))/d + (b^2*acot(c + d*x)^2*(c
 + d*x))/d
```

3.139 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{\left(a+b\cot^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}-\frac{ib(a+b\cot^{-1}(c$$

```
[Out] -(a+b*arccot(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^2*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-I*b*(a+b*arccot(d*x+c))*polylog(2,1-2
/(1-I*(d*x+c)))/f+I*b*(a+b*arccot(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-
c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*polylo
g(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A]
time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5156, 4969}

$$\frac{ib(a+b\cot^{-1}(c+dx))\operatorname{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{ib\operatorname{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{f}+\frac{b^2\operatorname{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{b^2\operatorname{Li}_3\left(1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f -
 (I*b*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f + (I*b
*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 4969

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
```

```
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2c}{(de+if-c}\right.}{f}$$

Mathematica [F]
time = 36.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot^{-1}(c+dx))^2}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x),x]
[Out] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.71, size = 2091, normalized size = 8.01

method	result	size
derivativedivides	Expression too large to display	2091
default	Expression too large to display	2091

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*d*c/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*
f)*(d*x+c+I)^2/(1+(d*x+c)^2))+a^2*d*ln(c*f-d*e-f*(d*x+c))/f-1/2*b^2*d^2/f*e
/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+
c)^2))+2*I*b^2*d/f*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+2
*I*b^2*d/f*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-I*b^2*d/
(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^
2/(1+(d*x+c)^2))+2*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)+I*a*b*d/f*di
log((I*f+f*(d*x+c))/(c*f-d*e+I*f))-I*a*b*d/f*dilog((I*f-f*(d*x+c))/(d*e+I*f
-c*f))-1/2*I*b^2*d/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d
```

```
*x+c+I)^2/(1+(d*x+c)^2))+2*I*b^2*d^2/f*e*arccot(d*x+c)*polylog(2,(d*e+I*f-c
*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-1/2*I*b^
2*d/f*Pi*arccot(d*x+c)^2*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I*(c*f*
(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+
I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2+1/2*I*b^2*d/f*Pi*a
rccot(d*x+c)^2*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I*(c*f*(d*x+c+I)^
2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d
*x+c)^2)-I*f))*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*
x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^
2)-1))-2*b^2*d/f*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-2*b^2*d/f*polylog
(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+I*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f+
f*(d*x+c))/(c*f-d*e+I*f))-I*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c)
)/(d*e+I*f-c*f))-1/2*I*b^2*d/f*Pi*arccot(d*x+c)^2*csgn(I*(c*f*(d*x+c+I)^2/(
1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+
c)^2)-I*f))*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c
)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-
1))^2-b^2*d^2/f*e/(-I*f+c*f-d*e)*arccot(d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d
*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-I*b^2*d*c/(-I*f+c*f-d*e)*arccot(d*x+c)*p
olylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+1/2*I*b^2*
d/f*Pi*arccot(d*x+c)^2*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^
2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+
(d*x+c)^2)-1))^3-b^2*d/f*arccot(d*x+c)^2*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2)
)-b^2*d/f*arccot(d*x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/2*b^2*d*c/(
-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)
^2))-b^2*d/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I
*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)^
2-b^2*d/f*arccot(d*x+c)^2*ln(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/
(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)+b^2*d/f*arccot(d*x
+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan2(1, d*x + c)^2 + b^2*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan2(1, d*x + c))/(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f*x + e), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**2/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^2/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acot}(c+d\,x))^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^2/(e + f*x),x)
```

```
[Out] int((a + b*acot(c + d*x))^2/(e + f*x), x)
```

3.140 $\int \frac{(a+b\cot^{-1}(c+dx))^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=567

$$\frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f(d^2e^2-2cdef+(1+c^2)f^2)}-\frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)}-\frac{2abd(de-cf)\text{ArcTan}}{f(f^2+(de-c}$$

```
[Out] I*b^2*d*arccot(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*ar
ccot(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^2/f/(f*
x+e)-2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-2*a*b*d*ln(f*x+e
)/(f^2+(-c*f+d*e)^2)+2*b^2*d*arccot(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-2*b^2*d*arccot(d*x+c)*ln(2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+a*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+
d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f
+(c^2+1)*f^2)
```

Rubi [A]

time = 1.05, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5154, 2007, 719, 31, 648, 632, 210, 642, 6873, 5166, 720, 649, 209, 266, 6820, 12, 6857, 4967, 2449, 2352, 2497, 5105, 5005, 5041, 4965}

$$-\frac{2abd\text{ArcTan}(c+dx)(de-cf)}{f((de-cf)^2+f^2)}-\frac{2abd\log(e+fx)}{(de-cf)^2+f^2}+\frac{abd\log((c+dx)^2+1)}{(de-cf)^2+f^2}-\frac{(a+b\cot^{-1}(c+dx))^2}{f(e+fx)}+\frac{ib^2d\text{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{ib^2d\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{ib^2d\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{ib^2d\cot^{-1}(c+dx)^2}{(c^2+1)f^2-2cdef+d^2e^2}+\frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f((c^2+1)f^2-2cdef+d^2e^2)}+\frac{2b^2d\log\left(\frac{2}{1-i(c+dx)}\right)\cot^{-1}(c+dx)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{2b^2d\cot^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{(c^2+1)f^2-2cdef+d^2e^2}-\frac{2b^2d\log\left(\frac{2}{1+i(c+dx)}\right)\cot^{-1}(c+dx)}{(c^2+1)f^2-2cdef+d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (I*b^2*d*ArcCot[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*
(d*e - c*f)*ArcCot[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) -
(a + b*ArcCot[c + d*x])^2/(f*(e + f*x)) - (2*a*b*d*(d*e - c*f)*ArcTan[c + d
*x])/(f*(f^2 + (d*e - c*f)^2)) - (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) + (2*b^2*d*ArcCot[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) - (2*b^2*d*ArcCot[c + d*x]*Log[(2*d*(e + f*x))/((d*e + I
*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (2*b
^2*d*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) + (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
```

```
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
```

```
EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\int \frac{a+b\cot^{-1}(c+dx)}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2b)\mathrm{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2b)\mathrm{Subst}\left(\int \frac{d(a+b\cot^{-1}(x))}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\mathrm{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2bd)\mathrm{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\cot^{-1}(x)}{(de-cf+fx)(1+x^2)}\right.\right.}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\mathrm{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} - $$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2b^2d)\,\mathrm{Subst}\left(\int \left(\frac{f^2\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \right.\right.}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} - \frac{(2b^2d)\,\mathrm{Subst}\left(\int \frac{(de-cf-fx)\cot^{-1}}{1+x^2}\right.}{f\left(d^2e^2-2cdef+(1\right.}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{2abd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\cot^{-1}(}{f(e+f}$$

$$= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\cot^{-1}(}{f(e+f}$$

$$= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\cot^{-1}(}{f(e+f}$$

$$= \frac{ib^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{b^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{(a+b\cot^{-1}(}{f(e+f}$$

Mathematica [**A**]
time = 6.16, size = 454, normalized size = 0.80

$$-\frac{a^2+\frac{2abf\left((-cde+f+c^2f-d^2ex+cdfx)\cot^{-1}(c+dx)+d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1+\frac{1}{(c+dx)^2}}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)(1+(c+dx)^2)\left(\frac{e^{i\text{ArcTan}\left(\frac{f}{de-cf}\right)}\cot^{-1}(c+dx)^2}{(-de+cf)\sqrt{1+\frac{f^2}{(de-cf)^2}}}+\frac{\cot^{-1}(c+dx)^2}{de+dfx}+\frac{f\left(i\pi\cot^{-1}(c+dx)+\pi\log\left(1+e^{-2i\cot^{-1}(c+dx)}\right)+2\cot^{-1}(c+dx)\log\left(1-e^{2i\left(\cot^{-1}(c+dx)+\text{ArcTan}\left(\frac{f}{de-cf}\right)\right)}\right)-\pi\log\left(\frac{1}{\sqrt{1+\frac{1}{(c+dx)^2}}}\right)+2\text{ArcTan}\left(\frac{f}{-de+cf}\right)\left(i\cot^{-1}(c+dx)-\log\left(1-e^{2i\left(\cot^{-1}(c+dx)+\text{ArcTan}\left(\frac{f}{de-cf}\right)\right)}\right)+\log\left(\sin\left(\cot^{-1}(c+dx)+\text{ArcTan}\left(\frac{f}{de-cf}\right)\right)\right)\right)-i\text{PolyLog}\left(2,e^{2i\left(\cot^{-1}(c+dx)+\text{ArcTan}\left(\frac{f}{de-cf}\right)\right)}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}\right)}{(c+dx)^2\left(1+\frac{1}{(c+dx)^2}\right)}}{f(e+fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a^2 + (2*a*b*f*((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcCot[c + d*
x] + d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]))]
))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(1 + (c + d*x)^
2)*((E^(I*ArcTan[f/(d*e - c*f)])*ArcCot[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1
+ f^2/(d*e - c*f)^2]) + ArcCot[c + d*x]^2/(d*e + d*f*x) + (f*(I*Pi*ArcCot[c
 + d*x] + Pi*Log[1 + E^((-2*I)*ArcCot[c + d*x])] + 2*ArcCot[c + d*x]*Log[1
- E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] - Pi*Log[1/Sqrt[1 +
(c + d*x)^(-2)]] + 2*ArcTan[f/(-(d*e) + c*f)]*(I*ArcCot[c + d*x] - Log[1 -
E^((2*I)*(ArcCot[c + d*x] + ArcTan[f/(d*e - c*f)]))] + Log[Sin[ArcCot[c + d
*x] + ArcTan[f/(d*e - c*f)]]]) - I*PolyLog[2, E^((2*I)*(ArcCot[c + d*x] + A
rcTan[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/((c + d*x
)^2*(1 + (c + d*x)^(-2))))/(f*(e + f*x)))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(555) = 1110.
time = 2.06, size = 1254, normalized size = 2.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a*b*d^2/(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)-2*a*b*d^3/f/(c^2*f^2-2*c
*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e-2*b^2*d^3/f*arccot(d*x+c)/(c^2*f^2-2*c*
d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+2*a*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2
)*arctan(d*x+c)*c+a^2*d^2/(c*f-d*e-f*(d*x+c))/f-1/2*I*b^2*d^2/(c^2*f^2-2*c*
d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+1/2*I*b^2*d^2/(c^2*f^2-2
*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-1/2*I*b^2*d^2/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(1+(d*x+c)^2)-b^2*d^3/f/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*e+I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2
+f^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))+1/2*I*b^2*d^2
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1+(d*x+c)^2)-I*b^2*d^2/(c^2
*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f+f*(d*x+c))/(c*f-d
*e+I*f))+2*b^2*d^2*arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x
+c)*c-1/4*I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)^2-1/2*I*b^2
*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(1/2*I*(d*x+c-I))-I*b^2*d^2/(c^2*
f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))+1/2*I*b^2*d
^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(-1/2*I*(d*x+c+I))+1/4*I*b^2*d^2/(c
```

```
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)^2+I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^
2*e^2+f^2)*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))+b^2*d^2/(c*f-d*e-f*(d*x+c))
/f*arccot(d*x+c)^2-2*b^2*d^2*arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*
ln(c*f-d*e-f*(d*x+c))+b^2*d^2*arccot(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)
*ln(1+(d*x+c)^2)+b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c-
2*a*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))+a*b*d^2/(c^
2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1)*f^
3 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2 + 1)
*f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2)) + 2
*arccot(d*x + c)/(f^2*x + f*e))*a*b - 1/16*(4*arctan2(1, d*x + c)^2 - 16*(f
^2*x + f*e)*integrate(1/16*(12*d^2*f*x^2*arctan2(1, d*x + c)^2 + 8*(3*c*arc
tan2(1, d*x + c)^2 - arctan2(1, d*x + c))*d*f*x - 8*d*arctan2(1, d*x + c)*e
 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
 + 12*(c^2*arctan2(1, d*x + c)^2 + arctan2(1, d*x + c)^2)*f - 4*(d^2*f*x^2
+ c*d*e + (c*d*f + d^2*e)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4
 + 2*(c*d*f^3 + d^2*f^2*e)*x^3 + (4*c*d*f^2*e + (c^2 + 1)*f^3 + d^2*f*e^2)*
x^2 + (c^2*e^2 + e^2)*f + 2*(c*d*f*e^2 + (c^2*e + e)*f^2)*x), x) - log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2)*b^2/(f^2*x + f*e) - a^2/(f^2*x + f*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)/(f^2*x^2 + 2
*f*x*e + e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**2/(f*x+e)**2,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2}{\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^2/(e + f*x)^2,x)
[Out] int((a + b*acot(c + d*x))^2/(e + f*x)^2, x)
```

3.141 $\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=565

$$\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3}+\frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}+\frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3}+\frac{3bf}{}$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccot(d*x+c)/d^3+1/2*b*f^2*(a+b*arccot(d*x
+c))^2/d^3+3*I*b*f*(-c*f+d*e)*(a+b*arccot(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(d
*x+c)*(a+b*arccot(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccot(d*x+c))^2/d
^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(d*x+c))^3/d^3-1/3
*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arccot(d*x+c))^3/d^3/f+1/
3*(f*x+e)^3*(a+b*arccot(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccot(d*x+c))*
ln(2/(1+I*(d*x+c)))/d^3-b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arccot(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3+1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3+3*I*b^3*f
*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3-1/2*b^3*(
3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A]

time = 0.69, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5156, 4975, 4931, 5041, 4965, 2449, 2352, 4947, 5037, 266, 5005, 5105, 5115, 6745}

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCot[c + d*x])/d^3 + (b*f^2*(a + b
*ArcCot[c + d*x])^2)/(2*d^3) + ((3*I)*b*f*(d*e - c*f)*(a + b*ArcCot[c + d*x
])^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCot[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCot[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcCot[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcCot[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcCot[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 + (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) + ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 - (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5037

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\cot^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^3}{3f} + \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)\left(a+b\cot^{-1}(x\right.}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\cot^{-1}(c+dx)\right)^3}{3f} + \frac{b\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-3f}{}\right.}{}$$

$$= \frac{3bf(de-cf)(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2 (a+}{2d}$$

$$= \frac{3ibf(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^3} + \frac{3bf(de-cf)(c+dx)(a+}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3} + \frac{3ibf(de-cf)\left(a+b\cot^{-1}(\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\cot^{-1}(c+dx)}{d^3} + \frac{bf^2\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2336 vs. $2(565) = 1130$.
time = 9.74, size = 2336, normalized size = 4.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[c +
 d*x] + ((-3*a^2*b*c*d^2*e^2 - 3*a^2*b*d*e*f + 3*a^2*b*c^2*d*e*f + 3*a^2*b*
```

```
c*f^2 - a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + ((3*a^2*b*d^2*e^2 - 6*a^2*b*c
*d*e*f - a^2*b*f^2 + 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*
d^3) + (a*b^2*f^2*x^2*(1 + (c + d*x)^2)*((c + d*x)*(1 - 6*c*ArcCot[c + d*x]
 + 3*ArcCot[c + d*x]^2 + 3*c^2*ArcCot[c + d*x]^2) - (c + d*x)*Sqrt[1 + (c +
 d*x)^(-2)]*(1 - 6*c*ArcCot[c + d*x] - ArcCot[c + d*x]^2 + 3*c^2*ArcCot[c +
 d*x]^2)*Cos[3*ArcCot[c + d*x]] - 2*(-2*ArcCot[c + d*x] + I*ArcCot[c + d*x]
^2 + 6*c*ArcCot[c + d*x]^2 - (3*I)*c^2*ArcCot[c + d*x]^2 + 2*(-1 + 3*c^2)*A
rcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] - 6*c*Log[1/((c + d*x)*Sq
rt[1 + (c + d*x)^(-2)])]) + Cos[2*ArcCot[c + d*x]]*(I*(-1 + 3*c^2)*ArcCot[c
+ d*x]^2 + (2 - 6*c^2)*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] +
 6*c*Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])])) + ((4*I)*(-1 + 3*c^2)*Po
lyLog[2, E^((2*I)*ArcCot[c + d*x])])/((c + d*x)^2*(1 + (c + d*x)^(-2)))))/(
4*d*(c + d*x)^2*(1 + (c + d*x)^(-2))*(1/Sqrt[1 + (c + d*x)^(-2)] - c/((c +
d*x)*Sqrt[1 + (c + d*x)^(-2)]))^2) - (3*a*b^2*e^2*(1 + (c + d*x)^2)*(-((c +
 d*x)*ArcCot[c + d*x]^2) + 2*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*
x])] - I*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])))/(d*(
c + d*x)^2*(1 + (c + d*x)^(-2))) + (6*a*b^2*e*f*(1 + (c + d*x)^2)*(((c + d*
x)*ArcCot[c + d*x])/d^2 - (c*(c + d*x)*ArcCot[c + d*x]^2)/d^2 + ((c + d*x)^
2*(1 + (c + d*x)^(-2))*ArcCot[c + d*x]^2)/(2*d^2) - Log[1/((c + d*x)*Sqrt[1
 + (c + d*x)^(-2)])]/d^2 + (2*c*(ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c
+ d*x])] - (I/2)*(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])]
)))/d^2))/((c + d*x)^2*(1 + (c + d*x)^(-2))) - (b^3*e^2*(1 + (c + d*x)^2)*(
(-1/8*I)*Pi^3 + I*ArcCot[c + d*x]^3 - (c + d*x)*ArcCot[c + d*x]^3 + 3*ArcCo
t[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] + (3*I)*ArcCot[c + d*x]*Po
lyLog[2, E^((-2*I)*ArcCot[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcCot[c + d
*x])])/2))/(d*(c + d*x)^2*(1 + (c + d*x)^(-2))) + (b^3*e*f*(1 + (c + d*x)^2
)*((-I)*c*Pi^3 + (12*I)*ArcCot[c + d*x]^2 + 12*(c + d*x)*ArcCot[c + d*x]^2
+ (8*I)*c*ArcCot[c + d*x]^3 - 8*c*(c + d*x)*ArcCot[c + d*x]^3 + 4*(c + d*x)
^2*(1 + (c + d*x)^(-2))*ArcCot[c + d*x]^3 + 24*c*ArcCot[c + d*x]^2*Log[1 -
E^((-2*I)*ArcCot[c + d*x])] - 24*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c
+ d*x])] + (24*I)*c*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])]
+ (12*I)*PolyLog[2, E^((2*I)*ArcCot[c + d*x])] + 12*c*PolyLog[3, E^((-2*I)*
ArcCot[c + d*x])]))/(4*d^2*(c + d*x)^2*(1 + (c + d*x)^(-2))) - (b^3*f^2*(1
+ (c + d*x)^2)*(I*(-1 + 3*c^2)*ArcCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[
c + d*x])] + ((c + d*x)^3*(1 + (c + d*x)^(-2))^(3/2)*(((3*I)*Pi^3)/((c + d*
x)*Sqrt[1 + (c + d*x)^(-2)]) - ((9*I)*c^2*Pi^3)/((c + d*x)*Sqrt[1 + (c + d*
x)^(-2)]) - (24*ArcCot[c + d*x])/Sqrt[1 + (c + d*x)^(-2)] + (72*c*ArcCot[c
+ d*x]^2)/Sqrt[1 + (c + d*x)^(-2)] - (48*ArcCot[c + d*x]^2)/((c + d*x)*Sqrt
[1 + (c + d*x)^(-2)]) + ((216*I)*c*ArcCot[c + d*x]^2)/((c + d*x)*Sqrt[1 + (
c + d*x)^(-2)]) - (24*ArcCot[c + d*x]^3)/Sqrt[1 + (c + d*x)^(-2)] - (24*c^2
*ArcCot[c + d*x]^3)/Sqrt[1 + (c + d*x)^(-2)] - ((24*I)*ArcCot[c + d*x]^3)/(
(c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + (96*c*ArcCot[c + d*x]^3)/((c + d*x)*S
qrt[1 + (c + d*x)^(-2)]) + ((72*I)*c^2*ArcCot[c + d*x]^3)/((c + d*x)*Sqrt[1
 + (c + d*x)^(-2)]) + 24*ArcCot[c + d*x]*Cos[3*ArcCot[c + d*x]] - 72*c*ArcC
ot[c + d*x]^2*Cos[3*ArcCot[c + d*x]] - 8*ArcCot[c + d*x]^3*Cos[3*ArcCot[c +
```

```
d*x]] + 24*c^2*ArcCot[c + d*x]^3*Cos[3*ArcCot[c + d*x]] - (72*ArcCot[c + d
*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])])/((c + d*x)*Sqrt[1 + (c + d*x)^(-
2)]) + (216*c^2*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])])/((c
+ d*x)*Sqrt[1 + (c + d*x)^(-2)]) - (432*c*ArcCot[c + d*x]*Log[1 - E^((2*I)*
ArcCot[c + d*x])])/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + (72*Log[1/((c + d
*x)*Sqrt[1 + (c + d*x)^(-2)])])/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)]) + ((28
8*I)*c*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])/((c + d*x)^3*(1 + (c + d*x)^(
-2))^(3/2)) + (48*(-1 + 3*c^2)*PolyLog[3, E^((-2*I)*ArcCot[c + d*x])])/((c
+ d*x)^3*(1 + (c + d*x)^(-2))^(3/2)) - I*Pi^3*Sin[3*ArcCot[c + d*x]] + (3*I
)*c^2*Pi^3*Sin[3*ArcCot[c + d*x]] - (72*I)*c*ArcCot[c + d*x]^2*Sin[3*ArcCot
[c + d*x]] + (8*I)*ArcCot[c + d*x]^3*Sin[3*ArcCot[c + d*x]] - (24*I)*c^2*Ar
cCot[c + d*x]^3*Sin[3*ArcCot[c + d*x]] + 24*ArcCot[c + d*x]^2*Log[1 - E^((-
2*I)*ArcCot[c + d*x])]*Sin[3*ArcCot[c + d*x]] - 72*c^2*ArcCot[c + d*x]^2*Lo
g[1 - E^((-2*I)*ArcCot[c + d*x])]*Sin[3*ArcCot[c + d*x]] + 144*c*ArcCot[c +
 d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])]*Sin[3*ArcCot[c + d*x]] - 24*Log[1/
((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]*Sin[3*ArcCot[c + d*x]]))/96))/(d^3*(c
 + d*x)^2*(1 + (c + d*x)^(-2)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 52.64, size = 12407, normalized size = 21.96

method	result	size
derivativedivides	Expression too large to display	12407
default	Expression too large to display	12407

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*f^2*x^3*arctan2(1, d*x + c)^3 + 1/8*b^3*f*x^2*arctan2(1, d*x + c)^
3*e + 1/3*a^3*f^2*x^3 + 1/8*b^3*x*arctan2(1, d*x + c)^3*e^2 + a^3*f*x^2*e +
 1/2*(2*x^3*arccot(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan
((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*
a^2*b*f^2 + 3*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c
*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f*e + a^3*x*e^2
 + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e^2/d - 1
```

```
/32*(b^3*f^2*x^3*arctan2(1, d*x + c) + 3*b^3*f*x^2*arctan2(1, d*x + c)*e +
3*b^3*x*arctan2(1, d*x + c)*e^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + integ
rate(1/32*(4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)
*d^2*f^2*x^4 + 28*b^3*arctan2(1, d*x + c)^3*e^2 + 96*a*b^2*arctan2(1, d*x +
 c)^2*e^2 + 4*(2*(7*b^3*arctan2(1, d*x + c)^3*e + 24*a*b^2*arctan2(1, d*x +
 c)^2*e)*d^2*f + (b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f^2)*x^3 + 4*(7*b^3*arctan2(1, d*x
 + c)^3*e^2 + 24*a*b^2*arctan2(1, d*x + c)^2*e^2)*c^2 + 4*((7*b^3*arctan2(1
, d*x + c)^3*e^2 + 24*a*b^2*arctan2(1, d*x + c)^2*e^2)*d^2 + (3*b^3*arctan2
(1, d*x + c)^2*e + 4*(7*b^3*arctan2(1, d*x + c)^3*e + 24*a*b^2*arctan2(1, d
*x + c)^2*e)*c)*d*f + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*
x + c)^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c
^2)*f^2)*x^2 + (3*b^3*d^2*f^2*x^4*arctan2(1, d*x + c) + 3*b^3*c^2*arctan2(1
, d*x + c)*e^2 + 3*b^3*arctan2(1, d*x + c)*e^2 + (6*b^3*d^2*f*arctan2(1, d*
x + c)*e + (6*b^3*c*arctan2(1, d*x + c) - b^3)*d*f^2)*x^3 + 3*(b^3*d^2*arct
an2(1, d*x + c)*e^2 + (4*b^3*c*arctan2(1, d*x + c)*e - b^3*e)*d*f + (b^3*c^
2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f^2)*x^2 + 3*((2*b^3*c*arc
tan2(1, d*x + c)*e^2 - b^3*e^2)*d + 2*(b^3*c^2*arctan2(1, d*x + c)*e + b^3*
arctan2(1, d*x + c)*e)*f)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*((3*b^3
*arctan2(1, d*x + c)^2*e^2 + 2*(7*b^3*arctan2(1, d*x + c)^3*e^2 + 24*a*b^2*
arctan2(1, d*x + c)^2*e^2)*c)*d + 2*(7*b^3*arctan2(1, d*x + c)^3*e + 24*a*b
^2*arctan2(1, d*x + c)^2*e + (7*b^3*arctan2(1, d*x + c)^3*e + 24*a*b^2*arct
an2(1, d*x + c)^2*e)*c^2)*f)*x + 4*(b^3*d^2*f^2*x^4*arctan2(1, d*x + c) + 3
*b^3*c*d*x*arctan2(1, d*x + c)*e^2 + (b^3*c*d*f^2*arctan2(1, d*x + c) + 3*b
^3*d^2*f*arctan2(1, d*x + c)*e)*x^3 + 3*(b^3*c*d*f*arctan2(1, d*x + c)*e +
b^3*d^2*arctan2(1, d*x + c)*e^2)*x^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*f*x*e + (b^3*f^2*x^2 + 2*b^3*f*x*e + b^3*e^2)*
arccot(d*x + c)^3 + a^3*e^2 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*f*x*e + a*b^2*e^2)
*arccot(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e + a^2*b*e^2)*arccot(d
*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acot}(c + dx))^3 (e + fx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acot(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acot(c + d*x))**3*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^2\,(a+b\,\mathrm{acot}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acot(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^2*(a + b*acot(c + d*x))^3, x)
```

3.142 $\int (e+fx) \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=337

$$\frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2}-\frac{(de+f}{}$$

```
[Out] 3/2*I*b*f*(a+b*arccot(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccot(d*x+c))^2/d
^2+I*(-c*f+d*e)*(a+b*arccot(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a
+b*arccot(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccot(d*x+c))^3/f-3*b^2*f*(a+
b*arccot(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2-3*b*(-c*f+d*e)*(a+b*arccot(d*x+c))
^2*ln(2/(1+I*(d*x+c)))/d^2+3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3*I
*b^2*(-c*f+d*e)*(a+b*arccot(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2-3/2*b^
3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Rubi [A]

time = 0.46, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5156, 4975, 4931, 5041, 4965, 2449, 2352, 5105, 5005, 5115, 6745}

$$\frac{3ib^2(de-cf)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2}-\frac{3b^2f\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)}{d^2}+\frac{i(de-cf)\left(a+b\cot^{-1}(c+dx)\right)^3}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)\left(a+b\cot^{-1}(c+dx)\right)^3}{2d^2f}-\frac{3b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)\left(a+b\cot^{-1}(c+dx)\right)^2}{d^2}+\frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2}+\frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f}-\frac{3b^3(de-cf)\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d^2}+\frac{3ib^3f\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (((3*I)/2)*b*f*(a + b*ArcCot[c + d*x])^2)/d^2 + (3*b*f*(c + d*x)*(a + b*Arc
Cot[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcCot[c + d*x])^3)/d^2 -
((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcCot[c + d*x])^3)/(2*d^2*f) + (
(e + f*x)^2*(a + b*ArcCot[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCot[c + d
*x])*Log[2/(1 + I*(c + d*x))])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCot[c + d*x
])^2*Log[2/(1 + I*(c + d*x))])/d^2 + (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1 +
 I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcCot[c + d*x])*PolyLo
g[2, 1 - 2/(1 + I*(c + d*x))])/d^2 - (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4975

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])^p/(e*(q + 1))), x] + D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCot[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\cot^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\cot^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} + \frac{(3b)\text{Subst}\left(\int \left(\frac{f^2\left(a+b\cot^{-1}(x)\right)^2}{d^2} + \cdots\right.\right.}{\cdots}$$

$$= \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\right)^3}{2f} + \frac{(3b)\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)+2\cdots}{1\cdots}\right.}{2\cdots}$$

$$= \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\cot^{-1}(c+dx)\cdots\right.}{2f}$$

$$= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \cdots$$

$$= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{i\cdots}{\cdots}$$

$$= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{i\cdots}{\cdots}$$

$$= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{i\cdots}{\cdots}$$

$$= \frac{3ibf\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^2}{2d^2} + \frac{i\cdots}{\cdots}$$

Mathematica [**A**]
time = 0.78, size = 630, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 - 3*a^2*b*(c
 + d*x)*(c*f - d*(2*e + f*x))*ArcCot[c + d*x] - 3*a^2*b*f*ArcTan[c + d*x] +
 6*a*b^2*f*((c + d*x)*ArcCot[c + d*x] + ((1 + (c + d*x)^2)*ArcCot[c + d*x]^
2)/2 - Log[1/((c + d*x)*Sqrt[1 + (c + d*x)^(-2)])]) + 3*a^2*b*(d*e - c*f)*L
og[1 + (c + d*x)^2] + 6*a*b^2*d*e*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c
+ d*x] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcC
ot[c + d*x])]) - 6*a*b^2*c*f*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x
] - 2*Log[1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c
+ d*x])]) + b^3*f*(3*(c + d*x)*ArcCot[c + d*x]^2 + (1 + (c + d*x)^2)*ArcCot
[c + d*x]^3 - 6*ArcCot[c + d*x]*Log[1 - E^((2*I)*ArcCot[c + d*x])] + (3*I)*
(ArcCot[c + d*x]^2 + PolyLog[2, E^((2*I)*ArcCot[c + d*x])])) + 2*b^3*d*e*((
I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 - 3*ArcCot[c
+ d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c + d*x]*PolyLo
g[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*ArcCot[c + d*x])
])/2) - 2*b^3*c*f*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c +
d*x]^3 - 3*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*Ar
cCot[c + d*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2
*I)*ArcCot[c + d*x])])/2))/(2*d^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.66, size = 18833, normalized size = 55.88

method	result	size
derivativedivides	Expression too large to display	18833
default	Expression too large to display	18833

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*f*x^2*arctan2(1, d*x + c)^3 + 1/8*b^3*x*arctan2(1, d*x + c)^3*e +
1/2*a^3*f*x^2 + 3/2*(x^2*arccot(d*x + c) + d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*f + a^3*x
*e + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((d*x + c)^2 + 1))*a^2*b*e/d - 3
/64*(b^3*f*x^2*arctan2(1, d*x + c) + 2*b^3*x*arctan2(1, d*x + c)*e)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + integrate(1/64*(8*(7*b^3*arctan2(1, d*x + c)^
3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*f*x^3 + 56*b^3*arctan2(1, d*x + c)^
3*e + 192*a*b^2*arctan2(1, d*x + c)^2*e + 8*(7*b^3*arctan2(1, d*x + c)^3*e
+ 24*a*b^2*arctan2(1, d*x + c)^2*e)*c^2 + 4*(2*(7*b^3*arctan2(1, d*x + c)^3
*e + 24*a*b^2*arctan2(1, d*x + c)^2*e)*d^2 + (3*b^3*arctan2(1, d*x + c)^2 +
 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f)*x
^2 + 3*(2*b^3*d^2*f*x^3*arctan2(1, d*x + c) + 2*b^3*c^2*arctan2(1, d*x + c)
*e + 2*b^3*arctan2(1, d*x + c)*e + (2*b^3*d^2*arctan2(1, d*x + c)*e + (4*b^
3*c*arctan2(1, d*x + c) - b^3)*d*f)*x^2 + 2*((2*b^3*c*arctan2(1, d*x + c)*e
 - b^3*e)*d + (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*x)
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8*((3*b^3*arctan2(1, d*x + c)^2*e + 2
*(7*b^3*arctan2(1, d*x + c)^3*e + 24*a*b^2*arctan2(1, d*x + c)^2*e)*c)*d +
(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7*b^3*arct
an2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f)*x + 12*(b^3*d^2
*f*x^3*arctan2(1, d*x + c) + 2*b^3*c*d*x*arctan2(1, d*x + c)*e + (b^3*c*d*f
*arctan2(1, d*x + c) + 2*b^3*d^2*arctan2(1, d*x + c)*e)*x^2)*log(d^2*x^2 +
2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f*x + (b^3*f*x + b^3*e)*arccot(d*x + c)^3 + a^3*e + 3*(a*b^2*f
*x + a*b^2*e)*arccot(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccot(d*x + c),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acot}(c + dx))^3 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acot(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acot(c + d*x))**3*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)*(b*arccot(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acot(c + d*x))^3,x)
[Out] int((e + f*x)*(a + b*acot(c + d*x))^3, x)
```

3.143 $\int (a+b\cot^{-1}(c+dx))^3\,dx$

Optimal. Leaf size=143

$$\frac{i(a+b\cot^{-1}(c+dx))^3}{d}+\frac{(c+dx)\,(a+b\cot^{-1}(c+dx))^3}{d}-\frac{3b(a+b\cot^{-1}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3ib^2(a+}{}$$

```
[Out] I*(a+b*arccot(d*x+c))^3/d+(d*x+c)*(a+b*arccot(d*x+c))^3/d-3*b*(a+b*arccot(d
*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arccot(d*x+c))*polylog(2,1-2/(1
+I*(d*x+c)))/d-3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5148, 4931, 5041, 4965, 5005, 5115, 6745}

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\cot^{-1}(c+dx))}{d}+\frac{(c+dx)\,(a+b\cot^{-1}(c+dx))^3}{d}+\frac{i(a+b\cot^{-1}(c+dx))^3}{d}-\frac{3b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{d}-\frac{3b^3\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcCot[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcCot[c + d*x])^3)/d -
 (3*b*(a + b*ArcCot[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcCot[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\cot^{-1}(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int \left(a+b\cot^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(3b)\operatorname{Subst}\left(\int \frac{x\left(a+b\cot^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\operatorname{Subst}\left(\int \frac{(c}{}\right.}{}$$

$$=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}-\frac{3b(a+b\cot^{-1}(}{}$$

$$=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}-\frac{3b(a+b\cot^{-1}(}{}$$

$$=\frac{i\left(a+b\cot^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cot^{-1}(c+dx)\right)^3}{d}-\frac{3b(a+b\cot^{-1}(}{}$$

Mathematica [A]
time = 0.23, size = 228, normalized size = 1.59

$$\frac{2a^3(c+dx)+6a^2b(c+dx)\cot^{-1}(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(\cot^{-1}(c+dx)\left((i+c+dx)\cot^{-1}(c+dx)-2\log\left(1-e^{2i\cot^{-1}(c+dx)}\right)\right)+i\text{PolyLog}\left(2,e^{2i\cot^{-1}(c+dx)}\right)\right)+2b^3\left(\frac{i\pi^3}{8}-i\cot^{-1}(c+dx)^3+(c+dx)\cot^{-1}(c+dx)^3-3\cot^{-1}(c+dx)^2\log\left(1-e^{-2i\cot^{-1}(c+dx)}\right)-3i\cot^{-1}(c+dx)\text{PolyLog}\left(2,e^{-2i\cot^{-1}(c+dx)}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\cot^{-1}(c+dx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCot[c + d*x] + 3*a^2*b*Log[1 + (c +
 d*x)^2] + 6*a*b^2*(ArcCot[c + d*x]*((I + c + d*x)*ArcCot[c + d*x] - 2*Log[
1 - E^((2*I)*ArcCot[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcCot[c + d*x])])
+ 2*b^3*((I/8)*Pi^3 - I*ArcCot[c + d*x]^3 + (c + d*x)*ArcCot[c + d*x]^3 - 3
*ArcCot[c + d*x]^2*Log[1 - E^((-2*I)*ArcCot[c + d*x])] - (3*I)*ArcCot[c + d
*x]*PolyLog[2, E^((-2*I)*ArcCot[c + d*x])] - (3*PolyLog[3, E^((-2*I)*ArcCot
[c + d*x])])/2))/(2*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(136) = 272.
time = 1.10, size = 430, normalized size = 3.01

method	result
derivativedivides	$\frac{(dx+c)a^3+3i\,\text{arccot}(dx+c)^2 a\,b^2+\text{arccot}(dx+c)^3 b^3 (dx+c)-3\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)\text{arccot}(dx+c)^2 b^3-3\text{arccot}(}{}$
default	$\frac{(dx+c)a^3+3i\,\text{arccot}(dx+c)^2 a\,b^2+\text{arccot}(dx+c)^3 b^3 (dx+c)-3\ln\left(1-\frac{dx+c+i}{\sqrt{1+(dx+c)^2}}\right)\text{arccot}(dx+c)^2 b^3-3\text{arccot}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+3*I*arccot(d*x+c)^2*a*b^2+arccot(d*x+c)^3*b^3*(d*x+c)-3*ln
(1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*arccot(d*x+c)^2*b^3-3*arccot(d*x+c)^2*ln(
1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*b^3+6*I*polylog(2,(d*x+c+I)/(1+(d*x+c)^2)^
(1/2))*a*b^2+6*I*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*a*b^2-6*polylog(
3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))*b^3-6*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(
1/2))*b^3+I*arccot(d*x+c)^3*b^3+3*arccot(d*x+c)^2*a*b^2*(d*x+c)-6*ln(1-(d*x
+c+I)/(1+(d*x+c)^2)^(1/2))*arccot(d*x+c)*a*b^2-6*arccot(d*x+c)*ln(1+(d*x+c+
I)/(1+(d*x+c)^2)^(1/2))*a*b^2+6*I*arccot(d*x+c)*polylog(2,(d*x+c+I)/(1+(d*x
+c)^2)^(1/2))*b^3+6*I*arccot(d*x+c)*polylog(2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2
))*b^3+3*a^2*b*arccot(d*x+c)*(d*x+c)+3/2*a^2*b*ln(1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x*arctan2(1, d*x + c)^3 - 3/32*b^3*x*arctan2(1, d*x + c)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2 + a^3*x + 3/2*(2*(d*x + c)*arccot(d*x + c) + log((
d*x + c)^2 + 1))*a^2*b/d + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 4
*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2*x^2 + 9
6*a*b^2*arctan2(1, d*x + c)^2 + 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*a
rctan2(1, d*x + c)^2)*c^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3*arcta
n2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*x + 3*(b^3*d^2*x^2*
arctan2(1, d*x + c) + b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1, d*x + c)
 + (2*b^3*c*arctan2(1, d*x + c) - b^3)*d*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2 + 12*(b^3*d^2*x^2*arctan2(1, d*x + c) + b^3*c*d*x*arctan2(1, d*x + c))*
log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arccot
(d*x + c) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acot}{(c + dx)})^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acot(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acot}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^3,x)
```

```
[Out] int((a + b*acot(c + d*x))^3, x)
```

3.144 $\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$

Optimal. Leaf size=372

$$-\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}-\frac{3ib(a+b\cot^{-1}(\ldots}{}$$

```
[Out] -(a+b*arccot(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arccot(d*x+c))^3*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arccot(d*x+c))^2*polylog
(2,1-2/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arccot(d*x+c))^2*polylog(2,1-2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2
/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arccot(d*x+c))*polylog(3,1-2*d*(f*x+e)/(d*e+
I*f-c*f)/(1-I*(d*x+c)))/f+3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f-3/4*I*b^
3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A]
time = 0.15, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5156, 4971}

$$\frac{3b^2(a+b\cot^{-1}(c+dx))\,\mathrm{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{3b^2\mathrm{Li}_3\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))}{2f}+\frac{3ib(a+b\cot^{-1}(c+dx))^2\,\mathrm{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}+\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{3ib\mathrm{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^2}{2f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\cot^{-1}(c+dx))^3}{f}-\frac{3ib^3\mathrm{Li}_4\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{4f}+\frac{3ib^3\mathrm{Li}_4\left(1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCot[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcCot[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f -
 (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f + (((3*I)/2)*b*(a + b*ArcCot[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcCot[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCot[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f - (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 4971

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[3
*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp
[3*I*b*(a + b*ArcCot[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCot[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] + Simp[3*I*b^3*(PolyLog
```

```
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] - Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\cot^{-1}(c+dx))^3\log\left(\frac{2c}{(de+if-c}\right.}{f}$$

Mathematica [F]
time = 37.89, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cot^{-1}(c+dx))^3}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x),x]
[Out] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.31, size = 4291, normalized size = 11.53

method	result	size
derivativedivides	Expression too large to display	4291
default	Expression too large to display	4291

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*I*b^3*d/f*Pi*arccot(d*x+c)^3*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))
*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*
```

```
e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f))*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)
-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/(
(d*x+c+I)^2/(1+(d*x+c)^2)-1))+6*I*a*b^2*d^2/f*e*arccot(d*x+c)*polylog(2,(d*
e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-3
/2*I*a*b^2*d/f*Pi*arccot(d*x+c)^2*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e
*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f))*csgn
(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f
*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2-3/2*I*a*b^
2*d/f*Pi*arccot(d*x+c)^2*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I*(c*f*
(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+
I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2-6*I*b^3*d/f*polylo
g(4,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+3*a*b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arcco
t(d*x+c)^2-3*a*b^2*d/f*arccot(d*x+c)^2*ln(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e
*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)+3*a*b
^2*d/f*arccot(d*x+c)^2*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)-3*a*b^2*d/f*arccot(d
*x+c)^2*ln(1+(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-3*a*b^2*d/f*arccot(d*x+c)^2*ln(
1-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+3/2*a*b^2*d*c/(-I*f+c*f-d*e)*polylog(3,(d*
e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-3*a*b^2*d/(-I*f+c*f-d*
e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x
+c)^2))-3/2*I*a*b^2*d/(-I*f+c*f-d*e)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)
*(d*x+c+I)^2/(1+(d*x+c)^2))+3*a^2*b*d*ln(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)
+3/2*I*a^2*b*d/f*dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))-3/2*I*a^2*b*d/f*dilog
((I*f-f*(d*x+c))/(d*e+I*f-c*f))+b^3*d*c/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1
-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+3/2*b^3*d*c/(-I*f+
c*f-d*e)*arccot(d*x+c)*polylog(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(
1+(d*x+c)^2))-I*b^3*d/(-I*f+c*f-d*e)*arccot(d*x+c)^3*ln(1-(d*e+I*f-c*f)/(-c
*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+3*I*b^3*d/f*arccot(d*x+c)^2*polylog(
2,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+3*I*b^3*d/f*arccot(d*x+c)^2*polylog(2,(d*
x+c+I)/(1+(d*x+c)^2)^(1/2))-3/2*I*b^3*d/(-I*f+c*f-d*e)*arccot(d*x+c)*polylo
g(3,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+3/4*I*b^3*d*c/(
-I*f+c*f-d*e)*polylog(4,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)
^2))+a^3*d*ln(c*f-d*e-f*(d*x+c))/f+3/4*b^3*d/(-I*f+c*f-d*e)*polylog(4,(d*e+
I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))+3/2*I*a*b^2*d/f*Pi*arcco
t(d*x+c)^2*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*csgn(I*(c*f*(d*x+c+I)^2/(1
+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c
)^2)-I*f))*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)
^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1
))-1/2*I*b^3*d/f*Pi*arccot(d*x+c)^3*csgn(I/((d*x+c+I)^2/(1+(d*x+c)^2)-1))*c
sgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-
I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^2+3*I*b^3
*d^2/f*e*arccot(d*x+c)^2*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2
/(1+(d*x+c)^2))/(-2*I*f+2*c*f-2*d*e)-3*a*b^2*d^2/f*e/(-I*f+c*f-d*e)*arccot(
d*x+c)^2*ln(1-(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)^2/(1+(d*x+c)^2))-3*I*a
*b^2*d*c/(-I*f+c*f-d*e)*arccot(d*x+c)*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f
)*(d*x+c+I)^2/(1+(d*x+c)^2))+3/2*I*a*b^2*d/f*Pi*arccot(d*x+c)^2*csgn(I*(c*f
```

```
*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c
+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)^2)-1))^3-6*a*b^2*d/f*polyl
og(3,-(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*a*b^2*d/f*polylog(3,(d*x+c+I)/(1+(d*
x+c)^2)^(1/2))+b^3*d*ln(c*f-d*e-f*(d*x+c))/f*arccot(d*x+c)^3-3/2*b^3*d/(-I*
f+c*f-d*e)*arccot(d*x+c)^2*polylog(2,(d*e+I*f-c*f)/(-c*f+d*e-I*f)*(d*x+c+I)
^2/(1+(d*x+c)^2))+b^3*d/f*arccot(d*x+c)^3*ln((d*x+c+I)^2/(1+(d*x+c)^2)-1)-6
*b^3*d/f*arccot(d*x+c)*polylog(3,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))-b^3*d/f*arc
cot(d*x+c)^3*ln(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)
-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)-b^3*d/f*arccot(d*x+c)^3*ln(1+(d
*x+c+I)/(1+(d*x+c)^2)^(1/2))-6*b^3*d/f*arccot(d*x+c)*polylog(3,-(d*x+c+I)/(
1+(d*x+c)^2)^(1/2))-b^3*d/f*arccot(d*x+c)^3*ln(1-(d*x+c+I)/(1+(d*x+c)^2)^(1
/2))-6*I*b^3*d/f*polylog(4,(d*x+c+I)/(1+(d*x+c)^2)^(1/2))+1/2*I*b^3*d/f*Pi*
arccot(d*x+c)^3*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d
*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2/(1+(d*x+c)
^2)-1))^3+3/2*I*a^2*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f+f*(d*x+c))/(c*f-d*e
+I*f))-3/2*I*a^2*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*
f))-1/2*I*b^3*d/f*Pi*arccot(d*x+c)^3*csgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-
d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f))*c
sgn(I*(c*f*(d*x+c+I)^2/(1+(d*x+c)^2)-d*e*(d*x+c+I)^2/(1+(d*x+c)^2)-c*f+d*e-
I*f*(d*x+c+I)^2/(1+(d*x+c)^2)-I*f)/((d*x+c+I)^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan2(1, d*x + c)^3 + 3*b^3*a
rctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan2(1,
 d*x + c)^2 + 96*a^2*b*arctan2(1, d*x + c))/(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**3/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acot}(c+d\,x))^3}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^3/(e + f*x),x)
```

```
[Out] int((a + b*acot(c + d*x))^3/(e + f*x), x)
```

3.145 $\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1233

$$\frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}+\frac{ib^3d\cot^{-1}(c+dx)^3}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^3d(de-cf)\cot^{-1}}{f\left(d^2e^2-2cdef+\right.}$$

```
[Out] -3*I*b^3*d*arccot(d*x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)
))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*a*b^2*d*(-c*f+d*e)*arccot(d*x+c)^2/f/(
d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^3*d*(-c*f+d*e)*arccot(d*x+c)^3/f/(d^2*e^2-2*
c*d*e*f+(c^2+1)*f^2)-(a+b*arccot(d*x+c))^3/f/(f*x+e)-3*a^2*b*d*(-c*f+d*e)*a
rctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-3*a^2*b*d*ln(f*x+e)/(f^2+(-c*f+d*e)^2)+6*
a*b^2*d*arccot(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3
*b^3*d*arccot(d*x+c)^2*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-
6*a*b^2*d*arccot(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arccot(d*x+c)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*
f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-6*a*b^2*d*arccot(d*x+c)*l
n(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*b^3*d*arccot(d*x+c)^2*
ln(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*a^2*b*d*ln(1+(d*x+c
)^2)/(f^2+(-c*f+d*e)^2)+3*I*a*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2
*c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-
I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*arccot(d*x+c)^2/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^3*d*arccot(d*x+c)^3/(d^2*e^2-2*c*d*e*f+(c^
2+1)*f^2)+3*I*b^3*d*arccot(d*x+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arccot(d*x+c)*polylog(2,1-2/(1+I*(d*x+c)))/(d
^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2/(1-I*(d*x+c)))/(d^2*e
^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(
1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A]
time = 1.73, antiderivative size = 1233, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5154, 6873, 5166, 6820, 12, 6857, 720, 31, 649, 209, 266, 4967, 2449, 2352, 2497, 5105, 5005, 5041, 4965, 4969, 5115, 6745}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] ((3*I)*a*b^2*d*ArcCot[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (
3*a*b^2*d*(d*e - c*f)*ArcCot[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
```

```
)*f^2)) + (I*b^3*d*ArcCot[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
 + (b^3*d*(d*e - c*f)*ArcCot[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2)) - (a + b*ArcCot[c + d*x])^3/(f*(e + f*x)) - (3*a^2*b*d*(d*e - c*f)*
ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) - (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) + (6*a*b^2*d*ArcCot[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcCot[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (6*a*b^2*d*ArcCot[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcCot[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) - (6*a*b^2*d*ArcCot[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcCot[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcCot[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcCot[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcCot[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) + (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) - (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
```

```
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4969

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcCot[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Cot[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] - Simp[I
*b*(a + b*ArcCot[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] + Simp[I*b*(a
+ b*ArcCot[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5105

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCot[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5154

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCot[c + d*x])^p/(f*(m +
```

```
1))), x] + Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5166

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCot[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^2}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3b)\text{Subst}\left(\int \frac{d\left(a+b\cot^{-1}(x)\right)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\text{Subst}\left(\int \frac{\left(a+b\cot^{-1}(x)\right)^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3bd)\text{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)(1+x^2)} + \frac{2ab\cot^{-1}(x)}{(de-cf+fx)(1+x^2}\right.\right.}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} -$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(6ab^2d)\,\text{Subst}\left(\int \left(\frac{f^2\cot^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)}\right.\right.}{f}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} - \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)}{1+x^2}\right.}{f\left(d^2e^2-2cdef+\right.}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{3a^2bd\log(e+f}{f^2+(de-cf)}$$

$$= -\frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\left(f^2+(de-cf)^2\right)} - \frac{3a^2bd\log(e+f}{f^2+(de-cf)}$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{ib^3d\cot}{d^2e^2-2cde}$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{ib^3d\cot}{d^2e^2-2cde}$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{ib^3d\cot}{d^2e^2-2cde}$$

$$= \frac{3iab^2d\cot^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)f^2} + \frac{3ab^2d(de-cf)\cot^{-1}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{ib^3d\cot}{d^2e^2-2cde}$$

Mathematica [F]
time = 44.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 21.65, size = 6740, normalized size = 5.47

method	result	size
derivativedivides	Expression too large to display	6740
default	Expression too large to display	6740

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1
)*f^3 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2
+ 1)*f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2))
 + 2*arccot(d*x + c)/(f^2*x + f*e))*a^2*b - a^3/(f^2*x + f*e) - 1/32*(4*b^3
*arctan2(1, d*x + c)^3 - 3*b^3*arctan2(1, d*x + c)*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2 - 32*(f^2*x + f*e)*integrate(-1/32*(12*b^3*d*arctan2(1, d*x + c)
^2*e - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*d^2
*f*x^2 + 4*(3*b^3*arctan2(1, d*x + c)^2 - 2*(7*b^3*arctan2(1, d*x + c)^3 +
24*a*b^2*arctan2(1, d*x + c)^2)*c)*d*f*x - 3*(b^3*d^2*f*x^2*arctan2(1, d*x
+ c) + b^3*d*e + (2*b^3*c*arctan2(1, d*x + c) + b^3)*d*f*x + (b^3*c^2*arcta
n2(1, d*x + c) + b^3*arctan2(1, d*x + c))*f)*log(d^2*x^2 + 2*c*d*x + c^2 +
1)^2 - 4*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + (7
*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2)*c^2)*f + 12*(b
^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*arctan2(1, d*x + c)*e + (b^3*c*d
```

```
*f*arctan2(1, d*x + c) + b^3*d^2*arctan2(1, d*x + c)*e)*x)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(d^2*f^3*x^4 + 2*(c*d*f^3 + d^2*f^2*e)*x^3 + (4*c*d*f^2*e
 + (c^2 + 1)*f^3 + d^2*f*e^2)*x^2 + (c^2*e^2 + e^2)*f + 2*(c*d*f*e^2 + (c^2
*e + e)*f^2)*x), x))/(f^2*x + f*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acot}\left(c+d\,x\right)\right)^3}{\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acot(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*acot(c + d*x))^3/(e + f*x)^2, x)
```

3.146 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx$

Optimal. Leaf size=177

$$\frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)}+\frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)}-\frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,\right.}{2f(de-(i+c)f}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arccot(d*x+c))/f/(1+m)+1/2*I*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)-1/2*I*
b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d*e-
(I+c)*f)/(1+m)/(2+m)
```

Rubi [A]
time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5156, 4973, 726, 70}

$$\frac{(e+fx)^{m+1}\left(a+b\cot^{-1}(c+dx)\right)}{f(m+1)}+\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}-\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-(c+i)f}\right)}{2f(m+1)(m+2)(de-(c+i)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCot[c + d*x]))/(f*(1 + m)) + ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) - ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 4973

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCot[c*x])/(e*(q + 1))), x] + Dist[b*(
```

```
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\cot^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} + \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2} dx,\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} + \frac{b\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)} +\right.\right.}{f(}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} + \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x} d\right.}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\cot^{-1}(c+dx)\right)}{f(1+m)} + \frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m\right.}{2f(de+(i-c)f)(1+}$$

Mathematica [A]
time = 0.29, size = 162, normalized size = 0.92

$$\frac{(e+fx)^{1+m}\left(2\left(a+b\cot^{-1}(c+dx)\right)+\frac{bd(e+fx)\left((de-(i+c)f)\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c)f)\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-(i+c)f}\right)\right)}{(-ide+f+icf)(de-(i+c)f)(2+m)}\right)}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcCot[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
```

```
d*e - (I + c)*f)]))/(((-I)*d*e + f + I*c*f)*(d*e - (I + c)*f)*(2 + m))))/(2
*f*(1 + m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (fx+e)^m (a+b\,\mathrm{arccot}(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x*arctan2(1, d*x + c) + arctan2(1, d*x + c)*e)*(f*x + e)^m + 2*(f*m
 + f)*integrate(1/2*((c^2*arctan2(1, d*x + c) + arctan2(1, d*x + c))*f*m +
(d^2*f*m*arctan2(1, d*x + c) + d^2*f*arctan2(1, d*x + c))*x^2 + (c^2*arctan
2(1, d*x + c) + arctan2(1, d*x + c))*f + (2*c*d*f*m*arctan2(1, d*x + c) + (
2*c*arctan2(1, d*x + c) + 1)*d*f)*x + d*e)*(f*x + e)^m/((c^2 + 1)*f*m + (d^
2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))*b/(f*m + f)
+ (f*x + e)^(m + 1)*a/(f*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)*(f*x + e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e+f\,x)^m\,(a+b\,\mathrm{acot}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x)),x)
```

```
[Out] int((e + f*x)^m*(a + b*acot(c + d*x)), x)
```

3.147 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCot[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx = \frac{\mathrm{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\cot^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 4.00, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^2, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arccot}(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^2/(f*(m + 1)) - 1/16*((b^2*f*x + b^2*e)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 4*(b^2*f*x*arctan2(1, d*x + c)^2 + b^2*ar
ctan2(1, d*x + c)^2*e)*(f*x + e)^m - 16*(f*m + f)*integrate(1/16*(((b^2*c^2
 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*
d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 4*(b
^2*d^2*f*x^2 + b^2*c*d*e + (b^2*c*d*f + b^2*d^2*e)*x)*(f*x + e)^m*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) + 4*(2*b^2*d*arctan2(1, d*x + c)*e + (3*b^2*arctan2
(1, d*x + c)^2 + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*
c^2 + 8*a*b*arctan2(1, d*x + c))*f*m + ((3*b^2*arctan2(1, d*x + c)^2 + 8*a*
b*arctan2(1, d*x + c))*d^2*f*m + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arcta
n2(1, d*x + c))*d^2*f)*x^2 + (3*b^2*arctan2(1, d*x + c)^2 + (3*b^2*arctan2(
1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c^2 + 8*a*b*arctan2(1, d*x + c))
*f + 2*((3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1, d*x + c))*c*d*f*m +
 (b^2*arctan2(1, d*x + c) + (3*b^2*arctan2(1, d*x + c)^2 + 8*a*b*arctan2(1,
 d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 +
 (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccot(d*x + c)^2 + 2*a*b*arccot(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^2*(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{acot}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*acot(c + d*x))^2, x)
```

3.148 $\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCot[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\cot^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\cot^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCot[c + d*x])^3, x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arccot}(dx+c)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccot(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^3/(f*(m + 1)) - 1/32*(3*(b^3*f*x*arctan2(1, d*x + c) +
b^3*arctan2(1, d*x + c)*e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 -
 4*(b^3*f*x*arctan2(1, d*x + c)^3 + b^3*arctan2(1, d*x + c)^3*e)*(f*x + e)^
m - 32*(f*m + f)*integrate(-1/32*(3*(b^3*d*e - (b^3*c^2*arctan2(1, d*x + c)
 + b^3*arctan2(1, d*x + c))*f*m - (b^3*d^2*f*m*arctan2(1, d*x + c) + b^3*d^
2*f*arctan2(1, d*x + c))*x^2 - (b^3*c^2*arctan2(1, d*x + c) + b^3*arctan2(1
, d*x + c))*f - (2*b^3*c*d*f*m*arctan2(1, d*x + c) + (2*b^3*c*arctan2(1, d*
x + c) - b^3)*d*f)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 12*(
b^3*d^2*f*x^2*arctan2(1, d*x + c) + b^3*c*d*arctan2(1, d*x + c)*e + (b^3*c*
d*f*arctan2(1, d*x + c) + b^3*d^2*arctan2(1, d*x + c)*e)*x)*(f*x + e)^m*log
(d^2*x^2 + 2*c*d*x + c^2 + 1) - 4*(3*b^3*d*arctan2(1, d*x + c)^2*e + (7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(
1, d*x + c) + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2
 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*f*m + ((7*b^3*arctan2(1, d*x + c)^3 +
 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*d^2*f*m + (
7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arc
tan2(1, d*x + c))*d^2*f)*x^2 + (7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arct
an2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c) + (7*b^3*arctan2(1, d*x +
c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(1, d*x + c))*c^2)*
f + (2*(7*b^3*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a
^2*b*arctan2(1, d*x + c))*c*d*f*m + (3*b^3*arctan2(1, d*x + c)^2 + 2*(7*b^3
*arctan2(1, d*x + c)^3 + 24*a*b^2*arctan2(1, d*x + c)^2 + 24*a^2*b*arctan2(
1, d*x + c))*c)*d*f)*x)*(f*x + e)^m)/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2
 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccot(d*x + c)^3 + 3*a*b^2*arccot(d*x + c)^2 + 3*a^2*b*arcco
t(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acot(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccot(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(d*x + c) + a)^3*(f*x + e)^m, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e+f\,x\right)}^m\,{\left(a+b\,\mathrm{acot}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acot(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^m*(a + b*acot(c + d*x))^3, x)
```

3.149 $\int x^3 \cot^{-1}(a+bx^4)\, dx$

Optimal. Leaf size=42

$$\frac{(a+bx^4)\cot^{-1}(a+bx^4)}{4b}+\frac{\log\left(1+(a+bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arccot(b*x^4+a)/b+1/8*ln(1+(b*x^4+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6847, 5148, 4931, 266}

$$\frac{\log\left((a+bx^4)^2+1\right)}{8b}+\frac{(a+bx^4)\cot^{-1}(a+bx^4)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCot[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCot[a + b*x^4])/(4*b) + Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \cot^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \cot^{-1}(a+bx)\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\cot^{-1}\left(a+bx^4\right)}{4b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\cot^{-1}\left(a+bx^4\right)}{4b} + \frac{\log\left(1+\left(a+bx^4\right)^2\right)}{8b}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.88

$$\frac{2(a+bx^4)\cot^{-1}\left(a+bx^4\right)+\log\left(1+\left(a+bx^4\right)^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCot[a + b*x^4],x]
```

```
[Out] (2*(a + b*x^4)*ArcCot[a + b*x^4] + Log[1 + (a + b*x^4)^2])/(8*b)
```

Maple [A]
time = 0.13, size = 37, normalized size = 0.88

method	result
derivativedivides	$\frac{\text{arccot}\left(b\,x^4+a\right)\left(b\,x^4+a\right)+\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
default	$\frac{\text{arccot}\left(b\,x^4+a\right)\left(b\,x^4+a\right)+\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
risch	$\frac{i x^4\ln\left(1+i\left(b\,x^4+a\right)\right)}{8}-\frac{i x^4\ln\left(1-i\left(b\,x^4+a\right)\right)}{8}+\frac{\pi\,x^4}{8}-\frac{a\arctan\left(\frac{x^4 b}{a^2+1}+\frac{a^2 b\,x^4}{a^2+1}+\frac{a^3}{a^2+1}+\frac{a}{a^2+1}\right)}{4b}+\frac{a\arctan(a)}{4b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccot(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(arccot(b*x^4+a)*(b*x^4+a)+1/2*ln(1+(b*x^4+a)^2))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.83

$$\frac{2\left(bx^4+a\right)\text{arccot}\left(bx^4+a\right)+\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arccot(b*x^4 + a) + log((b*x^4 + a)^2 + 1))/b
```

Fricas [A]
time = 2.85, size = 51, normalized size = 1.21

$$\frac{2\,bx^4\,\operatorname{arccot}\left(bx^4+a\right)-2\,a\arctan\left(bx^4+a\right)+\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x^4*arccot(b*x^4 + a) - 2*a*arctan(b*x^4 + a) + log(b^2*x^8 + 2*a*
b*x^4 + a^2 + 1))/b
```

Sympy [A]
time = 0.77, size = 60, normalized size = 1.43

$$\begin{cases} \frac{a\operatorname{acot}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acot}\left(a+bx^4\right)}{4}+\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for}\ b\neq 0 \\ \frac{x^4\operatorname{acot}\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acot(b*x**4+a),x)
```

```
[Out] Piecewise((a*acot(a + b*x**4)/(4*b) + x**4*acot(a + b*x**4)/4 + log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*acot(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(38) = 76.
time = 0.47, size = 127, normalized size = 3.02

$$-\frac{\arctan\left(\frac{1}{bx^4+a}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2+\log\left(\frac{16\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2}{\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^4+2\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)^2+1}\right)\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)-\arctan\left(\frac{1}{bx^4+a}\right)}{8\,b\tan\left(\frac{1}{2}\arctan\left(\frac{1}{bx^4+a}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccot(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(arctan(1/(b*x^4 + a))*tan(1/2*arctan(1/(b*x^4 + a)))^2 + log(16*tan(1
/2*arctan(1/(b*x^4 + a)))^2/(tan(1/2*arctan(1/(b*x^4 + a)))^4 + 2*tan(1/2*a
rctan(1/(b*x^4 + a)))^2 + 1))*tan(1/2*arctan(1/(b*x^4 + a))) - arctan(1/(b*
x^4 + a)))/(b*tan(1/2*arctan(1/(b*x^4 + a))))
```

Mupad [B]
time = 0.76, size = 230, normalized size = 5.48

$$\frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}+\frac{x^4\,\mathrm{acot}\left(b\,x^4+a\right)}{4}-\frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^4\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{a^6\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acot(a + b*x^4),x)
```

```
[Out] log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (x^4*acot(a + b*x^4))/4 - (a*ata
n(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^5)
/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*a^2
 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*b*x^
4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))/(4*b
)
```

3.150 $\int x^{-1+n} \cot^{-1}(a+bx^n)\, dx$

Optimal. Leaf size=45

$$\frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn}+\frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

[Out] (a+b*x^n)*arccot(a+b*x^n)/b/n+1/2*ln(1+(a+b*x^n)^2)/b/n

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5148, 4931, 266}

$$\frac{\log\left((a+bx^n)^2+1\right)}{2bn}+\frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*ArcCot[a + b*x^n],x]

[Out] ((a + b*x^n)*ArcCot[a + b*x^n])/(b*n) + Log[1 + (a + b*x^n)^2]/(2*b*n)

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5148

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\cot^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \cot^{-1}(a+bx)\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \cot^{-1}(x)\,dx, x, a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\cot^{-1}(a+bx^n)}{bn} + \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.89

$$\frac{2(a+bx^n)\cot^{-1}(a+bx^n)+\log\left(1+(a+bx^n)^2\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCot[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcCot[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(2*b*n)
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 149, normalized size = 3.31

method	result
risch	$\frac{i x^n \ln(1+i(a+b x^n))}{2n} - \frac{i x^n \ln(1-i(a+b x^n))}{2n} + \frac{\pi x^n}{2n} + \frac{i \ln\left(x^n - \frac{i-a}{b}\right) a}{2bn} - \frac{i \ln\left(\frac{i+a}{b} + x^n\right) a}{2bn} + \frac{\ln\left(x^n - \frac{i-a}{b}\right)}{2bn} + \frac{\ln\left(\frac{i+a}{b} + x^n\right)}{2bn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arccot(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/n*x^n*ln(1+I*(a+b*x^n))-1/2*I/n*x^n*ln(1-I*(a+b*x^n))+1/2/n*Pi*x^n+1/
2*I/b/n*ln(x^n-(I-a)/b)*a-1/2*I/b/n*ln((I+a)/b+x^n)*a+1/2/b/n*ln(x^n-(I-a)/
b)+1/2/b/n*ln((I+a)/b+x^n)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.84

$$\frac{2\,(bx^n+a)\,\text{arccot}\,(bx^n+a)+\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arccot(b*x^n + a) + log((b*x^n + a)^2 + 1))/(b*n)
```

Fricas [A]
time = 1.94, size = 56, normalized size = 1.24

$$\frac{2\,bx^n\operatorname{arccot}\left(bx^n+a\right)-2\,a\arctan\left(bx^n+a\right)+\log\left(b^2x^{2\,n}+2\,abx^n+a^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arccot(b*x^n + a) - 2*a*arctan(b*x^n + a) + log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acot(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.43, size = 60, normalized size = 1.33

$$\frac{b\left(\frac{2\,(bx^n+a)\arctan\left(\frac{1}{bx^n+a}\right)}{b^2}+\frac{\log\left(\frac{1}{(bx^n+a)^2}+1\right)}{b^2}-\frac{\log\left(\frac{1}{(bx^n+a)^2}\right)}{b^2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccot(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*b*(2*(b*x^n + a)*arctan(1/(b*x^n + a))/b^2 + log(1/(b*x^n + a)^2 + 1)/b
^2 - log((b*x^n + a)^(-2))/b^2)/n
```

Mupad [B]
time = 1.38, size = 58, normalized size = 1.29

$$\frac{\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)}{2}+a\,\mathrm{acot}\left(a+b\,x^n\right)}{b\,n}+\frac{x^n\,\mathrm{acot}\left(a+b\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acot(a + b*x^n),x)
```

```
[Out] (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)/2 + a*acot(a + b*x^n))/(b*n) + (x^n
*acot(a + b*x^n))/n
```

3.151 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
[Out] -integrate((b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
[Out] integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
[Out] -Integral((a + b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^n}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.152 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=488

$$-\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{3ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,\right.}{2c}$$

```
[Out] -2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arccoth(1-2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arccot((-
c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c+3/2*b^2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+
1)^(1/2)))*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b
*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/(I+(-c*
x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c-3/4*I*b^3*polylog(4,1-2*I/(I+(-c
*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,1-2*(-c*x+1)^(1/2)/(I+(-c
*x+1)^(1/2)/(c*x+1)^(1/2))/(c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.37, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 4943, 5109, 5005, 5113, 5117, 6745}

$$\frac{3b^2\text{Li}_3\left(1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{Li}_3\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{Li}_2\left(1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3ib\text{Li}_2\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}-\frac{3ib^3\text{Li}_4\left(1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{4c}+\frac{3ib^3\text{Li}_4\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcCot[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1
- (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c +
 (3*b^2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - (2*I)/(I
 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcCot[Sqrt[1 - c*x
]/Sqrt[1 + c*x]])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt
[1 - c*x]/Sqrt[1 + c*x]))])/(2*c) - (((3*I)/4)*b^3*PolyLog[4, 1 - (2*I)/(I
+ Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*PolyLog[4, 1 - (2*Sqrt[
1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5117

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k +
 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &
& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(6b)\text{S}}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)\text{S}}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(}{}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(402) = 804$.
time = 0.58, size = 1605, normalized size = 3.29

method	result	size
default	Expression too large to display	1605

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] 3*a^2*b/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-3
*a*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((I+(-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3*a*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-
3*I*b^3/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*I*a*b^2/c*arccot((-c*x+1)^(
1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(
c*x+1)+1)^(1/2))+3*a*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(I+(
-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+b^3/c*arccot((-c*x
+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/
(c*x+1)+1)^(1/2))+6*b^3/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(
I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6*I*b^3/c*polyl
og(4,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^3/c*ar
ccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln((I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/
((-c*x+1)/(c*x+1)+1)+1)-3/2*b^3/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*poly
log(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/4*I*b^3/c
*polylog(4,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+b^3/c*
arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)/((-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*
polylog(3,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6*I*
b^3/c*polylog(4,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)+3/2*I*b^3/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3*I*b^3/c*arccot((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c
*x+1)+1)^(1/2))-3*a^2*b/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((I+(-c*x+
1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2*I*a^2*b/c*dilog((I+(-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-3/2*I*a^2*b/c*dilog(1
-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-6*I*a*b^2/c*arcco
t((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/
((-c*x+1)/(c*x+1)+1)^(1/2))+6*a*b^2/c*polylog(3,-(I+(-c*x+1)^(1/2)/(c*x+1)^
(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3/2*a*b^2/c*polylog(3,-(I+(-c*x+1)^(1/2)
```

```
/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+6*a*b^2/c*polylog(3,(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*I*a*b^2/c*arccot((-c*x+1)^(
1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)
/(c*x+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x
 + 1)) + 64*c*integrate(-1/128*(112*b^3*arctan2(sqrt(c*x + 1), sqrt(-c*x +
1))^3 + 384*a*b^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 + 3*(b^3*log(2)^
2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c
*x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1
)))/(c^2*x^2 - 1), x))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccot(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.153 $\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=321

$$-\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{ib\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,1\right.}{c}$$

```
[Out] -2*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arccoth(1-2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arccot((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+1)^(
1/2))/(c*x+1)^(1/2))/c+1/2*b^2*polylog(3,1-2*I/(I+(-c*x+1)^(1/2)/(c*x+1)^(1
/2)))/c-1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2
))/(c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.24, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6813, 4943, 5109, 5005, 5113, 6745}

$$\frac{ib\text{Li}_2\left(1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\text{Li}_2\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}+\frac{b^2\text{Li}_3\left(1-\frac{2i}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i}\right)}{2c}-\frac{b^2\text{Li}_3\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+i\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (I*b*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (I*b*(a
 + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/
(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c + (b^2*PolyLog[3, 1 -
 (2*I)/(I + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (b^2*PolyLog[3, 1 - (2*S
qrt[1 - c*x])/(Sqrt[1 + c*x]*(I + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c)
```

Rule 4943

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Dist[2*b*c*p, Int[(a + b*
ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5109

```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegrand
[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b, c
, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)
))^2, 0]
```

Rule 5113

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cot^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(4b)\mathrm{S}}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\mathrm{S}}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\Big(a}{}$$

$$= -\frac{2\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\Big(a}{}$$

Mathematica [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 822 vs. 2(268) = 536.
time = 0.39, size = 823, normalized size = 2.56

method	result

default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}+\frac{b^2\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\frac{i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{2ib^2\operatorname{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{po}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln(1+(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)
)-2*I*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3,-(I+(-c*x+1
)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-b^2/c*arccot((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*ln((I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1
)+1)+1)+I*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(I+(-c*x+1)
^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2*b^2/c*polylog(3,-(I+(-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+b^2/c*arccot((-c*x+1)^(1/2
)/(c*x+1)^(1/2))^2*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+
1)^(1/2))-2*I*b^2/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(I+(-c*x
+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3,(I+(
-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2*a*b/c*arccot((-c
*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1
)/(c*x+1)+1))-2*a*b/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((I+(-c*x+1)^(
1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+I*a*b/c*dilog((I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-I*a*b/c*dilog(1-(I+(-c*x+1)^(1/
2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
ctan2(sqrt(c*x + 1), sqrt(-c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 + 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(c*x + 1)/sqrt(-c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) - 64*b^2*integr
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(c*x + 1)/sqrt(-c*x + 1))*
```

```
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(c*x +
1)/sqrt(-c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(c*x + 1)/sqrt(-c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccot(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.154 $\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{ib\text{PolyLog}\left(2,-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\text{PolyLog}\left(2,\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*I*b*polylog(2,-I*(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c-1/2*I*b*polylog(2,I*(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {212, 6813, 4941, 2438}

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}+\frac{ib\text{Li}_2\left(-\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\text{Li}_2\left(\frac{i\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + ((I/2)*b*PolyLog[2, ((-I)*Sqrt[1 + c*x])/Sqrt[1 - c*x]])/c - ((I/2)*b*PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1 - c*x]])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x] + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b\cot^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{(ib)\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{(ib)\text{Subs}}{}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{ib\text{Li}_2\left(-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}-\frac{ib\text{Li}_2\left(\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 0.74

$$\frac{a\tanh^{-1}(cx)}{c}+\frac{ib\left(\text{PolyLog}\left(2,-\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)-\text{PolyLog}\left(2,\frac{i\sqrt{1+cx}}{\sqrt{1-cx}}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c + ((I/2)*b*(PolyLog[2, ((-I)*Sqrt[1 + c*x])/Sqrt[1 - c*x
]] - PolyLog[2, (I*Sqrt[1 + c*x])/Sqrt[1 - c*x]]))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(78) = 156.
time = 0.26, size = 259, normalized size = 2.64

method	result

default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{\left(i+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}}{\frac{-cx+1}{cx+1}+1}\right)}{c}-\frac{b\,\mathrm{arccot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+b/c*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*ln(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-b/c*arccot
((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x
+1)/(c*x+1)+1)+1)+1/2*I*b/c*dilog((I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x
+1)/(c*x+1)+1)+1)-1/2*I*b/c*dilog(1-(I+(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c
*x+1)/(c*x+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.155 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)\left(a+b \operatorname{arccot}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a c^{2} x^{2}-a+b c^{2} x^{2} \operatorname{acot}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)-b \operatorname{acot}\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acot(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.156 $\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cot^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCot[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccot}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] -2*(2*(b^2*c^2*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c^2)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2
)*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
 + 1)/((b^2*c*arctan2(sqrt(c*x + 1), sqrt(-c*x + 1)) + a*b*c)*sqrt(c*x + 1)
*sqrt(-c*x + 1))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccot(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}c^{2}x^{2}-a^{2}+2abc^{2}x^{2}\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\operatorname{acot}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^{2}c^{2}x^{2}\operatorname{acot}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^{2}\operatorname{acot}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acot((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acot(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*acot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*ac
ot(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acot(sqrt(-c*x + 1)/sqrt(c*x + 1
))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccot((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccot(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{acot}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acot((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.157 $\int \cot^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arctan(tan(b*x+a)))^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = $\{2188, 30\}$

$$-\frac{\cot^{-1}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Tan[a + b*x]],x]
```

```
[Out] -1/2*ArcCot[Tan[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(\tan(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \cot^{-1}(\tan(a+bx))\right)}{b}\\ &= -\frac{\cot^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$\frac{bx^2}{2} + x\cot^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Tan[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcCot[Tan[a + b*x]]
```

Maple [A]
time = 0.15, size = 20, normalized size = 1.25

method	result
default	$\frac{\pi x}{2}-\frac{\arctan(\tan(bx+a))^2}{2b}$
derivativedivides	$\frac{\pi\arctan(\tan(bx+a))-\arctan(\tan(bx+a))^2}{2b}$
risch	$\frac{\pi x}{2}+ix\ln\left(e^{i(bx+a)}\right)+\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{4}-\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{cs}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*x-1/2*arctan(tan(b*x+a))^2/b
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.06

$$\frac{1}{2}\,\pi x-\frac{(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*pi*x - 1/2*(b*x + a)^2/b
```

Fricas [A]
time = 2.23, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 0.08, size = 48, normalized size = 3.00

$$\frac{\pi x}{2}-\begin{cases}\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for}\ b\neq 0\\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-atan(tan(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2
/(2*b), Ne(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Giac [A]
time = 0.40, size = 30, normalized size = 1.88

$$-\frac{1}{2}bx^2 + \pi x\left\lfloor\frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2}\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + pi*x*floor((b*x + a)/pi + 1/2) + 1/2*pi*x - a*x
```

Mupad [B]
time = 0.08, size = 21, normalized size = 1.31

$$\frac{\Pi\,x}{2} + \frac{b\,x^2}{2} - x\,\mathrm{atan}(\tan(a + b\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/2 - atan(tan(a + b*x)),x)
```

```
[Out] (Pi*x)/2 + (b*x^2)/2 - x*atan(tan(a + b*x))
```

3.158 $\int x^2 \cot^{-1}(c + d\tan(a + bx))\, dx$

Optimal. Leaf size=403

$$\frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx))-\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{x}{-}$$

```
[Out] 1/3*x^3*arccot(c+d*tan(b*x+a))-1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))+1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*x
^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x^2*polylog(2,-
(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/4*I*x*polylog(3,-(1+I*c+d)*
exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2+1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2*I*a+
2*I*b*x)/(c+I*(1+d)))/b^2+1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*
c-d))/b^3-1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b^3
```

Rubi [A]

time = 0.39, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5284, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}-\frac{\text{Li}_4\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}+\frac{ix\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}+\frac{x^2\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)+\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)+\frac{1}{3}x^3\cot^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tan[a + b*x]])/3 - (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] + (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] - (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) + (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) - ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 + ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 + PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) - PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5284

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{3}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x^{3}}{1-ic+d+(1-ic-\cdots}$$
$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}\cdots$$
$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}\cdots$$
$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}\cdots$$
$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}\cdots$$
$$= \frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{6}\cdots$$

Mathematica [A]
time = 0.63, size = 363, normalized size = 0.90

$$\frac{1}{3}x^3\cot^{-1}(c+d\tan(a+bx)) + \frac{-4ib^3x^3\log\left(1+\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)+4ib^3x^3\log\left(1+\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)-6b^2x^2\text{PolyLog}\left(2,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)+6b^2x^2\text{PolyLog}\left(2,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)-6ibx\text{PolyLog}\left(3,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)+6ibx\text{PolyLog}\left(3,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)+3\text{PolyLog}\left(4,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)-3\text{PolyLog}\left(4,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tan[a + b*x]])/3 + ((-4*I)*b^3*x^3*Log[1 + ((c - I*(1 + d
))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] + (4*I)*b^3*x^3*Log[1 + ((I + c -
 I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] - 6*b^2*x^2*PolyLog[2, -(((c -
I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + 6*b^2*x^2*PolyLog[2, -
(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] - (6*I)*b*x*PolyLog[
3, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + (6*I)*b*x*P
olyLog[3, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + 3*PolyL
og[4, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - 3*PolyLo
g[4, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 28.52, size = 8070, normalized size = 20.02

method	result	size
risch	Expression too large to display	8070

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c*
cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2(-(d - 1)
*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d -
1)*sin(2*b*x + 2*a) - c) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(
2*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b
*x + 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x +
 2*a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3
*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))
/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*c
os(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x +
 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*
(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c
^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x
+ 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4
*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*
(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x
+ 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(290) = 580$.
time = 3.96, size = 1965, normalized size = 4.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arccot(d*tan(b*x + a) + c) - 6*b^2*x^2*dilog(2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
+ 1) + 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
(I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog(2*((-I*c*d - d^2
```

```
 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) + 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 4*I*a^3*log(((I*c*d + d^2 + d)*
tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d
 - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2
 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x
 + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d
 + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1))
+ 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 6*I*b*x*poly
log(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*
(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)
*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
+ 1)) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
- 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3,
((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2
+ 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((I*c*d - d^2 + d
)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) +
 d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 4*(
I*b^3*x^3 + I*a^3)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((-I*c
*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*t
an(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*
d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2
+ 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c
*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)
^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*polylog(4
, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c
^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x +
a)^2 + c^2 + d^2 - 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(
b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x +
a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acot}(c+d\tan(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*acot(c + d*tan(a + b*x)), x)
```

3.159 $\int x\cot^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=305

$$\frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{x}{-}$$

```
[Out] 1/2*x^2*arccot(c+d*tan(b*x+a))-1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))+1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*x
*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*x*polylog(2,-(c+I
*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b-1/8*I*polylog(3,-(1+I*c+d)*exp(2*
I*a+2*I*b*x)/(1+I*c-d))/b^2+1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)
/(c+I*(1+d)))/b^2
```

Rubi [A]
time = 0.31, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5284, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}+\frac{i\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}-\frac{x\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}+\frac{x\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)+\frac{1}{2}x^2\cot^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] + (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] - (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) + (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) - ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 + ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5284

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{4}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{4}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{4}$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{4}$$

Mathematica [A]
time = 0.46, size = 272, normalized size = 0.89

$$\frac{1}{2}x^2\cot^{-1}(c+d\tan(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)-2b^2x^2\log\left(1+\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)-2ibx\text{PolyLog}\left(2,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)+2ibx\text{PolyLog}\left(2,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)+\text{PolyLog}\left(3,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)-\text{PolyLog}\left(3,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + ((c - I*(1 +
 d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - 2*b^2*x^2*Log[1 + ((I + c - I
*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] - (2*I)*b*x*PolyLog[2, -(((c - I*
(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + (2*I)*b*x*PolyLog[2, -((
(I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + PolyLog[3, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - PolyLog[3, -(((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.47, size = 7714, normalized size = 25.29

method	result	size
risch	Expression too large to display	7714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(-(d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, -c*
cos(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2(-(d - 1)
*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, -c*cos(2*b*x + 2*a) - (d -
1)*sin(2*b*x + 2*a) - c) - 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*
x + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
 2*a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a
) - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos
(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^
4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4
*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a
```

```
)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
 d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*
a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^
4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d
^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*
a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(217) = 434$.
time = 4.35, size = 1545, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arccot(d*tan(b*x + a) + c) - 2*b*x*dilog(2*((I*c*d - d^2 +
d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a)
+ d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
+ 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 -
2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a
)^2 + c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(2*((-I*c*d - d^2 + d)*tan(b*x
 + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/
((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) + 2*b*x*d
ilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d +
 I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1) + 1) + 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 -
c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x +
a)^2 + 1)) - 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) + 2
*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2
 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*a^2*log(((I
*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*
tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2
*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 +
 I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)
^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2
+ d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(I*b^2*x^2 - I*
a^2)*log(-2*((-I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*
c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2*((-I*c*d - d^2 -
d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a)
 - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I
```

```
*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2
 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*
tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c*d - d^2
 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*
tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d +
 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c
*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2
*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*c
*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d
^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acot}(c+d\tan(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*acot(c + d*tan(a + b*x)), x)
```

3.160 $\int \cot^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=198

$$x\cot^{-1}(c+d\tan(a+bx))-\frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)+\frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)-\frac{\text{Poly}}{}$$

```
[Out] x*arccot(c+d*tan(b*x+a))-1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d
))+1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))-1/4*polylog(2,-
(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b+1/4*polylog(2,-(c+I*(1-d))*exp(2*
I*a+2*I*b*x)/(c+I*(1+d)))/b
```

Rubi [A]
time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5276, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}+\frac{\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}-\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)+\frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)+x\cot^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tan[a + b*x]] - (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] + (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] - PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) + PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5276

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + (-Dist[b*(1 - I*c - d), Int[x*(E^(2*I*a + 2*I*
b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[b*(1
 + I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I
*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(c+d\tan(a+bx))\,dx &= x\cot^{-1}(c+d\tan(a+bx)) - (b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic-d)e^{2ic}} \\ &= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{2}ix\log \\ &= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{2}ix\log \\ &= x\cot^{-1}(c+d\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) + \frac{1}{2}ix\log\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 555 vs. $2(198)=396$.
time = 1.28, size = 555, normalized size = 2.80

$$x\cot^{-1}(c+d\tan(a+bx)) - \frac{x\left(-4ad\,\mathrm{ArcTan}(c+d\tan(a+bx)) - i\sqrt{-d^2}\left(\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right)+\mathrm{PolyLog}\left(2,\frac{d^2(1-i\tan(a+bx))}{icd+d^2-i\sqrt{-d^2}}\right)\right) + i\sqrt{-d^2}\left(\log(1-i\tan(a+bx))\log\left(\frac{cd+\sqrt{-d^2}+d^2\tan(a+bx)}{cd-id^2+\sqrt{-d^2}}\right)+\mathrm{PolyLog}\left(2,\frac{d^2(1-i\tan(a+bx))}{icd+d^2+i\sqrt{-d^2}}\right)\right) + i\sqrt{-d^2}\left(\log(1+i\tan(a+bx))\log\left(\frac{cd-\sqrt{-d^2}+d^2\tan(a+bx)}{cd+id^2-\sqrt{-d^2}}\right)+\mathrm{PolyLog}\left(2,\frac{d^2(1+i\tan(a+bx))}{-icd+d^2+i\sqrt{-d^2}}\right)\right) - i\sqrt{-d^2}\left(\log(1+i\tan(a+bx))\log\left(\frac{cd+\sqrt{-d^2}+d^2\tan(a+bx)}{cd+id^2+\sqrt{-d^2}}\right)+\mathrm{PolyLog}\left(2,\frac{d^2(1+i\tan(a+bx))}{d^2-i\left(cd+\sqrt{-d^2}\right)}\right)\right)\right)}{2d(2a-i\log(1-i\tan(a+bx))+i\log(1+i\tan(a+bx)))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tan[a + b*x]] - (x*(-4*a*d*ArcTan[c + d*Tan[a + b*x]] - I*Sq
rt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))/
(I*c*d + d^2 - I*Sqrt[-d^2])]) + I*Sqrt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[
(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[-d^2])] + PolyLog
[2, (d^2*(1 - I*Tan[a + b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])]) + I*Sqrt[-d^2
]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d +
 I*d^2 - Sqrt[-d^2])] + PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/((-I)*c*d + d
^2 + I*Sqrt[-d^2])]) - I*Sqrt[-d^2]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqr
t[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(
1 + I*Tan[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))])))/(2*d*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(168) = 336$.
time = 2.14, size = 1138, normalized size = 5.75

method	result	size
derivativedivides	Expression too large to display	1138
default	Expression too large to display	1138
risch	Expression too large to display	4945

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arccot(c+d*tan(b*x+a))+d^2*(-1/d*arctan(d*((c+d
*tan(b*x+a))/d-c/d)+c)*arctan(-(c+d*tan(b*x+a))/d+c/d)-1/d^2*(-1/2*I*d*arct
an(d*((c+d*tan(b*x+a))/d-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*tan(b*x+a))/d
-c/d)+c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d
*((c+d*tan(b*x+a))/d-c/d)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*tan(
b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))+1/2*I
*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan(b*
x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)/(1+I*c
+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d
*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)
/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d
)+c))^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*ta
n(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*tan(b*x+a))/d-c/d)+c)^2/(1+I*c+d
)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((
c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d*ta
n(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*tan(b*x+a)
)/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c)
)^2/((d*((c+d*tan(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(c-I*d-
I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*tan(b*x+a))/d-c/d)+c))^2/((d*((c+d*tan
(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(141) = 282$.
time = 0.56, size = 433, normalized size = 2.19

$$-\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d}-\frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1},\frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right)-4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+a)}{c^2+d^2-2\,d+1},\frac{cd\tan(bx+a)+c^2-d+1}{c^2+d^2-2\,d+1}\right)+\log\left(\tan(bx+a)^2+1\right)\log\left(\frac{d^2\tan(bx+a)^2+2\,cd\tan(bx+a)+c^2+1}{c^2+d^2+2\,d+1}\right)-\log\left(\tan(bx+a)^2+1\right)\log\left(\frac{d^2\tan(bx+a)^2+2\,cd\tan(bx+a)+c^2+1}{c^2+d^2-2\,d+1}\right)+2\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d+1}\right)-2\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d-1}\right)+2\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d+1}\right)-2\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d-1}\right)}{d}\right)-8\,(bx+a)\,\mathrm{arccot}\left(d\tan(bx+a)+c\right)-8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*ar
ctan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x +
```

```
a) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2
- d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/
(c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 +
2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
 1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*
d + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) - 8*(b*x + a)*arc
cot(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(141) = 282$.
time = 3.74, size = 1101, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arccot(d*tan(b*x + a) + c) - 2*(-I*b*x - I*a)*log(-2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
 - 2*(I*b*x + I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log(-2*((-I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x
 + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
) - 2*(-I*b*x - I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*
d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*I*a*log(((I*c*d + d^2 + d)*ta
n(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d -
 1)/(tan(b*x + a)^2 + 1)) + 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a
)^2 + 1)) - 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*
c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*
a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*
I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - dilog(2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) + dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*
c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1) + 1) - dilog(2*((-I*c*d - d^2 + d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) + dilog(2*((-I*c*
d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
```

```
n(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
 + 1) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(c+d\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*tan(b*x+a)),x)
[Out] Integral(acot(c + d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(d*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(c+d\tan(a+b\,x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*tan(a + b*x)),x)
[Out] int(acot(c + d*tan(a + b*x)), x)
```

3.161 $\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\cot^{-1}(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCot[c + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c+d\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arccot(c+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccot(d*tan(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(c + d\tan\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(acot(c + d*tan(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(d*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acot}(c+d\,\tan(a+b\,x))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*tan(a + b*x))/x,x)
[Out] int(acot(c + d*tan(a + b*x))/x, x)
```

3.162 $\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=154

$$\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{ix\text{PolyLog}(3,}{4b^2}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccot(c+(1+I*c)*tan(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,I
*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5280, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{i(1+ic)+c+ce}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2ia}}{i(1+ic)+}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

Mathematica [A]
time = 0.17, size = 136, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\cot^{-1}(c+(1+ic)\tan(a+bx)) + 4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.99, size = 1526, normalized size = 9.91

method	result	size
risch	Expression too large to display	1526

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2-1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a
))*c)*a^3+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/4*x^2*polylog
```

```
(2,I*c*exp(2*I*(b*x+a)))/b-1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*x*a^2+1/2*I
/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*I*x^3*ln(1-I*exp(2*I*(b*
x+a))*c)-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/
12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/12*b*x^4-1/12*x^3*Pi*csgn(
I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+1/3*I*x^3*ln(exp(I*(b*x+a)
))+1/12*Pi*x^3*csgn(I*exp(2*I*(b*x+a)))^3-1/6*Pi*x^3*csgn(I*exp(I*(b*x+a)))
*csgn(I*exp(2*I*(b*x+a)))^2+1/12*Pi*x^3*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp
(2*I*(b*x+a)))-1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*a^3*ln(1+I*
exp(I*(b*x+a))*(-I*c)^(1/2))+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp
(2*I*(b*x+a))+1))^3+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2
/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^3*a^2*dilog(1+I*exp(I
*(b*x+a))*(-I*c)^(1/2))-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)-1/8*polylog(4,I*
c*exp(2*I*(b*x+a)))/b^3-1/4/b^3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2+1/12*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*
x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1
/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*
csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(
c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x
+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a
))+1))^2-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+1/
12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*
(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*c
sgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(c-
I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1)
)^2+1/6*I*x^3*ln(c-I)-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(
2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1/12
*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*
exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn(I/(exp(2*I*(b
*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))-1/12*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I
)/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.
time = 0.29, size = 310, normalized size = 2.01

$$\frac{\frac{4\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arccot}((ic+1)\tan(bx+a)+c)}{b^2}-\frac{\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(c\cos(2bx+2a),c\sin(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2\left(ice^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left(c^2\cos(2bx+2a)^2+c^2\sin(2bx+2a)^2+2c\sin(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3\left(ice^{(2ibx+2ia)}\right)+6i\operatorname{Li}_4\left(ice^{(2ibx+2ia)}\right)\right)(-ic-1)}{b^2(c-i)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((I*c + 1)*
tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
```

```
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*
I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*b
*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(2
*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(-I*c - 1)/(b^2
*(c - I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(107) = 214$.
time = 2.88, size = 321, normalized size = 2.08

$$\frac{b^4x^4 - 2i\,b^3x^3\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{4i\,c}}{2\,c}\right) - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - 2\left(-i\,b^3x^3 - i\,a^3\right)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 2\left(-i\,b^3x^3 - i\,a^3\right)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2*x
^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e^(
I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 1
2*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^3*x^3 - I*a^
3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1
/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e^(I*b*
x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp
(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acot}(c+\tan(a+b\,x)\,\,(1+c\,1\mathrm{i}))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.163 $\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=123

$$\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{i\text{PolyLog}(3,ic}{8b^2}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arccot(c+(1+I*c)*tan(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*a
+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*ex
p(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5280, 2215, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(c+(1+ic)\tan(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{i(1+ic)+c+ce^{2ia\cdots}}\\
&=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia+2i\cdots}}{i(1+ic)+c\cdots}\\
&=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\
&=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\
&=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\
&=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.
\end{aligned}
$$

Mathematica [A]

time = 0.07, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2 \cot^{-1}(c+(1+ic)\tan(a+bx)) + \frac{i\left(2b^2x^2 \log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) + 2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right) + \text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (1 + I*c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.02, size = 1491, normalized size = 12.12

method	result	size
risch	Expression too large to display	1491

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2+1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*
c+I)-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I
*exp(I*(b*x+a))*(-I*c)^(1/2))+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(
I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+
a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))+1))+1/2*I/b*ln(1-I*exp(2*I*(b*x+a))*c)*x*a-1/2*I/b*a*ln(1-I*ex
p(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*b*x^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*
c+I)/(exp(2*I*(b*x+a))+1))^3+1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b-1/8*x^
2*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*e
xp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*
x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(ex
p(2*I*(b*x+a))+1))^2+1/2*I*x^2*ln(exp(I*(b*x+a)))+1/8*Pi*x^2*csgn(I*exp(I*(
b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*csgn(
I*exp(2*I*(b*x+a)))^2+1/8*Pi*x^2*csgn(I*exp(2*I*(b*x+a)))^3+1/8*x^2*Pi*csgn
(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/4/b^2*polylog(2,I*c*exp
(2*I*(b*x+a)))*a-1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^2*a
*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I
*c)^(1/2))*x-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3
+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*x^2*l
n(c-I)+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*ln(exp(2*I*(b*
x+a))*c+I)*x^2+1/4*I*x^2*ln(1-I*exp(2*I*(b*x+a))*c)+1/8*I*polylog(3,I*c*exp
(2*I*(b*x+a)))/b^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a
))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(ex
p(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn((exp(2*I*(b*x+
```

```
a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*cs
gn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I*(exp(
2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/
8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1
))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*
(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1
))^2-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((e
xp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+
a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2
*I*(b*x+a))+1))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(85) = 170$.
time = 0.29, size = 218, normalized size = 1.77

$$\frac{\frac{6\left((bx+a)^2-2(bx+a)a\right)\operatorname{arccot}((ic+1)\tan(bx+a)+c)}{b}-\frac{\left(-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left(ice^{(2ibx+2ia)}\right)-6\left(i(bx+a)^2-2i(bx+a)a\right)\arctan(c\cos(2bx+2a),c\sin(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left(c^2\cos(2bx+2a)^2+c^2\sin(2bx+2a)^2+2c\sin(2bx+2a)+1\right)+3\operatorname{Li}_3\left(ice^{(2ibx+2ia)}\right)\right)(-ic-1)}{b(c-i)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((I*c + 1)*tan(b*x + a) + c)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c*
sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x +
 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3,
 I*c*e^(2*I*b*x + 2*I*a)))*(-I*c - 1)/(b*(c - I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(85) = 170$.
time = 3.57, size = 270, normalized size = 2.20

$$\frac{2b^3x^3-3ib^2x^2\log\left(\frac{\left(ce^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{c-i}\right)+2a^3+6bx\operatorname{Li}_2\left(\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}\right)+6bx\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}\right)+3ia^2\log\left(\frac{2ce^{(ibx+ia)}+i\sqrt{4ic}}{2c}\right)+3ia^2\log\left(\frac{2ce^{(ibx+ia)}-i\sqrt{4ic}}{2c}\right)-3\left(-ib^2x^2+ia^2\right)\log\left(\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}+1\right)-3\left(-ib^2x^2+ia^2\right)\log\left(-\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}+1\right)+6i\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}\right)+6i\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{4ic}\,e^{(ibx+ia)}\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6
*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*b*
x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*sqr
t(4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) +
 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 6*
I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp
(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x*acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.164 $\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=85

$$\frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccot(c+(1+I*c)*tan(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*x
))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.294$, Rules used = {5272, 2215, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5272

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+(1+ic)\tan(a+bx))\,dx = x\cot^{-1}(c+(1+ic)\tan(a+bx))+(ib)\int \frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))-(bc)\int \frac{e^{2ia+2ibx}x}{i(1+ic)+c+ce}$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) -$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) -$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right) -$$

Mathematica [A]
time = 0.47, size = 75, normalized size = 0.88

$$x\cot^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)-\frac{\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (1 + I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I/(c*E^((2*I)*(a + b
*x)))] - PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(69) = 138.
time = 0.65, size = 599, normalized size = 7.05

method	result

derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c^2}{2i-2c}+\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c}{2i-2c}+\operatorname{arccot}(c-}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c^2}{2i-2c}+\frac{2i\operatorname{arccot}(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c}{2i-2c}+\operatorname{arccot}(c-}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(1+I*c)*(-arccot(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*
x+a))*c^2+2*I*arccot(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*
x+a))*c+arccot(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*x+a))+
arccot(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c^2-2*I*
arccot(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c-arccot
(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)+(1+I*c)^2*(-1/
8*I/(I-c)*ln(-I+c+(1+I*c)*tan(b*x+a))^2+1/4*I/(I-c)*ln(-I+c+(1+I*c)*tan(b*x
+a))*ln(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))+1/4*I/(I-c)*dilog(-1/2*I*(c+(1+I*c
)*tan(b*x+a)+I))-1/4*I/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln(1/2*(c+(1+I*c)*
tan(b*x+a)+I)/c)-1/4*I/(I-c)*dilog(1/2*(c+(1+I*c)*tan(b*x+a)+I)/c)+1/4*I/(I
-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln((-I+c+(1+I*c)*tan(b*x+a))/(-2*I+2*c))+1/
4*I/(I-c)*dilog((-I+c+(1+I*c)*tan(b*x+a))/(-2*I+2*c))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(60) = 120$.
time = 0.50, size = 455, normalized size = 5.35

$$-\frac{(-ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{ic+1}-\frac{i\left(4(bx+a)\left(\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)-\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\right)+i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)^2-2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\log\left(-\frac{1}{2}(c-i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)+2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\log\left(-\frac{(ic+1)\tan(bx+a)+c+i}{2c}+1\right)-2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)-2i\operatorname{Li}_2\left(\frac{1}{2}(c-i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)+2i\operatorname{Li}_2\left(\frac{(ic+1)\tan(bx+a)+c+i}{2c}\right)-2i\operatorname{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic+1}\right)-8(bx+a)\operatorname{arccot}\left((ic+1)\tan(bx+a)+c\right)-\frac{4(bx+a)(c-i)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{ic+1}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((-I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x
+ a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c +
1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)
 - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2
- 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)
*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*log
(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x +
 a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c
 + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a) -
 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*d
ilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arccot((I*c + 1)*t
```

```
an(b*x + a) + c) - 4*(b*x + a)*(c - I)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*t
an(b*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/
(I*c + 1))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(60) = 120$.
time = 6.28, size = 199, normalized size = 2.34

$$\frac{b^2x^2 - i\,bx\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\sqrt{4i\,c}}{2\,c}\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\sqrt{4i\,c}}{2\,c}\right) + \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(
c - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + (I
*b*x + I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*e^
(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I*sq
rt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*I + 2*c*exp(2*I*a
) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I
*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(acot(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.165 $\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c+(ic+1)\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arccot(c+(1+I*c)*tan(b*x+a))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acot}(c + \tan(a + b\,x)\ (1 + c\,1\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tan(a + b*x)*(c*1i + 1))/x,x)
[Out] int(acot(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.166 $\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal. Leaf size=155

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)-\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{ix\text{PolyLo}}{}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arccot(c-(1-I*c)*tan(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(3
,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5280, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[
1 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I
)*b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 +
 PolyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{i(-1+ic)+c+}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{3}(bc)\int \frac{e}{i(-1+i}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^2\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^2\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^2\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^2\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice^2\right.$$

Mathematica [A]
time = 0.17, size = 141, normalized size = 0.91

$$\frac{1}{3}x^3\cot^{-1}(c+i(i+c)\tan(a+bx)) - \frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right) + 3\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + I*(I + c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6*
I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*(
a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.01, size = 1527, normalized size = 9.85

method	result	size
risch	Expression too large to display	1527

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi
*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/4*x^2*polylog(2,-I*e
```

```
xp(2*I*(b*x+a))*c)/b+1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)*a^2-1/12*x^3*
Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))
*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+
1))+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1
))^2-1/6*I*x^3*ln(1+I*c*exp(2*I*(b*x+a)))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+
a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-1/6*I*x^3*ln(
I+c)-1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2-1/2*I/b^3*a^3*ln(1-I*exp(
I*(b*x+a))*(I*c)^(1/2))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*
(b*x+a))+1))^3-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/12*b*x^4
+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2
*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a)
)*c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp
(2*I*(b*x+a))+1))^3-1/12*Pi*x^3*csgn(I*exp(2*I*(b*x+a)))^3+1/6*Pi*x^3*csgn(
I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*Pi*x^3*csgn(I*exp(I*(b*x+
a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/3*I*x^3*ln(exp(I*(b*x+a)))+1/12*x^3*Pi*cs
gn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/2/b^3*a^2*dilog(1-I*exp(
I*(b*x+a))*(I*c)^(1/2))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+
c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))
*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*
(b*x+a))+1))+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+
a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))+1/8*polylog(4
,-I*exp(2*I*(b*x+a))*c)/b^3+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(
2*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a
))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*a^2*
ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I
*c)^(1/2))*x+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1)
)*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^3*a^3*ln(1+I*ex
p(I*(b*x+a))*(I*c)^(1/2))-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I
*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I
*(b*x+a))+1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/6*I/b^3*a^3*ln(-exp(2*
I*(b*x+a))*c+I)+1/3*I/b^3*ln(1+I*c*exp(2*I*(b*x+a)))*a^3+1/2*I/b^2*ln(1+I*c
*exp(2*I*(b*x+a)))*x*a^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(108) = 216$.
time = 0.30, size = 312, normalized size = 2.01

$$-\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{arccot}\left((-i\,c+1)\tan(bx+a)-c\right)}{b^2}+\frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+a)^2a-9i\,(bx+a)a^2\right)\arctan\left(c\cos(2\,bx+2\,a),-c\sin(2\,bx+2\,a)+1\right)-3\left(4i\,(bx+a)^2-6i\,(bx+a)a+3i\,a^2\right)\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(4\,(bx+a)^3-9\,(bx+a)^2a+9\,(bx+a)a^2\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2-2\,c\sin(2\,bx+2\,a)+1\right)+3\,(4\,bx+a)\mathrm{Li}_3\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,\mathrm{Li}_4\left(-i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c-1)}{b^2(c+i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccot((-I*c + 1
)*tan(b*x + a) - c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
```

```
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)
*arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2
- 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)
^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*si
n(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*
c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1
)/(b^2*(c + I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(108) = 216$.
time = 7.42, size = 321, normalized size = 2.07

$$-\frac{b^4x^4 + 2i\,b^3x^3\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,c}}{2\,c}\right) - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 2\,(i\,b^3x^3+i\,a^3)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right) + 2\,(i\,b^3x^3+i\,a^3)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 + 2*I*b^3*x^3*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3*x^3 +
 I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I*a^3)*l
og(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I*c)*
e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acot}(c + \tan(a + b\,x)\,(-1 + c\,1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(x^2*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.167 $\int x \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal. Leaf size=124

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{PolyLog}(3$$

```
[Out] -1/6*b*x^3+1/2*x^2*arccot(c-(1-I*c)*tan(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I*
a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I*c
*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5280, 2215, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCot[c - (1 - I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1
 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5280

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c-(1-ic)\tan(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{i(-1+ic)+c+ce^{2}}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ic}}{i(-1+ic)}$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+}\right.$$
$$=-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+}\right.$$

Mathematica [A]

time = 0.07, size = 111, normalized size = 0.90

$$\frac{1}{2}x^2\cot^{-1}(c+i(i+c)\tan(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + I*(I + c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.69, size = 1492, normalized size = 12.03

method	result	size
risch	Expression too large to display	1492

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*x^3-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1
/4/b^2*polylog(2,-I*exp(2*I*(b*x+a))*c)*a+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a)
)*(I*c)^(1/2))+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/8*x^2*Pi*c
sgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b
*x+a))+1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c)/(exp(2*I*
(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^
2+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2*Pi
*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*
(I+c)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(
I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*(exp(2*I
*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/8*x
^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+
a))+1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*
c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(I
*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(
exp(2*I*(b*x+a))+1))-1/8*Pi*x^2*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*
x+a)))+1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/2*I*x
^2*ln(exp(I*(b*x+a)))-1/8*Pi*x^2*csgn(I*exp(2*I*(b*x+a)))^3-1/4*I*x^2*ln(I+
c)+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp
(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))+1/4*I*ln(exp(2*I*(b*x+a))*c-I)*x^2
-1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I
)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))+1))^2+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*x^2*Pi*c
sgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c
```

```
-I)/(exp(2*I*(b*x+a))+1))^2-1/4*x*polylog(2,-I*exp(2*I*(b*x+a))*c)/b-1/8*I*
polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1)
)*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))-1/8*x^2*Pi*csgn(I*exp(2*
I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c
)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(
b*x+a))+1))^3-1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/8*x^2*Pi*cs
gn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/2*I/b*ln(1+I*c*exp(2*
I*(b*x+a)))*x*a+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*I/b*a*ln
(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2
-1/4*I/b^2*a^2*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a)
)*(I*c)^(1/2))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(86) = 172$.
time = 0.28, size = 220, normalized size = 1.77

$$-\frac{\frac{6\left((bx+a)^2-2(bx+a)a\right)\operatorname{arccot}((-ic+1)\tan(bx+a)-c)}{b}+\frac{\left(-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left(-ice^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(c\cos(2bx+2a),-c\sin(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left(c^2\cos(2bx+2a)^2+c^2\sin(2bx+2a)^2-2c\sin(2bx+2a)+1\right)+3\operatorname{Li}_3\left(-ice^{(2ibx+2ia)}\right)\right)(ic-1)}{b(c+i)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arccot((-I*c + 1)*tan(b*x + a) - c)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x
+ 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a)
, -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*
b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polyl
og(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(86) = 172$.
time = 2.79, size = 270, normalized size = 2.18

$$-\frac{2b^3x^3+3ib^2x^2\log\left(\frac{(c+i)e^{(2ibx+2ia)}}{ce^{(2ibx+2ia)}-i}\right)+2a^3+6bx\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)+6bx\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)+3ia^2\log\left(\frac{2ce^{(ibx+ia)}+i\sqrt{-4ic}}{2c}\right)+3ia^2\log\left(\frac{2ce^{(ibx+ia)}-i\sqrt{-4ic}}{2c}\right)+3\left(ib^2x^2-ia^2\right)\log\left(\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}+1\right)+3\left(ib^2x^2-ia^2\right)\log\left(-\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}+1\right)+6i\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)+6i\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{-4ic}\,e^{(ibx+ia)}\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*
a) + 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1)
+ 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(c + \tan(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.168 $\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx$

Optimal. Leaf size=86

$$-\frac{bx^2}{2}+x\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*arccot(c-(1-I*c)*tan(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*
x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5272, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\cot^{-1}(c-(1-ic)\tan(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCot[c - (1 - I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 + I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5272

```
Int[ArcCot[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcC
ot[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c-(1-ic)\tan(a+bx))\,dx = x\cot^{-1}(c-(1-ic)\tan(a+bx)) + (ib)\int \frac{x}{i(-1+ic)+c+ce^{2ia+2i}}$$

$$= -\frac{bx^2}{2} + x\cot^{-1}(c-(1-ic)\tan(a+bx)) + (bc)\int \frac{e^{2ia+2ib}}{i(-1+ic)+c-}$$

$$= -\frac{bx^2}{2} + x\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)$$

$$= -\frac{bx^2}{2} + x\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)$$

$$= -\frac{bx^2}{2} + x\cot^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)$$

Mathematica [**A**]
time = 0.60, size = 76, normalized size = 0.88

$$x\cot^{-1}(c+i(i+c)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) + \frac{\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcCot[c + I*(I + c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I/(c*E^((2*I)*(a + b
*x)))] + PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(70) = 140.
time = 0.68, size = 650, normalized size = 7.56

method	result

derivativedivides	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(ic-}{}}{}$
default	$\frac{\frac{\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\operatorname{arccot}(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\operatorname{arccot}(c+(ic-}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(1-I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(-1+I*c)*(arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+(-1+I*c)*tan(b*x
+a)+c)*c^2+2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+(-1+I*c)*tan(b*
x+a)+c)*c-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+(-1+I*c)*tan(b*x+a)+
c)-arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(-(-1+I*c)*tan(b*x+a)+c+I)*c^2
-2*I*arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(-(-1+I*c)*tan(b*x+a)+c+I)*c
+arccot(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(-(-1+I*c)*tan(b*x+a)+c+I)-(-1+I
*c)^2*(-1/4*I/(I+c)*ln(I+(-1+I*c)*tan(b*x+a)+c)*ln(-1/2*I*(I-c-(-1+I*c)*tan
(b*x+a)))+1/4*I/(I+c)*ln(-1/2*I*(I+(-1+I*c)*tan(b*x+a)+c))*ln(-1/2*I*(I-c-(
-1+I*c)*tan(b*x+a)))+1/4*I/(I+c)*dilog(-1/2*I*(I+(-1+I*c)*tan(b*x+a)+c))+1/
8*I/(I+c)*ln(I+(-1+I*c)*tan(b*x+a)+c)^2-1/4*I/(I+c)*ln(-(-1+I*c)*tan(b*x+a)
+c+I)*ln((-I-(-1+I*c)*tan(b*x+a)-c)/(-2*I-2*c))-1/4*I/(I+c)*dilog((-I-(-1+I
*c)*tan(b*x+a)-c)/(-2*I-2*c))+1/4*I/(I+c)*ln(-(-1+I*c)*tan(b*x+a)+c+I)*ln(-
1/2*(I-c-(-1+I*c)*tan(b*x+a))/c)+1/4*I/(I+c)*dilog(-1/2*(I-c-(-1+I*c)*tan(b
*x+a))/c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(61) = 122$.
time = 0.52, size = 450, normalized size = 5.23

$$\frac{(ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)}{2ic^2-2\left(c^2+2ic-1\right)\tan(bx+a)-4c-2i}\right)}{ic-1}+\frac{i\left(4(bx+a)\left(\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)+2c+i\right)-\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)\right)+i\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)+2c+i\right)^2-2i\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)\log\left(\frac{1}{2}(c+i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)+2i\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)\log\left(-\frac{(ic-1)\tan(bx+a)+c-i}{2c}+1\right)-2i\log\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)+2c+i\right)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)-2i\,\mathrm{Li}_2\left(-\frac{1}{2}(c+i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)+2i\,\mathrm{Li}_2\left(\frac{(ic-1)\tan(bx+a)+c-i}{2c}\right)-2i\,\mathrm{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic-1}\right)-8(bx+a)\operatorname{arccot}\left((-ic+1)\tan(bx+a)-c\right)+4(-ibx-ia)\log\left(-\frac{2\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)}{2ic^2-2\left(c^2+2ic-1\right)\tan(bx+a)-4c-2i}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x +
a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c - 1)
 + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I) -
 log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 +
2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*t
an(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log(-I
*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x + a)
 + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c +
I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a) + 1
```

```
/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*dil
og(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arccot((-I*c + 1)*ta
n(b*x + a) - c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b
*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(61) = 122$.
time = 7.27, size = 201, normalized size = 2.34

$$-\frac{b^2x^2 + i\,bx\log\left(\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 - (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)} + 1\right) - (-i\,bx - i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)} + 1\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,c}}{2\,c}\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,c}}{2\,c}\right) + \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 + I*b*x*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) -
(-I*b*x - I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*
sqrt(-4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(1-I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 - 3*_t0**2*I*c*e
xp(2*I*a) + _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(c + \tan(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(acot(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.169 $\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c-(1-ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (1 - I*c)*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c-(-ic+1)\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arccot(c-(1-I*c)*tan(b*x+a))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*I*log((c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(1-I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(1-I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot(-(-I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acot}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tan(a + b*x)*(c*1i - 1))/x,x)
[Out] int(acot(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.170 $\int \cot^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

```
[Out] 1/2*arccot(cot(b*x+a))^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\cot^{-1}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Cot[a + b*x]],x]
```

```
[Out] ArcCot[Cot[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(\cot(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \cot^{-1}(\cot(a+bx))\right)}{b}\\ &= \frac{\cot^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2}+x\cot^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Cot[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcCot[Cot[a + b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.
time = 0.18, size = 45, normalized size = 2.81

method	result
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^{2}}{2}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^{2}}{2}}{b}$
risch	$-ix\ln\left(e^{i(bx+a)}\right)-\frac{\pi x \operatorname{csgn}\left(ie^{i(bx+a)}\right)^{2}\operatorname{csgn}\left(ie^{2i(bx+a)}\right)}{4}+\frac{\pi x \operatorname{csgn}\left(ie^{i(bx+a)}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)^{2}}{2}-\frac{\pi x \operatorname{csgn}(ie$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-arccot(cot
(b*x+a)))^2)
```

Maxima [A]
time = 0.25, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 3.88, size = 10, normalized size = 0.62

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] 1/2*x^2*b + x*a
```

Sympy [A]
time = 0.08, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{acot}^{2}(\cot(a+bx))}{2b} & \text{for}\ b \neq 0 \\ x\operatorname{acot}(\cot(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(cot(b*x+a)),x)
```

```
[Out] Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a)), True))
```

Giac [A]
time = 0.41, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.64, size = 16, normalized size = 1.00

$$x\,\mathrm{acot}(\cot(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(cot(a + b*x)),x)
```

```
[Out] x*acot(cot(a + b*x)) - (b*x^2)/2
```

3.171 $\int x^2 \cot^{-1}(c + d\cot(a + bx))\, dx$

Optimal. Leaf size=399

$$\frac{1}{3}x^3 \cot^{-1}(c+d\cot(a+bx))-\frac{1}{6}ix^3 \log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+\frac{1}{6}ix^3 \log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)-$$

```
[Out] 1/3*x^3*arccot(c+d*cot(b*x+a))-1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))+1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*x
^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x^2*polylog(2,(c
+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/4*I*x*polylog(3,(1+I*c-d)*exp
(2*I*a+2*I*b*x)/(1+I*c+d))/b^2+1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*
b*x)/(c+I*(1-d)))/b^2+1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))
/b^3-1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

Rubi [A]
time = 0.38, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5286, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}-\frac{\text{Li}_4\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}-\frac{ix\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}+\frac{ix\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}+\frac{x^2\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)+\frac{1}{6}ix^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{1}{3}x^3\cot^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Cot[a + b*x]])/3 - (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] + (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] - (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) + (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) - ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 + ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 + Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) - P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{3}(b(1+ic-d))\int \frac{e^{2ia+2ibx}}{1+ic+d+(-1-}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+$$

Mathematica [A]
time = 0.62, size = 359, normalized size = 0.90

$$\frac{1}{3}x^3\cot^{-1}(c+d\cot(a+bx))+\frac{-4ib^3x^3\log\left(1-\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+4ib^3x^3\log\left(1-\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)-6b^2x^2\mathrm{PolyLog}\left(2,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+6b^2x^2\mathrm{PolyLog}\left(2,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)-6ibx\mathrm{PolyLog}\left(3,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+6ibx\mathrm{PolyLog}\left(3,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)+3\mathrm{PolyLog}\left(4,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-3\mathrm{PolyLog}\left(4,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Cot[a + b*x]])/3 + ((-4*I)*b^3*x^3*Log[1 - ((c + I*(-1 +
d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (4*I)*b^3*x^3*Log[1 - ((c + I*(
1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - 6*b^2*x^2*PolyLog[2, ((c + I*
(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + 6*b^2*x^2*PolyLog[2, ((c
+ I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - (6*I)*b*x*PolyLog[3, ((c
 + I*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (6*I)*b*x*PolyLog[3,
 ((c + I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + 3*PolyLog[4, ((c +
I*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 3*PolyLog[4, ((c + I*(1
 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 33.05, size = 7900, normalized size = 19.80

method	result	size
risch	Expression too large to display	7900

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*co
s(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/6*x^3*arctan2((d - 1)*co
s(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1)*s
in(2*b*x + 2*a) - c) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a)
 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.
time = 2.92, size = 1589, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arccot(d*cot(b*x + a) + c) - 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
```

```
 + 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 4*I*a^3
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*co
s(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(
c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2
*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d
^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 -
 I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 4*(I*b^3*x^3 + I*a
^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 +
2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + 3*p
olylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 3*polylog(4, ((c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a))/(c^2 + d^2 - 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos
(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 +
 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acot}(c+d\cot(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + d*cot(a + b*x)),x)
```

```
[Out] int(x^2*acot(c + d*cot(a + b*x)), x)
```

3.172 $\int x \cot^{-1}(c + d \cot(a + bx))\, dx$

Optimal. Leaf size=303

$$\frac{1}{2}x^2 \cot^{-1}(c+d\cot(a+bx))-\frac{1}{4}ix^2 \log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+\frac{1}{4}ix^2 \log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)-$$

```
[Out] 1/2*x^2*arccot(c+d*cot(b*x+a))-1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))+1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*x
*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*x*polylog(2,(c+I*(
1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b-1/8*I*polylog(3,(1+I*c-d)*exp(2*I*a
+2*I*b*x)/(1+I*c+d))/b^2+1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+
I*(1-d)))/b^2
```

Rubi [A]
time = 0.30, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5286, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}+\frac{i\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}-\frac{x\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}+\frac{x\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)+\frac{1}{4}ix^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{1}{2}x^2\cot^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] + (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] - (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) + (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) - ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 + ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (-Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 + Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*
I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ
[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}(b(1+ic-d))\int \frac{e^{2ia+2ibx}x^2}{1+ic+d+(-1-ic$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}i$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}i$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}i$$

$$= \frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{4}i$$

Mathematica [A]
time = 0.47, size = 270, normalized size = 0.89

$$\frac{1}{2}x^2\cot^{-1}(c+d\cot(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-2b^2x^2\log\left(1-\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)-2ibx\text{PolyLog}\left(2,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+2ibx\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)+\text{PolyLog}\left(3,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - ((c + I*(-1
+ d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 2*b^2*x^2*Log[1 - ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - (2*I)*b*x*PolyLog[2, ((c + I*(-
1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (2*I)*b*x*PolyLog[2, ((c +
I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + PolyLog[3, ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - PolyLog[3, ((c + I*(1 + d))*E^((
2*I)*(a + b*x)))/(I + c - I*d)]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.36, size = 7532, normalized size = 24.86

method	result	size
risch	Expression too large to display	7532

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2((d + 1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1, c*co
s(2*b*x + 2*a) - (d + 1)*sin(2*b*x + 2*a) - c) - 1/4*x^2*arctan2((d - 1)*co
s(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d + 1, c*cos(2*b*x + 2*a) - (d - 1)*s
in(2*b*x + 2*a) - c) - 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x + 2
*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*a)
^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (
c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*
x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d
^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*x
+ 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
```

```
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^4
- 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) -
4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d
^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^3 +
(c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))*s
in(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(213) = 426$.
time = 2.82, size = 1289, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arccot(d*cot(b*x + a) + c) - 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a^2*log(-1/2*c
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(I*b^2*x^2 - I*a^2)*log((c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b^2*x^2
+ I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c
^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1))
 - 2*(-I*b^2*x^2 + I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b
*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
 d^2 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d -
d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) -
2*d + 1)/(c^2 + d^2 - 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2
+ 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I
```

```
*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) + I*poly
log(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^
2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x +
2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+d*cot(b*x+a)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot(d*cot(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acot}(c+d\cot(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + d*cot(a + b*x)),x)
[Out] int(x*acot(c + d*cot(a + b*x)), x)
```

3.173 $\int \cot^{-1}(c+d\cot(a+bx))\,dx$

Optimal. Leaf size=198

$$x\cot^{-1}(c+d\cot(a+bx))-\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)+\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)-\frac{\text{PolyL}}{}$$

```
[Out] x*arccot(c+d*cot(b*x+a))-1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d
))+1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))-1/4*polylog(2,(
1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b+1/4*polylog(2,(c+I*(1+d))*exp(2*I*
a+2*I*b*x)/(c+I*(1-d)))/b
```

Rubi [A]

time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5278, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}+\frac{\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}-\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)+\frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)+x\cot^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Cot[a + b*x]] - (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] + (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] - PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) + PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5278

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + (-Dist[b*(1 + I*c - d), Int[x*(E^(2*I*a + 2*I*
b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[b*(1
 - I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I
*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+d\cot(a+bx))\,dx = x\cot^{-1}(c+d\cot(a+bx)) - (b(1+ic-d))\int \frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)\ldots}$$

$$= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{2}ix\log\ldots$$

$$= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{2}ix\log\ldots$$

$$= x\cot^{-1}(c+d\cot(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) + \frac{1}{2}ix\log\ldots$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1649 vs. 2(198) = 396.
time = 12.61, size = 1649, normalized size = 8.33

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCot[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Cot[a + b*x]] - (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
```

```
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(168) = 336$.
time = 2.14, size = 1146, normalized size = 5.79

method	result	size
derivativedivides	Expression too large to display	1146
default	Expression too large to display	1146
risch	Expression too large to display	4952

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccot(c+d*cot(b*x+a))-d^2*(-1/d*arct
an(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(1/2
*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(
b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)/(1+I
*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c
+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+
c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c
```

```
/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c
+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*
((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*cot(b*x+
a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+
c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(c-I*
d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*c
ot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c+d*cot(b*x+a))/d
-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*co
t(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d*cot(b*x+a))/d-c/d
)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d
*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c)))))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(140) = 280$.
time = 0.55, size = 532, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*tan
(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d +
 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan((c
*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a))
/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d
+ 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) -
log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2 +
 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) -
2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)*
tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*d
)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))/
d) + 8*(b*x + a)*arccot(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c^
2 + 1)*tan(b*x + a))/d))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(140) = 280$.
time = 4.54, size = 965, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arccot(d*cot(b*x + a) + c) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x
+ 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^
2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)
*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d
- d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a)
 - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2
- 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b
*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - dilog(-(c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 - 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 + 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(c + d \cot(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*cot(a + b*x)),x)
[Out] int(acot(c + d*cot(a + b*x)), x)
```

3.174 $\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{\cot^{-1}(c+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*cot(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCot[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + d*Cot[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCot[c + d*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccot}(c+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*cot(b*x+a))/x,x)
[Out] int(arccot(c+d*cot(b*x+a))/x,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Timed out
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*cot(b*x+a))/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*cot(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acot}(c+d\cot(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*cot(a + b*x))/x,x)
[Out] int(acot(c + d*cot(a + b*x))/x, x)
```

3.175 $\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=154

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{ix\text{PolyLog}(3,}{4b^}$$

```
[Out] -1/12*b*x^4+1/3*x^3*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/6*I*x^3*ln(1-I*c*e
xp(2*I*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*poly
log(3,I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5282, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[
1 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyL
og[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{-i(1-ic)+c-ce$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{3}(bc)\int \frac{e^{2i}}{-i(1-ic)$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia-}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia-}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia-}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia-}\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2ia-}\right.$$

Mathematica [A]
time = 0.15, size = 140, normalized size = 0.91

$$\frac{1}{3}x^3\cot^{-1}(c+(1-ic)\cot(a+bx)) - \frac{4ib^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right) + 3\mathrm{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.01, size = 1526, normalized size = 9.91

method	result	size
risch	Expression too large to display	1526

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2*polylog(2,I*c*exp(2*I*(b*x+a)))/b+1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a)
)*c)*x*a^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a)
```

```
)-1)*(I+c))^2+1/12*x^3*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^
2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(e
xp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(ex
p(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*
x+a))-1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))-1/12*x^3*Pi*csgn
(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))-1))+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*c
sgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a
))-1))-1/6*I*x^3*ln(I+c)+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)+1/12*x^3*Pi*csg
n(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-1/12*b*x^4-1/12*Pi*x^3*c
sgn(I*exp(2*I*(b*x+a)))^3+1/6*Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*
(b*x+a)))^2-1/12*Pi*x^3*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1
/3*I*x^3*ln(exp(I*(b*x+a)))+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(
2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+1/12*x^3
*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^3-1/2/
b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^3*a^2*dilog(1+I*exp(I*
(b*x+a))*(-I*c)^(1/2))-1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)+1/12*x^3*Pi*csg
n(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2
+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+
c)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I
*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1
))^3-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+1/8*po
lylog(4,I*c*exp(2*I*(b*x+a)))/b^3-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)
/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-
1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*
I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*
(I+c)/(exp(2*I*(b*x+a))-1))^3+1/4/b^3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2-1
/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(I
*(b*x+a))*(-I*c)^(1/2))+1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3+1/6*I/b^3*
a^3*ln(exp(2*I*(b*x+a))*c+I)-1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2-1/
2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*exp(
I*(b*x+a))*(-I*c)^(1/2))*x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.21, size = 174, normalized size = 1.13

$$-\frac{2\,b^{4}x^{4}-8\,\pi b^{3}x^{3}-4i\,b^{3}x^{3}\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right)+6\,b^{2}x^{2}\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^{4}-4i\,a^{3}\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+6i\,bx\mathrm{polylog}\left(3,i\,ce^{(2i\,bx+2i\,a)}\right)+4\left(i\,b^{3}x^{3}+i\,a^{3}\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)-3\,\mathrm{polylog}\left(4,i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 8*pi*b^3*x^3 - 4*I*b^3*x^3*log((c*e^(2*I*b*x + 2*I*a) +
I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a))
 - 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3,
I*c*e^(2*I*b*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*
a) + 1) - 3*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(\Pi+\mathrm{acot}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
```

```
[Out] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.176 $\int x \cot^{-1}(c + (1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=123

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{PolyLog}\left(3,\right.}{8b^2}$$

```
[Out] -1/6*b*x^3+1/2*x^2*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/4*I*x^2*ln(1-I*c*ex
p(2*I*a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3
,I*c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5282, 2215, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{-i(1-ic)+c-ce^{2i}}$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2ia}}{-i(1-ic)}$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+}\right.$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+}\right.$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+}\right.$$
$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+}\right.$$

Mathematica [A]

time = 0.07, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.88, size = 1491, normalized size = 12.12

method	result	size
risch	Expression too large to display	1491

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))-1))^2-1/6*b*x^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c
)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-1
/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I
*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+
I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*
(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))
-1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b
+1/4*I*ln(exp(2*I*(b*x+a))*c+I)*x^2-1/8*I*polylog(3,I*c*exp(2*I*(b*x+a)))/b
^2-1/4*I*x^2*ln(1-I*exp(2*I*(b*x+a))*c)-1/8*Pi*x^2*csgn(I*exp(I*(b*x+a)))^2
*csgn(I*exp(2*I*(b*x+a)))+1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*
(b*x+a)))^2-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/8*Pi*x^2*csgn(I*exp(2*I*(b*x+a))
)^3-1/4*I*x^2*ln(I+c)+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*
x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*cs
gn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2+1/8*x^2*Pi*
csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2-1/8*x^2*Pi*csgn(I/(exp(2
*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/8*x
^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/4/b^2*polylog(2,I*c*exp(2*I*(b*x+a)))*a+1/
2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^2*a*dilog(1+I*exp(I*(b
```

```
*x+a))*(-I*c)^(1/2))-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c))*
csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(
I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn
(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/2*I/b*a*ln(1-I*exp(I*(b*x
+a))*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/
b*ln(1-I*exp(2*I*(b*x+a))*c)*x*a+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(
exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^3-1/8*
x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/4*I/b^2*ln(1
-I*exp(2*I*(b*x+a))*c)*a^2-1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^2
*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a)
)*(-I*c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 5.00, size = 152, normalized size = 1.24

$$-\frac{4\,b^3x^3-12\,\pi b^2x^2-6i\,b^2x^2\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+6\left(i\,b^2x^2-i\,a^2\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)+3i\,\mathrm{polylog}\left(3,i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 12*pi*b^2*x^2 - 6*I*b^2*x^2*log((c*e^(2*I*b*x + 2*I*a) +
 I)*e^(-2*I*b*x - 2*I*a)/(c + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*
I*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*(I*b^2*x^2 - I*a^2)*
log(-I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a))
)/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-acot(-c-(1-I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c-(1-I*c)*cot(b*x+a))),x, algorithm="giac")
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(\Pi+\mathrm{acot}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))),x)
[Out] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i - 1))), x)
```

3.177 $\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=85

$$-\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*(Pi-arccot(-c-(1-I*c)*cot(b*x+a)))-1/2*I*x*ln(1-I*c*exp(2*I*a+
2*I*b*x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.294$, Rules used = {5274, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 - I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5274

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\cot^{-1}(c+(1-ic)\cot(a+bx))+(ib)\int \frac{x}{-i(1-ic)+c-ce^{2ia+2ib}}$$

$$= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-(bc)\int \frac{e^{2ia+2ibx}}{-i(1-ic)+c-}$$

$$= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$

$$= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$

$$= -\frac{bx^2}{2}+x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$

Mathematica [**A**]
time = 0.44, size = 75, normalized size = 0.88

$$x\cot^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+\frac{\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (1 - I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I/(c*E^((2*I)*(a + b
*x)))] + PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1497 vs. 2(76) = 152.
time = 0.85, size = 1498, normalized size = 17.62

method	result	size

risch	Expression too large to display	1244
default	Expression too large to display	1498
derivativedivides	Expression too large to display	1673

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi-arccot(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x+2*I/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(
-1+I*c)+c+I)*c-2*I/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(-I
+cot(b*x+a)*(-1+I*c)-c)*c-1/4*I/b/(-1+I*c)/(I+c)*ln(-I+cot(b*x+a)*(-1+I*c)-
c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*c^2-1/4*I/b/(-1+I*c)/(I+c)*ln(cot(b
*x+a)*(-1+I*c)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))*c^2+1/4*I/b/(
-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)
/c)*c^2-1/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*
(-1+I*c)+c+I)+1/4*I/b/(-1+I*c)/(I+c)*ln(-I+cot(b*x+a)*(-1+I*c)-c)*ln(-1/2*I
*(cot(b*x+a)*(-1+I*c)-c+I))+1/4*I/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)+c
+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))-1/4*I/b/(-1+I*c)/(I+c)*ln(cot
(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)+1/8*I/b/(-1+I*c)
/(I+c)*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2*c^2-1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2
*I*(cot(b*x+a)*(-1+I*c)-c+I))*c^2-1/4*I/b/(-1+I*c)/(I+c)*dilog((-I+cot(b*x+
a)*(-1+I*c)-c)/(-2*I-2*c))*c^2+1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*(cot(b*x+a
)*(-1+I*c)-c+I)/c)*c^2+1/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)
*ln(cot(b*x+a)*(-1+I*c)+c+I)*c^2-1/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)
/(2*I+2*c)*ln(-I+cot(b*x+a)*(-1+I*c)-c)*c^2+1/2/b/(-1+I*c)/(I+c)*ln(-I+cot(
b*x+a)*(-1+I*c)-c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*c+1/2/b/(-1+I*c)/(I
+c)*ln(cot(b*x+a)*(-1+I*c)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))*c
-1/2/b/(-1+I*c)/(I+c)*ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I
*c)-c+I)/c)*c+1/b/(-1+I*c)*arccot(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(-I+co
t(b*x+a)*(-1+I*c)-c)-1/4/b/(-1+I*c)/(I+c)*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2*c+
1/2/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))*c+1/2/b/(-1+I*
c)/(I+c)*dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))*c-1/2/b/(-1+I*c)/(I+c
)*dilog(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*c-1/8*I/b/(-1+I*c)/(I+c)*ln(-I+co
t(b*x+a)*(-1+I*c)-c)^2+1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*I*(cot(b*x+a)*(-1+
I*c)-c+I))+1/4*I/b/(-1+I*c)/(I+c)*dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*
c))-1/4*I/b/(-1+I*c)/(I+c)*dilog(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.02, size = 116, normalized size = 1.36

$$-\frac{2\,b^{2}x^{2}-4\,\pi bx-2i\,bx\log\left(\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c+i}\right)-2\,a^{2}+2\,(i\,bx+i\,a)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2 - 4*pi*b*x - 2*I*b*x*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/(c + I)) - 2*a^2 + 2*(I*b*x + I*a)*log(-I*c*e^(2*I*b*x + 2*I
*a) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x
+ 2*I*a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-acot(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**4 + 3*_t0**2*I*c*e
xp(2*I*a) - _t0**2*exp(2*I*a) + 2*c**2*exp(4*I*a) + I*c*exp(4*I*a) of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(pi - arccot(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \Pi+\mathrm{acot}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(Pi + acot(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.178 $\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\pi-\mathrm{arccot}(-c-(-ic+1)\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

```
[Out] int((Pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*pi + I*log((c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c + I)))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-acot(-c-(1-I*c)*cot(b*x+a)))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c-(1-I*c)*cot(b*x+a)))/x,x, algorithm="giac")
```

```
[Out] integrate((pi - arccot(-(-I*c + 1)*cot(b*x + a) - c))/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\Pi + \operatorname{acot}(c - \cot(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x,x)
[Out] int((Pi + acot(c - cot(a + b*x)*(c*1i - 1)))/x, x)
```

3.179 $\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{bx^4}{12}+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{ix\text{PolyLog}(3$$

```
[Out] 1/12*b*x^4+1/3*x^3*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/6*I*x^3*ln(1+I*c*ex
p(2*I*a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*poly
log(3,-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b
^3
```

Rubi [A]
time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5282, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{-i(-1-ic)+c-}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{3}(bc)\int \frac{e^{2}}{-i(-1-ic}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\cot^{-1}(c-(1+ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2ia+}\right.$$

Mathematica [A]
time = 0.17, size = 136, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\cot^{-1}(c+(-1-ic)\cot(a+bx)) + 4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I
)*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x
*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(
a + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.04, size = 1527, normalized size = 9.85

method	result	size
risch	Expression too large to display	1527

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi-arccot(-c+(1+I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2
*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)
```

```
))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I
*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)-1))^2-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/4*x^2*polylog(2,-I*exp(2*I*(b*
x+a))*c)/b-1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)*a^2-1/3*I/b^3*ln(1+I*c*
exp(2*I*(b*x+a)))*a^3-1/6*I/b^3*a^3*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*a^3
*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(I*
c)^(1/2))+1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/2/b^3*a^2*dilo
g(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*b*x^4+1/3*I*x^3*ln(exp(I*(b*x+a)))-1
/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+1/12*Pi*x^
3*csgn(I*exp(2*I*(b*x+a)))^3-1/6*Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2
*I*(b*x+a)))^2+1/12*Pi*x^3*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a))
)+1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2*I/b^2*ln(1+I*c*exp(
2*I*(b*x+a)))*x*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/2*
I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*x^3*Pi*csgn(I/(exp(2*I*
(b*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(c-I))
*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(
c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*polylog(4,-I*exp(2*I*(b*x+a))*c)/b^3+1/6*I
*x^3*ln(1+I*c*exp(2*I*(b*x+a)))+1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^
2-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*
Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(ex
p(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1/12*x^3*Pi*csg
n(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I
)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(ex
p(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1/
12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(
I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/12*x^3*Pi*csgn(exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)
/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c
-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))+1/6*I*x^3*ln(c-I)+1/12*x^3*Pi*csgn(
I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))
^2+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)
/(exp(2*I*(b*x+a))-1))^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 2.67, size = 174, normalized size = 1.12

$$\frac{2\,b^4x^4+8\,\pi b^3x^3+4i\,b^3x^3\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+6i\,bx\mathrm{polylog}\left(3,-i\,ce^{(2i\,bx+2i\,a)}\right)-4\left(-i\,b^3x^3-i\,a^3\right)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-3\,\mathrm{polylog}\left(4,-i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 8*pi*b^3*x^3 + 4*I*b^3*x^3*log((c - I)*e^(2*I*b*x + 2*I*a
)/(c*e^(2*I*b*x + 2*I*a) - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a))
- 2*a^4 - 4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -
I*c*e^(2*I*b*x + 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(I*c*e^(2*I*b*x + 2*I*
a) + 1) - 3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,ex
p(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(\Pi+\mathrm{acot}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
```

```
[Out] int(x^2*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.180 $\int x \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal. Leaf size=124

$$\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\text{PolyLog}(3,-}{8b^2}$$

```
[Out] 1/6*b*x^3+1/2*x^2*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/4*I*x^2*ln(1+I*c*exp
(2*I*a+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3
,-I*c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5282, 2215, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCot[c - (1 + I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5282

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c-(1+ic)\cot(a+bx))\,dx=\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{-i(-1-ic)+c-$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia}}{-i(-1-ic)}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2}\right.$$

Mathematica [A]

time = 0.08, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\cot^{-1}(c+(-1-ic)\cot(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (-1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.98, size = 1492, normalized size = 12.03

method	result	size
risch	Expression too large to display	1492

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi-arccot(-c+(1+I*c)*cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b*x^3-1/4*I*ln(exp(2*I*(b*x+a))*c-I)*x^2+1/4/b^2*polylog(2,-I*exp(2*I*(
b*x+a))*c)*a-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^2*a*dilo
g(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))+1/8*
I*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2+1/2*I/b*ln(1+I*c*exp(2*I*(b*x+a)))*x
*a-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(I*(b
*x+a))*(I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*
x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I
)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1
/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*P
i*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))-1))^2+1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2+1/4*I/b^2*a^2*ln(-exp(
2*I*(b*x+a))*c+I)-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I*x^
2*ln(exp(I*(b*x+a)))+1/8*Pi*x^2*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*
x+a)))-1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*Pi*
x^2*csgn(I*exp(2*I*(b*x+a)))^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(c-I)/(exp(
2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/4*
I*x^2*ln(c-I)+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^
2-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+1/4*x*poly
log(2,-I*exp(2*I*(b*x+a))*c)/b-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp
(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I
```

```
*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1/8*x^2
*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a
))-1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1
))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn(exp(
2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a)
)-1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*
x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(
I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I*(exp(2*I
*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(
b*x+a))-1))^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 3.29, size = 152, normalized size = 1.23

$$\frac{4\,b^3x^3+12\,\pi b^2x^2+6i\,b^2x^2\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)-6\left(-i\,b^2x^2+i\,a^2\right)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)+3i\,\mathrm{polylog}\left(3,-i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 12*pi*b^2*x^2 + 6*I*b^2*x^2*log((c - I)*e^(2*I*b*x + 2*I*
a)/(c*e^(2*I*b*x + 2*I*a) - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I
*a)) + 6*I*a^2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) - 6*(-I*b^2*x^2 + I*a^2)*
log(I*c*e^(2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a))
)/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-acot(-c+(1+I*c)*cot(b*x+a))),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type &lt;class 'sympy.core.add.Add'&gt; to QQ_I[x,b,c,_t0,ex
p(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(pi-arccot(-c+(1+I*c)*cot(b*x+a))),x, algorithm="giac")
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(\Pi+\mathrm{acot}\left(c-\mathrm{cot}\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))),x)
[Out] int(x*(Pi + acot(c - cot(a + b*x)*(c*1i + 1))), x)
```

3.181 $\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx$

Optimal. Leaf size=86

$$\frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*(Pi-arccot(-c+(1+I*c)*cot(b*x+a)))+1/2*I*x*ln(1+I*c*exp(2*I*a+2
*I*b*x))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {5274, 2215, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCot[c - (1 + I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5274

```
Int[ArcCot[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcC
ot[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c-(1+ic)\cot(a+bx))\,dx = x\cot^{-1}(c-(1+ic)\cot(a+bx))+(ib)\int \frac{x}{-i(-1-ic)+c-ce^{2ia+2i}}$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+(bc)\int \frac{e^{2ia+2ibx}x}{-i(-1-ic)+c-}$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-$$
$$= \frac{bx^2}{2}+x\cot^{-1}(c-(1+ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+$$

Mathematica [A]
time = 0.50, size = 75, normalized size = 0.87

$$x\cot^{-1}(c+(-1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)-\frac{\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I/(c*E^((2*I)*(a +
b*x)))] - PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1755 vs. 2(75) = 150.
time = 0.94, size = 1756, normalized size = 20.42

method	result	size

risch	Expression too large to display	1244
default	Expression too large to display	1756
derivativedivides	Expression too large to display	1898

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi-arccot(-c+(1+I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x-1/4*I/(1+I*c)/b/(I-c)*ln(I-c+(1+I*c)*cot(b*x+a))*ln(-1/2*I*(I+c-(1+I*c
)*cot(b*x+a)))*c^2+1/4*I/(1+I*c)/b/(I-c)*ln(-1/2*I*(I-c+(1+I*c)*cot(b*x+a))
)*ln(-1/2*I*(I+c-(1+I*c)*cot(b*x+a)))*c^2-1/4*I/(1+I*c)/b/(I-c)*ln(-c-(1+I*
c)*cot(b*x+a)+I)*ln((-I+c-(1+I*c)*cot(b*x+a))/(-2*I+2*c))*c^2+1/4*I/(1+I*c)
/b/(I-c)*ln(-c-(1+I*c)*cot(b*x+a)+I)*ln(1/2*(I+c-(1+I*c)*cot(b*x+a))/c)*c^2
-2*I/(1+I*c)/b*arccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I-c+(1+I*c)*cot(b
*x+a))*c+2*I/(1+I*c)/b*arccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-c-(1+I*c
)*cot(b*x+a)+I)*c-1/2/(1+I*c)/b/(I-c)*ln(I-c+(1+I*c)*cot(b*x+a))*ln(-1/2*I*
(I+c-(1+I*c)*cot(b*x+a)))*c+1/2/(1+I*c)/b/(I-c)*ln(-1/2*I*(I-c+(1+I*c)*cot(
b*x+a)))*ln(-1/2*I*(I+c-(1+I*c)*cot(b*x+a)))*c-1/2/(1+I*c)/b/(I-c)*ln(-c-(1
+I*c)*cot(b*x+a)+I)*ln((-I+c-(1+I*c)*cot(b*x+a))/(-2*I+2*c))*c+1/2/(1+I*c)/
b/(I-c)*ln(-c-(1+I*c)*cot(b*x+a)+I)*ln(1/2*(I+c-(1+I*c)*cot(b*x+a))/c)*c+1/
(1+I*c)/b*arccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I-c+(1+I*c)*cot(b*x+a)
)*c^2-1/(1+I*c)/b*arccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-c-(1+I*c)*cot
(b*x+a)+I)*c^2+1/4*I/(1+I*c)/b/(I-c)*dilog(1/2*(I+c-(1+I*c)*cot(b*x+a))/c)*
c^2+1/4*I/(1+I*c)/b/(I-c)*ln(-c-(1+I*c)*cot(b*x+a)+I)*ln((-I+c-(1+I*c)*cot(
b*x+a))/(-2*I+2*c))-1/4*I/(1+I*c)/b/(I-c)*ln(-c-(1+I*c)*cot(b*x+a)+I)*ln(1/
2*(I+c-(1+I*c)*cot(b*x+a))/c)+1/4*I/(1+I*c)/b/(I-c)*ln(I-c+(1+I*c)*cot(b*x+
a))*ln(-1/2*I*(I+c-(1+I*c)*cot(b*x+a)))-1/4*I/(1+I*c)/b/(I-c)*ln(-1/2*I*(I-
c+(1+I*c)*cot(b*x+a)))*ln(-1/2*I*(I+c-(1+I*c)*cot(b*x+a)))+1/8*I/(1+I*c)/b/
(I-c)*ln(I-c+(1+I*c)*cot(b*x+a))^2*c^2+1/4*I/(1+I*c)/b/(I-c)*dilog(-1/2*I*(
I-c+(1+I*c)*cot(b*x+a)))*c^2-1/4*I/(1+I*c)/b/(I-c)*dilog((-I+c-(1+I*c)*cot(
b*x+a))/(-2*I+2*c))*c^2+1/2/(1+I*c)/b/(I-c)*dilog(-1/2*I*(I-c+(1+I*c)*cot(b
*x+a)))*c-1/2/(1+I*c)/b/(I-c)*dilog((-I+c-(1+I*c)*cot(b*x+a))/(-2*I+2*c))*c
+1/2/(1+I*c)/b/(I-c)*dilog(1/2*(I+c-(1+I*c)*cot(b*x+a))/c)*c-1/(1+I*c)/b*ar
ccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I-c+(1+I*c)*cot(b*x+a))+1/(1+I*c)/
b*arccot(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-c-(1+I*c)*cot(b*x+a)+I)-1/4*I
/(1+I*c)/b/(I-c)*dilog(1/2*(I+c-(1+I*c)*cot(b*x+a))/c)-1/8*I/(1+I*c)/b/(I-c
)*ln(I-c+(1+I*c)*cot(b*x+a))^2-1/4*I/(1+I*c)/b/(I-c)*dilog(-1/2*I*(I-c+(1+I
*c)*cot(b*x+a)))+1/4*I/(1+I*c)/b/(I-c)*dilog((-I+c-(1+I*c)*cot(b*x+a))/(-2*
I+2*c))+1/4/(1+I*c)/b/(I-c)*ln(I-c+(1+I*c)*cot(b*x+a))^2*c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 2.83, size = 116, normalized size = 1.35

$$\frac{2\,b^2x^2+4\,\pi bx+2i\,bx\log\left(\frac{(c-i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)-2\,a^2-2\,(-i\,bx-i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 4*pi*b*x + 2*I*b*x*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(2
*I*b*x + 2*I*a) - I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(I*c*e^(2*I*b*x + 2*I*a
) + 1) - 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x +
 2*I*a)))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-acot(-c+(1+I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert -_t0**2*I + 2*c*exp(2*I*
a) - I*exp(2*I*a) of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(
I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi-arccot(-c+(1+I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(pi - arccot((I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \Pi+\mathrm{acot}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(Pi + acot(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.182 $\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate((Pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx = \int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c-(1+ic)\cot(a+bx))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (1 + I*c)*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\pi - \operatorname{arccot}(-c+(ic+1)\cot(bx+a))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

```
[Out] int((Pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*pi + I*log((c - I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*
a) - I)))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-acot(-c+(1+I*c)*cot(b*x+a)))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((pi-arccot(-c+(1+I*c)*cot(b*x+a)))/x,x, algorithm="giac")
```

```
[Out] integrate((pi - arccot((I*c + 1)*cot(b*x + a) - c))/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\Pi + \mathrm{acot}(c - \cot(a + b\,x)\,\left(1 + c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x,x)
[Out] int((Pi + acot(c - cot(a + b*x)*(c*1i + 1)))/x, x)
```

3.183 $\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=299

$$\frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f}+\frac{(e+fx)^4 \text{ArcTan}\left(e^{2a+2bx}\right)}{4f}-\frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^3\text{I}}{}$$

```
[Out] 1/4*(f*x+e)^4*arccot(tanh(b*x+a))/f+1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+3/16*
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A]
time = 0.16, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5292, 4265, 2611, 6744, 2320, 6724}

$$\frac{(e+fx)^4 \text{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{3if^3\text{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}-\frac{3if^3\text{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}-\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}-\frac{3if(e+fx)^2\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^4\cot^{-1}(\tanh(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCot[Tanh[a + b*x]])/(4*f) + ((e + f*x)^4*ArcTan[E^(2*a + 2*
b*x)])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{b\int (e+fx)^4 \mathrm{sech}(2a+2bx)\,dx}{4f}$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{1}{2}i\int (\ldots$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^4 \cot^{-1}(\tanh(a+bx))}{4f} + \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} - \frac{i(e+\ldots}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.
time = 0.22, size = 600, normalized size = 2.01

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Tanh[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.63, size = 7275, normalized size = 24.33

method	result	size
risch	Expression too large to display	7275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*f^2*x^3
*e^(2*a + 1) + 6*b*f*x^2*e^(2*a + 2) + 4*b*x*e^(2*a + 3))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(244) = 488.
time = 3.17, size = 2700, normalized size = 9.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*
f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*p
olylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^
4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3
+ 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*arctan(cosh(b*x + a)/si
nh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*co
sh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*s
inh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*d
ilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3
*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*si
```

```
nh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*
f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x
*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1)
)*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1)
)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3
+ 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3
*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b
^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x +
a) + sinh(b*x + a))) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x + a*b^3)*cos
h(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4
*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^
4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 - I*a^4*f
^3 + 4*I*(b^4*x + a*b^3)*cosh(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b
^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a
*b^3)*cosh(1))*sinh(1)^2 + 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4
*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2
*f)*cosh(1))*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x + a*b^3)*cosh(1)^3 - 4*I*(b^4
*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 - 6*I*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 - 4*I*(b^4*f^2*x^3 +
 a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x
 + a*b^3)*cosh(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*
b^2*f)*cosh(1)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*
sinh(1)^2 - 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*
b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*si
nh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*a^4*f^
3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^3 -
 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I*(a
^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^4*f^3 - 4*I*a^3*b*f^2*cosh(1)
 + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^3 - 4*I*a*b^3*sinh(1)^3 + 6*
I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*co
sh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*
sinh(b*x + a)) + (-I*a^4*f^3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1
)^2 + 4*I*a*b^3*cosh(1)^3 + 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*
cosh(1))*sinh(1)^2 + 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)
^2)*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^
4*f^3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)
^3 + 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*
I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqr
```

```
t(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(-I*b*f^3*x - I*b*f^2*cos
h(1) - I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*(I*b*f^3*x + I*b*f^2*cosh
(1) + I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 24*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, -1/2
*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*
f^2*x*cosh(1) + I*b^2*f*cosh(1)^2 + I*b^2*f*sinh(1)^2 + 2*I*(b^2*f^2*x + b^
2*f*cosh(1))*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*f^2*x*cosh(1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{acot}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*acot(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*acot(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccot(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acot}(\tanh(a+b\,x))\,{\left(e+f\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(tanh(a + b*x))*(e + f*x)^3,x)
```

```
[Out] int(acot(tanh(a + b*x))*(e + f*x)^3, x)
```

3.184 $\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=229

$$\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}+\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}-\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^2\text{I}}{}$$

```
[Out] 1/3*(f*x+e)^3*arccot(tanh(b*x+a))/f+1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A]

time = 0.12, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5292, 4265, 2611, 6744, 2320, 6724}

$$\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}-\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^2\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^3\cot^{-1}(\tanh(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCot[Tanh[a + b*x]])/(3*f) + ((e + f*x)^3*ArcTan[E^(2*a + 2*
b*x)])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \cot^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{b\int (e+fx)^3 \mathrm{sech}(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} - \frac{1}{2}i\int (\cdots$$
$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+\cdots}{}$$
$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+\cdots}{}$$
$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+\cdots}{}$$
$$= \frac{(e+fx)^3 \cot^{-1}(\tanh(a+bx))}{3f} + \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} - \frac{i(e+\cdots}{}$$

Mathematica [A]
time = 0.11, size = 375, normalized size = 1.64

$$\frac{1}{3}x(3e^2+3efx+f^2x^2)\cot^{-1}(\tanh(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)-12b^3efx^2\log\left(1+ie^{2(a+bx)}\right)-4b^3f^2x^3\log\left(1+ie^{2(a+bx)}\right)-6b^2(e+fx)^2\mathrm{PolyLog}(2,-ie^{2(a+bx)})+6b^2(e+fx)^2\mathrm{PolyLog}(2,ie^{2(a+bx)})+6bef\mathrm{PolyLog}(3,-ie^{2(a+bx)})+6bf^2x\mathrm{PolyLog}(3,-ie^{2(a+bx)})-6bef\mathrm{PolyLog}(3,ie^{2(a+bx)})-6bf^2x\mathrm{PolyLog}(3,ie^{2(a+bx)})-3f^2\mathrm{PolyLog}(4,-ie^{2(a+bx)})+3f^2\mathrm{PolyLog}(4,ie^{2(a+bx)})\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Tanh[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.66, size = 5425, normalized size = 23.69

method	result	size
risch	Expression too large to display	5425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*f*x^2*e^(2*a + 1) + 3*b
*x*e^(2*a + 2))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1511 vs. 2(186) = 372.
time = 2.90, size = 1511, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 +
2*b^3*x*cosh(1))*sinh(1))*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I*b^2*f
^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*
sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh
(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^
2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^
2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh
(1))*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (I*b
^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b^2
)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*f
+ 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1
)^2 + 3*I*(b^3*x + a*b^2)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3
*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt
```

```
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - I*a^3*f^2 -
3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b^2)*sinh(1)^2 - 3*I*(b^3*f*
x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(
1))*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*
b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b^
2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*f
 + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*cosh(1
)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*
sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^3*f^2 + 3*I*a^2*b*f*
cosh(1) - 3*I*a*b^2*cosh(1)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^
2*cosh(1))*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
 (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cosh(1)^2 + 3*I*a*b^2*sinh(1)
^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x
 + a) + 2*sinh(b*x + a)) + (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cos
h(1)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(I*b*f^2*x + I*b*f*
cosh(1) + I*b*f*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
 + a))) - 6*(I*b*f^2*x + I*b*f*cosh(1) + I*b*f*sinh(1))*polylog(3, -1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(-I*b*f^2*x - I*b*f*cosh(1) - I
*b*f*sinh(1))*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
6*(-I*b*f^2*x - I*b*f*cosh(1) - I*b*f*sinh(1))*polylog(3, -1/2*sqrt(-4*I)*(
cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \operatorname{acot}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acot(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*acot(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acot}(\tanh(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(tanh(a + b*x))*(e + f*x)^2,x)
[Out] int(acot(tanh(a + b*x))*(e + f*x)^2, x)
```

3.185 $\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=159

$$\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\text{ArcTan}\left(e^{2a+2bx}\right)}{2f}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\text{Po}}{}$$

```
[Out] 1/2*(f*x+e)^2*arccot(tanh(b*x+a))/f+1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f-
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A]

time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5292, 4265, 2611, 2320, 6724}

$$\frac{(e+fx)^2\text{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{if\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}-\frac{if\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCot[Tanh[a + b*x]])/(2*f) + ((e + f*x)^2*ArcTan[E^(2*a + 2*
b*x)])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5292

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Tanh[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (e+fx)\cot^{-1}(\tanh(a+bx))\,dx &= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{b\int (e+fx)^2\mathrm{sech}(2a+2bx)\,dx}{2f}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}-\frac{1}{2}i\int (e+\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}-\frac{i(e+fx}{}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}-\frac{i(e+fx}{}\\
&= \frac{(e+fx)^2\cot^{-1}(\tanh(a+bx))}{2f}+\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}-\frac{i(e+fx}{}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 278, normalized size = 1.75

$$ex\cot^{-1}(\tanh(a+bx))+\frac{1}{2}fx^2\cot^{-1}(\tanh(a+bx))+\frac{e\left(-\left(\left(-4ia+\pi-4ibx\right)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+\left(-4ia+\pi\right)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\mathrm{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\mathrm{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)\right)}{8b}+\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\mathrm{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\mathrm{PolyLog}\left(2,ie^{2(a+bx)}\right)+\mathrm{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\mathrm{PolyLog}\left(3,ie^{2(a+bx)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcCot[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcCot[Tanh[a + b*x]] + (f*x^2*ArcCot[Tanh[a + b*x]])/2 + (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
```

```
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) + ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 2688, normalized size = 16.91

method	result	size
risch	Expression too large to display	2688

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*I*f*ln(1
+I*exp(2*b*x+2*a))*x^2-1/2*I*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*e*ln(1-e
xp(b*x+a)*(-1)^(3/4))*x+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/2*I/b*e*dilog
(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a)
)/(-I)^(1/2))-1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*dilog(1-ex
p(b*x+a)*(-1)^(3/4))-1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2*a)+I)+1/4*Pi*x*e*
csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn((1-I)
*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*
x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1)
)^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(
exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1
/2*I*f/b*ln(1+I*exp(2*b*x+2*a))*x*a+1/2*I*f/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))
*x+1/2*I*f/b*a*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I/b*f*a*ln(((-I)^(1/2)-exp
(b*x+a))/(-I)^(1/2))*x-1/2*I/b*f*a*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x
+1/2*I/b*f*ln(1-I*exp(2*b*x+2*a))*x*a+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))
-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-
1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a))*
x-1/4*I*f/b^2*polylog(2,-I*exp(2*b*x+2*a))*a-1/2*I*e/b*ln(1+exp(b*x+a)*(-1)
^(3/4))*a-1/2*I/b^2*f*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I/b^2*
f*a^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*I/b^2*f*ln(1-I*exp(2*b*x+2
*a))*a^2+1/4*I/b*f*polylog(2,I*exp(2*b*x+2*a))*x+1/4*I/b^2*f*polylog(2,I*ex
```

```
p(2*b*x+2*a))*a+1/2*I/b*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a*e+1/2*I/b*
ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a*e-1/2*I*e/b*ln(1-exp(b*x+a)*(-1)^(
3/4))*a+1/2*I*f/b^2*a^2*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a^2*ln(1-ex
p(b*x+a)*(-1)^(3/4))-1/4*I*f/b^2*ln(1+I*exp(2*b*x+2*a))*a^2-1/4*Pi*x*e*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*
I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a
)-I)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+
2*a)+1))^3-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+1/4*I
*ln(exp(2*b*x+2*a)-I)*f*x^2+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x+1/4*I/b^2*f*a^2*
ln(exp(2*b*x+2*a)+I)-1/2*I/b*e*a*ln(exp(2*b*x+2*a)+I)+1/8*Pi*x^2*f*csgn((1+
I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))
+1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(b*x
+a)*(-1)^(3/4))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2
*I/b^2*f*a*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/4*Pi*x*e*csgn((1+I)*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a
)+I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)+1))^3-1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3
-1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*f*
x^2+1/4*Pi*e*x-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*
(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-
I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/4*I*f/b^2*a^2*ln(-exp
(2*b*x+2*a)+I)+1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/4*Pi*x*e*csgn(I*(exp(2*b
*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a
)+1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)
*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-1/8*I*f*polylog(3,I*exp(2*b*x+2*a))
/b^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+1/4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2+1
/2*I*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x*e+1/2*I*ln(((-I)^(1/2)-exp(b*
x+a))/(-I)^(1/2))*x*e+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a
)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+1/4*Pi*x*e*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*x*e^(2*a + 1))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(134) = 268$.
time = 2.37, size = 717, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*arctan(cosh(b*x + a)
/sinh(b*x + a)) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(1/2*sqrt(4
*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh
(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b*f*x + I
*b*cosh(1) + I*b*sinh(1))*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cos
h(1) + 2*I*(b^2*x + a*b)*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b
*x + a)) + 1) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b
^2*x + a*b)*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*(b^2*x + a*b)
*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2
*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*(b^2*x + a*b)*sinh(1))*l
og(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*a^2*f - 2*I*a*
b*cosh(1) - 2*I*a*b*sinh(1))*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
 + a)) + (I*a^2*f - 2*I*a*b*cosh(1) - 2*I*a*b*sinh(1))*log(-I*sqrt(4*I) + 2
*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^2*f + 2*I*a*b*cosh(1) + 2*I*a*b*s
inh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^2*f +
 2*I*a*b*cosh(1) + 2*I*a*b*sinh(1))*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
 2*I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*p
olylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{acot}(\tanh(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acot(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acot(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(\tanh(a + b\,x))\,\left(e + f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(tanh(a + b*x))*(e + f*x),x)
```

```
[Out] int(acot(tanh(a + b*x))*(e + f*x), x)
```

3.186 $\int \cot^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=73

$$x\cot^{-1}(\tanh(a+bx))+x\text{ArcTan}\left(e^{2a+2bx}\right)-\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arccot(tanh(b*x+a))+x*arctan(exp(2*b*x+2*a))-1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5288, 4265, 2317, 2438}

$$x\text{ArcTan}\left(e^{2a+2bx}\right)-\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+x\cot^{-1}(\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[Tanh[a + b*x]] + x*ArcTan[E^(2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5288

```
Int[ArcCot[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Tanh[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{-1}(\tanh(a+bx))\,dx &= x\cot^{-1}(\tanh(a+bx)) + b\int x\operatorname{sech}(2a+2bx)\,dx \\
&= x\cot^{-1}(\tanh(a+bx)) + x\tan^{-1}\left(e^{2a+2bx}\right) - \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx + \frac{1}{2}i\int \\
&= x\cot^{-1}(\tanh(a+bx)) + x\tan^{-1}\left(e^{2a+2bx}\right) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} + \\
&= x\cot^{-1}(\tanh(a+bx)) + x\tan^{-1}\left(e^{2a+2bx}\right) - \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 132, normalized size = 1.81

$$x\cot^{-1}(\tanh(a+bx)) + \frac{-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[Tanh[a + b*x]] + (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(62) = 124.
time = 0.64, size = 184, normalized size = 2.52

method	result
derivativedivides	$\frac{\mathrm{arctanh}(\tanh(bx+a))\mathrm{arccot}(\tanh(bx+a))+\mathrm{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\tanh}{\tanh^2(b}\right.}{2}}{b}$
default	$\frac{\mathrm{arctanh}(\tanh(bx+a))\mathrm{arccot}(\tanh(bx+a))+\mathrm{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))+\frac{\arctan(\tanh(bx+a))\ln\left(1+\frac{i(1+i\tanh}{\tanh^2(b}\right.}{2}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arccot(tanh(b*x+a))+arctanh(tanh(b*x+a))*arctan(t
anh(b*x+a))+1/2*arctan(tanh(b*x+a))*ln(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)
^2+1))-1/2*arctan(tanh(b*x+a))*ln(1-I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1)
)-1/4*I*dilog(1+I*(1+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1))+1/4*I*dilog(1-I*(1
+I*tanh(b*x+a))^2/(tanh(b*x+a)^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*
b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(56) = 112$.
time = 2.42, size = 334, normalized size = 4.58

$$\frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)-i\,a\log\left(i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)-i\,a\log\left(-i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,a\log\left(i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,a\log\left(-i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)-i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)-i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(tanh(b*x+a)),x)
```

```
[Out] Integral(acot(tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arccot(tanh(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(tanh(a + b*x)),x)
[Out] int(acot(tanh(a + b*x)), x)
```

3.187 $\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\cot^{-1}(\tanh(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccot(tanh(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcCot[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx = \int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Mathematica [A]
time = 5.54, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(\tanh(a+bx))}{e+fx} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcCot[Tanh[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(\tanh(bx+a))}{fx+e} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(tanh(b*x+a))/(f*x+e),x)
[Out] int(arccot(tanh(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccot(tanh(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccot(tanh(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(\tanh\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(tanh(b*x+a))/(f*x+e),x)
[Out] Integral(acot(tanh(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acot}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(tanh(a + b*x))/(e + f*x),x)
[Out] int(acot(tanh(a + b*x))/(e + f*x), x)
```

3.188 $\int x^2 \cot^{-1}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=355

$$\frac{1}{3}x^3 \cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3 \log\left(1 + \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix^2\mathrm{Pol}}{}$$

```
[Out] 1/3*x^3*arccot(c+d*tanh(b*x+a))-1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/
(I+c-d))/b+1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I*x*p
olylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,-(I-c-d)*exp(
2*b*x+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

Rubi [A]
time = 0.34, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5308, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\mathrm{Li}_4\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3} + \frac{i\mathrm{Li}_4\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3} + \frac{ix\mathrm{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2} - \frac{ix\mathrm{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2} - \frac{ix^2\mathrm{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{ix^2\mathrm{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{6}ix^3\log\left(1 + \frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{6}ix^3\log\left(1 + \frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + \frac{1}{3}x^3\cot^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b + ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b + ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 - ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 - ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
+ ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5308

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2bx}}{i+c-d+(i+}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}$$

$$= \frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{6}$$

Mathematica [A]
time = 4.44, size = 305, normalized size = 0.86

$$\frac{1}{3}x^3\cot^{-1}(c+d\tanh(a+bx)) - \frac{i\left(4b^3x^3\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 4b^3x^3\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) + 6b^2x^2\text{PolyLog}\left(2,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) - 6bx\text{PolyLog}\left(3,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) + 6bx\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) + 3\text{PolyLog}\left(4,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 3\text{PolyLog}\left(4,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Tanh[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(-I + c - d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
 + b*x)))/(I + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x
)))/(-I + c - d))] - 6*b^2*x^2*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(
I + c - d))] - 6*b*x*PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c -
d))] + 6*b*x*PolyLog[3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))] + 3*P
olyLog[4, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] - 3*PolyLog[4, -(
((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.01, size = 6939, normalized size = 19.55

method	result	size
risch	Expression too large to display	6939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
- d) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.
time = 1.17, size = 1289, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*
x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x
+ a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a)
 + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)
*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2
*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*l
og(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh
(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
```

```
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a))) + (-I*b^3*x^3 - I*a^3)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*l
og(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^
3 + I*a^3)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - 6*I*polylog(4, sqrt(-(c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylo
g(4, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a))) + 6*I*polylog(4, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*
c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt(-(c
^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a))))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(d*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acot}(c+d\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*acot(c + d*tanh(a + b*x)), x)
```

3.189 $\int x\cot^{-1}(c+d\tanh(a+bx))\,dx$

Optimal. Leaf size=267

$$\frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{ix\mathrm{Po}}{}$$

```
[Out] 1/2*x^2*arccot(c+d*tanh(b*x+a))-1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,-(I-c-
d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c
-d))/b+1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polylog(3
,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A]
time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {5308, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}-\frac{i\text{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}+\frac{ix\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)+\frac{1}{4}ix^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)+\frac{1}{2}x^2\cot^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b + ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b + ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
- ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5308

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\cot^{-1}(c+d\tanh(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}}{i+c-d+(i+c} \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{4} \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{4} \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{4} \\
&= \frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{4}
\end{aligned}$$

Mathematica [A]
time = 2.71, size = 229, normalized size = 0.86

$$\frac{1}{2}x^2\cot^{-1}(c+d\tanh(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2b^2x^2\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)+2bx\text{PolyLog}\left(2,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2bx\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)-\text{PolyLog}\left(3,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)+\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Tanh[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(-I + c - d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(I + c - d)] + 2*b*x*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x)))/(
-I + c - d))] - 2*b*x*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d
))] - PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] + PolyLog[
3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.16, size = 6589, normalized size = 24.68

method	result	size
risch	Expression too large to display	6589

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c
- d) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(197) = 394.
time = 1.52, size = 1067, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) -
2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
```

```
b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(sq
rt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 +
 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
- 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c
^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 -
I*a^2)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a)) + 1) + 2*I*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3,
 -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a))) - 2*I*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt(-(c^2 -
 d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
)/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acot}(c+d\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x*acot(c + d*tanh(a + b*x)), x)
```

3.190 $\int \cot^{-1}(c+d\tanh(a+bx))\,dx$

Optimal. Leaf size=174

$$x\cot^{-1}(c+d\tanh(a+bx))-\frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{i\text{PolyLog}(\ldots}{\ldots}$$

```
[Out] x*arccot(c+d*tanh(b*x+a))-1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2*
I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,-(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b+1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A]
time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5300, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}+\frac{i\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}-\frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)+\frac{1}{2}ix\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)+x\cot^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tanh[a + b*x]] - (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] + (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] - ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5300

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*
x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*b*(I + c + d
), Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+d\tanh(a+bx))\,dx = x\cot^{-1}(c+d\tanh(a+bx)) - (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(i+c+d)$$

$$= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\,\mathrm{lo}$$

$$= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\,\mathrm{lo}$$

$$= x\cot^{-1}(c+d\tanh(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\,\mathrm{lo}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 363 vs. 2(174) = 348.
time = 0.84, size = 363, normalized size = 2.09

$$x\cot^{-1}(c+d\tanh(a+bx)) - \frac{i\left(-2ia\,\mathrm{ArcTan}\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+(a+bx)\log\left(1-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right)+(a+bx)\log\left(1+\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right)-(a+bx)\log\left(1-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right)-(a+bx)\log\left(1+\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right)+\mathrm{PolyLog}\left(2,-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right)+\mathrm{PolyLog}\left(2,\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right)-\mathrm{PolyLog}\left(2,-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Tanh[a + b*x]] - ((I/2)*((-2*I)*a*ArcTan[(1 + c^2 - d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + (a + b*x)*Log[1 - (Sqrt[-I
+ c + d]*E^(a + b*x))/Sqrt[I - c + d]] + (a + b*x)*Log[1 + (Sqrt[-I + c + d
]*E^(a + b*x))/Sqrt[I - c + d]] - (a + b*x)*Log[1 - (Sqrt[I + c + d]*E^(a +
 b*x))/Sqrt[-I - c + d]] - (a + b*x)*Log[1 + (Sqrt[I + c + d]*E^(a + b*x))/
Sqrt[-I - c + d]] + PolyLog[2, -((Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[I - c
+ d])] + PolyLog[2, (Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[I - c + d]] - PolyL
og[2, -((Sqrt[I + c + d]*E^(a + b*x))/Sqrt[-I - c + d])] - PolyLog[2, (Sqrt
[I + c + d]*E^(a + b*x))/Sqrt[-I - c + d]]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(150) = 300.
time = 0.94, size = 365, normalized size = 2.10

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i}{}\right.}{2d}\right.$
default	$-\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i}{}\right.}{2d}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arccot(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arccot(c+d*t
anh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*d^2*(1/2*I/d*ln(-d*tanh(b*x+a)+d)*ln
((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I/d*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(b*x
+a)-c)/(I-c-d))+1/2*I/d*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I/d*dilog((I
-d*tanh(b*x+a)-c)/(I-c-d))-1/2*I/d*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b*x+a)
+c)/(I+c-d))+1/2*I/d*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d))-1
/2*I/d*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))+1/2*I/d*dilog((I-d*tanh(b*x+a)-c)
/(I-c+d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) + 1, (c*e^(2*a) +
 d*e^(2*a))*e^(2*b*x) + c - d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(128) = 256$.
time = 1.70, size = 825, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/(c*cosh(b*x + a) + d*sinh(b*x + a))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(
c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d
^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I
*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*
sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*
a)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*
a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - I*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-sqrt(-(c^2 - d^2
 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I
*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) + I*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(c+d\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(acot(c + d*tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}\left(c+d\,\tanh\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*tanh(a + b*x)),x)
```

```
[Out] int(acot(c + d*tanh(a + b*x)), x)
```

3.191 $\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\operatorname{Int}\left(\frac{\cot^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.84, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c+d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*tanh(b*x+a))/x,x)
[Out] int(arccot(c+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*tanh(b*x + a) + c)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*tanh(b*x+a))/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acot}(c + d \tanh(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*tanh(a + b*x))/x,x)
[Out] int(acot(c + d*tanh(a + b*x))/x, x)
```

3.192 $\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=142

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{ix\text{PolyLog}$$

[Out] 1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*tanh(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5304, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\tanh(a+bx))+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]

[Out] (I/12)*b*x^4 + (x^3*ArcCot[c + (I + c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1 + I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*c*E^(2*a + 2*b*x)])/b^3

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{-i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+}}{-i+c}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1+ice\right.$$

Mathematica [A]
time = 0.15, size = 128, normalized size = 0.90

$$\frac{1}{3}x^3\cot^{-1}(c+(i+c)\tanh(a+bx)) - \frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\mathrm{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right) - 3\mathrm{PolyLog}\left(4,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (I + c)*Tanh[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*Po
lyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.87, size = 1473, normalized size = 10.37

method	result	size
risch	Expression too large to display	1473

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3+1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a)
)*x*a^2+1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)+1))^2+1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)+1/12*Pi*x^3*csgn((2*exp(2
*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a
```

```
^3+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1/2*I/b^3*a^3*ln(1+I*exp(b*
x+a)*(I*c)^(1/2))-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/12*b/(I+c)
*x^4+1/12/b^3/(I+c)*a^4-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^2*a^2*ln
(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x
^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)+1))^2-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2
*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/4*I*x*polylog(3,-I*c*exp(2*b
*x+2*a))/b^2-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)+1))^3-1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))^3-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/6*I
*x^3*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)+1/12*Pi*x^3*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)
/(exp(2*b*x+2*a)+1))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/12*Pi*x^3
*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/6*I*x^3*ln(2*exp(2*b
*x+2*a)*c-2*I)-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2
*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(
I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-1/12*I/b^3*c/(I+c)*a^4+1/1
2*I*b*c/(I+c)*x^4-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*
x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-
2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csg
n(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2
```

<u>Maxima</u> [A]
time = 1.21, size = 129, normalized size = 0.91

$$\frac{1}{3}\,x^{3}\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c-4}-\frac{4\,b^{3}x^{3}\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}\left(-i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_{4}\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^{4}\left(-i\,c+1\right)}\right)b(c+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c + I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

<u>Fricas</u> [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

time = 5.10, size = 292, normalized size = 2.06

$$\frac{i b^4 x^4 + 2i b^3 x^3 \log\left(\frac{(ce^{(2bx+2a)}-i)e^{(-2bx-2a)}}{c+i}\right) - 6i b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right) - 6i b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right) - i a^4 + 2i a^3 \log\left(\frac{2ce^{(bx+a)}+i\sqrt{-4i c}}{2c}\right) + 2i a^3 \log\left(\frac{2ce^{(bx+a)}-i\sqrt{-4i c}}{2c}\right) + 12i bx \mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right) + 12i bx \mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right) - 2\left(i b^3 x^3 + i a^3\right)\log\left(\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)} + 1\right) - 2\left(i b^3 x^3 + i a^3\right)\log\left(-\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)} + 1\right) - 12i \,\mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right) - 12i \,\mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4i c}\, e^{(bx+a)}\right)}{12\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/
(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog
(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-4*
I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*x
+ a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*polylog(4,
 -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}(c+\tanh(a+b\,x)\ (c+1\mathrm{i}))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.193 $\int x \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=113

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}(3,-}{8b^2}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*tanh(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = $\{5304, 2215, 2221, 2611, 2320, 6724\}$

$$\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\tanh(a+bx))+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCot[c + (I + c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(c+(i+c)\tanh(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{-i+ce^{2a+2bx}}\,dx \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}}{-i+ce^{2}} \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right. \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right. \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right. \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 102, normalized size = 0.90

$$
\frac{1}{2}x^2\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (I + c)*Tanh[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I/
(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.72, size = 1437, normalized size = 12.72

method	result	size
risch	Expression too large to display	1437

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))
^2-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^
2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+
2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/4*I*x^
2*ln(1+I*c*exp(2*b*x+2*a))-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)-1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*x*a+1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^
(1/2))*x+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2
*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*
(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn
((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/8*I*polylo
g(3,-I*c*exp(2*b*x+2*a))/b^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(
2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))
^2+1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c-2*I)+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c
-2*I)/(exp(2*b*x+2*a)+1))^2+1/6*I*b*c/(I+c)*x^3+1/6*I/b^2*c/(I+c)*a^3+1/2*I
/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)
+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/8*
Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csg
n(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/4*I/b^2
*ln(1+I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a+1/
2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*
(I*c)^(1/2))+1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a*dilo
g(1-I*exp(b*x+a)*(I*c)^(1/2))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/6*
b/(I+c)*x^3-1/6/b^2/(I+c)*a^3+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
```

```
c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*
b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-1/4*I/b^2*a^
2*ln(-exp(2*b*x+2*a)*c+I)
```

Maxima [A]
time = 1.22, size = 107, normalized size = 0.95

$$-\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)
))*b*(c + I) + 1/2*x^2*arccot((c + I)*tanh(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(83) = 166$.
time = 1.72, size = 246, normalized size = 2.18

$$\frac{2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c+i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a
)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x
^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqr
t(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.194 $\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccot(c+(I+c)*tanh(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5296, 2215, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\cot^{-1}(c+(c+i)\tanh(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCot[c + (I + c)*Tanh[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5296

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+(i+c)\tanh(a+bx))\,dx = x\cot^{-1}(c+(i+c)\tanh(a+bx))+b\int \frac{x}{-i+ce^{2a+2bx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-(ibc)\int \frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}\,dx$$
$$=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+$$
$$=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+$$
$$=\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-$$

Mathematica [A]
time = 0.42, size = 71, normalized size = 0.90

$$x\cot^{-1}(c+(i+c)\tanh(a+bx))-\frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(65) = 130.
time = 0.59, size = 597, normalized size = 7.56

method	result

derivativedivides	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i\cdots}{}$
default	$-\frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i\cdots}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(-arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))
+2*I*arccot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+arcc
ot(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arccot(c+(I
+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arccot(c+(I+c)*tan
h(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arccot(c+(I+c)*tanh(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2+(I+c)^2*(-1/4*I/(I+c)*ln(I+c+(I+c)
*tanh(b*x+a))*ln(-1/2*I*(I-c-(I+c)*tanh(b*x+a)))+1/4*I/(I+c)*ln(-1/2*I*(I+c
+(I+c)*tanh(b*x+a)))*ln(-1/2*I*(I-c-(I+c)*tanh(b*x+a)))+1/4*I/(I+c)*dilog(-
1/2*I*(I+c+(I+c)*tanh(b*x+a)))+1/8*I/(I+c)*ln(I+c+(I+c)*tanh(b*x+a))^2-1/4*
I/(I+c)*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))-1
/4*I/(I+c)*dilog((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))+1/4*I/(I+c)*ln(c-(I+c
)*tanh(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)+1/4*I/(I+c)*dilog(-1/2*
(I-c-(I+c)*tanh(b*x+a))/c)))
```

Maxima [A]
time = 1.20, size = 80, normalized size = 1.01

$$-2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\operatorname{arccot}\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*tanh(b*x +
a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.
time = 2.56, size = 186, normalized size = 2.35

$$\frac{i\,b^2x^2+i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c+i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I)
) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
- I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
+ a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) - 3*_t0**2*I*c*exp(2*a) + _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(acot(c + tanh(a + b*x)*(c + 1i)), x)
```

3.195 $\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (I + c)*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c+(i+c)\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

```
[Out] int(arccot(c+(I+c)*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(1/c) + I*log(c^2 + 1)
)*log(x) + 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integrate
(log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log((c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(I+c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acot}(c + \tanh(a + b\,x)\;(c + 1\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tanh(a + b*x)*(c + 1i))/x,x)
[Out] int(acot(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.196 $\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=145

$$-\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\text{PolyLog}(}{4}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*tanh(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c*e
xp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5304, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcCot[c - (I - c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log
[1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b -
 ((I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2
*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{3}b\int\frac{x^3}{i+ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{3}(ibc)\int\frac{e^{2a+2}}{i+ce}$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ic\right.$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ic\right.$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ic\right.$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ic\right.$$
$$= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1-ic\right.$$

Mathematica [A]
time = 0.15, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\cot^{-1}(c+(-i+c)\tanh(a+bx))+\frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6b^2x^2\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6bx\mathrm{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)-3\mathrm{PolyLog}\left(4,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (-I + c)*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*
x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x
)))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.85, size = 1486, normalized size = 10.25

method	result	size
risch	Expression too large to display	1486

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/6*I*x^3*ln(-2*exp(2*b
*x+2*a)*c-2*I)-1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)+1))^2-1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*x*a^2+1/2*I/b^2*a^2*
```

```
ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c)^(1
/2))*x+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+1/
4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))
/b^3-1/12/b^3/(I-c)*a^4+1/12*b/(I-c)*x^4+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))
+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(ex
p(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn((-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/3*Pi*x^3-1/3*I/
b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^
2+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^3*ln(1+I*exp(b*
x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b
^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/4*I*x*polylog(3,I*c*exp(2*b*x+2
*a))/b^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2
*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1
))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(
-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(-2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-1/12*Pi*x^3*csgn(I*(-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/12*Pi*x^3*csgn((2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn((2*exp(2*b*x+
2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)-1/12
*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-1/12*I/b^3*c*a^4/(I-c)+1/12*I*
b*c/(I-c)*x^4+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)+1))-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+
2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))
```

Maxima [A]
time = 1.22, size = 129, normalized size = 0.89

$$\frac{1}{3}\,x^{3}\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c+4}-\frac{4\,b^{3}x^{3}\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_{4}\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^{4}(-i\,c-1)}\right)b(c-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c - I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

time = 1.22, size = 292, normalized size = 2.01

$$-\frac{i b^4 x^4 + 2 i b^3 x^3 \log\left(\frac{(c-i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right) + 6 i b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) + 6 i b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) + i a^4 - 2 i a^3 \log\left(\frac{2ce^{(bx+a)}+i\sqrt{4ic}}{2c}\right) - 2 i a^3 \log\left(\frac{2ce^{(bx+a)}-i\sqrt{4ic}}{2c}\right) - 12 i bx\mathrm{polylog}\left(3, \frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) - 12 i bx\mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) - 2\left(-i b^3 x^3 - i a^3\right)\log\left(\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)} + 1\right) - 2\left(-i b^3 x^3 - i a^3\right)\log\left(-\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)} + 1\right) + 12 i\,\mathrm{polylog}\left(4, \frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right) + 12 i\,\mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{4ic}\,e^{(bx+a)}\right)}{12 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*
a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*dil
og(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3, -
1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4*I*c)*e
^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) +
 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/2*s
qrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}\left(c + \tanh(a + b\,x)\,\left(c - \mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.197 $\int x \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=116

$$-\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(3,\right.}{8b}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*tanh(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(2*
b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5304, 2215, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcCot[c - (I - c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[
1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((
I/8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}(c-(i-c)\tanh(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}}{i+ce^{2a-}}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.\end{aligned}$$

Mathematica [A]
time = 0.08, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\cot^{-1}(c+(-i+c)\tanh(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (-I + c)*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3
, (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.61, size = 1450, normalized size = 12.50

method	result	size
risch	Expression too large to display	1450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+
I)+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x^2*ln(-2*exp(2*b*x+2*a)*c
-2*I)+1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x*a-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*
c)^(1/2))*x-1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/4*I*x^2*ln(1-I*c*
exp(2*b*x+2*a))-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1)
)^2-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x
^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*
a)*c+2*I)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*c
sgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I*(-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/
8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1)
)*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*P
i*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*c
sgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)
+1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/6/b^2/(I-c)*a^3+1/6*b/(I-c)*x
^3+1/2*Pi*x^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1)
)^3+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+
2*a)+1))-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I
*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/
2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a
)-2*exp(2*b*x+2*a)*c)+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*poly
log(2,I*c*exp(2*b*x+2*a))*a-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*
b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*ex
```

```
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*
x+2*a)+1))+1/6*I*b*c/(I-c)*x^3+1/6*I/b^2*c/(I-c)*a^3
```

Maxima [A]
time = 1.20, size = 106, normalized size = 0.91

$$\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^2\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)))
*b*(c - I) + 1/2*x^2*arccot((c - I)*tanh(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.
time = 3.27, size = 246, normalized size = 2.12

$$\frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*x
*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) + I
*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - 3
*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2*x^2
+ I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt(4*
I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acot}(c + \tanh(a + b\,x)\,(c - \mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + tanh(a + b*x)*(c - 1i)),x)
[Out] int(x*acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.198 $\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\tanh(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arccot(c-(I-c)*tanh(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A]

time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5296, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\cot^{-1}(c-(-c+i)\tanh(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcCot[c - (I - c)*Tanh[a + b*x]] + (I/2)*x*Log[1 - I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5296

```
Int[ArcCot[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Cot[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c-(i-c)\tanh(a+bx))\,dx = x\cot^{-1}(c-(i-c)\tanh(a+bx)) + b\int \frac{x}{i+ce^{2a+2bx}}\,dx$$

$$= -\frac{1}{2}ibx^2 + x\cot^{-1}(c-(i-c)\tanh(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,d$$

$$= -\frac{1}{2}ibx^2 + x\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$

$$= -\frac{1}{2}ibx^2 + x\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$

$$= -\frac{1}{2}ibx^2 + x\cot^{-1}(c-(i-c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$

Mathematica [A]
time = 0.40, size = 71, normalized size = 0.87

$$x\cot^{-1}(c+(-i+c)\tanh(a+bx)) + \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(68) = 136.
time = 0.63, size = 554, normalized size = 6.76

method	result

derivativedivides	$\frac{-\frac{\operatorname{arccot}((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}((c-i)\tanh}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}((c-i)\tanh}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arccot((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)
-2*I*arccot((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c+arcc
ot((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c^2+arccot((c-I
)*tanh(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)+2*I*arccot((c-I)*tanh
(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)*c-arccot((c-I)*tanh(b*x+a)+
c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)*c^2-(I-c)^2*(-1/8*I/(I-c)*ln(-I+(c-
I)*tanh(b*x+a)+c)^2+1/4*I/(I-c)*ln(-I+(c-I)*tanh(b*x+a)+c)*ln(-1/2*I*((c-I)
*tanh(b*x+a)+c+I))+1/4*I/(I-c)*dilog(-1/2*I*((c-I)*tanh(b*x+a)+c+I))-1/4*I/
(I-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(1/2*((c-I)*tanh(b*x+a)+c+I)/c)-1/4*I/(I-
c)*dilog(1/2*((c-I)*tanh(b*x+a)+c+I)/c)+1/4*I/(I-c)*ln((c-I)*tanh(b*x+a)-c+
I)*ln((-I+(c-I)*tanh(b*x+a)+c)/(-2*I+2*c))+1/4*I/(I-c)*dilog((-I+(c-I)*tanh
(b*x+a)+c)/(-2*I+2*c))))
```

Maxima [A]
time = 1.21, size = 80, normalized size = 0.98

$$2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\operatorname{arccot}\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*tanh(b*x + a
) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.
time = 2.15, size = 186, normalized size = 2.27

$$\frac{-i\,b^2x^2+i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I)
) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x + I
*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*di
log(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) + I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,ex
p(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(c + \tanh(a + b\,x)\ (c - \mathrm{i}))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(acot(c + tanh(a + b*x)*(c - 1i)), x)
```

3.199 $\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx=\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c-(i-c)\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
[Out] int(arccot(c-(I-c)*tanh(b*x+a))/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] I*b*x - 1/4*(-4*I*a - 2*arctan(1/c) - I*log(c^2 + 1))*log(x) + 1/2*integrat
e(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x + 4*a
) + 1)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(I-c)*tanh(b*x+a))/x,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot((c - I)*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acot}(c+\tanh(a+b\,x)\,(c-\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + tanh(a + b*x)*(c - 1i))/x,x)
[Out] int(acot(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.200 $\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=299

$$\frac{(e+fx)^4\cot^{-1}(\coth(a+bx))}{4f}-\frac{(e+fx)^4\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{i(e+fx)^3\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^3\mathrm{P}}{}$$

```
[Out] 1/4*(f*x+e)^4*arccot(coth(b*x+a))/f-1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-3/16*
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A]

time = 0.16, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5294, 4265, 2611, 6744, 2320, 6724}

$$-\frac{(e+fx)^4\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{4f}-\frac{3if^3\mathrm{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}+\frac{3if^3\mathrm{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}+\frac{3if^2(e+fx)\mathrm{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx)\mathrm{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2\mathrm{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}+\frac{3if(e+fx)^2\mathrm{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)^3\mathrm{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^3\mathrm{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^4\cot^{-1}(\coth(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCot[Coth[a + b*x]])/(4*f) - ((e + f*x)^4*ArcTan[E^(2*a + 2*
b*x)])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \cot^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{b\int (e+fx)^4 \mathrm{sech}(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{1}{2}i\int (e$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+f}{}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+f}{}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+f}{}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+f}{}$$

$$= \frac{(e+fx)^4 \cot^{-1}(\coth(a+bx))}{4f} - \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{i(e+f}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.
time = 0.21, size = 600, normalized size = 2.01

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCot[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCot[Coth[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.67, size = 7275, normalized size = 24.33

method	result	size
risch	Expression too large to display	7275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*f^2*x
^3*e^(2*a + 1) + 6*b*f*x^2*e^(2*a + 2) + 4*b*x*e^(2*a + 3))*e^(2*b*x)/(e^(4
*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(244) = 488.
time = 1.37, size = 2700, normalized size = 9.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^4
*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 +
 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*arctan(sinh(b*x + a)/cos
h(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh
(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sin
h(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dil
og(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*
b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(
```

```
1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x
*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*c
osh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*
sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*
dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 -
 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*
sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^
3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a
) + sinh(b*x + a))) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x + a*b^3)*cos
h(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 - 4
*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^
4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 + I*a^4*
f^3 - 4*I*(b^4*x + a*b^3)*cosh(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(
b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x +
a*b^3)*cosh(1))*sinh(1)^2 - 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^
4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^
2*f)*cosh(1))*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x + a*b^3)*cosh(1)^3 + 4*I*(b^4
*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*I*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*I*(b^4*f^2*x^3 +
 a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x
+ a*b^3)*cosh(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b
^2*f)*cosh(1)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*s
inh(1)^2 + 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b
*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sin
h(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*a^4*f^
3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)^3 +
 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*I*(a
^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^4*f^3 + 4*I*a^3*b*f^2*cosh(1
) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)^3 + 4*I*a*b^3*sinh(1)^3 - 6
*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*c
osh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + (I*a^4*f^3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1
)^2 - 4*I*a*b^3*cosh(1)^3 - 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*
cosh(1))*sinh(1)^2 - 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)
^2)*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^4
*f^3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^
3 - 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I
*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt
```

```
(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(I*b*f^3*x + I*b*f^2*cosh(
1) + I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 24*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, -1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1
) - I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -1/2*
sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*
f^2*x*cosh(1) - I*b^2*f*cosh(1)^2 - I*b^2*f*sinh(1)^2 - 2*I*(b^2*f^2*x + b^
2*f*cosh(1))*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*f^2*x*cosh(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{acot}(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)**3*acot(coth(b*x+a)),x)

[Out] Integral((e + f*x)**3*acot(coth(a + b*x)), x)

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^3*arccot(coth(b*x+a)),x, algorithm="giac")

[Out] Timed out

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acot}(\coth(a+b\,x))\,{(e+f\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acot(coth(a + b*x))*(e + f*x)^3,x)

[Out] int(acot(coth(a + b*x))*(e + f*x)^3, x)

3.201 $\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=229

$$\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}-\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^2\text{F}}{}$$

```
[Out] 1/3*(f*x+e)^3*arccot(coth(b*x+a))/f-1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A]

time = 0.12, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5294, 4265, 2611, 6744, 2320, 6724}

$$-\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}+\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^2\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCot[Coth[a + b*x]])/(3*f) - ((e + f*x)^3*ArcTan[E^(2*a + 2*
b*x)])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \cot^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{b\int (e+fx)^3 \text{sech}(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{1}{2}i\int (e$$
$$= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+f}{}$$
$$= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+f}{}$$
$$= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+f}{}$$
$$= \frac{(e+fx)^3\cot^{-1}(\coth(a+bx))}{3f} - \frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{i(e+f}{}$$

<u>**Mathematica**</u> [**A**]
time = 0.13, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\cot^{-1}(\coth(a+bx)) - \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)-12b^3efx^2\log\left(1+ie^{2(a+bx)}\right)-4b^3f^2x^3\log\left(1+ie^{2(a+bx)}\right)-6b^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+6b^2(e+fx)^2\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+6bef\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)+6bf^2x\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-6bef\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-6bf^2x\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-3f^2\text{PolyLog}\left(4,-ie^{2(a+bx)}\right)+3f^2\text{PolyLog}\left(4,ie^{2(a+bx)}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCot[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCot[Coth[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

<u>**Maple**</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.64, size = 5425, normalized size = 23.69

method	result	size
risch	Expression too large to display	5425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*f*x^2*e^(2*a + 1) + 3
*b*x*e^(2*a + 2))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1511 vs. $2(186) = 372$.
time = 1.98, size = 1511, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 +
 2*b^3*x*cosh(1))*sinh(1))*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I*b^2*f
^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*s
inh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^
2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b
^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cos
h(1))*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-I
*b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b
^2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*
f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cos
h(1)^2 - 3*I*(b^3*x + a*b^2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1)
- 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*s
```

```
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + I*a^3*f^2
+ 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b^2)*sinh(1)^2 + 3*I*(b^3*
f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cos
h(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I
*b^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b
^2)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*
f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cosh(1
)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*
sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^3*f^2 - 3*I*a^2*b*f*c
osh(1) + 3*I*a*b^2*cosh(1)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2
*cosh(1))*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
(-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*cosh(1)^2 - 3*I*a*b^2*sinh(1)
^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x
 + a) + 2*sinh(b*x + a)) + (-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*co
sh(1)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*lo
g(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(-I*b*f^2*x - I*b*
f*cosh(1) - I*b*f*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b
*x + a))) - 6*(-I*b*f^2*x - I*b*f*cosh(1) - I*b*f*sinh(1))*polylog(3, -1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(I*b*f^2*x + I*b*f*cosh(1) +
 I*b*f*sinh(1))*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 6*(I*b*f^2*x + I*b*f*cosh(1) + I*b*f*sinh(1))*polylog(3, -1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \operatorname{acot}(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acot(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*acot(coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccot(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acot}(\coth(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(coth(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(acot(coth(a + b*x))*(e + f*x)^2, x)
```

3.202 $\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=159

$$\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}-\frac{(e+fx)^2\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{i(e+fx)\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\mathrm{Po}}{}$$

```
[Out] 1/2*(f*x+e)^2*arccot(coth(b*x+a))/f-1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5294, 4265, 2611, 2320, 6724}

$$-\frac{(e+fx)^2\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{2f}-\frac{if\mathrm{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}+\frac{if\mathrm{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\mathrm{Li}_2\left(ie^{2a+2bx}\right)}{4b}+\frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCot[Coth[a + b*x]])/(2*f) - ((e + f*x)^2*ArcTan[E^(2*a + 2*
b*x)])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5294

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCot[Coth[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)\cot^{-1}(\coth(a+bx))\,dx &= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{b\int (e+fx)^2\operatorname{sech}(2a+2bx)\,dx}{2f}\\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{1}{2}i\int (e+\\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{i(e+fx)}{}\\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{i(e+fx)}{}\\
&= \frac{(e+fx)^2\cot^{-1}(\coth(a+bx))}{2f} - \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{i(e+fx)}{}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 278, normalized size = 1.75

$$ex\cot^{-1}(\coth(a+bx))+\frac{1}{2}fx^2\cot^{-1}(\coth(a+bx))-\frac{e\left(-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)\right)}{8b}-\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(3,ie^{2(a+bx)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcCot[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcCot[Coth[a + b*x]] + (f*x^2*ArcCot[Coth[a + b*x]])/2 - (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
```

```
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) - ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.60, size = 2688, normalized size = 16.91

method	result	size
risch	Expression too large to display	2688

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)-1))^2-1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))
^3+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3-1/4*Pi*x*e*cs
gn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-1/4*Pi*x*e*csgn((1-I)*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/
4))+1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e*dilog(((-I)^(1/2)+ex
p(b*x+a))/(-I)^(1/2))-1/2*I/b*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1
/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*ln(exp(2*b*x+2*a)-I)*f*x^2-1/
2*I*ln(exp(2*b*x+2*a)-I)*e*x+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*
b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/2*I*f/b^
2*a^2*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a^2*ln(1-exp(b*x+a)*(-1)^(3/4
))+1/2*I/b^2*f*a^2*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b^2*f*a^2*l
n(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I/b*ln(((-I)^(1/2)+exp(b*x+a))/(-
I)^(1/2))*a*e-1/2*I/b*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*a*e-1/4*I/b^2*
f*polylog(2,I*exp(2*b*x+2*a))*a-1/4*I/b^2*f*ln(1-I*exp(2*b*x+2*a))*a^2-1/4*
I/b*f*polylog(2,I*exp(2*b*x+2*a))*x+1/4*I*f/b*polylog(2,-I*exp(2*b*x+2*a))*
x+1/4*I*f/b^2*polylog(2,-I*exp(2*b*x+2*a))*a+1/2*I*e/b*ln(1+exp(b*x+a)*(-1)
^(3/4))*a+1/2*I*e/b*ln(1-exp(b*x+a)*(-1)^(3/4))*a+1/4*I*f/b^2*ln(1+I*exp(2*
b*x+2*a))*a^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*cs
gn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I*(exp(2*
b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)-1))+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))
*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b
*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x
+2*a)-1))+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1)
)*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*f*csgn(I*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)-1))+1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*Pi*f*x^2+1/4*Pi*e*x-1/2*
I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)^(3/4))+1/2*I/b^2*f*a*dilog(((-I)^(1/2)+ex
```

```
p(b*x+a))/(-I)^(1/2))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-
1))^3+1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*
Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b^2*f*a*di
log(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*a*dilog(1+exp(b*x+a)*(-
1)^(3/4))+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(
exp(2*b*x+2*a)-1))^2+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2*a)+I)-1/8*Pi*x^2*
f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^
2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(
2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(e
xp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-
1))^3-1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+1/8*
Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csg
n((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b*f*a*ln(((-I)^(1/2)
+exp(b*x+a))/(-I)^(1/2))*x-1/2*I/b*f*ln(1-I*exp(2*b*x+2*a))*x*a+1/2*I/b*f*a
*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x+1/2*I*f/b*ln(1+I*exp(2*b*x+2*a))*
x*a-1/2*I*f/b*a*ln(1+exp(b*x+a)*(-1)^(3/4))*x-1/2*I*f/b*a*ln(1-exp(b*x+a)*(
-1)^(3/4))*x+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn(
(1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x
+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-
1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*
b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp
(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(
exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/8*
Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*
x+2*a)-I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)-1))*csgn(
I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2
*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b*e*a*ln(exp(
2*b*x+2*a)+I)+1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)-1/2*I*ln(((-I)^(1/2)+ex
p(b*x+a))/(-I)^(1/2))*x*e-1/2*I*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x*e-
1/4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2-1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)-1/2
*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/4*I*f*ln(1+I*exp(2*b*x+2*a))*x^2+1/2*I*e*l
n(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e*ln(1-exp(b*x+a)*(-1)^(3/4))*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*x*e^(2*a + 1))*e^(2*b*x)/(e^(4*b*x + 4*a) +
 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(134) = 268$.
time = 3.83, size = 717, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*arctan(sinh(b*x + a)
/cosh(b*x + a)) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1
))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I*b*f*x - I*
b*cosh(1) - I*b*sinh(1))*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*co
sh(1) - 2*I*(b^2*x + a*b)*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*
(b^2*x + a*b)*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b
)*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2
*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b)*sinh(1))*l
og(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*a^2*f + 2*I*a
*b*cosh(1) + 2*I*a*b*sinh(1))*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (-I*a^2*f + 2*I*a*b*cosh(1) + 2*I*a*b*sinh(1))*log(-I*sqrt(4*I) +
 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^2*f - 2*I*a*b*cosh(1) - 2*I*a*b*
sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^2*f -
 2*I*a*b*cosh(1) - 2*I*a*b*sinh(1))*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) + 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 2*I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*p
olylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{acot}(\coth(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acot(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acot(coth(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccot(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(\coth(a+b\,x))\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(coth(a + b*x))*(e + f*x),x)
```

```
[Out] int(acot(coth(a + b*x))*(e + f*x), x)
```

3.203 $\int \cot^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=74

$$x\cot^{-1}(\coth(a+bx))-x\text{ArcTan}\left(e^{2a+2bx}\right)+\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arccot(coth(b*x+a))-x*arctan(exp(2*b*x+2*a))+1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5290, 4265, 2317, 2438}

$$-x\text{ArcTan}\left(e^{2a+2bx}\right)+\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}+x\cot^{-1}(\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[Coth[a + b*x]] - x*ArcTan[E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5290

```
Int[ArcCot[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCot[Coth[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \cot^{-1}(\coth(a+bx))\,dx = x\cot^{-1}(\coth(a+bx)) - b\int x\text{sech}(2a+2bx)\,dx$$
$$= x\cot^{-1}(\coth(a+bx)) - x\tan^{-1}\left(e^{2a+2bx}\right) + \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i\int$$
$$= x\cot^{-1}(\coth(a+bx)) - x\tan^{-1}\left(e^{2a+2bx}\right) + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} -$$
$$= x\cot^{-1}(\coth(a+bx)) - x\tan^{-1}\left(e^{2a+2bx}\right) + \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} - \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A]
time = 0.02, size = 132, normalized size = 1.78

$$x\cot^{-1}(\coth(a+bx)) - \frac{-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[Coth[a + b*x]] - (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(63) = 126.
time = 0.68, size = 184, normalized size = 2.49

method	result
derivativedivides	$\frac{\text{arctanh}(\coth(bx+a))\text{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\,\text{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\coth(}{\coth^2(bx}\right.}{2}}{b}$
default	$\frac{\text{arctanh}(\coth(bx+a))\text{arccot}(\coth(bx+a))+\arctan(\coth(bx+a))\,\text{arctanh}(\coth(bx+a))+\frac{\arctan(\coth(bx+a))\ln\left(1+\frac{i(1+i\coth(}{\coth^2(bx}\right.}{2}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arccot(coth(b*x+a))+arctan(coth(b*x+a))*arctanh(c
oth(b*x+a))+1/2*arctan(coth(b*x+a))*ln(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+a)
^2+1))-1/2*arctan(coth(b*x+a))*ln(1-I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1)
)-1/4*I*dilog(1+I*(1+I*coth(b*x+a))^2/(coth(b*x+a)^2+1))+1/4*I*dilog(1-I*(1
+I*coth(b*x+a))^2/(coth(b*x+a)^2+1)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(57) = 114$.
time = 2.32, size = 334, normalized size = 4.51

$$\frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (-i\,bx - i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + i\,a\log\left(i\sqrt{4i} + 2\cosh(bx+a) + 2\sinh(bx+a)\right) + i\,a\log\left(-i\sqrt{4i} + 2\cosh(bx+a) + 2\sinh(bx+a)\right) - i\,a\log\left(i\sqrt{-4i} + 2\cosh(bx+a) + 2\sinh(bx+a)\right) - i\,a\log\left(-i\sqrt{-4i} + 2\cosh(bx+a) + 2\sinh(bx+a)\right) - i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) - i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) + i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) + i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(\coth\left(a+bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(coth(b*x+a)),x)
```

```
[Out] Integral(acot(coth(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(coth(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(\coth(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(coth(a + b*x)),x)
```

```
[Out] int(acot(coth(a + b*x)), x)
```

3.204 $\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\cot^{-1}(\coth(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccot(coth(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcCot[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[Coth[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcCot[Coth[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(\coth(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(coth(b*x+a))/(f*x+e),x)
[Out] int(arccot(coth(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccot(coth(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccot(coth(b*x + a))/(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(coth(b*x+a))/(f*x+e),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(coth(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acot}(\coth(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(coth(a + b*x))/(e + f*x),x)
[Out] int(acot(coth(a + b*x))/(e + f*x), x)
```

3.205 $\int x^2 \cot^{-1}(c + d \coth(a + bx))\, dx$

Optimal. Leaf size=351

$$\frac{1}{3}x^3 \cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3 \log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{6}ix^3 \log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{ix^2\mathrm{Po}}{}$$

```
[Out] 1/3*x^3*arccot(c+d*coth(b*x+a))-1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x^2*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I
+c-d))/b+1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/4*I*x*poly
log(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2-1/8*I*polylog(4,(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b^3+1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^3
```

Rubi [A]
time = 0.34, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5310, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\mathrm{Li}_4\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}+\frac{i\mathrm{Li}_4\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}+\frac{ix\mathrm{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}-\frac{ix\mathrm{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}-\frac{ix^2\mathrm{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}+\frac{ix^2\mathrm{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)+\frac{1}{6}ix^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)+\frac{1}{3}x^3\cot^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b + ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b + ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 - (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 - ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 + ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5310

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cot^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))+\frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+2}}{i+c-d+(-i} \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+ \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+ \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+ \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+ \\
&= \frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+
\end{aligned}
$$

Mathematica [A]
time = 3.40, size = 299, normalized size = 0.85

$$
\frac{1}{3}x^3\cot^{-1}(c+d\coth(a+bx))-\frac{i\left(4b^3x^3\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{i-c+d}\right)-4b^3x^3\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{-i-c+d}\right)+6b^2x^2\text{PolyLog}\left(2,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-6b^2x^2\text{PolyLog}\left(2,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)-6bx\text{PolyLog}\left(3,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)+6bx\text{PolyLog}\left(3,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)+3\text{PolyLog}\left(4,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-3\text{PolyLog}\left(4,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{24b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + d*Coth[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(I - c + d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(-I - c + d)] + 6*b^2*x^2*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x))
)/(-I + c - d)] - 6*b^2*x^2*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c
 - d)] - 6*b*x*PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + 6*
b*x*PolyLog[3, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] + 3*PolyLog[4, ((
-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] - 3*PolyLog[4, ((I + c + d)*E^(2
*(a + b*x)))/(I + c - d)]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 26.91, size = 6888, normalized size = 19.62

method	result	size
risch	Expression too large to display	6888

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
+ d) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(259) = 518.
time = 2.26, size = 1269, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^
2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^3*log(2*(c^2 + 2
*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2
*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
 d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d
 + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(
b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d +
 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^3*log(2*(c
^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x +
a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1
```

```
))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt((c^2 - d^2 -
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (-I
*b^3*x^3 - I*a^3)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(-sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
) + 1) + (I*b^3*x^3 + I*a^3)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 6*I*polylog(4, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + 6*I*polylog(4, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acot}(c+d\,\mathrm{coth}(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*acot(c + d*coth(a + b*x)), x)
```

3.206 $\int x \cot^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=265

$$\frac{1}{2}x^2 \cot^{-1}(c+d\coth(a+bx)) - \frac{1}{4}ix^2 \log\left(1 - \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{4}ix^2 \log\left(1 - \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) - \frac{ix\text{Poly}}{}$$

```
[Out] 1/2*x^2*arccot(c+d*coth(b*x+a))-1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))+1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*x*polylog(2,(I-c-d
)*exp(2*b*x+2*a)/(I-c+d))/b+1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d
))/b+1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2-1/8*I*polylog(3,(I
+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A]

time = 0.30, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5310, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2} - \frac{i\text{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2} - \frac{ix\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b} + \frac{ix\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b} - \frac{1}{4}ix^2\log\left(1 - \frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right) + \frac{1}{4}ix^2\log\left(1 - \frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right) + \frac{1}{2}x^2\cot^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] + (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] - ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b + ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b +
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 - ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5310

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\cot^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))+\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2b}}{i+c-d+(-i\cdots}\\
&= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\cdots\\
&= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\cdots\\
&= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\cdots\\
&= \frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\cdots
\end{aligned}$$

Mathematica [A]
time = 2.70, size = 225, normalized size = 0.85

$$\frac{1}{2}x^2\cot^{-1}(c+d\coth(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{i-c+d}\right)-2b^2x^2\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{-i-c+d}\right)+2bx\text{PolyLog}\left(2,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2bx\text{PolyLog}\left(2,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)-\text{PolyLog}\left(3,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)+\text{PolyLog}\left(3,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + d*Coth[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(I - c + d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a +
 b*x)))/(-I - c + d)] + 2*b*x*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x)))/(-I
 + c - d)] - 2*b*x*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] -
PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + PolyLog[3, ((I +
c + d)*E^(2*(a + b*x)))/(I + c - d)]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.31, size = 6508, normalized size = 24.56

method	result	size
risch	Expression too large to display	6508

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c
+ d) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(195) = 390.
time = 3.58, size = 1051, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) -
2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
```

```
*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(sqrt
((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*
cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 - 2*
I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b
*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^
2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 -
d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log(2*(c^2 + 2*c*d + d^2
 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^
2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (-I
*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(-sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
) + 1) + (I*b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 2*I*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*I*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(d*coth(b*x + a) + c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acot}(c + d \coth(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + d*coth(a + b*x)),x)
```

```
[Out] int(x*acot(c + d*coth(a + b*x)), x)
```

3.207 $\int \cot^{-1}(c+d\coth(a+bx))\,dx$

Optimal. Leaf size=174

$$x\cot^{-1}(c+d\coth(a+bx))-\frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)+\frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)-\frac{i\text{PolyLog}}{}$$

```
[Out] x*arccot(c+d*coth(b*x+a))-1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))+1/2*
I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))-1/4*I*polylog(2,(I-c-d)*exp(2*b*x+
2*a)/(I-c+d))/b+1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5302, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}+\frac{i\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}-\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)+\frac{1}{2}ix\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)+x\cot^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Coth[a + b*x]] - (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] + (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] - ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5302

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*x
)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b*(I + c + d)
, Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \cot^{-1}(c+d\coth(a+bx))\,dx = x\cot^{-1}(c+d\coth(a+bx)) + (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(-i-c-d)}$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log$$

$$= x\cot^{-1}(c+d\coth(a+bx)) - \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) + \frac{1}{2}ix\log$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 363 vs. $2(174) = 348$.
time = 0.78, size = 363, normalized size = 2.09

$$x\cot^{-1}(c+d\coth(a+bx)) - \frac{i\left(2ia\,\mathrm{ArcTan}\left(\frac{-1-c^2+d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+(a+bx)\log\left(1-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)+(a+bx)\log\left(1+\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)-(a+bx)\log\left(1-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)-(a+bx)\log\left(1+\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)+\mathrm{PolyLog}\left(2,-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)+\mathrm{PolyLog}\left(2,\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)-\mathrm{PolyLog}\left(2,-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)-\mathrm{PolyLog}\left(2,\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + d*Coth[a + b*x]] - ((I/2)*((2*I)*a*ArcTan[(-1 - c^2 + d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + (a + b*x)*Log[1 - (Sqrt[-I
+ c + d]*E^(a + b*x))/Sqrt[-I + c - d]] + (a + b*x)*Log[1 + (Sqrt[-I + c +
d]*E^(a + b*x))/Sqrt[-I + c - d]] - (a + b*x)*Log[1 - (Sqrt[I + c + d]*E^(a
 + b*x))/Sqrt[I + c - d]] - (a + b*x)*Log[1 + (Sqrt[I + c + d]*E^(a + b*x))
/Sqrt[I + c - d]] + PolyLog[2, -((Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[-I + c
 - d])] + PolyLog[2, (Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[-I + c - d]] - Pol
yLog[2, -((Sqrt[I + c + d]*E^(a + b*x))/Sqrt[I + c - d])] - PolyLog[2, (Sqr
t[I + c + d]*E^(a + b*x))/Sqrt[I + c - d]]))/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(150) = 300$.
time = 0.96, size = 365, normalized size = 2.10

method	result
derivativedivides	$-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln(}{2d}\right.}{}$
default	$-\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccot}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln(}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arccot(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccot(c+d*c
oth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*d^2*(1/2*I/d*ln(-d*coth(b*x+a)+d)*ln
((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I/d*ln(-d*coth(b*x+a)+d)*ln((I-d*coth(b*x
+a)-c)/(I-c-d))+1/2*I/d*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I/d*dilog((I
-d*coth(b*x+a)-c)/(I-c-d))-1/2*I/d*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(b*x+a)
+c)/(I+c-d))+1/2*I/d*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+d))-1
/2*I/d*dilog((I+d*coth(b*x+a)+c)/(I+c-d))+1/2*I/d*dilog((I-d*coth(b*x+a)-c)
/(I-c+d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2(e^(2*b*x + 2*a) - 1, (c*e^(2*a)
+ d*e^(2*a))*e^(2*b*x) - c + d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(128) = 256.
time = 3.83, size = 813, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/(d*cosh(b*x + a) + c*sinh(b*x + a))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c
^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2
+ 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*lo
g(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-sqrt((c^2 - d^2 + 2*I*d +
 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(sqr
t((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + I*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(c+d\coth(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*coth(a + b*x)),x)
[Out] int(acot(c + d*coth(a + b*x)), x)
```

3.208 $\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\cot^{-1}(c+d\coth(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arccot(c+d*coth(b*x+a))/x,x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcCot[c + d*Coth[a + b*x]]/x,x]

[Out] Defer[Int][ArcCot[c + d*Coth[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.84, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCot[c + d*Coth[a + b*x]]/x,x]

[Out] Integrate[ArcCot[c + d*Coth[a + b*x]]/x, x]

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c+d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+d*coth(b*x+a))/x,x)
[Out] int(arccot(c+d*coth(b*x+a))/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arccot(d*coth(b*x + a) + c)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+d*coth(b*x+a))/x,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccot(d*coth(b*x + a) + c)/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acot}(c+d\coth(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + d*coth(a + b*x))/x,x)
[Out] int(acot(c + d*coth(a + b*x))/x, x)
```

3.209 $\int x^2 \cot^{-1}(c + (i + c)\coth(a + bx))\,dx$

Optimal. Leaf size=142

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\text{PolyLog}(3,}{4b^2}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccot(c+(I+c)*coth(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*ex
p(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5306, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCot[c + (I + c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
- I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((
I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a
+ 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{-i-ce^{2a+2bx}}\,dx$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{3}(ibc)\int \frac{e^{2a+2bx}}{-i-ce^{2}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a}\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a}\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a}\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a}\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice^{2a}\right.$$

Mathematica [A]
time = 0.15, size = 128, normalized size = 0.90

$$\frac{1}{3}x^3\cot^{-1}(c+(i+c)\coth(a+bx)) - \frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (I + c)*Coth[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*x
*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x)
))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.90, size = 1472, normalized size = 10.37

method	result	size
risch	Expression too large to display	1472

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1
))^3-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/4*I*
x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+
```

```
2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/12*Pi*x^3*csgn
((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*
csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/12*b/(
I+c)*x^4+1/12/b^3/(I+c)*a^4-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))+1/12*Pi*x^3*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2
*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)-1
/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^
2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/
12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2
*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(2*exp(
2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*
b*x+2*a)-1))-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1)
)*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I*(2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/2*I/b^3*a^2*dilog(1-I*exp
(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/3*
I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*
a^2-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(
b*x+a)*(-I*c)^(1/2))+1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/6*I*x^3*ln
(2*exp(2*b*x+2*a)*c+2*I)-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/
2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/12*Pi*x^3*csgn(I*(2*exp(2*b
*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*x*a
^2+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/12*I*b
*c/(I+c)*x^4-1/12*I/b^3*c/(I+c)*a^4
```

Maxima [A]
time = 1.29, size = 129, normalized size = 0.91

$$\frac{1}{3}\,x^{3}\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c-4}-\frac{4\,b^{3}x^{3}\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_{4}\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^{4}(-i\,c+1)}\right)b(c+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c + I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

time = 2.72, size = 292, normalized size = 2.06

$$\frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c+i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - i\,a^4 + 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right) + 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) + 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - 2\left(i\,b^3x^3+i\,a^3\right)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right) - 2\left(i\,b^3x^3+i\,a^3\right)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right) - 12i\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - 12i\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/
(c + I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*dil
og(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3, -
1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*I*c)*e^
(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1
) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -1/2*sqr
t(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.210 $\int x \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=113

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}(3,ic$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccot(c+(I+c)*coth(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2*b
*x+2*a))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5306, 2215, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCot[c + (I + c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(c+(i+c)\coth(a+bx))\,dx &= \frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{-i-ce^{2a+2bx}}\,dx \\
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x^{
-i-ce^{2a+
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2
&= \frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 0.90

$$
\frac{1}{2}x^2\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (I + c)*Coth[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.61, size = 1436, normalized size = 12.71

method	result	size
risch	Expression too large to display	1436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+
I)-1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2
*I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I
*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-
1))^2+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(
(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*I*polylog
(3,I*c*exp(2*b*x+2*a))/b^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1
))-1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2,I*c*exp(2*b*x
+2*a))*a+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I
*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/
2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a
))*x*a+1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(b*x
+a)*(-I*c)^(1/2))*x+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*
c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*
b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*
a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)+1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+2*I)-1/6*b/(I+c)*x^3-1/6/b^2/(I+c)
*a^3+1/6*I*b*c/(I+c)*x^3+1/6*I/b^2*c/(I+c)*a^3+1/8*Pi*x^2*csgn((2*exp(2*b*x
+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*
I)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+
2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-1/8*
Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*
```

```
x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))
```

Maxima [A]
time = 1.21, size = 107, normalized size = 0.95

$$-\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\,\mathrm{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arccot((c + I)*coth(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.
time = 2.07, size = 246, normalized size = 2.18

$$\frac{2i\,b^3x^3+3i\,b^2x^2\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-3\,(i\,b^2x^2-i\,a^2)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-3\,(i\,b^2x^2-i\,a^2)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a
)/(c + I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*x
*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) + I
*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - 3
*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x^2 -
I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqrt(4*I*
c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + coth(a + b*x)*(c + 1i)),x)
[Out] int(x*acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.211 $\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccot(c+(I+c)*coth(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5298, 2215, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+x\cot^{-1}(c+(c+i)\coth(a+bx))+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCot[c + (I + c)*Coth[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5298

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^{-1}(c+(i+c)\coth(a+bx))\,dx &= x\cot^{-1}(c+(i+c)\coth(a+bx))+b\int \frac{x}{-i-ce^{2a+2bx}}\,dx\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))+(ibc)\int \frac{e^{2a+2bx}x}{-i-ce^{2a+2bx}}\,d\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)\\
&= \frac{1}{2}ibx^2+x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 71, normalized size = 0.90

$$
x\cot^{-1}(c+(i+c)\coth(a+bx))-\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(65) = 130$.
time = 0.70, size = 597, normalized size = 7.56

method	result

derivativedivides	$\frac{-\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(i+c+(i+c)\coth(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(i+c+(i+c)\coth(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i+c)}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(i+c+(i+c)\coth(bx+a))}{2i+2c}+\frac{2i\operatorname{arccot}(c+(i+c)\coth(bx+a))\ln(i+c+(i+c)\coth(bx+a))c}{2i+2c}+\frac{\operatorname{arccot}(c+(i+c)}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(-arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))
+2*I*arccot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arcc
ot(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2+arccot(c+(I
+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)-2*I*arccot(c+(I+c)*cot
h(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arccot(c+(I+c)*coth(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2+(I+c)^2*(-1/4*I/(I+c)*ln(I+c+(I+c)
*coth(b*x+a))*ln(-1/2*I*(I-c-(I+c)*coth(b*x+a)))+1/4*I/(I+c)*ln(-1/2*I*(I+c
+(I+c)*coth(b*x+a)))*ln(-1/2*I*(I-c-(I+c)*coth(b*x+a)))+1/4*I/(I+c)*dilog(-
1/2*I*(I+c+(I+c)*coth(b*x+a)))+1/8*I/(I+c)*ln(I+c+(I+c)*coth(b*x+a))^2-1/4*
I/(I+c)*ln(c-(I+c)*coth(b*x+a)+I)*ln((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))-1
/4*I/(I+c)*dilog((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))+1/4*I/(I+c)*ln(c-(I+c
)*coth(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*coth(b*x+a))/c)+1/4*I/(I+c)*dilog(-1/2*
(I-c-(I+c)*coth(b*x+a))/c)))
```

Maxima [A]
time = 1.27, size = 80, normalized size = 1.01

$$-2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\,\operatorname{arccot}\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arccot((c + I)*coth(b*x +
a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.
time = 2.73, size = 186, normalized size = 2.35

$$\frac{i\,b^2x^2+i\,bx\log\left(\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c+i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I)
) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x -
 I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I*
dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))
/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**4*c**2*exp(4*a) +
 _t0**4*I*c*exp(4*a) + 3*_t0**2*I*c*exp(2*a) - _t0**2*exp(2*a) - 1 of type
<class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acot}(c + \coth(a + b\,x)\ (c + 1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c + 1i)), x)
```

3.212 $\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.41, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c + (I + c)*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccot}(c+(i+c)\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] int(arccot(c+(I+c)*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-4*I*a + 2*arctan(1/c) - I*log(c^2 + 1))*log(x) - 1/2*integra
te(arctan(e^(-2*b*x - 2*a)/c)/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x + 4*
a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log((c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c + I))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c + I)*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acot}(c+\coth(a+b\,x)\ (c+1\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + coth(a + b*x)*(c + 1i))/x,x)
[Out] int(acot(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.213 $\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=145

$$-\frac{1}{12}ibx^4+\frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\text{PolyL}}{}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arccot(c-(I-c)*coth(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5306, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{3}x^3\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcCot[c - (I - c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log
[1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/
b - ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-
I)*c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+}}{i-c}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1+i\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1+i\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1+i\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1+i\right.$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\cot^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1+i\right.$$

Mathematica [A]
time = 0.15, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\cot^{-1}(c+(-i+c)\coth(a+bx)) + \frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCot[c + (-I + c)*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 - I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*P
olyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^
3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.92, size = 1487, normalized size = 10.26

method	result	size
risch	Expression too large to display	1487

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))^2-1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/3*I/b^3*ln(1+I*c*exp(2*b
*x+2*a))*a^3-1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2+1/2*I/b^3*a^3*ln(
```

```
1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)^(1/2))+1/
2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b
*x+a)*(I*c)^(1/2))-1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)-1))^3+1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/12*Pi*x^3*csgn
(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/12/b^3/
(I-c)*a^4+1/12*b/(I-c)*x^4-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)-1))^2-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*Pi*x^3*csgn(I*(2
*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(e
xp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)-1))^2+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(
4,-I*c*exp(2*b*x+2*a))/b^3-1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c+2*I)+1/6*I*x^3*
ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/3*Pi*x^3+1/2*I/b^2*a^2*ln(1-I*e
xp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*a^2+1/2*I/b^2
*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a
)-1))^3-1/12*I/b^3*c*a^4/(I-c)+1/12*I*b*c/(I-c)*x^4+1/12*Pi*x^3*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1
))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x+
2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+
2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))
```

Maxima [**A**]
time = 1.23, size = 129, normalized size = 0.89

$$\frac{1}{3}\,x^{3}\,\mathrm{arccot}\left(\left(c-i\right)\coth\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c+4}-\frac{4\,b^{3}x^{3}\log\left(i\,ce^{\left(2\,bx+2\,a\right)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_2\left(-i\,ce^{\left(2\,bx+2\,a\right)}\right)-6\,bx\mathrm{Li}_3\left(-i\,ce^{\left(2\,bx+2\,a\right)}\right)+3\,\mathrm{Li}_4\left(-i\,ce^{\left(2\,bx+2\,a\right)}\right)}{-2\,b^{4}\left(-i\,c-1\right)}\right)b\left(c-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccot((c - I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(105) = 210$.

time = 2.51, size = 292, normalized size = 2.01

$$\frac{-i\,b^4x^4+2i\,b^3x^3\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+i\,a^4-2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)-2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-2\,(-i\,b^3x^3-i\,a^3)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-2\,(-i\,b^3x^3-i\,a^3)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+12i\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+12i\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*
a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog
(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-4
*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*
x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polylog(
4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c-\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.214 $\int x \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=116

$$-\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}(3,$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arccot(c-(I-c)*coth(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5306, 2215, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}x^2\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcCot[c - (I - c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[
1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b -
 ((I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5306

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCot[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\cot^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx$$

$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2b}}{i-ce^{2a}}$$

$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice\right.$$

$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice\right.$$

$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice\right.$$

$$= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1+ice\right.$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\cot^{-1}(c+(-i+c)\coth(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCot[c + (-I + c)*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I
/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.81, size = 1451, normalized size = 12.51

method	result	size
risch	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+1/2*I/b*ln(1+I*c*exp(2*b*x
+2*a))*x*a-1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(
b*x+a)*(I*c)^(1/2))*x+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(ex
p(2*b*x+2*a)-1))-1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/4*I*x^2*ln(1+I*
c*exp(2*b*x+2*a))-1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-
1))^2+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2
*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a
)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2
+1/6*I*b*c/(I-c)*x^3+1/6*I/b^2*c/(I-c)*a^3+1/6*b/(I-c)*x^3+1/6/b^2/(I-c)*a^
3+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2
*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a
)-1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn
((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+1/2*Pi*x^2+1/4*I*x^2*ln(2*I*e
xp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a
^2+1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a-1/2*I/b^2*a^2*ln(1+I*exp(b*x+
a)*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*di
log(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2
))-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^
2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3-1/4*I*x^2*ln(-2*exp(2
```

```
*b*x+2*a)*c+2*I)+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)+1/8*Pi*x^2*csgn(I*(2
*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3
```

Maxima [A]
time = 1.25, size = 106, normalized size = 0.91

$$\left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arccot((c - I)*coth(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(83) = 166.
time = 3.67, size = 246, normalized size = 2.12

$$\frac{-2i\,b^3x^3 + 3i\,b^2x^2\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) - 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right) + 3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right) + 3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right) - 3\left(-i\,b^2x^2+i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right) - 3\left(-i\,b^2x^2+i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right) - 6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right) - 6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2
*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*s
qrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b
^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[x,b,c,_t0,
exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(c+\coth(a+b\,x)\,(c-\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.215 $\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=82

$$-\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arccot(c-(I-c)*coth(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5298, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+x\cot^{-1}(c-(-c+i)\coth(a+bx))-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcCot[c - (I - c)*Coth[a + b*x]] + (I/2)*x*Log[1 + I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5298

```
Int[ArcCot[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Cot[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}(c-(i-c)\coth(a+bx))\,dx &= x\cot^{-1}(c-(i-c)\coth(a+bx))+b\int \frac{x}{i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))-(ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx \\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) \\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) \\
&= -\frac{1}{2}ibx^2+x\cot^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)
\end{aligned}$$

Mathematica [A]
time = 0.40, size = 71, normalized size = 0.87

$$x\cot^{-1}(c+(-i+c)\coth(a+bx))+\frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcCot[c + (-I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(68) = 136.
time = 0.68, size = 554, normalized size = 6.76

method	result

derivativedivides	$\frac{-\frac{\operatorname{arccot}((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}((c-i)}{}}{}$
default	$\frac{-\frac{\operatorname{arccot}((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)}{2i-2c}-\frac{2i\operatorname{arccot}((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)c}{2i-2c}+\frac{\operatorname{arccot}((c-i)}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arccot((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)
-2*I*arccot((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c+arcc
ot((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c^2+arccot((c-I
)*coth(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)+2*I*arccot((c-I)*coth
(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)*c-arccot((c-I)*coth(b*x+a)+
c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)*c^2-(I-c)^2*(-1/8*I/(I-c)*ln(-I+(c-
I)*coth(b*x+a)+c)^2+1/4*I/(I-c)*ln(-I+(c-I)*coth(b*x+a)+c)*ln(-1/2*I*((c-I)
*coth(b*x+a)+c+I))+1/4*I/(I-c)*dilog(-1/2*I*((c-I)*coth(b*x+a)+c+I))-1/4*I/
(I-c)*ln((c-I)*coth(b*x+a)-c+I)*ln(1/2*((c-I)*coth(b*x+a)+c+I)/c)-1/4*I/(I-
c)*dilog(1/2*((c-I)*coth(b*x+a)+c+I)/c)+1/4*I/(I-c)*ln((c-I)*coth(b*x+a)-c+
I)*ln((-I+(c-I)*coth(b*x+a)+c)/(-2*I+2*c))+1/4*I/(I-c)*dilog((-I+(c-I)*coth
(b*x+a)+c)/(-2*I+2*c))))
```

Maxima [A]
time = 1.22, size = 80, normalized size = 0.98

$$2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\operatorname{arccot}\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arccot((c - I)*coth(b*x + a
) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(58) = 116.
time = 3.12, size = 186, normalized size = 2.27

$$\frac{-i\,b^2x^2+i\,bx\log\left(\frac{(c-i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I)
) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert 2*_t0**2*c*exp(2*a) - _t
0**2*I*exp(2*a) - I of type <class 'sympy.core.add.Add'> to QQ_I[b,c,_t0,ex
p(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}(c+\coth(a+b\,x)\,\,(c-\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(acot(c + coth(a + b*x)*(c - 1i)), x)
```

3.216 $\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCot[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.39, size = 0, normalized size = 0.00

$$\int \frac{\cot^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCot[c - (I - c)*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccot}(c-(i-c)\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] int(arccot(c-(I-c)*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x + 1/2*pi*log(x) - 1/4*(2*pi - 4*I*a - 2*arctan(1/c) - I*log(c^2 + 1))
*log(x) - 1/2*integrate(arctan(e^(-2*b*x - 2*a)/c)/x, x) - 1/4*I*integrate(
log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log((c - I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccot((c - I)*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acot}(c+\coth(a+b\,x)\,\,(c-\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(c + coth(a + b*x)*(c - 1i))/x,x)
[Out] int(acot(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.217 $\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal. Leaf size=187

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\log(fx^m)\text{PolyLog}\left(2,-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\text{PolyLog}\left(2,\frac{ix^{-n}}{c}\right)}{2n}$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*I*b*d*polylog(2,-I/c/(x^n))/n-1/2*I*b*e
*ln(f*x^m)*polylog(2,-I/c/(x^n))/n+1/2*I*b*d*polylog(2,I/c/(x^n))/n+1/2*I*b
*e*ln(f*x^m)*polylog(2,I/c/(x^n))/n-1/2*I*b*e*m*polylog(3,-I/c/(x^n))/n^2+1
/2*I*b*e*m*polylog(3,I/c/(x^n))/n^2
```

Rubi [A]
time = 0.46, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 6874, 4945, 4941, 2438, 5128, 5126, 2421, 6724}

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}-\frac{ibd\text{Li}_2\left(-\frac{ix^{-n}}{c}\right)}{2n}+\frac{ibd\text{Li}_2\left(\frac{ix^{-n}}{c}\right)}{2n}-\frac{ibe\text{Li}_2\left(-\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{ibe\text{Li}_2\left(\frac{ix^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{ibem\text{Li}_3\left(-\frac{ix^{-n}}{c}\right)}{2n^2}+\frac{ibem\text{Li}_3\left(\frac{ix^{-n}}{c}\right)}{2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - ((I/2)*b*d*PolyLog[2, (-I)/(c*x^n)]
)/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)/(c*x^n)])/n + ((I/2)*b*d*PolyLo
g[2, I/(c*x^n)])/n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I/(c*x^n)])/n - ((I/2
)*b*e*m*PolyLog[3, (-I)/(c*x^n)])/n^2 + ((I/2)*b*e*m*PolyLog[3, I/(c*x^n)])
/n^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4945

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5126

```
Int[(ArcCot[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[Log[d*x^m]*(Log[1 - I/(c*x^n)]/x), x], x] - Dist[I/2, Int[Log[d*
x^m]*(Log[1 + I/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 5128

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcCot[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcC
ot[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+b\cot^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = \int \left(\frac{d(a+b\cot^{-1}(cx^n))}{x}+\frac{e(a+b\cot^{-1}(cx^n))\log(fx^m)}{x}\right) dx$$

$$= d\int \frac{a+b\cot^{-1}(cx^n)}{x}\,dx + e\int \frac{(a+b\cot^{-1}(cx^n))\log(fx^m)}{x}\,dx$$

$$= (ae)\int \frac{\log(fx^m)}{x}\,dx + (be)\int \frac{\cot^{-1}(cx^n)\log(fx^m)}{x}\,dx + \frac{d\text{Sub}}{}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} + \frac{1}{2}(ibe)\int \frac{\log(fx^m)\log\left(1-\frac{ix^{-n}}{c}\right)}{x}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} - \frac{ibd\text{Li}_2\left(-\frac{ix^{-n}}{c}\right)}{2n} - \frac{ibe\log(fx^m)\,\text{Li}}{2n}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} - \frac{ibd\text{Li}_2\left(-\frac{ix^{-n}}{c}\right)}{2n} - \frac{ibe\log(fx^m)\,\text{Li}}{2n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.24, size = 132, normalized size = 0.71

$$\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2} - \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)(d+e\log(fx^m))}{n} - \frac{1}{2}\left(a+b\cot^{-1}(cx^n)+b\text{ArcTan}(cx^n)\right)\log(x)\,(em\log(x)-2(d+e\log(fx^m)))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCot[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] (b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2*x
^(2*n))])/n^2 - (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(c^2
*x^(2*n))]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCot[c*x^n] + b*ArcTan[c*x^n])
*Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 1058, normalized size = 5.66

method	result	size
risch	Expression too large to display	1058

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccot(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n)-1/2*I*e*b/n*dilog(-I*(c*x^n+I))*m*ln
(x)+1/4*e/m*ln(x^m)^2*b*Pi+1/2/n*ln(x^n)*Pi*b*d+1/4/n*Pi*dilog(1+I*c*x^n)*b
```

```
*e*csgn(I*f)*csgn(I*f*x^m)^2+1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csgn(I*x^m)*csgn
(I*f*x^m)^2-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x^m)^3-1/4/n*Pi*dilog(1-I*c*x^n
)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I*e*b*m/n^2*polylog(3,-I*c*x^n)+1/2*I
*e*b*ln(-I*(-c*x^n+I))*ln(x)^2*m-1/2*I*e*b*ln(x^m)*ln(-I*(-c*x^n+I))*ln(x)+
1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)-1/2*I*e*b*m/n*ln(x)*polylog(2,-I*c*x^n)
+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m)-1/4*I/n*ln(x^n)*Pi^2*b*
e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I/n*ln(x^n)*Pi^2*b*e*csgn(I*f*x^m
)^3-1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)^2-1/4/n*Pi*dilog(
1+I*c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*c
sgn(I*f)*csgn(I*f*x^m)^2+1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f)*csgn(I*x^m
)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*
ln(f)*dilog(1-I*c*x^n)*b*e+1/2/n*ln(x^n)*ln(f)*Pi*b*e+1/4*I/n*ln(x^n)*Pi^2*
b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I/n*ln(x^n)*Pi^2*b*e*csgn(I*x^m)*csgn(I*f
*x^m)^2-1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)-1/2*I*e*b/n*dilo
g(-I*c*x^n)*m*ln(x)+1/2*e*a/m*ln(x^m)^2+1/2*I*e*b*ln(1-I*c*x^n)*m*ln(x)^2-1
/2*I*e*b*ln(x^m)*ln(1-I*c*x^n)*ln(x)-1/2*I*e*b*m/n^2*polylog(3,I*c*x^n)+1/2
*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)-1/2*I*e*b*ln(-I*(c*x^n+I))*ln(x)^2*m+1
/2*I*e*b*ln(-I*(c*x^n+I))*ln(x^m)*ln(x)-1/2*I/n*ln(f)*dilog(1+I*c*x^n)*b*e+
1/n*ln(f)*ln(x^n)*a*e-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f
*x^m)+1/n*ln(x^n)*a*d-1/2*I*e*b*ln(1+I*c*x^n)*m*ln(x)^2+1/2*I*e*b*ln(1+I*c*
x^n)*ln(x^m)*ln(x)-1/2*I/n*dilog(1+I*c*x^n)*b*d+1/2*I/n*dilog(1-I*c*x^n)*b*
d+1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f*x^m)^3-1/4/n*Pi*dilog(1+I*c*x^n)*b
*e*csgn(I*f*x^m)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*m*e*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(1/(c*x^n)) + integrate(-1/2*(b
*c*m*n*e^(n*log(x) + 1)*log(x)^2 - 2*b*c*n*e^(n*log(x) + 1)*log(x)*log(x^m)
 - 2*(b*c*e*log(f) + b*c*d)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

Fricas [A]
time = 3.11, size = 264, normalized size = 1.41

$$\frac{2\,amn^2e\log(x)^2 - 2i\,bme\operatorname{polylog}(3, i\,cx^n) + 2i\,bme\operatorname{polylog}(3, -i\,cx^n) + 2\left(bmn^2e\log(x)^2 + 2\left(bn^2e\log(f) + bdn^2\right)\log(x)\right)\operatorname{arccot}(cx^n) - 2\left(-i\,bmne\log(x) - i\,bne\log(f) - i\,bdn\right)\mathrm{Li}_2(i\,cx^n) - 2\left(i\,bmne\log(x) + i\,bne\log(f) + i\,bdn\right)\mathrm{Li}_2(-i\,cx^n) + \left(-i\,bmn^2e\log(x)^2 - 2\left(i\,bn^2e\log(f) + i\,bdn^2\right)\log(x)\right)\log(i\,cx^n + 1) + \left(i\,bmn^2e\log(x)^2 - 2\left(-i\,bn^2e\log(f) - i\,bdn^2\right)\log(x)\right)\log(-i\,cx^n + 1) + 4\left(an^2e\log(f) + adn^2\right)\log(x)}{4\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*m*n^2*e*log(x)^2 - 2*I*b*m*e*polylog(3, I*c*x^n) + 2*I*b*m*e*polyl
og(3, -I*c*x^n) + 2*(b*m*n^2*e*log(x)^2 + 2*(b*n^2*e*log(f) + b*d*n^2)*log(
x))*arccot(c*x^n) - 2*(-I*b*m*n*e*log(x) - I*b*n*e*log(f) - I*b*d*n)*dilog(
I*c*x^n) - 2*(I*b*m*n*e*log(x) + I*b*n*e*log(f) + I*b*d*n)*dilog(-I*c*x^n)
+ (-I*b*m*n^2*e*log(x)^2 - 2*(I*b*n^2*e*log(f) + I*b*d*n^2)*log(x))*log(I*c
*x^n + 1) + (I*b*m*n^2*e*log(x)^2 - 2*(-I*b*n^2*e*log(f) - I*b*d*n^2)*log(x
))*log(-I*c*x^n + 1) + 4*(a*n^2*e*log(f) + a*d*n^2)*log(x))/n^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acot(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccot(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccot(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acot}(c\,x^n))\,\left(d + e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

```
[Out] int(((a + b*acot(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

3.218 $\int \cot^{-1}(e^x)\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}i\text{PolyLog}(2,-ie^{-x})+\frac{1}{2}i\text{PolyLog}(2,ie^{-x})$$

```
[Out] -1/2*I*polylog(2,-I/exp(x))+1/2*I*polylog(2,I/exp(x))
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {2320, 4941, 2438}

$$\frac{1}{2}i\text{Li}_2(ie^{-x})-\frac{1}{2}i\text{Li}_2(-ie^{-x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*PolyLog[2, (-I)/E^x] + (I/2)*PolyLog[2, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \cot^{-1}\left(e^x\right)\,dx = \text{Subst}\left(\int \frac{\cot^{-1}(x)}{x}\,dx, x, e^x\right)$$
$$= \frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x}\,dx, x, e^x\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x}\,dx, x, e^x\right)$$
$$= -\frac{1}{2}i\text{Li}_2\left(-ie^{-x}\right) + \frac{1}{2}i\text{Li}_2\left(ie^{-x}\right)$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 1.69

$$x\cot^{-1}\left(e^x\right) + \frac{1}{2}i(x(\log\left(1-ie^x\right) - \log\left(1+ie^x\right)) - \text{PolyLog}(2,-ie^x) + \text{PolyLog}(2,ie^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^x],x]
```

```
[Out] x*ArcCot[E^x] + (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.
time = 0.11, size = 59, normalized size = 1.69

method	result	size
derivativedivides	$\ln\left(e^x\right)\mathrm{arccot}(e^x) - \frac{i\ln(e^x)\ln(1+ie^x)}{2} + \frac{i\ln(e^x)\ln(1-ie^x)}{2} - \frac{i\,\mathrm{dilog}(1+ie^x)}{2} + \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	59
default	$\ln\left(e^x\right)\mathrm{arccot}(e^x) - \frac{i\ln(e^x)\ln(1+ie^x)}{2} + \frac{i\ln(e^x)\ln(1-ie^x)}{2} - \frac{i\,\mathrm{dilog}(1+ie^x)}{2} + \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	59
risch	$\frac{ix\ln(1+ie^x)}{2} + \frac{\pi x}{2} + \frac{i\,\mathrm{dilog}(1-ie^x)}{2} + \frac{i\ln(-ie^x)\ln(-i(-e^x+i))}{2} - \frac{i\ln(-i(-e^x+i))x}{2} + \frac{i\,\mathrm{dilog}(-ie^x)}{2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x))*arccot(exp(x))-1/2*I*ln(exp(x))*ln(1+I*exp(x))+1/2*I*ln(exp(x))*
ln(1-I*exp(x))-1/2*I*dilog(1+I*exp(x))+1/2*I*dilog(1-I*exp(x))
```

Maxima [A]
time = 0.53, size = 34, normalized size = 0.97

$$x\,\mathrm{arccot}\left(e^x\right) + \frac{1}{4}\pi\log\left(e^{(2x)}+1\right) + \frac{1}{2}i\,\mathrm{Li}_2(i\,e^x+1) - \frac{1}{2}i\,\mathrm{Li}_2(-i\,e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x)),x, algorithm="maxima")
```

```
[Out] x*arccot(e^x) + 1/4*pi*log(e^(2*x) + 1) + 1/2*I*dilog(I*e^x + 1) - 1/2*I*di
log(-I*e^x + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 2.99, size = 40, normalized size = 1.14

$$x \operatorname{arccot}(e^x) - \frac{1}{2} i x \log (i e^x + 1) + \frac{1}{2} i x \log (-i e^x + 1) + \frac{1}{2} i \operatorname{Li}_2(i e^x) - \frac{1}{2} i \operatorname{Li}_2(-i e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] x*arccot(e^x) - 1/2*I*x*log(I*e^x + 1) + 1/2*I*x*log(-I*e^x + 1) + 1/2*I*di
log(I*e^x) - 1/2*I*dilog(-I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}(e^x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(x)),x)
```

```
[Out] Integral(acot(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arccot(e^x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{acot}(\mathrm{e}^x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(exp(x)),x)
```

```
[Out] int(acot(exp(x)), x)
```

3.219 $\int x \cot^{-1}(e^x)\, dx$

Optimal. Leaf size=71

$$-\frac{1}{2}ix\text{PolyLog}(2,-ie^{-x})+\frac{1}{2}ix\text{PolyLog}(2,ie^{-x})-\frac{1}{2}i\text{PolyLog}(3,-ie^{-x})+\frac{1}{2}i\text{PolyLog}(3,ie^{-x})$$

```
[Out] -1/2*I*x*polylog(2,-I/exp(x))+1/2*I*x*polylog(2,I/exp(x))-1/2*I*polylog(3,-
I/exp(x))+1/2*I*polylog(3,I/exp(x))
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5252, 2611, 2320, 6724}

$$-\frac{1}{2}ix\text{Li}_2(-ie^{-x})+\frac{1}{2}ix\text{Li}_2(ie^{-x})-\frac{1}{2}i\text{Li}_3(-ie^{-x})+\frac{1}{2}i\text{Li}_3(ie^{-x})$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*x*PolyLog[2, (-I)/E^x] + (I/2)*x*PolyLog[2, I/E^x] - (I/2)*PolyLog
[3, (-I)/E^x] + (I/2)*PolyLog[3, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}(e^x)\,dx &= \frac{1}{2}i\int x\log\left(1-ie^{-x}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^{-x}\right)\,dx\\ &= -\frac{1}{2}ix\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix\text{Li}_2\left(ie^{-x}\right)+\frac{1}{2}i\int \text{Li}_2\left(-ie^{-x}\right)\,dx-\frac{1}{2}i\int \text{Li}_2\left(ie^{-x}\right)\,dx\\ &= -\frac{1}{2}ix\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix\text{Li}_2\left(ie^{-x}\right)-\frac{1}{2}i\text{Subst}\left(\int \frac{\text{Li}_2(-ix)}{x}\,dx,x,e^{-x}\right)+\frac{1}{2}i\text{Subst}\left(\int\right.\\ &= -\frac{1}{2}ix\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix\text{Li}_2\left(ie^{-x}\right)-\frac{1}{2}i\text{Li}_3\left(-ie^{-x}\right)+\frac{1}{2}i\text{Li}_3\left(ie^{-x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 0.82

$$-\frac{1}{2}i\left(x\text{PolyLog}\left(2,-ie^{-x}\right)-x\text{PolyLog}\left(2,ie^{-x}\right)+\text{PolyLog}\left(3,-ie^{-x}\right)-\text{PolyLog}\left(3,ie^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[E^x],x]
[Out] (-1/2*I)*(x*PolyLog[2, (-I)/E^x] - x*PolyLog[2, I/E^x] + PolyLog[3, (-I)/E^
x] - PolyLog[3, I/E^x])
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.70

method	result	size
risch	$\frac{\pi x^2}{4}+\frac{ix\,\text{polylog}(2,ie^x)}{2}-\frac{i\,\text{polylog}(3,ie^x)}{2}-\frac{ix\,\text{polylog}(2,-ie^x)}{2}+\frac{i\,\text{polylog}(3,-ie^x)}{2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*Pi*x^2+1/2*I*polylog(2,I*exp(x))*x-1/2*I*polylog(3,I*exp(x))-1/2*I*poly
log(2,-I*exp(x))*x+1/2*I*polylog(3,-I*exp(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(-x)) + integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Fricas [A]
time = 8.99, size = 65, normalized size = 0.92

$$\frac{1}{2}x^2\operatorname{arccot}\left(e^x\right)-\frac{1}{4}ix^2\log\left(ie^x+1\right)+\frac{1}{4}ix^2\log\left(-ie^x+1\right)+\frac{1}{2}ix\mathrm{Li}_2(ie^x)-\frac{1}{2}ix\mathrm{Li}_2(-ie^x)-\frac{1}{2}i\,\mathrm{polylog}(3,ie^x)+\frac{1}{2}i\,\mathrm{polylog}(3,-ie^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arccot(e^x) - 1/4*I*x^2*log(I*e^x + 1) + 1/4*I*x^2*log(-I*e^x + 1)
+ 1/2*I*x*dilog(I*e^x) - 1/2*I*x*dilog(-I*e^x) - 1/2*I*polylog(3, I*e^x) +
1/2*I*polylog(3, -I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acot}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(exp(x)),x)
```

```
[Out] Integral(x*acot(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(e^x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}(\mathrm{e}^x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(exp(x)),x)
```

```
[Out] int(x*acot(exp(x)), x)
```

3.220 $\int x^2 \cot^{-1}(e^x)\, dx$

Optimal. Leaf size=103

$$-\frac{1}{2}ix^2\text{PolyLog}(2,-ie^{-x})+\frac{1}{2}ix^2\text{PolyLog}(2,ie^{-x})-ix\text{PolyLog}(3,-ie^{-x})+ix\text{PolyLog}(3,ie^{-x})-i\text{PolyLog}(4,$$

```
[Out] -1/2*I*x^2*polylog(2,-I/exp(x))+1/2*I*x^2*polylog(2,I/exp(x))-I*x*polylog(3
,-I/exp(x))+I*x*polylog(3,I/exp(x))-I*polylog(4,-I/exp(x))+I*polylog(4,I/ex
p(x))
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5252, 2611, 6744, 2320, 6724}

$$-\frac{1}{2}ix^2\text{Li}_2(-ie^{-x})+\frac{1}{2}ix^2\text{Li}_2(ie^{-x})-ix\text{Li}_3(-ie^{-x})+ix\text{Li}_3(ie^{-x})-i\text{Li}_4(-ie^{-x})+i\text{Li}_4(ie^{-x})$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*x^2*PolyLog[2, (-I)/E^x] + (I/2)*x^2*PolyLog[2, I/E^x] - I*x*PolyL
og[3, (-I)/E^x] + I*x*PolyLog[3, I/E^x] - I*PolyLog[4, (-I)/E^x] + I*PolyLo
g[4, I/E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

```
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\cot^{-1}\left(e^x\right)\,dx &= \frac{1}{2}i\int x^2\log\left(1-ie^{-x}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^{-x}\right)\,dx\\ &= -\frac{1}{2}ix^2\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix^2\text{Li}_2\left(ie^{-x}\right)+i\int x\text{Li}_2\left(-ie^{-x}\right)\,dx-i\int x\text{Li}_2\left(ie^{-x}\right)\,dx\\ &= -\frac{1}{2}ix^2\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix^2\text{Li}_2\left(ie^{-x}\right)-ix\text{Li}_3\left(-ie^{-x}\right)+ix\text{Li}_3\left(ie^{-x}\right)+i\int \text{Li}_3\left(-ie^{-x}\right)\,d\\ &= -\frac{1}{2}ix^2\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix^2\text{Li}_2\left(ie^{-x}\right)-ix\text{Li}_3\left(-ie^{-x}\right)+ix\text{Li}_3\left(ie^{-x}\right)-i\text{Subst}\left(\int\frac{\text{Li}_3(-}{x}\right.\\ &= -\frac{1}{2}ix^2\text{Li}_2\left(-ie^{-x}\right)+\frac{1}{2}ix^2\text{Li}_2\left(ie^{-x}\right)-ix\text{Li}_3\left(-ie^{-x}\right)+ix\text{Li}_3\left(ie^{-x}\right)-i\text{Li}_4\left(-ie^{-x}\right)+i\text{L}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 103, normalized size = 1.00

$$-\frac{1}{2}ix^2\text{PolyLog}\left(2,-ie^{-x}\right)+\frac{1}{2}ix^2\text{PolyLog}\left(2,ie^{-x}\right)-ix\text{PolyLog}\left(3,-ie^{-x}\right)+ix\text{PolyLog}\left(3,ie^{-x}\right)-i\text{PolyLog}\left(4,-ie^{-x}\right)+i\text{PolyLog}\left(4,ie^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[E^x],x]
```

```
[Out] (-1/2*I)*x^2*PolyLog[2, (-I)/E^x] + (I/2)*x^2*PolyLog[2, I/E^x] - I*x*PolyL
og[3, (-I)/E^x] + I*x*PolyLog[3, I/E^x] - I*PolyLog[4, (-I)/E^x] + I*PolyLo
g[4, I/E^x]
```

Maple [A]
time = 0.09, size = 76, normalized size = 0.74

method	result
risch	$\frac{\pi x^3}{6} + \frac{i \operatorname{polylog}(2, ie^x)x^2}{2} - ix \operatorname{polylog}(3, ie^x) + i \operatorname{polylog}(4, ie^x) - \frac{i \operatorname{polylog}(2, -ie^x)x^2}{2} + i \operatorname{polylog}(3, -ie$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*Pi*x^3+1/2*I*polylog(2,I*exp(x))*x^2-I*x*polylog(3,I*exp(x))+I*polylog(
4,I*exp(x))-1/2*I*polylog(2,-I*exp(x))*x^2+I*polylog(3,-I*exp(x))*x-I*polyl
og(4,-I*exp(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(-x)) + integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

Fricas [A]
time = 7.06, size = 87, normalized size = 0.84

$$\frac{1}{3} x^3 \operatorname{arccot}(e^x) - \frac{1}{6} i x^3 \log(i e^x + 1) + \frac{1}{6} i x^3 \log(-i e^x + 1) + \frac{1}{2} i x^2 \operatorname{Li}_2(i e^x) - \frac{1}{2} i x^2 \operatorname{Li}_2(-i e^x) - i x \operatorname{polylog}(3, i e^x) + i x \operatorname{polylog}(3, -i e^x) + i \operatorname{polylog}(4, i e^x) - i \operatorname{polylog}(4, -i e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arccot(e^x) - 1/6*I*x^3*log(I*e^x + 1) + 1/6*I*x^3*log(-I*e^x + 1)
+ 1/2*I*x^2*dilog(I*e^x) - 1/2*I*x^2*dilog(-I*e^x) - I*x*polylog(3, I*e^x)
+ I*x*polylog(3, -I*e^x) + I*polylog(4, I*e^x) - I*polylog(4, -I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acot}(e^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(exp(x)),x)
```

```
[Out] Integral(x**2*acot(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(e^x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acot}\left(\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(exp(x)),x)
```

```
[Out] int(x^2*acot(exp(x)), x)
```

3.221 $\int \cot^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=51

$$-\frac{i\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{i\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}$$

```
[Out] -1/2*I*polylog(2,-I*exp(-b*x-a))/b+1/2*I*polylog(2,I*exp(-b*x-a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 4941, 2438}

$$\frac{i\text{Li}_2\left(ie^{-a-bx}\right)}{2b}-\frac{i\text{Li}_2\left(-ie^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*PolyLog[2, I*E^(-a - b*
x)])/b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \cot^{-1}\left(e^{a+bx}\right) dx = \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{x} dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log\left(1-\frac{i}{x}\right)}{x} dx, x, e^{a+bx}\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log\left(1+\frac{i}{x}\right)}{x} dx, x, e^{a+bx}\right)}{2b}$$
$$= -\frac{i\text{Li}_2\left(-ie^{-a-bx}\right)}{2b} + \frac{i\text{Li}_2\left(ie^{-a-bx}\right)}{2b}$$

Mathematica [A]
time = 0.12, size = 83, normalized size = 1.63

$$x\cot^{-1}\left(e^{a+bx}\right) + \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^(a + b*x)],x]
```

```
[Out] x*ArcCot[E^(a + b*x)] + ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(41) = 82.
time = 0.10, size = 95, normalized size = 1.86

method	result
derivativedivides	$\frac{\ln\left(e^{bx+a}\right)\text{arccot}\left(e^{bx+a}\right)-\frac{i\ln\left(e^{bx+a}\right)\ln\left(1+ie^{bx+a}\right)}{2}+\frac{i\ln\left(e^{bx+a}\right)\ln\left(1-ie^{bx+a}\right)}{2}-\frac{i\,\text{dilog}\left(1+ie^{bx+a}\right)}{2}+\frac{i\,\text{dilog}\left(1-ie^{bx+a}\right)}{2}}{b}$
default	$\frac{\ln\left(e^{bx+a}\right)\text{arccot}\left(e^{bx+a}\right)-\frac{i\ln\left(e^{bx+a}\right)\ln\left(1+ie^{bx+a}\right)}{2}+\frac{i\ln\left(e^{bx+a}\right)\ln\left(1-ie^{bx+a}\right)}{2}-\frac{i\,\text{dilog}\left(1+ie^{bx+a}\right)}{2}+\frac{i\,\text{dilog}\left(1-ie^{bx+a}\right)}{2}}{b}$
risch	$\frac{ix\ln\left(1+ie^{bx+a}\right)}{2}+\frac{\pi x}{2}+\frac{i\,\text{dilog}\left(1-ie^{bx+a}\right)}{2b}+\frac{i\ln\left(-ie^{bx+a}\right)\ln\left(-i\left(-e^{bx+a}+i\right)\right)}{2b}-\frac{i\ln\left(-i\left(-e^{bx+a}+i\right)\right)x}{2}-\frac{i\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(exp(b*x+a))*arccot(exp(b*x+a))-1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x+
a))+1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))-1/2*I*dilog(1+I*exp(b*x+a))+1/2
*I*dilog(1-I*exp(b*x+a)))
```

Maxima [A]
time = 0.52, size = 63, normalized size = 1.24

$$\frac{(bx+a)\,\text{arccot}\left(e^{(bx+a)}\right)}{b} + \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\,\text{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\,\text{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arccot(e^(b*x + a))/b + 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(35) = 70$.
time = 5.38, size = 103, normalized size = 2.02

$$\frac{2\,bx\operatorname{arccot}\left(e^{(bx+a)}\right) - i\,a\log\left(e^{(bx+a)} + i\right) + i\,a\log\left(e^{(bx+a)} - i\right) + \left(-i\,bx - i\,a\right)\log\left(i\,e^{(bx+a)} + 1\right) + \left(i\,bx + i\,a\right)\log\left(-i\,e^{(bx+a)} + 1\right) + i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) - i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arccot(e^(b*x + a)) - I*a*log(e^(b*x + a) + I) + I*a*log(e^(b*x
+ a) - I) + (-I*b*x - I*a)*log(I*e^(b*x + a) + 1) + (I*b*x + I*a)*log(-I*e^
(b*x + a) + 1) + I*dilog(I*e^(b*x + a)) - I*dilog(-I*e^(b*x + a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(e^{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(b*x+a)),x)
```

```
[Out] Integral(acot(exp(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccot(e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(exp(a + b*x)),x)
```

```
[Out] int(acot(exp(a + b*x)), x)
```

3.222 $\int x\cot^{-1}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=103

$$-\frac{ix\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}-\frac{i\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(3,ie^{-a-bx}\right)}{2b^2}$$

```
[Out] -1/2*I*x*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x*polylog(2,I*exp(-b*x-a))/b-1/2
*I*polylog(3,-I*exp(-b*x-a))/b^2+1/2*I*polylog(3,I*exp(-b*x-a))/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {5252, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-ie^{-a-bx}\right)}{2b^2}+\frac{i\text{Li}_3\left(ie^{-a-bx}\right)}{2b^2}-\frac{ix\text{Li}_2\left(-ie^{-a-bx}\right)}{2b}+\frac{ix\text{Li}_2\left(ie^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*x*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*x*PolyLog[2, I*E^(-a
- b*x)])/b - ((I/2)*PolyLog[3, (-I)*E^(-a - b*x)])/b^2 + ((I/2)*PolyLog[3,
I*E^(-a - b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

```
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}\left(e^{a+bx}\right)\,dx &= \frac{1}{2}i\int x\log\left(1-ie^{-a-bx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ie^{-a-bx}\right)\,dx\\ &= -\frac{ix\mathrm{Li}_2\left(-ie^{-a-bx}\right)}{2b}+\frac{ix\mathrm{Li}_2\left(ie^{-a-bx}\right)}{2b}+\frac{i\int\mathrm{Li}_2\left(-ie^{-a-bx}\right)\,dx}{2b}-\frac{i\int\mathrm{Li}_2\left(ie^{-a-bx}\right)\,dx}{2b}\\ &= -\frac{ix\mathrm{Li}_2\left(-ie^{-a-bx}\right)}{2b}+\frac{ix\mathrm{Li}_2\left(ie^{-a-bx}\right)}{2b}-\frac{i\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2(-ix)}{x}\,dx,x,e^{-a-bx}\right)}{2b^2}+\frac{i\mathrm{Subst}\left(\ldots\right.}{\ldots}\\ &= -\frac{ix\mathrm{Li}_2\left(-ie^{-a-bx}\right)}{2b}+\frac{ix\mathrm{Li}_2\left(ie^{-a-bx}\right)}{2b}-\frac{i\mathrm{Li}_3\left(-ie^{-a-bx}\right)}{2b^2}+\frac{i\mathrm{Li}_3\left(ie^{-a-bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.81

$$-\frac{i\left(bx\mathrm{PolyLog}\left(2,-ie^{-a-bx}\right)-bx\mathrm{PolyLog}\left(2,ie^{-a-bx}\right)+\mathrm{PolyLog}\left(3,-ie^{-a-bx}\right)-\mathrm{PolyLog}\left(3,ie^{-a-bx}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*(b*x*PolyLog[2, (-I)*E^(-a - b*x)] - b*x*PolyLog[2, I*E^(-a - b*x
)] + PolyLog[3, (-I)*E^(-a - b*x)] - PolyLog[3, I*E^(-a - b*x)]))/b^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(83) = 166.
time = 0.28, size = 355, normalized size = 3.45

method	result
risch	$\frac{\pi x^2}{4}+\frac{i\ln(-i(-\mathrm{e}^{bx+a}+i))xa}{2b}+\frac{i\ln(-i(-\mathrm{e}^{bx+a}+i))a^2}{2b^2}+\frac{i\,\mathrm{polylog}(3,-i\mathrm{e}^{bx+a})}{2b^2}-\frac{i\,\mathrm{dilog}(-i(\mathrm{e}^{bx+a}+i))a}{2b^2}+\frac{i\ln(1-i\mathrm{e}^{bx+a}\ldots}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*x^2+1/2*I/b*polylog(2,I*exp(b*x+a))*x-1/2*I/b*polylog(2,-I*exp(b*x+a
))*x-1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b*ln(-I*(-exp(b*x+a)+I))*x*
a+1/2*I/b^2*polylog(3,-I*exp(b*x+a))+1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*a^2-1
/2*I/b^2*dilog(-I*exp(b*x+a))*a-1/2*I/b^2*dilog(-I*(exp(b*x+a)+I))*a+1/2*I/
b*ln(1-I*exp(b*x+a))*x*a-1/2*I/b^2*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))
*a-1/2*I/b^2*polylog(3,I*exp(b*x+a))-1/2*I/b*ln(1+I*exp(b*x+a))*x*a-1/2*I/b
^2*ln(-I*(exp(b*x+a)+I))*a^2-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a))+1/2*I/b^2*pol
ylog(2,I*exp(b*x+a))*a+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a))-1/2*I/b*ln(-I*(exp(
b*x+a)+I))*x*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(-b*x - a)) + b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x +
2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.
time = 2.52, size = 151, normalized size = 1.47

$$\frac{2\,b^2x^2\,\mathrm{arccot}\left(e^{(bx+a)}\right)+2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right)-2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)+i\,a^2\log\left(e^{(bx+a)}+i\right)-i\,a^2\log\left(e^{(bx+a)}-i\right)+\left(-i\,b^2x^2+i\,a^2\right)\log\left(i\,e^{(bx+a)}+1\right)+\left(i\,b^2x^2-i\,a^2\right)\log\left(-i\,e^{(bx+a)}+1\right)-2i\,\mathrm{polylog}\left(3,i\,e^{(bx+a)}\right)+2i\,\mathrm{polylog}\left(3,-i\,e^{(bx+a)}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arccot(e^(b*x + a)) + 2*I*b*x*dilog(I*e^(b*x + a)) - 2*I*b*x
*dilog(-I*e^(b*x + a)) + I*a^2*log(e^(b*x + a) + I) - I*a^2*log(e^(b*x + a)
 - I) + (-I*b^2*x^2 + I*a^2)*log(I*e^(b*x + a) + 1) + (I*b^2*x^2 - I*a^2)*l
og(-I*e^(b*x + a) + 1) - 2*I*polylog(3, I*e^(b*x + a)) + 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acot}\left(e^{a}e^{bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(exp(b*x+a)),x)
```

```
[Out] Integral(x*acot(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccot(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(exp(a + b*x)),x)
```

```
[Out] int(x*acot(exp(a + b*x)), x)
```

3.223 $\int x^2 \cot^{-1}\left(e^{a+bx}\right)\, dx$

Optimal. Leaf size=151

$$-\frac{ix^2\text{PolyLog}\left(2,-ie^{-a-bx}\right)}{2b}+\frac{ix^2\text{PolyLog}\left(2,ie^{-a-bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(3,-ie^{-a-bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{-a-bx}\right)}{b^2}-\frac{i\text{P}}{}$$

```
[Out] -1/2*I*x^2*polylog(2,-I*exp(-b*x-a))/b+1/2*I*x^2*polylog(2,I*exp(-b*x-a))/b
-I*x*polylog(3,-I*exp(-b*x-a))/b^2+I*x*polylog(3,I*exp(-b*x-a))/b^2-I*polyl
og(4,-I*exp(-b*x-a))/b^3+I*polylog(4,I*exp(-b*x-a))/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5252, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-ie^{-a-bx}\right)}{b^3}+\frac{i\text{Li}_4\left(ie^{-a-bx}\right)}{b^3}-\frac{ix\text{Li}_3\left(-ie^{-a-bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{-a-bx}\right)}{b^2}-\frac{ix^2\text{Li}_2\left(-ie^{-a-bx}\right)}{2b}+\frac{ix^2\text{Li}_2\left(ie^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*x^2*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*x^2*PolyLog[2, I*E^
(-a - b*x)])/b - (I*x*PolyLog[3, (-I)*E^(-a - b*x)])/b^2 + (I*x*PolyLog[3,
I*E^(-a - b*x)])/b^2 - (I*PolyLog[4, (-I)*E^(-a - b*x)])/b^3 + (I*PolyLog[4
, I*E^(-a - b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
```

```
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cot^{-1}\left(e^{a+bx}\right) dx &= \frac{1}{2} i \int x^2 \log\left(1-ie^{-a-bx}\right) dx - \frac{1}{2} i \int x^2 \log\left(1+ie^{-a-bx}\right) dx \\
&= -\frac{ix^2 \text{Li}_2\left(-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \text{Li}_2\left(ie^{-a-bx}\right)}{2b} + \frac{i \int x \text{Li}_2\left(-ie^{-a-bx}\right) dx}{b} - \frac{i \int x \text{Li}_2\left(ie^{-a-\right.}}{b} \\
&= -\frac{ix^2 \text{Li}_2\left(-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \text{Li}_2\left(ie^{-a-bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{-a-bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{-a-bx}\right)}{b^2} + \frac{i \int}{} \\
&= -\frac{ix^2 \text{Li}_2\left(-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \text{Li}_2\left(ie^{-a-bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{-a-bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{-a-bx}\right)}{b^2} - \frac{iS}{} \\
&= -\frac{ix^2 \text{Li}_2\left(-ie^{-a-bx}\right)}{2b} + \frac{ix^2 \text{Li}_2\left(ie^{-a-bx}\right)}{2b} - \frac{ix \text{Li}_3\left(-ie^{-a-bx}\right)}{b^2} + \frac{ix \text{Li}_3\left(ie^{-a-bx}\right)}{b^2} - \frac{iL}{}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 151, normalized size = 1.00

$$-\frac{ix^2 \text{PolyLog}(2,-ie^{-a-bx})}{2b}+\frac{ix^2 \text{PolyLog}(2,ie^{-a-bx})}{2b}-\frac{ix \text{PolyLog}(3,-ie^{-a-bx})}{b^2}+\frac{ix \text{PolyLog}(3,ie^{-a-bx})}{b^2}-\frac{i \text{PolyLog}(4,-ie^{-a-bx})}{b^3}+\frac{i \text{PolyLog}(4,ie^{-a-bx})}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*x^2*PolyLog[2, (-I)*E^(-a - b*x)])/b + ((I/2)*x^2*PolyLog[2, I*E^
(-a - b*x)])/b - (I*x*PolyLog[3, (-I)*E^(-a - b*x)])/b^2 + (I*x*PolyLog[3,
I*E^(-a - b*x)])/b^2 - (I*PolyLog[4, (-I)*E^(-a - b*x)])/b^3 + (I*PolyLog[4
, I*E^(-a - b*x)])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(129) = 258$.
time = 0.28, size = 413, normalized size = 2.74

method	result
risch	$\frac{\pi x^3}{6} + \frac{ia^3 \ln(1+ie^{bx+a})}{2b^3} + \frac{i\,\mathrm{dilog}(-ie^{bx+a})a^2}{2b^3} - \frac{ia^3 \ln(1-ie^{bx+a})}{2b^3} - \frac{i\ln(-i(-e^{bx+a}+i))x\,a^2}{2b^2} + \frac{i\ln(-ie^{bx+a})\ln(-i(-e^{bx+}}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*Pi*x^3+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a))-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)
)+1/2*I/b*polylog(2,I*exp(b*x+a))*x^2-1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*x*a^
2+1/2*I/b^3*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(1+I*e
xp(b*x+a))*x*a^2+1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2+I/b^2*polylog(3,-I*exp(
b*x+a))*x+1/2*I/b^3*dilog(-I*(exp(b*x+a)+I))*a^2-1/2*I/b^2*ln(1-I*exp(b*x+a
))*x*a^2-I/b^2*polylog(3,I*exp(b*x+a))*x+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*x*
a^2+1/2*I/b^3*ln(-I*(exp(b*x+a)+I))*a^3+I/b^3*polylog(4,I*exp(b*x+a))-1/2*I
/b*polylog(2,-I*exp(b*x+a))*x^2-I/b^3*polylog(4,-I*exp(b*x+a))-1/2*I/b^3*ln
(-I*(-exp(b*x+a)+I))*a^3-1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/2*I/b^3*po
lylog(2,-I*exp(b*x+a))*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(-b*x - a)) + b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x +
2*a) + 1), x)
```

Fricas [A]
time = 4.37, size = 187, normalized size = 1.24

$$\frac{2\,b^3x^3\operatorname{arccot}\left(e^{(bx+a)}\right) + 3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) - 3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^3\log\left(e^{(bx+a)} + i\right) + i\,a^3\log\left(e^{(bx+a)} - i\right) - 6i\,bx\mathrm{polylog}\left(3, i\,e^{(bx+a)}\right) + 6i\,bx\mathrm{polylog}\left(3, -i\,e^{(bx+a)}\right) + \left(-i\,b^3x^3 - i\,a^3\right)\log\left(i\,e^{(bx+a)} + 1\right) + \left(i\,b^3x^3 + i\,a^3\right)\log\left(-i\,e^{(bx+a)} + 1\right) + 6i\,\mathrm{polylog}\left(4, i\,e^{(bx+a)}\right) - 6i\,\mathrm{polylog}\left(4, -i\,e^{(bx+a)}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arccot(e^(b*x + a)) + 3*I*b^2*x^2*dilog(I*e^(b*x + a)) - 3*I
*b^2*x^2*dilog(-I*e^(b*x + a)) - I*a^3*log(e^(b*x + a) + I) + I*a^3*log(e^(
b*x + a) - I) - 6*I*b*x*polylog(3, I*e^(b*x + a)) + 6*I*b*x*polylog(3, -I*e
^(b*x + a)) + (-I*b^3*x^3 - I*a^3)*log(I*e^(b*x + a) + 1) + (I*b^3*x^3 + I*
```

```
a^3)*log(-I*e^(b*x + a) + 1) + 6*I*polylog(4, I*e^(b*x + a)) - 6*I*polylog(
4, -I*e^(b*x + a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acot}\left(e^a e^{bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(exp(b*x+a)),x)
[Out] Integral(x**2*acot(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccot(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acot}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(exp(a + b*x)),x)
[Out] int(x^2*acot(exp(a + b*x)), x)
```

3.224 $\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=196

$$-\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}-\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i(}\right.}{2d\log(f)}$$

```
[Out] -arccot(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arccot(a+b*f^(d*
x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2
,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a)/(
1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2320, 5156, 4967, 2449, 2352, 2497}

$$-\frac{i\text{Li}_2\left(1-\frac{2}{1-i(bf^{c+dx}+a)}\right)}{2d\log(f)}+\frac{i\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(i-a)(1-i(bf^{c+dx}+a))}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)\cot^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+dx}}{(-a+i)(1-i(a+bf^{c+dx}))}\right)\cot^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcCot[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcCot[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4967

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcCot[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (-Dist[b*(c/e), Int[
Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Dist[b*(c/e), Int[Log[2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcCot
[c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5156

```
Int[((a_.) + ArcCot[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCot[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot^{-1}\left(a+bf^{c+dx}\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}\\
&= \frac{\text{Subst}\left(\int \frac{\cot^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf\cdots}\right.}{d\log(f)}\\
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf\cdots}\right.}{d\log(f)}\\
&= -\frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\cot^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf\cdots}\right.}{d\log(f)}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 167, normalized size = 0.85

$$x\cot^{-1}\left(a+bf^{c+dx}\right)+\frac{b\left(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)}{2\sqrt{-b^2}\,d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcCot[a + b*f^(c + d*x)] + (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

Maple [A]

time = 0.35, size = 162, normalized size = 0.83

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccot}\left(a+b\,f^{dx+c}\right)+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx+c}}{i+a}\right.}{2}}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccot}\left(a+b\,f^{dx+c}\right)+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}+\frac{i\,\text{dilog}\left(\frac{i+b\,f^{dx+c}}{i+a}\right.}{2}}{d\ln(f)}$
risch	$\frac{ix\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{2}+\frac{\pi x}{2}-\frac{i\ln\left(1-ia-ib\,f^{dx+c}\right)\ln\left(-\frac{if^{dx+c}b}{ia-1}\right)}{2d\ln(f)}-\frac{i\,\text{dilog}\left(-\frac{if^{dx+c}b}{ia-1}\right)}{2d\ln(f)}-\frac{i\,\text{dilog}\left(\frac{b\,f^{dx}f^{c}+a-}{a-i}\right.}{2d\ln(f)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arccot(a+b*f^(d*x+c))+1/2*I*ln(-b*f^(d*x+c))*ln
((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-a)
)+1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))-1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-a))
)
```

Maxima [A]

time = 0.54, size = 189, normalized size = 0.96

$$\frac{(dx+c)\operatorname{arccot}\left(bf^{dx+c}+a\right)}{d}+\frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log\left(f\right)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\arctan\left(bf^{dx+c}+a\right)\log\left(\frac{b^2f^{2\,dx+2\,c}}{a^2+1}\right)+i\,\text{Li}_2\left(\frac{i\,bf^{dx+c}+i\,a+1}{i\,a+1}\right)-i\,\text{Li}_2\left(\frac{i\,bf^{dx+c}+i\,a-1}{i\,a-1}\right)}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arccot(b*f^(d*x + c) + a)/d + 1/2*(2*(d*x + c)*arctan((b^2*f^(d*x
 + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a*b
```

```
*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2*c)
/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b
*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

Fricas [A]
time = 2.79, size = 212, normalized size = 1.08

$$\frac{2\,dx\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(-i\,dx-i\,c\right)\log\left(f\right)\log\left(\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}\right)+\left(i\,dx+i\,c\right)\log\left(f\right)\log\left(\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}\right)-i\,\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)+i\,\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x*arccot(b*f^(d*x + c) + a)*log(f) - I*c*log(b*f^(d*x + c) + a + I
)*log(f) + I*c*log(b*f^(d*x + c) + a - I)*log(f) + (-I*d*x - I*c)*log(f)*lo
g((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) - I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) + I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acot}\left(a+bf^{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(acot(a + b*f**(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(arccot(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(a + b*f^(c + d*x)),x)
```

```
[Out] int(acot(a + b*f^(c + d*x)), x)
```

3.225 $\int x \cot^{-1}\left(a + bf^{c+dx}\right)\,dx$

Optimal. Leaf size=250

$$-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

```
[Out] -1/4*I*x^2*ln(1-b*f^(d*x+c)/(I-a))+1/4*I*x^2*ln(1+b*f^(d*x+c)/(I+a))+1/4*I*
x^2*ln(1-I/(a+b*f^(d*x+c)))-1/4*I*x^2*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x*polyl
og(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x*polylog(2,-b*f^(d*x+c)/(I+a))/d/ln(
f)+1/2*I*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-1/2*I*polylog(3,-b*f^(d*x
+c)/(I+a))/d^2/ln(f)^2
```

Rubi [A]
time = 2.05, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5252, 2631, 12, 6874, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}-\frac{i\text{Li}_3\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)}-\frac{ix\text{Li}_2\left(\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix\text{Li}_2\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/4*I)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/4)*x^2*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/4)*x^2*Log[1 - I/(a + b*f^(c + d*x))] - (I/4)*x^2*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/
(d*Log[f]) + ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) +
((I/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - ((I/2)*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\cot^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{1}{2}i\int x\log\left(1-\frac{i}{a+bf^{c+dx}}\right)\,dx-\frac{1}{2}i\int x\log\left(1+\frac{i}{a+bf^{c+dx}}\right)\,dx$$

$$=\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{4}\int\frac{bdf^{c+}}{(i(1-ia)+b}$$

$$=\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int\frac{}{(i(}$$

$$=\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int\left(\frac{}{a}\right.$$

$$=\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}ix^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{4}(ibd\log(f))\int\frac{}{-}$$

$$=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right.$$

$$=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right.$$

$$=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right.$$

$$=-\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{4}ix^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right.$$

Mathematica [A]
time = 0.48, size = 261, normalized size = 1.04

$$\frac{1}{2}x^2\cot^{-1}\left(a+bf^{c+dx}\right)+\frac{b\left(d^2x^2\log^2(f)\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-d^2x^2\log^2(f)\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)+2dx\log(f)\mathrm{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-2dx\log(f)\mathrm{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)-2\mathrm{PolyLog}\left(3,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)+2\mathrm{PolyLog}\left(3,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)}{4\sqrt{-b^2}\,d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (x^2*ArcCot[a + b*f^(c + d*x)])/2 + (b*(d^2*x^2*Log[f]^2*Log[1 + (b^2*f^(c
 + d*x))/(a*b - Sqrt[-b^2])] - d^2*x^2*Log[f]^2*Log[1 + (b^2*f^(c + d*x))/(a
*b + Sqrt[-b^2])] + 2*d*x*Log[f]*PolyLog[2, -((b^2*f^(c + d*x))/(a*b - Sqrt
[-b^2]))] - 2*d*x*Log[f]*PolyLog[2, -((b^2*f^(c + d*x))/(a*b + Sqrt[-b^2]))
] - 2*PolyLog[3, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] + 2*PolyLog[3, -(
(b^2*f^(c + d*x))/(a*b + Sqrt[-b^2]))]))/(4*Sqrt[-b^2]*d^2*Log[f]^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(218) = 436$.
time = 0.16, size = 678, normalized size = 2.71

method	result
risch	$-\frac{i c^{2} \ln\left(\frac{b f^{dx} f^{c}+i+a}{i+a}\right)}{2 d^{2}}+\frac{\pi x^{2}}{4}+\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a+1}\right) x^{2}}{4}+\frac{i \operatorname{polylog}\left(2, \frac{i b f^{dx} f^{c}}{-i a+1}\right) x}{2 d \ln(f)}+\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-i a+1}\right) x c}{2 d}-\frac{i c \operatorname{dilog}\left(\frac{b f^{dx} f}{i+}\right.}{2 d^{2} \ln(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/d^2/ln(f)*c*dilog((b*f^(d*x)*f^c+I+a)/(I+a))+1/4*Pi*x^2-1/2*I/d^2*c^
2*ln((b*f^(d*x)*f^c+I+a)/(I+a))+1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/2
*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x+1/2*I/d*ln(1-I*b/(1-I*a)*f^
(d*x)*f^c)*x*c+1/2*I/d^2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)-1/2*I/
d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x+1/2*I/d*c*ln((b*f^(d*x)*f^c+a
-I)/(a-I))*x-1/4*I/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)-1/2*I/d^2/ln(f)*polylo
g(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c-1/2*I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(
d*x)*f^c)+1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)+1/2*I/d^2/ln(f)*c*dilog((
b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/d^2/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c
)*c-1/4*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2+1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(
d*x)*f^c)*c^2-1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))+1/2*I/d^2*c^2*ln((b*f^(d*x)
*f^c+a-I)/(a-I))+1/4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))-1/2*I/d*c*ln((b*f^(d*x)*
f^c+I+a)/(I+a))*x-1/4*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^2-1/2*I/d*ln(1
-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f^
c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(1/(b*f^(d*x)*f^c + a))
```

Fricas [A]
time = 3.28, size = 304, normalized size = 1.22

$$\frac{2 d^{2} x^{2} \operatorname{arccot}\left(b f^{dx+c}+a\right) \log(f)^{2}+i c^{2} \log\left(b f^{dx+c}+a+i\right) \log(f)^{2}-i c^{2} \log\left(b f^{dx+c}+a-i\right) \log(f)^{2}-2 i d x \operatorname{Li}_{2}\left(-\frac{a^{2}+(ab+ib) f^{dx+c}+1}{a^{2}+1}+1\right) \log(f)+2 i d x \operatorname{Li}_{2}\left(-\frac{a^{2}+(ab-ib) f^{dx+c}+1}{a^{2}+1}+1\right) \log(f)+\left(-i d^{2} x^{2}+i c^{2}\right) \log(f)^{2} \log\left(\frac{a^{2}+(ab+ib) f^{dx+c}+1}{a^{2}+1}\right)+\left(i d^{2} x^{2}-i c^{2}\right) \log(f)^{2} \log\left(\frac{a^{2}+(ab-ib) f^{dx+c}+1}{a^{2}+1}\right)+2 i \operatorname{polylog}\left(3,-\frac{(ab+ib) f^{dx+c}}{a^{2}+1}\right)-2 i \operatorname{polylog}\left(3,-\frac{(ab-ib) f^{dx+c}}{a^{2}+1}\right)}{4 d^{2} \log(f)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arccot(b*f^(d*x + c) + a)*log(f)^2 + I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 - I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 - 2*I*d*x*di
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + 2*I*d*x*di
```

```
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (-I*d^2*x^
2 + I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (I
*d^2*x^2 - I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) + 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acot}\left(a + bf^c f^{dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(a+b*f**(d*x+c)),x)
[Out] Integral(x*acot(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(x*arccot(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acot}\left(a + b\, f^{c+d\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(a + b*f^(c + d*x)),x)
[Out] int(x*acot(a + b*f^(c + d*x)), x)
```

3.226 $\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=313

$$-\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

```
[Out] -1/6*I*x^3*ln(1-b*f^(d*x+c)/(I-a))+1/6*I*x^3*ln(1+b*f^(d*x+c)/(I+a))+1/6*I*
x^3*ln(1-I/(a+b*f^(d*x+c)))-1/6*I*x^3*ln(1+I/(a+b*f^(d*x+c)))-1/2*I*x^2*pol
ylog(2,b*f^(d*x+c)/(I-a))/d/ln(f)+1/2*I*x^2*polylog(2,-b*f^(d*x+c)/(I+a))/d
/ln(f)+I*x*polylog(3,b*f^(d*x+c)/(I-a))/d^2/ln(f)^2-I*x*polylog(3,-b*f^(d*x
+c)/(I+a))/d^2/ln(f)^2-I*polylog(4,b*f^(d*x+c)/(I-a))/d^3/ln(f)^3+I*polylog
(4,-b*f^(d*x+c)/(I+a))/d^3/ln(f)^3
```

Rubi [A]
time = 1.97, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5252, 2631, 12, 6874, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}+\frac{i\text{Li}_4\left(-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)}+\frac{ix\text{Li}_3\left(\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{ix\text{Li}_3\left(-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}-\frac{ix^2\text{Li}_2\left(\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}+\frac{ix^2\text{Li}_2\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}-\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{a+i}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCot[a + b*f^(c + d*x)],x]
```

```
[Out] (-1/6*I)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] + (I/6)*x^3*Log[1 + (b*f^(c +
 d*x))/(I + a)] + (I/6)*x^3*Log[1 - I/(a + b*f^(c + d*x))] - (I/6)*x^3*Log[
1 + I/(a + b*f^(c + d*x))] - ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)]
)/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]
) + (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) - (I*x*PolyLog
[3, -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) - (I*PolyLog[4, (b*f^(c + d
*x))/(I - a)])/(d^3*Log[f]^3) + (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/
(d^3*Log[f]^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 5252

```
Int[ArcCot[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I/(a + b*f^(c + d*x))], x], x] - Dist[I/2, Int[
x^m*Log[1 + I/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cot^{-1}\left(a+bf^{c+dx}\right)\,dx &= \frac{1}{2}i\int x^2\log\left(1-\frac{i}{a+bf^{c+dx}}\right)dx-\frac{1}{2}i\int x^2\log\left(1+\frac{i}{a+bf^{c+dx}}\right)dx\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{6}\int\frac{bdf\cdots}{(i(1-ia)-\cdots}\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\cdots\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\cdots\\
&= \frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}ix^3\log\left(1+\frac{i}{a+bf^{c+dx}}\right)-\frac{1}{6}(ibd\log(f))\int\cdots\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+\cdots}}\right.\cdots\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+\cdots}}\right.\cdots\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+\cdots}}\right.\cdots\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+\cdots}}\right.\cdots\\
&= -\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{i-a}\right)+\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{i+a}\right)+\frac{1}{6}ix^3\log\left(1-\frac{i}{a+bf^{c+\cdots}}\right.\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 345, normalized size = 1.10

$$
\frac{1}{3}x^3\cot^{-1}\left(a+bf^{c+dx}\right)+\frac{b\left(d^3x^3\log^3(f)\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-d^3x^3\log^3(f)\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)+3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)-6dx\log(f)\text{PolyLog}\left(3,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)+6dx\log(f)\text{PolyLog}\left(3,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)+6\text{PolyLog}\left(4,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-6\text{PolyLog}\left(4,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)}{6\sqrt{-b^2}\,d^3\log^3(f)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCot[a + b*f^(c + d*x)],x]

[Out] (x^3*ArcCot[a + b*f^(c + d*x)])/3 + (b*(d^3*x^3*Log[f]^3*Log[1 + (b^2*f^(c
+ d*x))/(a*b - Sqrt[-b^2])] - d^3*x^3*Log[f]^3*Log[1 + (b^2*f^(c + d*x))/(a
```

```
*b + Sqrt[-b^2])] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b^2*f^(c + d*x))/(a*b
- Sqrt[-b^2]))] - 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b^2*f^(c + d*x))/(a*b +
Sqrt[-b^2]))] - 6*d*x*Log[f]*PolyLog[3, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^
2]))] + 6*d*x*Log[f]*PolyLog[3, -((b^2*f^(c + d*x))/(a*b + Sqrt[-b^2]))] +
6*PolyLog[4, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - 6*PolyLog[4, -((b^2
*f^(c + d*x))/(a*b + Sqrt[-b^2]))]))/(6*Sqrt[-b^2]*d^3*Log[f]^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(281) = 562$.
time = 0.16, size = 764, normalized size = 2.44

method	result
risch	$-\frac{i\ln\left(1-\frac{ib f^{dx} f^{c}}{-ia+1}\right) x c^{2}}{2d^{2}}-\frac{i\,\mathrm{polylog}\left(2,\frac{ib f^{dx} f^{c}}{-ia-1}\right) x^{2}}{2d\ln\left(f\right)}+\frac{i\,\mathrm{polylog}\left(2,\frac{ib f^{dx} f^{c}}{-ia-1}\right) c^{2}}{2d^{3}\ln\left(f\right)}+\frac{ic^{2}\ln\left(\frac{b f^{dx} f^{c}+i+a}{i+a}\right) x}{2d^{2}}-\frac{i\ln\left(1-\frac{ib f^{dx} f^{c}}{-ia+1}\right)}{3d^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccot(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+I+a)/(I+a))*x-1/2*I/d^2*ln(1-I*b/(1-I*a)*f^
(d*x)*f^c)*x*c^2+1/6*I/d^3*c^3*ln(I*f^(d*x)*f^c*b+I*a+1)-1/3*I/d^3*ln(1-I*b
/(1-I*a)*f^(d*x)*f^c)*c^3-1/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c*b)+1/2*I/d^3
*c^3*ln((b*f^(d*x)*f^c+I+a)/(I+a))+1/3*I/d^3*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)
*c^3-I/d^3/ln(f)^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)+I/d^2/ln(f)^2*polylo
g(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/d/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x
)*f^c)*x^2+1/2*I/d^3/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2-I/d^2/ln
(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^c)*x+1/2*I/d^3/ln(f)*c^2*dilog((b*f^(
d*x)*f^c+I+a)/(I+a))-1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^c+a-I)/(a-I))-1
/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^
(d*x)*f^c)*x*c^2+1/2*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2-1/2*I
/d^3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+1/6*Pi*x^3+I/d^3/ln(f)^3*
polylog(4,I*b/(1-I*a)*f^(d*x)*f^c)-1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a-I)/(a-
I))-1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))-1/6*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*
x^3+1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))+1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x
^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f^
c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(1/(b*f^(d*x)*f^c + a))
```

Fricas [A]
time = 2.03, size = 378, normalized size = 1.21

$$\frac{2\,d^3x^3\,\mathrm{arccot}\left(bf^{dx+c}+a\right)\log\left(f\right)^3-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2-i\,c^3\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^3+i\,c^3\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^3+\left(-i\,d^3x^3-i\,c^3\right)\log\left(f\right)^3\log\left(\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}\right)+\left(i\,d^3x^3+i\,c^3\right)\log\left(f\right)^3\log\left(\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}\right)+6i\,dx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{(ab+i\,b)f^{dx+c}}{a^2+1}\right)-6i\,dx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{(ab-i\,b)f^{dx+c}}{a^2+1}\right)-6i\,\mathrm{polylog}\left(4,-\frac{(ab+i\,b)f^{dx+c}}{a^2+1}\right)+6i\,\mathrm{polylog}\left(4,-\frac{(ab-i\,b)f^{dx+c}}{a^2+1}\right)}{6\,d^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arccot(b*f^(d*x + c) + a)*log(f)^3 - 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 + I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^
2 + 1)) + (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) + 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) - 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{acot}\left(a+bf^cf^{dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acot(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x**2*acot(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccot(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccot(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acot}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acot(a + b*f^(c + d*x)),x)
```

```
[Out] int(x^2*acot(a + b*f^(c + d*x)), x)
```

3.227 $\int e^{-x} \cot^{-1}(e^x)\, dx$

Optimal. Leaf size=27

$$-x-e^{-x}\cot^{-1}(e^x)+\frac{1}{2}\log\left(1+e^{2x}\right)$$

```
[Out] -x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2225, 5316, 2320, 36, 29, 31}

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\int e^{-x} \cot^{-1}\left(e^x\right) dx &= -e^{-x} \cot^{-1}\left(e^x\right) - \int \frac{1}{1+e^{2x}}\, dx \\
&= -e^{-x} \cot^{-1}\left(e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\, dx, x, e^{2x}\right) \\
&= -e^{-x} \cot^{-1}\left(e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\, dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\, dx, x, e^{2x}\right) \\
&= -x - e^{-x} \cot^{-1}\left(e^x\right) + \frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.00

$$
-x - e^{-x} \cot^{-1}\left(e^x\right) + \frac{1}{2}\log\left(1+e^{2x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^x]/E^x,x]
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.93

method	result	size
derivativedivides	$-\mathrm{arccot}(\mathrm{e}^x)\,\mathrm{e}^{-x} + \frac{\ln(1+\mathrm{e}^{2x})}{2} - \ln\left(\mathrm{e}^x\right)$	25
default	$-\mathrm{arccot}(\mathrm{e}^x)\,\mathrm{e}^{-x} + \frac{\ln(1+\mathrm{e}^{2x})}{2} - \ln\left(\mathrm{e}^x\right)$	25
risch	$-\frac{i\mathrm{e}^{-x}\ln(1+i\mathrm{e}^x)}{2} - \frac{(-\ln(1+\mathrm{e}^{2x})\mathrm{e}^x - i\ln(1-i\mathrm{e}^x) + 2x\,\mathrm{e}^x + \pi)\mathrm{e}^{-x}}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(exp(x))/exp(x)+1/2*ln(exp(x)^2+1)-ln(exp(x))
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.70

$$-\operatorname{arccot}\left(e^{x}\right)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Fricas [A]
time = 1.40, size = 28, normalized size = 1.04

$$-\frac{1}{2}\left(2\,xe^{x}-e^{x}\log\left(e^{(2x)}+1\right)+2\operatorname{arccot}\left(e^{x}\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A]
time = 1.44, size = 19, normalized size = 0.70

$$-x+\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\operatorname{acot}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(x))/exp(x),x)
```

```
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

Giac [A]
time = 0.54, size = 21, normalized size = 0.78

$$-\arctan\left(e^{(-x)}\right)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Mupad [B]
time = 0.09, size = 22, normalized size = 0.81

$$\frac{\ln\left(e^{2\,x}+1\right)}{2}-x-\mathrm{acot}(e^x)\ e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(exp(x))*exp(-x),x)
```

```
[Out] log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.228 $\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx$

Optimal. Leaf size=17

$$\frac{\log(1-2\cot^{-1}(x))}{2ab}$$

```
[Out] 1/2*ln(1-2*arccot(x))/a/b
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5003}

$$\frac{\log(1-2\cot^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
```

```
[Out] Log[1 - 2*ArcCot[x]]/(2*a*b)
```

Rule 5003

```
Int[1/(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[-Log[RemoveContent[a + b*ArcCot[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{(a+ax^2)(b-2b\cot^{-1}(x))}\,dx=\frac{\log(1-2\cot^{-1}(x))}{2ab}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\log(-1+2\cot^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcCot[x])),x]
```

```
[Out] Log[-1 + 2*ArcCot[x]]/(2*a*b)
```

Maple [A]
time = 0.14, size = 19, normalized size = 1.12

method	result	size
default	$\frac{\ln(2b\,\mathrm{arccot}(x)-b)}{2ab}$	19
risch	$\frac{\ln(\ln(ix+1)+i(i\ln(-ix+1)-\pi+1))}{2ab}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a)/(b-2*b*arccot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*ln(2*b*arccot(x)-b)/b
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.00

$$\frac{\log\left(|2\arctan\left(1,x\right)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(abs(2*arctan2(1, x) - 1))/(a*b)
```

Fricas [A]
time = 3.99, size = 15, normalized size = 0.88

$$\frac{\log\left(2\,\mathrm{arccot}\left(x\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*arccot(x) - 1)/(a*b)
```

Sympy [A]
time = 0.23, size = 12, normalized size = 0.71

$$\frac{\log\left(\mathrm{acot}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a)/(b-2*b*acot(x)),x)
```

```
[Out] log(acot(x) - 1/2)/(2*a*b)
```

Giac [A]
time = 0.40, size = 18, normalized size = 1.06

$$\frac{\log\left(\left|2\arctan\left(\frac{1}{x}\right)-1\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arccot(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*arctan(1/x) - 1))/(a*b)
```

Mupad [B]
time = 0.13, size = 15, normalized size = 0.88

$$\frac{\ln\left(2\,\mathrm{acot}(x)-1\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x^2)*(b - 2*b*acot(x))),x)
```

```
[Out] log(2*acot(x) - 1)/(2*a*b)
```

3.229 $\int e^{c(a+bx)} \cot^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=47

$$\frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

[Out] exp(b*c*x+a*c)*arccot(sinh(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2225, 5316, 2320, 12, 266}

$$\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}+\frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Sinh[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cot^{-1}(\sinh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\cot^{-1}(\sinh(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{2x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 1.30

$$\frac{-e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Sinh[a*c + b*c*x]],x]
[Out] (-(E^(c*(a + b*x))*ArcCot[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2]) + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 1281, normalized size = 27.26

method	result	size
risch	Expression too large to display	1281

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
[Out] -1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x
+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a
```

```
))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp
(c*(b*x+a))+I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(
b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4
/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*
x+a))-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-2*a/b+1/2/b/c*Pi*csgn(I*(ex
p(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*
(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csg
n(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a)
)+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(I*(ex
p(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csg
n(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2
)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x
+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+ln(1+exp(
2*c*(b*x+a)))/b/c+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3
*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp
(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi
*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csg
n(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(
exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(e
xp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))
-I)
```

Maxima [A]
time = 0.49, size = 47, normalized size = 1.00

$$\frac{\operatorname{arccot}\left(\sinh\left(bcx + ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{\left(2\,bcx + 2\,ac\right)} + 1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.
time = 1.37, size = 131, normalized size = 2.79

$$\frac{\left(\cosh\left(bcx + ac\right) + \sinh\left(bcx + ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\,\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right) + \log\left(\frac{2\,\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acot}\left(\sinh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acot(sinh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(sinh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.56, size = 66, normalized size = 1.40

$$\frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.73, size = 65, normalized size = 1.38

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{acot}\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acot(sinh(a*c + b*c*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*acot((exp(b*c
*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.230 $\int e^{c(a+bx)} \cot^{-1}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2c(a+}\right.}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(cosh(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 5316, 2320, 12, 1261, 646, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Cosh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cot^{-1}(\cosh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\cot^{-1}(\cosh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{e^x\sinh(x)}{1+\cosh^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx, x,\right.}{2bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bcx}\right.}{2bc}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.08, size = 146, normalized size = 1.42

$$\frac{4c(a+bx)+2e^{c(a+bx)}\cot^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)-7ac\#1^2-7bcx\#1^2+7\log\left(e^{c(a+bx)}-\#1\right)\#1^2}{1+3\#1^2}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Cosh[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.27, size = 1358, normalized size = 13.18

method	result	size
risch	Expression too large to display	1358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/b/c*
Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))
*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*
x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp
(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))
))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))
*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(
I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b
*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*
(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))
^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))
)*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*
x+a))))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a)
)+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c
*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+
a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a
))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/c*Pi*cs
gn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*
(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c
*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*
```

```
(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b
*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b
*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+
a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*ex
p(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*
x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(e
xp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2
))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/
b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)
-1)^2)
```

Maxima [A]
time = 0.48, size = 131, normalized size = 1.27

$$\frac{\operatorname{arccot}\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\,\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc}+\frac{2\left(bcx+ac\right)}{bc}+\frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac)}+1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) + 2*(b*c*x + a*c)/(b*c) + 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.
time = 2.33, size = 276, normalized size = 2.68

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^{2}+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}+1}\right)+\sqrt{2}\,\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^{2}-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}+3\right)\sinh(bcx+ac)^{2}+2\sqrt{2}+3}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}+3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}+3\right)}{\cosh(bcx+ac)^{2}-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acot}\left(\cosh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acot(cosh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(cosh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.45, size = 154, normalized size = 1.50

$$-\frac{\left(\sqrt{2}\,e^{(-ac)}\log\left(-\frac{2\sqrt{2}\,e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}\,e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)-2\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.86, size = 133, normalized size = 1.29

$$\frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}+1\right)}{2\,b\,c}-\frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}-1\right)}{2\,b\,c}+\frac{e^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acot(cosh(a*c + b*c*x)),x)
```

```
[Out] (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1
/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6*2^(1/2)*exp(2
*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (exp(a*c + b*c*x)*acot((exp(b*c*x)*
exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.231 $\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=180

$$\frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\log\left(1+e^{2c(a+bx)}-\right.}{2\sqrt{2}\,bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(tanh(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2225, 5316, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\text{ArcTan}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}+\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}+\frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Tanh[c*(a + b*x)]])/(b*c) - ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b
*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
- Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
```

```
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\cot^{-1}(\tanh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\cot^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc} + \text{Subs}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,e^{ac+bcx}\right)}{2bc}$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}\,e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}\,bc} -$$
$$= \frac{e^{ac+bcx}\cot^{-1}(\tanh(c(a+bx)))}{bc} - \frac{\tan^{-1}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc} + \frac{\tan^{-1}\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 89, normalized size = 0.49

$$\frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Tanh[a*c + b*c*x]],x]
[Out] (2*E^(c*(a + b*x))*ArcCot[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.45, size = 1323, normalized size = 7.35

method	result	size
risch	Expression too large to display	1323

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(
b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I)
/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))
-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4
/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2
*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(ex
p(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))
+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+
a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*
x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*
(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x
+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I
)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+
a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b
*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*
c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*ln(exp(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))
*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2)-1/4/b/c*ln(
exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))-(1/2+
1/2*I)*2^(1/2))*2^(1/2)+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2
*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+ex
p(2*c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I
)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*
c*(b*x+a))+I)+1/4/b/c*exp(c*(b*x+a))*Pi+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x
+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a))*ln(
exp(2*c*(b*x+a))-I)-1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2
)+1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp
(c*(b*x+a))-(1/2+1/2*I)*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+(-1/2+
1/2*I)*2^(1/2))*2^(1/2)-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(
b*x+a))))^3*exp(c*(b*x+a))
```

Maxima [A]
time = 0.49, size = 167, normalized size = 0.93

$$\frac{\operatorname{arccot}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bcx+ac)}\right)\right)}{2bc}+\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bcx+ac)}\right)\right)}{2bc}-\frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}+\frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="maxima")

[Out] arccot(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(153) = 306$.
time = 2.28, size = 431, normalized size = 2.39

$$-\frac{4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)-4\arctan\left(\frac{e^{(2\,bcx+2\,ac)}+1}{e^{(2\,bcx+2\,ac)}-1}\right)e^{(bcx+ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="fricas")

[Out] -1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c)) - 4*arctan((e^(2*b*c*x + 2*a*c) + 1)/(e^(2*b*c*x + 2*a*c) - 1))*e^(b*c*x + a*c))/(b*c)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acot}\left(\tanh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*acot(tanh(b*c*x+a*c)),x)

[Out] exp(a*c)*Integral(exp(b*c*x)*acot(tanh(a*c + b*c*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.56, size = 164, normalized size = 0.91

$$\frac{4\,{\mathrm{e}}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{{\mathrm{e}}^{2\,b\,c\,x}\,{\mathrm{e}}^{2\,a\,c}-1}{{\mathrm{e}}^{2\,b\,c\,x}\,{\mathrm{e}}^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4{}\mathrm{i}\right)-{\mathrm{e}}^{b\,c\,x}\,{\mathrm{e}}^{a\,c}\,8{}\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4{}\mathrm{i}\right)+{\mathrm{e}}^{b\,c\,x}\,{\mathrm{e}}^{a\,c}\,8{}\mathrm{i}\right)\,\left(-1+1{}\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4-4{}\mathrm{i}\right)+{\mathrm{e}}^{b\,c\,x}\,{\mathrm{e}}^{a\,c}\,8{}\mathrm{i}\right)\,\left(1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4+4{}\mathrm{i}\right)-{\mathrm{e}}^{b\,c\,x}\,{\mathrm{e}}^{a\,c}\,8{}\mathrm{i}\right)\,\left(1+1{}\mathrm{i}\right)}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acot(tanh(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2^(
1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(2*b*
c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.232 $\int e^{c(a+bx)} \cot^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=180

$$\frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}+\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\log\left(1+e^{2c(a+bx)}-\cdots\right.}{2\sqrt{2}\,bc}$$

[Out] exp(b*c*x+a*c)*arccot(coth(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2225, 5316, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\text{ArcTan}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}+\frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]

[Out] (E^(a*c + b*c*x)*ArcCot[Coth[c*(a + b*x)]])/(b*c) + ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c) + Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
```

```
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)}\cot^{-1}(\coth(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\cot^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx,x,e^{ac+bcx}\right)}{bc}+\frac{\text{Sub}}{}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,e^{ac+bcx}\right)}{2bc}-\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}-\frac{\log\left(1-\sqrt{2}\,e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}\,bc}+\frac{\text{l}}{}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\coth(c(a+bx)))}{bc}+\frac{\tan^{-1}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\tan^{-1}\left(1\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 89, normalized size = 0.49

$$
\frac{2e^{c(a+bx)}\cot^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Coth[a*c + b*c*x]],x]
[Out] (2*E^(c*(a + b*x))*ArcCot[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 1323, normalized size = 7.35

method	result	size
risch	Expression too large to display	1323

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(e
xp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(
exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(
exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I
)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1
))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(
b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a)
)+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn
(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b/c*ln(exp
(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+(1/2-1/
2*I)*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^(1/2
)-1/4/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I)*2^(1/2))*2^(1/2)+1/2*I/b/c*exp(c*(b
*x+a))*ln(exp(2*c*(b*x+a))+I)-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*cs
gn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1)
)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*
(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2
*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2
*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*cs
gn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi
*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4
/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a
))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c
*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^
2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a
))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2+1/2*I)*2^(1/2))*2^
(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+(-1/2+1/2*I)*2^(1/2))*2^(1/2)-1/2*I/b/c*e
xp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)+1/4*I/b/c*ln(exp(c*(b*x+a))-(1/2+1/2*I
)*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+(1/2-1/2*I)*2^(1/2))*2^(1/2)
+1/4/b/c*exp(c*(b*x+a))*Pi
```

Maxima [A]

time = 0.48, size = 167, normalized size = 0.93

$$\frac{\operatorname{arccot}\left(\coth\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}-\frac{\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,e^{\left(bcx+ac\right)}\right)\right)}{2\,bc}-\frac{\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,e^{\left(bcx+ac\right)}\right)\right)}{2\,bc}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\,e^{\left(bcx+ac\right)}+e^{\left(2\,bcx+2\,ac\right)}+1\right)}{4\,bc}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,e^{\left(bcx+ac\right)}+e^{\left(2\,bcx+2\,ac\right)}+1\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(153) = 306$.
time = 2.03, size = 431, normalized size = 2.39

$$\frac{4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac34}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2bcx+2ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}-1\right)+4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}e^{(bcx+ac)}+\sqrt{2}\sqrt{-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac34}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2bcx+2ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}+1\right)+\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}\log\left(\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac34}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2bcx+2ac)}\right)-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac14}\log\left(-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac34}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2bcx+2ac)}\right)+4\arctan\left(\frac{e^{(2bcx+2ac)}-1}{e^{(2bcx+2ac)}+1}\right)e^{(bcx+ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1
/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b
*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*
c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1
/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*c
^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c
))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt(
2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
+ e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*c
^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b
*c*x + 2*a*c)) + 4*arctan((e^(2*b*c*x + 2*a*c) - 1)/(e^(2*b*c*x + 2*a*c) +
1))*e^(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acot}\left(\coth\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acot(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(coth(a*c + b*c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.37, size = 164, normalized size = 0.91

$$\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{acot}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4\mathrm{i}\right)+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4\mathrm{i}\right)-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4-4\mathrm{i}\right)-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4+4\mathrm{i}\right)+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(1+1\mathrm{i}\right)}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acot(coth(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(exp
(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*acot((exp(2*b*
c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.233 $\int e^{c(a+bx)} \cot^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccot(sech(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 5316, 2320, 12, 1261, 646, 31}

$$-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Sech[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\cot^{-1}(\operatorname{sech}(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^{x}\cot^{-1}(\operatorname{sech}(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\operatorname{Subst}\left(\int \frac{e^{x}\operatorname{sech}(x)\tanh(x)}{1+\operatorname{sech}^2(x)}\,dx,x,ac+\right.}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\operatorname{Subst}\left(\int \frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{2\operatorname{Subst}\left(\int \frac{x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\operatorname{Subst}\left(\int \frac{-1+x}{1+6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{3-2\sqrt{2}+x}\,dx,\right.}{2bc}\\
&= \frac{e^{ac+bcx}\cot^{-1}(\operatorname{sech}(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2b\cdots}\right.}{2bc}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.08, size = 145, normalized size = 1.41

$$\frac{-4c(a+bx)+2e^{c(a+bx)}\cot^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)+7ac\#1^2+7bcx\#1^2-7\log\left(e^{c(a+bx)}-\#1\right)\#1^2}{1+3\#1^2}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Sech[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(1 + E^(2*c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 859, normalized size = 8.34

method	result
risch	$\frac{i e^{c(bx+a)}\ln\left(e^{2c(bx+a)}+1+2i e^{c(bx+a)}\right)}{2bc}-\frac{\pi\,\mathrm{csgn}\left(i\left(-e^{2c(bx+a)}-1+2i e^{c(bx+a)}\right)\right)\mathrm{csgn}\left(\frac{i}{1+e^{2c(bx+a)}}\right)\mathrm{csgn}\left(\frac{i\left(-e^{2c(bx+a)}-1+2i e^{c(bx+}\right.}{1+e^{2c(bx+a)}}\right.}{4bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))-1/4/b/c*
Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x+
a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))
*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))
*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+
a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi
*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x+a))
))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp
(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a)
)+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(
I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*
x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*
(b*x+a))))^3*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-
2*I*exp(c*(b*x+a)))+1/2/b/c*exp(c*(b*x+a))*Pi+1/2/b/c*ln(exp(2*c*(b*x+a))+(
2^(1/2)-1)^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)*2^(1/2)+2*
a/b-1/2/b/c*ln(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)-1/2/b/c*ln(exp(2*c*(b*x+a))+
(1+2^(1/2))^2)
```

Maxima [A]
time = 0.52, size = 169, normalized size = 1.64

$$\frac{\operatorname{arccot}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(2\,bcx+2\,ac\right)}-3}{2\sqrt{2}+e^{\left(2\,bcx+2\,ac\right)}+3}\right)}{8\,bc}-\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(-2\,bcx-2\,ac\right)}-3}{2\sqrt{2}+e^{\left(-2\,bcx-2\,ac\right)}+3}\right)}{8\,bc}-\frac{\log\left(e^{\left(4\,bcx+4\,ac\right)}+6\,e^{\left(2\,bcx+2\,ac\right)}+1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
- 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) - 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.
time = 1.85, size = 221, normalized size = 2.15

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^{2}-4\left(3\sqrt{2}-4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}-3\right)\sinh(bcx+ac)^{2}+2\sqrt{2}-3}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}+3}\right)-\log\left(\frac{2\left(\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}+3\right)}{\cosh(bcx+ac)^{2}-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cos
h(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 +
2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(co
sh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(
b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acot(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 154, normalized size = 1.50

$$\frac{\left(\sqrt{2}\,e^{(-ac)}\log\left(-\frac{2\sqrt{2}\,e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}\,e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}+\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.83, size = 135, normalized size = 1.31

$$\frac{e^{ac+bcx}\,\mathrm{acot}\left(\frac{1}{\frac{e^{bcx}\,e^{ac}}{2}+\frac{e^{-bcx}\,e^{-ac}}{2}}\right)}{b\,c}+\frac{\ln\left(-8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*acot(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))
/2)))/(b*c) + (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(
a + b*x)))*(2^(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) +
 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c)
```

3.234 $\int e^{c(a+bx)} \cot^{-1}(\mathbf{csch}(ac+bcx))\,dx$

Optimal. Leaf size=48

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc}-\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

[Out] exp(b*c*x+a*c)*arccot(csch(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5316, 2320, 12, 266}

$$\frac{e^{ac+bcx}\cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCot[Csch[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$
\begin{aligned}
\int e^{c(a+bx)} \cot^{-1}(\operatorname{csch}(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x \cot^{-1}(\operatorname{csch}(x))\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int e^x \operatorname{sech}(x)\,dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \cot^{-1}(\operatorname{csch}(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.23

$$
\frac{e^{c(a+bx)} \cot^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) - \log\left(1+e^{2c(a+bx)}\right)}{bc}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCot[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcCot[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] - Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 903, normalized size = 18.81

method	result	size
risch	Expression too large to display	903

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(
```

```
c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csg
n(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)+I)^2)*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*cs
gn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1)
)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))
+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-
1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x
+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(I*(e
xp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*
csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*
Pi*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+I/b/c
*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/2/b/c*exp(c*(b*x+a))*Pi+2*a/b-ln(1+e
xp(2*c*(b*x+a)))/b/c
```

Maxima [A]
time = 0.50, size = 48, normalized size = 1.00

$$\frac{\operatorname{arccot}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccot(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [A]
time = 1.74, size = 75, normalized size = 1.56

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\sinh\left(bcx+ac\right)\right)-\log\left(\frac{2\cosh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acot}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acot(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acot(csch(a*c + b*c*x)), x)
```

Giac [A]
time = 0.44, size = 65, normalized size = 1.35

$$\frac{\left(\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}-\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccot(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.71, size = 68, normalized size = 1.42

$$\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{acot}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c}-\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(b*c*x)*exp(a*c)*acot(1/((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*
c))/2)))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```