Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/153-5.3.7-Inverse-tangent-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [153]. This is test number [153].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (153)	0.00 (0)
Mathematica	100.00 (153)	0.00 (0)
Fricas	93.46 (143)	6.54 (10)
Maple	87.58 (134)	12.42 (19)
Maxima	56.86 (87)	43.14 (66)
Giac	38.56 (59)	61.44 (94)
Mupad	35.95 (55)	64.05 (98)
Sympy	33.99 (52)	66.01 (101)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

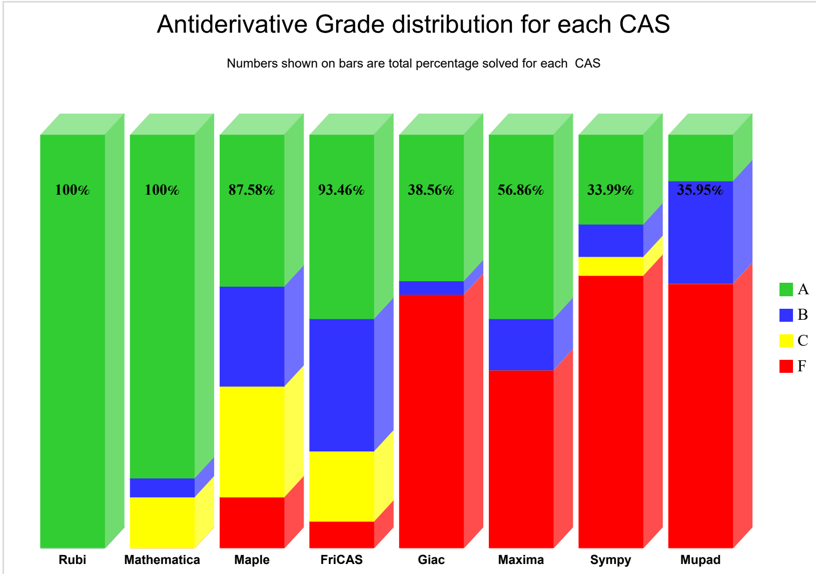
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

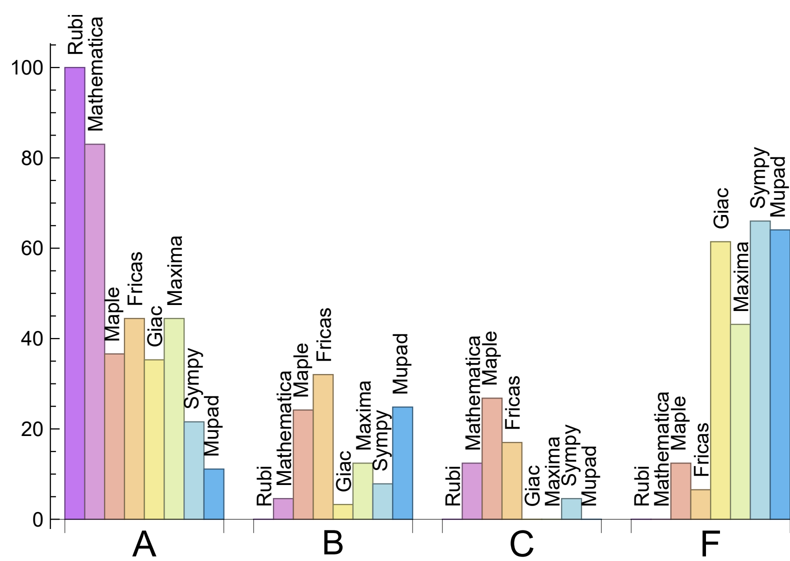
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.01	4.58	12.42	0.00
Fricas	44.44	32.03	16.99	6.54
Maxima	44.44	12.42	0.00	43.14
Maple	36.60	24.18	26.80	12.42
Giac	35.29	3.27	0.00	61.44
Sympy	21.57	7.84	4.58	66.01
Mupad	N/A	24.84	0.00	64.05

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	19	100.00 %	0.00 %	0.00 %
Fricas	10	100.00 %	0.00 %	0.00 %
Giac	94	98.94 %	1.06 %	0.00 %
Maxima	66	51.52 %	1.52 %	46.97 %
Sympy	101	40.59 %	31.68 %	27.72 %
Mupad	98	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

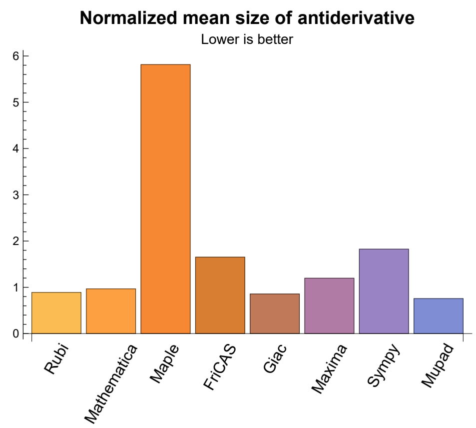
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

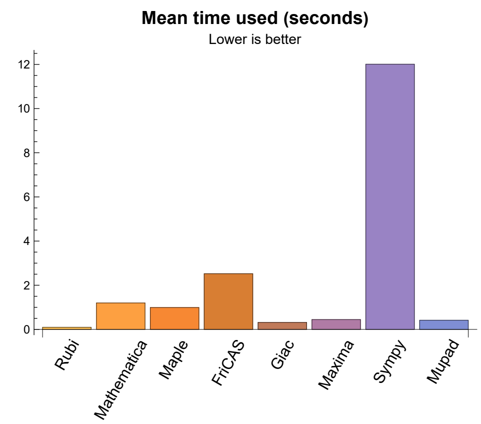
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	111.39	0.89	85.00	1.00
Mathematica	1.20	122.42	0.97	75.00	0.90
Maple	0.99	1046.39	5.81	163.00	2.10
Maxima	0.45	104.30	1.20	48.00	0.94
Fricas	2.52	266.77	1.65	85.00	1.12
Sympy	12.01	100.42	1.83	60.00	0.99
Giac	0.32	50.63	0.86	28.00	0.80
Mupad	0.41	37.20	0.76	21.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{50, 63\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

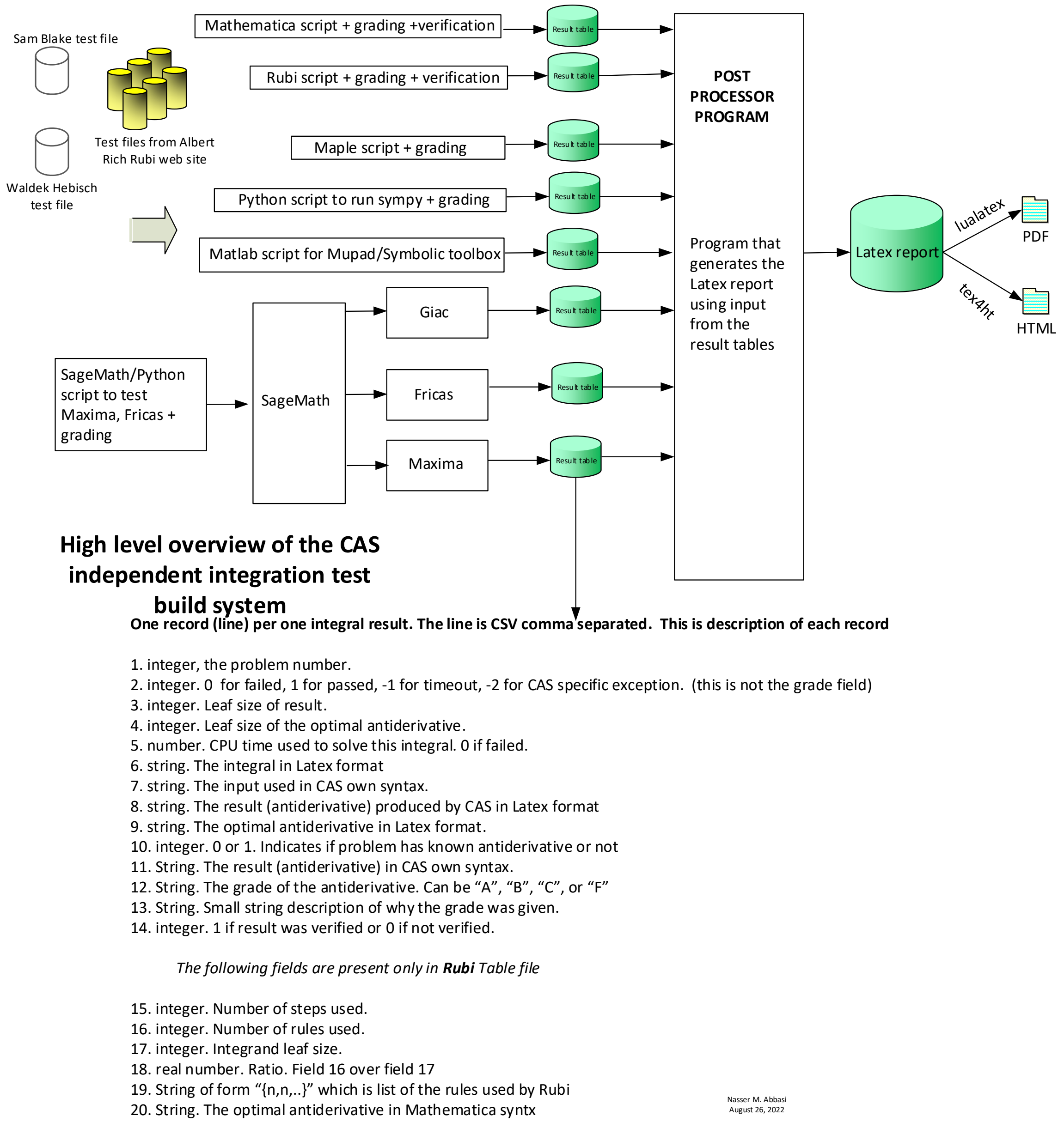
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152 }

B grade: { 50, 63, 75, 76, 83, 93, 100 }

C grade: { 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 121, 148, 149, 150, 151, 153 }

F grade: { }

2.1.3 Maple

A grade: { 1, 8, 9, 10, 15, 16, 17, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 51, 55, 59, 64, 68, 72, 73, 80, 84, 88, 92, 97, 101, 105, 109, 111, 112, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 3, 4, 5, 7, 11, 12, 13, 14, 32, 33, 34, 43, 44, 45, 50, 54, 58, 60, 63, 67, 71, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 114, 115, 117, 118, 130, 131 }

C grade: { 2, 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 146, 147, 148, 149, 150, 151, 152, 153 }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 140, 141, 142, 143, 144, 145 }

2.1.4 Maxima

A grade: { 1, 2, 7, 8, 9, 10, 11, 12, 13, 30, 31, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 51, 60, 80, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 101, 102, 103, 104, 105, 106, 107, 108, 109, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 139, 147, 148, 149, 150, 151, 152 }

B grade: { 14, 38, 39, 50, 52, 53, 54, 56, 57, 58, 63, 65, 66, 67, 69, 70, 71, 110, 113 }

C grade: { }

F grade: { 3, 4, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 48, 49, 55, 59, 61, 62, 64, 68, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 93, 94, 95, 96, 98, 99, 100, 111, 112, 114, 115, 117, 118, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 153 }

2.1.5 FriCAS

A grade: { 1, 2, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 80, 84, 88, 92, 97, 101, 105, 109, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 143, 144, 145, 147 }

B grade: { 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 73, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 113, 114, 134, 148, 149, 150, 151, 152, 153 }

C grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 }

F grade: { 6, 32, 33, 34, 130, 135, 136, 141, 142, 146 }

2.1.6 Sympy

A grade: { 1, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 38, 41, 43, 44, 45, 46, 51, 60, 80, 119, 122, 123, 124, 127, 128, 129 }

B grade: { 9, 10, 37, 39, 40, 42, 47, 120, 121, 125, 131, 132 }

C grade: { 19, 20, 21, 22, 26, 27, 28 }

F grade: { 2, 6, 18, 23, 24, 25, 29, 32, 33, 34, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 126, 130, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 30, 31, 35, 36, 40, 42, 43, 44, 45, 46, 47, 51, 55, 59, 60, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109, 119, 120, 122, 123, 124, 126, 127, 128, 129, 131, 132, 133, 147, 148, 151, 152 }

B grade: { 7, 8, 9, 10, 125 }

C grade: { }

F grade: { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 153 }

2.1.8 Mupad

A grade: { 31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109 }

B grade: { 1, 2, 14, 30, 38, 39, 40, 43, 44, 45, 47, 60, 110, 113, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 137, 138, 139, 147, 148, 149, 150, 151, 152 }

C grade: { }

F grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 41, 42, 46, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 111, 112, 114, 115, 116, 117, 118, 130, 135, 136, 140, 141, 142, 143, 144, 145, 146, 153 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	37	37	45	60	37	230
N.S.	1	1.00	0.88	0.88	0.88	1.07	1.43	0.88	5.48
time (sec)	N/A	0.030	0.015	0.102	0.252	3.049	0.858	0.409	0.686

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	140	40	58	0	40	58
N.S.	1	1.00	0.89	3.11	0.89	1.29	0.00	0.89	1.29
time (sec)	N/A	0.031	0.030	0.084	0.263	3.337	0.000	0.411	0.632

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	86	260	0	85	138	94	-1
N.S.	1	1.00	0.60	1.81	0.00	0.59	0.96	0.65	-0.01
time (sec)	N/A	0.059	0.057	0.016	0.000	3.266	1.663	0.499	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	74	212	0	75	107	80	-1
N.S.	1	1.00	0.64	1.83	0.00	0.65	0.92	0.69	-0.01
time (sec)	N/A	0.032	0.041	0.011	0.000	3.398	0.590	0.485	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	164	0	61	73	63	-1
N.S.	1	1.00	0.67	1.86	0.00	0.69	0.83	0.72	-0.01
time (sec)	N/A	0.023	0.030	0.011	0.000	4.385	0.326	0.497	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	171	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	1.827	0.034	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	122	64	56	53	106	-1
N.S.	1	1.00	0.95	2.14	1.12	0.98	0.93	1.86	-0.02
time (sec)	N/A	0.015	0.027	0.013	0.282	6.190	2.015	0.486	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	67	69	75	69	83	202	-1
N.S.	1	1.00	0.79	0.81	0.88	0.81	0.98	2.38	-0.01
time (sec)	N/A	0.021	0.033	0.012	0.279	4.395	2.421	0.518	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	78	117	116	79	352	281	-1
N.S.	1	1.00	0.69	1.04	1.03	0.70	3.12	2.49	-0.01
time (sec)	N/A	0.028	0.043	0.011	0.270	3.367	3.131	0.512	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	89	165	137	89	575	361	-1
N.S.	1	1.00	0.63	1.17	0.97	0.63	4.08	2.56	-0.01
time (sec)	N/A	0.037	0.050	0.013	0.261	2.772	4.389	0.514	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	83	231	178	86	136	137	-1
N.S.	1	1.00	0.67	1.86	1.44	0.69	1.10	1.10	-0.01
time (sec)	N/A	0.050	0.078	0.018	0.261	2.450	1.958	0.485	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	72	183	148	76	105	101	-1
N.S.	1	1.00	0.73	1.85	1.49	0.77	1.06	1.02	-0.01
time (sec)	N/A	0.044	0.066	0.013	0.262	3.274	0.832	0.461	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	135	118	66	75	64	-1
N.S.	1	1.00	0.81	1.82	1.59	0.89	1.01	0.86	-0.01
time (sec)	N/A	0.027	0.061	0.012	0.288	2.592	0.433	0.465	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	86	82	53	41	40	35
N.S.	1	1.00	1.00	2.00	1.91	1.23	0.95	0.93	0.81
time (sec)	N/A	0.008	0.013	0.016	0.264	2.750	0.324	0.436	0.658

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	86	90	0	181	60	53	-1
N.S.	1	1.00	1.46	1.53	0.00	3.07	1.02	0.90	-0.02
time (sec)	N/A	0.023	0.042	0.010	0.000	3.646	2.785	0.476	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	130	0	225	82	87	-1
N.S.	1	1.00	1.11	1.43	0.00	2.47	0.90	0.96	-0.01
time (sec)	N/A	0.033	0.065	0.010	0.000	3.003	5.696	0.487	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	114	178	0	253	148	116	-1
N.S.	1	1.00	0.96	1.50	0.00	2.13	1.24	0.97	-0.01
time (sec)	N/A	0.043	0.082	0.012	0.000	3.766	10.067	0.476	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	170	0	0	92	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.44	0.00	0.00	-0.00
time (sec)	N/A	0.094	10.413	0.029	0.000	0.658	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	158	0	0	82	75	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.45	0.41	0.00	-0.01
time (sec)	N/A	0.072	0.305	0.030	0.000	0.574	51.932	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	0	0	69	75	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.45	0.49	0.00	-0.01
time (sec)	N/A	0.057	0.205	0.026	0.000	0.659	2.181	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	0	0	50	71	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.41	0.58	0.00	-0.01
time (sec)	N/A	0.049	0.090	0.027	0.000	0.513	2.824	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	0	0	76	78	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.49	0.50	0.00	-0.01
time (sec)	N/A	0.060	0.191	0.029	0.000	0.438	36.691	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	162	0	0	90	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.234	0.026	0.000	0.668	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	171	0	0	100	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.088	0.397	0.029	0.000	0.511	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	139	0	0	90	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.156	10.112	0.027	0.000	1.911	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	119	0	0	76	75	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.26	0.25	0.00	-0.00
time (sec)	N/A	0.130	0.082	0.028	0.000	1.419	6.080	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	89	0	0	52	71	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.20	0.27	0.00	-0.00
time (sec)	N/A	0.109	0.077	0.027	0.000	0.512	4.575	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	121	0	0	83	78	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.28	0.26	0.00	-0.00
time (sec)	N/A	0.129	0.099	0.030	0.000	1.072	5.523	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	137	0	0	97	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.29	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.083	0.027	0.000	1.389	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	43	42	47	41	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.94	0.82	0.86	0.84
time (sec)	N/A	0.108	0.015	0.075	0.469	2.223	0.300	0.426	0.759

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.071	0.105	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	530	1631	0	0	0	0	-1
N.S.	1	1.00	1.23	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.139	0.280	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	354	837	0	0	0	0	-1
N.S.	1	1.00	1.25	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.088	0.076	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	263	0	0	0	0	-1
N.S.	1	1.00	0.74	2.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.063	0.066	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.074	0.112	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.588	0.116	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	33	158	0	-1
N.S.	1	1.00	0.92	1.11	1.03	0.89	4.27	0.00	-0.03
time (sec)	N/A	0.017	0.044	0.120	0.264	1.674	0.955	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	81	13	32	0	19
N.S.	1	1.00	0.87	0.87	3.52	0.57	1.39	0.00	0.83
time (sec)	N/A	0.006	0.014	0.042	0.264	2.674	0.110	0.000	0.120

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	57	13	76	0	19
N.S.	1	1.00	0.87	0.87	2.48	0.57	3.30	0.00	0.83
time (sec)	N/A	0.006	0.013	0.036	0.265	2.632	0.096	0.000	0.061

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.002	0.006	0.031	0.259	1.841	0.059	0.396	0.039

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	42	8	34	0	-1
N.S.	1	1.00	0.90	1.10	2.00	0.38	1.62	0.00	-0.05
time (sec)	N/A	0.022	0.012	0.043	0.458	0.608	0.494	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	56	40	42	160	62	-1
N.S.	1	1.00	0.86	1.56	1.11	1.17	4.44	1.72	-0.03
time (sec)	N/A	0.016	0.037	0.147	0.262	2.011	3.114	0.415	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	65	17	17	26	19	25
N.S.	1	1.00	0.87	2.83	0.74	0.74	1.13	0.83	1.09
time (sec)	N/A	0.006	0.014	0.084	0.261	1.396	0.118	0.440	0.542

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	54	17	17	26	19	25
N.S.	1	1.00	0.87	2.35	0.74	0.74	1.13	0.83	1.09
time (sec)	N/A	0.005	0.017	0.079	0.268	0.705	0.081	0.404	0.072

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	15	15	24	15	21
N.S.	1	1.00	1.12	3.19	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.003	0.006	0.036	0.261	1.928	0.061	0.436	0.071

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	35	14	14	27	15	-1
N.S.	1	1.00	1.00	1.84	0.74	0.74	1.42	0.79	-0.05
time (sec)	N/A	0.023	0.016	0.073	0.263	0.578	2.736	0.405	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.002	0.001	0.000	0.260	1.569	0.065	0.404	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	363	8112	0	1965	0	0	-1
N.S.	1	1.00	0.90	20.13	0.00	4.88	0.00	0.00	-0.00
time (sec)	N/A	0.386	0.720	19.223	0.000	0.818	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	272	7720	0	1545	0	0	-1
N.S.	1	1.00	0.89	25.31	0.00	5.07	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.476	1.539	0.000	2.006	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	555	1001	433	1101	0	0	-1
N.S.	1	1.00	2.80	5.06	2.19	5.56	0.00	0.00	-0.01
time (sec)	N/A	0.176	5.020	0.779	0.530	0.645	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.102	3.574	0.059	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	140	1532	310	322	0	0	-1
N.S.	1	1.00	0.91	9.95	2.01	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.338	0.806	0.283	0.887	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	218	271	0	0	-1
N.S.	1	1.00	0.89	12.17	1.77	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.253	0.494	0.313	1.166	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	600	448	202	0	0	-1
N.S.	1	1.00	0.88	7.06	5.27	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.091	6.299	0.259	0.525	1.209	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.105	0.512	0.066	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	137	1533	310	322	0	0	-1
N.S.	1	1.00	0.88	9.89	2.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.358	0.779	0.308	1.403	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	1498	218	271	0	0	-1
N.S.	1	1.00	0.90	12.08	1.76	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.269	0.510	0.291	0.648	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	649	448	200	0	0	-1
N.S.	1	1.00	0.88	7.55	5.21	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.090	6.222	0.263	0.514	1.547	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.120	0.831	0.074	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	51	15	15	24	15	21
N.S.	1	1.00	1.12	3.19	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.003	0.007	0.001	0.275	1.224	0.057	0.418	0.002

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	359	7924	0	1589	0	0	-1
N.S.	1	1.00	0.90	19.86	0.00	3.98	0.00	0.00	-0.00
time (sec)	N/A	0.385	0.710	20.123	0.000	1.078	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	270	7556	0	1289	0	0	-1
N.S.	1	1.00	0.89	24.94	0.00	4.25	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.467	1.535	0.000	1.562	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	1648	1145	532	965	0	0	-1
N.S.	1	1.00	8.32	5.78	2.69	4.87	0.00	0.00	-0.01
time (sec)	N/A	0.182	20.562	0.774	0.559	1.503	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.106	3.505	0.084	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1532	309	166	0	0	-1
N.S.	1	1.00	0.88	9.95	2.01	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.254	0.754	0.286	2.467	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	1497	217	144	0	0	-1
N.S.	1	1.00	0.89	12.17	1.76	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.155	0.503	0.274	2.256	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	611	458	112	0	0	-1
N.S.	1	1.00	0.88	7.19	5.39	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.096	6.050	0.270	0.503	2.849	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	0.531	0.079	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	140	1533	311	166	0	0	-1
N.S.	1	1.00	0.90	9.89	2.01	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.257	0.757	0.297	3.293	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	1498	219	144	0	0	-1
N.S.	1	1.00	0.89	12.08	1.77	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.161	0.508	0.278	3.406	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	75	690	458	112	0	0	-1
N.S.	1	1.00	0.87	8.02	5.33	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.101	5.955	0.266	0.493	2.377	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	0.556	0.082	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	64	52	0	58	0	0	-1
N.S.	1	1.00	1.64	1.33	0.00	1.49	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.020	0.248	0.000	1.874	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	105	732	0	93	0	0	-1
N.S.	1	1.00	1.42	9.89	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.016	0.125	0.000	2.074	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	356	758	0	125	0	0	-1
N.S.	1	1.00	3.30	7.02	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.075	0.110	0.000	1.604	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	2700	0	0	-1
N.S.	1	1.00	2.01	24.33	0.00	9.03	0.00	0.00	-0.00
time (sec)	N/A	0.166	3.248	8.211	0.000	2.898	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	1511	0	0	-1
N.S.	1	1.00	1.64	23.69	0.00	6.60	0.00	0.00	-0.00
time (sec)	N/A	0.116	1.678	6.053	0.000	3.692	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2414	0	717	0	0	-1
N.S.	1	1.00	1.75	15.18	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.076	1.216	0.533	0.000	5.139	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	132	160	0	334	0	0	-1
N.S.	1	1.00	1.78	2.16	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.031	0.244	0.000	6.286	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.027	6.042	0.075	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	305	6990	0	1289	0	0	-1
N.S.	1	1.00	0.86	19.69	0.00	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.341	3.375	16.481	0.000	6.065	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	229	6640	0	1067	0	0	-1
N.S.	1	1.00	0.86	24.87	0.00	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	2.688	1.429	0.000	6.477	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	363	365	0	825	0	0	-1
N.S.	1	1.00	2.09	2.10	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.159	2.587	0.513	0.000	11.050	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.101	7.446	0.073	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1479	129	293	0	0	-1
N.S.	1	1.00	0.90	10.42	0.91	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.153	3.837	0.801	1.250	3.725	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1443	106	247	0	0	-1
N.S.	1	1.00	0.90	12.77	0.94	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.138	3.884	0.474	1.176	3.473	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	598	80	187	0	0	-1
N.S.	1	1.00	0.90	7.57	1.01	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.087	1.106	0.286	1.183	3.492	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.093	2.694	0.081	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1486	129	293	0	0	-1
N.S.	1	1.00	0.88	10.25	0.89	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.158	4.085	0.829	1.157	2.837	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1450	107	247	0	0	-1
N.S.	1	1.00	0.88	12.50	0.92	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.140	3.976	0.516	1.175	2.125	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	553	80	187	0	0	-1
N.S.	1	1.00	0.87	6.74	0.98	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.104	0.287	1.160	3.166	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.092	2.783	0.075	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	600	7275	0	2700	0	0	-1
N.S.	1	1.00	2.01	24.33	0.00	9.03	0.00	0.00	-0.00
time (sec)	N/A	0.154	3.251	8.608	0.000	4.513	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	375	5425	0	1511	0	0	-1
N.S.	1	1.00	1.64	23.69	0.00	6.60	0.00	0.00	-0.00
time (sec)	N/A	0.117	1.707	5.795	0.000	5.940	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	278	2415	0	717	0	0	-1
N.S.	1	1.00	1.75	15.19	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.078	1.439	0.490	0.000	2.623	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	132	160	0	334	0	0	-1
N.S.	1	1.00	1.81	2.19	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.035	0.240	0.000	5.369	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.029	3.126	0.078	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	299	6864	0	1269	0	0	-1
N.S.	1	1.00	0.85	19.56	0.00	3.62	0.00	0.00	-0.00
time (sec)	N/A	0.345	3.291	16.426	0.000	3.565	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	225	6514	0	1051	0	0	-1
N.S.	1	1.00	0.85	24.58	0.00	3.97	0.00	0.00	-0.00
time (sec)	N/A	0.303	2.777	1.459	0.000	5.610	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	363	365	0	813	0	0	-1
N.S.	1	1.00	2.09	2.10	0.00	4.67	0.00	0.00	-0.01
time (sec)	N/A	0.189	2.424	0.533	0.000	3.431	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.104	7.585	0.068	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	1478	129	293	0	0	-1
N.S.	1	1.00	0.90	10.41	0.91	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.163	1.224	0.797	1.198	2.638	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	1442	106	247	0	0	-1
N.S.	1	1.00	0.90	12.76	0.94	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.143	1.173	0.528	1.198	2.987	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	598	80	187	0	0	-1
N.S.	1	1.00	0.90	7.57	1.01	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.425	0.310	1.162	2.417	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.092	2.819	0.097	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	1487	129	293	0	0	-1
N.S.	1	1.00	0.88	10.26	0.89	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.164	1.302	0.826	1.161	3.480	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	1451	107	247	0	0	-1
N.S.	1	1.00	0.88	12.51	0.92	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.175	0.506	1.176	4.660	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	553	80	187	0	0	-1
N.S.	1	1.00	0.87	6.74	0.98	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.417	0.299	1.162	3.388	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.095	2.787	0.102	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	59	59	34	40	0	0	21
N.S.	1	1.00	1.90	1.90	1.10	1.29	0.00	0.00	0.68
time (sec)	N/A	0.018	0.046	0.071	0.498	3.342	0.000	0.000	0.688

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	44	0	65	0	0	-1
N.S.	1	1.00	0.79	0.70	0.00	1.03	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.008	0.032	0.000	2.600	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	70	0	87	0	0	-1
N.S.	1	1.00	1.00	0.77	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.007	0.031	0.000	2.449	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	83	95	63	103	0	0	37
N.S.	1	1.00	1.84	2.11	1.40	2.29	0.00	0.00	0.82
time (sec)	N/A	0.022	0.136	0.039	0.506	2.389	0.000	0.000	0.745

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	71	349	0	151	0	0	-1
N.S.	1	1.00	0.78	3.84	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.010	0.079	0.000	2.332	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	407	0	187	0	0	-1
N.S.	1	1.00	1.00	3.06	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.009	0.056	0.000	2.241	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	167	162	189	212	0	0	-1
N.S.	1	1.00	0.85	0.83	0.96	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.305	0.142	0.544	2.310	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	250	236	672	0	304	0	0	-1
N.S.	1	1.08	1.02	2.90	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.062	0.058	0.000	3.171	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	313	334	758	0	378	0	0	-1
N.S.	1	1.04	1.11	2.51	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.013	0.056	0.000	2.992	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	19	28	19	20	20
N.S.	1	1.00	1.00	0.92	0.76	1.12	0.76	0.80	0.80
time (sec)	N/A	0.016	0.013	0.029	0.290	2.911	1.466	0.430	0.089

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	32	31	50	153	32	31
N.S.	1	1.00	0.78	0.71	0.69	1.11	3.40	0.71	0.69
time (sec)	N/A	0.022	0.027	0.112	0.494	2.334	0.243	0.446	0.124

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	54	54	77	223	0	46
N.S.	1	1.00	1.27	0.84	0.84	1.20	3.48	0.00	0.72
time (sec)	N/A	0.049	0.032	0.123	0.481	1.487	0.258	0.000	0.700

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	24	18	26	24	24
N.S.	1	1.00	0.73	0.83	0.80	0.60	0.87	0.80	0.80
time (sec)	N/A	0.008	0.018	0.008	0.482	1.934	0.087	0.413	0.075

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.018	0.024	0.092	0.267	2.336	0.125	0.424	0.225

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	16	15	14	16	15
N.S.	1	1.00	1.00	1.12	0.94	0.88	0.82	0.94	0.88
time (sec)	N/A	0.027	0.024	0.084	0.296	2.150	0.244	0.434	0.135

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	26	53	104	16
N.S.	1	1.00	1.00	0.94	0.89	1.44	2.94	5.78	0.89
time (sec)	N/A	0.110	0.023	0.118	0.570	2.580	0.280	0.413	0.127

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	45	44	40	0	44	72
N.S.	1	1.00	0.85	0.66	0.65	0.59	0.00	0.65	1.06
time (sec)	N/A	0.020	0.041	0.016	0.562	2.734	0.000	0.403	1.653

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	40	39	35	46	39	65
N.S.	1	1.00	0.90	0.68	0.66	0.59	0.78	0.66	1.10
time (sec)	N/A	0.020	0.023	0.018	0.537	3.380	163.509	0.436	0.940

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	28	39	34	58
N.S.	1	1.00	0.96	0.70	0.68	0.56	0.78	0.68	1.16
time (sec)	N/A	0.014	0.021	0.015	0.544	2.654	96.429	0.416	0.850

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	28	26	22	29	27	40
N.S.	1	1.00	0.84	0.76	0.70	0.59	0.78	0.73	1.08
time (sec)	N/A	0.008	0.078	0.014	0.595	4.932	43.083	0.396	0.894

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	84	194	43	0	0	0	-1
N.S.	1	1.00	2.00	4.62	1.02	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.134	0.200	0.479	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	57	29	28	537	28	44
N.S.	1	1.00	0.98	1.39	0.71	0.68	13.10	0.68	1.07
time (sec)	N/A	0.016	0.027	0.017	0.535	4.704	33.697	0.413	1.409

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	35	34	35	755	34	49
N.S.	1	1.00	0.96	0.70	0.68	0.70	15.10	0.68	0.98
time (sec)	N/A	0.017	0.021	0.025	0.537	5.289	129.503	0.434	1.359

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	40	39	40	0	39	56
N.S.	1	1.00	0.86	0.68	0.66	0.68	0.00	0.66	0.95
time (sec)	N/A	0.018	0.029	0.026	0.514	9.510	0.000	0.439	0.939

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	59	0	126	0	0	57
N.S.	1	1.00	1.00	0.94	0.00	2.00	0.00	0.00	0.90
time (sec)	N/A	0.080	0.045	0.274	0.000	9.771	0.000	0.000	0.727

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	-1
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.021	0.094	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	-1
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.018	0.092	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	55	0	83	0	0	49
N.S.	1	1.00	1.00	1.00	0.00	1.51	0.00	0.00	0.89
time (sec)	N/A	0.075	0.039	0.625	0.000	3.053	0.000	0.000	0.599

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	57	29	82	0	0	51
N.S.	1	1.00	1.00	1.00	0.51	1.44	0.00	0.00	0.89
time (sec)	N/A	0.074	0.020	0.085	0.410	4.174	0.000	0.000	0.602

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	29	82	0	0	51
N.S.	1	1.00	1.00	0.97	0.49	1.39	0.00	0.00	0.86
time (sec)	N/A	0.072	0.019	0.083	0.435	4.189	0.000	0.000	0.614

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	0	111	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.103	0.060	0.000	3.149	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.058	0.058	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.033	0.057	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	62	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.97	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.048	0.057	0.000	1.918	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	0	0	61	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.92	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.046	0.055	0.000	2.401	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	62	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.042	0.058	0.000	2.579	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	166	1087	0	0	0	0	-1
N.S.	1	1.00	1.64	10.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	2.090	0.941	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	61	1299	48	75	0	65	66
N.S.	1	1.00	1.27	27.06	1.00	1.56	0.00	1.35	1.38
time (sec)	N/A	0.058	0.061	0.267	0.520	3.440	0.000	0.436	0.769

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	146	1375	131	221	0	154	133
N.S.	1	1.00	1.42	13.35	1.27	2.15	0.00	1.50	1.29
time (sec)	N/A	0.120	0.090	0.421	0.486	2.431	0.000	0.444	0.314

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	431	0	0	164
N.S.	1	1.00	0.49	7.53	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.131	0.067	0.441	0.496	3.067	0.000	0.000	1.416

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	89	1355	167	431	0	0	164
N.S.	1	1.00	0.49	7.53	0.93	2.39	0.00	0.00	0.91
time (sec)	N/A	0.139	0.060	0.339	0.486	3.410	0.000	0.000	1.656

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	842	169	276	0	154	135
N.S.	1	1.00	1.41	8.17	1.64	2.68	0.00	1.50	1.31
time (sec)	N/A	0.110	0.089	0.290	0.526	3.334	0.000	0.462	0.819

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	885	47	131	0	66	67
N.S.	1	1.00	1.21	18.83	1.00	2.79	0.00	1.40	1.43
time (sec)	N/A	0.062	0.051	0.171	0.546	2.869	0.000	0.448	0.697

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	116	896	0	264	0	0	-1
N.S.	1	1.00	0.71	5.50	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.431	0.186	0.187	0.000	3.316	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [32] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	12	0.333
2	A	4	4	1.00	14	0.286
3	A	6	4	1.00	25	0.160
4	A	5	4	1.00	25	0.160
5	A	4	4	1.00	23	0.174
6	A	8	8	1.00	25	0.320
7	A	2	2	1.00	25	0.080
8	A	3	3	1.00	25	0.120
9	A	4	3	1.00	25	0.120
10	A	5	3	1.00	25	0.120
11	A	4	3	1.00	25	0.120
12	A	4	3	1.00	25	0.120
13	A	4	3	1.00	25	0.120
14	A	2	2	1.00	21	0.095
15	A	4	4	1.00	25	0.160
16	A	5	5	1.00	25	0.200
17	A	6	5	1.00	25	0.200
18	A	6	4	1.00	27	0.148
19	A	5	4	1.00	27	0.148
20	A	4	4	1.00	27	0.148
21	A	3	3	1.00	27	0.111
22	A	4	4	1.00	27	0.148
23	A	5	4	1.00	27	0.148
24	A	6	4	1.00	27	0.148
25	A	7	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	27	0.222
27	A	5	5	1.00	27	0.185
28	A	6	6	1.00	27	0.222
29	A	7	6	1.00	27	0.222
30	A	8	7	1.00	11	0.636
31	A	0	0	0.00	0	0.000
32	A	9	7	1.00	40	0.175
33	A	7	6	1.00	40	0.150
34	A	4	4	1.00	38	0.105
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	11	0.182
39	A	2	2	1.00	9	0.222
40	A	2	2	1.00	7	0.286
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	9	0.222
45	A	2	2	1.00	7	0.286
46	A	2	2	1.00	11	0.182
47	A	2	2	1.00	7	0.286
48	A	11	6	1.00	15	0.400
49	A	9	5	1.00	13	0.385
50	A	7	4	1.00	11	0.364
51	A	0	0	0.00	0	0.000
52	A	7	7	1.00	21	0.333
53	A	6	6	1.00	19	0.316
54	A	5	5	1.00	17	0.294
55	A	0	0	0.00	0	0.000
56	A	7	7	1.00	21	0.333
57	A	6	6	1.00	19	0.316
58	A	5	5	1.00	17	0.294
59	A	0	0	0.00	0	0.000
60	A	2	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	11	6	1.00	15	0.400
62	A	9	5	1.00	13	0.385
63	A	7	4	1.00	11	0.364
64	A	0	0	0.00	0	0.000
65	A	7	7	1.00	21	0.333
66	A	6	6	1.00	19	0.316
67	A	5	5	1.00	17	0.294
68	A	0	0	0.00	0	0.000
69	A	7	7	1.00	21	0.333
70	A	6	6	1.00	19	0.316
71	A	5	5	1.00	17	0.294
72	A	0	0	0.00	0	0.000
73	A	6	4	1.00	3	1.333
74	A	8	5	1.00	5	1.000
75	A	10	6	1.00	7	0.857
76	A	12	6	1.00	15	0.400
77	A	10	6	1.00	15	0.400
78	A	8	5	1.00	13	0.385
79	A	6	4	1.00	7	0.571
80	A	0	0	0.00	0	0.000
81	A	11	6	1.00	15	0.400
82	A	9	5	1.00	13	0.385
83	A	7	4	1.00	11	0.364
84	A	0	0	0.00	0	0.000
85	A	7	7	1.00	19	0.368
86	A	6	6	1.00	17	0.353
87	A	5	5	1.00	15	0.333
88	A	0	0	0.00	0	0.000
89	A	7	7	1.00	22	0.318
90	A	6	6	1.00	20	0.300
91	A	5	5	1.00	18	0.278
92	A	0	0	0.00	0	0.000
93	A	12	6	1.00	15	0.400
94	A	10	6	1.00	15	0.400
95	A	8	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	4	1.00	7	0.571
97	A	0	0	0.00	0	0.000
98	A	11	6	1.00	15	0.400
99	A	9	5	1.00	13	0.385
100	A	7	4	1.00	11	0.364
101	A	0	0	0.00	0	0.000
102	A	7	7	1.00	19	0.368
103	A	6	6	1.00	17	0.353
104	A	5	5	1.00	15	0.333
105	A	0	0	0.00	0	0.000
106	A	7	7	1.00	22	0.318
107	A	6	6	1.00	20	0.300
108	A	5	5	1.00	18	0.278
109	A	0	0	0.00	0	0.000
110	A	4	3	1.00	4	0.750
111	A	7	4	1.00	6	0.667
112	A	9	5	1.00	8	0.625
113	A	4	3	1.00	8	0.375
114	A	7	4	1.00	10	0.400
115	A	9	5	1.00	12	0.417
116	A	6	6	1.00	12	0.500
117	A	9	5	1.08	14	0.357
118	A	11	6	1.04	16	0.375
119	A	5	6	1.00	10	0.600
120	A	5	4	1.00	8	0.500
121	A	9	8	1.00	14	0.571
122	A	4	4	1.00	8	0.500
123	A	1	1	1.00	14	0.071
124	A	1	1	1.00	19	0.053
125	A	5	3	1.00	26	0.115
126	A	9	6	1.00	21	0.286
127	A	8	6	1.00	21	0.286
128	A	7	6	1.00	19	0.316
129	A	6	6	1.00	18	0.333
130	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	6	6	1.00	21	0.286
132	A	7	6	1.00	21	0.286
133	A	8	6	1.00	21	0.286
134	A	2	2	1.00	39	0.051
135	A	2	2	1.00	39	0.051
136	A	2	2	1.00	37	0.054
137	A	2	2	1.00	39	0.051
138	A	2	2	1.00	39	0.051
139	A	2	2	1.00	39	0.051
140	A	2	2	1.00	40	0.050
141	A	2	2	1.00	40	0.050
142	A	2	2	1.00	38	0.053
143	A	2	2	1.00	40	0.050
144	A	2	2	1.00	40	0.050
145	A	2	2	1.00	40	0.050
146	A	9	7	1.00	24	0.292
147	A	5	5	1.00	20	0.250
148	A	8	7	1.00	20	0.350
149	A	13	10	1.00	20	0.500
150	A	13	10	1.00	20	0.500
151	A	8	7	1.00	20	0.350
152	A	5	5	1.00	20	0.250
153	A	13	9	1.00	24	0.375

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3 \text{ArcTan}(a+bx^4)\,dx$

Optimal. Leaf size=42

$$\frac{(a+bx^4)\,\text{ArcTan}(a+bx^4)}{4b}-\frac{\log\left(1+(a+bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arctan(b*x^4+a)/b-1/8*ln(1+(b*x^4+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 5147, 4930, 266}

$$\frac{(a+bx^4)\,\text{ArcTan}(a+bx^4)}{4b}-\frac{\log\left((a+bx^4)^2+1\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTan[a + b*x^4])/(4*b) - Log[1 + (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^3 \tan^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \tan^{-1}(a+bx)\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\ &= \frac{(a+bx^4)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx^4\right)}{4b}\\ &= \frac{(a+bx^4)\tan^{-1}\left(a+bx^4\right)}{4b} - \frac{\log\left(1+(a+bx^4)^2\right)}{8b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.88

$$-\frac{-2(a+bx^4)\,\text{ArcTan}(a+bx^4)+\log\left(1+(a+bx^4)^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x^4],x]
[Out] -1/8*(-2*(a + b*x^4)*ArcTan[a + b*x^4] + Log[1 + (a + b*x^4)^2])/b
```

Maple [A]
time = 0.10, size = 37, normalized size = 0.88

method	result
derivativedivides	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+(b\,x^4+a)^2\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+(b\,x^4+a)^2\right)}{2}}{4b}$
risch	$-\frac{ix^4\ln(1+i(b\,x^4+a))}{8}+\frac{ix^4\ln(1-i(b\,x^4+a))}{8}-\frac{a\arctan(a)}{4b}+\frac{a\arctan\left(\frac{x^4 b}{a^2+1}+\frac{a^2 b\,x^4}{a^2+1}+\frac{a^3}{a^2+1}+\frac{a}{a^2+1}\right)}{4b}-\frac{\ln(a^6 b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arctan(b*x^4+a)-1/2*ln(1+(b*x^4+a)^2))
```

Maxima [A]
time = 0.25, size = 37, normalized size = 0.88

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

Fricas [A]
time = 3.05, size = 45, normalized size = 1.07

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/
b
```

Sympy [A]
time = 0.86, size = 60, normalized size = 1.43

$$\begin{cases}\frac{a\operatorname{atan}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{atan}\left(a+bx^4\right)}{4}-\frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for } b\neq 0\\ \frac{x^4\operatorname{atan}\left(a\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x**4+a),x)
```

```
[Out] Piecewise((a*atan(a + b*x**4)/(4*b) + x**4*atan(a + b*x**4)/4 - log(a**2 +
2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*atan(a)/4, True))
```

Giac [A]
time = 0.41, size = 37, normalized size = 0.88

$$\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

Mupad [B]
time = 0.69, size = 230, normalized size = 5.48

$$\frac{x^4\operatorname{atan}(b\,x^4+a)}{4}-\frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}+\frac{a\operatorname{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^4\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{a^6\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a + b*x^4),x)
```

```
[Out] (x^4*atan(a + b*x^4))/4 - log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (a*ata
n(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^5)
/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*a^2
 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*b*x^
4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))/(4*b
)
```

3.2 $\int x^{-1+n}\textbf{ArcTan}(a+bx^n)\,dx$

Optimal. Leaf size=45

$$\frac{(a+bx^n)\,\mathrm{ArcTan}(a+bx^n)}{bn}-\frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

```
[Out] (a+b*x^n)*arctan(a+b*x^n)/b/n-1/2*ln(1+(a+b*x^n)^2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6847, 5147, 4930, 266}

$$\frac{(a+bx^n)\,\mathrm{ArcTan}(a+bx^n)}{bn}-\frac{\log\left((a+bx^n)^2+1\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTan[a + b*x^n])/(b*n) - Log[1 + (a + b*x^n)^2]/(2*b*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\tan^{-1}(a+bx^n)\,dx = \frac{\text{Subst}\left(\int \tan^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx^n\right)}{bn}$$
$$= \frac{(a+bx^n)\tan^{-1}(a+bx^n)}{bn} - \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.89

$$-\frac{-2(a+bx^n)\,\text{ArcTan}(a+bx^n)+\log\left(1+(a+bx^n)^2\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTan[a + b*x^n],x]
[Out] -1/2*(-2*(a + b*x^n)*ArcTan[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(b*n)
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 140, normalized size = 3.11

method	result	size
risch	$-\frac{i x^n \ln(1+i(a+b\,x^n))}{2n}+\frac{i x^n \ln(1-i(a+b\,x^n))}{2n}-\frac{\ln\left(x^n-\frac{i-a}{b}\right)}{2bn}-\frac{\ln\left(\frac{i+a}{b}+x^n\right)}{2bn}-\frac{i\ln\left(x^n-\frac{i-a}{b}\right)a}{2bn}+\frac{i\ln\left(\frac{i+a}{b}+x^n\right)a}{2bn}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arctan(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] -1/2*I/n*x^n*ln(1+I*(a+b*x^n))+1/2*I/n*x^n*ln(1-I*(a+b*x^n))-1/2/b/n*ln(x^n
-(I-a)/b)-1/2/b/n*ln((I+a)/b+x^n)-1/2*I/b/n*ln(x^n-(I-a)/b)*a+1/2*I/b/n*ln(
(I+a)/b+x^n)*a
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.89

$$\frac{2\,(bx^n+a)\arctan(bx^n+a)-\log\left((bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

Fricas [A]
time = 3.34, size = 58, normalized size = 1.29

$$\frac{2\,bx^n\arctan\left(bx^n+a\right)+2\,a\arctan\left(bx^n+a\right)-\log\left(b^2x^{2\,n}+2\,abx^n+a^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^n*arctan(b*x^n + a) + 2*a*arctan(b*x^n + a) - log(b^2*x^(2*n) +
2*a*b*x^n + a^2 + 1))/(b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atan(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.41, size = 40, normalized size = 0.89

$$\frac{2\left(bx^n+a\right)\arctan\left(bx^n+a\right)-\log\left(\left(bx^n+a\right)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

Mupad [B]
time = 0.63, size = 58, normalized size = 1.29

$$\frac{x^n\,\mathrm{atan}(a+b\,x^n)}{n}-\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)-2\,a\,\mathrm{atan}(a+b\,x^n)}{2\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*atan(a + b*x^n),x)
```

```
[Out] (x^n*atan(a + b*x^n))/n - (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1) - 2*a*ata
n(a + b*x^n))/(2*b*n)
```

3.3 $\int x^5 \mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=144

$$\frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{1}{6}x^6\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{5d^3\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}$$

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+5/96*d^3*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(7/2)+5/96*d^2*x*(e*x^2+d)^(1/2)/(-e)^(5/2)+5/14
4*d*x^3*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/36*x^5*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5259, 327, 223, 212}

$$\frac{1}{6}x^6\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{5d^3\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}+\frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}}+\frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}}+\frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (5*d^2*x*Sqrt[d + e*x^2])/(96*(-e)^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*
(-e)^(3/2)) + (x^5*Sqrt[d + e*x^2])/(36*Sqrt[-e]) + (x^6*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/6 + (5*d^3*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(96*e^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^5 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \frac{x^6}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(5d)\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{36\sqrt{-e}}$$

$$= \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(5d^2)\int \frac{}{\sqrt{d}}}{48(-e}$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex}}\right.$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex}}\right.$$

$$= \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex}}\right.$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.60

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\left(-15d^2+10dex^2-8e^2x^4\right)+3\left(5d^3+16e^3x^6\right)\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 3*(5*d^3 +
 16*e^3*x^6)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(288*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(110) = 220$.

time = 0.02, size = 260, normalized size = 1.81

method	result
default	$\frac{x^{6} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{6}+\frac{\sqrt{-e}\, e \left(\frac{x^{7}\sqrt{e\,x^{2}+d}}{8e}-\frac{7d\left(\frac{x^{5}\sqrt{e\,x^{2}+d}}{6e}-\frac{5d\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\cdots\right.}{\cdots}\right.}{6e}\right.}{8e}\right.}{6d}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)*e/d*(1/8*x^7/e*
(e*x^2+d)^(1/2)-7/8*d/e*(1/6*x^5/e*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^
2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x
^2+d)^(1/2))))))-1/6*(-e)^(1/2)/d*(1/8*x^5*(e*x^2+d)^(3/2)/e-5/8*d/e*(1/6*x
^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2
+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.27, size = 85, normalized size = 0.59

$$-\frac{1}{576}\left(2\left(8i\,x^5e^2-10i\,dx^3e+15i\,d^2x\right)\sqrt{x^2e+d}\,e^{\frac{1}{2}}+3\left(-16i\,x^6e^3-5i\,d^3\right)\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/576*(2*(8*I*x^5*e^2 - 10*I*d*x^3*e + 15*I*d^2*x)*sqrt(x^2*e + d)*e^(1/2)
 + 3*(-16*I*x^6*e^3 - 5*I*d^3)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) +
 d)/d))*e^(-3)
```

Sympy [A]
time = 1.66, size = 138, normalized size = 0.96

$$\begin{cases}\frac{5d^3\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{96e^3}-\frac{5d^2x\sqrt{-e}\,\sqrt{d+ex^2}}{96e^3}+\frac{5dx^3\sqrt{-e}\,\sqrt{d+ex^2}}{144e^2}+\frac{x^6\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6}-\frac{x^5\sqrt{-e}\,\sqrt{d+ex^2}}{36e} & \text{for}\ e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*d**3*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*sq
rt(-e)*sqrt(d + e*x**2)/(96*e**3) + 5*d*x**3*sqrt(-e)*sqrt(d + e*x**2)/(144
*e**2) + x**6*atan(x*sqrt(-e)/sqrt(d + e*x**2))/6 - x**5*sqrt(-e)*sqrt(d +
e*x**2)/(36*e), Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.50, size = 94, normalized size = 0.65

$$\frac{1}{6}\,x^6\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\frac{1}{288}\,\sqrt{-e^2x^2-de}\,\left(2\,x^2\left(\frac{4\,x^2}{e}-\frac{5\,d}{e^2}\right)+\frac{15\,d^2}{e^3}\right)x-\frac{5\,d^3\arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{96\,e^2|e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/288*sqrt(-e^2*x^2 - d*e)*(2*
x^2*(4*x^2/e - 5*d/e^2) + 15*d^2/e^3)*x - 5/96*d^3*arcsin(e*x/sqrt(-d*e))*s
gn(e)/(e^2*abs(e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\operatorname{atan}\left(\frac{\sqrt{-e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.4 $\int x^3 \text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=116

$$\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}+\frac{1}{4}x^4\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)-\frac{3d^2\sqrt{-e}\, \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}$$

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-3/32*d^2*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))*(-e)^(1/2)/e^(5/2)+3/32*d*x*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/16*x
^3*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5259, 327, 223, 212}

$$\frac{1}{4}x^4\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)-\frac{3d^2\sqrt{-e}\, \tanh^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}+\frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}}+\frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*(-e)^(3/2)) + (x^3*Sqrt[d + e*x^2])/(16*Sqrt[-e
]) + (x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (3*d^2*Sqrt[-e]*ArcTanh
[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\int \frac{x^4}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d)\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{16\sqrt{-e}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\int \frac{1}{\sqrt{d+}}}{32(-e)^{3}}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\,\mathrm{Subst}\Big(\int}{}$$

$$= \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\,\tanh}{3}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.64

$$\frac{\sqrt{-e}\,x(3d-2ex^2)\sqrt{d+ex^2} + (-3d^2+8e^2x^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[-e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + (-3*d^2 + 8*e^2*x^4)*ArcTan[(
Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(32*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(88) = 176.
time = 0.01, size = 212, normalized size = 1.83

method	result
default	$\frac{x^4 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{4}+\frac{\sqrt{-e}\, e\left(\frac{x^5\sqrt{e x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2}\right.}{2e^{\frac{3}{2}}}\right.}{4e}\right.}{6e}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*(-e)^(1/2)*e/d*(1/6*x^5/e*
(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+
d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))-1/4*(-e)^(1/2)/d*(1
/6*x^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e
*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.40, size = 75, normalized size = 0.65

$$-\frac{1}{64}\left(2\left(2i\,x^3e-3i\,dx\right)\sqrt{x^2e+d}\,e^{\frac{1}{2}}-\left(8i\,x^4e^2-3i\,d^2\right)\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/64*(2*(2*I*x^3*e - 3*I*d*x)*sqrt(x^2*e + d)*e^(1/2) - (8*I*x^4*e^2 - 3*I
*d^2)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d))*e^(-2)
```

Sympy [A]
time = 0.59, size = 107, normalized size = 0.92

$$\begin{cases} -\frac{3d^2 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{3dx\sqrt{-e}\sqrt{d+ex^2}}{32e^2} + \frac{x^4 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4} - \frac{x^3\sqrt{-e}\sqrt{d+ex^2}}{16e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*d**2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqrt
(-e)*sqrt(d + e*x**2)/(32*e**2) + x**4*atan(x*sqrt(-e)/sqrt(d + e*x**2))/4
- x**3*sqrt(-e)*sqrt(d + e*x**2)/(16*e), Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.48, size = 80, normalized size = 0.69

$$\frac{1}{4}\,x^4 \arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right) - \frac{1}{32}\,\sqrt{-e^2x^2-de}\,x\left(\frac{2\,x^2}{e} - \frac{3\,d}{e^2}\right) + \frac{3\,d^2 \arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{32\,e|e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/32*sqrt(-e^2*x^2 - d*e)*x*(2
*x^2/e - 3*d/e^2) + 3/32*d^2*arcsin(e*x/sqrt(-d*e))*sgn(e)/(e*abs(e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atan}\left(\frac{\sqrt{-e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.5 $\int x \mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=88

$$\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{d\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}$$

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*d*arctanh(x*e^(1/2)/(e*x^2
+d)^(1/2))*(-e)^(1/2)/e^(3/2)+1/4*x*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5259, 327, 223, 212}

$$\frac{1}{2}x^2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{d\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}+\frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(4*Sqrt[-e]) + (x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Sqrt[-e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{2}\sqrt{-e}\int\frac{x^2}{\sqrt{d+ex^2}}\,dx\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{d\int\frac{1}{\sqrt{d+ex^2}}\,dx}{4\sqrt{-e}}\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{d\text{Subst}\left(\int\frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e}}\\
&= \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}}+\frac{1}{2}x^2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-e^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.67

$$\frac{-\sqrt{-e}\,x\sqrt{d+ex^2}+(d+2ex^2)\,\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-(Sqrt[-e]*x*Sqrt[d + e*x^2]) + (d + 2*e*x^2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/(4*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.
time = 0.01, size = 164, normalized size = 1.86

method	result

default	$\frac{x^{2} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{2}+\frac{\sqrt{-e}\, e \left(\frac{x^{3} \sqrt{e\,x^{2}+d}}{4 e}-\frac{3 d \left(\frac{x \sqrt{e\,x^{2}+d}}{2 e}-\frac{d \ln\left(x \sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2 e^{\frac{3}{2}}}\right)}{4 e}\right)}{2 d}-\frac{\sqrt{-e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)*e/d*(1/4*x^3/e*
(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)
+(e*x^2+d)^(1/2))))-1/2*(-e)^(1/2)/d*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*
x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 4.39, size = 61, normalized size = 0.69

$$\frac{1}{8} \left(-2 i \sqrt{x^{2} e+d}\, x e^{\frac{1}{2}}+\left(2 i\, x^{2} e+i\, d\right) \log\left(\frac{2\, x^{2} e+2\, \sqrt{x^{2} e+d}\, x e^{\frac{1}{2}}+d}{d}\right)\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*(-2*I*sqrt(x^2*e + d)*x*e^(1/2) + (2*I*x^2*e + I*d)*log((2*x^2*e + 2*sq
rt(x^2*e + d)*x*e^(1/2) + d)/d))*e^(-1)
```

Sympy [A]
time = 0.33, size = 73, normalized size = 0.83

$$\begin{cases} \frac{d \operatorname{atan}\left(\frac{x \sqrt{-e}}{\sqrt{d+e x^{2}}}\right)}{4 e}+\frac{x^{2} \operatorname{atan}\left(\frac{x \sqrt{-e}}{\sqrt{d+e x^{2}}}\right)}{2}-\frac{x \sqrt{-e}\, \sqrt{d+e x^{2}}}{4 e} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((d*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*e) + x**2*atan(x*sqrt(-e)
/sqrt(d + e*x**2))/2 - x*sqrt(-e)*sqrt(d + e*x**2)/(4*e), Ne(e, 0)), (0, Tr
ue))
```

Giac [A]
time = 0.50, size = 63, normalized size = 0.72

$$\frac{1}{2}\,x^2\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)-\frac{d\arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{4\,|e|}-\frac{\sqrt{-e^2x^2-de}\,x}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/4*d*arcsin(e*x/sqrt(-d*e))*s
gn(e)/abs(e) - 1/4*sqrt(-e^2*x^2 - d*e)*x/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.6 $\int \frac{\textbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal. Leaf size=288

$$-\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}}+\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\,\sqrt{d+ex^2}}$$

```
[Out] arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2
*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(
1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1
/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln
(x)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+1/2*polylo
g(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)
^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5257, 2364, 2362, 5775, 3797, 2221, 2317, 2438}

$$\log(x)\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\,\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}}+\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\,\sqrt{d+ex^2}}-\frac{\sqrt{d}\,\sqrt{-e}\,\log(x)\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -1/2*(Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/
(Sqrt[e]*Sqrt[d + e*x^2]) + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(
Sqrt[e]*x)/Sqrt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(Sqrt[e]*S
qrt[d + e*x^2]) - (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)
/Sqrt[d]]*Log[x])/(Sqrt[e]*Sqrt[d + e*x^2]) + ArcTan[(Sqrt[-e]*x)/Sqrt[d +
e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*PolyLog[2, E^(2*ArcS
inh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[e]*Sqrt[d + e*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2364

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5257

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Simp
[ArcTan[c*(x/Sqrt[a + b*x^2])]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b*x
^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{-e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\,\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x)$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\,\sqrt{d+ex^2}} + \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x)$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}} - \frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\Big(\sqrt{}}{\sqrt{e}\,\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}} + \frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\Big(\sqrt{}}{\sqrt{e}\,\sqrt{d-}}$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}} + \frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\Big(\sqrt{}}{\sqrt{e}\,\sqrt{d-}}$$

$$= -\frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{e}\,\sqrt{d+ex^2}} + \frac{\sqrt{d}\,\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,\sinh^{-1}\Big(\sqrt{}}{\sqrt{e}\,\sqrt{d-}}$$

Mathematica [A]
time = 1.83, size = 171, normalized size = 0.59

$$\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\left(\sinh^{-1}\left(\sqrt{\frac{e}{d}}\,x\right)^2 + 2\sinh^{-1}\left(\sqrt{\frac{e}{d}}\,x\right)\log\left(1-e^{-2\sinh^{-1}\left(\sqrt{\frac{e}{d}}\,x\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{e}{d}}\,x+\sqrt{1+\frac{ex^2}{d}}\right) - \text{PolyLog}\left(2,e^{-2\sinh^{-1}\left(\sqrt{\frac{e}{d}}\,x\right)}\right)\right)}{2\sqrt{\frac{e}{d}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[-e]*Sqrt[1 + (e*x^2)/d]
*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqr
t[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2,
E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

```
[Out] Integral(atan(x*sqrt(-e)/sqrt(d + e*x**2))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

3.7 $\int \frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*(e*x^2+d)^(1/2
)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5259, 270}

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2} + \frac{1}{2}\sqrt{-e}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{2dx} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Mathematica [**A**]
time = 0.03, size = 54, normalized size = 0.95

$$-\frac{\sqrt{-e}\,x\sqrt{d+ex^2} + d\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[-e]*x*Sqrt[d + e*x^2] + d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/
(d*x^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.
time = 0.01, size = 122, normalized size = 2.14

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2} - \frac{\sqrt{-e}\,\sqrt{e}\,\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d} + \frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx} + \frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2} + \frac{d\ln}{}\right.}{}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*e^(1/2)/d*ln(x
*e^(1/2)+(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)/d*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1
/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Maxima [**A**]
time = 0.28, size = 64, normalized size = 1.12

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{2\,x^2} - \frac{x^2\sqrt{-e}\,e + d\sqrt{-e}}{2\,\sqrt{x^2e+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^2 - 1/2*(x^2*sqrt(-e)*e + d*sqrt(
-e))/(sqrt(x^2*e + d)*d*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 6.19, size = 56, normalized size = 0.98

$$\frac{-2i\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-i\,d\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*sqrt(x^2*e + d)*x*e^(1/2) - I*d*log((2*x^2*e + 2*sqrt(x^2*e + d)*
x*e^(1/2) + d)/d))/(d*x^2)
```

Sympy [A]
time = 2.02, size = 53, normalized size = 0.93

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{e}\,\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(2*x**2) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(2*d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(45) = 90.
time = 0.49, size = 106, normalized size = 1.86

$$-\frac{e^4x}{4\left(\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|\right)d|e|}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{2\,x^2}+\frac{\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|}{4\,dx|e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*e^4*x/((sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))*d*abs(e)) - 1/2*ar
ctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 + 1/4*(sqrt(-d*e)*e - sqrt(-e^2*x^2 -
d*e)*abs(e))/(d*x*abs(e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

3.8 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4-1/6*(-e)^(3/2)*(e*x^2+d)^(1/2
)/d^2/x-1/12*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5259, 277, 270}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -1/12*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^3) - ((-e)^(3/2)*Sqrt[d + e*x^2])/(6*
d^2*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
```

```
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{1}{4}\sqrt{-e}\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}+\frac{(-e)^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{6d}$$
$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{12dx^3}-\frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Mathematica [**A**]
time = 0.03, size = 67, normalized size = 0.79

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\,(-d+2ex^2)-3d^2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTan[(Sqrt[-e]*x)/Sqrt
[d + e*x^2]])/(12*d^2*x^4)
```

Maple [**A**]
time = 0.01, size = 69, normalized size = 0.81

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{\sqrt{-e}\,e\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{-e}\,(e\,x^2+d)^{\frac{3}{2}}}{12d^2x^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*(-e)^(1/2)*e/d^2/x*(e*x^2
+d)^(1/2)-1/12*(-e)^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.88

$$\frac{\sqrt{x^2e+d}\,\sqrt{-e}\,e}{4\,d^2x}-\frac{(x^2e+d)^{\frac{3}{2}}\sqrt{-e}}{12\,d^2x^3}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^2*e + d)*sqrt(-e)*e/(d^2*x) - 1/12*(x^2*e + d)^(3/2)*sqrt(-e)/(d
^2*x^3) - 1/4*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^4
```

Fricas [C] Result contains complex when optimal does not.
time = 4.39, size = 69, normalized size = 0.81

$$\frac{-3i\,d^2\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\,(-2i\,x^3e+i\,dx)\sqrt{x^2e+d}\,e^{\frac{1}{2}}}{24\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-3*I*d^2*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*(-2*I
*x^3*e + I*d*x)*sqrt(x^2*e + d)*e^(1/2))/(d^2*x^4)
```

Sympy [A]
time = 2.42, size = 83, normalized size = 0.98

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4x^4}-\frac{\sqrt{e}\,\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{12dx^2}+\frac{e^{\frac{3}{2}}\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*x**4) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(12*d*x**2) + e**(3/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(6*d**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(67) = 134.
time = 0.52, size = 202, normalized size = 2.38

$$\frac{\left(e^3+\frac{9\left(\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|\right)^2}{ex^2}\right)e^6x^3}{96\left(\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|\right)^3d^2|e|}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{4\,x^4}-\frac{\frac{9\left(\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|\right)d^4e^6}{x}+\frac{\left(\sqrt{-de}\,e-\sqrt{-e^2x^2-de}\,|e|\right)^3d^4e^2}{x^3}}{96\,d^6e^5|e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/96*(e^3 + 9*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^2/(e*x^2))*e^6*x
^3/((sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^2*abs(e)) - 1/4*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4 - 1/96*(9*(sqrt(-d*e)*e - sqrt(-e^2*x^2 -
d*e)*abs(e))*d^4*e^6/x + (sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^4
*e^2/x^3)/(d^6*e^5*abs(e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

3.9 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{30dx^5}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-2/45*(-e)^(3/2)*(e*x^2+d)^(1/
2)/d^2/x^3-4/45*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5259, 277, 270}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{30dx^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -1/30*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^5) - (2*(-e)^(3/2)*Sqrt[d + e*x^2])/(
45*d^2*x^3) - (4*(-e)^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTan[(Sqrt[-e]*
x)/Sqrt[d + e*x^2]]/(6*x^6)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{1}{6}\sqrt{-e}\int\frac{1}{x^6\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{30dx^5}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{\left(2(-e)^{3/2}\right)\int\frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{30dx^5}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{6x^6}+\frac{(4(-e)^{5/}}{}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{30dx^5}-\frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x}-\frac{\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{6x}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.69

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\,(-3d^2+4dex^2-8e^2x^4)-15d^3\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

Maple [A]
time = 0.01, size = 117, normalized size = 1.04

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*(-e)^(1/2)*e/d*(-1/3/d/x^
3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)/d*(-1/5/d/x^5
*(e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

Maxima [A]
time = 0.27, size = 116, normalized size = 1.03

$$-\frac{(2\,x^4e^2+dx^2e-d^2)\sqrt{-e}\,e}{18\,\sqrt{x^2e+d}\,d^3x^3}-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)}{6\,x^6}+\frac{(2\,x^4e^2-dx^2e-3\,d^2)\sqrt{x^2e+d}\,\sqrt{-e}}{90\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*x^4*e^2 + d*x^2*e - d^2)*sqrt(-e)*e/(sqrt(x^2*e + d)*d^3*x^3) - 1/
6*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^6 + 1/90*(2*x^4*e^2 - d*x^2*e - 3*d^
2)*sqrt(x^2*e + d)*sqrt(-e)/(d^3*x^5)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.37, size = 79, normalized size = 0.70

$$\frac{-15i\,d^3\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\,(8i\,x^5e^2-4i\,dx^3e+3i\,d^2x)\sqrt{x^2e+d}\,e^{\frac{1}{2}}}{180\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] 1/180*(-15*I*d^3*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*(8*
I*x^5*e^2 - 4*I*d*x^3*e + 3*I*d^2*x)*sqrt(x^2*e + d)*e^(1/2))/(d^3*x^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(102) = 204.
time = 3.13, size = 352, normalized size = 3.12

$$-\frac{d^4e^{\frac{9}{2}}\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8}-\frac{d^3e^{\frac{11}{2}}x^2\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8}-\frac{d^2e^{\frac{13}{2}}x^4\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8}-\frac{2de^{\frac{15}{2}}x^6\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8}-\frac{4e^{\frac{17}{2}}x^8\sqrt{-e}\,\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] -d**4*e**(9/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e
**5*x**6 + 30*d**3*e**6*x**8) - d**3*e**(11/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2
) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x**6 + 45*d**3*e**6*x**8) - d**2*e
**(13/2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*e*
*5*x**6 + 30*d**3*e**6*x**8) - 2*d*e**(15/2)*x**6*sqrt(-e)*sqrt(d/(e*x**2)
+ 1)/(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8) - 4*e**(17
```

```
/2)*x**8*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x*
*6 + 45*d**3*e**6*x**8) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(6*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(89) = 178$.
time = 0.51, size = 281, normalized size = 2.49

$$-\frac{\left(3 e^{4} + \frac{25 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right)^{2}}{x^{2}} + \frac{150 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right)^{4}}{e^{4} x^{4}}\right) e^{10} x^{5}}{2880 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right)^{5} d^{3} |e|} - \frac{\arctan\left(\frac{\sqrt{-e}\, x}{\sqrt{e x^{2} + d}}\right)}{6 x^{6}} + \frac{\frac{150 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right) d^{12} e^{16}}{x} + \frac{25 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right)^{3} d^{12} e^{12}}{x^{3}} + \frac{3 \left(\sqrt{-de}\, e - \sqrt{-e^{2} x^{2} - de}\, |e|\right)^{5} d^{12} e^{8}}{x^{5}}}{2880 d^{15} e^{14} |e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] -1/2880*(3*e^4 + 25*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^2/x^2 + 15
0*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^4*x^4))*e^10*x^5/((sqrt
(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^3*abs(e)) - 1/6*arctan(sqrt(-e)
*x/sqrt(e*x^2 + d))/x^6 + 1/2880*(150*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*
abs(e))*d^12*e^16/x + 25*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^1
2*e^12/x^3 + 3*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^12*e^8/x^5)
/(d^15*e^14*abs(e))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\, x^2 + d}}\right)}{x^7} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

3.10 $\int \frac{\textbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-3/140*(-e)^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^5-1/35*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3-2/35*(-e)^(7/2)*(e*x^2+
d)^(1/2)/d^4/x-1/56*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5259, 277, 270}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8}-\frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x}-\frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -1/56*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^7) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(
140*d^2*x^5) - ((-e)^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) - (2*(-e)^(7/2)*Sq
rt[d + e*x^2])/(35*d^4*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
```

```
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{-e}\int \frac{1}{x^8\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3(-e)^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{(3(-e)^{5/}}{}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{d}}\right.}{8x^8}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{2(-e)^{7/2}\sqrt{d}}{35d^4}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 0.63

$$\frac{\sqrt{-e}\,x\sqrt{d+ex^2}\left(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6\right)-35d^4\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[-e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^
6) - 35*d^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

Maple [A]
time = 0.01, size = 165, normalized size = 1.17

method	result

default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{8x^{8}}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^{2}+d}}{5d\,x^{5}}-\frac{4e\left(-\frac{\sqrt{e\,x^{2}+d}}{3d\,x^{3}}+\frac{2e\sqrt{e\,x^{2}+d}}{3d^{2}x}\right)}{5d}\right)}{8d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)}{7d\,x^{7}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*(-e)^(1/2)*e/d*(-1/5/d/x^
5*(e*x^2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)
^(1/2)))+1/8*(-e)^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(
e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

Maxima [A]
time = 0.26, size = 137, normalized size = 0.97

$$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^{2}e+d}}\right)}{8\,x^{8}}+\frac{\left(8\,x^{6}e^{3}+4\,dx^{4}e^{2}-d^{2}x^{2}e+3\,d^{3}\right)\sqrt{-e}\,e}{120\,\sqrt{x^{2}e+d}\,d^{4}x^{5}}-\frac{\left(8\,x^{6}e^{3}-4\,dx^{4}e^{2}+3\,d^{2}x^{2}e+15\,d^{3}\right)\sqrt{x^{2}e+d}\,\sqrt{-e}}{840\,d^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*arctan(x*sqrt(-e)/sqrt(x^2*e + d))/x^8 + 1/120*(8*x^6*e^3 + 4*d*x^4*e^
2 - d^2*x^2*e + 3*d^3)*sqrt(-e)*e/(sqrt(x^2*e + d)*d^4*x^5) - 1/840*(8*x^6*
e^3 - 4*d*x^4*e^2 + 3*d^2*x^2*e + 15*d^3)*sqrt(x^2*e + d)*sqrt(-e)/(d^4*x^7
)
```

Fricas [C] Result contains complex when optimal does not.
time = 2.77, size = 89, normalized size = 0.63

$$\frac{-35i\,d^{4}\log\left(\frac{2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\left(-16i\,x^{7}e^{3}+8i\,dx^{5}e^{2}-6i\,d^{2}x^{3}e+5i\,d^{3}x\right)\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}}{560\,d^{4}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] 1/560*(-35*I*d^4*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*(-1
6*I*x^7*e^3 + 8*I*d*x^5*e^2 - 6*I*d^2*x^3*e + 5*I*d^3*x)*sqrt(x^2*e + d)*e^
(1/2))/(d^4*x^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(128) = 256.
time = 4.39, size = 575, normalized size = 4.08

$$-\frac{5d^{6}e^{\frac{19}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{280d^{7}e^{9}x^{6}+840d^{6}e^{10}x^{8}+840d^{5}e^{11}x^{10}+280d^{4}e^{12}x^{12}}-\frac{9d^{5}e^{\frac{21}{2}}x^{2}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{280d^{7}e^{9}x^{6}+840d^{6}e^{10}x^{8}+840d^{5}e^{11}x^{10}+280d^{4}e^{12}x^{12}}-\frac{5d^{4}e^{\frac{23}{2}}x^{4}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{280d^{7}e^{9}x^{6}+840d^{6}e^{10}x^{8}+840d^{5}e^{11}x^{10}+280d^{4}e^{12}x^{12}}+\frac{5d^{3}e^{\frac{25}{2}}x^{6}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{280d^{7}e^{9}x^{6}+840d^{6}e^{10}x^{8}+840d^{5}e^{11}x^{10}+280d^{4}e^{12}x^{12}}+\frac{15d^{2}e^{\frac{27}{2}}x^{8}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{140d^{7}e^{9}x^{6}+420d^{6}e^{10}x^{8}+420d^{5}e^{11}x^{10}+140d^{4}e^{12}x^{12}}+\frac{5de^{\frac{29}{2}}x^{10}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{35d^{7}e^{9}x^{6}+105d^{6}e^{10}x^{8}+105d^{5}e^{11}x^{10}+35d^{4}e^{12}x^{12}}+\frac{2e^{\frac{31}{2}}x^{12}\sqrt{-e}\sqrt{\frac{d}{ex^{2}}+1}}{35d^{7}e^{9}x^{6}+105d^{6}e^{10}x^{8}+105d^{5}e^{11}x^{10}+35d^{4}e^{12}x^{12}}-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] -5*d**6*e**(19/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d
**6*e**10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 9*d**5*e**(
21/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**
10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 5*d**4*e**(23/2)*x
**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8
 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 5*d**3*e**(25/2)*x**6*sqr
t(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8 + 840*
d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 15*d**2*e**(27/2)*x**8*sqrt(-e)*
sqrt(d/(e*x**2) + 1)/(140*d**7*e**9*x**6 + 420*d**6*e**10*x**8 + 420*d**5*e
**11*x**10 + 140*d**4*e**12*x**12) + 5*d*e**(29/2)*x**10*sqrt(-e)*sqrt(d/(e
*x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10
 + 35*d**4*e**12*x**12) + 2*e**(31/2)*x**12*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(
35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10 + 35*d**4*e*
*12*x**12) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(8*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(111) = 222.

time = 0.51, size = 361, normalized size = 2.56

$$\frac{\left(5\,e^{5}+\frac{49\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{2}e}{x^{2}}+\frac{245\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{4}}{e^{3}x^{4}}+\frac{1225\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{6}}{e^{7}x^{6}}\right)e^{14}x^{7}}{35840\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{7}d^{4}|e|}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^{2}+d}}\right)}{8\,x^{8}}-\frac{\frac{1225\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)d^{24}e^{30}}{x}+\frac{245\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{3}d^{24}e^{26}}{x^{3}}+\frac{49\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{5}d^{24}e^{22}}{x^{5}}+\frac{5\left(\sqrt{-de}\,e-\sqrt{-e^{2}x^{2}-de}\,|e|\right)^{7}d^{24}e^{18}}{x^{7}}}{35840\,d^{28}e^{27}|e|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] 1/35840*(5*e^5 + 49*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^2*e/x^2 +
245*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^3*x^4) + 1225*(sqrt(-
d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^6/(e^7*x^6))*e^14*x^7/((sqrt(-d*e)*e
- sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^4*abs(e)) - 1/8*arctan(sqrt(-e)*x/sqrt(e
*x^2 + d))/x^8 - 1/35840*(1225*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))
*d^24*e^30/x + 245*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^24*e^26
/x^3 + 49*(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^24*e^22/x^5 + 5*
(sqrt(-d*e)*e - sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^24*e^18/x^7)/(d^28*e^27*ab
s(e))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

3.11 $\int x^6 \textbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=124

$$\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}}+\frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}}-\frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}}+\frac{1}{7}x^7\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/7*d^2*(e*x^2+d)^(3/2)/(-e)^(7/2)+3/35*d*(e*x^2+d)^(5/2)/(-e)^(7/2)-1/49*
(e*x^2+d)^(7/2)/(-e)^(7/2)+1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/7
*d^3*(e*x^2+d)^(1/2)/(-e)^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5259, 272, 45}

$$\frac{1}{7}x^7\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}}-\frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}}-\frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}}+\frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*(-e)^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*(-e)^(7/2
)) + (3*d*(d + e*x^2)^(5/2))/(35*(-e)^(7/2)) - (d + e*x^2)^(7/2)/(49*(-e)^(
7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
```

```
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^6 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{-e}\int \frac{x^7}{\sqrt{d+ex^2}}\,dx \\
&= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{d+ex}}\,dx, x, x^2\right) \\
&= \frac{1}{7}x^7 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\,\text{Subst}\left(\int \left(-\frac{d^3}{e^3\sqrt{d+ex}} + \frac{3d^2\sqrt{d+}}{e^3}\right.\right. \\
&= \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}} - \frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}} + \frac{1}{7}x^7 \tan
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 83, normalized size = 0.67

$$\frac{\sqrt{d+ex^2}\,(16d^3 - 8d^2ex^2 + 6de^2x^4 - 5e^3x^6)}{245(-e)^{7/2}} + \frac{1}{7}x^7 \text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*(-e
)^(7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(94) = 188.
time = 0.02, size = 231, normalized size = 1.86

method	result

default	$\frac{x^7 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{7}+\frac{\sqrt{-e}\, e \left(\frac{x^8\sqrt{e x^2+d}}{9e}-\frac{8d\left(\frac{x^6\sqrt{e x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e}\right.}{5e}\right.}{7e}\right.}{9e}\right.}{7d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/7*(-e)^(1/2)*e/d*(1/9*x^8/e*
(e*x^2+d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^
2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))
-1/7*(-e)^(1/2)/d*(1/9*x^6*(e*x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/
2)/e-4/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))
```

Maxima [A]
time = 0.26, size = 178, normalized size = 1.44

$$\frac{1}{7}\,x^7\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{\left(35\,(x^2e+d)^{\frac{9}{2}}-135\,(x^2e+d)^{\frac{7}{2}}d+189\,(x^2e+d)^{\frac{5}{2}}d^2-105\,(x^2e+d)^{\frac{3}{2}}d^3\right)\sqrt{-e}\,e^{(-4)}}{2205\,d}+\frac{\left(35\,(x^2e+d)^{\frac{9}{2}}-180\,(x^2e+d)^{\frac{7}{2}}d+378\,(x^2e+d)^{\frac{5}{2}}d^2-420\,(x^2e+d)^{\frac{3}{2}}d^3+315\,\sqrt{x^2e+d}\,d^4\right)\sqrt{-e}\,e^{(-4)}}{2205\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/2205*(35*(x^2*e + d)^(9/2) -
 135*(x^2*e + d)^(7/2)*d + 189*(x^2*e + d)^(5/2)*d^2 - 105*(x^2*e + d)^(3/2
)*d^3)*sqrt(-e)*e^(-4)/d + 1/2205*(35*(x^2*e + d)^(9/2) - 180*(x^2*e + d)^(
7/2)*d + 378*(x^2*e + d)^(5/2)*d^2 - 420*(x^2*e + d)^(3/2)*d^3 + 315*sqrt(x
^2*e + d)*d^4)*sqrt(-e)*e^(-4)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 2.45, size = 86, normalized size = 0.69

$$\frac{1}{490}\left(35i\,x^7e^4\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\left(5i\,x^6e^3-6i\,dx^4e^2+8i\,d^2x^2e-16i\,d^3\right)\sqrt{x^2e+d}\,e^{\frac{1}{2}}\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/490*(35*I*x^7*e^4*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*
(5*I*x^6*e^3 - 6*I*d*x^4*e^2 + 8*I*d^2*x^2*e - 16*I*d^3)*sqrt(x^2*e + d)*e^
(1/2))*e^(-4)
```

Sympy [A]
time = 1.96, size = 136, normalized size = 1.10

$$\begin{cases} \frac{16d^3\sqrt{-e}\sqrt{d+ex^2}}{245e^4} - \frac{8d^2x^2\sqrt{-e}\sqrt{d+ex^2}}{245e^3} + \frac{6dx^4\sqrt{-e}\sqrt{d+ex^2}}{245e^2} + \frac{x^7\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{-e}\sqrt{d+ex^2}}{49e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*d**3*sqrt(-e)*sqrt(d + e*x**2)/(245*e**4) - 8*d**2*x**2*sqrt(
-e)*sqrt(d + e*x**2)/(245*e**3) + 6*d*x**4*sqrt(-e)*sqrt(d + e*x**2)/(245*e
**2) + x**7*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**6*sqrt(-e)*sqrt(d + e*
x**2)/(49*e), Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.49, size = 137, normalized size = 1.10

$$\frac{1}{7}x^7\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right) + \frac{\sqrt{-e^2x^2-de}\,d^3}{7\,e^4} + \frac{35\left(-e^2x^2-de\right)^{\frac{3}{2}}d^2e^2 + 21\left(e^2x^2+de\right)^2\sqrt{-e^2x^2-de}\,de - 5\left(e^2x^2+de\right)^3\sqrt{-e^2x^2-de}}{245\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/7*sqrt(-e^2*x^2 - d*e)*d^3/e
^4 + 1/245*(35*(-e^2*x^2 - d*e)^(3/2)*d^2*e^2 + 21*(e^2*x^2 + d*e)^2*sqrt(-
e^2*x^2 - d*e)*d*e - 5*(e^2*x^2 + d*e)^3*sqrt(-e^2*x^2 - d*e))/e^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.12 $\int x^4 \text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=99

$$\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}-\frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}}+\frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}}+\frac{1}{5}x^5\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -2/15*d*(e*x^2+d)^(3/2)/(-e)^(5/2)+1/25*(e*x^2+d)^(5/2)/(-e)^(5/2)+1/5*x^5*
arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*d^2*(e*x^2+d)^(1/2)/(-e)^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5259, 272, 45}

$$\frac{1}{5}x^5\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)+\frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}}+\frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}}-\frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^2*Sqrt[d + e*x^2])/(5*(-e)^(5/2)) - (2*d*(d + e*x^2)^(3/2))/(15*(-e)^(5/
2)) + (d + e*x^2)^(5/2)/(25*(-e)^(5/2)) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) dx &= \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{5}\sqrt{-e} \int \frac{x^5}{\sqrt{d+ex^2}}\, dx \\
&= \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{-e}\, \text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex}}\, dx, x, x^2\right) \\
&= \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) - \frac{1}{10}\sqrt{-e}\, \text{Subst}\left(\int \left(\frac{d^2}{e^2\sqrt{d+ex}} - \frac{2d\sqrt{d+ex}}{e^2} \right.\right. \\
&= \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}} - \frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}} + \frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}} + \frac{1}{5}x^5 \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.73

$$
\frac{\sqrt{d+ex^2}\,(8d^2 - 4dex^2 + 3e^2x^4)}{75(-e)^{5/2}} + \frac{1}{5}x^5 \text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*(-e)^(5/2)) + (x^5*Ar
cTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(75) = 150.
time = 0.01, size = 183, normalized size = 1.85

method	result
default	$\frac{x^5 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{5} + \frac{\sqrt{-e}\, e \left(\frac{x^6\sqrt{e x^2+d}}{7e} - \frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e} - \frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right.}{7e}\right.}{5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*(-e)^(1/2)*e/d*(1/7*x^6/e*
(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^
2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*(-e)^(1/2)/d*(1/7*x^4*(e*x^2+d)
^(3/2)/e-4/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

Maxima [A]
time = 0.26, size = 148, normalized size = 1.49

$$\frac{1}{5}\,x^5\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{\left(15\,(x^2e+d)^{\frac{7}{2}}-42\,(x^2e+d)^{\frac{5}{2}}d+35\,(x^2e+d)^{\frac{3}{2}}d^2\right)\sqrt{-e}\,e^{(-3)}}{525\,d}+\frac{\left(5\,(x^2e+d)^{\frac{7}{2}}-21\,(x^2e+d)^{\frac{5}{2}}d+35\,(x^2e+d)^{\frac{3}{2}}d^2-35\,\sqrt{x^2e+d}\,d^3\right)\sqrt{-e}\,e^{(-3)}}{175\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/525*(15*(x^2*e + d)^(7/2) -
42*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2)*sqrt(-e)*e^(-3)/d + 1/17
5*(5*(x^2*e + d)^(7/2) - 21*(x^2*e + d)^(5/2)*d + 35*(x^2*e + d)^(3/2)*d^2
- 35*sqrt(x^2*e + d)*d^3)*sqrt(-e)*e^(-3)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 3.27, size = 76, normalized size = 0.77

$$\frac{1}{150}\left(15i\,x^5e^3\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\left(3i\,x^4e^2-4i\,dx^2e+8i\,d^2\right)\sqrt{x^2e+d}\,e^{\frac{1}{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/150*(15*I*x^5*e^3*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*
(3*I*x^4*e^2 - 4*I*d*x^2*e + 8*I*d^2)*sqrt(x^2*e + d)*e^(1/2))*e^(-3)
```

Sympy [A]
time = 0.83, size = 105, normalized size = 1.06

$$\begin{cases} -\frac{8d^2\sqrt{-e}\,\sqrt{d+ex^2}}{75e^3}+\frac{4dx^2\sqrt{-e}\,\sqrt{d+ex^2}}{75e^2}+\frac{x^5\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^4\sqrt{-e}\,\sqrt{d+ex^2}}{25e} & \text{for}\ e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*d**2*sqrt(-e)*sqrt(d + e*x**2)/(75*e**3) + 4*d*x**2*sqrt(-e)*
sqrt(d + e*x**2)/(75*e**2) + x**5*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**
4*sqrt(-e)*sqrt(d + e*x**2)/(25*e), Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.46, size = 101, normalized size = 1.02

$$\frac{1}{5}\,x^{5}\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^{2}+d}}\right)-\frac{\sqrt{-e^{2}x^{2}-de}\,d^{2}}{5\,e^{3}}-\frac{10\,(-e^{2}x^{2}-de)^{\frac{3}{2}}de+3\,(e^{2}x^{2}+de)^{2}\sqrt{-e^{2}x^{2}-de}}{75\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/5*sqrt(-e^2*x^2 - d*e)*d^2/e
^3 - 1/75*(10*(-e^2*x^2 - d*e)^(3/2)*d*e + 3*(e^2*x^2 + d*e)^2*sqrt(-e^2*x^
2 - d*e))/e^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{atan}\left(\frac{\sqrt{-e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.13 $\int x^2 \mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=74

$$\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}-\frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}}+\frac{1}{3}x^3\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/9*(e*x^2+d)^(3/2)/(-e)^(3/2)+1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)
)+1/3*d*(e*x^2+d)^(1/2)/(-e)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5259, 272, 45}

$$\frac{1}{3}x^3\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}}+\frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*(-e)^(3/2)) - (d + e*x^2)^(3/2)/(9*(-e)^(3/2)) + (x^
3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{3}x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{3}\sqrt{-e}\int \frac{x^3}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{1}{3}x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int \frac{x}{\sqrt{d+ex}}\,dx, x, x^2\right)$$
$$= \frac{1}{3}x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int \left(-\frac{d}{e\sqrt{d+ex}} + \frac{\sqrt{d+ex}}{e}\right.\right.$$
$$= \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}} - \frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}} + \frac{1}{3}x^3 \tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Mathematica [**A**]
time = 0.06, size = 60, normalized size = 0.81

$$\frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}} + 3x^3\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 3*x^3*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]])/9
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(56) = 112$.
time = 0.01, size = 135, normalized size = 1.82

method	result
default	$\frac{x^3 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{3} + \frac{\sqrt{-e}\,e\left(\frac{x^4\sqrt{e\,x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e} - \frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{3d} - \frac{\sqrt{-e}\left(\frac{x^2(e\,x^2}{5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/3*(-e)^(1/2)*e/d*(1/5*x^4/e*
(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2
)))-1/3*(-e)^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.59

$$\frac{1}{3}\,x^3\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{\left(3\,(x^2e+d)^{\frac{5}{2}}-5\,(x^2e+d)^{\frac{3}{2}}d\right)\sqrt{-e}\,e^{(-2)}}{45\,d}+\frac{\left(3\,(x^2e+d)^{\frac{5}{2}}-10\,(x^2e+d)^{\frac{3}{2}}d+15\,\sqrt{x^2e+d}\,d^2\right)\sqrt{-e}\,e^{(-2)}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/45*(3*(x^2*e + d)^(5/2) - 5*
(x^2*e + d)^(3/2)*d)*sqrt(-e)*e^(-2)/d + 1/45*(3*(x^2*e + d)^(5/2) - 10*(x^
2*e + d)^(3/2)*d + 15*sqrt(x^2*e + d)*d^2)*sqrt(-e)*e^(-2)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 2.59, size = 66, normalized size = 0.89

$$\frac{1}{18}\left(3i\,x^3e^2\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2\,\sqrt{x^2e+d}\,\left(i\,x^2e-2i\,d\right)e^{\frac{1}{2}}\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*I*x^3*e^2*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*sq
rt(x^2*e + d)*(I*x^2*e - 2*I*d)*e^(1/2))*e^(-2)
```

Sympy [A]
time = 0.43, size = 75, normalized size = 1.01

$$\begin{cases}\frac{2d\sqrt{-e}\,\sqrt{d+ex^2}}{9e^2}+\frac{x^3\,\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3}-\frac{x^2\sqrt{-e}\,\sqrt{d+ex^2}}{9e} & \text{for } e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*d*sqrt(-e)*sqrt(d + e*x**2)/(9*e**2) + x**3*atan(x*sqrt(-e)/sq
rt(d + e*x**2))/3 - x**2*sqrt(-e)*sqrt(d + e*x**2)/(9*e), Ne(e, 0)), (0, Tr
ue))
```

Giac [A]
time = 0.47, size = 64, normalized size = 0.86

$$\frac{1}{3}\,x^3\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)+\frac{\sqrt{-e^2x^2-de}\,d}{3\,e^2}+\frac{(-e^2x^2-de)^{\frac{3}{2}}}{9\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/3*sqrt(-e^2*x^2 - d*e)*d/e^2
 + 1/9*(-e^2*x^2 - d*e)^(3/2)/e^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}\left(\frac{\sqrt{-e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.14 $\int \text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)\, dx$

Optimal. Leaf size=43

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)$$

```
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(e*x^2+d)^(1/2)/(-e)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5255, 267}

$$x\text{ArcTan}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) + \frac{\sqrt{d+ex^2}}{\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5255

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*Ar
cTan[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)\, dx &= x\tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right) - \sqrt{-e}\int \frac{x}{\sqrt{d+ex^2}}\, dx\\ &= \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\tan^{-1}\left(\frac{\sqrt{-e}\, x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$\frac{\sqrt{d+ex^2}}{\sqrt{-e}}+x\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.
time = 0.02, size = 86, normalized size = 2.00

method	result	size
default	$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)+\frac{\sqrt{-e}\,e\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{3de}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(-e)^(1/2)*e/d*(1/3*x^2/e*(e*x^2+d)^
(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))-1/3*(-e)^(1/2)/d/e*(e*x^2+d)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.
time = 0.26, size = 82, normalized size = 1.91

$$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{x^2e+d}}\right)-\frac{\left(x^2e+d\right)^{\frac{3}{2}}\sqrt{-e}\,e^{(-1)}}{3\,d}+\frac{\left(\left(x^2e+d\right)^{\frac{3}{2}}-3\,\sqrt{x^2e+d}\,d\right)\sqrt{-e}\,e^{(-1)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(x*sqrt(-e)/sqrt(x^2*e + d)) - 1/3*(x^2*e + d)^(3/2)*sqrt(-e)*e^(-1
)/d + 1/3*((x^2*e + d)^(3/2) - 3*sqrt(x^2*e + d)*d)*sqrt(-e)*e^(-1)/d
```

Fricas [C] Result contains complex when optimal does not.
time = 2.75, size = 53, normalized size = 1.23

$$\frac{1}{2}\left(i\,xe\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-2i\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*x*e*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 2*I*sqrt(x^
2*e + d)*e^(1/2))*e^(-1)
```

Sympy [A]
time = 0.32, size = 41, normalized size = 0.95

$$\begin{cases} x \operatorname{atan}{\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}} \right)} - \frac{\sqrt{-e}\,\sqrt{d+ex^2}}{e} & \text{for}\: e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((x*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - sqrt(-e)*sqrt(d + e*x**2)/
e, Ne(e, 0)), (0, True))
```

Giac [A]
time = 0.44, size = 40, normalized size = 0.93

$$x \arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right) - \frac{\sqrt{-e^2x^2-de}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(-e^2*x^2 - d*e)/e
```

Mupad [B]
time = 0.66, size = 35, normalized size = 0.81

$$\frac{\sqrt{e\,x^2+d}}{\sqrt{-e}} + x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] (d + e*x^2)^(1/2)/(-e)^(1/2) + x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))
```

3.15 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*(-
e)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5259, 272, 65, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x^
2]/Sqrt[d]])/Sqrt[d]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\sqrt{-e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\frac{1}{2}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{\sqrt{-e}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 86, normalized size = 1.46

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x}+\frac{i\sqrt{e}\,\log\left(\frac{2i\sqrt{d}}{\sqrt{e}\,x}-\frac{2\sqrt{-e}\,\sqrt{d+ex^2}}{ex}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) + (I*Sqrt[e]*Log[((2*I)*Sqrt[d])/
(Sqrt[e]*x) - (2*Sqrt[-e]*Sqrt[d + e*x^2])/(e*x)])/Sqrt[d]
```

Maple [A]
time = 0.01, size = 90, normalized size = 1.53

method	result	s

default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x}+\frac{\sqrt{-e}\,\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{d}-\frac{\sqrt{-e}\,\sqrt{e\,x^{2}+d}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x+(-e)^(1/2)/d*((e*x^2+d)^(1/2)-d^(1/
2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))-(-e)^(1/2)/d*(e*x^2+d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.65, size = 181, normalized size = 3.07

$$-\frac{x\sqrt{-\frac{e}{d}}\log\left(-\frac{2\left(\sqrt{x^{2}e+d}\sqrt{-\frac{e}{d}}+ie^{\frac{1}{2}}\right)}{x}\right)-x\sqrt{-\frac{e}{d}}\log\left(\frac{2\left(\sqrt{x^{2}e+d}\sqrt{-\frac{e}{d}}-ie^{\frac{1}{2}}\right)}{x}\right)+(-ix+i)\log\left(\frac{2x^{2}e+2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)+ix\log\left(\frac{2\left(xe+\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)-ix\log\left(\frac{2\left(xe-\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*sqrt(-e/d)*log(-2*(sqrt(x^2*e + d)*sqrt(-e/d) + I*e^(1/2))/x) - x*s
qrt(-e/d)*log(2*(sqrt(x^2*e + d)*sqrt(-e/d) - I*e^(1/2))/x) + (-I*x + I)*lo
g((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) + I*x*log(2*(x*e + sqrt(x^
2*e + d)*e^(1/2))/x) - I*x*log(2*(x*e - sqrt(x^2*e + d)*e^(1/2))/x))/x
```

Sympy [A]
time = 2.78, size = 60, normalized size = 1.02

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{x}+\frac{\sqrt{-e}\,\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{d}}\,\sqrt{d+ex^{2}}}\right)}{d\sqrt{-\frac{1}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/x + sqrt(-e)*atan(1/(sqrt(-1/d)*sqrt(d +
 e*x**2)))/(d*sqrt(-1/d))
```

Giac [A]
time = 0.48, size = 53, normalized size = 0.90

$$-\frac{e \arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -e*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/sqrt(d*e) - arctan(sqrt(-e)*x/sqr
t(e*x^2 + d))/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

3.16 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{6dx^2}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}$$

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3-1/6*(-e)^(3/2)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/d^(3/2)-1/6*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5259, 272, 44, 65, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{6dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^
2]]/(3*x^3) - ((-e)^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{-e}\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{(-e)^{3/2}\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d}\right.}{6d}\\
&= -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{(-e)^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 1.11

$$
-\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{6d^{3/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) + (e^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-e]
)/(Sqrt[e]*Sqrt[d + e*x^2])])/(6*d^(3/2)) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*
x^2]]/(3*x^3)
```

Maple [A]
time = 0.01, size = 130, normalized size = 1.43

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{\sqrt{-e}\,e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\mathrm{l}\right.}{}\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3+1/3*(-e)^(1/2)*e/d^(3/2)*ln((
2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/3*(-e)^(1/2)/d*(-1/2/d/x^2*(e*x^2+d)^(3
/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.00, size = 225, normalized size = 2.47

$$\frac{dx^{3}\sqrt{-\frac{e^{3}}{d^{3}}}\log\left(\frac{\sqrt{x^{2}e+d}\,d\sqrt{-\frac{e^{3}}{d^{3}}}+ie^{\frac{3}{2}}}{3\,dx}\right)-dx^{3}\sqrt{-\frac{e^{3}}{d^{3}}}\log\left(-\frac{\sqrt{x^{2}e+d}\,d\sqrt{-\frac{e^{3}}{d^{3}}}-ie^{\frac{3}{2}}}{3\,dx}\right)-2i\,dx^{3}\log\left(\frac{2\left(xe+\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)+2i\,dx^{3}\log\left(\frac{2\left(xe-\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)-2i\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-2\left(-i\,dx^{3}+i\,d\right)\log\left(\frac{2x^{2}e+2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{12\,dx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(d*x^3*sqrt(-e^3/d^3)*log(1/3*(sqrt(x^2*e + d)*d*sqrt(-e^3/d^3) + I*e^
(3/2))/(d*x)) - d*x^3*sqrt(-e^3/d^3)*log(-1/3*(sqrt(x^2*e + d)*d*sqrt(-e^3/
d^3) - I*e^(3/2))/(d*x)) - 2*I*d*x^3*log(2*(x*e + sqrt(x^2*e + d)*e^(1/2))/
```

```
x) + 2*I*d*x^3*log(2*(x*e - sqrt(x^2*e + d)*e^(1/2))/x) - 2*I*sqrt(x^2*e +
d)*x*e^(1/2) - 2*(-I*d*x^3 + I*d)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2
) + d)/d))/(d*x^3)
```

Sympy [A]
time = 5.70, size = 82, normalized size = 0.90

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6dx}+\frac{e\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\,x}\right)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**3) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(6*d*x) + e*sqrt(-e)*asinh(sqrt(d)/(sqrt(e)*x))/(6*d**(3/2))
```

Giac [A]
time = 0.49, size = 87, normalized size = 0.96

$$\frac{\frac{e^3\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}\,d}-\frac{\sqrt{-e^2x^2-de}\,e}{dx^2}}{6\,e}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^2+d}}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(e^3*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d) - sqrt(-e^2*x
^2 - d*e)*e/(d*x^2))/e - 1/3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

3.17 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*(-e)^(5/2)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))/d^(5/2)-3/40*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*(-
e)^(1/2)*(e*x^2+d)^(1/2)/d/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5259, 272, 44, 65, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3(-e)^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -1/20*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^4) - (3*(-e)^(3/2)*Sqrt[d + e*x^2])/(
40*d^2*x^2) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*(-e)^(5/2)*
ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{1}{5}\sqrt{-e}\int \frac{1}{x^5\sqrt{d+ex^2}}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{1}{10}\sqrt{-e}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{d+ex}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{(3(-e)^{3/2})\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\right.}{40d}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} + \frac{(3(-e)^{5/2}}{}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{(3(-e)^{3/2}}{}$$

$$= -\frac{\sqrt{-e}\,\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3(-e)^{5/2}\,\text{ta}}{}$$

Mathematica [A]

time = 0.08, size = 114, normalized size = 0.96

$$\sqrt{-e}\left(-\frac{1}{20dx^4}+\frac{3e}{40d^2x^2}\right)\sqrt{d+ex^2}-\frac{3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{40d^{5/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] Sqrt[-e]*(-1/20*1/(d*x^4) + (3*e)/(40*d^2*x^2))*Sqrt[d + e*x^2] - (3*e^(5/2
)*ArcTan[(Sqrt[d]*Sqrt[-e])/(Sqrt[e]*Sqrt[d + e*x^2])])/(40*d^(5/2)) - ArcT
an[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5)
```

Maple [A]
time = 0.01, size = 178, normalized size = 1.50

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{2x^2d}+\frac{e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{4d\,x^4}-\cdots e\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-1/5*(-e)^(1/2)*e/d*(-1/2*(e*x
^2+d)^(1/2)/x^2/d+1/2*e/d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))+1/5*
(-e)^(1/2)/d*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(3/2
)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.77, size = 253, normalized size = 2.13

$$-\frac{3\,d^{2}x^{5}\sqrt{-\frac{e^{5}}{d^{5}}}\log\left(-\frac{3\left(\sqrt{x^{2}e+d}\,d^{2}\sqrt{-\frac{e^{5}}{d^{5}}}+ie^{\frac{5}{2}}\right)}{20\,d^{2}x}\right)-3\,d^{2}x^{5}\sqrt{-\frac{e^{5}}{d^{5}}}\log\left(\frac{3\left(\sqrt{x^{2}e+d}\,d^{2}\sqrt{-\frac{e^{5}}{d^{5}}}-ie^{\frac{5}{2}}\right)}{20\,d^{2}x}\right)+8i\,d^{2}x^{5}\log\left(\frac{2\left(xe+\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)-8i\,d^{2}x^{5}\log\left(\frac{2\left(xe-\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}\right)}{x}\right)-2\left(3i\,x^{3}e-2i\,dx\right)\sqrt{x^{2}e+d}\,e^{\frac{1}{2}}+8\left(-i\,d^{2}x^{5}+i\,d^{2}\right)\log\left(\frac{2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{80\,d^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] -1/80*(3*d^2*x^5*sqrt(-e^5/d^5)*log(-3/20*(sqrt(x^2*e + d)*d^2*sqrt(-e^5/d^
5) + I*e^(5/2))/(d^2*x)) - 3*d^2*x^5*sqrt(-e^5/d^5)*log(3/20*(sqrt(x^2*e +
d)*d^2*sqrt(-e^5/d^5) - I*e^(5/2))/(d^2*x)) + 8*I*d^2*x^5*log(2*(x*e + sqrt
(x^2*e + d)*e^(1/2))/x) - 8*I*d^2*x^5*log(2*(x*e - sqrt(x^2*e + d)*e^(1/2))
/x) - 2*(3*I*x^3*e - 2*I*d*x)*sqrt(x^2*e + d)*e^(1/2) + 8*(-I*d^2*x^5 + I*d
^2)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d))/(d^2*x^5)
```

Sympy [A]
time = 10.07, size = 148, normalized size = 1.24

$$-\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{5x^{5}}-\frac{\sqrt{-e}}{20\sqrt{e}\,x^{5}\sqrt{\frac{d}{ex^{2}}+1}}+\frac{\sqrt{e}\,\sqrt{-e}}{40dx^{3}\sqrt{\frac{d}{ex^{2}}+1}}+\frac{3e^{\frac{3}{2}}\sqrt{-e}}{40d^{2}x\sqrt{\frac{d}{ex^{2}}+1}}-\frac{3e^{2}\sqrt{-e}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}\,x}\right)}{40d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

```
[Out] -atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**5) - sqrt(-e)/(20*sqrt(e)*x**5*sqr
t(d/(e*x**2) + 1)) + sqrt(e)*sqrt(-e)/(40*d*x**3*sqrt(d/(e*x**2) + 1)) + 3*
e**(3/2)*sqrt(-e)/(40*d**2*x*sqrt(d/(e*x**2) + 1)) - 3*e**2*sqrt(-e)*asinh(
sqrt(d)/(sqrt(e)*x))/(40*d**(5/2))
```

Giac [A]
time = 0.48, size = 116, normalized size = 0.97

$$-\frac{\frac{3\,e^{4}\arctan\left(\frac{\sqrt{-e^{2}x^{2}-de}}{\sqrt{de}}\right)}{\sqrt{de}\,d^{2}}+\frac{5\,\sqrt{-e^{2}x^{2}-de}\,de^{5}+3\,(-e^{2}x^{2}-de)^{\frac{3}{2}}\,e^{4}}{d^{2}e^{4}x^{4}}}{40\,e}-\frac{\arctan\left(\frac{\sqrt{-e}\,x}{\sqrt{ex^{2}+d}}\right)}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] -1/40*(3*e^4*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d^2) + (5*sq
rt(-e^2*x^2 - d*e)*d*e^5 + 3*(-e^2*x^2 - d*e)^(3/2)*e^4)/(d^2*e^4*x^4))/e -
 1/5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

3.18 $\int x^{9/2}\mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=211

$$\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{\right.}}{}$$

```
[Out] 2/11*x^(11/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+
d)^(1/2)/(-e)^(3/2)+4/121*x^(9/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)+60/847*d^2*x^(
1/2)*(e*x^2+d)^(1/2)/(-e)^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(13/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 327, 335, 226}

$$\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}+\frac{2}{11}x^{11/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*(-e)^(5/2)) + (36*d*x^(5/2)*Sqrt[d +
e*x^2])/(847*(-e)^(3/2)) + (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[-e]) + (2*
x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*Sqrt[-e]*(
Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*
ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(13/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{11}\left(2\sqrt{-e}\right)\int\frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx$$

$$=\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx}{121\sqrt{-e}}$$

$$=\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-$$

$$=\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\,\mathrm{ta}$$

$$=\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\,\mathrm{ta}$$

$$=\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\,\mathrm{ta}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.41, size = 170, normalized size = 0.81

$$\frac{4\sqrt{x}\,\sqrt{d+ex^2}\,(15d^2-9dex^2+7e^2x^4)}{847(-e)^{5/2}}+\frac{2}{11}x^{11/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{60id^3\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{847\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,(-e)^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/(847*(-e)^(5/2
)) + (2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (((60*I)/847)*d
^3*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt
[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*(-e)^(5/2)*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{9}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 92, normalized size = 0.44

$$\frac{1}{847}\left(77i\,x^{\frac{11}{2}}e^3\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)+60i\,d^3\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)-4\left(7i\,x^4e^2-9i\,dx^2e+15i\,d^2\right)\sqrt{x^2e+d}\,\sqrt{x}\,e^{\frac{1}{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 1/847*(77*I*x^(11/2)*e^3*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d)
 + 60*I*d^3*weierstrassPInverse(-4*d*e^(-1), 0, x) - 4*(7*I*x^4*e^2 - 9*I*d
*x^2*e + 15*I*d^2)*sqrt(x^2*e + d)*sqrt(x)*e^(1/2))*e^(-3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8855 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.19 $\int x^{5/2}\mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=181

$$\frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e}{\left(\sqrt{d}+\right.}}}{147e^{9/4}\sqrt{d}}$$

```
[Out] 2/7*x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/49*x^(5/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/(-e)^(3/2)-10/147*d^(7/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-
e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4
)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 327, 335, 226}

$$-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}+\frac{2}{7}x^{7/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*(-e)^(3/2)) + (4*x^(5/2)*Sqrt[d + e*x^2
])/(49*Sqrt[-e]) + (2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (10
*d^(7/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]
*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(9/4)*Sq
rt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\left(2\sqrt{-e}\right)\int \frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(10d)\int \frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx}{49\sqrt{-e}}$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147(-e)^{3/2}} + \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(1}{}$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147(-e)^{3/2}} + \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(2}{}$$

$$10$$

$$= \frac{20d\sqrt{x}\,\sqrt{d+ex^2}}{147(-e)^{3/2}} + \frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}} + \frac{2}{7}x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - $$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 158, normalized size = 0.87

$$\frac{2}{147}\sqrt{x}\left(\frac{2(5d-3ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}} + 21x^3\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right) - \frac{20id^2\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{147\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,(-e)^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 21*x^3*ArcTan[
(Sqrt[-e]*x)/Sqrt[d + e*x^2]]))/147 - (((20*I)/147)*d^2*Sqrt[1 + d/(e*x^2)]
*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sq
rt[d])/Sqrt[e]]*(-e)^(3/2)*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 82, normalized size = 0.45

$$\frac{1}{147}\left(21i\,x^{\frac{7}{2}}e^2\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4\,\sqrt{x^2e+d}\,(3i\,x^2e-5i\,d)\,\sqrt{x}\,e^{\frac{1}{2}}-20i\,d^2\text{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 1/147*(21*I*x^(7/2)*e^2*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d)
- 4*sqrt(x^2*e + d)*(3*I*x^2*e - 5*I*d)*sqrt(x)*e^(1/2) - 20*I*d^2*weierstr
assPInverse(-4*d*e^(-1), 0, x))*e^(-2)
```

Sympy [C] Result contains complex when optimal does not.
time = 51.93, size = 75, normalized size = 0.41

$$\frac{2x^{\frac{7}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^{\frac{9}{2}}\sqrt{-e}\,\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{7\sqrt{d}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(7/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**(9/2)*sqrt(-e)*gamma(9/
4)*hyper((1/2, 9/4), (13/4,), e*x**2*exp_polar(I*pi)/d)/(7*sqrt(d)*gamma(13
/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.20 $\int \sqrt{x}\,\mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=153

$$\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\right.}{9e^{5/4}\sqrt{d+ex^2}}$$

```
[Out] 2/3*x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/9*x^(1/2)*(e*x^2+d)^(1/2
)/(-e)^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)
+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 327, 335, 226}

$$\frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}+\frac{2}{3}x^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*
x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)]
, 1/2])/(9*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx=\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\left(2\sqrt{-e}\right)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}\,dx$$

$$=\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(2d)\int\frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,d}{9\sqrt{-e}}$$

$$=\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(4d)\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\right.}{9\sqrt{-e}}$$

$$=\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 147, normalized size = 0.96

$$\frac{4\sqrt{x}\,\sqrt{d+ex^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{4id\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{9\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{-e}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/S
qrt[d + e*x^2]])/3 - (((4*I)/9)*d*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[-
e]*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 69, normalized size = 0.45

$$\frac{1}{9}\left(3i\,x^{\frac{3}{2}}e\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4i\,\sqrt{x^2e+d}\,\sqrt{x}\,e^{\frac{1}{2}}+4i\,d\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 1/9*(3*I*x^(3/2)*e*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 4*I
*sqrt(x^2*e + d)*sqrt(x)*e^(1/2) + 4*I*d*weierstrassPInverse(-4*d*e^(-1), 0
, x))*e^(-1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.18, size = 75, normalized size = 0.49

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3}-\frac{x^{\frac{5}{2}}\sqrt{-e}\,\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(3/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/3 - x**(5/2)*sqrt(-e)*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(9/4
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.21 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}+\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}$$

```
[Out] -2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*(cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(si
n(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(
1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/e^(1/4)/(e*x^2+d)^(1/
2)
```

Rubi [A]

time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5259, 335, 226}

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^
(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{-e}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}\,dx$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\,\text{t}\ldots\right.}{\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{d+ex^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 115, normalized size = 0.94

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{-e}\,\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[-e]*Sqrt[1
+ d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 50, normalized size = 0.41

$$\frac{4i\,x\text{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)-i\,\sqrt{x}\,\log\left(\frac{2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas
")
```

```
[Out] (4*I*x*weierstrassPInverse(-4*d*e^(-1), 0, x) - I*sqrt(x)*log((2*x^2*e + 2*
sqrt(x^2*e + d)*x*e^(1/2) + d)/d))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 2.82, size = 71, normalized size = 0.58

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^{2}}}\right)}{\sqrt{x}}+\frac{\sqrt{x}\,\sqrt{-e}\,\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{ex^{2}e^{i\pi}}{d}\right)}{\sqrt{d}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/sqrt(x) + sqrt(x)*sqrt(-e)*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

3.22 $\int \frac{\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{2\sqrt{-e}\,e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\operatorname{ArcTan}\left(\right.\right.}{15d^{5/4}\sqrt{d+ex^2}}$$

```
[Out] -2/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(3/2)-2/15*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1
/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x
^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^
(1/2)+x*e^(1/2))^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 331, 335, 226}

$$-\frac{2\sqrt{-e}\,e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\operatorname{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{2\operatorname{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[
d + e*x^2]])/(5*x^(5/2)) - (2*Sqrt[-e]*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(
d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^
(1/4)], 1/2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{1}{5}\left(2\sqrt{-e}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{\left(2(-e)^{3/2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{d+ex^2}}}{15d}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+e}}\right.}{15d}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{2(-e)^{3/2}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\cdots}}{15\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 150, normalized size = 0.96

$$-\frac{2\left(2\sqrt{-e}\,x\sqrt{d+ex^2}+3d\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}}+\frac{4i(-e)^{3/2}\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{15d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[-e]*x*Sqrt[d + e*x^2] + 3*d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]
]))/(15*d*x^(5/2)) + (((4*I)/15)*(-e)^(3/2)*Sqrt[1 + d/(e*x^2)]*x*EllipticF
[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d*Sqrt[(I*Sqrt[d])/Sqr
t[e]]*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 76, normalized size = 0.49

$$\frac{-4i\,x^3e\text{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)-4i\,\sqrt{x^2e+d}\,x^{\frac{3}{2}}e^{\frac{1}{2}}-3i\,d\sqrt{x}\,\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{15\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas
")
```

```
[Out] 1/15*(-4*I*x^3*e*weierstrassPInverse(-4*d*e^(-1), 0, x) - 4*I*sqrt(x^2*e +
d)*x^(3/2)*e^(1/2) - 3*I*d*sqrt(x)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/
2) + d)/d))/(d*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 36.69, size = 78, normalized size = 0.50

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^{\frac{5}{2}}}+\frac{\sqrt{-e}\,\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\,x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**(5/2)) + sqrt(-e)*gamma(-3/4)*hy
per((-3/4, 1/2), (1/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*x**(3/2)*gamm
a(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

3.23 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ \ldots}{\left(\sqrt{d}+ \ldots\right.}}}{189d^{9/4}\ldots}$$

```
[Out] -2/9*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)-20/189*(-e)^(3/2)*(e*x^2+
d)^(1/2)/d^2/x^(3/2)-4/63*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(7/
4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2
))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d
^(9/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 331, 335, 226}

$$\frac{10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) - (20*(-e)^(3/2)*Sqrt[d + e*x^
2])/(189*d^2*x^(3/2)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)
) + (10*Sqrt[-e]*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(
9/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx &= -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{1}{9}\left(2\sqrt{-e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx\\
&= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{\left(10(-e)^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}}{63d}\\
&= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{(10(\cdots}{\cdots}\\
&= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{(20(\cdots}{\cdots}\\
&= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}+\frac{10\sqrt{\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.23, size = 162, normalized size = 0.87

$$\frac{4\sqrt{-e}\,x\sqrt{d+ex^2}\,(-3d+5ex^2)-42d^2\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}}+\frac{20i(-e)^{5/2}\sqrt{1+\frac{d}{ex^2}}\,xF\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{189d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTan[(Sqrt[-e]*x)
/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (((20*I)/189)*(-e)^(5/2)*Sqrt[1 + d/
(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d^
2*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 90, normalized size = 0.48

$$\frac{20i\,x^5e^2\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)-21i\,d^2\sqrt{x}\,\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4\,(-5i\,x^3e+3i\,dx)\sqrt{x^2e+d}\,\sqrt{x}\,e^{\frac{1}{2}}}{189\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 1/189*(20*I*x^5*e^2*weierstrassPInverse(-4*d*e^(-1), 0, x) - 21*I*d^2*sqrt(
x)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 4*(-5*I*x^3*e + 3*I
*d*x)*sqrt(x^2*e + d)*sqrt(x)*e^(1/2))/(d^2*x^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

3.24 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{143dx^{11/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}-\frac{30\sqrt{-e}\,e^{11/4}\left(\sqrt{\right.}}{}$$

```
[Out] -2/13*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)-36/1001*(-e)^(3/2)*(e*x
^2+d)^(1/2)/d^2/x^(7/2)-60/1001*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/14
3*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(11/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5259, 331, 335, 226}

$$-\frac{30\sqrt{-e}\,e^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}-\frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}-\frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{143dx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) - (36*(-e)^(3/2)*Sqrt[d + e*
x^2])/(1001*d^2*x^(7/2)) - (60*(-e)^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2
)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*Sqrt[-e]*
e^(11/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*El
lipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d +
e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{-e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(18(-e)^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}}{143d}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{(90(-}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-}}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-}}{}$$

$$= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tan^{-}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.40, size = 171, normalized size = 0.79

$$\frac{2\left(-\frac{2\sqrt{-e}\,\sqrt{d+ex^2}\,(7d^2x-9dex^3+15e^2x^5)}{d^3}-77\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{30i(-e)^{7/2}\sqrt{1+\frac{d}{ex^2}}\,x^{15/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^3\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,\sqrt{d+ex^2}}\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[-e]*Sqrt[d + e*x^2]*(7*d^2*x - 9*d*e*x^3 + 15*e^2*x^5))/d^3 -
77*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + ((30*I)*(-e)^(7/2)*Sqrt[1 + d/(e*
x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])
/(d^3*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 100, normalized size = 0.46

$$\frac{-60i\,x^7e^3\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)-77i\,d^3\sqrt{x}\,\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4\,(15i\,x^5e^2-9i\,dx^3e+7i\,d^2x)\sqrt{x^2e+d}\,\sqrt{x}\,e^{\frac{1}{2}}}{1001\,d^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="frica
s")
```

```
[Out] 1/1001*(-60*I*x^7*e^3*weierstrassPInverse(-4*d*e^(-1), 0, x) - 77*I*d^3*sqr
t(x)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 4*(15*I*x^5*e^2 -
 9*I*d*x^3*e + 7*I*d^2*x)*sqrt(x^2*e + d)*sqrt(x)*e^(1/2))/(d^3*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac"
)
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

3.25 $\int x^{7/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=326

$$\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{9}x^{9/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{28d^{9/4}\sqrt{-e}}{}$$

```
[Out] 2/9*x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)
^(1/2)/(-e)^(3/2)+4/81*x^(7/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)-28/135*d^2*(-e)^(
1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(5/2)/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e
)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4
)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)
^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4
)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/
(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5259, 327, 335, 311, 226, 1210}

$$-\frac{14d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}+\frac{2}{9}x^{9/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{28d^2\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*(-e)^(3/2)) + (4*x^(7/2)*Sqrt[d + e*x^2
])/(81*Sqrt[-e]) - (28*d^2*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^(5/2)*(
Sqrt[d] + Sqrt[e]*x)) + (2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9
+ (28*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sq
rt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(11
/4)*Sqrt[d + e*x^2]) - (14*d^(9/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(135*e^(11/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{9}(2\sqrt{-e})\int\frac{x^{9/2}}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx}{81\sqrt{-e}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(14c}{}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(28c}{}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}+\frac{2}{9}x^{9/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{(28c}{}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{9}x^{9/}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 139, normalized size = 0.43

$$\frac{2x^{3/2}\left(2\sqrt{-e}\,(7d^2+2dex^2-5e^2x^4)+45e^2x^3\sqrt{d+ex^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-14d^2\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)\right)}{405e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(2*Sqrt[-e]*(7*d^2 + 2*d*e*x^2 - 5*e^2*x^4) + 45*e^2*x^3*Sqrt[d
+ e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[-e]*Sqrt[1 + (e
*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^2*Sqrt[d +
 e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.91, size = 90, normalized size = 0.28

$$\frac{1}{405}\left(45i\,x^{\frac{9}{2}}e^{2}\log\left(\frac{2\,x^{2}e+2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4\left(5i\,x^{3}e-7i\,dx\right)\sqrt{x^{2}e+d}\,\sqrt{x}\,e^{\frac{1}{2}}+84i\,d^{2}\text{weierstrassZeta}\left(-4\,de^{(-1)},0,\text{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 1/405*(45*I*x^(9/2)*e^2*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d)
- 4*(5*I*x^3*e - 7*I*d*x)*sqrt(x^2*e + d)*sqrt(x)*e^(1/2) + 84*I*d^2*weiers
trassZeta(-4*d*e^(-1), 0, weierstrassPInverse(-4*d*e^(-1), 0, x)))*e^(-2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.26 $\int x^{3/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)\,dx$

Optimal. Leaf size=296

$$\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}+\frac{12d\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{25e^{3/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}x^{5/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{12d^{5/4}\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{(\cdots}}{25\cdots}$$

```
[Out] 2/5*x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/25*x^(3/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+12/25*d*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)/(d^(1/2)+x
*e^(1/2))-12/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+
x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1
/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellip
ticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2
)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2
)
```

Rubi [A]
time = 0.13, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5259, 327, 335, 311, 226, 1210}

$$\frac{6d^{5/4}\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}}+\frac{2}{5}x^{5/2}\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{12d\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{25e^{3/2}\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[-e]) + (12*d*Sqrt[-e]*Sqrt[x]*Sqrt[d +
 e*x^2])/(25*e^(3/2)*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/5 - (12*d^(5/4)*Sqrt[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
+ e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2]) + (6*d^(5/4)*Sqrt[-e]*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(7/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{5}(2\sqrt{-e})\int\frac{x^{5/2}}{\sqrt{d+ex^2}}dx$$

$$= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} + \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}dx}{25\sqrt{-e}}$$

$$= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} + \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(12d)\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex}\right.}{25\sqrt{-e}}$$

$$= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} + \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{(12d^{3/2})\,\text{Subst}\left(\int\frac{1}{\sqrt{d+}\right.}{25\sqrt{-e}}$$

$$= \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} - \frac{12d\sqrt{x}\sqrt{d+ex^2}}{25\sqrt{-e^2}\left(\sqrt{d}+\sqrt{e}\,x\right)} + \frac{2}{5}x^{5/2}\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 119, normalized size = 0.40

$$\frac{2x^{3/2}\left(-2\sqrt{-e}\,(d+ex^2)+5ex\sqrt{d+ex^2}\,\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+2d\sqrt{-e}\,\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)\right)}{25e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*Sqrt[-e]*(d + e*x^2) + 5*e*x*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e
]*x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[
1/2, 3/4, 7/4, -((e*x^2)/d)]))/(25*e*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.42, size = 76, normalized size = 0.26

$$\frac{1}{25}\left(5i\,x^{\frac{5}{2}}e\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4i\,\sqrt{x^2e+d}\,x^{\frac{3}{2}}e^{\frac{1}{2}}-12i\,d\mathrm{weierstrassZeta}\left(-4\,de^{(-1)},0,\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas
")
```

```
[Out] 1/25*(5*I*x^(5/2)*e*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) - 4*
I*sqrt(x^2*e + d)*x^(3/2)*e^(1/2) - 12*I*d*weierstrassZeta(-4*d*e^(-1), 0,
weierstrassPInverse(-4*d*e^(-1), 0, x)))*e^(-1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 6.08, size = 75, normalized size = 0.25

$$\frac{2x^{\frac{5}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^{\frac{7}{2}}\sqrt{-e}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] 2*x**(5/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**(7/2)*sqrt(-e)*gamma(7/
4)*hyper((1/2, 7/4), (11/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*gamma(11
/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] integrate(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \operatorname{atan}\left(\frac{\sqrt{-e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.27 $\int \frac{\mathbf{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=260

$$-\frac{4\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)}+2\sqrt{x}\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt[4]{d}\,\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\Big(2\ldots}{e^{3/4}\sqrt{d+ex^2}}$$

```
[Out] 2*x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-4*(-e)^(1/2)*x^(1/2)*(e*x^2+
d)^(1/2)/e^(1/2)/(d^(1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2
*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2
))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(3/4)/(e*x^2+d)^(1/2)-2*d^(1/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^
(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5259, 335, 311, 226, 1210}

$$-\frac{2\sqrt[4]{d}\,\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\,\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}+2\sqrt{x}\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)-\frac{4\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) + 2*S
qrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*Sqrt[-e]*(Sqrt[d]
+ Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*Sqr
t[-e]*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \left(2\sqrt{-e}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx$$

$$= 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \left(4\sqrt{-e}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$

$$= 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(4\sqrt{d}\,\sqrt{-e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{e}}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)} + 2\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) + \frac{4\sqrt[4]{d}\,\sqrt{-e}\left(\sqrt{d}+\sqrt{\ldots}\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 89, normalized size = 0.34

$$2\sqrt{x}\,\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{-e}\,x^{3/2}\sqrt{1+\frac{ex^2}{d}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[-e]*x^(3/2)*Sqrt[1
 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e
*x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 52, normalized size = 0.20

$$i\sqrt{x}\log\left(\frac{2x^2e + 2\sqrt{x^2e+d}\,xe^{\frac{1}{2}} + d}{d}\right) + 4i\,\mathrm{weierstrassZeta}\left(-4\,de^{(-1)}, 0, \mathrm{weierstrassPInverse}\left(-4\,de^{(-1)}, 0, x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas
")
```

```
[Out] I*sqrt(x)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d) + 4*I*weierstr
assZeta(-4*d*e^(-1), 0, weierstrassPInverse(-4*d*e^(-1), 0, x))
```

Sympy [C] Result contains complex when optimal does not.
time = 4.57, size = 71, normalized size = 0.27

$$2\sqrt{x}\,\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right) - \frac{x^{\frac{3}{2}}\sqrt{-e}\,\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - x**(3/2)*sqrt(-e)*gamma(3/4)*
hyper((1/2, 3/4), (7/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

3.28 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}}+\frac{4\sqrt{-e^2}\,\sqrt{x}\,\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}-\frac{4\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{\cdots}{\left(\sqrt{d}\cdots\right.}}\cdots}{3d^{3/\cdots}}$$

```
[Out] -2/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*(-e)^(1/2)*(e*x^2+d)^
(1/2)/d/x^(1/2)+4/3*(-e^2)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/
2))-4/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arct
an(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)
)),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2
))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)+2/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/
2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5259, 331, 335, 311, 226, 1210}

$$\frac{2\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{-e}\,\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}+\frac{4\sqrt{-e^2}\,\sqrt{x}\,\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*Sqrt[-e^2]*Sqrt[x]*Sqrt[d
+ e*x^2])/(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x
^2]])/(3*x^(3/2)) - (4*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x
^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2]) + (2*Sqrt[-e]*e^(1/4)*(Sqrt[d] + Sqrt[e]
*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*S
qrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{-e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(2(-e)^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+e}}\right.}{3d}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{\left(4(-e)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+e}}\right.}{3\sqrt{d}\,\sqrt{e}}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{4(-e)^{3/2}\sqrt{x}\,\sqrt{d+ex^2}}{3d\sqrt{e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{4(}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 121, normalized size = 0.41

$$-\frac{2\left(6\sqrt{-e}\,x(d+ex^2)+3d\sqrt{d+ex^2}\,\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)+2(-e)^{3/2}x^3\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)\right)}{9dx^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-2*(6*Sqrt[-e]*x*(d + e*x^2) + 3*d*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]] + 2*(-e)^(3/2)*x^3*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((e*x^2)/d)]))/(9*d*x^(3/2)*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.07, size = 83, normalized size = 0.28

$$\frac{-4i\,x^2e\mathrm{weierstrassZeta}\left(-4\,de^{(-1)},0,\mathrm{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)-4i\,\sqrt{x^2e+d}\,x^{\frac{3}{2}}e^{\frac{1}{2}}-i\,d\sqrt{x}\,\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)}{3\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(-4*I*x^2*e*weierstrassZeta(-4*d*e^(-1), 0, weierstrassPInverse(-4*d*e^
(-1), 0, x)) - 4*I*sqrt(x^2*e + d)*x^(3/2)*e^(1/2) - I*d*sqrt(x)*log((2*x^2
*e + 2*sqrt(x^2*e + d)*x*e^(1/2) + d)/d))/(d*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.52, size = 78, normalized size = 0.26

$$-\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^{\frac{3}{2}}}+\frac{\sqrt{-e}\,\Gamma\left(-\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\,\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] -2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**(3/2)) + sqrt(-e)*gamma(-1/4)*hy
per((-1/4, 1/2), (3/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*sqrt(x)*gamma
(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

3.29 $\int \frac{\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}}-\frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{12\sqrt{-e}\,e^{3/2}\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}+\frac{12\sqrt{-e}\,e}{}$$

```
[Out] -2/7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(5/2)-12/35*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^(3/2)
*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(5/4)*(
cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(
-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/
4)/(e*x^2+d)^(1/2)-6/35*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^
(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(
d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5259, 331, 335, 311, 226, 1210}

$$-\frac{6e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}-\frac{12e^{3/2}\sqrt{-e}\,\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[-e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) - (12*(-e)^(3/2)*Sqrt[d + e*x^
2])/(35*d^2*Sqrt[x]) - (12*Sqrt[-e]*e^(3/2)*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^
2*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7
/2)) + (12*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d
^(7/4)*Sqrt[d + e*x^2]) - (6*Sqrt[-e]*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d
 + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(
1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5259

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{-e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{\left(6(-e)^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{(6(-e}{}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{(12(-}{}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{(12\sqrt{-}}{}$$

$$= -\frac{4\sqrt{-e}\,\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12\sqrt{-e}\,e^{3/2}\sqrt{x}\,\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}\,x\right)} - \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 137, normalized size = 0.41

$$\frac{4\sqrt{-e}\,x(-d^2+2dex^2+3e^2x^4) - 10d^2\sqrt{d+ex^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{-e}\,x}{\sqrt{d+ex^2}}\right) - 4(-e)^{5/2}x^5\sqrt{1+\frac{ex^2}{d}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
[Out] (4*Sqrt[-e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTa
n[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*(-e)^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2]
)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-%e)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.39, size = 97, normalized size = 0.29

$$\frac{12i\,x^4e^2\text{weierstrassZeta}\left(-4\,de^{(-1)},0,\text{weierstrassPInverse}\left(-4\,de^{(-1)},0,x\right)\right)-5i\,d^2\sqrt{x}\,\log\left(\frac{2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}+d}{d}\right)-4\left(-3i\,x^3e+i\,dx\right)\sqrt{x^2e+d}\,\sqrt{x}\,e^{\frac{1}{2}}}{35\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas
")
```

```
[Out] 1/35*(12*I*x^4*e^2*weierstrassZeta(-4*d*e^(-1), 0, weierstrassPInverse(-4*d
*e^(-1), 0, x)) - 5*I*d^2*sqrt(x)*log((2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2
) + d)/d) - 4*(-3*I*x^3*e + I*d*x)*sqrt(x^2*e + d)*sqrt(x)*e^(1/2))/(d^2*x^
4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
```

```
[Out] int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

3.30 $\int \frac{\mathbf{ArcTan}(1+x+x^2)}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}\text{ArcTan}(1+x)-\frac{\text{ArcTan}(1+x+x^2)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)$$

```
[Out] 1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Rubi [A]
time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.636$, Rules used = {5313, 6874, 266, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}(x^2+x+1)}{x}+\frac{1}{2}\text{ArcTan}(x+1)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(1+x+x^2\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \frac{1+2x}{x\left(2+2x+3x^2+2x^3+x^4\right)}\,dx\\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\int \left(\frac{1}{2x}-\frac{x}{1+x^2}+\frac{2+x}{2\left(2+2x+x^2\right)}\right)dx\\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}+\frac{1}{2}\int \frac{2+x}{2+2x+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\int \frac{2+2x}{2+2x+x^2}\,dx+\frac{1}{2}\int \frac{}{2-}\\
&= -\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)-\frac{1}{2}\text{Subst}\\
&= \frac{1}{2}\tan^{-1}(1+x)-\frac{\tan^{-1}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.00

$$\frac{1}{2}\text{ArcTan}(1+x)-\frac{\text{ArcTan}\left(1+x+x^2\right)}{x}+\frac{\log(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{4}\log\left(2+2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x + x^2]/x^2,x]
```

```
[Out] ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[2
 + 2*x + x^2]/4
```

Maple [A]
time = 0.08, size = 43, normalized size = 0.86

method	result
default	$\frac{\arctan(1+x)}{2}-\frac{\arctan(x^2+x+1)}{x}+\frac{\ln(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{\ln(x^2+2x+2)}{4}$
risch	$\frac{i\ln(1+i(x^2+x+1))}{2x}-\frac{i(2i\ln(x)x+i\ln(1+i+x)x+i\ln(1-i+x)x-2i\ln(x^2+1)x-\ln(1+i+x)x+\ln(1-i+x)x+2\ln(1-i(x^2+x+1)))}{4x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^2+x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2)
```

Maxima [A]
time = 0.47, size = 42, normalized size = 0.84

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\arctan\left(x+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(x)
```

Fricas [A]
time = 2.22, size = 47, normalized size = 0.94

$$\frac{2x\arctan\left(x+1\right)+x\log\left(x^2+2x+2\right)-2x\log\left(x^2+1\right)+2x\log\left(x\right)-4\arctan\left(x^2+x+1\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*arctan(x + 1) + x*log(x^2 + 2*x + 2) - 2*x*log(x^2 + 1) + 2*x*log(
x) - 4*arctan(x^2 + x + 1))/x
```

Sympy [A]
time = 0.30, size = 41, normalized size = 0.82

$$\frac{\log\left(x\right)}{2}-\frac{\log\left(x^2+1\right)}{2}+\frac{\log\left(x^2+2x+2\right)}{4}+\frac{\operatorname{atan}\left(x+1\right)}{2}-\frac{\operatorname{atan}\left(x^2+x+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**2+x+1)/x**2,x)
```

```
[Out] log(x)/2 - log(x**2 + 1)/2 + log(x**2 + 2*x + 2)/4 + atan(x + 1)/2 - atan(x
**2 + x + 1)/x
```

Giac [A]
time = 0.43, size = 43, normalized size = 0.86

$$-\frac{\arctan\left(x^2+x+1\right)}{x}+\frac{1}{2}\arctan\left(x+1\right)+\frac{1}{4}\log\left(x^2+2x+2\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^2+x+1)/x^2,x, algorithm="giac")
```

```
[Out] -arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*l
og(x^2 + 1) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.76, size = 42, normalized size = 0.84

$$\frac{\mathrm{atan}(x+1)}{2}+\frac{\ln\left(x^2+2x+2\right)}{4}-\frac{\ln\left(x^2+1\right)}{2}+\frac{\ln\left(x\right)}{2}-\frac{\mathrm{atan}(x^2+x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x + x^2 + 1)/x^2,x)
```

```
[Out] atan(x + 1)/2 + log(2*x + x^2 + 2)/4 - log(x^2 + 1)/2 + log(x)/2 - atan(x +
 x^2 + 1)/x
```

3.31 $\int \frac{\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate((b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.32 $\int \frac{\left(a+b\mathbf{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=431

$$-\frac{2\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{3ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}}{2c}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arctan((-
c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(
1/2)))/c+3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(
1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2)))*polylog(3,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*I*b^3
*polylog(4,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*polylog(4,-1
+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A]
time = 0.36, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 4942, 5108, 5004, 5114, 5118, 6745}

$$\frac{3b^2\text{Li}_3\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b^2\text{Li}_3\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{Li}_2\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{3ib\text{Li}_2\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}-\frac{3ib^3\text{Li}_4\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{4c}+\frac{3ib^3\text{Li}_4\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/S
qrt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
 - (((3*I)/2)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1
 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (3*b^2*(a + b*ArcTan[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c
*x])])/(2*c) - (3*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3
, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/(2*c) - (((3*I)/4)*b^3*Pol
yLog[4, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c + (((3*I)/4)*b^3*Po
lyLog[4, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/c
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
```

```
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)S\ldots}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)S\ldots}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(\ldots}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(\ldots}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3ib\Big(\ldots}{}$$

Mathematica [A]
time = 0.14, size = 530, normalized size = 1.23

$$-\frac{8\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2i}{i-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)+6ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)-6ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)+6b^2\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)-6b^2\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)-3ib^3\text{PolyLog}\left(4,-\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)+3ib^3\text{PolyLog}\left(4,\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(8*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - (2*I)/(I
- Sqrt[1 - c*x]/Sqrt[1 + c*x])] + (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])^2*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x]
- I*Sqrt[1 + c*x]))] - (6*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^
```

```
2*PolyLog[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 +
c*x])] + 6*b^2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sq
rt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] - 6*b^2*
(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + I*S
qrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x])] - (3*I)*b^3*PolyLog[4, -((
Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]))] + (3*I
)*b^3*PolyLog[4, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[
1 + c*x])])/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1630 vs. 2(360) = 720.
time = 0.28, size = 1631, normalized size = 3.78

method	result	size
default	Expression too large to display	1631

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+3*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*
x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+
1)+1)^(1/2))+3*a^2*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1
)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-3/2*I*a^2*b/c*dilog((1+I*(
-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2*I*a^2*b/c*dilog(
1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3*a^2*b/c*arct
an((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/
((-c*x+1)/(c*x+1)+1))-b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+
I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*b^3/c*arctan(
(-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))
/((-c*x+1)/(c*x+1)+1)^(1/2))-6*I*b^3/c*polylog(4,-(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(
1/2))^3*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+3/2
*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)/
(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3/4*I*b^3/c*polylog(4,-(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-b^3/c*arctan((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1
)^(1/2))-6*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,(1+I*(-c*x+
1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*I*b^3/c*polylog(4,(1+
I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3*a*b^2/c*arcta
n((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/(
(-c*x+1)/(c*x+1)+1)+1)-3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(
1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3/2*I*b^3/
c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*
x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3*I*b^3/c*arctan((-c*x+1)^(1/2)/(c*x+1)
```

```
^(1/2))^2*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)
^(1/2))-3*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6*I*a*b^2/c*arctan((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*
x+1)/(c*x+1)+1)^(1/2))-3*I*a*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*pol
ylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+6*I*a*b^
2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x
+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*polylog(3,-(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+3/2*a*b^2/c*polylog(3,-(1+I
*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-6*a*b^2/c*polylog(3,
(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^3
*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^3 - 3*(b^3*log(2)^2*
log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x
 + 1)) - 64*c*integrate(1/128*(112*b^3*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1
))^3 + 384*a*b^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - 3*(b^3*log(2)^2
*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^3*log(
-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2)*sqrt(c*x + 1)*sqrt(-c*
x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)
))/(c^2*x^2 - 1), x))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctan(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{\left(a+b\mathbf{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=283

$$-\frac{2\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}+\frac{ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2\right.}{c}$$

```
[Out] 2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arctanh(-1+2/(1+I*(-c*x+1)^(
1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2
*b^2*polylog(3,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*polylog(3,
-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A]
time = 0.23, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6813, 4942, 5108, 5004, 5114, 6745}

$$\frac{ib\text{Li}_2\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{ib\text{Li}_2\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}+\frac{b^2\text{Li}_3\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{2c}-\frac{b^2\text{Li}_3\left(\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])]])/c + (I*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 +
c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])]])/c - (I*b*(a
 + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])]])/c + (b^2*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/S
qrt[1 + c*x])]])/(2*c) - (b^2*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[
1 + c*x])]])/(2*c)
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\text{S}}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\text{S}}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\Big(a}{}$$

$$= -\frac{2\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\Big(a}{}$$

Mathematica [A]
time = 0.09, size = 354, normalized size = 1.25

$$-\frac{4\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2i}{i-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)+2ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)-2ib\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)+b^2\text{PolyLog}\left(3,-\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)-b^2\text{PolyLog}\left(3,\frac{\sqrt{1-cx}+i\sqrt{1+cx}}{\sqrt{1-cx}-i\sqrt{1+cx}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(4*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - (2*I)/(I
- Sqrt[1 - c*x]/Sqrt[1 + c*x])] + (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])*PolyLog[2, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] -
I*Sqrt[1 + c*x]))] - (2*I)*b*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Po
lyLog[2, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*Sqrt[1 + c*x]
)] + b^2*PolyLog[3, -((Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1 - c*x] - I*
Sqrt[1 + c*x]))] - b^2*PolyLog[3, (Sqrt[1 - c*x] + I*Sqrt[1 + c*x])/(Sqrt[1
 - c*x] - I*Sqrt[1 + c*x])])/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(240) = 480.
time = 0.08, size = 837, normalized size = 2.96

method	result
default	$-\frac{a^2 \ln(cx-1)}{2c}+\frac{a^2 \ln(cx+1)}{2c}-\frac{b^2 \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1+\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}+\frac{2ib^2 \arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{p}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/
2))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c*polylog(3,-(1+I*(
-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+b^2/c*arctan((-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1
)/(c*x+1)+1)+1)-I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+
I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*b^2/c*polylog(3
,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-b^2/c*arctan((
-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c
*x+1)/(c*x+1)+1)^(1/2))+2*I*b^2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*poly
log(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/
c*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-
2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+2*a*b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/
2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)-I*a*b/c
*dilog((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1)+1)+I*a*b/c
*dilog(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1)
 - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*ar
ctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x - 1
)/c)*log(2)^2 - 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(s
qrt(-c*x + 1)/sqrt(c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) + 64*b^2*integr
```

```
ate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))*
log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt(-c*x +
 1)/sqrt(c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*arctan(sqr
t(-c*x + 1)/sqrt(c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctan(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{a+b\mathbf{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{ib\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{ib\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*I*b*polylog(2,-I*(-c*x+1)^(1/2)/(
c*x+1)^(1/2))/c+1/2*I*b*polylog(2,I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {212, 6813, 4940, 2438}

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}-\frac{ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}+\frac{ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - ((I/2)*b*PolyLog[2, ((-I)*Sqrt[
1 - c*x])/Sqrt[1 + c*x]])/c + ((I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1
+ c*x]])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{(ib)\text{Subs}}{}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{ib\text{Li}_2\left(-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{ib\text{Li}_2\left(\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 0.74

$$\frac{a\tanh^{-1}(cx)}{c}-\frac{ib\left(\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)-\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c - ((I/2)*b*(PolyLog[2, ((-I)*Sqrt[1 - c*x])/Sqrt[1 + c*x
]] - PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]]))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(78) = 156.
time = 0.07, size = 263, normalized size = 2.68

method	result

default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{\left(1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{\frac{-cx+1}{cx+1}+1}\right)}{c}+\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2
))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+b/c*arct
an((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((
-c*x+1)/(c*x+1)+1)+1)-1/2*I*b/c*dilog((1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/
((-c*x+1)/(c*x+1)+1)+1)+1/2*I*b/c*dilog(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2/((-c*x+1)/(c*x+1)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x + 1
))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) - 2*c*integrate(1/2*(e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*log(-
c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x - 1)
), x))*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="giac")
```

```
[Out] integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

`[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)`

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]`

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]`

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \arctan \left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a c^{2} x^{2} - a + b c^{2} x^{2} \operatorname{atan}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)} - b \operatorname{atan}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.36 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tan^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcTan}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \arctan \left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] 2*(2*(b^2*c^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c^2)*sqrt(c*x +
1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)
*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
+ 1)/((b^2*c*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c)*sqrt(c*x + 1)*
sqrt(-c*x + 1))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctan(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2c^2x^2 - a^2 + 2abc^2x^2 \operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab \operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^2c^2x^2 \operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^2 \operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atan(sqrt(-c*x + 1)/sq
rt(c*x + 1)) - 2*a*b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*at
an(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atan(sqrt(-c*x + 1)/sqrt(c*x + 1
))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a + b \operatorname{atan}\left(\frac{\sqrt{1 - c\,x}}{\sqrt{c\,x + 1}}\right)\right)^2 \left(c^2\,x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.37 $\int x^m \textbf{ArcTan}(\tan(a+bx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\mathrm{ArcTan}(\tan(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctan(tan(b*x+a))/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$\frac{x^{m+1}\mathrm{ArcTan}(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\tan^{-1}(\tan(a+bx))\,dx&=\frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\tan(a+bx))}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.92

$$x^m\left(\frac{bx^2}{2+m}+\frac{x(-bx+\text{ArcTan}(\tan(a+bx)))}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Tan[a + b*x]],x]
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTan[Tan[a + b*x]]))/(1 + m))
```

Maple [A]
time = 0.12, size = 41, normalized size = 1.11

method	result
default	$\frac{b\,x^{2} e^{m \ln(x)}}{2+m}+\frac{(\arctan(\tan(bx+a))-bx)x\, e^{m\ln(x)}}{1+m}$
risch	$-\frac{ix\,x^{m}\ln\left(e^{i(bx+a)}\right)}{1+m}-\frac{x\left(-\pi m\,\text{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\text{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)^{2}+\pi m\text{csgn}\left(ie^{i(bx+a)}\right)^{2}\text{csgn}\left(ie^{2i(bx+a)}\right)-2\pi m\,\text{csgn}\left(ie^{i(bx-\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctan(tan(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\arctan\left(\tan\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctan(tan(b*x + a))/(m + 1)
```

Fricas [A]
time = 1.67, size = 33, normalized size = 0.89

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(31) = 62$.
time = 0.96, size = 158, normalized size = 4.27

$$\begin{cases} b\log(x) - \frac{\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor}{x} & \text{for } m=-2 \\ -bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + 2\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x) & \text{for } m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} + \frac{2xx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atan(tan(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/
x, Eq(m, -2)), (-b*x*log(x) + b*x + (atan(tan(a + b*x)) + 2*pi*floor((a + b
*x - pi/2)/pi))*log(x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x*
*m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m + 2) +
2*x*x**m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m +
 2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m \operatorname{atan}(\tan(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atan(tan(a + b*x)),x)
```

```
[Out] int(x^m*atan(tan(a + b*x)), x)
```

3.38 $\int x^2 \mathbf{ArcTan}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(\tan(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\text{ArcTan}(\tan(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTan[Tan[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\tan(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(\tan(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3(bx - 4\text{ArcTan}(\tan(a + bx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Tan[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcTan[Tan[a + b*x]]))
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^4}{12} + \frac{x^3 \arctan(\tan(bx+a))}{3}$
risch	$-\frac{ix^3 \ln\left(e^{i(bx+a)}\right)}{3} - \frac{\pi\,x^3 \operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right) \operatorname{csgn}\left(ie^{2i(bx+a)}\right) \operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{12} + \frac{\pi\,x^3 \operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right) \operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(19) = 38.
time = 0.26, size = 81, normalized size = 3.52

$$\frac{\frac{4\left((bx+a)^3 - 3\,(bx+a)^2 a + 3\,(bx+a)a^2\right)\arctan(\tan(bx+a))}{b^2} - \frac{(bx+a)^4 - 4\,(bx+a)^3 a + 6\,(bx+a)^2 a^2}{b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan(tan(b*x +
a))/b^2 - ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2)/b^2)/b
```

Fricas [A]
time = 2.67, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.11, size = 32, normalized size = 1.39

$$-\frac{bx^4}{12}+\frac{x^3\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(tan(b*x+a)),x)
[Out] -b*x**4/12 + x**3*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(tan(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.12, size = 19, normalized size = 0.83

$$\frac{x^3\,\mathrm{atan}\left(\tan\left(a+b\,x\right)\right)}{3}-\frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(tan(a + b*x)),x)
[Out] (x^3*atan(tan(a + b*x)))/3 - (b*x^4)/12
```

3.39 $\int x\textbf{ArcTan}(\tan(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(\tan(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = $\{5279, 30\}$

$$\frac{1}{2}x^2\text{ArcTan}(\tan(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[Tan[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(\tan(a+bx))\,dx&=\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(\tan(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2(bx-3\text{ArcTan}(\tan(a+bx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Tan[a + b*x]],x]
```

```
[Out] -1/6*(x^2*(b*x - 3*ArcTan[Tan[a + b*x]]))
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\arctan\left(\tan\left(bx+a\right)\right)}{2}$
risch	$-\frac{i x^{2} \ln\left(\mathrm{e}^{i(bx+a)}\right)}{2}-\frac{\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{8}+\frac{\pi\,x^{2}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(19) = 38$.
time = 0.27, size = 57, normalized size = 2.48

$$\frac{\frac{3\left(\left(bx+a\right)^{2}-2\left(bx+a\right)a\right)\arctan\left(\tan\left(bx+a\right)\right)}{b}-\frac{\left(bx+a\right)^{3}-3\left(bx+a\right)^{2}a}{b}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*((b*x + a)^2 - 2*(b*x + a)*a)*arctan(tan(b*x + a))/b - ((b*x + a)^3
- 3*(b*x + a)^2*a)/b)/b
```

Fricas [A]
time = 2.63, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(19) = 38$.

time = 0.10, size = 76, normalized size = 3.30

$$\begin{cases} \frac{x\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^{2}}{2b} - \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^{3}}{6b^{2}} & \text{for}\ b\neq 0 \\ \frac{x^{2}\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(tan(b*x+a)),x)
```

```
[Out] Piecewise((x*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b)
- (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**3/(6*b**2), Ne(b, 0
)), (x**2*(atan(tan(a)) + pi*floor((a - pi/2)/pi))/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{atan}(\tan(a+b\,x))}{2}-\frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(tan(a + b*x)),x)
```

```
[Out] (x^2*atan(tan(a + b*x)))/2 - (b*x^3)/6
```

3.40 $\int \textbf{ArcTan}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(\tan(a+bx))^2}{2b}$$

[Out] 1/2*arctan(tan(b*x+a))^2/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\text{ArcTan}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[Tan[a + b*x]],x]

[Out] ArcTan[Tan[a + b*x]]^2/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2} + x\text{ArcTan}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right) - \frac{\pi x\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4} + \frac{\pi x\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(b*x + a)^2/b
```

Fricas [A]
time = 1.84, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.
time = 0.06, size = 42, normalized size = 2.62

$$\begin{cases} \frac{\left(\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for}\ b \neq 0 \\ x\left(\operatorname{atan}(\tan(a)) + \pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
```

```
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Giac [A]
time = 0.40, size = 26, normalized size = 1.62

$$\frac{1}{2}\,bx^2 - \pi x \left\lfloor \frac{bx+a}{\pi} + \frac{1}{2} \right\rfloor + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

Mupad [B]
time = 0.04, size = 16, normalized size = 1.00

$$x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x)),x)
```

```
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.41 $\int \frac{\text{ArcTan}(\tan(a+bx))}{x} dx$

Optimal. Leaf size=21

$$bx - (bx - \text{ArcTan}(\tan(a + bx))) \log(x)$$

```
[Out] b*x-(b*x-arctan(tan(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$bx - \log(x)(bx - \text{ArcTan}(\tan(a + bx)))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\tan(a + bx))}{x} dx &= bx - \left(bx - \tan^{-1}(\tan(a + bx))\right) \int \frac{1}{x} dx \\ &= bx - \left(bx - \tan^{-1}(\tan(a + bx))\right) \log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$bx + (-bx + \text{ArcTan}(\tan(a + bx))) \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]]/x,x]
```

[Out] b*x + (-(b*x) + ArcTan[Tan[a + b*x]])*Log[x]

Maple [A]
time = 0.04, size = 23, normalized size = 1.10

method	result
default	$\ln(x)\arctan(\tan(bx+a)) - b(x\ln(x) - x)$
risch	$-i\ln(x)\ln\left(e^{i(bx+a)}\right) - \ln(x)\,xb + bx - \frac{\ln(x)\pi\,\mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{4} + \frac{\ln(x)\pi\,\mathrm{csg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctan(tan(b*x+a))/x,x,method=_RETURNVERBOSE)

[Out] ln(x)*arctan(tan(b*x+a))-b*(x*ln(x)-x)

Maxima [A]
time = 0.46, size = 42, normalized size = 2.00

$$\frac{b\arctan(\tan(bx+a))\log(bx) + (bx - (bx+a)\log(bx) + a\log(bx) + a)b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="maxima")

[Out] (b*arctan(tan(b*x + a))*log(b*x) + (b*x - (b*x + a)*log(b*x) + a*log(b*x) + a)*b)/b

Fricas [A]
time = 0.61, size = 8, normalized size = 0.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="fricas")

[Out] b*x + a*log(x)

Sympy [A]
time = 0.49, size = 34, normalized size = 1.62

$$-bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atan(tan(b*x+a))/x,x)

```
[Out] -b*x*log(x) + b*x + (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))*lo
g(x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atan}(\tan(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x))/x,x)
```

```
[Out] int(atan(tan(a + b*x))/x, x)
```

3.42 $\int x^m \textbf{ArcTan}(\cot(a+bx))\,dx$

Optimal. Leaf size=36

$$\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\text{ArcTan}(\cot(a+bx))}{1+m}$$

```
[Out] b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*(1/2*Pi-arccot(cot(b*x+a)))/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{x^{m+1}\text{ArcTan}(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^(2 + m))/(2 + 3*m + m^2) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m \tan^{-1}(\cot(a+bx))\,dx &= \frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}+\frac{b\int x^{1+m}\,dx}{1+m}\\ &= \frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tan^{-1}(\cot(a+bx))}{1+m}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 31, normalized size = 0.86

$$\frac{x^{1+m}(bx+(2+m)\text{ArcTan}(\cot(a+bx)))}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTan[Cot[a + b*x]],x]
[Out] (x^(1 + m)*(b*x + (2 + m)*ArcTan[Cot[a + b*x]]))/((1 + m)*(2 + m))
```

Maple [**A**]
time = 0.15, size = 56, normalized size = 1.56

method	result
default	$\frac{\pi x^{1+m}}{2m+2}-\frac{b\,x^{2}\mathrm{e}^{m\ln(x)}}{2+m}-\frac{(\mathrm{arccot}(\cot(bx+a))-bx)x\,\mathrm{e}^{m\ln(x)}}{1+m}$
risch	$\frac{ix\,x^{m}\ln\left(\mathrm{e}^{i(bx+a)}\right)}{1+m}+\frac{x\left(4\pi-\pi m\,\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}-1}\right)^{2}-\pi m\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}-1}\right)^{2}+2\pi\,\mathrm{csgn}(i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*x^(1+m)/(1+m)-b/(2+m)*x^2*exp(m*ln(x))-(arccot(cot(b*x+a))-b*x)/(1+m
)*x*exp(m*ln(x))
```

Maxima [**A**]
time = 0.26, size = 40, normalized size = 1.11

$$-\frac{bx^{m+2}}{m+2}+\frac{\pi x^{m+1}}{2\,(m+1)}-\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
[Out] -b*x^(m + 2)/(m + 2) + 1/2*pi*x^(m + 1)/(m + 1) - a*x^(m + 1)/(m + 1)
```

Fricas [**A**]
time = 2.01, size = 42, normalized size = 1.17

$$-\frac{(2\,(bm+b)x^2-(\pi(m+2)-2\,am-4\,a)x)x^m}{2\,(m^2+3\,m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*m + b)*x^2 - (pi*(m + 2) - 2*a*m - 4*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.
time = 3.11, size = 160, normalized size = 4.44

$$\begin{cases} -b\log(x) + \frac{\operatorname{acot}(\cot(a+bx))}{x} - \frac{\pi}{2x} & \text{for } m = -2 \\ bx\log(x) - bx - \log(x)\operatorname{acot}(\cot(a+bx)) + \frac{\pi\log(x)}{2} & \text{for } m = -1 \\ \frac{2bx^2x^m}{2m^2+6m+4} - \frac{2mxx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4} + \frac{\pi mxx^m}{2m^2+6m+4} - \frac{4xx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4} + \frac{2\pi xx^m}{2m^2+6m+4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] Piecewise((-b*log(x) + acot(cot(a + b*x))/x - pi/(2*x), Eq(m, -2)), (b*x*lo
g(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2, Eq(m, -1)), (2*b*x**2
*x**m/(2*m**2 + 6*m + 4) - 2*m*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6*m + 4)
 + pi*m*x*x**m/(2*m**2 + 6*m + 4) - 4*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6
*m + 4) + 2*pi*x*x**m/(2*m**2 + 6*m + 4), True))
```

Giac [A]
time = 0.42, size = 62, normalized size = 1.72

$$-\frac{2\,bmx^2x^m - \pi mxx^m + 2\,amxx^m + 2\,bx^2x^m - 2\,\pi xx^m + 4\,axx^m}{2\,(m^2 + 3\,m + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*m*x^2*x^m - pi*m*x*x^m + 2*a*m*x*x^m + 2*b*x^2*x^m - 2*pi*x*x^m +
 4*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^m\left(\frac{\Pi}{2} - \operatorname{acot}(\cot(a + b\,x))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(Pi/2 - acot(cot(a + b*x))),x)
```

```
[Out] int(x^m*(Pi/2 - acot(cot(a + b*x))), x)
```

3.43 $\int x^2 \mathbf{ArcTan}(\cot(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(\cot(a+bx))$$

```
[Out] 1/12*b*x^4+1/3*x^3*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\text{ArcTan}(\cot(a+bx))+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(\cot(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(\cot(a+bx))+\frac{1}{3}b\int x^3\,dx\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tan^{-1}(\cot(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{12}x^3(bx + 4\text{ArcTan}(\cot(a + bx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Cot[a + b*x]],x]
[Out] (x^3*(b*x + 4*ArcTan[Cot[a + b*x]]))/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.
time = 0.08, size = 65, normalized size = 2.83

method	result
default	$\frac{\pi x^3}{6} - \frac{x^3 \operatorname{arccot}(\cot(bx+a))}{3} - \frac{-\frac{(bx+a)^4}{4} + a(bx+a)^3 - \frac{3(bx+a)^2 a^2}{2} + a^3(bx+a)}{3b^3}$
risch	$\frac{ix^3 \ln(\mathrm{e}^{i(bx+a)})}{3} + \frac{\pi x^3 \operatorname{csgn}(i\mathrm{e}^{i(bx+a)})^2 \operatorname{csgn}(i\mathrm{e}^{2i(bx+a)})}{12} - \frac{\pi x^3 \operatorname{csgn}(i\mathrm{e}^{i(bx+a)}) \operatorname{csgn}(i\mathrm{e}^{2i(bx+a)})^2}{6} + \frac{\pi x^3 \operatorname{csgn}(i\mathrm{e}^{2i(bx+a)})^3}{12} + \frac{\pi}{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
[Out] 1/6*Pi*x^3-1/3*x^3*arccot(cot(b*x+a))-1/3/b^3*(-1/4*(b*x+a)^4+a*(b*x+a)^3-3
/2*(b*x+a)^2*a^2+a^3*(b*x+a))
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.74

$$-\frac{1}{4}bx^4 + \frac{1}{6}(\pi - 2a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Fricas [A]
time = 1.40, size = 17, normalized size = 0.74

$$-\frac{1}{4}bx^4 + \frac{1}{6}(\pi - 2a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

Sympy [A]
time = 0.12, size = 26, normalized size = 1.13

$$\frac{bx^4}{12} - \frac{x^3 \operatorname{acot}\left(\cot\left(a + bx\right)\right)}{3} + \frac{\pi x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1/2*pi-acot(cot(b*x+a))),x)
[Out] b*x**4/12 - x**3*acot(cot(a + b*x))/3 + pi*x**3/6
```

Giac [A]
time = 0.44, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,bx^4 + \frac{1}{6}\,\pi x^3 - \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
[Out] -1/4*b*x^4 + 1/6*pi*x^3 - 1/3*a*x^3
```

Mupad [B]
time = 0.54, size = 25, normalized size = 1.09

$$\frac{\Pi\,x^3}{6} + \frac{b\,x^4}{12} - \frac{x^3\,\operatorname{acot}(\cot(a + b\,x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(Pi/2 - acot(cot(a + b*x))),x)
[Out] (Pi*x^3)/6 + (b*x^4)/12 - (x^3*acot(cot(a + b*x)))/3
```

3.44 $\int x\mathbf{ArcTan}(\cot(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(\cot(a+bx))$$

```
[Out] 1/6*b*x^3+1/2*x^2*(1/2*Pi-arccot(cot(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5281, 30}

$$\frac{1}{2}x^2\text{ArcTan}(\cot(a+bx))+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(\cot(a+bx))\,dx&=\frac{1}{2}x^2\tan^{-1}(\cot(a+bx))+\frac{1}{2}b\int x^2\,dx\\&=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(\cot(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.87

$$\frac{1}{6}x^2(bx+3\text{ArcTan}(\cot(a+bx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Cot[a + b*x]],x]
```

```
[Out] (x^2*(b*x + 3*ArcTan[Cot[a + b*x]]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.
time = 0.08, size = 54, normalized size = 2.35

method	result
default	$\frac{\pi x^2}{4} - \frac{x^2 \operatorname{arccot}(\cot(bx+a))}{2} - \frac{-\frac{(bx+a)^3}{3}+(bx+a)^2 a-a^2(bx+a)}{2b^2}$
risch	$\frac{ix^2 \ln(e^{i(bx+a)})}{2} + \frac{\pi x^2 \operatorname{csgn}(ie^{i(bx+a)})^2 \operatorname{csgn}(ie^{2i(bx+a)})}{8} - \frac{\pi x^2 \operatorname{csgn}(ie^{i(bx+a)}) \operatorname{csgn}(ie^{2i(bx+a)})^2}{4} + \frac{\pi x^2 \operatorname{csgn}(ie^{2i(bx+a)})^3}{8} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*x^2-1/2*x^2*arccot(cot(b*x+a))-1/2/b^2*(-1/3*(b*x+a)^3+(b*x+a)^2*a-a
^2*(b*x+a))
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.74

$$-\frac{1}{3}\,bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Fricas [A]
time = 0.71, size = 17, normalized size = 0.74

$$-\frac{1}{3}\,bx^3 + \frac{1}{4}\,(\pi - 2\,a)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

```
[Out] -1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

Sympy [A]
time = 0.08, size = 26, normalized size = 1.13

$$\frac{bx^3}{6} - \frac{x^2 \operatorname{acot}\left(\cot\left(a + bx\right)\right)}{2} + \frac{\pi x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-acot(cot(b*x+a))),x)
```

```
[Out] b*x**3/6 - x**2*acot(cot(a + b*x))/2 + pi*x**2/4
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.83

$$-\frac{1}{3}\,bx^3+\frac{1}{4}\,\pi x^2-\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

```
[Out] -1/3*b*x^3 + 1/4*pi*x^2 - 1/2*a*x^2
```

Mupad [B]
time = 0.07, size = 25, normalized size = 1.09

$$\frac{\Pi\,x^2}{4}+\frac{b\,x^3}{6}-\frac{x^2\,\mathrm{acot}(\cot(a+b\,x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(Pi/2 - acot(cot(a + b*x))),x)
```

```
[Out] (Pi*x^2)/4 + (b*x^3)/6 - (x^2*acot(cot(a + b*x)))/2
```

3.45 $\int \mathbf{ArcTan}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\text{ArcTan}(\cot(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$-\frac{\text{ArcTan}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]],x]
```

```
[Out] -1/2*ArcTan[Cot[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$\frac{bx^2}{2} + x\text{ArcTan}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

time = 0.04, size = 51, normalized size = 3.19

method	result
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)-\operatorname{arccot}(\cot(bx+a))^2}{2b}$
default	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$\frac{\pi x}{2}+ix\ln\left(e^{i(bx+a)}\right)+\frac{\pi x \operatorname{csgn}\left(ie^{i(bx+a)}\right)^2\operatorname{csgn}\left(ie^{2i(bx+a)}\right)}{4}-\frac{\pi x\operatorname{csgn}\left(ie^{i(bx+a)}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)^2}{2}+\frac{\pi x \operatorname{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

Maxima [A]

time = 0.26, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Fricas [A]

time = 1.93, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A]

time = 0.06, size = 24, normalized size = 1.50

$$\frac{\pi x}{2}-\begin{cases}\frac{\operatorname{acot}^2\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Giac [A]
time = 0.44, size = 15, normalized size = 0.94

$$-\frac{1}{2}bx^2 + \frac{1}{2}\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Mupad [B]
time = 0.07, size = 21, normalized size = 1.31

$$\frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a + b\,x)) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
```

```
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.46 $\int \frac{\textbf{ArcTan}(\cot(a+bx))}{x} \, dx$

Optimal. Leaf size=19

$$-bx + (bx + \text{ArcTan}(\cot(a + bx))) \log(x)$$

```
[Out] -b*x+(b*x+1/2*Pi-arccot(cot(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$\log(x)(\text{ArcTan}(\cot(a + bx)) + bx) - bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]]/x,x]
[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(\cot(a+bx))}{x} \, dx &= -bx - \left(-bx - \tan^{-1}(\cot(a+bx))\right) \int \frac{1}{x} \, dx \\ &= -bx + \left(bx + \tan^{-1}(\cot(a+bx))\right) \log(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-bx + (bx + \text{ArcTan}(\cot(a + bx))) \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]]/x,x]
```

[Out] -(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]

Maple [A]
time = 0.07, size = 35, normalized size = 1.84

method	result
default	$\frac{\pi \ln(x)}{2} - bx - \ln(x)\, a - \ln(x)\left(\operatorname{arccot}(\cot(bx+a)) - bx - a\right)$
risch	$i \ln(x) \ln\left(\mathrm{e}^{i(bx+a)}\right) + \ln(x)\, xb - bx + \frac{\ln(x)\pi \mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^{2} \mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{4} - \frac{\ln(x)\pi\, \mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/2*Pi-arccot(cot(b*x+a)))/x,x,method=_RETURNVERBOSE)

[Out] 1/2*Pi*ln(x)-b*x-ln(x)*a-ln(x)*(arccot(cot(b*x+a))-b*x-a)

Maxima [A]
time = 0.26, size = 14, normalized size = 0.74

$$-bx + \frac{1}{2}(\pi - 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="maxima")

[Out] -b*x + 1/2*(pi - 2*a)*log(x)

Fricas [A]
time = 0.58, size = 14, normalized size = 0.74

$$-bx + \frac{1}{2}(\pi - 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="fricas")

[Out] -b*x + 1/2*(pi - 2*a)*log(x)

Sympy [A]
time = 2.74, size = 27, normalized size = 1.42

$$bx\log(x) - bx - \log(x)\operatorname{acot}(\cot(a+bx)) + \frac{\pi\log(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/2*pi-acot(cot(b*x+a)))/x,x)

[Out] b*x*log(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2

Giac [A]
time = 0.41, size = 15, normalized size = 0.79

$$-bx + \frac{1}{2}(\pi - 2a)\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(pi - 2*a)*log(abs(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\frac{\Pi}{2} - \operatorname{acot}(\cot(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi/2 - acot(cot(a + b*x)))/x,x)
```

```
[Out] int((Pi/2 - acot(cot(a + b*x)))/x, x)
```

3.47 $\int \textbf{ArcTan}(\tan(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\text{ArcTan}(\tan(a+bx))^2}{2b}$$

```
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\text{ArcTan}(\tan(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tan[a + b*x]],x]
```

```
[Out] ArcTan[Tan[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\tan(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\tan(a+bx))\right)}{b}\\ &= \frac{\tan^{-1}(\tan(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2} + x\text{ArcTan}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tan[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

Maple [A]
time = 0.00, size = 15, normalized size = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
risch	$-ix\ln\left(e^{i(bx+a)}\right)-\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right)}{4}+\frac{\pi x\,\mathrm{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(tan(b*x+a))^2/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
[Out] 1/2*(b*x + a)^2/b
```

Fricas [A]
time = 1.57, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12)=24$.
time = 0.07, size = 42, normalized size = 2.62

$$\begin{cases}\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for}\ b\neq 0\\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tan(b*x+a)),x)
```

```
[Out] Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b), N
e(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

Giac [A]
time = 0.40, size = 26, normalized size = 1.62

$$\frac{1}{2}bx^2 - \pi x \left\lfloor \frac{bx+a}{\pi} + \frac{1}{2} \right\rfloor + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

Mupad [B]
time = 0.00, size = 16, normalized size = 1.00

$$x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tan(a + b*x)),x)
```

```
[Out] x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.48 $\int x^2 \mathbf{ArcTan}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=403

$$\frac{1}{3}x^3\text{ArcTan}(c+d\tan(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)+$$

```
[Out] 1/3*x^3*arctan(c+d*tan(b*x+a))+1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x^2*polylog(2,-
(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/4*I*x*polylog(3,-(1+I*c+d)*
exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2-1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2*I*a+
2*I*b*x)/(c+I*(1+d)))/b^2-1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*
c-d))/b^3+1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b^3
```

Rubi [A]
time = 0.39, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {5283, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(d\tan(a+bx)+c)-\frac{\text{Li}_4\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^3}+\frac{\text{Li}_4\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x^2\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{6}ix^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/6)*x^3*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x^2*PolyLog[2, -(((1 + I*c + d)*E
^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/4)*x*PolyL
og[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/
4)*x*PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))
)])/b^2 - PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d
))]/(8*b^3) + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 + d)))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5283

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c + d\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{6}$$

Mathematica [A]
time = 0.72, size = 363, normalized size = 0.90

$$\frac{1}{3}x^3\text{ArcTan}(c+d\tan(a+bx)) + \frac{4ib^3x^3\log\left(1+\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right) - 4ib^3x^3\log\left(1+\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right) + 6b^2x^2\text{PolyLog}\left(2,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right) + 6ibx\text{PolyLog}\left(3,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right) - 6ibx\text{PolyLog}\left(3,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right) - 3\text{PolyLog}\left(4,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right) + 3\text{PolyLog}\left(4,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tan[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 + ((c - I*(1 + d)
)*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - (4*I)*b^3*x^3*Log[1 + ((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] + 6*b^2*x^2*PolyLog[2, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - 6*b^2*x^2*PolyLog[2, -(
((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + (6*I)*b*x*PolyLog[3
, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - (6*I)*b*x*Po
lyLog[3, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] - 3*PolyLo
g[4, -(((c - I*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + 3*PolyLog
[4, -(((I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.22, size = 8112, normalized size = 20.13

method	result	size
risch	Expression too large to display	8112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/6*x^3*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/6*x^3*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2*b
*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*x
+ 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*
a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*co
s(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c
^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(
4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*
a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^
4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4
+ d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2
*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c
^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*
d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2
*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(290) = 580.
time = 0.82, size = 1965, normalized size = 4.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*tan(b*x + a) + c) + 6*b^2*x^2*dilog(2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
+ a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)
+ 1) - 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
(I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*ta
n(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(2*((-I*c*d - d^2
 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x +
```

```
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) - 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 4*I*a^3*log(((I*c*d + d^2 + d)*
tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d
 - 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d + d^2 - d)*tan(b*x + a)^2
 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x
 + a)^2 + 1)) + 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d
 + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1))
- 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) + 6*I*b*x*poly
log(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*
(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^
2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)
*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
+ 1)) - 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2
- 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + 6*I*b*x*polylog(3,
((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2
+ 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 - 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((I*c*d - d^2 + d)
*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) +
d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) - 4*(-
I*b^3*x^3 - I*a^3)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 4*(-I*b^3*x^3 - I*a^3)*log(-2*((-I*
c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*
tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2
*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 +
d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2
+ 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c
*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(b*x + a)
^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x + a) - 1)/
((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 3*polylog(4
, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2 - 2*(-I*c
^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x +
a)^2 + c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*tan(
b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*tan(b*x +
a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy **[F(-1)]** Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atan}(c + d \tan(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*tan(a + b*x)), x)
```

3.49 $\int x\mathbf{ArcTan}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=305

$$\frac{1}{2}x^2\text{ArcTan}(c{+}d\tan(a{+}bx)){+}\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right){-}\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right){+}$$

```
[Out] 1/2*x^2*arctan(c+d*tan(b*x+a))+1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/
(1+I*c-d))-1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*x
*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x*polylog(2,-(c+I
*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/8*I*polylog(3,-(1+I*c+d)*exp(2*
I*a+2*I*b*x)/(1+I*c-d))/b^2-1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)
/(c+I*(1+d)))/b^2
```

Rubi [A]
time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5283, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(d\tan(a+bx)+c)+\frac{i\text{Li}_3\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{8b^2}-\frac{i\text{Li}_3\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{8b^2}+\frac{x\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{x\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((1 + I*c + d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c - d)] - (I/4)*x^2*Log[1 + ((c + I*(1 - d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 + d))] + (x*PolyLog[2, -(((1 + I*c + d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(4*b) - (x*PolyLog[2, -(((c + I*(1 -
 d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(4*b) + ((I/8)*PolyLog[3,
-(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/b^2 - ((I/8)*Pol
yLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5283

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 - I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 + I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{2}(b(1-ic-d))\int\frac{e^{2ia+2ibx}}{1-ic+d+(1-ic}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-$$

Mathematica [A]
time = 0.48, size = 272, normalized size = 0.89

$$\frac{1}{2}x^2\mathrm{ArcTan}(c+d\tan(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)-2b^2x^2\log\left(1+\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)-2ibx\mathrm{PolyLog}\left(2,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)+2ibx\mathrm{PolyLog}\left(2,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)+\mathrm{PolyLog}\left(3,-\frac{(c-i(1+d))e^{2i(a+bx)}}{c+i(-1+d)}\right)-\mathrm{PolyLog}\left(3,-\frac{(i+c-id)e^{2i(a+bx)}}{c+i(1+d)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((c - I*(1 +
 d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d))] - 2*b^2*x^2*Log[1 + ((I + c - I
*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d))] - (2*I)*b*x*PolyLog[2, -(((c - I*
(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] + (2*I)*b*x*PolyLog[2, -((
(I + c - I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))] + PolyLog[3, -(((c - I
*(1 + d))*E^((2*I)*(a + b*x)))/(c + I*(-1 + d)))] - PolyLog[3, -(((I + c -
I*d)*E^((2*I)*(a + b*x)))/(c + I*(1 + d)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.54, size = 7720, normalized size = 25.31

method	result	size
risch	Expression too large to display	7720

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)*
cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/4*x^2*arctan2(c*cos(2*b*
x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*sin(
2*b*x + 2*a) + d + 1) + 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b*x +
 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x + 2*
a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) -
 (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*
b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 +
 d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*b*
x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)^2
```

```
 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 +
d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 + d^
4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a)
- 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 -
 d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c)*d - 2*(c*d^3
+ (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a))
*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(217) = 434$.
time = 2.01, size = 1545, normalized size = 5.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*tan(b*x + a) + c) + 2*b*x*dilog(2*((I*c*d - d^2 +
d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a)
+ d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1)
- 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 -
2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a
)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(2*((-I*c*d - d^2 + d)*tan(b*x
 + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/
((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*d
ilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d +
 I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*d + 1) + 1) - 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*x + a)^2 -
c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)/(tan(b*x +
a)^2 + 1)) + 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d +
(I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a)^2 + 1)) - 2
*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2
 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((I
*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*
tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*
((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 +
I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
- 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)
^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2
+ d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(-I*b^2*x^2 + I
*a^2)*log(-2*((-I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2
*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)
^2 + c^2 + d^2 - 2*d + 1)) - 2*(I*b^2*x^2 - I*a^2)*log(-2*((-I*c*d - d^2 -
d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a)
 - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + I
```

```
*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c*d + d^2
 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*
tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - I*polylog(3, ((c^2 - 2*I*c*d - d^2
 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d^2 + I)*
tan(b*x + a) - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d +
 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 - 2*I*c
*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2
*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*c
*d - d^2 + 1)*tan(b*x + a)^2 - c^2 + 2*I*c*d + d^2 - 2*(I*c^2 + 2*c*d - I*d
^2 + I)*tan(b*x + a) - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2
 - 2*d + 1)))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tan(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(c+d\tan(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*tan(a + b*x)), x)
```

3.50 $\int \textbf{ArcTan}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=198

$$x\text{ArcTan}(c+d\tan(a+bx))+\frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)+\frac{\text{Po}}{}$$

```
[Out] x*arctan(c+d*tan(b*x+a))+1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d
))-1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*polylog(2,-
(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*polylog(2,-(c+I*(1-d))*exp(2*
I*a+2*I*b*x)/(c+I*(1+d)))/b
```

Rubi [A]
time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5275, 2221, 2317, 2438}

$$x\text{ArcTan}(d\tan(a+bx)+c)+\frac{\text{Li}_2\left(-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b}-\frac{\text{Li}_2\left(-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (I/2)*x*Log[1 + ((1 + I*c + d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c - d)] - (I/2)*x*Log[1 + ((c + I*(1 - d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 + d))] + PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*
I)*b*x))/(1 + I*c - d))]/(4*b) - PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a +
 (2*I)*b*x))/(c + I*(1 + d)))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5275

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[x*(E^(2*I*a + 2*I*b
*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Dist[b*(1
+ I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*
a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\tan(a+bx))\,dx = x\tan^{-1}(c+d\tan(a+bx)) + (b(1-ic-d))\int \frac{e^{2ia+2ibx}x}{1-ic+d+(1-ic-d)e^{2i}}$$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\tan(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) - \frac{1}{2}ix\log$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 555 vs. 2(198) = 396.
time = 5.02, size = 555, normalized size = 2.80

$$x\text{ArcTan}(c+d\tan(a+bx)) + \frac{x\left(-4ad\text{ArcTan}(c+d\tan(a+bx)) - i\sqrt{-d^2}\left(\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right)+\text{PolyLog}\left(2,\frac{d^2(1-i\tan(a+bx))}{icd+d^2-i\sqrt{-d^2}}\right)\right)+i\sqrt{-d^2}\left(\log(1-i\tan(a+bx))\log\left(\frac{cd+\sqrt{-d^2}+d^2\tan(a+bx)}{cd-id^2+\sqrt{-d^2}}\right)+\text{PolyLog}\left(2,\frac{d^2(1-i\tan(a+bx))}{icd+d^2+i\sqrt{-d^2}}\right)\right)+i\sqrt{-d^2}\left(\log(1+i\tan(a+bx))\log\left(\frac{cd-\sqrt{-d^2}+d^2\tan(a+bx)}{cd+id^2-\sqrt{-d^2}}\right)+\text{PolyLog}\left(2,\frac{d^2(1+i\tan(a+bx))}{-icd+d^2+i\sqrt{-d^2}}\right)\right)-i\sqrt{-d^2}\left(\log(1+i\tan(a+bx))\log\left(\frac{cd+\sqrt{-d^2}+d^2\tan(a+bx)}{cd+id^2+\sqrt{-d^2}}\right)+\text{PolyLog}\left(2,\frac{d^2(1+i\tan(a+bx))}{d^2-i\left(cd+\sqrt{-d^2}\right)}\right)\right)\right)}{2d(2a-i\log(1-i\tan(a+bx))+i\log(1+i\tan(a+bx)))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tan[a + b*x]] + (x*(-4*a*d*ArcTan[c + d*Tan[a + b*x]] - I*Sq
rt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(1 - I*Tan[a + b*x]))/
(I*c*d + d^2 - I*Sqrt[-d^2])]) + I*Sqrt[-d^2]*(Log[1 - I*Tan[a + b*x]]*Log[
(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 + Sqrt[-d^2])] + PolyLog
[2, (d^2*(1 - I*Tan[a + b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])]) + I*Sqrt[-d^2
]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d +
 I*d^2 - Sqrt[-d^2])] + PolyLog[2, (d^2*(1 + I*Tan[a + b*x]))/((-I)*c*d + d
^2 + I*Sqrt[-d^2])]) - I*Sqrt[-d^2]*(Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqr
t[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] + PolyLog[2, (d^2*(
1 + I*Tan[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))])))/(2*d*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(168) = 336.
time = 0.78, size = 1001, normalized size = 5.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arctan(c+d*tan(b*x+a))-d^2*(1/2*I/d*arctan(-(c+
d*tan(b*x+a))/d+c/d)*ln(1-(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d
*tan(b*x+a))/d-c/d)^2+1)/(-I*d+I-c))-1/2/d*arctan(-(c+d*tan(b*x+a))/d+c/d)^
2-1/4/d*polylog(2,(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x
+a))/d-c/d)^2+1)/(-I*d+I-c))+1/2/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*
x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c+d*tan
(b*x+a))/d+c/d)+1/2/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d
))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c+d*tan(b*x+a))/d+
c/d)-1/2*I/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c
+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c*arctan(-(c+d*tan(b*x+a))/d+c/d)+1/
2*I/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/(I+c+I*d)*polylog(2,(
c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-
I*d-I-c))+1/2*I/d/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/2/d/(I+c+I*
d)*c*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/d/(I+c+I*d)*polylog(2,(c-I*d+I
)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c
))+1/4/d/(I+c+I*d)*polylog(2,(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((
c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(141) = 282.
time = 0.53, size = 433, normalized size = 2.19

$$\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d}-\frac{4\,(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2\,d+1},\frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2\,d+1}\right)-4\,(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+a)}{c^2+d^2-2\,d+1},\frac{cd\tan(bx+a)+c^2-d+1}{c^2+d^2-2\,d+1}\right)+\log\left(\tan(bx+a)^2+1\right)\log\left(\frac{d^2\tan(bx+a)^2+2\,cd\tan(bx+a)+c^2+1}{c^2+d^2+2\,d+1}\right)-\log\left(\tan(bx+a)^2+1\right)\log\left(\frac{d^2\tan(bx+a)^2+2\,cd\tan(bx+a)+c^2+1}{c^2+d^2-2\,d+1}\right)+2\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d+1}\right)-2\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d-1}\right)+2\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d+1}\right)-2\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d-1}\right)}{d}\right)+8\,(bx+a)\arctan\left(d\tan\left(bx+a\right)+c\right)-8\,(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*arc
tan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x + a
) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^2 -
 d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d + 1)/(
c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2
*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a)^2 +
1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 - 2*d
 + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-(I*d*tan
(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) + 8*(b*x + a)*arct
an(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(141) = 282.
time = 0.65, size = 1101, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*tan(b*x + a) + c) - 2*(I*b*x + I*a)*log(-2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1))
- 2*(-I*b*x - I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d +
 (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*t
an(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log(-2*((-I*c*d -
d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*
x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1
)) - 2*(I*b*x + I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*
d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 2*I*a*log(((I*c*d + d^2 + d)*ta
n(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d -
 1)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x + a
)^2 + 1)) + 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*
c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) - 2*I*
a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*d^2 + 2*
I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) + dilog(2*((I*c*d - d^
2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x +
 a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
 1) - dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*
c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^
2 + c^2 + d^2 + 2*d + 1) + 1) + dilog(2*((-I*c*d - d^2 + d)*tan(b*x + a)^2
- c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) + d - 1)/((c^2 +
d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) - dilog(2*((-I*c*
d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
n(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d
 + 1) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(c + d\tan\left(a + bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a)),x)
[Out] Integral(atan(c + d*tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(c + d\tan(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tan(a + b*x)),x)
[Out] int(atan(c + d*tan(a + b*x)), x)
```

3.51 $\int \frac{\text{ArcTan}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\text{ArcTan}(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.57, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(d*tan(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(c + d\tan\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atan(c + d*tan(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}(c + d\tan(a + b\,x))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tan(a + b*x))/x,x)
[Out] int(atan(c + d*tan(a + b*x))/x, x)
```

3.52 $\int x^2 \mathbf{ArcTan}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=154

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{ix\text{PolyLog}}{}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctan(c+(1+I*c)*tan(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(3,
I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5279, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c+(1+ic)\tan(a+bx))+\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/6)*x^3*Log[
1 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 + PolyL
og[4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(1+ic)+c+ce}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) + \frac{1}{3}(bc)\int \frac{e}{i(1+ic)}$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

Mathematica [A]
time = 0.34, size = 140, normalized size = 0.91

$$\frac{1}{3}x^3\mathrm{ArcTan}(c+(1+ic)\tan(a+bx)) - \frac{4ib^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right) + 3\mathrm{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E^
((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(c*E
^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 1532, normalized size = 9.95

method	result	size
risch	Expression too large to display	1532

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3-1/4*x^2*
polylog(2,I*c*exp(2*I*(b*x+a)))/b-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(
```

```
exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(
b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi
*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a
))+1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a)
)*c+I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csg
n(I*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(e
xp(2*I*(b*x+a))+1))^2-1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1/6*I*x^3*ln(c-I
)-1/12*b*x^4+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1)
)*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))-1/4*I*x*polylog(3,I*c*e
xp(2*I*(b*x+a)))/b^2+1/6*Pi*x^3-1/12*Pi*x^3*csgn(I*exp(2*I*(b*x+a)))^3+1/6*
Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*Pi*x^3*csgn(I
*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/3*I*x^3*ln(exp(I*(b*x+a)))+1/
12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*
(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b
*x+a))+1))^3+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)+1/12*x^3*Pi*csgn(I/(exp(2*I
*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/
(exp(2*I*(b*x+a))+1))-1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/
2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/12*x^3*Pi*csgn(I*exp(2*I
*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*
(b*x+a))+1))+1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a))*c)*a^3+1/6*I/b^3*a^3*ln(exp(
2*I*(b*x+a))*c+I)-1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b
^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a
))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(ex
p(2*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))+1))^3+1/8*polylog(4,I*c*exp(2*I*(b*x+a)))/b^3-1/12*x^3*Pi*csgn(I*exp(
2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c
-I)/(exp(2*I*(b*x+a))+1))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(
c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*
(c-I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*x*a^2-1/
2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*exp(
I*(b*x+a))*(-I*c)^(1/2))*x+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(
2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*
I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/4/b^
3*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.
time = 0.28, size = 310, normalized size = 2.01

$$\frac{\frac{4\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\arctan((ic+1)\tan(bx+a)+c)}{b^2}-\frac{\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(c\cos(2bx+2a),c\sin(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2\left(ice^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left(c^2\cos(2bx+2a)^2+c^2\sin(2bx+2a)^2+2c\sin(2bx+2a)+1\right)+3(4bx+a)\mathrm{Li}_3(ice^{(2ibx+2ia)})+6i\,\mathrm{Li}_4(ice^{(2ibx+2ia)})\right)(ic+1)}{b^2(c-i)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c + 1)*
tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
```

```
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*
I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*b
*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(2
*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b^2*
(c - I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(107) = 214$.
time = 0.89, size = 322, normalized size = 2.09

$$-\frac{b^4x^4 - 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\sqrt{4i\,c}}{2\,c}\right) - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\sqrt{4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3, \frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 12i\,bx\mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + 2\,(i\,b^3x^3+i\,a^3)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) + 2\,(i\,b^3x^3+i\,a^3)\log\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 12\,\mathrm{polylog}\left(4, \frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) - 12\,\mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(b^4*x^4 - 2*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x -
 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b^2
*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3*x^3 + I*a
^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I*a^3)*log(-1
/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)*e^(I*b*
x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}(c + \tan(a + b\,x)\ (1 + c\,1\mathrm{i}))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.53 $\int x\textbf{ArcTan}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=123

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{PolyLog}(3}{8}$$

```
[Out] -1/6*b*x^3+1/2*x^2*arctan(c+(1+I*c)*tan(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5279, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c+(1+ic)\tan(a+bx))-\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/4)*x^2*Log[1
 - I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) - ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(c+(1+ic)\tan(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{i(1+ic)+c+ce^{2ic}} \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia}}{i(1+ic)+} \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia-}\right. \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia-}\right. \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia-}\right. \\
&= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1+ic)\tan(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2ia-}\right.
\end{aligned}$$

Mathematica [A]

time = 0.25, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\mathrm{ArcTan}(c+(1+ic)\tan(a+bx)) - \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 1497, normalized size = 12.17

method	result	size
risch	Expression too large to display	1497

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*exp(
I*(b*x+a))*(-I*c)^(1/2))+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*
(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))-1/4*I/b^2*ln
(1-I*exp(2*I*(b*x+a))*c)*a^2-1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*c+I)-1/8*x^2
*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a
))+1))-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1
))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))-1/6*b*x^3-1/8*x^2*Pi
*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c
-I)/(exp(2*I*(b*x+a))+1))^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2
*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2+1/8*x^2
*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x
^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(
b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*
x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b-
1/8*I*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2+1/4*Pi*x^2-1/8*Pi*x^2*csgn(I*exp(
I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*cs
gn(I*exp(2*I*(b*x+a)))^2-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/8*Pi*x^2*csgn(I*exp
(2*I*(b*x+a)))^3+1/8*x^2*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1)
)^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))+1))^2+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp
(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-1/4/b^2*polylog(2,I*c*exp(2*I*(
b*x+a)))*a+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2/b^2*a*dilog
(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/4*I*ln(exp(2*I*(b*x+a))*c+I)*x^2-1/8*x^
2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+
```

```
a))*(c-I)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(e
xp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+1/8
*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*
(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+1/2*I/b*a*ln(1-I*exp(I*(b*x+a)
)*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/4*I*x^2
*ln(c-I)+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+1/8
*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*x^2*ln(
1-I*exp(2*I*(b*x+a))*c)+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))+1))^3-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-
1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b*ln(1
-I*exp(2*I*(b*x+a))*c)*x*a
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(85) = 170$.
time = 0.31, size = 218, normalized size = 1.77

$$\frac{\frac{6\left(\left(bx+a\right)^{2}-2\left(bx+a\right)a\right)\arctan\left(\left(ic+1\right)\tan\left(bx+a\right)+c\right)}{b}-\frac{\left(-4i\left(bx+a\right)^{3}+12i\left(bx+a\right)^{2}a-6ibx\mathrm{Li}_{2}\left(ice^{\left(2ibx+2ia\right)}\right)-6\left(i\left(bx+a\right)^{2}-2i\left(bx+a\right)a\right)\arctan\left(c\cos\left(2bx+2a\right),c\sin\left(2bx+2a\right)+1\right)+3\left(\left(bx+a\right)^{2}-2\left(bx+a\right)a\right)\log\left(c^{2}\cos\left(2bx+2a\right)^{2}+c^{2}\sin\left(2bx+2a\right)^{2}+2c\sin\left(2bx+2a\right)+1\right)+3\mathrm{Li}_{3}\left(ice^{\left(2ibx+2ia\right)}\right)\right)\left(ic+1\right)}{b\left(c-i\right)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c + 1)*tan(b*x + a) + c)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x + 2
*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c*
sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x +
 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3,
 I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(85) = 170$.
time = 1.17, size = 271, normalized size = 2.20

$$-\frac{2b^{3}x^{3}-3ib^{2}x^{2}\log\left(-\frac{\left(ce^{\left(2ibx+2ia\right)}+i\right)e^{\left(-2ibx-2ia\right)}}{c-i}\right)+2a^{3}+6bx\mathrm{Li}_{2}\left(\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}\right)+6bx\mathrm{Li}_{2}\left(-\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}\right)+3ia^{2}\log\left(\frac{2ce^{\left(ibx+ia\right)}+i\sqrt{4ic}}{2c}\right)+3ia^{2}\log\left(\frac{2ce^{\left(ibx+ia\right)}-i\sqrt{4ic}}{2c}\right)+3\left(ib^{2}x^{2}-ia^{2}\right)\log\left(\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}+1\right)+3\left(ib^{2}x^{2}-ia^{2}\right)\log\left(-\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{4ic}e^{\left(ibx+ia\right)}\right)}{12b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) +
 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*s
qrt(4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)
+ 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 6*
I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*sqrt(4*
I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(1+I*c)*tan(b*x+a)),x)
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i + 1)),x)
[Out] int(x*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.54 $\int \textbf{ArcTan}(c+(1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=85

$$-\frac{bx^2}{2}+x\text{ArcTan}(c+(1+ic)\tan(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2+x*arctan(c+(1+I*c)*tan(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5271, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c+(1+ic)\tan(a+bx))-\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 - I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5271

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c+(1+ic)\tan(a+bx)) - (ib)\int \frac{x}{i(1+ic)+c+ce^{2ia+2ibx}}$$

$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) + (bc)\int \frac{e^{2ia+2ibx}}{i(1+ic)+c+}$$

$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right.$$

$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right.$$

$$= -\frac{bx^2}{2} + x\tan^{-1}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right.$$

Mathematica [**A**]
time = 6.30, size = 75, normalized size = 0.88

$$x\text{ArcTan}(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ix\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) + \frac{\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - (I/2)*x*Log[1 + I/(c*E^((2*I)*(a + b
*x)))] + PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(69) = 138.
time = 0.26, size = 600, normalized size = 7.06

method	result

derivativedivides	$-\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c^2}{2i-2c}+\frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c}{2i-2c}+\frac{\arctan(c}{}$
default	$-\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c^2}{2i-2c}+\frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-i+c+(ic+1)\tan(bx+a))c}{2i-2c}+\frac{\arctan(c}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(1+I*c)*(-arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*
x+a))*c^2+2*I*arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*
x+a))*c+arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(1+I*c)*tan(b*x+a))+
arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c^2-2*I*
arctan(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*c-arctan
(c+(1+I*c)*tan(b*x+a))/(2*I-2*c)*ln(-c+(1+I*c)*tan(b*x+a)+I)-(1+I*c)^2*(-1/
8*I/(I-c)*ln(-I+c+(1+I*c)*tan(b*x+a))^2+1/4*I/(I-c)*ln(-I+c+(1+I*c)*tan(b*x
+a))*ln(-1/2*I*(c+(1+I*c)*tan(b*x+a)+I))+1/4*I/(I-c)*dilog(-1/2*I*(c+(1+I*c
)*tan(b*x+a)+I))-1/4*I/(I-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln(1/2*(c+(1+I*c)*
tan(b*x+a)+I)/c)-1/4*I/(I-c)*dilog(1/2*(c+(1+I*c)*tan(b*x+a)+I)/c)+1/4*I/(I
-c)*ln(-c+(1+I*c)*tan(b*x+a)+I)*ln((-I+c+(1+I*c)*tan(b*x+a))/(-2*I+2*c))+1/
4*I/(I-c)*dilog((-I+c+(1+I*c)*tan(b*x+a))/(-2*I+2*c))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(60) = 120$.
time = 0.52, size = 448, normalized size = 5.27

$$-\frac{(ic+1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{ic+1}-\frac{i\left(4(bx+a)\left(\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)-\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\right)+i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)^2-2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\log\left(-\frac{1}{2}(c-i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)+2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-i\right)\log\left(-\frac{(ic+1)\tan(bx+a)+c+i}{2c}+1\right)-2i\log\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)-2i\,\mathrm{Li}_2\left(\frac{1}{2}(c-i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)+2i\,\mathrm{Li}_2\left(\frac{(ic+1)\tan(bx+a)+c+i}{2c}\right)-2i\,\mathrm{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic+1}\right)-8(bx+a)\arctan\left((ic+1)\tan(bx+a)+c\right)+4(-ibx-ia)\log\left(-\frac{2\left(-ic^2+\left(c^2-2ic-1\right)\tan(bx+a)-2c+i\right)}{2ic^2-2\left(c^2-2ic-1\right)\tan(bx+a)+2i}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x +
 a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c + 1
) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)
- log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 -
 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*
tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*I*log(
-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan(b*x +
a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c
+ I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*x + a) -
1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c) - 2*I*di
log(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arctan((I*c + 1)*ta
```

```
n(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b
*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(60) = 120$.
time = 1.21, size = 202, normalized size = 2.38

$$-\frac{b^2x^2 - i\,bx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 - (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - (-i\,bx - i\,a)\log\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\sqrt{4i\,c}}{2\,c}\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\sqrt{4i\,c}}{2\,c}\right) + \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right) + \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)
/(c - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) -
 (-I*b*x - I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) -
I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sqr
t(4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + \tan(a + b\,x)\ (1 + c\,1\mathrm{i}))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.55 $\int \frac{\text{ArcTan}(c+(1+ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\text{ArcTan}(c+(1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+(ic+1)\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c + 1)*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c+\tan(a+b\,x)\,\,(1+c\,1\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i + 1))/x,x)
[Out] int(atan(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.56 $\int x^2 \mathbf{ArcTan}(c+(-1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(c-(1-ic)\tan(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{ix\text{PolyLog}}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctan(c-(1-I*c)*tan(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*I*
a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5279, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c-(1-ic)\tan(a+bx))-\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/6)*x^3*Log[1 +
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) + ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - Po
lyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{i(-1+ic)+c\cdots}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) - \frac{1}{3}(bc)\int \frac{\cdots}{i(-1+i\cdots}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2\cdots}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2\cdots}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2\cdots}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2\cdots}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1-ic)\tan(a+bx)) + \frac{1}{6}ix^3\log\left(1+ice^{2\cdots}\right.$$

Mathematica [A]
time = 0.36, size = 137, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\mathrm{ArcTan}(c+i(i+c)\tan(a+bx))+4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)-\frac{6x^2\mathrm{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{b}+\frac{6ix\mathrm{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2}+\frac{3\mathrm{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*x*
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)*(a
 + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 1533, normalized size = 9.89

method	result	size
risch	Expression too large to display	1533

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/6*I*x^3*
ln(I+c)+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/6
```

```
*I*x^3*ln(exp(2*I*(b*x+a))*c-I)+1/4*x^2*polylog(2,-I*exp(2*I*(b*x+a))*c)/b-
1/4/b^3*polylog(2,-I*exp(2*I*(b*x+a))*c)*a^2+1/12*x^3*Pi*csgn(I*(I+c)/(exp(
2*I*(b*x+a))+1))^3+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+
a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/3*I/b^3*ln(1+I*
c*exp(2*I*(b*x+a)))*a^3-1/6*I/b^3*a^3*ln(-exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*a
^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(
I*c)^(1/2))+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))
*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/
12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/2/b^3*a^2
*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*b*x^4+1/3*I*x^3*ln(exp(I*(b*x+a
)))-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(ex
p(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn((exp(2*I*(b*x
+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(
exp(2*I*(b*x+a))+1))^3+1/6*Pi*x^3+1/12*Pi*x^3*csgn(I*exp(2*I*(b*x+a)))^3-1/
6*Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*Pi*x^3*csgn
(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/12*x^3*Pi*csgn(I*(I+c))*csg
n(I*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(I*c
)^(1/2))+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I*(I+c
)/(exp(2*I*(b*x+a))+1))+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(
exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/
2*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*x*a^2+1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a)
)*(I*c)^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-1/8*polylog(4,-I*exp(2*I*(b*x+a))
*c)/b^3-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+1
/6*I*x^3*ln(1+I*c*exp(2*I*(b*x+a)))+1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c
)/b^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I
*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I*(exp(2*I
*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(
b*x+a))+1))+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x
+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*
csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(108) = 216$.
time = 0.31, size = 310, normalized size = 2.00

$$\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b^2}+\frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(bx+a)^2a-9i\,(bx+a)a^2\right)\arctan(c\cos(2\,bx+2\,a),-c\sin(2\,bx+2\,a)+1)-3\left(4i\,(bx+a)^2-6i\,(bx+a)a+3i\,a^2\right)\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+\left(4\,(bx+a)^3-9\,(bx+a)^2a+9\,(bx+a)a^2\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2-2\,c\sin(2\,bx+2\,a)+1\right)+3\,(4\,bx+a)\mathrm{Li}_3\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,\mathrm{Li}_4\left(-i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c-1)}{b^2(c+i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c - 1)*
tan(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
```

```
+ a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 -
6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3
 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(
2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c*
e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/
(b^2*(c + I)))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(108) = 216$.
time = 1.40, size = 322, normalized size = 2.08

$$\frac{b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,c}}{2\,c}\right) - 2i\,a^3\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - 2\left(-i\,b^3x^3 - i\,a^3\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 2\left(-i\,b^3x^3 - i\,a^3\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
+ 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^2*
x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*c*e
^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)
) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^3*x^3
- I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 - I*a^3)
*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(-4*I*c
)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}(c + \tan(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.57 $\int x\mathbf{ArcTan}(c+(-1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=124

$$\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\text{PolyLog}(3$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctan(c-(1-I*c)*tan(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*I*a
+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,-I*c*
exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5279, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c-(1-ic)\tan(a+bx))+\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/4)*x^2*Log[1 +
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5279

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c + I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{i(-1+ic)+c+}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))-\frac{1}{2}(bc)\int\frac{e^{2i}}{i(-1+ic)}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia}\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia}\right.$$

Mathematica [A]

time = 0.27, size = 111, normalized size = 0.90

$$\frac{1}{2}x^2\text{ArcTan}(c+i(i+c)\tan(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + I*(I + c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 1498, normalized size = 12.08

method	result	size
risch	Expression too large to display	1498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b*x^3+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-1/
4*I*ln(exp(2*I*(b*x+a))*c-I)*x^2+1/4/b^2*polylog(2,-I*exp(2*I*(b*x+a))*c)*a
-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/2/b^2*a*dilog(1-I*exp(I*
(b*x+a))*(I*c)^(1/2))-1/8*x^2*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(I*
exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))+1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*(I+c))*c
sgn(I*(I+c)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/
(exp(2*I*(b*x+a))+1))^2+1/4*I*x^2*ln(1+I*c*exp(2*I*(b*x+a)))+1/8*I*polylog(
3,-I*exp(2*I*(b*x+a))*c)/b^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(
2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/8*x^
2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b
*x+a))+1))^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x
+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(
I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+1/2*I/b*ln(1+I*c*exp(2*I*(
b*x+a)))*x*a-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-
I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csg
n(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))
+1))-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/8*x^2*Pi*csgn(I*exp
(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(
2*I*(b*x+a))+1))+1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2+1/4*I/b^2*a^2*ln(
-exp(2*I*(b*x+a))*c+I)-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/2
*I*x^2*ln(exp(I*(b*x+a)))+1/4*Pi*x^2+1/8*Pi*x^2*csgn(I*exp(I*(b*x+a)))^2*cs
gn(I*exp(2*I*(b*x+a)))-1/4*Pi*x^2*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*
x+a)))^2+1/8*Pi*x^2*csgn(I*exp(2*I*(b*x+a)))^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(
```

```
b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*
x+a))+1))+1/4*I*x^2*ln(I+c)+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I
*(b*x+a))+1))^3-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1)
)^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+1/4*x*polylog(2,-I*exp(2*I*(b*x
+a))*c)/b+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I*(I+c
)/(exp(2*I*(b*x+a))+1))+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(e
xp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))-1/8
*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*cs
gn(I*(I+c)/(exp(2*I*(b*x+a))+1))^3+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)
/(exp(2*I*(b*x+a))+1))^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(86) = 172$.
time = 0.29, size = 218, normalized size = 1.76

$$\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right)\arctan(c\cos(2\,bx+2\,a),-c\sin(2\,bx+2\,a)+1)+3\left((bx+a)^2-2\,(bx+a)a\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2-2\,c\sin(2\,bx+2\,a)+1\right)+3\,\mathrm{Li}_3\left(-i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c-1)}{b(c+i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c - 1)*tan(b*x + a) + c)/b
+ (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
-c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*
x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog
(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(86) = 172$.
time = 0.65, size = 271, normalized size = 2.19

$$\frac{2\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right)+2\,a^3+6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)+6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)+3i\,a^2\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) +
6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^(I*
b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) - I*
sqrt(-4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I
*a) + 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1
) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*
sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(-1+I*c)*tan(b*x+a)),x)
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(c + \tan(a + b\,x)\,(-1 + c\,1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tan(a + b*x)*(c*1i - 1)),x)
[Out] int(x*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.58 $\int \text{ArcTan}(c+(-1+ic)\tan(a+bx))\,dx$

Optimal. Leaf size=86

$$\frac{bx^2}{2}+x\text{ArcTan}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arctan(c-(1-I*c)*tan(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*x
))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5271, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c-(1-ic)\tan(a+bx))+\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + (I/2)*x*Log[1 + I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5271

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1+ic)\tan(a+bx))\,dx = x\tan^{-1}(c-(1-ic)\tan(a+bx))-(ib)\int \frac{x}{i(-1+ic)+c+ce^{2ia}}$$

$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))-(bc)\int \frac{e^{2ia+2ib}}{i(-1+ic)+c}$$

$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right.$$

$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right.$$

$$= \frac{bx^2}{2}+x\tan^{-1}(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right.$$

Mathematica [A]
time = 6.22, size = 76, normalized size = 0.88

$$x\text{ArcTan}(c+i(i+c)\tan(a+bx))+\frac{1}{2}ix\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)-\frac{\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

```
[Out] x*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (I/2)*x*Log[1 - I/(c*E^((2*I)*(a + b
*x)))] - PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(70) = 140.
time = 0.26, size = 649, normalized size = 7.55

method	result

derivativedivides	$-\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(c-(ic-1)\tan(bx+a)+i)c^2}{2i+2c}-\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(c-(ic-1)\tan(bx+a)+i)c}{2i+2c}+\arctan(c+($
default	$-\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(c-(ic-1)\tan(bx+a)+i)c^2}{2i+2c}-\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(c-(ic-1)\tan(bx+a)+i)c}{2i+2c}+\arctan(c+($
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(-1+I*c)*(-arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*
x+a)+I)*c^2-2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b
*x+a)+I)*c+arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)
+I)+arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a))*c^2
+2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a))*c-
arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*x+a))+(-1+I*c
)^2*(-1/4*I/(I+c)*ln(I+c+(-1+I*c)*tan(b*x+a))*ln(-1/2*I*(I-c-(-1+I*c)*tan(b
*x+a)))+1/4*I/(I+c)*ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))*ln(-1/2*I*(I-c-(-1
+I*c)*tan(b*x+a)))+1/4*I/(I+c)*dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))+1/8*
I/(I+c)*ln(I+c+(-1+I*c)*tan(b*x+a))^2-1/4*I/(I+c)*ln(c-(-1+I*c)*tan(b*x+a)+
I)*ln((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c))-1/4*I/(I+c)*dilog((-I-c-(-1+I*
c)*tan(b*x+a))/(-2*I-2*c))+1/4*I/(I+c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*
(I-c-(-1+I*c)*tan(b*x+a))/c)+1/4*I/(I+c)*dilog(-1/2*(I-c-(-1+I*c)*tan(b*x+a
))/c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(61) = 122$.
time = 0.51, size = 448, normalized size = 5.21

$$-\frac{(ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)}{2ic^2-2\left(c^2+2ic-1\right)\tan(bx+a)-4c-2i}\right)}{ic-1}+\frac{i\left(4(bx+a)\left(\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)+2c+i\right)-\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)-i\right)\right)+i\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)+2c+i\right)^2-2i\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)-i\right)\log\left(\frac{1}{2}(c+i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)+2i\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)-i\right)\log\left(-\frac{(ic-1)\tan(bx+a)+c-i}{2c}+1\right)-2i\log\left(-ic^2+(c^2+2ic-1)\tan(bx+a)+2c+i\right)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)-2i\,\mathrm{Li}_2\left(-\frac{1}{2}(c+i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)+2i\,\mathrm{Li}_2\left(\frac{(ic-1)\tan(bx+a)+c-i}{2c}\right)-2i\,\mathrm{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic-1}\right)-8(bx+a)\arctan\left((ic-1)\tan(bx+a)+c\right)+4(-ibx-ia)\log\left(-\frac{2\left(-ic^2+\left(c^2+2ic-1\right)\tan(bx+a)-i\right)}{2ic^2-2\left(c^2+2ic-1\right)\tan(bx+a)-4c-2i}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x +
 a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c - 1
) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)
- log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c^2 +
 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*
tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I*log(-
I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(b*x + a
) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c +
 I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*x + a) +
```

```
1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c) - 2*I*di
log(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arctan((I*c - 1)*ta
n(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b
*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(61) = 122$.
time = 1.55, size = 200, normalized size = 2.33

$$\frac{b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)} + 1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)} + 1\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,c}}{2\,c}\right) - i\,a\log\left(\frac{2\,ce^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,c}}{2\,c}\right) + \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right) + \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a
) - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) + (
I*b*x + I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2*c*
e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a) - I
*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1/2*sq
rt(-4*I*c)*e^(I*b*x + I*a)))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + \tan(a + b\,x)\,(-1 + c\,1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.59 $\int \frac{\text{ArcTan}(c+(-1+ic)\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\text{ArcTan}(c+(-1+ic)\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(-1+ic)\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+(ic-1)\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I)
)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((I*c - 1)*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c + \tan(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tan(a + b*x)*(c*1i - 1))/x,x)
[Out] int(atan(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.60 $\int \mathbf{ArcTan}(\cot(a+bx))\,dx$

Optimal. Leaf size=16

$$-\frac{\text{ArcTan}(\cot(a+bx))^2}{2b}$$

```
[Out] -1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$-\frac{\text{ArcTan}(\cot(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Cot[a + b*x]],x]
```

```
[Out] -1/2*ArcTan[Cot[a + b*x]]^2/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(\cot(a+bx))\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \tan^{-1}(\cot(a+bx))\right)}{b}\\ &= -\frac{\tan^{-1}(\cot(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$\frac{bx^2}{2}+x\text{ArcTan}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.
time = 0.00, size = 51, normalized size = 3.19

method	result
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)-\operatorname{arccot}(\cot(bx+a))^{2}}{2b}$
default	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)^{2}}{2}}{b}$
risch	$\frac{\pi x}{2}+ix\ln\left(\mathrm{e}^{i(bx+a)}\right)+\frac{\pi x \operatorname{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^{2}\operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{4}-\frac{\pi x\operatorname{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^{2}}{2}+\frac{\pi x \operatorname{csgn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Pi*x-1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccot(cot(b*x+a))-1/2*(1/2*Pi-a
rccot(cot(b*x+a)))^2)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,\pi x-ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Fricas [A]
time = 1.22, size = 15, normalized size = 0.94

$$-\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

Sympy [A]
time = 0.06, size = 24, normalized size = 1.50

$$\frac{\pi x}{2}-\begin{cases}\frac{\operatorname{acot}^{2}\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-acot(cot(b*x+a)),x)
```

```
[Out] pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a))
, True))
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.94

$$-\frac{1}{2}bx^2 + \frac{1}{2}\pi x - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*pi*x - a*x
```

Mupad [B]
time = 0.00, size = 21, normalized size = 1.31

$$\frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a + b\,x)) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/2 - acot(cot(a + b*x)),x)
```

```
[Out] (Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.61 $\int x^2 \textbf{ArcTan}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=399

$$\frac{1}{3}x^3\text{ArcTan}(c+d\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+$$

```
[Out] 1/3*x^3*arctan(c+d*cot(b*x+a))+1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x^2*polylog(2,(c
+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/4*I*x*polylog(3,(1+I*c-d)*exp
(2*I*a+2*I*b*x)/(1+I*c+d))/b^2-1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*
b*x)/(c+I*(1-d)))/b^2-1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))
/b^3+1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

Rubi [A]
time = 0.39, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5285, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(d\cot(a+bx)+c)-\frac{\text{Li}_4\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^3}+\frac{\text{Li}_4\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}+\frac{ix\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x^2\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/6)*x^3*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x^2*PolyLog[2, ((1 + I*c - d)*E^(
(2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x^2*PolyLog[2, ((c + I*(1 +
d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/4)*x*PolyLog[3,
((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/4)*x*Poly
Log[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2 - Po
lyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(8*b^3) + P
olyLog[4, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(8*b^3
)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5285

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c + d\cot(a + bx))\,dx = \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{3}(b(1 + ic - d)) \int \frac{e^{2ia+2ibx}x}{1 + ic + d + (-1 - i}$$

$$= \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}$$

$$= \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}$$

$$= \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}$$

$$= \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}$$

$$= \frac{1}{3}x^3 \tan^{-1}(c + d\cot(a + bx)) + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) - \frac{1}{6}$$

Mathematica [A]
time = 0.71, size = 359, normalized size = 0.90

$$\frac{1}{3}x^3\mathrm{ArcTan}(c+d\cot(a+bx))+\frac{4ib^3x^3\log\left(1-\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-4ib^3x^3\log\left(1-\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)+6b^2x^2\mathrm{PolyLog}\left(2,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-6b^2x^2\mathrm{PolyLog}\left(2,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)+6ibx\mathrm{PolyLog}\left(3,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-6ibx\mathrm{PolyLog}\left(3,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)-3\mathrm{PolyLog}\left(4,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+3\mathrm{PolyLog}\left(4,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Cot[a + b*x]])/3 + ((4*I)*b^3*x^3*Log[1 - ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (4*I)*b^3*x^3*Log[1 - ((c + I*(1
 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + 6*b^2*x^2*PolyLog[2, ((c + I*(
-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 6*b^2*x^2*PolyLog[2, ((c +
 I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + (6*I)*b*x*PolyLog[3, ((c
+ I*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - (6*I)*b*x*PolyLog[3,
((c + I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - 3*PolyLog[4, ((c + I
*(-1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + 3*PolyLog[4, ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 20.12, size = 7924, normalized size = 19.86

method	result	size
risch	Expression too large to display	7924

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*x^3*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/6*x^3*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*cos(2
*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2*b*
x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x +
2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*
cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/
(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*co
s(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x +
2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(
c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^
4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x +
 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*
(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(
c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x +
 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.
time = 1.08, size = 1589, normalized size = 3.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(16*b^3*x^3*arctan(d*cot(b*x + a) + c) + 6*b^2*x^2*dilog(-(c^2 + d^2 -
 (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*
sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I
*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b^2*x^
2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1)
```

```
 - 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a)
 + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d
 + 1) + 1) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1
/2) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) + 4
*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x
 + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 4*I*a^3
*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a
) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 6*I*b*x*polyl
og(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 +
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a))/(c^2 + d^2 - 2*d + 1)) - 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2
+ 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2
 + d^2 + 2*d + 1)) + 6*I*b*x*polylog(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d
+ 1)) - 4*(-I*b^3*x^3 - I*a^3)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/
(c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^2 - (c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2
*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(I*b^3*x^3 + I*a^3)*log((c^2 + d^
2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 -
I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 4*(-I*b^3*x^3 - I*a
^3)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 +
2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 3*p
olylog(4, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + 3*polylog(4, ((c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x +
 2*a))/(c^2 + d^2 - 2*d + 1)) - 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos
(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 +
 2*d + 1)) + 3*polylog(4, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctan(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(c+d\cot(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*cot(a + b*x)),x)
[Out] int(x^2*atan(c + d*cot(a + b*x)), x)
```

3.62 $\int x\mathbf{ArcTan}(c+d\cot(a+bx))\,dx$

Optimal. Leaf size=303

$$\frac{1}{2}x^2\text{ArcTan}(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+$$

```
[Out] 1/2*x^2*arctan(c+d*cot(b*x+a))+1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/
(1+I*c+d))-1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*x
*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x*polylog(2,(c+I*(
1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/8*I*polylog(3,(1+I*c-d)*exp(2*I*a
+2*I*b*x)/(1+I*c+d))/b^2-1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+
I*(1-d)))/b^2
```

Rubi [A]
time = 0.31, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5285, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(d\cot(a+bx)+c)+\frac{i\text{Li}_3\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{8b^2}-\frac{i\text{Li}_3\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}+\frac{x\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{x\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)] - (I/4)*x^2*Log[1 - ((c + I*(1 + d))*E^(
(2*I)*a + (2*I)*b*x))/(c + I*(1 - d))] + (x*PolyLog[2, ((1 + I*c - d)*E^((2
*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(4*b) - (x*PolyLog[2, ((c + I*(1 + d))*
E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(4*b) + ((I/8)*PolyLog[3, ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/b^2 - ((I/8)*PolyLog[3, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5285

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Dist[b*((1 + I*c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2
*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
- Dist[b*((1 - I*c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2*I
*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[
{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx))+\frac{1}{2}(b(1+ic-d))\int\frac{e^{2ia+2ibx}}{1+ic+d+(-1-i}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-$$

Mathematica [A]
time = 0.47, size = 270, normalized size = 0.89

$$\frac{1}{2}x^2\text{ArcTan}(c+d\cot(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-2b^2x^2\log\left(1-\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)-2ibx\text{PolyLog}\left(2,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)+2ibx\text{PolyLog}\left(2,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)+\text{PolyLog}\left(3,\frac{(c+i(-1+d))e^{2i(a+bx)}}{c-i(1+d)}\right)-\text{PolyLog}\left(3,\frac{(c+i(1+d))e^{2i(a+bx)}}{i+c-id}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - ((c + I*(-1
+ d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - 2*b^2*x^2*Log[1 - ((c + I*(1
+ d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] - (2*I)*b*x*PolyLog[2, ((c + I*(-
1 + d))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] + (2*I)*b*x*PolyLog[2, ((c +
I*(1 + d))*E^((2*I)*(a + b*x)))/(I + c - I*d)] + PolyLog[3, ((c + I*(-1 + d
))*E^((2*I)*(a + b*x)))/(c - I*(1 + d))] - PolyLog[3, ((c + I*(1 + d))*E^((
2*I)*(a + b*x)))/(I + c - I*d)]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.54, size = 7556, normalized size = 24.94

method	result	size
risch	Expression too large to display	7556

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^2*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1
)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/4*x^2*arctan2(-c*cos(2
*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) - c*s
in(2*b*x + 2*a) - d - 1) + 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2*b*x
 + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x +
2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a)
 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(
2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4
 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*cos(4*
b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*x + 2*a)
```

```
^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4
+ d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 + 2*(c^4 +
d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a
) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4
 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 + c)*d + 2*(c*d^
3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*sin(2*b*x + 2*a
))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(213) = 426$.
time = 1.56, size = 1289, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b^2*x^2*arctan(d*cot(b*x + a) + c) + 2*b*x*dilog(-(c^2 + d^2 - (c^2
 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2
*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2 + d^2
 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)
*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(-(c^2
 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*b*x*dilo
g(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) + 2*I*
a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2
*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log
(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a^2*log(-1/2*c
^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*
c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 2*I*a^2*log(-1/2*c^2 + I
*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 +
I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(-I*b^2*x^2 + I*a^2)*log((c^
2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*
d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(I*b^2*x^2
- I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c
^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1))
- 2*(I*b^2*x^2 - I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
d^2 - 2*d + 1)) - 2*(-I*b^2*x^2 + I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d -
d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) -
2*d + 1)/(c^2 + d^2 - 2*d + 1)) + I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*c
os(2*b*x + 2*a) + (I*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2
+ 2*d + 1)) - I*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I
```

```
*c^2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*d + 1)) - I*poly
log(3, ((c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^
2 - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) + I*polylog(3, ((c^2 - 2*I*
c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*c*d + I*d^2 - I)*sin(2*b*x +
2*a))/(c^2 + d^2 - 2*d + 1)))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*cot(b*x+a)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(d*cot(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(c+d\cot(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*cot(a + b*x)),x)
[Out] int(x*atan(c + d*cot(a + b*x)), x)
```

3.63 $\int \textbf{ArcTan}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=198

$$x\text{ArcTan}(c+d\cot(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)+\frac{\text{Po}}{}$$

```
[Out] x*arctan(c+d*cot(b*x+a))+1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d
))-1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*polylog(2,(
1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*polylog(2,(c+I*(1+d))*exp(2*I*
a+2*I*b*x)/(c+I*(1-d)))/b
```

Rubi [A]
time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5277, 2221, 2317, 2438}

$$x\text{ArcTan}(d\cot(a+bx)+c)+\frac{\text{Li}_2\left(\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b}-\frac{\text{Li}_2\left(\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (I/2)*x*Log[1 - ((1 + I*c - d)*E^((2*I)*a +
(2*I)*b*x))/(1 + I*c + d)] - (I/2)*x*Log[1 - ((c + I*(1 + d))*E^((2*I)*a +
(2*I)*b*x))/(c + I*(1 - d))] + PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)
*b*x))/(1 + I*c + d)]/(4*b) - PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I
)*b*x))/(c + I*(1 - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5277

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] + (Dist[b*(1 + I*c - d), Int[x*(E^(2*I*a + 2*I*b
*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Dist[b*(1
- I*c + d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*
a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\cot(a+bx))\,dx = x\tan^{-1}(c+d\cot(a+bx)) + (b(1+ic-d))\int \frac{e^{2ia+2ibx}x}{1+ic+d+(-1-ic+d)e}$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1648 vs. $2(198) = 396$.
time = 20.56, size = 1648, normalized size = 8.32

$$x\text{ArcTan}(c+d\cot(a+bx)) + \frac{d\left(4a\sqrt{-d^2}\,\text{ArcTan}\left(\frac{cd+\tan(a+bx)+c^2\tan(a+bx)}{d}\right) + id\log(1+i\tan(a+bx))\log\left(\frac{cd-\sqrt{-d^2}+\tan(a+bx)+c^2\tan(a+bx)}{i+ic^2+cd-\sqrt{-d^2}}\right) + \ldots\right)}{[illegible]}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Cot[a + b*x]] + (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*x
] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d^2
] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I*d*
Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a +
b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c
*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[
-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*
Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1 + c^2)*(
1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyLog[2, ((1
 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] - I*d*PolyL
og[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])] +
I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-
```

```
d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)]
 - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a + b*x))/(b*(-1 -
 c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)]
 - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/
(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d
*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])]
*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sqrt[-d^2] + Tan[a +
b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(
1 - I*Tan[a + b*x]) - (d*Log[(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x])
/(I + I*c^2 - c*d + Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) - (d*
Log[1 - ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt[-d^2])]*
Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[1 - ((1 + c^2)*(1 + I*Tan[a +
 b*x]))/(1 + c^2 - I*c*d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*
x]) - (d*Log[(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^
2 + c*d - Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (d*Log[(c*d +
 Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d + Sqrt[-d^2
])]*Sec[a + b*x]^2)/(1 + I*Tan[a + b*x]) + (I*d*Log[1 + I*Tan[a + b*x]]*(Se
c[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d - Sqrt[-d^2] + Tan[a + b*x] + c^2*
Tan[a + b*x]) + (I*d*Log[1 - I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a +
b*x]^2))/(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x]) - (I*d*Log[1
+ I*Tan[a + b*x]]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(c*d + Sqrt[-d^2]
+ Tan[a + b*x] + c^2*Tan[a + b*x]) + (I*(1 + c^2)*d*Log[1 - I*Tan[a + b*x]]
*Sec[a + b*x]^2)/(-(c*d) + Sqrt[-d^2] - (1 + c^2)*Tan[a + b*x]) + (4*a*Sqrt
[-d^2]*(Sec[a + b*x]^2 + c^2*Sec[a + b*x]^2))/(d*(1 + (c*d + Tan[a + b*x] +
 c^2*Tan[a + b*x])^2/d^2)))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(168) = 336$.
time = 0.77, size = 1145, normalized size = 5.78

method	result	size
derivativedivides	Expression too large to display	1145
default	Expression too large to display	1145
risch	Expression too large to display	4968

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arctan(c+d*cot(b*x+a))+d^2*(-1/d*arct
an(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(1/2
*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(
b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)/(1+I
*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c
+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a))/d-c/d)+
c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c
```

```
/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c
+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*
((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/2*d*arctan(d*((c+d*
cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c*arctan(d*((c+d*cot(b*x+
a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+
c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d)+1/4*d/(c-I*
d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*c
ot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c-1/2*I*d*arctan(d*((c+d*cot(b*x+a))/d
-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*co
t(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-1/2*d*arctan(d*((c+d*cot(b*x+a))/d-c/d
)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d
*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c)))))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(140) = 280$.
time = 0.56, size = 532, normalized size = 2.69

$$-\frac{d\left(\frac{8(bx+a)\arctan\left(\frac{cd+(c^2+1)\tan(bx+a)}{d}\right)}{d} - \frac{8(bx+a)\arctan\left(\frac{cd+(c^2+1)\tan(bx+a)}{d}\right) - 4\arctan\left(\frac{cd+(c^2+1)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+(c^2+d+1)\tan(bx+a)}{c^2+d^2+2d+1}, -\frac{cd\tan(bx+a)-c^2-d-1}{c^2+d^2+2d+1}\right) + 4\arctan\left(\frac{cd+(c^2+1)\tan(bx+a)}{d}\right)\arctan\left(-\frac{cd+(c^2-d+1)\tan(bx+a)}{c^2+d^2-2d+1}, -\frac{cd\tan(bx+a)-c^2+d-1}{c^2+d^2-2d+1}\right) - \left(\log\left(\frac{(c^2+1)\tan(bx+a)^2+c^2+1}{c^2+d^2+2d+1}\right) - \log\left(\frac{(c^2+1)\tan(bx+a)^2+c^2+1}{c^2+d^2-2d+1}\right)\right)\log\left((c^2+1)d^2+2(c^3+c)d\tan(bx+a)+(c^4+2c^2+1)\tan(bx+a)^2\right) - 2\operatorname{Li}_2\left(\frac{(ic-1)\tan(bx+a)+id}{c+id+i}\right) + 2\operatorname{Li}_2\left(\frac{(ic+1)\tan(bx+a)+id}{c+id-i}\right) + 2\operatorname{Li}_2\left(-\frac{(ic-1)\tan(bx+a)+id}{c-id+i}\right) - 2\operatorname{Li}_2\left(-\frac{(ic+1)\tan(bx+a)+id}{c-id-i}\right)}{d}\right) - 8(bx+a)\arctan\left(c+\frac{d}{\tan(bx+a)}\right) - 8(bx+a)\arctan\left(\frac{cd+(c^2+1)\tan(bx+a)}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x +
 a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*ta
n(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2*d
+ 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arctan((
c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x + a)
)/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2 - 2*d
 + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) -
 log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*log((c^2
+ 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x + a)^2) -
 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(((I*c + 1)
*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*x + a) + I*
d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c - I*d - I)))
/d) - 8*(b*x + a)*arctan(c + d/tan(b*x + a)) - 8*(b*x + a)*arctan((c*d + (c
^2 + 1)*tan(b*x + a))/d))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(140) = 280$.
time = 1.50, size = 965, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*x*arctan(d*cot(b*x + a) + c) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2
 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d
+ I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c
^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(
2*b*x + 2*a) + 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d +
 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1
/2) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x
 + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d
^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)
*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)
/(c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d -
 d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a)
- 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2
- 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b
*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + dilog(-(c^2 + d^2 - (c^2 + 2*
I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c^2 + d^2 - (c^2 - 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
+ 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 + 2
*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - dilog(-(c^2 + d^2 - (c^2 -
2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x
 + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(c + d\cot(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*cot(a + b*x)),x)
```

```
[Out] int(atan(c + d*cot(a + b*x)), x)
```

3.64 $\int \frac{\text{ArcTan}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\text{ArcTan}(c+d\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arctan(c+d*cot(b*x+a))/x,x)

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{ArcTan}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcTan[c + d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c + d*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcTan[c + d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c + d*Cot[a + b*x]]/x, x]

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*cot(b*x+a))/x,x)
[Out] int(arctan(c+d*cot(b*x+a))/x,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="maxima")
[Out] Timed out
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*cot(b*x + a) + c)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*cot(b*x+a))/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*cot(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}(c+d\cot(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*cot(a + b*x))/x,x)
[Out] int(atan(c + d*cot(a + b*x))/x, x)
```

3.65 $\int x^2 \textbf{ArcTan}(c + (1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=154

$$\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{x^2\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{ix\text{PolyLog}(\ldots}{4\ldots}$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctan(-c-(1-I*c)*cot(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*I
*a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(3,
I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5281, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c+(1-ic)\cot(a+bx))-\frac{\text{Li}_4\left(ice^{2ia+2ibx}\right)}{8b^3}+\frac{ix\text{Li}_3\left(ice^{2ia+2ibx}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/6)*x^3*Log[1 -
 I*c*E^((2*I)*a + (2*I)*b*x)] + (x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)
])/(4*b) + ((I/4)*x*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2 - PolyLog[
4, I*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{-i(1-ic)+c-}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{3}(bc)\int \frac{e^{2i}}{-i(1-ic)}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia}\right.$$

Mathematica [A]
time = 0.25, size = 136, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\text{ArcTan}(c+(1-ic)\cot(a+bx)) + 4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I)
*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)
*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c*E^
((2*I)*(a + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 1532, normalized size = 9.95

method	result	size
risch	Expression too large to display	1532

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*x*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2-1/3*I/b^3*ln(1-I*exp(2*I*(b*x+a
))*c)*a^3+1/2*I/b^3*a^3*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/6*I*x^3*ln(I+
```

```
c)+1/4*x^2*polylog(2,I*c*exp(2*I*(b*x+a)))/b-1/2*I/b^2*ln(1-I*exp(2*I*(b*x+
a))*c)*x*a^2+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/12*x^3*P
i*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2-1/12*x^
3*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2+1/12*x^3*Pi*csgn(I/
(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2
+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(
exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c
))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*
csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(
b*x+a))-1))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a
))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))+1/6*I*x^3*ln(1
-I*exp(2*I*(b*x+a))*c)-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(
b*x+a))-1))^3+1/12*b*x^4+1/3*I*x^3*ln(exp(I*(b*x+a)))+1/6*Pi*x^3+1/12*Pi*x^
3*csgn(I*exp(2*I*(b*x+a)))^3-1/6*Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2
*I*(b*x+a)))^2+1/12*Pi*x^3*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a))
)-1/6*I/b^3*a^3*ln(exp(2*I*(b*x+a))*c+I)+1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a)
)*(-I*c)^(1/2))-1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))
-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-1/12*x^3*Pi*csgn((ex
p(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+
a))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^3+1/2*I/b^2*a^2*ln
(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/2/b^3*a^2*dilog(1-I*exp(I*(b*x+a))*(-
I*c)^(1/2))+1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/12*x^3*Pi*
csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1)
)^2-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*
(I+c)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(
2*I*(b*x+a))-1))^2-1/6*I*x^3*ln(exp(2*I*(b*x+a))*c+I)+1/12*x^3*Pi*csgn(exp(
2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn((exp(2*I*(b*x+a
))*c+I)/(exp(2*I*(b*x+a))-1))^3-1/8*polylog(4,I*c*exp(2*I*(b*x+a)))/b^3+1/1
2*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*
(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(
I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1)
)+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/4/b^3
*polylog(2,I*c*exp(2*I*(b*x+a)))*a^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(110) = 220$.
time = 0.29, size = 309, normalized size = 2.01

$$\frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan\left((-i\,c+1)\cot(bx+a)+c\right)}{b^2}+\frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a)^2a+9i\,(bx+a)a^2\right)\arctan\left(c\cos(2\,bx+2\,a),c\sin(2\,bx+2\,a)+1\right)-3\left(4i\,(bx+a)^2-6i\,(bx+a)a+3i\,a^2\right)\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+\left(4\,(bx+a)^3-9\,(bx+a)^2a+9\,(bx+a)a^2\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2+2\,c\sin(2\,bx+2\,a)+1\right)+3\,(4\,bx+a)\mathrm{Li}_3\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,\mathrm{Li}_4\left(i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c-1)}{b^2(c+i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c + 1)
*cot(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
```

```
 + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*a
rctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6
*I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 -
 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin(2*
b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I*c*e^(
2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b^2
*(c + I)))/b
```

Fricas [A]
time = 2.47, size = 166, normalized size = 1.08

$$\frac{2\,b^4x^4+4i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+6i\,bx\mathrm{polylog}\left(3,i\,ce^{(2i\,bx+2i\,a)}\right)-4\left(-i\,b^3x^3-i\,a^3\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)-3\,\mathrm{polylog}\left(4,i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*b^4*x^4 + 4*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*I*a
^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3, I*c*e^(2*I*b*x +
 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*pol
ylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(-1+c\,1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)),x)
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.66 $\int x\textbf{ArcTan}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=123

$$\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{x\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}+\frac{i\text{PolyLog}(3,}{8b^2}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctan(-c-(1-I*c)*cot(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I*
a+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c*e
xp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5281, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{i\text{Li}_3\left(ice^{2ia+2ibx}\right)}{8b^2}+\frac{x\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/4)*x^2*Log[1 -
I*c*E^((2*I)*a + (2*I)*b*x)] + (x*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/
(4*b) + ((I/8)*PolyLog[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(c+(1-ic)\cot(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))-\frac{1}{2}(ib)\int\frac{x^2}{-i(1-ic)+c-ce^{2}\cdots}\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}(bc)\int\frac{e^{2ia+\cdots}}{-i(1-ic)+\cdots}\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\\ &= \frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2ia+2i\cdots}\right.\end{aligned}$$

Mathematica [A]

time = 0.16, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\mathrm{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 1497, normalized size = 12.17

method	result	size
risch	Expression too large to display	1497

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/b^2*ln(1-I*exp(2*I*(b*x+a))*c)*a^2+1/4*I/b^2*a^2*ln(exp(2*I*(b*x+a))*
c+I)-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/8*x^2*Pi*csgn(I*(e
xp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp
(2*I*(b*x+a))-1))-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))+1/2*I/b
*ln(1-I*exp(2*I*(b*x+a))*c)*x*a-1/2*I/b*a*ln(1-I*exp(I*(b*x+a))*(-I*c)^(1/2
))*x-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/6*b*x
^3+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(
2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a)
)*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-
1))^2+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^
2*Pi*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x+a)
)*(I+c)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(
I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1
))+1/4*x*polylog(2,I*c*exp(2*I*(b*x+a)))/b+1/2*I*x^2*ln(exp(I*(b*x+a)))+1/4
*Pi*x^2+1/8*Pi*x^2*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*Pi
*x^2*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/8*Pi*x^2*csgn(I*ex
p(2*I*(b*x+a)))^3-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I
/(exp(2*I*(b*x+a))-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2-1/8*x^2*Pi*csgn
(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(
b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*P
i*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))-1))^2-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+
```

```
c)/(exp(2*I*(b*x+a))-1))^2+1/4/b^2*polylog(2,I*c*exp(2*I*(b*x+a)))*a-1/2/b^
2*a*dilog(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2/b^2*a*dilog(1+I*exp(I*(b*x+a
))*(-I*c)^(1/2))+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c))*csgn
(I/(exp(2*I*(b*x+a))-1)*(I+c))-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2)
)*x-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(
I+c)/(exp(2*I*(b*x+a))-1))^2+1/4*I*x^2*ln(I+c)+1/8*x^2*Pi*csgn(exp(2*I*(b*x
+a))*(I+c)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)
/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(I+c))^3-1/
4*I*ln(exp(2*I*(b*x+a))*c+I)*x^2+1/4*I*x^2*ln(1-I*exp(2*I*(b*x+a))*c)+1/8*I
*polylog(3,I*c*exp(2*I*(b*x+a)))/b^2+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a))*(I+
c)/(exp(2*I*(b*x+a))-1))^3
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(88) = 176$.
time = 0.27, size = 217, normalized size = 1.76

$$\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan\left((-i\,c+1)\cot(bx+a)+c\right)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\arctan\left(c\cos(2\,bx+2\,a),c\sin(2\,bx+2\,a)+1\right)+3\left((bx+a)^2-2\,(bx+a)a\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2+2\,c\sin(2\,bx+2\,a)+1\right)+3\,\mathrm{Li}_3\left(i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c-1)}{b(c+i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c + 1)*cot(b*x + a) + c)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x +
2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a), c
*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b*x
+ 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polylog(3
, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

Fricas [A]
time = 2.26, size = 144, normalized size = 1.17

$$\frac{4\,b^3x^3+6i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)-6\left(-i\,b^2x^2+i\,a^2\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)+3i\,\mathrm{polylog}\left(3,i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*x^3 + 6*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*
x + 2*I*a) + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^2*l
og((c*e^(2*I*b*x + 2*I*a) + I)/c) - 6*(-I*b^2*x^2 + I*a^2)*log(-I*c*e^(2*I*
b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(x*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.67 $\int \textbf{ArcTan}(c+(1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=85

$$\frac{bx^2}{2}+x\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*b*x^2-x*arctan(-c-(1-I*c)*cot(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b*
x))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5273, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{\text{Li}_2\left(ice^{2ia+2ibx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 - I*c*E^((
2*I)*a + (2*I)*b*x)] + PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5273

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c+(1-ic)\cot(a+bx))-(ib)\int \frac{x}{-i(1-ic)+c-ce^{2ia+2i}$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c+(1-ic)\cot(a+bx))+(bc)\int \frac{e^{2ia+2ibx}x}{-i(1-ic)+c-}$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$
$$= \frac{bx^2}{2}+x\tan^{-1}(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)$$

Mathematica [**A**]
time = 6.05, size = 75, normalized size = 0.88

$$x\text{ArcTan}(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)-\frac{\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
[Out] x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (I/2)*x*Log[1 + I/(c*E^((2*I)*(a + b
*x)))] - PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(73) = 146.
time = 0.27, size = 611, normalized size = 7.19

method	result

derivativedivides	$-\frac{\frac{\arctan(\cot(bx+a)(ic-1)-c)\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(\cot(bx+a)(ic-1)-c)\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\text{arctan(co}}{}$
default	$-\frac{\frac{\arctan(\cot(bx+a)(ic-1)-c)\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(\cot(bx+a)(ic-1)-c)\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\text{arctan(co}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(-1+I*c)*(arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(-I+cot(b*x+a)*(-1
+I*c)-c)*c^2+2*I*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(-I+cot(b*x+a)*(
-1+I*c)-c)*c-arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(-I+cot(b*x+a)*(-1+I
*c)-c)-arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)+c+I)*
c^2-2*I*arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)+c+I)
*c+arctan(cot(b*x+a)*(-1+I*c)-c)/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*c)+c+I)-(-1+
I*c)^2*(1/8*I/(I+c)*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2-1/4*I/(I+c)*ln(-I+cot(b*
x+a)*(-1+I*c)-c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I))-1/4*I/(I+c)*dilog(-1/
2*I*(cot(b*x+a)*(-1+I*c)-c+I))-1/4*I/(I+c)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-
2*I-2*c))*ln(cot(b*x+a)*(-1+I*c)+c+I)-1/4*I/(I+c)*dilog((-I+cot(b*x+a)*(-1+
I*c)-c)/(-2*I-2*c))+1/4*I/(I+c)*ln(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)*ln(cot
(b*x+a)*(-1+I*c)+c+I)+1/4*I/(I+c)*dilog(-1/2*(cot(b*x+a)*(-1+I*c)-c+I)/c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(64) = 128$.
time = 0.50, size = 458, normalized size = 5.39

$$-\frac{(ic-1)\left(\frac{4i(bx+a)\log\left(-\frac{2(ic^2-(c^2+1)\tan(bx+a)-2c-i)}{-2ic^2+2(c^2+1)\tan(bx+a)-2i}\right)}{ic-1}-\frac{i\left(4(bx+a)(\log(-ic^2+(c^2+1)\tan(bx+a)+2c+i)-\log(-ic^2+(c^2+1)\tan(bx+a)-i))-2i\log(-ic^2+(c^2+1)\tan(bx+a)+2c+i)\log\left(-\frac{(ic+1)\tan(bx+a)+c+i}{2c}+1\right)+2i\log(-ic^2+(c^2+1)\tan(bx+a)+2c+i)\log(\tan(bx+a)-i)-2i\log\left(\frac{1}{2}(c-i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)\log(\tan(bx+a)-i)-i\log(\tan(bx+a)-i)^2-2i\log(c^2+1)\log(i\tan(bx+a)+1)+2i\log(\tan(bx+a)-i)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)+2i\log(c^2+1)\log(-i\tan(bx+a)+1)-2i\,\mathrm{Li}_2\left(-\frac{1}{2}(c-i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)-2i\,\mathrm{Li}_2\left(\frac{(ic+1)\tan(bx+a)+c+i}{2c}\right)+2i\,\mathrm{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic-1}\right)-8(bx+a)\arctan\left(c+\frac{-ic+1}{\tan(bx+a)}\right)+4(-ibx-ia)\log\left(-\frac{2(ic^2-(c^2+1)\tan(bx+a)-2c-i)}{-2ic^2+2(c^2+1)\tan(bx+a)-2i}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*c
 - I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I))/(I*c - 1) - I*(4*(b*x +
a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I) - log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I)*lo
g(-1/2*((I*c + 1)*tan(b*x + a) + c + I)/c + 1) + 2*I*log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) + 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(1/2*(c - I)*tan(b*
x + a) - 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)^2 -
 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*log(-
1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) - 2*I
*dilog(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c + 1
)*tan(b*x + a) + c + I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)
```

```
) - 8*(b*x + a)*arctan(c + (-I*c + 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*log(
-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*c - I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b
*x + a) - 2*I)))/b
```

Fricas [A]
time = 2.85, size = 112, normalized size = 1.32

$$\frac{2\,b^2x^2+2i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)-2\,a^2-2\left(-i\,bx-i\,a\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)+\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2 + 2*I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2
*I*a) + I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) -
2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x + 2*I*a)))/
b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.68 $\int \frac{\text{ArcTan}(c+(1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\text{ArcTan}(c+(1-ic)\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-c-(-ic+1)\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) + I)
)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c - \cot(a + b\,x)\,\left(-1 + c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i - 1))/x,x)
[Out] int(atan(c - cot(a + b*x)*(c*1i - 1))/x, x)
```

3.69 $\int x^2 \mathbf{ArcTan}(c+(-1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=155

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\text{ArcTan}(c-(1+ic)\cot(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)-\frac{x^2\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{ix\text{PolyL}}{}$$

```
[Out] -1/12*b*x^4-1/3*x^3*arctan(-c+(1+I*c)*cot(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
I*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(
3,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5281, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c-(1+ic)\cot(a+bx))+\frac{\text{Li}_4\left(-ice^{2ia+2ibx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/6)*x^3*Log[
1 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I
)*b*x)])/(4*b) - ((I/4)*x*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2 +
 PolyLog[4, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(ib)\int \frac{x^3}{-i(-1-ic)+}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{3}(bc)\int \frac{}{-i(-1}$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= -\frac{bx^4}{12} + \frac{1}{3}x^3\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$

Mathematica [A]
time = 0.26, size = 140, normalized size = 0.90

$$\frac{1}{3}x^3\text{ArcTan}(c+(-1-ic)\cot(a+bx)) - \frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right) + 3\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] + (6
*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*I)*
(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 1533, normalized size = 9.89

method	result	size
risch	Expression too large to display	1533

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2*polylog(2,-I*exp(2*I*(b*x+a))*c)/b+1/4/b^3*polylog(2,-I*exp(2*I*(b
*x+a))*c)*a^2-1/12*x^3*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/2*I/b^2*a^
```

```
2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*
(c-I)/(exp(2*I*(b*x+a))-1))^3+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(ex
p(2*I*(b*x+a))-1))^3-1/6*I*x^3*ln(c-I)+1/12*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))
*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1
))-1/2/b^3*a^2*dilog(1+I*exp(I*(b*x+a))*(I*c)^(1/2))+1/12*x^3*Pi*csgn((exp(
2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-1/12*b*x^4+1/6*Pi*x^3-1/12*Pi*x^3
*csgn(I*exp(2*I*(b*x+a)))^3+1/6*Pi*x^3*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*
I*(b*x+a)))^2-1/12*Pi*x^3*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))
-1/3*I*x^3*ln(exp(I*(b*x+a)))+1/6*I*x^3*ln(exp(2*I*(b*x+a))*c-I)-1/2/b^3*a^
2*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))-1/6*I*x^3*ln(1+I*c*e
xp(2*I*(b*x+a)))-1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-
1))^2+1/12*x^3*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/12*
x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(I*e
xp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))+1/2*I/b^2*ln(1+I*c*exp(2*I*(b*x
+a)))*x*a^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a
))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1/8*polylog(4,
-I*exp(2*I*(b*x+a))*c)/b^3-1/12*x^3*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I
*(b*x+a))-1))^2-1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/12*x^3
*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2
*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I)/(ex
p(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a
))-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*cs
gn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/12*x^3*Pi*csgn(I*(exp(2
*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/1
2*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*
(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/12*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*
csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/2*I/b^3*a^3*ln(1-I*
exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*x*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2-1/
12*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2
*I*(b*x+a))-1))^2+1/3*I/b^3*ln(1+I*c*exp(2*I*(b*x+a)))*a^3+1/6*I/b^3*a^3*ln
(-exp(2*I*(b*x+a))*c+I)-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/
12*x^3*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*
(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.
time = 0.30, size = 311, normalized size = 2.01

$$\frac{\frac{4\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\arctan((-ic-1)\cot(bx+a)+c)}{b^2}-\frac{\left(-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(c\cos(2bx+2a),-c\sin(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2\left(-ice^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left(c^2\cos(2bx+2a)^2+c^2\sin(2bx+2a)^2-2c\sin(2bx+2a)+1\right)+3(4bx+a)\mathrm{Li}_3\left(-ice^{(2ibx+2ia)}\right)+6i\,\mathrm{Li}_4\left(-ice^{(2ibx+2ia)}\right)\right)(ic+1)}{b^2(c-i)}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c - 1)
*cot(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
```

```
 + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*
arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 -
 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^
3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*sin
(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, -I*c
*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)
/(b^2*(c - I)))/b
```

Fricas [A]
time = 3.29, size = 166, normalized size = 1.07

$$-\frac{2\,b^4x^4-4i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}-i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+6i\,bx\mathrm{polylog}\left(3,-i\,ce^{(2i\,bx+2i\,a)}\right)+4\left(i\,b^3x^3+i\,a^3\right)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-3\,\mathrm{polylog}\left(4,-i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(2*b^4*x^4 - 4*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*
I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -I*c*e^(2*I*b
*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) - 3*p
olylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)),x)
[Out] int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.70 $\int x\mathbf{ArcTan}(c+(-1-ic)\cot(a+bx))\,dx$

Optimal. Leaf size=124

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\text{ArcTan}(c-(1+ic)\cot(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{x\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}-\frac{i\text{PolyLog}}{}$$

```
[Out] -1/6*b*x^3-1/2*x^2*arctan(-c+(1+I*c)*cot(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*I
*a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-I*
c*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5281, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c-(1+ic)\cot(a+bx))-\frac{i\text{Li}_3\left(-ice^{2ia+2ibx}\right)}{8b^2}-\frac{x\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/4)*x^2*Log[1
 + I*c*E^((2*I)*a + (2*I)*b*x)] - (x*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b
*x)])/(4*b) - ((I/8)*PolyLog[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2*
I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && Eq
Q[(c - I*d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{2}(ib)\int \frac{x^2}{-i(-1-ic)+c-}$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{2}(bc)\int \frac{}{-i(-1-}$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2i}\right.$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2i}\right.$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2i}\right.$$

$$= -\frac{bx^3}{6}+\frac{1}{2}x^2\tan^{-1}(c-(1+ic)\cot(a+bx)) - \frac{1}{4}ix^2\log\left(1+ice^{2i}\right.$$

Mathematica [A]

time = 0.16, size = 110, normalized size = 0.89

$$\frac{1}{2}x^2\mathrm{ArcTan}(c+(-1-ic)\cot(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\mathrm{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\mathrm{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
 PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 1498, normalized size = 12.08

method	result	size
risch	Expression too large to display	1498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b*x^3+1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I
)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))*csg
n(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn(I/(exp(2
*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-1/4/b
^2*polylog(2,-I*exp(2*I*(b*x+a))*c)*a+1/2/b^2*a*dilog(1+I*exp(I*(b*x+a))*(I
*c)^(1/2))+1/2/b^2*a*dilog(1-I*exp(I*(b*x+a))*(I*c)^(1/2))+1/8*x^2*Pi*csgn(
I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*x^2*Pi*csgn(I*(exp(2*I
*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(
b*x+a))-1))^2+1/2*I/b^2*a^2*ln(1-I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*x^2*ln
(1+I*c*exp(2*I*(b*x+a)))+1/4*I*ln(exp(2*I*(b*x+a))*c-I)*x^2+1/4*Pi*x^2-1/8*
Pi*x^2*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*Pi*x^2*csgn(I*
exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/2*I*x^2*ln(exp(I*(b*x+a)))-1/8
*x^2*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^3-1/8*Pi*x^2*csgn(I*exp(2*I*(b*x
+a)))^3-1/4*I/b^2*ln(1+I*c*exp(2*I*(b*x+a)))*a^2-1/4*I/b^2*a^2*ln(-exp(2*I*
(b*x+a))*c+I)-1/8*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*
x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*cs
gn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+1/8*x^2*Pi*csgn(I*exp(2
*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*
I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))
-1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))+1/2*I/b*a*ln(1-I*exp
(I*(b*x+a))*(I*c)^(1/2))*x-1/4*I*x^2*ln(c-I)-1/8*x^2*Pi*csgn(exp(2*I*(b*x+a
))*(c-I)/(exp(2*I*(b*x+a))-1))^2-1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(ex
p(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a
))-1))^3-1/8*I*polylog(3,-I*exp(2*I*(b*x+a))*c)/b^2+1/8*x^2*Pi*csgn(I/(exp(
```

```
2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-
I)/(exp(2*I*(b*x+a))-1))-1/8*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(
exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I/(exp(2*I*(b
*x+a))-1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn(I*(c-I))*cs
gn(I*(c-I)/(exp(2*I*(b*x+a))-1))^2+1/8*x^2*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(
exp(2*I*(b*x+a))-1))^3-1/2*I/b*ln(1+I*c*exp(2*I*(b*x+a)))*x*a+1/2*I/b*a*ln(
1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/4*x*polylog(2,-I*exp(2*I*(b*x+a))*c)/b+
1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/8*x^2*Pi*csgn(I*exp(2*I*
(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(
b*x+a))-1))-1/8*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-I))*csgn(I*(c
-I)/(exp(2*I*(b*x+a))-1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(87) = 174$.
time = 0.28, size = 219, normalized size = 1.77

$$\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b}-\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right)\arctan(c\cos(2\,bx+2\,a),-c\sin(2\,bx+2\,a)+1)+3\left((bx+a)^2-2\,(bx+a)a\right)\log\left(c^2\cos(2\,bx+2\,a)^2+c^2\sin(2\,bx+2\,a)^2-2\,c\sin(2\,bx+2\,a)+1\right)+3\,\mathrm{Li}_3\left(-i\,ce^{(2i\,bx+2i\,a)}\right)\right)(i\,c+1)}{b(c-i)}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c - 1)*cot(b*x + a) + c)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x +
 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
 -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b
*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*polylo
g(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

Fricas [A]
time = 3.41, size = 144, normalized size = 1.16

$$-\frac{4\,b^3x^3-6i\,b^2x^2\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+6\left(i\,b^2x^2-i\,a^2\right)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)+3i\,\mathrm{polylog}\left(3,-i\,ce^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b^3*x^3 - 6*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x
 - 2*I*a)/(c - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^
2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*(I*b^2*x^2 - I*a^2)*log(I*c*e^(2*I
*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(c - \cot(a + b\,x)\,(1 + c\,1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(x*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.71 $\int \textbf{ArcTan}(c + (-1 - ic)\cot(a + bx))\,dx$

Optimal. Leaf size=86

$$-\frac{bx^2}{2}+x\text{ArcTan}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

```
[Out] -1/2*b*x^2-x*arctan(-c+(1+I*c)*cot(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b
*x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5273, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c-(1+ic)\cot(a+bx))-\frac{\text{Li}_2\left(-ice^{2ia+2ibx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 + I*c*E
^((2*I)*a + (2*I)*b*x)] - PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5273

```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*
x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(-1-ic)\cot(a+bx))\,dx = x\tan^{-1}(c-(1+ic)\cot(a+bx))-(ib)\int \frac{x}{-i(-1-ic)+c-ce^{2i}}$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-(bc)\int \frac{e^{2ia+}}{-i(-1-ic)+}$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2i}\right.$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2i}\right.$$

$$= -\frac{bx^2}{2}+x\tan^{-1}(c-(1+ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2ia+2i}\right.$$

Mathematica [A]
time = 5.96, size = 75, normalized size = 0.87

$$x\text{ArcTan}(c+(-1-ic)\cot(a+bx))-\frac{1}{2}ix\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+\frac{\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]] - (I/2)*x*Log[1 - I/(c*E^((2*I)*(a +
b*x)))] + PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(72) = 144.
time = 0.27, size = 690, normalized size = 8.02

method	result

derivativedivides	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\arctan}{}$
default	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\arctan}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(1+I*c)*(-arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x
+a)-c)*c^2+2*I*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x
+a)-c)*c+arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(I+(1+I*c)*cot(b*x+a)-c)
+arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c^2-2*
I*arctan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*c-arc
tan(-c+(1+I*c)*cot(b*x+a))/(2*I-2*c)*ln(-(1+I*c)*cot(b*x+a)-c+I)+(1+I*c)^2*
(-1/4*I/(I-c)*ln(-1/2*I*(I+(1+I*c)*cot(b*x+a)-c))*ln(-1/2*I*(I-(1+I*c)*cot(
b*x+a)+c))+1/4*I/(I-c)*ln(-1/2*I*(I-(1+I*c)*cot(b*x+a)+c))*ln(I+(1+I*c)*cot
(b*x+a)-c)-1/4*I/(I-c)*dilog(-1/2*I*(I+(1+I*c)*cot(b*x+a)-c))-1/8*I/(I-c)*l
n(I+(1+I*c)*cot(b*x+a)-c)^2-1/4*I/(I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln(1/2*
(I-(1+I*c)*cot(b*x+a)+c)/c)-1/4*I/(I-c)*dilog(1/2*(I-(1+I*c)*cot(b*x+a)+c)/
c)+1/4*I/(I-c)*ln(-(1+I*c)*cot(b*x+a)-c+I)*ln((-I-(1+I*c)*cot(b*x+a)+c)/(-2
*I+2*c))+1/4*I/(I-c)*dilog((-I-(1+I*c)*cot(b*x+a)+c)/(-2*I+2*c))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(63) = 126$.
time = 0.49, size = 458, normalized size = 5.33

$$-\frac{(ic+1)\left(\frac{4i(bx+a)\log\left(-\frac{2\left(ic^2-(c^2+1)\tan(bx+a)+i\right)}{-2ic^2+2(c^2+1)\tan(bx+a)-4c+2i}\right)}{ic+1}+\frac{i\left(4(bx+a)\left(\log\left(-ic^2+(c^2+1)\tan(bx+a)-2c+i\right)-\log\left(-ic^2+(c^2+1)\tan(bx+a)-i\right)\right)-2i\log\left(-ic^2+(c^2+1)\tan(bx+a)-2c+i\right)\log\left(-\frac{(ic-1)\tan(bx+a)+c-i}{2c}+1\right)+2i\log\left(-ic^2+(c^2+1)\tan(bx+a)-2c+i\right)\log(\tan(bx+a)-i)-2i\log\left(-\frac{1}{2}(c+i)\tan(bx+a)+\frac{1}{2}ic+\frac{1}{2}\right)\log(\tan(bx+a)-i)-i\log(\tan(bx+a)-i)^2-2i\log(c^2+1)\log(i\tan(bx+a)+1)+2i\log(\tan(bx+a)-i)\log\left(-\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)+2i\log(c^2+1)\log(-i\tan(bx+a)+1)-2i\,\mathrm{Li}_2\left(\frac{1}{2}(c+i)\tan(bx+a)-\frac{1}{2}ic+\frac{1}{2}\right)-2i\,\mathrm{Li}_2\left(\frac{(ic-1)\tan(bx+a)+c-i}{2c}\right)+2i\,\mathrm{Li}_2\left(\frac{1}{2}i\tan(bx+a)+\frac{1}{2}\right)\right)}{ic+1}\right)-8(bx+a)\arctan\left(c+\frac{-ic-1}{\tan(bx+a)}\right)+4(-ibx-ia)\log\left(-\frac{2\left(ic^2-(c^2+1)\tan(bx+a)+i\right)}{-2ic^2+2(c^2+1)\tan(bx+a)-4c+2i}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)/
(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 4*c + 2*I))/(I*c + 1) + I*(4*(b*x +
a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I) - log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I)*lo
g(-1/2*((I*c - 1)*tan(b*x + a) + c - I)/c + 1) + 2*I*log(-I*c^2 + (c^2 + 1)
*tan(b*x + a) - 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(-1/2*(c + I)*tan(b
*x + a) + 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) - I)^2
- 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) - I)*log(
-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) + 1) - 2*
I*dilog(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) - 2*I*dilog(1/2*((I*c - 1
```

```
)*tan(b*x + a) + c - I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)
) - 8*(b*x + a)*arctan(c + (-I*c - 1)/tan(b*x + a)) + 4*(-I*b*x - I*a)*log(
-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a
) - 4*c + 2*I)))/b
```

Fricas [A]
time = 2.38, size = 112, normalized size = 1.30

$$-\frac{2\,b^2x^2-2i\,bx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)-2\,a^2+2\,(i\,bx+i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right)-2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)}-i}{c}\right)+\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^2 - 2*I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*
I*a)/(c - I)) - 2*a^2 + 2*(I*b*x + I*a)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) -
2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x + 2*I*a)))
/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c-\cot\left(a+b\,x\right)\,\left(1+c\,1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

```
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.72 $\int \frac{\textbf{ArcTan}(c+(-1-ic)\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\text{ArcTan}(c+(-1-ic)\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int -\frac{\arctan\left(-c-(-ic-1)\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)

[Out] int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c-1>0)', see `assume?` for more det
ails)Is

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c - I
))/x, x)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-atan(-c-(-1-I*c)*cot(b*x+a))/x,x)

[Out] Timed out

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="giac")

[Out] integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c)/x, x)

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c - cot(a + b*x)*(c*1i + 1))/x,x)
[Out] int(atan(c - cot(a + b*x)*(c*1i + 1))/x, x)
```

3.73 $\int \textbf{ArcTan}(\sinh(x))\,dx$

Optimal. Leaf size=39

$$-2x\text{ArcTan}(e^x) + x\text{ArcTan}(\sinh(x)) + i\text{PolyLog}(2, -ie^x) - i\text{PolyLog}(2, ie^x)$$

```
[Out] -2*x*arctan(exp(x))+x*arctan(sinh(x))+I*polylog(2,-I*exp(x))-I*polylog(2,I*
exp(x))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {5311, 4265, 2317, 2438}

$$-2x\text{ArcTan}(e^x) + x\text{ArcTan}(\sinh(x)) + i\text{Li}_2(-ie^x) - i\text{Li}_2(ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sinh[x]],x]
```

```
[Out] -2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[2
, I*E^x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \tan^{-1}(\sinh(x))\,dx &= x\tan^{-1}(\sinh(x)) - \int x\mathrm{sech}(x)\,dx \\
&= -2x\tan^{-1}(e^x) + x\tan^{-1}(\sinh(x)) + i\int \log(1-ie^x)\,dx - i\int \log(1+ie^x)\,dx \\
&= -2x\tan^{-1}(e^x) + x\tan^{-1}(\sinh(x)) + i\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) - i\mathrm{Subst}\left(\int\right. \\
&= -2x\tan^{-1}(e^x) + x\tan^{-1}(\sinh(x)) + i\mathrm{Li}_2(-ie^x) - i\mathrm{Li}_2(ie^x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 1.64

$$x\mathrm{ArcTan}(\sinh(x)) + i\left(x\left(\log\left(1-ie^{-x}\right) - \log\left(1+ie^{-x}\right)\right) + \mathrm{PolyLog}\left(2,-ie^{-x}\right) - \mathrm{PolyLog}\left(2,ie^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sinh[x]],x]
[Out] x*ArcTan[Sinh[x]] + I*(x*(Log[1 - I/E^x] - Log[1 + I/E^x]) + PolyLog[2, (-I
)/E^x] - PolyLog[2, I/E^x])
```

Maple [A]
time = 0.25, size = 52, normalized size = 1.33

method	result
default	$x\arctan(\sinh(x)) - ix(\ln(1-ie^x) - \ln(1+ie^x)) + i\,\mathrm{dilog}(1+ie^x) - i\,\mathrm{dilog}(1-ie^x)$
risch	$\frac{\pi x}{2} - \frac{x\pi\,\mathrm{csgn}\left(i(e^x-i)^2\right)\mathrm{csgn}\left(ie^{-x}\right)\mathrm{csgn}\left(ie^{-x}(e^x-i)^2\right)}{4} + \frac{x\pi\,\mathrm{csgn}\left(i(e^x+i)^2\right)\mathrm{csgn}\left(ie^{-x}\right)\mathrm{csgn}\left(ie^{-x}(e^x+i)^2\right)}{4} - i\ln(e^x-i)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(sinh(x)),x,method=_RETURNVERBOSE)
[Out] x*arctan(sinh(x))-I*x*(ln(1-I*exp(x))-ln(1+I*exp(x)))+I*dilog(1+I*exp(x))-I
*dilog(1-I*exp(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.
time = 1.87, size = 58, normalized size = 1.49

$$x\arctan\left(\sinh\left(x\right)\right)+i\,x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)-i\,x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)-i\,{\rm Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))+i\,{\rm Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(sinh(x)) + I*x*log(I*cosh(x) + I*sinh(x) + 1) - I*x*log(-I*cosh(x)
 - I*sinh(x) + 1) - I*dilog(I*cosh(x) + I*sinh(x)) + I*dilog(-I*cosh(x) - I
*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}{\left(\sinh{\left(x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(sinh(x)),x)
```

```
[Out] Integral(atan(sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(sinh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{atan}\left(\mathrm{sinh}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(sinh(x)),x)
```

```
[Out] int(atan(sinh(x)), x)
```

3.74 $\int x\mathbf{ArcTan}(\sinh(x))\,dx$

Optimal. Leaf size=74

$$-x^2\text{ArcTan}(e^x)+\frac{1}{2}x^2\text{ArcTan}(\sinh(x))+ix\text{PolyLog}(2,-ie^x)-ix\text{PolyLog}(2,ie^x)-i\text{PolyLog}(3,-ie^x)+i\text{PolyLo}$$

```
[Out] -x^2*arctan(exp(x))+1/2*x^2*arctan(sinh(x))+I*x*polylog(2,-I*exp(x))-I*x*po
lylog(2,I*exp(x))-I*polylog(3,-I*exp(x))+I*polylog(3,I*exp(x))
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5313, 4265, 2611, 2320, 6724}

$$x^2(-\text{ArcTan}(e^x))+\frac{1}{2}x^2\text{ArcTan}(\sinh(x))+ix\text{Li}_2(-ie^x)-ix\text{Li}_2(ie^x)-i\text{Li}_3(-ie^x)+i\text{Li}_3(ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sinh[x]],x]
```

```
[Out] -(x^2*ArcTan[E^x]) + (x^2*ArcTan[Sinh[x]])/2 + I*x*PolyLog[2, (-I)*E^x] - I
*x*PolyLog[2, I*E^x] - I*PolyLog[3, (-I)*E^x] + I*PolyLog[3, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(\sinh(x))\,dx &= \frac{1}{2}x^2\tan^{-1}(\sinh(x))-\frac{1}{2}\int x^2\text{sech}(x)\,dx\\
&= -x^2\tan^{-1}\left(e^x\right)+\frac{1}{2}x^2\tan^{-1}(\sinh(x))+i\int x\log\left(1-ie^x\right)\,dx-i\int x\log\left(1+ie^x\right)\\
&= -x^2\tan^{-1}\left(e^x\right)+\frac{1}{2}x^2\tan^{-1}(\sinh(x))+ix\text{Li}_2(-ie^x)-ix\text{Li}_2(ie^x)-i\int\text{Li}_2(-ie^x)\\
&= -x^2\tan^{-1}\left(e^x\right)+\frac{1}{2}x^2\tan^{-1}(\sinh(x))+ix\text{Li}_2(-ie^x)-ix\text{Li}_2(ie^x)-i\text{Subst}\left(\int\frac{\text{Li}_2}{}\right.\\
&= -x^2\tan^{-1}\left(e^x\right)+\frac{1}{2}x^2\tan^{-1}(\sinh(x))+ix\text{Li}_2(-ie^x)-ix\text{Li}_2(ie^x)-i\text{Li}_3(-ie^x)+i\end{aligned}$$

Mathematica [A]
time = 0.02, size = 105, normalized size = 1.42

$$\frac{1}{2}x^2\text{ArcTan}(\sinh(x))-\frac{1}{2}i\left(-x^2\left(\log\left(1-ie^{-x}\right)-\log\left(1+ie^{-x}\right)\right)-2x\left(\text{PolyLog}\left(2,-ie^{-x}\right)-\text{PolyLog}\left(2,ie^{-x}\right)\right)-2\left(\text{PolyLog}\left(3,-ie^{-x}\right)-\text{PolyLog}\left(3,ie^{-x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sinh[x]],x]

[Out] (x^2*ArcTan[Sinh[x]])/2 - (I/2)*(-(x^2*(Log[1 - I/E^x] - Log[1 + I/E^x])) -
 2*x*(PolyLog[2, (-I)/E^x] - PolyLog[2, I/E^x]) - 2*(PolyLog[3, (-I)/E^x] -
 PolyLog[3, I/E^x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 732, normalized size = 9.89

method	result	size
risch	Expression too large to display	732

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x*polylog(2,I*exp(x))+1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*cs
gn(I*exp(-x)*(exp(x)+I)^2)+I*x*polylog(2,-I*exp(x))-1/8*Pi*x^2*csgn(I*(exp(
x)-I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)-1/2*I*x^2*ln(1-I*exp(
x))-1/2*I*x^2*ln(exp(x)-I)+1/2*I*x^2*ln(1+I*exp(x))+1/2*I*x^2*ln(exp(x)+I)-
1/8*Pi*x^2*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)^2)^2+1/8*Pi*x^2*c
sgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2-1/8*Pi*x^2*csgn(I*exp(-x))*cs
gn(I*exp(-x)*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(e
xp(-x)*(exp(x)+I)^2)^2-1/8*Pi*x^2*csgn(I*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)
+1/4*Pi*x^2*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-I)^2)^2+1/8*Pi*x^2*csgn(I*(ex
p(x)+I))^2*csgn(I*(exp(x)+I)^2)-1/4*Pi*x^2*csgn(I*(exp(x)+I))*csgn(I*(exp(x
)+I)^2)^2+1/8*Pi*x^2*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2+1/
8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2+1/8*Pi*x
^2*csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)-I*polylog(3,-I*e
xp(x))-1/8*Pi*x^2*csgn(exp(-x)*(exp(x)-I)^2)^2-1/8*Pi*x^2*csgn(exp(-x)*(exp
(x)+I)^2)^2+1/8*Pi*x^2*csgn(exp(-x)*(exp(x)+I)^2)^3+1/8*Pi*x^2*csgn(exp(-x)
*(exp(x)-I)^2)^3-1/8*Pi*x^2*csgn(I*(exp(x)-I)^2)^3+1/8*Pi*x^2*csgn(I*(exp(x
)+I)^2)^3-1/8*Pi*x^2*csgn(I*exp(-x)*(exp(x)-I)^2)^3+1/8*Pi*x^2*csgn(I*exp(-
x)*(exp(x)+I)^2)^3+1/4*Pi*x^2+I*polylog(3,I*exp(x))-1/8*Pi*x^2*csgn(I*exp(-
x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - integrate(x^2*e^x/(e^(2*x) + 1),
 x)
```

Fricas [A]
time = 2.07, size = 93, normalized size = 1.26

$$\frac{1}{2}x^2 \arctan(\sinh(x)) + \frac{1}{2}ix^2 \log(i\cosh(x) + i\sinh(x) + 1) - \frac{1}{2}ix^2 \log(-i\cosh(x) - i\sinh(x) + 1) - ix\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) + ix\mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) + i\,\mathrm{polylog}(3, i\cosh(x) + i\sinh(x)) - i\,\mathrm{polylog}(3, -i\cosh(x) - i\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arctan(sinh(x)) + 1/2*I*x^2*log(I*cosh(x) + I*sinh(x) + 1) - 1/2*I*
x^2*log(-I*cosh(x) - I*sinh(x) + 1) - I*x*dilog(I*cosh(x) + I*sinh(x)) + I*
x*dilog(-I*cosh(x) - I*sinh(x)) + I*polylog(3, I*cosh(x) + I*sinh(x)) - I*p
olylog(3, -I*cosh(x) - I*sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atan}\left(\sinh\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(sinh(x)),x)
```

```
[Out] Integral(x*atan(sinh(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(sinh(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(\sinh(x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(sinh(x)),x)
```

```
[Out] int(x*atan(sinh(x)), x)
```

3.75 $\int x^2 \mathbf{ArcTan}(\sinh(x))\,dx$

Optimal. Leaf size=108

$$-\frac{2}{3}x^3\text{ArcTan}(e^x)+\frac{1}{3}x^3\text{ArcTan}(\sinh(x))+ix^2\text{PolyLog}(2,-ie^x)-ix^2\text{PolyLog}(2,ie^x)-2ix\text{PolyLog}(3,-ie^x)+2$$

```
[Out] -2/3*x^3*arctan(exp(x))+1/3*x^3*arctan(sinh(x))+I*x^2*polylog(2,-I*exp(x))-
I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*x*polylog(3,I*exp(
x))+2*I*polylog(4,-I*exp(x))-2*I*polylog(4,I*exp(x))
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5313, 4265, 2611, 6744, 2320, 6724}

$$-\frac{2}{3}x^3\text{ArcTan}(e^x)+\frac{1}{3}x^3\text{ArcTan}(\sinh(x))+ix^2\text{Li}_2(-ie^x)-ix^2\text{Li}_2(ie^x)-2ix\text{Li}_3(-ie^x)+2ix\text{Li}_3(ie^x)+2i\text{Li}_4(-ie^x)-2i\text{Li}_4(ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (-2*x^3*ArcTan[E^x])/3 + (x^3*ArcTan[Sinh[x]])/3 + I*x^2*PolyLog[2, (-I)*E^
x] - I*x^2*PolyLog[2, I*E^x] - (2*I)*x*PolyLog[3, (-I)*E^x] + (2*I)*x*PolyL
og[3, I*E^x] + (2*I)*PolyLog[4, (-I)*E^x] - (2*I)*PolyLog[4, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5313

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\tan^{-1}(\sinh(x))\,dx &= \frac{1}{3}x^3\tan^{-1}(\sinh(x)) - \frac{1}{3}\int x^3\operatorname{sech}(x)\,dx \\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right) + \frac{1}{3}x^3\tan^{-1}(\sinh(x)) + i\int x^2\log\left(1-ie^x\right)\,dx - i\int x^2\log\left(1+\right. \\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right) + \frac{1}{3}x^3\tan^{-1}(\sinh(x)) + ix^2\mathrm{Li}_2(-ie^x) - ix^2\mathrm{Li}_2(ie^x) - 2i\int x\mathrm{Li}_2 \\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right) + \frac{1}{3}x^3\tan^{-1}(\sinh(x)) + ix^2\mathrm{Li}_2(-ie^x) - ix^2\mathrm{Li}_2(ie^x) - 2ix\mathrm{Li}_3(-i \\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right) + \frac{1}{3}x^3\tan^{-1}(\sinh(x)) + ix^2\mathrm{Li}_2(-ie^x) - ix^2\mathrm{Li}_2(ie^x) - 2ix\mathrm{Li}_3(-i \\
&= -\frac{2}{3}x^3\tan^{-1}\left(e^x\right) + \frac{1}{3}x^3\tan^{-1}(\sinh(x)) + ix^2\mathrm{Li}_2(-ie^x) - ix^2\mathrm{Li}_2(ie^x) - 2ix\mathrm{Li}_3(-i
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 356 vs. 2(108) = 216.

time = 0.08, size = 356, normalized size = 3.30

$$\frac{1}{192}i\left(7\pi^4+8i\pi^3x+24\pi^2x^2-32i\pi x^3-16x^4-64ix^3\text{ArcTan}(\sinh(x))+8i\pi^3\log\left(1+ie^{-x}\right)+48\pi^2x\log\left(1+ie^{-x}\right)-96i\pi x^2\log\left(1+ie^{-x}\right)-64x^3\log\left(1+ie^{-x}\right)-48\pi^2x\log\left(1-ie^{x}\right)+96i\pi x^2\log\left(1-ie^{x}\right)-8i\pi^3\log\left(1+ie^{x}\right)+64x^3\log\left(1+ie^{x}\right)+8i\pi^3\log\left(\tan\left(\frac{1}{4}(\pi+2ix)\right)\right)-48(\pi-2ix)^2\text{PolyLog}(2,-ie^{-x})+192x^2\text{PolyLog}(2,-ie^{x})-48\pi^2\text{PolyLog}(2,ie^{x})+192i\pi x\text{PolyLog}(2,ie^{x})+192i\pi\text{PolyLog}\left(3,-ie^{-x}\right)+384x\text{PolyLog}\left(3,-ie^{-x}\right)-384x\text{PolyLog}(3,-ie^{x})-192i\pi\text{PolyLog}(3,ie^{x})+384\text{PolyLog}\left(4,-ie^{-x}\right)+384\text{PolyLog}(4,-ie^{x})\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sinh[x]],x]
```

```
[Out] (I/192)*(7*Pi^4 + (8*I)*Pi^3*x + 24*Pi^2*x^2 - (32*I)*Pi*x^3 - 16*x^4 - (64
*I)*x^3*ArcTan[Sinh[x]] + (8*I)*Pi^3*Log[1 + I/E^x] + 48*Pi^2*x*Log[1 + I/E
^x] - (96*I)*Pi*x^2*Log[1 + I/E^x] - 64*x^3*Log[1 + I/E^x] - 48*Pi^2*x*Log[
1 - I*E^x] + (96*I)*Pi*x^2*Log[1 - I*E^x] - (8*I)*Pi^3*Log[1 + I*E^x] + 64*
x^3*Log[1 + I*E^x] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*x)/4]] - 48*(Pi - (2*I)
*x)^2*PolyLog[2, (-I)/E^x] + 192*x^2*PolyLog[2, (-I)*E^x] - 48*Pi^2*PolyLog
[2, I*E^x] + (192*I)*Pi*x*PolyLog[2, I*E^x] + (192*I)*Pi*PolyLog[3, (-I)/E^
x] + 384*x*PolyLog[3, (-I)/E^x] - 384*x*PolyLog[3, (-I)*E^x] - (192*I)*Pi*P
olyLog[3, I*E^x] + 384*PolyLog[4, (-I)/E^x] + 384*PolyLog[4, (-I)*E^x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 758, normalized size = 7.02

method	result	size
risch	Expression too large to display	758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))+I*x^2*polylog(2,-I*ex
p(x))+2*I*x*polylog(3,I*exp(x))-2*I*polylog(4,I*exp(x))+1/12*Pi*x^3*csgn(I*
exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)+1/12*Pi*x^3*csgn(I*exp(-x)
*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2+1/12*Pi*x^3*csgn(exp(-x)*(exp(x
)-I)^2)^3+1/12*Pi*x^3*csgn(exp(-x)*(exp(x)+I)^2)^3-1/12*Pi*x^3*csgn(exp(-x)
*(exp(x)-I)^2)^2-1/12*Pi*x^3*csgn(exp(-x)*(exp(x)+I)^2)^2+1/12*Pi*x^3*csgn(
I*(exp(x)+I)^2)^3-1/12*Pi*x^3*csgn(I*(exp(x)-I)^2)^3+1/12*Pi*x^3*csgn(I*exp
(-x)*(exp(x)+I)^2)^3-1/12*Pi*x^3*csgn(I*exp(-x)*(exp(x)-I)^2)^3-1/12*Pi*x^3
*csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)+1/12*Pi*
x^3*csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)+1/6*P
i*x^3+2*I*polylog(4,-I*exp(x))+1/6*Pi*x^3*csgn(I*(exp(x)-I))*csgn(I*(exp(x)
-I)^2)^2-1/6*Pi*x^3*csgn(I*(exp(x)+I))*csgn(I*(exp(x)+I)^2)^2-1/12*Pi*x^3*c
sgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I*exp(-
x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2+1/12*Pi*x^3*csgn(I*(exp(x)-I)
^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2+1/12*Pi*x^3*csgn(I*(exp(x)+I))^2*csgn(I*
(exp(x)+I)^2)+1/12*Pi*x^3*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2-1/
12*Pi*x^3*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-1/12*Pi*x^3*csgn(I
*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)-1/3*I*x^3*ln(exp(x)-I)-1/3*I*x^3*ln(1-I
```

```
*exp(x))+1/3*I*x^3*ln(exp(x)+I)+1/3*I*x^3*ln(1+I*exp(x))-1/12*Pi*x^3*csgn(I
*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(1/3*x^3*e^x/(e^(2*x)
 + 1), x)
```

Fricas [A]
time = 1.60, size = 125, normalized size = 1.16

$$\frac{1}{3}x^3\arctan(\sinh(x)) + \frac{1}{3}ix^3\log(i\cosh(x) + i\sinh(x) + 1) - \frac{1}{3}ix^3\log(-i\cosh(x) - i\sinh(x) + 1) - ix^2\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) + ix^2\mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) + 2ix\,\mathrm{polylog}(3, i\cosh(x) + i\sinh(x)) - 2ix\,\mathrm{polylog}(3, -i\cosh(x) - i\sinh(x)) - 2i\,\mathrm{polylog}(4, i\cosh(x) + i\sinh(x)) + 2i\,\mathrm{polylog}(4, -i\cosh(x) - i\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="fricas")
[Out] 1/3*x^3*arctan(sinh(x)) + 1/3*I*x^3*log(I*cosh(x) + I*sinh(x) + 1) - 1/3*I*
x^3*log(-I*cosh(x) - I*sinh(x) + 1) - I*x^2*dilog(I*cosh(x) + I*sinh(x)) +
I*x^2*dilog(-I*cosh(x) - I*sinh(x)) + 2*I*x*polylog(3, I*cosh(x) + I*sinh(x
)) - 2*I*x*polylog(3, -I*cosh(x) - I*sinh(x)) - 2*I*polylog(4, I*cosh(x) +
I*sinh(x)) + 2*I*polylog(4, -I*cosh(x) - I*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atan}(\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(sinh(x)),x)
[Out] Integral(x**2*atan(sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(sinh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}(\sinh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(sinh(x)),x)
[Out] int(x^2*atan(sinh(x)), x)
```

3.76 $\int (e+fx)^3 \textbf{ArcTan}(\tanh(a+bx))\,dx$

Optimal. Leaf size=299

$$-\frac{(e+fx)^4 \text{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4 \text{ArcTan}(\tanh(a+bx))}{4f}+\frac{i(e+fx)^3 \text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+f}{}$$

```
[Out] -1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2,I
*exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/8*I
*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(4,-I
*exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3-3/16
*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*b*x+2*
a))/b^4
```

Rubi [A]
time = 0.17, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5291, 4265, 2611, 6744, 2320, 6724}

$$-\frac{(e+fx)^4 \text{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4 \text{ArcTan}(\tanh(a+bx))}{4f}-\frac{3if^3 \text{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}+\frac{3if^3 \text{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}+\frac{3if^2(e+fx) \text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{3if^2(e+fx) \text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{3if(e+fx)^2 \text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}+\frac{3if(e+fx)^2 \text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)^3 \text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^3 \text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/4*((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^4*ArcTan[Tanh[a +
 b*x]])/(4*f) + ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b - (((3*I)/8)*f*(e + f*x)^
2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f*(e + f*x)^2*PolyLog[
3, I*E^(2*a + 2*b*x)])/b^2 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*
a + 2*b*x)])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/
b^3 - (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 + (((3*I)/16)*f
^3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \tan^{-1}(\tanh(a+bx))\,dx = \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} - \frac{b\int (e+fx)^4 \mathrm{sech}(2a+2bx)\,dx}{4f}$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{1}{2}i \int$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e}{}$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e}{}$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e}{}$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e}{}$$

$$= -\frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\tanh(a+bx))}{4f} + \frac{i(e}{}$$

<u>**Mathematica**</u> [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.
time = 3.25, size = 600, normalized size = 2.01

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Tanh[a + b*x]])/4 - (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.21, size = 7275, normalized size = 24.33

method	result	size
risch	Expression too large to display	7275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*arctan((e^(2*b*x + 2*a)
 - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*f^2*x
^3*e^(2*a + 1) + 6*b*f*x^2*e^(2*a + 2) + 4*b*x*e^(2*a + 3))*e^(2*b*x)/(e^(4
*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(244) = 488.
time = 2.90, size = 2700, normalized size = 9.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2
4*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f
^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3*po
lylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
+ 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^4
*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 +
 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*arctan(sinh(b*x + a)/cos
h(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh
(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sin
h(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dil
og(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*
b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(
```

```
1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x
*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*c
osh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*
sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*
dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 -
 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*
sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^
3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a
) + sinh(b*x + a))) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x + a*b^3)*cos
h(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 - 4
*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^
4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 + I*a^4*
f^3 - 4*I*(b^4*x + a*b^3)*cosh(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(
b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x +
a*b^3)*cosh(1))*sinh(1)^2 - 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^
4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^
2*f)*cosh(1))*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x + a*b^3)*cosh(1)^3 + 4*I*(b^4
*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*I*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*I*(b^4*f^2*x^3 +
 a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x
+ a*b^3)*cosh(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b
^2*f)*cosh(1)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*s
inh(1)^2 + 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b
*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sin
h(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*a^4*f^
3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)^3 +
 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*I*(a
^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^4*f^3 + 4*I*a^3*b*f^2*cosh(1
) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)^3 + 4*I*a*b^3*sinh(1)^3 - 6
*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*c
osh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + (I*a^4*f^3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1
)^2 - 4*I*a*b^3*cosh(1)^3 - 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*
cosh(1))*sinh(1)^2 - 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)
^2)*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^4
*f^3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^
3 - 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I
*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt
```

```
(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(I*b*f^3*x + I*b*f^2*cosh(
1) + I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 24*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, -1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1
) - I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -1/2*
sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*
f^2*x*cosh(1) - I*b^2*f*cosh(1)^2 - I*b^2*f*sinh(1)^2 - 2*I*(b^2*f^2*x + b^
2*f*cosh(1))*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 12*(-I*b^2*f^3*x^2 - 2*I*b^2*f^2*x*cosh(...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{atan}(\tanh(a+bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atan(tanh(a + b*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(\tanh(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^3,x)
```

```
[Out] int(atan(tanh(a + b*x))*(e + f*x)^3, x)
```

3.77 $\int (e+fx)^2 \mathbf{ArcTan}(\tanh(a+bx))\,dx$

Optimal. Leaf size=229

$$-\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\text{ArcTan}(\tanh(a+bx))}{3f}+\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+f}{}$$

```
[Out] -1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2,I
*exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*I*f
*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b*x+2
*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5291, 4265, 2611, 6744, 2320, 6724}

$$-\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\text{ArcTan}(\tanh(a+bx))}{3f}+\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}-\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}+\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}+\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)^2\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/3*((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^3*ArcTan[Tanh[a +
 b*x]])/(3*f) + ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((
I/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/4)*f*(e + f*x)*Poly
Log[3, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a
+ 2*b*x)])/b^2 + ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 - ((I/8)*
f^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e+fx)^2\tan^{-1}(\tanh(a+bx))\,dx &= \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} - \frac{b\int (e+fx)^3\text{sech}(2a+2bx)\,dx}{3f}\\ &= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{1}{2}i\int\\ &= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e}{}\\ &= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e}{}\\ &= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e}{}\\ &= -\frac{(e+fx)^3\tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3\tan^{-1}(\tanh(a+bx))}{3f} + \frac{i(e}{}\end{aligned}$$

Mathematica [A]
time = 1.68, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\text{ArcTan}(\tanh(a+bx)) - \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)-12b^3efx^2\log\left(1+ie^{2(a+bx)}\right)-4b^3f^2x^3\log\left(1+ie^{2(a+bx)}\right)-6b^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+6b^2(e+fx)^2\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+6bef\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)+6bf^2x\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-6bef\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-6bf^2x\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-3f^2\text{PolyLog}\left(4,-ie^{2(a+bx)}\right)+3f^2\text{PolyLog}\left(4,ie^{2(a+bx)}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Tanh[a + b*x]])/3 - ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.05, size = 5425, normalized size = 23.69

method	result	size
risch	Expression too large to display	5425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
+ 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*f*x^2*e^(2*a + 1) + 3
*b*x*e^(2*a + 2))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1511 vs. 2(186) = 372.
time = 3.69, size = 1511, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2
*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*polyl
og(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3
*b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 +
 2*b^3*x*cosh(1))*sinh(1))*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I*b^2*f
^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*s
inh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(
b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^
2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b
^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cos
h(1))*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (-I
*b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b
^2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*
f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cos
h(1)^2 - 3*I*(b^3*x + a*b^2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1)
- 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*s
```

```
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + I*a^3*f^2
+ 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b^2)*sinh(1)^2 + 3*I*(b^3*
f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cos
h(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I
*b^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b
^2)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*
f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a)
+ sinh(b*x + a)) + 1) + (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cosh(1
)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*
sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^3*f^2 - 3*I*a^2*b*f*c
osh(1) + 3*I*a*b^2*cosh(1)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2
*cosh(1))*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
(-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*cosh(1)^2 - 3*I*a*b^2*sinh(1)
^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x
 + a) + 2*sinh(b*x + a)) + (-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*co
sh(1)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*lo
g(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(-I*b*f^2*x - I*b*
f*cosh(1) - I*b*f*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b
*x + a))) - 6*(-I*b*f^2*x - I*b*f*cosh(1) - I*b*f*sinh(1))*polylog(3, -1/2*
sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(I*b*f^2*x + I*b*f*cosh(1) +
 I*b*f*sinh(1))*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 6*(I*b*f^2*x + I*b*f*cosh(1) + I*b*f*sinh(1))*polylog(3, -1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^2 \operatorname{atan}(\tanh(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(tanh(b*x+a)),x)
[Out] Integral((e + f*x)**2*atan(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atan}(\tanh(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x)^2,x)
[Out] int(atan(tanh(a + b*x))*(e + f*x)^2, x)
```

3.78 $\int (e+fx)\mathbf{ArcTan}(\tanh(a+bx))\,dx$

Optimal. Leaf size=159

$$-\frac{(e+fx)^2\text{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\text{ArcTan}(\tanh(a+bx))}{2f}+\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx}{}$$

```
[Out] -1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(tanh(b*x+a))/f
+1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*exp
(2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(3,I
*exp(2*b*x+2*a))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5291, 4265, 2611, 2320, 6724}

$$-\frac{(e+fx)^2\text{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\text{ArcTan}(\tanh(a+bx))}{2f}-\frac{if\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}+\frac{if\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}+\frac{i(e+fx)\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i(e+fx)\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -1/2*((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/f + ((e + f*x)^2*ArcTan[Tanh[a +
 b*x]])/(2*f) + ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b - ((I/
4)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b - ((I/8)*f*PolyLog[3, (-I)*E^
(2*a + 2*b*x)])/b^2 + ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5291

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (e+fx)\tan^{-1}(\tanh(a+bx))\,dx &= \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} - \frac{b\int (e+fx)^2\text{sech}(2a+2bx)\,dx}{2f} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{1}{2}i\int (\ldots \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+\ldots}{} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+\ldots}{} \\
&= -\frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\tan^{-1}(\tanh(a+bx))}{2f} + \frac{i(e+\ldots}{}
\end{aligned}$$

Mathematica [A]
time = 1.22, size = 278, normalized size = 1.75

$$ex\text{ArcTan}(\tanh(a+bx)) + \frac{1}{2}fx^2\text{ArcTan}(\tanh(a+bx)) - \frac{e\left(-\left(\left(-4ia+\pi-4ibx\right)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+\left(-4ia+\pi\right)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)\right)}{8b} - \frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(3,ie^{2(a+bx)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Tanh[a + b*x]] + (f*x^2*ArcTan[Tanh[a + b*x]])/2 - (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
```

```
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))])))/(8*b) - ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 2414, normalized size = 15.18

method	result	size
risch	Expression too large to display	2414

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^
(3/4))-1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I/b*e*dilog(
((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^
2-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*
x+2*a)+1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b
*x+2*a)+1))^2+1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)-1/8*Pi*x^2*f*csgn(I*(ex
p(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)+1))+1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3
-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn
(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+
2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1
))^2+1/2*I/b*e*a*ln(exp(2*b*x+2*a)+I)-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/
(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-1/2*I
*f/b^2*a*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f/b^2*a*(b*x+a)*ln(1-exp
(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(
1/2))+1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*Pi*x
*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))
^2+1/8*I*f*polylog(3,I*exp(2*b*x+2*a))/b^2+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b
*x+2*a)+I)-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2
*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*f*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)+1))-1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*
b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-1/4*I/b^2*f*a^2*
ln(exp(2*b*x+2*a)+I)-1/2*I*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/8*Pi*f*x^2+1/4*Pi*
e*x-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/2*I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)
^(3/4))+1/2*I*e/b*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b^2*f*a*dilog((
```

```
(-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(
3/4))+1/4*I*f/b^2*(b*x+a)^2*ln(1+I*exp(2*b*x+2*a))+1/4*I*f/b^2*(b*x+a)*poly
log(2,-I*exp(2*b*x+2*a))+1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(
1/2))-1/4*I*f/b^2*(b*x+a)^2*ln(1-I*exp(2*b*x+2*a))-1/4*I*f/b^2*(b*x+a)*poly
log(2,I*exp(2*b*x+2*a))-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(
1/2))-1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*f/b^2*
a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+1/8*Pi
*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+
2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2
*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+1/4*Pi*x*e*
csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*f*csgn(I*(ex
p(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))^3+1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x
+2*a)+1))^3+1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^
3-1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2
*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))
^2+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2
*b*x+2*a)+1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2
*a)-I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+1/4*Pi*x*e*c
sgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b
*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)-I)
/(exp(2*b*x+2*a)+1))^2-1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))^2-1/4*I*ln(exp(2*b*x+2*a)-I)*f*x^2-1/2*I*ln(exp(2*b*x+2*a)-I)*e*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - i
ntegrate((b*f*x^2*e^(2*a) + 2*b*x*e^(2*a + 1))*e^(2*b*x)/(e^(4*b*x + 4*a) +
 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(134) = 268.
time = 5.14, size = 717, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*arctan(sinh(b*x + a)
/cosh(b*x + a)) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1
))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I*b*f*x - I*
b*cosh(1) - I*b*sinh(1))*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a))) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*co
sh(1) - 2*I*(b^2*x + a*b)*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*
(b^2*x + a*b)*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
 1) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b
)*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2
*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b)*sinh(1))*l
og(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*a^2*f + 2*I*a
*b*cosh(1) + 2*I*a*b*sinh(1))*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*
x + a)) + (-I*a^2*f + 2*I*a*b*cosh(1) + 2*I*a*b*sinh(1))*log(-I*sqrt(4*I) +
 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^2*f - 2*I*a*b*cosh(1) - 2*I*a*b*
sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^2*f -
 2*I*a*b*cosh(1) - 2*I*a*b*sinh(1))*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2
*sinh(b*x + a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) + 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 2*I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*p
olylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{atan}(\tanh(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atan(tanh(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atan(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(\tanh(a + b\,x))\,\,(e + f\,x)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))*(e + f*x),x)
```

```
[Out] int(atan(tanh(a + b*x))*(e + f*x), x)
```

3.79 $\int \mathbf{ArcTan}(\tanh(a+bx))\,dx$

Optimal. Leaf size=74

$$-x\text{ArcTan}\left(e^{2a+2bx}\right)+x\text{ArcTan}(\tanh(a+bx))+\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] -x*arctan(exp(2*b*x+2*a))+x*arctan(tanh(b*x+a))+1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5287, 4265, 2317, 2438}

$$-x\text{ArcTan}\left(e^{2a+2bx}\right)+x\text{ArcTan}(\tanh(a+bx))+\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}-\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] -(x*ArcTan[E^(2*a + 2*b*x)]) + x*ArcTan[Tanh[a + b*x]] + ((I/4)*PolyLog[2,
(-I)*E^(2*a + 2*b*x)])/b - ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5287

```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Tanh[a + b
*x]], x] - Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \tan^{-1}(\tanh(a+bx))\,dx = x\tan^{-1}(\tanh(a+bx)) - b\int x\text{sech}(2a+2bx)\,dx$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx - \frac{1}{2}i$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b}$$
$$= -x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\tanh(a+bx)) + \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} - \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A]
time = 0.03, size = 132, normalized size = 1.78

$$x\text{ArcTan}(\tanh(a+bx)) - \frac{-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[Tanh[a + b*x]] - (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(63) = 126.
time = 0.24, size = 160, normalized size = 2.16

method	result
derivativedivides	$\frac{\text{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a)) - \frac{i\,\text{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\left(\tanh^2(bx+a)\right)}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\left(\tanh^2(bx+a)\right)}\right)\right)}{2} +}{b}$
default	$\frac{\text{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a)) - \frac{i\,\text{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\left(\tanh^2(bx+a)\right)}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\left(\tanh^2(bx+a)\right)}\right)\right)}{2} +}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arctan(tanh(b*x+a))-1/2*I*arctanh(tanh(b*x+a))*(l
n(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-ln(1+I*(tanh(b*x+a)+1)^2/(1-tanh
(b*x+a)^2)))+1/4*I*dilog(1+I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-1/4*I*dil
og(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^(
2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(57) = 114$.
time = 6.29, size = 334, normalized size = 4.51

$$\frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (-i\,bx - i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) + i\,a\log\left(i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right) + i\,a\log\left(-i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right) - i\,a\log\left(i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right) - i\,a\log\left(-i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right) - i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) - i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) + i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))\right) + i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-4*I
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
+ I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(\tanh\left(a + bx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a)),x)
```

```
[Out] Integral(atan(tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(tanh(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(\mathrm{tanh}(a+b\,x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x)),x)
```

```
[Out] int(atan(tanh(a + b*x)), x)
```

3.80 $\int \frac{\mathbf{ArcTan}(\tanh(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{\mathrm{ArcTan}(\tanh(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctan(tanh(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\mathrm{ArcTan}(\tanh(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\tan^{-1}(\tanh(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 6.04, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{ArcTan}(\tanh(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(tanh(b*x+a))/(f*x+e),x)
[Out] int(arctan(tanh(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(tanh(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\tanh\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(tanh(b*x+a))/(f*x+e),x)
[Out] Integral(atan(tanh(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(tanh(a + b*x))/(e + f*x),x)
[Out] int(atan(tanh(a + b*x))/(e + f*x), x)
```

3.81 $\int x^2 \mathbf{ArcTan}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=355

$$\frac{1}{3}x^3\text{ArcTan}(c+d\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{ix^2\text{P}}{}$$

```
[Out] 1/3*x^3*arctan(c+d*tanh(b*x+a))+1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/
(I+c-d))/b-1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*p
olylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,-(I-c-d)*exp(
2*b*x+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

Rubi [A]
time = 0.34, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5307, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(d\tanh(a+bx)+c)+\frac{i\text{Li}_4\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{Li}_4\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix^2\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{6}ix^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + (I/6)*x^3*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c
+ d))])/b - ((I/4)*x^2*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c -
d))])/b - ((I/4)*x*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]
)/b^2 + ((I/4)*x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/
b^2 + ((I/8)*PolyLog[4, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^3
- ((I/8)*PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5307

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c + d\tanh(a + bx))\,dx = \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{3}(b(1 - i(c + d)))\int \frac{e^{2a+2b}}{i + c - d + (i +}$$
$$= \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) -$$
$$= \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) -$$
$$= \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) -$$
$$= \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) -$$
$$= \frac{1}{3}x^3 \tan^{-1}(c + d\tanh(a + bx)) + \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) -$$

Mathematica [A]
time = 3.37, size = 305, normalized size = 0.86

$$\frac{1}{3}x^3\text{ArcTan}(c + d\tanh(a + bx)) + \frac{i\left(4b^3x^3\log\left(1 + \frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 4b^3x^3\log\left(1 + \frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) + 6b^2x^2\text{PolyLog}\left(2, -\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2, -\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) - 6bx\text{PolyLog}\left(3, -\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) + 6bx\text{PolyLog}\left(3, -\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) + 3\text{PolyLog}\left(4, -\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 3\text{PolyLog}\left(4, -\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(-I + c - d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
 + b*x)))/(I + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x
)))/(-I + c - d))] - 6*b^2*x^2*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(
I + c - d))] - 6*b*x*PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c -
d))] + 6*b*x*PolyLog[3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))] + 3*P
olyLog[4, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] - 3*PolyLog[4, -(
((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 16.48, size = 6990, normalized size = 19.69

method	result	size
risch	Expression too large to display	6990

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a)
 - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.
time = 6.06, size = 1289, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(c
osh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*
x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x
+ a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a)
 + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*
c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)
*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2
*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*l
og(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh
(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -
sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
```

```
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
+ a))) + (I*b^3*x^3 + I*a^3)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log
(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^
3 - I*a^3)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + 6*I*polylog(4, sqrt(-(c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylo
g(4, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a))) - 6*I*polylog(4, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*
c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt(-(c
^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
a))))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atan}(c + d \tanh(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*tanh(a + b*x)), x)
```

3.82 $\int x\,\mathbf{ArcTan}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=267

$$\frac{1}{2}x^2\text{ArcTan}(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{ix\text{F}}{}$$

```
[Out] 1/2*x^2*arctan(c+d*tanh(b*x+a))+1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,-(I-c-
d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c
-d))/b-1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3
,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A]
time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5307, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(d\tanh(a+bx)+c)-\frac{i\text{Li}_3\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{Li}_3\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{4}ix^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[1 + ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d))])/b - ((I/4)*x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
)/b - ((I/8)*PolyLog[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b^2
+ ((I/8)*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5307

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b
*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c -
 d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2bx}}{i+c-d+(i+c}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}$$

$$=\frac{1}{2}x^2\tan^{-1}(c+d\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}$$

Mathematica [A]
time = 2.69, size = 229, normalized size = 0.86

$$\frac{1}{2}x^2\text{ArcTan}(c+d\tanh(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2b^2x^2\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)+2bx\text{PolyLog}\left(2,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2bx\text{PolyLog}\left(2,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)-\text{PolyLog}\left(3,-\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)+\text{PolyLog}\left(3,-\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(-I + c - d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(I + c - d)] + 2*b*x*PolyLog[2, -(((-I + c + d)*E^(2*(a + b*x)))/(
-I + c - d))] - 2*b*x*PolyLog[2, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d
))] - PolyLog[3, -(((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d))] + PolyLog[
3, -(((I + c + d)*E^(2*(a + b*x)))/(I + c - d))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.43, size = 6640, normalized size = 24.87

method	result	size
risch	Expression too large to display	6640

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x + 2*a)
 + 1)) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d
^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(197) = 394.
time = 6.48, size = 1067, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(
```

```
b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(sq
rt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^
2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 +
 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
- 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^
2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-
(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c
^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d
^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-sq
rt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 +
I*a^2)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a)) + 1) - 2*I*polylog(3, sqrt(-(c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3,
 -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + s
inh(b*x + a))) + 2*I*polylog(3, sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d
+ d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt(-(c^2 -
 d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
)/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*tanh(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}(c+d\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*tanh(a + b*x)),x)
[Out] int(x*atan(c + d*tanh(a + b*x)), x)
```

3.83 $\int \textbf{ArcTan}(c + d \tanh(a + bx))\, dx$

Optimal. Leaf size=174

$$x\text{ArcTan}(c+d\tanh(a+bx))+\frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{i\text{PolyLog}}{}$$

```
[Out] x*arctan(c+d*tanh(b*x+a))+1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,-(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b-1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A]
time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5299, 2221, 2317, 2438}

$$x\text{ArcTan}(d\tanh(a+bx)+c)+\frac{i\text{Li}_2\left(-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{Li}_2\left(-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + (I/2)*x*Log[1 + ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b - ((
I/4)*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5299

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] + (Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*x
)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[I*b*(I + c + d)
, Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]
) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\tanh(a+bx))\,dx = x\tan^{-1}(c+d\tanh(a+bx)) + (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(i+c+d$$

$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\,\mathrm{l}$$

$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\,\mathrm{l}$$

$$= x\tan^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\,\mathrm{l}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 363 vs. $2(174) = 348$.
time = 2.59, size = 363, normalized size = 2.09

$$x\mathrm{ArcTan}(c+d\tanh(a+bx)) + \frac{i\left(-2ia\mathrm{ArcTan}\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right) + (a+bx)\log\left(1-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right) + (a+bx)\log\left(1+\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right) - (a+bx)\log\left(1-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right) - (a+bx)\log\left(1+\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right) + \mathrm{PolyLog}\left(2,-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right) + \mathrm{PolyLog}\left(2,\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{i-c+d}}\right) - \mathrm{PolyLog}\left(2,-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right) - \mathrm{PolyLog}\left(2,\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{-i-c+d}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Tanh[a + b*x]] + ((I/2)*((-2*I)*a*ArcTan[(1 + c^2 - d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + (a + b*x)*Log[1 - (Sqrt[-I
+ c + d]*E^(a + b*x))/Sqrt[I - c + d]] + (a + b*x)*Log[1 + (Sqrt[-I + c + d
]*E^(a + b*x))/Sqrt[I - c + d]] - (a + b*x)*Log[1 - (Sqrt[I + c + d]*E^(a +
 b*x))/Sqrt[-I - c + d]] - (a + b*x)*Log[1 + (Sqrt[I + c + d]*E^(a + b*x))/
Sqrt[-I - c + d]] + PolyLog[2, -((Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[I - c
+ d])] + PolyLog[2, (Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[I - c + d]] - PolyL
og[2, -((Sqrt[I + c + d]*E^(a + b*x))/Sqrt[-I - c + d])] - PolyLog[2, (Sqrt
[I + c + d]*E^(a + b*x))/Sqrt[-I - c + d]]))/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(150) = 300$.
time = 0.51, size = 365, normalized size = 2.10

method	result
derivativedivides	$-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i}{}\right.}{2d}\right.}{}$
default	$-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\frac{i}{}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arctan(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arctan(c+d*t
anh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*d^2*(1/2*I/d*ln(-d*tanh(b*x+a)+d)*ln
((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I/d*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(b*x
+a)-c)/(I-c-d))+1/2*I/d*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I/d*dilog((I
-d*tanh(b*x+a)-c)/(I-c-d))-1/2*I/d*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b*x+a)
+c)/(I+c-d))+1/2*I/d*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d))-1
/2*I/d*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))+1/2*I/d*dilog((I-d*tanh(b*x+a)-c)
/(I-c+d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*a)
 + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x
) + c - d)/(e^(2*b*x + 2*a) + 1))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(128) = 256.
time = 11.05, size = 825, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(
c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d
^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I
*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*
sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)
*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a)
 + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(
c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*
a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
a) + sinh(b*x + a)) + 1) + I*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-sqrt(-(c^2 - d^2
 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I
*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) - I*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(c + d\tanh\left(a + bx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a)),x)
[Out] Integral(atan(c + d*tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arctan(d*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+d\,\mathrm{tanh}\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tanh(a + b*x)),x)
[Out] int(atan(c + d*tanh(a + b*x)), x)
```

3.84 $\int \frac{\text{ArcTan}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\text{ArcTan}(c+d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 7.45, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*tanh(b*x+a))/x,x)
[Out] int(arctan(c+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*tanh(b*x + a) + c)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*tanh(b*x+a))/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atan}(c+d\tanh(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*tanh(a + b*x))/x,x)
[Out] int(atan(c + d*tanh(a + b*x))/x, x)
```

3.85 $\int x^2 \mathbf{ArcTan}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=142

$$-\frac{1}{12}ibx^4+\frac{1}{3}x^3\text{ArcTan}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{ix^2\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\text{Poly}}{}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*tanh(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5303, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c+(c+i)\tanh(a+bx))+\frac{i\text{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (I/6)*x^3*Log
[1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/
b - ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, (-
I)*c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(c+(i+c)\tanh(a+bx))\,dx &= \frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{3}b\int\frac{x^3}{-i+ce^{2a+2bx}}\,dx \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{3}(ibc)\int\frac{e^2}{-i-} \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log(1+ \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log(1+ \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log(1+ \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log(1+ \\ &= -\frac{1}{12}ibx^4+\frac{1}{3}x^3\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log(1+\end{aligned}$$

Mathematica [A]
time = 3.84, size = 128, normalized size = 0.90

$$\frac{1}{3}x^3\text{ArcTan}(c+(i+c)\tanh(a+bx))+\frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-6bx\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)-3\text{PolyLog}\left(4,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*Po
lyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 1479, normalized size = 10.42

method	result	size
risch	Expression too large to display	1479

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*
exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2
```

```
*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c-2*I)-1/12*Pi*x^3*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1
))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*
c-2*I)/(exp(2*b*x+2*a)+1))^2-1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a^3-1/4*I/b
^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)
^(1/2))+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+
I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1
/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(
2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/12*b/(I+c)*x^4-1/12/b^3/(I+c)*a^4+1
/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1
))^3+1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2
*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*
I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-
1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/4
*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a)
)/b^3+1/6*Pi*x^3-1/12*I*b*c/(I+c)*x^4+1/12*I/b^3*c/(I+c)*a^4+1/2*I/b^2*a^2*
ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*x*a^2+1
/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/12*Pi*x^3*csgn((2*exp(2*b*x
+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2
*I)/(exp(2*b*x+2*a)+1))^3-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*Pi*x^3
*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/12*Pi
*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*
exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x
+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))
```

Maxima [A]
time = 1.25, size = 129, normalized size = 0.91

$$\frac{1}{3}\,x^{3}\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)+\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c-4}-\frac{4\,b^{3}x^{3}\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}(-i\,ce^{(2\,bx+2\,a)})-6\,bx\mathrm{Li}_{3}(-i\,ce^{(2\,bx+2\,a)})+3\,\mathrm{Li}_{4}(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^{4}(-i\,c+1)}\right)b(c+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c - 2)))*b*(c + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

time = 3.72, size = 293, normalized size = 2.06

$$\frac{-i b^4 x^4 + 2 i b^3 x^3 \log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}-i}\right) + 6 i b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) + 6 i b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) + i a^4 - 2 i a^3 \log\left(\frac{2ce^{(bx+a)}+i\sqrt{-4ic}}{2c}\right) - 2 i a^3 \log\left(\frac{2ce^{(bx+a)}-i\sqrt{-4ic}}{2c}\right) - 12 i bx \mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - 12 i bx \mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - 2\left(-i b^3 x^3 - i a^3\right)\log\left(\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)} + 1\right) - 2\left(-i b^3 x^3 - i a^3\right)\log\left(-\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)} + 1\right) + 12 i \mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) + 12 i \mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right)}{12 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(-
4*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b
*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*polylog
(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\tanh(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.86 $\int x\mathbf{ArcTan}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=113

$$-\frac{1}{6}ibx^3+\frac{1}{2}x^2\text{ArcTan}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{i\text{PolyLog}(}{}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*tanh(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5303, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c+(c+i)\tanh(a+bx))-\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (I/4)*x^2*Log[
1 + I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b -
 ((I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x\tan^{-1}(c+(i+c)\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))-\frac{1}{2}b\int\frac{x^2}{-i+ce^{2a+2bx}}\,dx\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+}}{-i+c}\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ic\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ic\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ic\right.\\
&= -\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\tanh(a+bx))+\frac{1}{4}ix^2\log\left(1+ic\right.
\end{aligned}
$$

Mathematica [A]
time = 3.88, size = 102, normalized size = 0.90

$$
\frac{1}{2}x^2\mathrm{ArcTan}(c+(i+c)\tanh(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\mathrm{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}
$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]

[Out] (x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(c*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I/(c*E^(2*(a + b*x)))]))/b^2

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 1443, normalized size = 12.77

method	result	size
risch	Expression too large to display	1443

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)

[Out] 1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*x*a-1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b*a*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)+1/4*Pi*x^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,-I*c*exp(2*b*x+2*a))*a-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))+1/6*b/(I+c)*x^3+1/6/b^2/(I+c)*a^3+1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2-1/4*I*x^2*ln(2*exp(2*b

```
*x+2*a)*c-2*I)+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/8*Pi*x
^2*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/6*I
*b*c/(I+c)*x^3-1/6*I/b^2*c/(I+c)*a^3
```

Maxima [A]
time = 1.18, size = 106, normalized size = 0.94

$$\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(
-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2))
)*b*(c + I) + 1/2*x^2*arctan((c + I)*tanh(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.
time = 3.47, size = 247, normalized size = 2.19

$$\frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^
2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*
sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/
b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.87 $\int \textbf{ArcTan}(c+(i+c)\tanh(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}ibx^2+x\text{ArcTan}(c+(i+c)\tanh(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*tanh(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5295, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c+(c+i)\tanh(a+bx))+\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcTan[c + (I + c)*Tanh[a + b*x]] + (I/2)*x*Log[1 + I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5295

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\tanh(a+bx))\,dx = x\tan^{-1}(c+(i+c)\tanh(a+bx)) - b\int \frac{x}{-i+ce^{2a+2bx}}\,dx$$

$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{-i+ce^{2a+2bx}}$$

$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)$$

$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)$$

$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)$$

Mathematica [**A**]
time = 1.11, size = 71, normalized size = 0.90

$$x\text{ArcTan}(c+(i+c)\tanh(a+bx)) + \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(65) = 130$.
time = 0.29, size = 598, normalized size = 7.57

method	result

derivativedivides	$\frac{-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\arctan(c+(\cdots}{\cdots}}{\cdots}$
default	$\frac{-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\frac{\arctan(c+(\cdots}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(-arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))
+2*I*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+arct
an(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arctan(c+(I
+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arctan(c+(I+c)*tan
h(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arctan(c+(I+c)*tanh(b*x+a))
/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2-(I+c)^2*(1/8*I/(I+c)*ln(I+c+(I+c)*
tanh(b*x+a))^2-1/4*I/(I+c)*ln(I+c+(I+c)*tanh(b*x+a))*ln(-1/2*I*(I-c-(I+c)*t
anh(b*x+a)))+1/4*I/(I+c)*ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))*ln(-1/2*I*(I-c-
(I+c)*tanh(b*x+a)))+1/4*I/(I+c)*dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a)))-1/4*I
/(I+c)*ln(c-(I+c)*tanh(b*x+a)+I)*ln((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))-1/
4*I/(I+c)*dilog((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))+1/4*I/(I+c)*ln(c-(I+c)
*tanh(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*tanh(b*x+a))/c)+1/4*I/(I+c)*dilog(-1/2*(
I-c-(I+c)*tanh(b*x+a))/c)))
```

Maxima [A]
time = 1.18, size = 80, normalized size = 1.01

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dilo
g(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*tanh(b*x + a
) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.
time = 3.49, size = 187, normalized size = 2.37

$$\frac{-i\,b^2x^2+i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\sqrt{-4i\,c}}{2\,c}\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\sqrt{-4i\,c}}{2\,c}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x +
 I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c) +
 I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x +
 a)))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(c+\tanh\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c + 1i)),x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.88 $\int \frac{\text{ArcTan}(c+(i+c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\text{ArcTan}(c+(i+c)\tanh(a+bx))}{x},x\right)$$

`[Out] CannotIntegrate(arctan(c+(I+c)*tanh(b*x+a))/x,x)`

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

`[In] Int[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]`

`[Out] Defer[Int][ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]`

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(i+c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]`

`[Out] Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]`

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+(i+c)\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*log(x) + 1/2*inte
grate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x +
4*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atan}(c + \tanh(a + b\,x)\,\,(c + 1\mathrm{i}))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c + 1i))/x,x)
[Out] int(atan(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.89 $\int x^2\textbf{ArcTan}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=145

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\mathrm{ArcTan}(c-(i-c)\tanh(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{ix^2\mathrm{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\mathrm{PolyLog}(}{4}$$

[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*tanh(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3

Rubi [A]
time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5303, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\mathrm{ArcTan}(c-(-c+i)\tanh(a+bx))-\frac{i\mathrm{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}+\frac{ix\mathrm{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\mathrm{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]

[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (I/6)*x^3*Log[1 - I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, I*c*E^(2*a + 2*b*x)])/b^3

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}b\int \frac{x^3}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+2b}}{i+ce^{2a}}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{6}ix^3\log\left(1-ice\right.$$

Mathematica [A]
time = 4.08, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\text{ArcTan}(c+(-i+c)\tanh(a+bx)) - \frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Tanh[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 + I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*
x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x
)))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 1486, normalized size = 10.25

method	result	size
risch	Expression too large to display	1486

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*csgn((2*exp(2
*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*ex
```

```
p(2*b*x+2*a)*c)-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3-1/12*Pi*x^3*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+1/12*I/b^3*c*a^4/(I-c)-1/12*
I*b*c/(I-c)*x^4-1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))+1/12*Pi*x^3*csgn((-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn((2
*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/2*I/b^3*a^2*dilog(1-I*exp(b*
x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/3*I/b
^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2
-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*exp(b*x
+a)*(-I*c)^(1/2))+1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/6*Pi*x^3+1/6*
I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)+1/12/b^3/(I-c)*a^4-1/12*b/(I-c)*x^4-1/2*I
/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*
(-I*c)^(1/2))*x+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*x*a^2+1/12*Pi*x^3*csgn((
-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^3+1/12*Pi*x^3*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*
x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b
*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)+1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*
a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn
(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*
exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/12*Pi*x^3*csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))+1/6*I/b^3*a^3*ln(exp(2
*b*x+2*a)*c+I)-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)
/(exp(2*b*x+2*a)+1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x
+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))
```

Maxima [A]
time = 1.16, size = 129, normalized size = 0.89

$$\frac{1}{3}\,x^{3}\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^{4}}{4i\,c+4}-\frac{4\,b^{3}x^{3}\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_{4}\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^{4}(-i\,c-1)}\right)b(c-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

time = 2.84, size = 293, normalized size = 2.02

$$\frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}+i\right)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right) - i\,a^4 + 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}+i\sqrt{4i\,c}}{2\,c}\right) + 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}-i\sqrt{4i\,c}}{2\,c}\right) + 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right) + 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right) - 2\left(i\,b^3x^3+i\,a^3\right)\log\left(\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}+1\right) - 2\left(i\,b^3x^3+i\,a^3\right)\log\left(-\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}+1\right) - 12i\,\mathrm{polylog}\left(4,\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right) - 12i\,\mathrm{polylog}\left(4,-\frac{1}{2}\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*I*c)*e
^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) +
1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4, -1/2*sq
rt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}(c+\tanh(a+b\,x)\,(c-\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.90 $\int x \textbf{ArcTan}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=116

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\text{ArcTan}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ix\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(3,\right.}{8b}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*tanh(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2*b
*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5303, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c-(-c+i)\tanh(a+bx))+\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (I/4)*x^2*Log[1 -
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b + ((I/8
)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5303

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c-(i-c)\tanh(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}b\int\frac{x^2}{i+ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}x}{i+ce^{2a+2}}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a-}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a-}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a-}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a-}\right.$$

Mathematica [A]

time = 3.98, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\text{ArcTan}(c+(-i+c)\tanh(a+bx))-\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Tanh[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3
, (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 1450, normalized size = 12.50

method	result	size
risch	Expression too large to display	1450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a
)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+
1))-1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/6*b/(I-c)*x^3-1/6/b^2/(I-c)*a^3-
1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(
2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-1/4*I*x^2*ln(1-I*c*exp(2*
b*x+2*a))+1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2
*b*x+2*a)+1))^3+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)+1))^3+1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/8*Pi*x^2*csgn
(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a
)+1))^2-1/4*Pi*x^2-1/6*I*b*c/(I-c)*x^3-1/6*I/b^2*c/(I-c)*a^3-1/4*I/b^2*ln(1
-I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*polylog(2,I*c*exp(2*b*x+2*a))*a+1/2*I/b^
2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c
)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*a*dilog(1
+I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))*x*a+1/2*I/b*a*
ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))
*x-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I
*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+1/8*Pi*x^2*csgn(I
*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(e
xp(2*b*x+2*a)+1))^2-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)+1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)+1))-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4
*I*x^2*ln(-2*exp(2*b*x+2*a)*c-2*I)+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2
*a)+1))+1/8*Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x
```

```
+2*a)+1))^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+
1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3-1/8*Pi*x^2*c
sgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3
```

Maxima [A]
time = 1.17, size = 107, normalized size = 0.92

$$-\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^2\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2))
)*b*(c - I) + 1/2*x^2*arctan((c - I)*tanh(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.
time = 2.13, size = 247, normalized size = 2.13

$$\frac{2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) -
3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*x^2 -
 I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sqrt(4*I
*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atan}(c + \tanh(a + b\,x)\,\,(c - \mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.91 $\int \text{ArcTan}(c-(i-c)\tanh(a+bx))\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}ibx^2+x\text{ArcTan}(c-(i-c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*tanh(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5295, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c-(-c+i)\tanh(a+bx))-\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Tanh[a + b*x]] - (I/2)*x*Log[1 - I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5295

```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c-(i-c)\tanh(a+bx))\,dx = x\tan^{-1}(c-(i-c)\tanh(a+bx)) - b\int \frac{x}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{i+ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\tanh(a+bx)) - \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)$$

Mathematica [A]
time = 1.10, size = 71, normalized size = 0.87

$$x\text{ArcTan}(c+(-i+c)\tanh(a+bx)) - \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(68) = 136.
time = 0.29, size = 553, normalized size = 6.74

method	result

derivativedivides	$-\frac{\arctan((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)}{2i-2c}-\frac{2i\arctan((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)c}{2i-2c}+\frac{\arctan((c-i)}{}$
default	$-\frac{\arctan((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)}{2i-2c}-\frac{2i\arctan((c-i)\tanh(bx+a)+c)\ln((c-i)\tanh(bx+a)-c+i)c}{2i-2c}+\frac{\arctan((c-i)}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)
-2*I*arctan((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c+arct
an((c-I)*tanh(b*x+a)+c)/(2*I-2*c)*ln((c-I)*tanh(b*x+a)-c+I)*c^2+arctan((c-I
)*tanh(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)+2*I*arctan((c-I)*tanh
(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)*c-arctan((c-I)*tanh(b*x+a)+
c)/(2*I-2*c)*ln(-I+(c-I)*tanh(b*x+a)+c)*c^2+(I-c)^2*(-1/8*I/(I-c)*ln(-I+(c-
I)*tanh(b*x+a)+c)^2+1/4*I/(I-c)*ln(-I+(c-I)*tanh(b*x+a)+c)*ln(-1/2*I*((c-I)
*tanh(b*x+a)+c+I))+1/4*I/(I-c)*dilog(-1/2*I*((c-I)*tanh(b*x+a)+c+I))-1/4*I/
(I-c)*ln((c-I)*tanh(b*x+a)-c+I)*ln(1/2*((c-I)*tanh(b*x+a)+c+I)/c)-1/4*I/(I-
c)*dilog(1/2*((c-I)*tanh(b*x+a)+c+I)/c)+1/4*I/(I-c)*ln((c-I)*tanh(b*x+a)-c+
I)*ln((-I+(c-I)*tanh(b*x+a)+c)/(-2*I+2*c))+1/4*I/(I-c)*dilog((-I+(c-I)*tanh
(b*x+a)+c)/(-2*I+2*c))))
```

Maxima [A]
time = 1.16, size = 80, normalized size = 0.98

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*tanh(b*x +
a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.
time = 3.17, size = 187, normalized size = 2.28

$$\frac{i\,b^2x^2+i\,bx\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*x
- I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) - I
*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a))
)/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + \tanh(a + b\,x)\,(c - \mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i)),x)
```

```
[Out] int(atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.92 $\int \frac{\text{ArcTan}(c-(i-c)\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\text{ArcTan}(c-(i-c)\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c-(i-c)\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*
log(x) + 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(lo
g(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*tanh(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + tanh(a + b*x)*(c - 1i))/x,x)
[Out] int(atan(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.93 $\int (e+fx)^3 \textbf{ArcTan}(\coth(a+bx))\,dx$

Optimal. Leaf size=299

$$\frac{(e+fx)^4\text{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\text{ArcTan}(\coth(a+bx))}{4f}-\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,I*
exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8*I*
f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4,-I*
exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+3/16*
I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b*x+2*a
))/b^4
```

Rubi [A]
time = 0.15, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5293, 4265, 2611, 6744, 2320, 6724}

$$\frac{(e+fx)^4\text{ArcTan}\left(e^{2a+2bx}\right)}{4f}+\frac{(e+fx)^4\text{ArcTan}(\coth(a+bx))}{4f}+\frac{3if^3\text{Li}_5\left(-ie^{2a+2bx}\right)}{16b^4}-\frac{3if^3\text{Li}_5\left(ie^{2a+2bx}\right)}{16b^4}-\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{3if(e+fx)^2\text{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}-\frac{3if(e+fx)^2\text{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcTan[E^(2*a + 2*b*x)])/(4*f) + ((e + f*x)^4*ArcTan[Coth[a +
b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^3*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (((3*I)/8)*f*(e + f*x)^2
*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f*(e + f*x)^2*PolyLog[3
, I*E^(2*a + 2*b*x)])/b^2 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, (-I)*E^(2*a
 + 2*b*x)])/b^3 + (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/b
^3 + (((3*I)/16)*f^3*PolyLog[5, (-I)*E^(2*a + 2*b*x)])/b^4 - (((3*I)/16)*f^
3*PolyLog[5, I*E^(2*a + 2*b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} + \frac{b\int (e+fx)^4 \mathrm{sech}(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{1}{2}i\int (\ldots$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \tan^{-1}\left(e^{2a+2bx}\right)}{4f} + \frac{(e+fx)^4 \tan^{-1}(\coth(a+bx))}{4f} - \frac{i(e+\ldots}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. $2(299) = 598$.
time = 3.25, size = 600, normalized size = 2.01

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Coth[a + b*x]])/4 + (
(I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 - I
*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f^3*
x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))] - 1
2*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I*E^(2
*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e + f*x)^3
*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E^(2*(a
+ b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*f^2*x*Po
lyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(2*(a + b*
x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3
, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*e*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I)*E^(2*(a +
b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*PolyLog[4, I*E
^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*f^3*PolyLog[5,
 I*E^(2*(a + b*x))]))/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.61, size = 7275, normalized size = 24.33

method	result	size
risch	Expression too large to display	7275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*arctan2(e^(2*b*x + 2*a)
 + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*f^2*x^3
*e^(2*a + 1) + 6*b*f*x^2*e^(2*a + 2) + 4*b*x*e^(2*a + 3))*e^(2*b*x)/(e^(4*b
*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(244) = 488.
time = 4.51, size = 2700, normalized size = 9.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*
f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^3*p
olylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*x^4
 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^
4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3
+ 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*arctan(cosh(b*x + a)/si
nh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*co
sh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*s
inh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*d
ilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3
*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*si
```

```
nh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*
f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x
*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1)
)*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1)
)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3
+ 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3
*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b
^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x +
a) + sinh(b*x + a))) + (I*b^4*f^3*x^4 - I*a^4*f^3 + 4*I*(b^4*x + a*b^3)*cos
h(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4
*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^
4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 - I*a^4*f
^3 + 4*I*(b^4*x + a*b^3)*cosh(1)^3 + 4*I*(b^4*x + a*b^3)*sinh(1)^3 + 6*I*(b
^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a
*b^3)*cosh(1))*sinh(1)^2 + 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*I*(b^4
*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2
*f)*cosh(1))*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) +
1) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x + a*b^3)*cosh(1)^3 - 4*I*(b^4
*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 - 6*I*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 - 4*I*(b^4*f^2*x^3 +
 a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(1/2*sqrt(-4*I)*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 + I*a^4*f^3 - 4*I*(b^4*x
 + a*b^3)*cosh(1)^3 - 4*I*(b^4*x + a*b^3)*sinh(1)^3 - 6*I*(b^4*f*x^2 - a^2*
b^2*f)*cosh(1)^2 - 6*I*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*
sinh(1)^2 - 4*I*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) - 4*I*(b^4*f^2*x^3 + a^3*
b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*si
nh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*a^4*f^
3 - 4*I*a^3*b*f^2*cosh(1) + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^3 -
 4*I*a*b^3*sinh(1)^3 + 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I*(a
^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(I*sqrt(4*I
) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^4*f^3 - 4*I*a^3*b*f^2*cosh(1)
 + 6*I*a^2*b^2*f*cosh(1)^2 - 4*I*a*b^3*cosh(1)^3 - 4*I*a*b^3*sinh(1)^3 + 6*
I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*co
sh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*
sinh(b*x + a)) + (-I*a^4*f^3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1
)^2 + 4*I*a*b^3*cosh(1)^3 + 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*
cosh(1))*sinh(1)^2 + 4*I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)
^2)*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^
4*f^3 + 4*I*a^3*b*f^2*cosh(1) - 6*I*a^2*b^2*f*cosh(1)^2 + 4*I*a*b^3*cosh(1)
^3 + 4*I*a*b^3*sinh(1)^3 - 6*I*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 + 4*
I*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-I*sqr
```

```
t(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 24*(-I*b*f^3*x - I*b*f^2*cos
h(1) - I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 24*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*(I*b*f^3*x + I*b*f^2*cosh
(1) + I*b*f^2*sinh(1))*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 24*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, -1/2
*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*
f^2*x*cosh(1) + I*b^2*f*cosh(1)^2 + I*b^2*f*sinh(1)^2 + 2*I*(b^2*f^2*x + b^
2*f*cosh(1))*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x +
a))) - 12*(I*b^2*f^3*x^2 + 2*I*b^2*f^2*x*cosh(1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{atan}(\coth(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atan(coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(\coth(a+b\,x))\,{(e+f\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x)^3,x)
```

```
[Out] int(atan(coth(a + b*x))*(e + f*x)^3, x)
```

3.94 $\int (e+fx)^2 \mathbf{ArcTan}(\coth(a+bx))\,dx$

Optimal. Leaf size=229

$$\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\text{ArcTan}(\coth(a+bx))}{3f}-\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,I*
exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I*f*
(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*
a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5293, 4265, 2611, 6744, 2320, 6724}

$$\frac{(e+fx)^3\text{ArcTan}\left(e^{2a+2bx}\right)}{3f}+\frac{(e+fx)^3\text{ArcTan}(\coth(a+bx))}{3f}-\frac{if^2\text{Li}_4\left(-ie^{2a+2bx}\right)}{8b^3}+\frac{if^2\text{Li}_4\left(ie^{2a+2bx}\right)}{8b^3}+\frac{if(e+fx)\text{Li}_3\left(-ie^{2a+2bx}\right)}{4b^2}-\frac{if(e+fx)\text{Li}_3\left(ie^{2a+2bx}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)^2\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcTan[E^(2*a + 2*b*x)])/(3*f) + ((e + f*x)^3*ArcTan[Coth[a +
b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I
/4)*(e + f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/4)*f*(e + f*x)*PolyL
og[3, (-I)*E^(2*a + 2*b*x)])/b^2 - ((I/4)*f*(e + f*x)*PolyLog[3, I*E^(2*a +
 2*b*x)])/b^2 - ((I/8)*f^2*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/b^3 + ((I/8)*f
^2*PolyLog[4, I*E^(2*a + 2*b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \tan^{-1}(\coth(a+bx))\,dx = \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} + \frac{b\int (e+fx)^3 \text{sech}(2a+2bx)\,dx}{3f}$$

$$= \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} - \frac{1}{2}i\int (\ldots$$

$$= \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+\ldots}{\ldots}$$

$$= \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+\ldots}{\ldots}$$

$$= \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+\ldots}{\ldots}$$

$$= \frac{(e+fx)^3 \tan^{-1}\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \tan^{-1}(\coth(a+bx))}{3f} - \frac{i(e+\ldots}{\ldots}$$

Mathematica [A]
time = 1.71, size = 375, normalized size = 1.64

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\text{ArcTan}(\coth(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(1+ie^{2(a+bx)}\right)-12b^3efx^2\log\left(1+ie^{2(a+bx)}\right)-4b^3f^2x^3\log\left(1+ie^{2(a+bx)}\right)-6b^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+6b^2(e+fx)^2\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+6bef\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)+6bf^2x\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-6bef\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-6bf^2x\text{PolyLog}\left(3,ie^{2(a+bx)}\right)-3f^2\text{PolyLog}\left(4,-ie^{2(a+bx)}\right)+3f^2\text{PolyLog}\left(4,ie^{2(a+bx)}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Coth[a + b*x]])/3 + ((I/24)*(12*b^3*e
^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x))]
 + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(2*(
a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*Log[
1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b*x))]
 + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[3, (-I
)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*b*e*f*P
olyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x))] - 3*
f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a + b*x))])
)/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.80, size = 5425, normalized size = 23.69

method	result	size
risch	Expression too large to display	5425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x +
 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*f*x^2*e^(2*a + 1) + 3*b
*x*e^(2*a + 2))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1511 vs. 2(186) = 372.
time = 5.94, size = 1511, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*
polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*polylo
g(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3 + 3*
b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 +
2*b^3*x*cosh(1))*sinh(1))*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I*b^2*f
^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
+ a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 - I*b^2*
sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh
(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^
2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^
2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh
(1))*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (I*b
^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1)^2 + 3*I*(b^3*x + a*b^2
)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*I*(b^3*f*x^2 - a^2*b*f
+ 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + I*a^3*f^2 + 3*I*(b^3*x + a*b^2)*cosh(1
)^2 + 3*I*(b^3*x + a*b^2)*sinh(1)^2 + 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3
*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt
```

```
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - I*a^3*f^2 -
3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b^2)*sinh(1)^2 - 3*I*(b^3*f*
x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(
1))*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*
b^3*f^2*x^3 - I*a^3*f^2 - 3*I*(b^3*x + a*b^2)*cosh(1)^2 - 3*I*(b^3*x + a*b^
2)*sinh(1)^2 - 3*I*(b^3*f*x^2 - a^2*b*f)*cosh(1) - 3*I*(b^3*f*x^2 - a^2*b*f
 + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) +
 sinh(b*x + a)) + 1) + (-I*a^3*f^2 + 3*I*a^2*b*f*cosh(1) - 3*I*a*b^2*cosh(1
)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*
sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^3*f^2 + 3*I*a^2*b*f*
cosh(1) - 3*I*a*b^2*cosh(1)^2 - 3*I*a*b^2*sinh(1)^2 + 3*I*(a^2*b*f - 2*a*b^
2*cosh(1))*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
 (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cosh(1)^2 + 3*I*a*b^2*sinh(1)
^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x
 + a) + 2*sinh(b*x + a)) + (I*a^3*f^2 - 3*I*a^2*b*f*cosh(1) + 3*I*a*b^2*cos
h(1)^2 + 3*I*a*b^2*sinh(1)^2 - 3*I*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 6*(I*b*f^2*x + I*b*f*
cosh(1) + I*b*f*sinh(1))*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x
 + a))) - 6*(I*b*f^2*x + I*b*f*cosh(1) + I*b*f*sinh(1))*polylog(3, -1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*(-I*b*f^2*x - I*b*f*cosh(1) - I
*b*f*sinh(1))*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
6*(-I*b*f^2*x - I*b*f*cosh(1) - I*b*f*sinh(1))*polylog(3, -1/2*sqrt(-4*I)*(
cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^2 \operatorname{atan}(\coth(a+bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atan(coth(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atan(coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atan}(\coth(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atan(coth(a + b*x))*(e + f*x)^2, x)
```

3.95 $\int (e+fx)\mathbf{ArcTan}(\coth(a+bx))\,dx$

Optimal. Leaf size=159

$$\frac{(e+fx)^2\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\mathrm{ArcTan}(\coth(a+bx))}{2f}-\frac{i(e+fx)\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\mathrm{P}}{}$$

```
[Out] 1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(coth(b*x+a))/f-
1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*exp(
2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3,I*
exp(2*b*x+2*a))/b^2
```

Rubi [A]

time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5293, 4265, 2611, 2320, 6724}

$$\frac{(e+fx)^2\mathrm{ArcTan}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\mathrm{ArcTan}(\coth(a+bx))}{2f}+\frac{if\mathrm{Li}_3\left(-ie^{2a+2bx}\right)}{8b^2}-\frac{if\mathrm{Li}_3\left(ie^{2a+2bx}\right)}{8b^2}-\frac{i(e+fx)\mathrm{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i(e+fx)\mathrm{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(2*a + 2*b*x)])/(2*f) + ((e + f*x)^2*ArcTan[Coth[a +
b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b + ((I/4
)*(e + f*x)*PolyLog[2, I*E^(2*a + 2*b*x)])/b + ((I/8)*f*PolyLog[3, (-I)*E^(
2*a + 2*b*x)])/b^2 - ((I/8)*f*PolyLog[3, I*E^(2*a + 2*b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5293

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b
, e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (e+fx)\tan^{-1}(\coth(a+bx))\,dx &= \frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f}+\frac{b\int (e+fx)^2\text{sech}(2a+2bx)\,dx}{2f}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f}-\frac{1}{2}i\int (e+\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f}-\frac{i(e+fx}{}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f}-\frac{i(e+fx}{}\\ &= \frac{(e+fx)^2\tan^{-1}\left(e^{2a+2bx}\right)}{2f}+\frac{(e+fx)^2\tan^{-1}(\coth(a+bx))}{2f}-\frac{i(e+fx}{}\end{aligned}$$

Mathematica [A]
time = 1.44, size = 278, normalized size = 1.75

$$ex\text{ArcTan}(\coth(a+bx))+\frac{1}{2}fx^2\text{ArcTan}(\coth(a+bx))+\frac{e\left(-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)\right)}{8b}+\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\text{PolyLog}\left(2,ie^{2(a+bx)}\right)+\text{PolyLog}\left(3,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(3,ie^{2(a+bx)}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

```
[Out] e*x*ArcTan[Coth[a + b*x]] + (f*x^2*ArcTan[Coth[a + b*x]])/2 + (e*(-(((-4*I)
*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x)
```

```
)])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a + Pi + (4*I)*b*x)/4]] - (2*I)*(Poly
Log[2, (-I)*E^(2*(a + b*x))] - PolyLog[2, I*E^(2*(a + b*x))]))))/(8*b) + ((I
/8)*f*(2*b^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a +
 b*x))] - 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*
(a + b*x))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x
))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 2415, normalized size = 15.19

method	result	size
risch	Expression too large to display	2415

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*
b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp
(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/4*Pi*x*
e*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+
I)/(exp(2*b*x+2*a)-1))+1/8*Pi*x^2*f*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*
x+2*a)-1))^3+1/2*I*ln(exp(2*b*x+2*a)-I)*e*x+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*
a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)
-1))-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(
I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(b*
x+a))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I
*e/b*dilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(b*x+a)*(-1)^(3/4)
)+1/4*Pi*x*e*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2
*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-1/4*Pi*x*e*csgn(I
*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)-1))^3+1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e*
a*ln(exp(2*b*x+2*a)+I)-1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I
)^(1/2))-1/2*I*f/b^2*a*(b*x+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I
*f/b^2*a*(b*x+a)*ln(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*(b*x+a)*ln(1-exp
(b*x+a)*(-1)^(3/4))+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*Pi*x*e*csg
n((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*x*e*csgn((1-I)*(exp
(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/2*I*(-1/2*f*x^2-e*x)*ln(exp(2*b*x+2*
a)+I)+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+
2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*
a)-1))-1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)+1/2*I*e/b*a*ln(-exp(2*b*x+2*a)
+I)-1/8*Pi*x^2*f*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi
*x*e*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*
```

```
a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b
*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*
f*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*f*csgn((1-
I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)-1))^3-1/4*I*f/b^2*(b*x+a)^2*ln(1+I*exp(2*b*x+2*a))-1/
4*I*f/b^2*(b*x+a)*polylog(2,-I*exp(2*b*x+2*a))-1/2*I*e/b*(b*x+a)*ln(1+exp(b
*x+a)*(-1)^(3/4))-1/2*I*e/b*(b*x+a)*ln(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2
*a*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-exp(b*x+a)*(-1)^(3/
4))+1/2*I*e/b*(b*x+a)*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*e/b*(b*x
+a)*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/4*I*f/b^2*(b*x+a)^2*ln(1-I*exp
(2*b*x+2*a))+1/4*I*f/b^2*(b*x+a)*polylog(2,I*exp(2*b*x+2*a))-1/2*I/b^2*f*a*
dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+e
xp(b*x+a))/(-I)^(1/2))-1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(
2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I/(exp(2*b*x+2*a)-1))
*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2
*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2
*a)-1))^2+1/8*Pi*f*x^2+1/4*Pi*e*x-1/4*Pi*x*e*csgn(I*(exp(2*b*x+2*a)-I))*csg
n(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-1/8*I*f*polylog(3,I*exp(2*b*x+
2*a))/b^2+1/4*Pi*x*e*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+1/
4*I*ln(exp(2*b*x+2*a)-I)*f*x^2-1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I))*csgn
(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+
2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x*e*csgn(I*
(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-1/4*Pi*
x*e*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1)
)^2+1/4*Pi*x*e*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*
b*x+2*a)-1))^2+1/8*Pi*x^2*f*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(f*x^2 + 2*x*e)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + int
egrate((b*f*x^2*e^(2*a) + 2*b*x*e^(2*a + 1))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(134) = 268.
time = 2.62, size = 717, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="fricas")

[Out] 1/4*(2*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*arctan(cosh(b*x + a)/sinh(b*x + a)) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b)*sinh(1))*log(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 - I*a^2*f + 2*I*(b^2*x + a*b)*cosh(1) + 2*I*(b^2*x + a*b)*sinh(1))*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*(b^2*x + a*b)*sinh(1))*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 + I*a^2*f - 2*I*(b^2*x + a*b)*cosh(1) - 2*I*(b^2*x + a*b)*sinh(1))*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*a^2*f - 2*I*a*b*cosh(1) - 2*I*a*b*sinh(1))*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (I*a^2*f - 2*I*a*b*cosh(1) - 2*I*a*b*sinh(1))*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^2*f + 2*I*a*b*cosh(1) + 2*I*a*b*sinh(1))*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (-I*a^2*f + 2*I*a*b*cosh(1) + 2*I*a*b*sinh(1))*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*polylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{atan}(\coth(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*atan(coth(b*x+a)),x)

[Out] Integral((e + f*x)*atan(coth(a + b*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="giac")

[Out] sage0*x

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(\coth(a+b\,x))\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))*(e + f*x),x)
```

```
[Out] int(atan(coth(a + b*x))*(e + f*x), x)
```

3.96 $\int \textbf{ArcTan}(\coth(a+bx))\,dx$

Optimal. Leaf size=73

$$x\text{ArcTan}\left(e^{2a+2bx}\right)+x\text{ArcTan}(\coth(a+bx))-\frac{i\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

```
[Out] x*arctan(exp(2*b*x+2*a))+x*arctan(coth(b*x+a))-1/4*I*polylog(2,-I*exp(2*b*x
+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5289, 4265, 2317, 2438}

$$x\text{ArcTan}\left(e^{2a+2bx}\right)+x\text{ArcTan}(\coth(a+bx))-\frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[E^(2*a + 2*b*x)] + x*ArcTan[Coth[a + b*x]] - ((I/4)*PolyLog[2, (-I
)*E^(2*a + 2*b*x)])/b + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5289

```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Coth[a + b
*x]], x] + Dist[b, Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \tan^{-1}(\coth(a+bx))\,dx = x\tan^{-1}(\coth(a+bx)) + b\int x\text{sech}(2a+2bx)\,dx$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{1}{2}i\int \log\left(1-ie^{2a+2bx}\right)\,dx + \frac{1}{2}i\int$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{2a+2bx}\right)}{4b} +$$
$$= x\tan^{-1}\left(e^{2a+2bx}\right) + x\tan^{-1}(\coth(a+bx)) - \frac{i\text{Li}_2\left(-ie^{2a+2bx}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2a+2bx}\right)}{4b}$$

Mathematica [A]
time = 0.04, size = 132, normalized size = 1.81

$$x\text{ArcTan}(\coth(a+bx)) + \frac{-\left((-4ia+\pi-4ibx)\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)\right)+(-4ia+\pi)\log\left(\cot\left(\frac{1}{4}(4ia+\pi+4ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)-\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Coth[a + b*x]],x]
[Out] x*ArcTan[Coth[a + b*x]] + (-(((-4*I)*a + Pi - (4*I)*b*x)*(Log[1 - I*E^(2*(a
 + b*x))] - Log[1 + I*E^(2*(a + b*x))])) + ((-4*I)*a + Pi)*Log[Cot[((4*I)*a
 + Pi + (4*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(2*(a + b*x))] - PolyLog[
2, I*E^(2*(a + b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.
time = 0.24, size = 160, normalized size = 2.19

method	result
derivativedivides	$\frac{\text{arctanh}(\coth(bx+a))\arctan(\coth(bx+a)) - \frac{i\,\text{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\left(\coth^2(bx+a)\right)}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\left(\coth^2(bx+a)\right)}\right)\right)}{2} + \cdots}{b}$
default	$\frac{\text{arctanh}(\coth(bx+a))\arctan(\coth(bx+a)) - \frac{i\,\text{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\left(\coth^2(bx+a)\right)}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\left(\coth^2(bx+a)\right)}\right)\right)}{2} + \cdots}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arctan(coth(b*x+a))-1/2*I*arctanh(coth(b*x+a))*(l
n(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-ln(1+I*(coth(b*x+a)+1)^2/(1-coth
(b*x+a)^2)))+1/4*I*dilog(1+I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-1/4*I*dil
og(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2*
b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(56) = 112.
time = 5.37, size = 334, normalized size = 4.58

$$\frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right)+(i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right)-i\,a\log\left(i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)-i\,a\log\left(-i\sqrt{4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,a\log\left(i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,a\log\left(-i\sqrt{-4i}+2\cosh(bx+a)+2\sinh(bx+a)\right)+i\,{\rm Li}_2\left(\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)+i\,{\rm Li}_2\left(-\frac{1}{2}\sqrt{4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)-i\,{\rm Li}_2\left(\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)-i\,{\rm Li}_2\left(-\frac{1}{2}\sqrt{-4i}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*x + a
) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*a*log
(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) +
 sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}{\left(\coth{\left(a + b x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a)),x)
```

```
[Out] Integral(atan(coth(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(coth(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atan}(\coth(a+b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x)),x)
```

```
[Out] int(atan(coth(a + b*x)), x)
```

3.97 $\int \frac{\mathbf{ArcTan}(\coth(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\text{ArcTan}(\coth(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctan(coth(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(\coth(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[Coth[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx = \int \frac{\tan^{-1}(\coth(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 3.13, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(\coth(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(coth(b*x+a))/(f*x+e),x)
[Out] int(arctan(coth(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctan(coth(b*x + a))/(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(coth(b*x+a))/(f*x+e),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}(\coth(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(coth(a + b*x))/(e + f*x),x)
[Out] int(atan(coth(a + b*x))/(e + f*x), x)
```

3.98 $\int x^2 \textbf{ArcTan}(c + d\coth(a + bx))\, dx$

Optimal. Leaf size=351

$$\frac{1}{3}x^3\text{ArcTan}(c+d\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+ix^2$$

```
[Out] 1/3*x^3*arctan(c+d*coth(b*x+a))+1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I
+c-d))/b-1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*poly
log(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,(I-c-d)*exp(2*b*x
+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^3
```

Rubi [A]
time = 0.34, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5309, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(d\coth(a+bx)+c)+\frac{i\text{Li}_4\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^3}-\frac{i\text{Li}_4\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^3}-\frac{ix\text{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}+\frac{ix\text{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix^2\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{6}ix^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (I/6)*x^3*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/6)*x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c +
d)])/b - ((I/4)*x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/
b - ((I/4)*x*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + (
(I/4)*x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2 + ((I/8)
*PolyLog[4, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^3 - ((I/8)*PolyLo
g[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5309

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-i(c+d)))\int \frac{e^{2a+}}{i+c-d+(-}$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) -$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) -$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) -$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) -$$
$$= \frac{1}{3}x^3\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) -$$

Mathematica [A]
time = 3.29, size = 299, normalized size = 0.85

$$\frac{1}{3}x^3\text{ArcTan}(c+d\coth(a+bx)) + \frac{i\left(4b^3x^3\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{i-c+d}\right) - 4b^3x^3\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{-i-c+d}\right) + 6b^2x^2\text{PolyLog}\left(2,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) - 6bx\text{PolyLog}\left(3,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) + 6bx\text{PolyLog}\left(3,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right) + 3\text{PolyLog}\left(4,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right) - 3\text{PolyLog}\left(4,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + d*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + ((-I + c +
 d)*E^(2*(a + b*x)))/(I - c + d)] - 4*b^3*x^3*Log[1 + ((I + c + d)*E^(2*(a
+ b*x)))/(-I - c + d)] + 6*b^2*x^2*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x))
)/(-I + c - d)] - 6*b^2*x^2*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c
 - d)] - 6*b*x*PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + 6*
b*x*PolyLog[3, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] + 3*PolyLog[4, ((
-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] - 3*PolyLog[4, ((I + c + d)*E^(2
*(a + b*x)))/(I + c - d)]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 16.43, size = 6864, normalized size = 19.56

method	result	size
risch	Expression too large to display	6864

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
 d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. 2(259) = 518.
time = 3.56, size = 1269, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^
2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2
- 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log(2*(c^2 + 2
*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2
*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
 d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d
 + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(
b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d +
 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c
^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x +
a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*polylog(3, -sqrt((c^2
 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1
```

```
))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, -sqrt((c^2 - d^2 -
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + (I*
b^3*x^3 + I*a^3)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*x^3 + I*a^3)*log(-sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (-I*b^3*x^3 - I*a^3)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*x^3 - I*a^3)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + 6*I*polylog(4, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*polylog(4, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 - 6*I*polylog(4, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 6*I*polylog(4, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atan}(c+d\coth(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*atan(c + d*coth(a + b*x)), x)
```

3.99 $\int x\textbf{ArcTan}(c+d\coth(a+bx))\,dx$

Optimal. Leaf size=265

$$\frac{1}{2}x^2\text{ArcTan}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{ix\text{Po}}{}$$

```
[Out] 1/2*x^2*arctan(c+d*coth(b*x+a))+1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+
d))-1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,(I-c-d
)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d
))/b-1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(3,(I
+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

Rubi [A]
time = 0.30, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5309, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(d\coth(a+bx)+c)-\frac{i\text{Li}_3\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{8b^2}+\frac{i\text{Li}_3\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{8b^2}+\frac{ix\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{ix\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{4}ix^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (I/4)*x^2*Log[1 - ((I - c - d)*E^(2*a
 + 2*b*x))/(I - c + d)] - (I/4)*x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d)] + ((I/4)*x*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)
])/b - ((I/4)*x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b -
((I/8)*PolyLog[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b^2 + ((I/8)*
PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5309

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((I - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*
b*((I + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
- d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))-\frac{1}{2}(b(1-i(c+d)))\int\frac{e^{2a+2}}{i+c-d+(-i}\\ &= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\\ &= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\\ &= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\\ &= \frac{1}{2}x^2\tan^{-1}(c+d\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\end{aligned}$$

Mathematica [A]

time = 2.78, size = 225, normalized size = 0.85

$$\frac{1}{2}x^2\text{ArcTan}(c+d\coth(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{(-i+c+d)e^{2(a+bx)}}{i-c+d}\right)-2b^2x^2\log\left(1+\frac{(i+c+d)e^{2(a+bx)}}{-i-c+d}\right)+2bx\text{PolyLog}\left(2,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)-2bx\text{PolyLog}\left(2,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)-\text{PolyLog}\left(3,\frac{(-i+c+d)e^{2(a+bx)}}{-i+c-d}\right)+\text{PolyLog}\left(3,\frac{(i+c+d)e^{2(a+bx)}}{i+c-d}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + d*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + ((-I + c +
d)*E^(2*(a + b*x)))/(I - c + d)] - 2*b^2*x^2*Log[1 + ((I + c + d)*E^(2*(a +
 b*x)))/(-I - c + d)] + 2*b*x*PolyLog[2, ((-I + c + d)*E^(2*(a + b*x)))/(-I
 + c - d)] - 2*b*x*PolyLog[2, ((I + c + d)*E^(2*(a + b*x)))/(I + c - d)] -
PolyLog[3, ((-I + c + d)*E^(2*(a + b*x)))/(-I + c - d)] + PolyLog[3, ((I +
c + d)*E^(2*(a + b*x)))/(I + c - d)]))/b^2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.46, size = 6514, normalized size = 24.58

method	result	size
risch	Expression too large to display	6514

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
- 1) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a
) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2
*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(195) = 390.
time = 5.61, size = 1051, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
```

```
*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(sqrt
((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*
cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 - 2*
I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b
*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c
*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^
2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 -
d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2
 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^
2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (I*
b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2)*log(-sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))
+ 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
 d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 2*I*polylog(3, sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*polylog(3, -sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)))
 + 2*I*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 2*I*polylog(3, -sqrt((c^2 - d^2 - 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+d*coth(b*x+a)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arctan(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atan}(c + d \coth(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + d*coth(a + b*x)),x)
```

```
[Out] int(x*atan(c + d*coth(a + b*x)), x)
```

3.100 $\int \mathbf{ArcTan}(c + d\coth(a + bx))\, dx$

Optimal. Leaf size=174

$$x\text{ArcTan}(c+d\coth(a+bx))+\frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)-\frac{1}{2}ix\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)+\frac{i\text{PolyLo}}{}$$

```
[Out] x*arctan(c+d*coth(b*x+a))+1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2*
I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,(I-c-d)*exp(2*b*x+
2*a)/(I-c+d))/b-1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

Rubi [A]
time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5301, 2221, 2317, 2438}

$$x\text{ArcTan}(d\coth(a+bx)+c)+\frac{i\text{Li}_2\left(\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b}-\frac{i\text{Li}_2\left(\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b}+\frac{1}{2}ix\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)-\frac{1}{2}ix\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + (I/2)*x*Log[1 - ((I - c - d)*E^(2*a + 2*b*x
))/(I - c + d)] - (I/2)*x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
] + ((I/4)*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b - ((I/4
)*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5301

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] + (-Dist[I*b*(I - c - d), Int[x*(E^(2*a + 2*b*
x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[I*b*(I + c + d
), Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x
]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+d\coth(a+bx))\,dx = x\tan^{-1}(c+d\coth(a+bx)) - (b(1-i(c+d)))\int \frac{e^{2a+2bx}x}{i+c-d+(-i-c-d}$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log$$

$$= x\tan^{-1}(c+d\coth(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) - \frac{1}{2}ix\log$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 363 vs. $2(174) = 348$.
time = 2.42, size = 363, normalized size = 2.09

$$x\text{ArcTan}(c+d\coth(a+bx)) + \frac{i\left(2ia\text{ArcTan}\left(\frac{-1-c^2+d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+(a+bx)\log\left(1-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)+(a+bx)\log\left(1+\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)-(a+bx)\log\left(1-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)-(a+bx)\log\left(1+\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)+\text{PolyLog}\left(2,-\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{-i+c+d}\,e^{a+bx}}{\sqrt{-i+c-d}}\right)-\text{PolyLog}\left(2,-\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{i+c+d}\,e^{a+bx}}{\sqrt{i+c-d}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + d*Coth[a + b*x]] + ((I/2)*((2*I)*a*ArcTan[(-1 - c^2 + d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + (a + b*x)*Log[1 - (Sqrt[-I
+ c + d]*E^(a + b*x))/Sqrt[-I + c - d]] + (a + b*x)*Log[1 + (Sqrt[-I + c +
d]*E^(a + b*x))/Sqrt[-I + c - d]] - (a + b*x)*Log[1 - (Sqrt[I + c + d]*E^(a
 + b*x))/Sqrt[I + c - d]] - (a + b*x)*Log[1 + (Sqrt[I + c + d]*E^(a + b*x))
/Sqrt[I + c - d]] + PolyLog[2, -((Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[-I + c
 - d])] + PolyLog[2, (Sqrt[-I + c + d]*E^(a + b*x))/Sqrt[-I + c - d]] - Pol
yLog[2, -((Sqrt[I + c + d]*E^(a + b*x))/Sqrt[I + c - d])] - PolyLog[2, (Sqr
t[I + c + d]*E^(a + b*x))/Sqrt[I + c - d]]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(150) = 300$.
time = 0.53, size = 365, normalized size = 2.10

method	result
derivativedivides	$\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2} - \frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2} + \frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i-}{}\right.}{2d}\right.}{}$
default	$\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2} - \frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2} + \frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i-}{}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arctan(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*arctan(c+d*co
th(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*d^2*(1/2*I/d*ln(-d*coth(b*x+a)+d)*ln(
(I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I/d*ln(-d*coth(b*x+a)+d)*ln((I-d*coth(b*x+
a)-c)/(I-c-d))+1/2*I/d*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I/d*dilog((I-
d*coth(b*x+a)-c)/(I-c-d))-1/2*I/d*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(b*x+a)+
c)/(I+c-d))+1/2*I/d*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+d))-1/
2*I/d*dilog((I+d*coth(b*x+a)+c)/(I+c-d))+1/2*I/d*dilog((I-d*coth(b*x+a)-c)/
(I-c+d))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*d
*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a)
+ e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x)
 - c + d, e^(2*b*x + 2*a) - 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(128) = 256.
time = 3.43, size = 813, normalized size = 4.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) - I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c
^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
 d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d + d^2
+ 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2
 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*
log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sin
h(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(
-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2
*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(
-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) + I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2
+ 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-sqrt((c^2 - d^2 + 2*I*d +
 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(sqr
t((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*
x + a))) - I*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(d*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + d\,\coth(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*coth(a + b*x)),x)
[Out] int(atan(c + d*coth(a + b*x)), x)
```

3.101 $\int \frac{\mathbf{ArcTan}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\text{ArcTan}(c+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 7.59, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+d*coth(b*x+a))/x,x)
[Out] int(arctan(c+d*coth(b*x+a))/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctan(d*coth(b*x + a) + c)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+d*coth(b*x+a))/x,x)
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{atan}(c+d\coth(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + d*coth(a + b*x))/x,x)
[Out] int(atan(c + d*coth(a + b*x))/x, x)
```

3.102 $\int x^2 \mathbf{ArcTan}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=142

$$-\frac{1}{12}ibx^4+\frac{1}{3}x^3\text{ArcTan}(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)+\frac{ix^2\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\text{PolyLog}$$

```
[Out] -1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*coth(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c*e
xp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5305, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\text{ArcTan}(c+(c+i)\coth(a+bx))+\frac{i\text{Li}_4\left(ice^{2a+2bx}\right)}{8b^3}-\frac{ix\text{Li}_3\left(ice^{2a+2bx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right)-\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/12*I)*b*x^4 + (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (I/6)*x^3*Log
[1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x^2*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b -
 ((I/4)*x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2 + ((I/8)*PolyLog[4, I*c*E^(2
*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{3}b\int \frac{x^3}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) - \frac{1}{3}(ibc)\int \frac{e^{2a+}}{-i-}$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log(1-ic$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log(1-ic$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log(1-ic$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log(1-ic$$
$$= -\frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{6}ix^3\log(1-ic$$

Mathematica [A]
time = 1.22, size = 128, normalized size = 0.90

$$\frac{1}{3}x^3\text{ArcTan}(c+(i+c)\coth(a+bx)) + \frac{i\left(4b^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + ((I/24)*(4*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - 6*b*x
*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, (-I)/(c*E^(2*(a + b*x)
))]))/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 1478, normalized size = 10.41

method	result	size
risch	Expression too large to display	1478

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*Pi*x^3*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1)
)^3+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/2*I/b
^2*ln(1-I*c*exp(2*b*x+2*a))*x*a^2+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1
```

```
/2))*x+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/12*Pi*x^3*csgn(I*(
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/6*I*x^3*ln(2
*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a
))/b+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3-1/12*Pi*x^3*csgn((2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I/(exp(2*
b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/12*b/(I+c)*x^4-1/12/
b^3/(I+c)*a^4+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a))-1/12*Pi*x^3*csgn(I/(exp(2*
b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*
I)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I*(2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)
*c)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/
(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(
2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/
12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2
*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))
^2+1/12*Pi*x^3*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*
a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))+1/6
*Pi*x^3-1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/4*I/b^3*polylog(2,I*c*exp(
2*b*x+2*a))*a^2+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^3
*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I*c)^
(1/2))+1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/12*I*b*c/(I+c)*x^
4+1/12*I/b^3*c/(I+c)*a^4-1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/12*Pi*
x^3*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/12*Pi*x^3*csgn(
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/6*I*x^3*ln(2*exp(2*b*x+2*a
)*c+2*I)-1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)
```

Maxima [A]
time = 1.20, size = 129, normalized size = 0.91

$$\frac{1}{3}x^3\arctan\left((c+i)\coth(bx+a)+c\right)+\frac{4}{9}\left(\frac{3x^4}{4ic-4}-\frac{4b^3x^3\log\left(-ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(ice^{(2bx+2a)}\right)+3\,\mathrm{Li}_4\left(ice^{(2bx+2a)}\right)}{-2b^4(-ic+1)}\right)b(c+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c + I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^3
*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a)))
/(b^4*(2*I*c - 2)))*b*(c + I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

time = 2.64, size = 293, normalized size = 2.06

$$\frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) + i\,a^4 - 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right) - 2i\,a^3\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right) - 12i\,bx\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - 12i\,bx\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) - 2\,(-i\,b^3x^3 - i\,a^3)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)} + 1\right) - 2\,(-i\,b^3x^3 - i\,a^3)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)} + 1\right) + 12i\,\mathrm{polylog}\left(4,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right) + 12i\,\mathrm{polylog}\left(4,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2
*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*di
log(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylog(3,
-1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4*I*c)*
e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x + a)
+ 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(4, -1/2*
sqrt(4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}\left(c+\mathrm{coth}\left(a+b\,x\right)\,\left(c+1{}\mathrm{i}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x^2*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.103 $\int x\textbf{ArcTan}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=113

$$-\frac{1}{6}ibx^3+\frac{1}{2}x^2\text{ArcTan}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{ix\text{PolyLog}(2,ice^{2a+2bx})}{4b}-\frac{i\text{PolyLog}(\ldots}{8\ldots}$$

```
[Out] -1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*coth(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(2*
b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5305, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c+(c+i)\coth(a+bx))-\frac{i\text{Li}_3\left(ice^{2a+2bx}\right)}{8b^2}+\frac{ix\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/6*I)*b*x^3 + (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (I/4)*x^2*Log[
1 - I*c*E^(2*a + 2*b*x)] + ((I/4)*x*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b - ((
I/8)*PolyLog[3, I*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c+(i+c)\coth(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{-i-ce^{2a+2bx}}\,dx$$

$$=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))-\frac{1}{2}(ibc)\int\frac{e^{2a+2b}}{-i-ce^{2}}$$

$$=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.$$

$$=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.$$

$$=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.$$

$$=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a}\right.$$

Mathematica [A]
time = 1.17, size = 102, normalized size = 0.90

$$\frac{1}{2}x^2\mathrm{ArcTan}(c+(i+c)\coth(a+bx))+\frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-2bx\mathrm{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-\mathrm{PolyLog}\left(3,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(c*
E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 1442, normalized size = 12.76

method	result	size
risch	Expression too large to display	1442

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+
I)+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/2*I/b*ln(1-I*c*exp(2*b*x+2*a))
*x*a-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x-1/2*I/b*a*ln(1+I*exp(b*x+a
)*(-I*c)^(1/2))*x+1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))+1/8*Pi*x^2*csgn((2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+2*
I)-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2
*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*
I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I
*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a)+2*e
xp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/4*Pi*x^2+1/8*Pi*x^2*csgn(I/(exp(2*
b*x+2*a)-1))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*csgn(I/(exp
(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)-1))-1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x
+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(
exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*
b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/6*b/(I+c)*x^3+1/6/b^2/(I+c)*a^3-1/2
*I/b^2*a^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*
(-I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^2*a*di
log(1+I*exp(b*x+a)*(-I*c)^(1/2))+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*
x+2*a)*c)-1/6*I*b*c/(I+c)*x^3-1/6*I/b^2*c/(I+c)*a^3+1/4*I/b^2*ln(1-I*c*exp(
2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,I*c*exp(2*b*x+2*a))*a-1/8*Pi*x^2*csgn((
2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2-1/8*Pi*x^2*csgn(I*(2*exp(2*b*
x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x^2*csgn((2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+
2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*
a)-1))+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn(
```

```
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*I*exp(2
*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((2*I*exp(2*b*x+2*a)+
2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
```

Maxima [A]
time = 1.20, size = 106, normalized size = 0.94

$$\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)+\frac{1}{2}\,x^2\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] (2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)))
*b*(c + I) + 1/2*x^2*arctan((c + I)*coth(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.
time = 2.99, size = 247, normalized size = 2.19

$$\frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)+3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-3\left(-i\,b^2x^2+i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)-6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b*
x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) +
I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) -
3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^2*x^2
 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*sqrt(4
*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.104 $\int \mathbf{ArcTan}(c+(i+c)\coth(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{1}{2}ibx^2+x\text{ArcTan}(c+(i+c)\coth(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)+\frac{i\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

```
[Out] -1/2*I*b*x^2+x*arctan(c+(I+c)*coth(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5297, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c+(c+i)\coth(a+bx))+\frac{i\text{Li}_2\left(ice^{2a+2bx}\right)}{4b}+\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] (-1/2*I)*b*x^2 + x*ArcTan[c + (I + c)*Coth[a + b*x]] + (I/2)*x*Log[1 - I*c*
E^(2*a + 2*b*x)] + ((I/4)*PolyLog[2, I*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5297

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c+(i+c)\coth(a+bx))\,dx = x\tan^{-1}(c+(i+c)\coth(a+bx)) - b\int \frac{x}{-i-ce^{2a+2bx}}\,dx$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) - (ibc)\int \frac{e^{2a+2bx}x}{-i-ce^{2a+2b}}$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$
$$= -\frac{1}{2}ibx^2 + x\tan^{-1}(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2b}\right.$$

Mathematica [A]
time = 0.43, size = 71, normalized size = 0.90

$$x\text{ArcTan}(c+(i+c)\coth(a+bx)) + \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(65) = 130.
time = 0.31, size = 598, normalized size = 7.57

method	result

derivativedivides	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c)\,c}{}}{}$
default	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c)\,c}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I+c)*(arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)-
2*I*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arcta
n(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2-arctan(c+(I+
c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))+2*I*arctan(c+(I+c)*coth
(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arctan(c+(I+c)*coth(b*x+a))/
(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2-(I+c)^2*(1/8*I/(I+c)*ln(I+c+(I+c)*c
oth(b*x+a))^2+1/4*I/(I+c)*ln(-1/2*I*(I+c+(I+c)*coth(b*x+a)))*ln(-1/2*I*(I-c
-(I+c)*coth(b*x+a)))-1/4*I/(I+c)*ln(-1/2*I*(I-c-(I+c)*coth(b*x+a)))*ln(I+c+
(I+c)*coth(b*x+a))+1/4*I/(I+c)*dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a)))+1/4*I/
(I+c)*ln(-1/2*(I-c-(I+c)*coth(b*x+a))/c)*ln(c-(I+c)*coth(b*x+a)+I)+1/4*I/(I
+c)*dilog(-1/2*(I-c-(I+c)*coth(b*x+a))/c)-1/4*I/(I+c)*ln((-I-c-(I+c)*coth(b
*x+a))/(-2*I-2*c))*ln(c-(I+c)*coth(b*x+a)+I)-1/4*I/(I+c)*dilog((-I-c-(I+c)*
coth(b*x+a))/(-2*I-2*c))))
```

Maxima [A]
time = 1.16, size = 80, normalized size = 1.01

$$2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)+x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + dil
og(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*coth(b*x + a
) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.
time = 2.42, size = 187, normalized size = 2.37

$$\frac{-i\,b^2x^2+i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right)+i\,a^2+(i\,bx+i\,a)\log\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)+(i\,bx+i\,a)\log\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}+1\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{4i\,c}}{2\,c}\right)-i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{4i\,c}}{2\,c}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I
)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x +
I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a) +
 I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) + I*d
ilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)))/
b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + \coth(a + b\,x)\,(c + 1\mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c + 1i)),x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.105 $\int \frac{\text{ArcTan}(c+(i+c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\text{ArcTan}(c+(i+c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\tan^{-1}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c+(i+c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] I*b*x + 1/2*pi*log(x) - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*l
og(x) - 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log
(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c+(I+c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c + I)*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atan}(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c + 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.106 $\int x^2 \textbf{ArcTan}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=145

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\mathrm{ArcTan}(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)-\frac{ix^2\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{ix\mathrm{PolyL}}{}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*coth(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*b
*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*c*
exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5305, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{1}{3}x^3\mathrm{ArcTan}(c-(-c+i)\coth(a+bx))-\frac{i\mathrm{Li}_4\left(-ice^{2a+2bx}\right)}{8b^3}+\frac{ix\mathrm{Li}_3\left(-ice^{2a+2bx}\right)}{4b^2}-\frac{ix^2\mathrm{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (I/6)*x^3*Log[1
+ I*c*E^(2*a + 2*b*x)] - ((I/4)*x^2*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b +
 ((I/4)*x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2 - ((I/8)*PolyLog[4, (-I)*
c*E^(2*a + 2*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tan^{-1}(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) + \frac{1}{3}(ibc)\int \frac{e^{2a+2}}{i-ce^2}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{6}ix^3\log\left(1+ic\right.$$

Mathematica [A]
time = 1.30, size = 128, normalized size = 0.88

$$\frac{1}{3}x^3\text{ArcTan}(c+(-i+c)\coth(a+bx)) - \frac{i\left(4b^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right) - 6bx\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right) - 3\text{PolyLog}\left(4,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTan[c + (-I + c)*Coth[a + b*x]])/3 - ((I/24)*(4*b^3*x^3*Log[1 - I/(
c*E^(2*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - 6*b*x*P
olyLog[3, I/(c*E^(2*(a + b*x)))] - 3*PolyLog[4, I/(c*E^(2*(a + b*x)))]))/b^
3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 1487, normalized size = 10.26

method	result	size
risch	Expression too large to display	1487

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3-1/12*b/(I-c)*x^4+1/12/b^3/(I-c)*a
^4+1/12*Pi*x^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a
)-1))^2+1/12*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/1
```

```
2*Pi*x^3*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/2*I/b^2*ln(1
+I*c*exp(2*b*x+2*a))*x*a^2+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c+2*I)+1/3*I/b^3*
ln(1+I*c*exp(2*b*x+2*a))*a^3+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*a^2-1
/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)
*(I*c)^(1/2))-1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^2*a^2*ln(1-I*exp(b
*x+a)*(I*c)^(1/2))*x-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2
*a)*c)/(exp(2*b*x+2*a)-1))+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2
*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))
+1/12*I/b^3*c*a^4/(I-c)-1/12*I*b*c/(I-c)*x^4-1/2*I/b^3*a^2*dilog(1+I*exp(b*
x+a)*(I*c)^(1/2))-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/12*Pi*x
^3*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3-1/6*
I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I*x*polylog(3,-I*c*exp(
2*b*x+2*a))/b^2-1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)
-1))^3+1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b
*x+2*a)-1))^3-1/6*Pi*x^3-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/4
*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1
/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(
2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I*(-2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(2*b*x+2*a)+2*exp(
2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-1/12*Pi*x^3*csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*csgn(I*(-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b
*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I
))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/6*I/b^3*a^3*ln(-
exp(2*b*x+2*a)*c+I)+1/12*Pi*x^3*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+
2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2-1/12*Pi*x^3*cs
gn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*
I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2
```

Maxima [A]
time = 1.16, size = 129, normalized size = 0.89

$$\frac{1}{3}\,x^3\arctan\left((c-i)\coth\left(bx+a\right)+c\right)-\frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4}-\frac{4\,b^3x^3\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_4\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^4(-i\,c-1)}\right)b(c-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan((c - I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^3
*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2*a)
))/(b^4*(2*I*c + 2)))*b*(c - I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

time = 3.48, size = 293, normalized size = 2.02

$$\frac{i b^4 x^4 + 2i b^3 x^3 \log\left(-\frac{(c e^{(2bx+2a)} - i) e^{(-2bx-2a)}}{c-i}\right) - 6i b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - 6i b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - i a^4 + 2i a^3 \log\left(\frac{2c e^{(bx+a)} + i\sqrt{-4ic}}{2c}\right) + 2i a^3 \log\left(\frac{2c e^{(bx+a)} - i\sqrt{-4ic}}{2c}\right) + 12i bx \mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) + 12i bx \mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - 2(i b^3 x^3 + i a^3)\log\left(\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)} + 1\right) - 2(i b^3 x^3 + i a^3)\log\left(-\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)} + 1\right) - 12i \mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right) - 12i \mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4ic}\, e^{(bx+a)}\right)}{12 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)
/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*d
ilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*polylo
g(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(-4
*I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(b*x
 + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*polylog(4
, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atan}(c + \coth(a + b\,x)\,(c - \mathrm{i}))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x^2*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.107 $\int x\mathbf{ArcTan}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=116

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\text{ArcTan}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\text{PolyLog}(3}{8}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*coth(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b*
x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {5305, 2215, 2221, 2611, 2320, 6724}

$$\frac{1}{2}x^2\text{ArcTan}(c-(-c+i)\coth(a+bx))+\frac{i\text{Li}_3\left(-ice^{2a+2bx}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (I/4)*x^2*Log[1 +
 I*c*E^(2*a + 2*b*x)] - ((I/4)*x*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b + ((
I/8)*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/b^2
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5305

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d
)^2, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}(c-(i-c)\coth(a+bx))\,dx=\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))+\frac{1}{2}(ibc)\int\frac{e^{2a+2bx}}{i-ce^{2a+}}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tan^{-1}(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a}\right.$$

Mathematica [A]
time = 1.17, size = 102, normalized size = 0.88

$$\frac{1}{2}x^2\text{ArcTan}(c+(-i+c)\coth(a+bx))-\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-2bx\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(3,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTan[c + (-I + c)*Coth[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/(c
*E^(2*(a + b*x)))] - 2*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - PolyLog[3, I
/(c*E^(2*(a + b*x)))]))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 1451, normalized size = 12.51

method	result	size
risch	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp
(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*
b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-1/6*I*b*c/(I-c)*x^3-1/6*
I/b^2*c/(I-c)*a^3-1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b*ln(1+I*c*exp(2
*b*x+2*a))*x*a+1/2*I/b*a*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*x-1/8*Pi*x^2*csgn(I
*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2
*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))
^2+1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2-1/8*Pi*x^2*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b
*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+1/8*Pi*x^2*
csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*
c-2*I)/(exp(2*b*x+2*a)-1))-1/4*Pi*x^2-1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)
+1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csg
n(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))+1/2*I/b*a*ln(1-I*exp(b*x+a
)*(I*c)^(1/2))*x-1/8*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c
)/(exp(2*b*x+2*a)-1))-1/4*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2-1/4*I/b^2*poly
log(2,-I*c*exp(2*b*x+2*a))*a+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1
/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^2*a*dilog(1+I*exp(b*x+a
)*(I*c)^(1/2))+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))-1/4*I*x*polylo
g(2,-I*c*exp(2*b*x+2*a))/b-1/6/b^2/(I-c)*a^3-1/6*b/(I-c)*x^3+1/8*Pi*x^2*csg
n(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/8*Pi*x
^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^3+1/8*
Pi*x^2*csgn((-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-
1/8*Pi*x^2*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+1/4*I*x^2*
ln(-2*exp(2*b*x+2*a)*c+2*I)+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)-1))^2+1/8*Pi*x^2*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1)
```

```
)^3-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/8*Pi*x^2*csgn(I*(
-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))*csgn((-2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
```

Maxima [A]
time = 1.18, size = 107, normalized size = 0.92

$$-\left(\frac{2\,x^3}{3i\,c+3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i)+\frac{1}{2}\,x^2\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2)
))*b*(c - I) + 1/2*x^2*arctan((c - I)*coth(b*x + a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.
time = 4.66, size = 247, normalized size = 2.13

$$\frac{2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)+2i\,a^3-6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}+i\,\sqrt{-4i\,c}}{2\,c}\right)-3i\,a^2\log\left(\frac{2\,ce^{(bx+a)}-i\,\sqrt{-4i\,c}}{2\,c}\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)-3\left(i\,b^2x^2-i\,a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+6i\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)+6i\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*
a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b
*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
+ I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c
) - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*
x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sq
rt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^
2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(x*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.108 $\int \textbf{ArcTan}(c-(i-c)\coth(a+bx))\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}ibx^2+x\text{ArcTan}(c-(i-c)\coth(a+bx))-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)-\frac{i\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arctan(c-(I-c)*coth(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))-
1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5297, 2215, 2221, 2317, 2438}

$$x\text{ArcTan}(c-(-c+i)\coth(a+bx))-\frac{i\text{Li}_2\left(-ice^{2a+2bx}\right)}{4b}-\frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTan[c - (I - c)*Coth[a + b*x]] - (I/2)*x*Log[1 + I*c*E^(
2*a + 2*b*x)] - ((I/4)*PolyLog[2, (-I)*c*E^(2*a + 2*b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5297

```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)), x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

Rubi steps

$$\int \tan^{-1}(c-(i-c)\coth(a+bx))\,dx = x\tan^{-1}(c-(i-c)\coth(a+bx)) - b\int \frac{x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) + (ibc)\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) -$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) -$$
$$= \frac{1}{2}ibx^2 + x\tan^{-1}(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) -$$

Mathematica [A]
time = 0.42, size = 71, normalized size = 0.87

$$x\text{ArcTan}(c+(-i+c)\coth(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

```
[Out] x*ArcTan[c + (-I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(68) = 136.
time = 0.30, size = 553, normalized size = 6.74

method	result

derivativedivides	$\frac{-\frac{\arctan((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)}{2i-2c}-\frac{2i\arctan((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)c}{2i-2c}+\frac{\arctan((c-i}{}}{}$
default	$\frac{-\frac{\arctan((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)}{2i-2c}-\frac{2i\arctan((c-i)\coth(bx+a)+c)\ln((c-i)\coth(bx+a)-c+i)c}{2i-2c}+\frac{\arctan((c-i}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(c-I)*(-arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)
-2*I*arctan((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c+arct
an((c-I)*coth(b*x+a)+c)/(2*I-2*c)*ln((c-I)*coth(b*x+a)-c+I)*c^2+arctan((c-I
)*coth(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)+2*I*arctan((c-I)*coth
(b*x+a)+c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)*c-arctan((c-I)*coth(b*x+a)+
c)/(2*I-2*c)*ln(-I+(c-I)*coth(b*x+a)+c)*c^2+(I-c)^2*(1/4*I/(I-c)*ln(-1/2*I*
((c-I)*coth(b*x+a)+c+I))*ln(-I+(c-I)*coth(b*x+a)+c)+1/4*I/(I-c)*dilog(-1/2*
I*((c-I)*coth(b*x+a)+c+I))-1/8*I/(I-c)*ln(-I+(c-I)*coth(b*x+a)+c)^2-1/4*I/(
I-c)*ln(1/2*((c-I)*coth(b*x+a)+c+I)/c)*ln((c-I)*coth(b*x+a)-c+I)-1/4*I/(I-c
)*dilog(1/2*((c-I)*coth(b*x+a)+c+I)/c)+1/4*I/(I-c)*ln((-I+(c-I)*coth(b*x+a)
+c)/(-2*I+2*c))*ln((c-I)*coth(b*x+a)-c+I)+1/4*I/(I-c)*dilog((-I+(c-I)*coth(
b*x+a)+c)/(-2*I+2*c))))
```

Maxima [A]
time = 1.16, size = 80, normalized size = 0.98

$$-2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right)+x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*coth(b*x +
a) + c)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.
time = 3.39, size = 187, normalized size = 2.28

$$\frac{i\,b^2x^2+i\,bx\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}+1\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}+i\sqrt{-4i\,c}}{2\,c}\right)+i\,a\log\left(\frac{2\,ce^{(bx+a)}-i\sqrt{-4i\,c}}{2\,c}\right)-i\,\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)-i\,\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I
)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b*x
 - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/c)
 - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e^(b*x
 + a)))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: CoercionFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a)),x)
```

```
[Out] Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of t
ype <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}(c + \coth(a + b\,x)\,\left(c - \mathrm{i}\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c - 1i)),x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.109 $\int \frac{\text{ArcTan}(c-(i-c)\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\text{ArcTan}(c-(i-c)\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{ArcTan}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\tan^{-1}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.79, size = 0, normalized size = 0.00

$$\int \frac{\text{ArcTan}(c-(i-c)\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*int
egrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x +
 4*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*I*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c-(I-c)*coth(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan((c - I)*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(c + coth(a + b*x)*(c - 1i))/x,x)
```

```
[Out] int(atan(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.110 $\int \mathbf{ArcTan}(e^x)\, dx$

Optimal. Leaf size=31

$$\frac{1}{2}i\text{PolyLog}(2,-ie^x)-\frac{1}{2}i\text{PolyLog}(2,ie^x)$$

[Out] 1/2*I*polylog(2,-I*exp(x))-1/2*I*polylog(2,I*exp(x))

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {2320, 4940, 2438}

$$\frac{1}{2}i\text{Li}_2(-ie^x)-\frac{1}{2}i\text{Li}_2(ie^x)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[E^x],x]

[Out] (I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(e^x)\,dx &= \text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^x\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^x\right)\\ &= \frac{1}{2}i\text{Li}_2(-ie^x) - \frac{1}{2}i\text{Li}_2(ie^x)\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 59, normalized size = 1.90

$$x\text{ArcTan}(e^x) - \frac{1}{2}i(x(\log(1-ie^x) - \log(1+ie^x)) - \text{PolyLog}(2,-ie^x) + \text{PolyLog}(2,ie^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x],x]
```

```
[Out] x*ArcTan[E^x] - (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-I
)*E^x] + PolyLog[2, I*E^x])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.
time = 0.07, size = 59, normalized size = 1.90

method	result	size
derivativedivides	$\ln(\mathrm{e}^x)\arctan(\mathrm{e}^x) + \frac{i\ln(\mathrm{e}^x)\ln(1+i\mathrm{e}^x)}{2} - \frac{i\ln(\mathrm{e}^x)\ln(1-i\mathrm{e}^x)}{2} + \frac{i\,\mathrm{dilog}(1+i\mathrm{e}^x)}{2} - \frac{i\,\mathrm{dilog}(1-i\mathrm{e}^x)}{2}$	59
default	$\ln(\mathrm{e}^x)\arctan(\mathrm{e}^x) + \frac{i\ln(\mathrm{e}^x)\ln(1+i\mathrm{e}^x)}{2} - \frac{i\ln(\mathrm{e}^x)\ln(1-i\mathrm{e}^x)}{2} + \frac{i\,\mathrm{dilog}(1+i\mathrm{e}^x)}{2} - \frac{i\,\mathrm{dilog}(1-i\mathrm{e}^x)}{2}$	59
risch	$-\frac{ix\ln(1+i\mathrm{e}^x)}{2} - \frac{i\ln(-i\mathrm{e}^x)\ln(-i(-\mathrm{e}^x+i))}{2} + \frac{i\ln(-i(-\mathrm{e}^x+i))x}{2} - \frac{i\,\mathrm{dilog}(-i\mathrm{e}^x)}{2} - \frac{i\,\mathrm{dilog}(1-i\mathrm{e}^x)}{2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x))*arctan(exp(x))+1/2*I*ln(exp(x))*ln(1+I*exp(x))-1/2*I*ln(exp(x))*
ln(1-I*exp(x))+1/2*I*dilog(1+I*exp(x))-1/2*I*dilog(1-I*exp(x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.
time = 0.50, size = 34, normalized size = 1.10

$$x\arctan(e^x) - \frac{1}{4}\pi\log\left(e^{(2x)}+1\right) - \frac{1}{2}i\,\text{Li}_2(i\,e^x+1) + \frac{1}{2}i\,\text{Li}_2(-i\,e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(e^x) - 1/4*pi*log(e^(2*x) + 1) - 1/2*I*dilog(I*e^x + 1) + 1/2*I*di
log(-I*e^x + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.
time = 3.34, size = 40, normalized size = 1.29

$$x \arctan\left(e^{x}\right) + \frac{1}{2} i x \log\left(i e^{x} + 1\right) - \frac{1}{2} i x \log\left(-i e^{x} + 1\right) - \frac{1}{2} i \operatorname{Li}_2(i e^{x}) + \frac{1}{2} i \operatorname{Li}_2(-i e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(e^x) + 1/2*I*x*log(I*e^x + 1) - 1/2*I*x*log(-I*e^x + 1) - 1/2*I*di
log(I*e^x) + 1/2*I*dilog(-I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(e^{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x)),x)
```

```
[Out] Integral(atan(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^x), x)
```

Mupad [B]
time = 0.69, size = 21, normalized size = 0.68

$$\frac{\mathrm{polylog}(2, -\mathrm{e}^{x}\, 1\mathrm{i})\ 1\mathrm{i}}{2} - \frac{\mathrm{polylog}(2, \mathrm{e}^{x}\, 1\mathrm{i})\ 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(x)),x)
```

```
[Out] (polylog(2, -exp(x)*1i)*1i)/2 - (polylog(2, exp(x)*1i)*1i)/2
```

3.111 $\int x\mathbf{ArcTan}(e^x)\,dx$

Optimal. Leaf size=63

$$\frac{1}{2}ix\text{PolyLog}(2,-ie^x)-\frac{1}{2}ix\text{PolyLog}(2,ie^x)-\frac{1}{2}i\text{PolyLog}(3,-ie^x)+\frac{1}{2}i\text{PolyLog}(3,ie^x)$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5251, 2611, 2320, 6724}

$$\frac{1}{2}ix\text{Li}_2(-ie^x)-\frac{1}{2}ix\text{Li}_2(ie^x)-\frac{1}{2}i\text{Li}_3(-ie^x)+\frac{1}{2}i\text{Li}_3(ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*x*PolyLog[2, (-I)*E^x] - (I/2)*x*PolyLog[2, I*E^x] - (I/2)*PolyLog[3,
 (-I)*E^x] + (I/2)*PolyLog[3, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(e^x)\,dx &= \frac{1}{2}i\int x\log(1-ie^x)\,dx-\frac{1}{2}i\int x\log(1+ie^x)\,dx\\ &= \frac{1}{2}ix\text{Li}_2(-ie^x)-\frac{1}{2}ix\text{Li}_2(ie^x)-\frac{1}{2}i\int \text{Li}_2(-ie^x)\,dx+\frac{1}{2}i\int \text{Li}_2(ie^x)\,dx\\ &= \frac{1}{2}ix\text{Li}_2(-ie^x)-\frac{1}{2}ix\text{Li}_2(ie^x)-\frac{1}{2}i\text{Subst}\left(\int\frac{\text{Li}_2(-ix)}{x}\,dx,x,e^x\right)+\frac{1}{2}i\text{Subst}\left(\int\frac{\text{Li}_2(ix)}{x}\right.\\ &= \frac{1}{2}ix\text{Li}_2(-ie^x)-\frac{1}{2}ix\text{Li}_2(ie^x)-\frac{1}{2}i\text{Li}_3(-ie^x)+\frac{1}{2}i\text{Li}_3(ie^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.79

$$\frac{1}{2}i(x\text{PolyLog}(2,-ie^x)-x\text{PolyLog}(2,ie^x)-\text{PolyLog}(3,-ie^x)+\text{PolyLog}(3,ie^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^x],x]
```

```
[Out] (I/2)*(x*PolyLog[2, (-I)*E^x] - x*PolyLog[2, I*E^x] - PolyLog[3, (-I)*E^x]
+ PolyLog[3, I*E^x])
```

Maple [A]
time = 0.03, size = 44, normalized size = 0.70

method	result	size
risch	$\frac{ix\,\text{polylog}(2,-ie^x)}{2}-\frac{ix\,\text{polylog}(2,ie^x)}{2}-\frac{i\,\text{polylog}(3,-ie^x)}{2}+\frac{i\,\text{polylog}(3,ie^x)}{2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-I
*exp(x))+1/2*I*polylog(3,I*exp(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(e^x) - integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

Fricas [A]
time = 2.60, size = 65, normalized size = 1.03

$$\frac{1}{2}x^2\arctan\left(e^x\right)+\frac{1}{4}i\,x^2\log\left(i\,e^x+1\right)-\frac{1}{4}i\,x^2\log\left(-i\,e^x+1\right)-\frac{1}{2}i\,x\mathrm{Li}_2(i\,e^x)+\frac{1}{2}i\,x\mathrm{Li}_2(-i\,e^x)+\frac{1}{2}i\operatorname{polylog}(3,i\,e^x)-\frac{1}{2}i\operatorname{polylog}(3,-i\,e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="fricas")
[Out] 1/2*x^2*arctan(e^x) + 1/4*I*x^2*log(I*e^x + 1) - 1/4*I*x^2*log(-I*e^x + 1)
- 1/2*I*x*dilog(I*e^x) + 1/2*I*x*dilog(-I*e^x) + 1/2*I*polylog(3, I*e^x) -
1/2*I*polylog(3, -I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atan}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(x)),x)
[Out] Integral(x*atan(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(x)),x, algorithm="giac")
[Out] integrate(x*arctan(e^x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atan}\left(\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(exp(x)),x)
[Out] int(x*atan(exp(x)), x)
```

3.112 $\int x^2 \mathbf{ArcTan}(e^x)\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}ix^2\text{PolyLog}(2,-ie^x)-\frac{1}{2}ix^2\text{PolyLog}(2,ie^x)-ix\text{PolyLog}(3,-ie^x)+ix\text{PolyLog}(3,ie^x)+i\text{PolyLog}(4,-ie^x)-i\text{P}$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5251, 2611, 6744, 2320, 6724}

$$\frac{1}{2}ix^2\text{Li}_2(-ie^x)-\frac{1}{2}ix^2\text{Li}_2(ie^x)-ix\text{Li}_3(-ie^x)+ix\text{Li}_3(ie^x)+i\text{Li}_4(-ie^x)-i\text{Li}_4(ie^x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^x],x]
```

```
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

```
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \tan^{-1}(e^x)\,dx &= \frac{1}{2}i\int x^2\log(1-ie^x)\,dx-\frac{1}{2}i\int x^2\log(1+ie^x)\,dx\\ &= \frac{1}{2}ix^2\text{Li}_2(-ie^x)-\frac{1}{2}ix^2\text{Li}_2(ie^x)-i\int x\text{Li}_2(-ie^x)\,dx+i\int x\text{Li}_2(ie^x)\,dx\\ &= \frac{1}{2}ix^2\text{Li}_2(-ie^x)-\frac{1}{2}ix^2\text{Li}_2(ie^x)-ix\text{Li}_3(-ie^x)+ix\text{Li}_3(ie^x)+i\int \text{Li}_3(-ie^x)\,dx-i\int \text{I}\\ &= \frac{1}{2}ix^2\text{Li}_2(-ie^x)-\frac{1}{2}ix^2\text{Li}_2(ie^x)-ix\text{Li}_3(-ie^x)+ix\text{Li}_3(ie^x)+i\text{Subst}\left(\int \frac{\text{Li}_3(-ix)}{x}\,dx,\right.\\ &= \frac{1}{2}ix^2\text{Li}_2(-ie^x)-\frac{1}{2}ix^2\text{Li}_2(ie^x)-ix\text{Li}_3(-ie^x)+ix\text{Li}_3(ie^x)+i\text{Li}_4(-ie^x)-i\text{Li}_4(ie^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 91, normalized size = 1.00

$$\frac{1}{2}ix^2\text{PolyLog}(2,-ie^x)-\frac{1}{2}ix^2\text{PolyLog}(2,ie^x)-ix\text{PolyLog}(3,-ie^x)+ix\text{PolyLog}(3,ie^x)+i\text{PolyLog}(4,-ie^x)-i\text{PolyLog}(4,ie^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^x],x]
[Out] (I/2)*x^2*PolyLog[2, (-I)*E^x] - (I/2)*x^2*PolyLog[2, I*E^x] - I*x*PolyLog[
3, (-I)*E^x] + I*x*PolyLog[3, I*E^x] + I*PolyLog[4, (-I)*E^x] - I*PolyLog[4
, I*E^x]
```

Maple [A]
time = 0.03, size = 70, normalized size = 0.77

method	result
risch	$\frac{i x^2 \operatorname{polylog}(2,-i e^x)}{2}-\frac{i x^2 \operatorname{polylog}(2,i e^x)}{2}-i x \operatorname{polylog}(3,-i e^x)+i x \operatorname{polylog}(3,i e^x)+i \operatorname{polylog}(4,-i e^x)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3,
-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*exp
(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^x) - integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

Fricas [A]
time = 2.45, size = 87, normalized size = 0.96

$$\frac{1}{3}x^3\arctan(e^x)+\frac{1}{6}i x^3\log(i e^x+1)-\frac{1}{6}i x^3\log(-i e^x+1)-\frac{1}{2}i x^2\operatorname{Li}_2(i e^x)+\frac{1}{2}i x^2\operatorname{Li}_2(-i e^x)+i x\operatorname{polylog}(3,i e^x)-i x\operatorname{polylog}(3,-i e^x)-i\operatorname{polylog}(4,i e^x)+i\operatorname{polylog}(4,-i e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(e^x) + 1/6*I*x^3*log(I*e^x + 1) - 1/6*I*x^3*log(-I*e^x + 1)
- 1/2*I*x^2*dilog(I*e^x) + 1/2*I*x^2*dilog(-I*e^x) + I*x*polylog(3, I*e^x)
- I*x*polylog(3, -I*e^x) - I*polylog(4, I*e^x) + I*polylog(4, -I*e^x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atan}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(x)),x)
```

```
[Out] Integral(x**2*atan(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(e^x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}(e^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(exp(x)),x)
```

```
[Out] int(x^2*atan(exp(x)), x)
```

3.113 $\int \text{ArcTan}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=45

$$\frac{i\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{i\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

```
[Out] 1/2*I*polylog(2,-I*exp(b*x+a))/b-1/2*I*polylog(2,I*exp(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 4940, 2438}

$$\frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*PolyLog[2, I*E^(a + b*x)])/
b
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \tan^{-1}\left(e^{a+bx}\right)\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, e^{a+bx}\right)}{b}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{2b} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{a+bx}\right)}{2b}$$
$$= \frac{i\text{Li}_2\left(-ie^{a+bx}\right)}{2b} - \frac{i\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

Mathematica [A]
time = 0.14, size = 83, normalized size = 1.84

$$x\text{ArcTan}\left(e^{a+bx}\right) - \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^(a + b*x)],x]
```

```
[Out] x*ArcTan[E^(a + b*x)] - ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^(
a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(35) = 70.
time = 0.04, size = 95, normalized size = 2.11

method	result
derivativedivides	$\frac{\ln\left(e^{bx+a}\right)\arctan\left(e^{bx+a}\right)+\frac{i\ln\left(e^{bx+a}\right)\ln\left(1+ie^{bx+a}\right)}{2}-\frac{i\ln\left(e^{bx+a}\right)\ln\left(1-ie^{bx+a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+ie^{bx+a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(1-ie^{bx+a}\right)}{2}}{b}$
default	$\frac{\ln\left(e^{bx+a}\right)\arctan\left(e^{bx+a}\right)+\frac{i\ln\left(e^{bx+a}\right)\ln\left(1+ie^{bx+a}\right)}{2}-\frac{i\ln\left(e^{bx+a}\right)\ln\left(1-ie^{bx+a}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+ie^{bx+a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(1-ie^{bx+a}\right)}{2}}{b}$
risch	$-\frac{ix\ln\left(1+ie^{bx+a}\right)}{2}-\frac{i\ln\left(-ie^{bx+a}\right)\ln\left(-i\left(-e^{bx+a}+i\right)\right)}{2b}+\frac{i\ln\left(-i\left(-e^{bx+a}+i\right)\right)x}{2}+\frac{i\ln\left(-i\left(-e^{bx+a}+i\right)\right)a}{2b}-\frac{i\,\mathrm{dil}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(exp(b*x+a))*arctan(exp(b*x+a))+1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x+
a))-1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))+1/2*I*dilog(1+I*exp(b*x+a))-1/2
*I*dilog(1-I*exp(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.
time = 0.51, size = 63, normalized size = 1.40

$$\frac{(bx+a)\arctan\left(e^{(bx+a)}\right)}{b} - \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right)+2i\,\text{Li}_2\left(i\,e^{(bx+a)}+1\right)-2i\,\text{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctan(e^(b*x + a))/b - 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*di
log(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(29) = 58.
time = 2.39, size = 103, normalized size = 2.29

$$\frac{2\,bx\arctan\left(e^{(bx+a)}\right)+i\,a\log\left(e^{(bx+a)}+i\right)-i\,a\log\left(e^{(bx+a)}-i\right)+(i\,bx+i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\log\left(-i\,e^{(bx+a)}+1\right)-i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}\right)+i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*arctan(e^(b*x + a)) + I*a*log(e^(b*x + a) + I) - I*a*log(e^(b*x
+ a) - I) + (I*b*x + I*a)*log(I*e^(b*x + a) + 1) + (-I*b*x - I*a)*log(-I*e^
(b*x + a) + 1) - I*dilog(I*e^(b*x + a)) + I*dilog(-I*e^(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(e^{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(b*x+a)),x)
```

```
[Out] Integral(atan(exp(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctan(e^(b*x + a)), x)
```

Mupad [B]
time = 0.75, size = 37, normalized size = 0.82

$$-\frac{\mathrm{Li}_2\left(1-\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}+\frac{\mathrm{Li}_2\left(1+\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(a + b*x)),x)
```

```
[Out] (dilog(exp(b*x)*exp(a)*1i + 1)*1i)/(2*b) - (dilog(1 - exp(b*x)*exp(a)*1i)*1
i)/(2*b)
```

3.114 $\int x \mathbf{ArcTan}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=91

$$\frac{ix\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}-\frac{i\text{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2}+\frac{i\text{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}$$

```
[Out] 1/2*I*x*polylog(2,-I*exp(b*x+a))/b-1/2*I*x*polylog(2,I*exp(b*x+a))/b-1/2*I*
polylog(3,-I*exp(b*x+a))/b^2+1/2*I*polylog(3,I*exp(b*x+a))/b^2
```

Rubi [A]

time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5251, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{2b^2}+\frac{i\text{Li}_3\left(ie^{a+bx}\right)}{2b^2}+\frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x*PolyLog[2, I*E^(a + b*x
)])/b - ((I/2)*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + ((I/2)*PolyLog[3, I*E^(a
 + b*x)])/b^2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

```
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}\left(e^{a+bx}\right)dx &= \frac{1}{2}i\int x\log\left(1-ie^{a+bx}\right)dx-\frac{1}{2}i\int x\log\left(1+ie^{a+bx}\right)dx\\ &= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\int\text{Li}_2\left(-ie^{a+bx}\right)dx}{2b}+\frac{i\int\text{Li}_2\left(ie^{a+bx}\right)dx}{2b}\\ &= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\text{Subst}\left(\int\frac{\text{Li}_2(-ix)}{x}dx,x,e^{a+bx}\right)}{2b^2}+\frac{i\text{Subst}\left(\int\frac{\text{Li}_2(i\ldots}{x}\ldots\right.}{2\ldots}\\ &= \frac{ix\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\text{Li}_3\left(-ie^{a+bx}\right)}{2b^2}+\frac{i\text{Li}_3\left(ie^{a+bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 0.78

$$\frac{i\left(bx\text{PolyLog}\left(2,-ie^{a+bx}\right)-bx\text{PolyLog}\left(2,ie^{a+bx}\right)-\text{PolyLog}\left(3,-ie^{a+bx}\right)+\text{PolyLog}\left(3,ie^{a+bx}\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[E^(a + b*x)],x]
[Out] ((I/2)*(b*x*PolyLog[2, (-I)*E^(a + b*x)] - b*x*PolyLog[2, I*E^(a + b*x)] -
PolyLog[3, (-I)*E^(a + b*x)] + PolyLog[3, I*E^(a + b*x)]))/b^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(71) = 142.
time = 0.08, size = 349, normalized size = 3.84

method	result
risch	$-\frac{ix\,\text{polylog}(2,ie^{bx+a})}{2b}+\frac{ia^2\ln(1+ie^{bx+a})}{2b^2}-\frac{ia^2\ln(1-ie^{bx+a})}{2b^2}+\frac{ix\,\text{polylog}(2,-ie^{bx+a})}{2b}-\frac{i\,\text{polylog}(2,ie^{bx+a})a}{2b^2}-\frac{i\ln(1-ie\ldots}{2\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/b^2*a^2*ln(1+I*exp(b*x+a))-1/2*I*polylog(3,-I*exp(b*x+a))/b^2-1/2*I/b
^2*a^2*ln(1-I*exp(b*x+a))-1/2*I*x*polylog(2,I*exp(b*x+a))/b+1/2*I/b^2*dilog
(-I*exp(b*x+a))*a+1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a-1/2*I/b*ln(1-I*exp(b
*x+a))*x*a-1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*a^2+1/2*I/b^2*ln(-I*exp(b*x+a))
*ln(-I*(-exp(b*x+a)+I))*a-1/2*I/b^2*polylog(2,I*exp(b*x+a))*a+1/2*I*x*polyl
og(2,-I*exp(b*x+a))/b+1/2*I/b*ln(1+I*exp(b*x+a))*x*a+1/2*I/b*ln(-I*(exp(b*x
+a)+I))*x*a+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I*polylog(3,I*exp(b*x+a
))/b^2+1/2*I/b^2*dilog(-I*(exp(b*x+a)+I))*a-1/2*I/b*ln(-I*(-exp(b*x+a)+I))*
x*a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(e^(b*x + a)) - b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(61) = 122$.
time = 2.33, size = 151, normalized size = 1.66

$$\frac{2\,b^2x^2\arctan\left(e^{(bx+a)}\right) - 2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) + 2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^2\log\left(e^{(bx+a)} + i\right) + i\,a^2\log\left(e^{(bx+a)} - i\right) + \left(i\,b^2x^2 - i\,a^2\right)\log\left(i\,e^{(bx+a)} + 1\right) + \left(-i\,b^2x^2 + i\,a^2\right)\log\left(-i\,e^{(bx+a)} + 1\right) + 2i\,\mathrm{polylog}\left(3, i\,e^{(bx+a)}\right) - 2i\,\mathrm{polylog}\left(3, -i\,e^{(bx+a)}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*x^2*arctan(e^(b*x + a)) - 2*I*b*x*dilog(I*e^(b*x + a)) + 2*I*b*x
*dilog(-I*e^(b*x + a)) - I*a^2*log(e^(b*x + a) + I) + I*a^2*log(e^(b*x + a)
 - I) + (I*b^2*x^2 - I*a^2)*log(I*e^(b*x + a) + 1) + (-I*b^2*x^2 + I*a^2)*l
og(-I*e^(b*x + a) + 1) + 2*I*polylog(3, I*e^(b*x + a)) - 2*I*polylog(3, -I*
e^(b*x + a)))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atan}\left(e^{a}e^{bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(exp(b*x+a)),x)
```

```
[Out] Integral(x*atan(exp(a)*exp(b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(exp(a + b*x)),x)
```

```
[Out] int(x*atan(exp(a + b*x)), x)
```

3.115 $\int x^2 \text{ArcTan}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=133

$$\frac{ix^2\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}-\frac{ix\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}+\frac{ix\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}+\frac{i\text{PolyLo}}{}$$

```
[Out] 1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b-1/2*I*x^2*polylog(2,I*exp(b*x+a))/b-I*
x*polylog(3,-I*exp(b*x+a))/b^2+I*x*polylog(3,I*exp(b*x+a))/b^2+I*polylog(4,
-I*exp(b*x+a))/b^3-I*polylog(4,I*exp(b*x+a))/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5251, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(-ie^{a+bx}\right)}{b^3}-\frac{i\text{Li}_4\left(ie^{a+bx}\right)}{b^3}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
```

```
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}\left(e^{a+bx}\right)\,dx &= \frac{1}{2}i\int x^2\log\left(1-ie^{a+bx}\right)\,dx-\frac{1}{2}i\int x^2\log\left(1+ie^{a+bx}\right)\,dx\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{i\int x\text{Li}_2\left(-ie^{a+bx}\right)\,dx}{b}+\frac{i\int x\text{Li}_2\left(ie^{a+bx}\right)\,dx}{b}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\int \text{Li}_3\left(-i\right.}{b^2}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\text{Subst}\left(\int\right.}{}\\ &= \frac{ix^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b}-\frac{ix^2\text{Li}_2\left(ie^{a+bx}\right)}{2b}-\frac{ix\text{Li}_3\left(-ie^{a+bx}\right)}{b^2}+\frac{ix\text{Li}_3\left(ie^{a+bx}\right)}{b^2}+\frac{i\text{Li}_4\left(-ie^{a+\right.}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 133, normalized size = 1.00

$$\frac{ix^2\text{PolyLog}(2,-ie^{a+bx})}{2b}-\frac{ix^2\text{PolyLog}(2,ie^{a+bx})}{2b}-\frac{ix\text{PolyLog}(3,-ie^{a+bx})}{b^2}+\frac{ix\text{PolyLog}(3,ie^{a+bx})}{b^2}+\frac{i\text{PolyLog}(4,-ie^{a+bx})}{b^3}-\frac{i\text{PolyLog}(4,ie^{a+bx})}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[E^(a + b*x)],x]
```

```
[Out] ((I/2)*x^2*PolyLog[2, (-I)*E^(a + b*x)])/b - ((I/2)*x^2*PolyLog[2, I*E^(a +
 b*x)])/b - (I*x*PolyLog[3, (-I)*E^(a + b*x)])/b^2 + (I*x*PolyLog[3, I*E^(a
 + b*x)])/b^2 + (I*PolyLog[4, (-I)*E^(a + b*x)])/b^3 - (I*PolyLog[4, I*E^(a
 + b*x)])/b^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(111) = 222$.
time = 0.06, size = 407, normalized size = 3.06

method	result
risch	$-\frac{ix \operatorname{polylog}(3,-i e^{bx+a})}{b^2} - \frac{ix^2 \operatorname{polylog}(2,i e^{bx+a})}{2b} + \frac{ix \operatorname{polylog}(3,i e^{bx+a})}{b^2} - \frac{i \operatorname{dilog}(-i e^{bx+a}) a^2}{2b^3} - \frac{i \operatorname{dilog}(-i(e^{bx+a}+i)) a^2}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b^3*dilog(-I*(exp(b*x+a)+I))*a^2-1/2*I/b^3*polylog(2,-I*exp(b*x+a))*
a^2-I*x*polylog(3,-I*exp(b*x+a))/b^2+1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b+1
/2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3+I*polylog(4,-I*exp(b*x+a))/b^3+I*x*poly
log(3,I*exp(b*x+a))/b^2-1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2-1/2*I*x^2*polylo
g(2,I*exp(b*x+a))/b+1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*x*a^2+1/2*I/b^3*polylo
g(2,I*exp(b*x+a))*a^2-1/2*I/b^3*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a^
2-I*polylog(4,I*exp(b*x+a))/b^3-1/2*I/b^2*ln(1+I*exp(b*x+a))*x*a^2-1/2*I/b^
3*ln(-I*(exp(b*x+a)+I))*a^3-1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*x*a^2-1/2*I/b^3
*a^3*ln(1+I*exp(b*x+a))+1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2+1/2*I/b^3*a^3*ln
(1-I*exp(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(e^(b*x + a)) - b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x + 2
*a) + 1), x)
```

Fricas [A]
time = 2.24, size = 187, normalized size = 1.41

$$\frac{2\,b^3x^3\arctan\left(e^{(bx+a)}\right) - 3i\,b^2x^2\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) + 3i\,b^2x^2\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) + i\,a^3\log\left(e^{(bx+a)}+i\right) - i\,a^3\log\left(e^{(bx+a)}-i\right) + 6i\,bx\mathrm{polylog}\left(3,i\,e^{(bx+a)}\right) - 6i\,bx\mathrm{polylog}\left(3,-i\,e^{(bx+a)}\right) + \left(i\,b^3x^3+i\,a^3\right)\log\left(i\,e^{(bx+a)}+1\right) + \left(-i\,b^3x^3-i\,a^3\right)\log\left(-i\,e^{(bx+a)}+1\right) - 6i\,\mathrm{polylog}\left(4,i\,e^{(bx+a)}\right) + 6i\,\mathrm{polylog}\left(4,-i\,e^{(bx+a)}\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3*arctan(e^(b*x + a)) - 3*I*b^2*x^2*dilog(I*e^(b*x + a)) + 3*I
*b^2*x^2*dilog(-I*e^(b*x + a)) + I*a^3*log(e^(b*x + a) + I) - I*a^3*log(e^(
b*x + a) - I) + 6*I*b*x*polylog(3, I*e^(b*x + a)) - 6*I*b*x*polylog(3, -I*e
^(b*x + a)) + (I*b^3*x^3 + I*a^3)*log(I*e^(b*x + a) + 1) + (-I*b^3*x^3 - I*
```

```
a^3)*log(-I*e^(b*x + a) + 1) - 6*I*polylog(4, I*e^(b*x + a)) + 6*I*polylog(
4, -I*e^(b*x + a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atan}\left(e^a e^{bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(exp(b*x+a)),x)
[Out] Integral(x**2*atan(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arctan(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(exp(a + b*x)),x)
[Out] int(x^2*atan(exp(a + b*x)), x)
```

3.116 $\int \text{ArcTan}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=196

$$-\frac{\text{ArcTan}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\text{ArcTan}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}+\frac{i\text{PolyLog}\left(2,1-\right.}{2d\log}$$

```
[Out] -arctan(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arctan(a+b*f^(d*
x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog(2
,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a)/(
1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 5155, 4966, 2449, 2352, 2497}

$$-\frac{\text{ArcTan}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)}+\frac{\text{ArcTan}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(-a+i)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)}+\frac{i\text{Li}_2\left(1-\frac{2}{1-i(bf^{c+dx}+a)}\right)}{2d\log(f)}-\frac{i\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(i-a)(1-i(bf^{c+dx}+a))}\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))])/(d*Log[f])
) + (ArcTan[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b
*f^(c + d*x))))])/(d*Log[f]) + ((I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c +
 d*x)))])/(d*Log[f]) - ((I/2)*PolyLog[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1
- I*(a + b*f^(c + d*x))))])/(d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \tan^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{\text{Subst}\left(\int\frac{\tan^{-1}(a+bx)}{x}\,dx,x,f^{c+dx}\right)}{d\log(f)}$$

$$=\frac{\text{Subst}\left(\int\frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx,x,a+bf^{c+dx}\right)}{bd\log(f)}$$

$$=-\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)}+\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c}\right.\right.}\right.}{d\log(f)}$$

$$=-\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)}+\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c}\right.\right.}\right.}{d\log(f)}$$

$$=-\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i\left(a+bf^{c+dx}\right)}\right)}{d\log(f)}+\frac{\tan^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(a+bf^{c}\right.\right.}\right.}{d\log(f)}$$

Mathematica [A]
time = 0.30, size = 167, normalized size = 0.85

$$x\text{ArcTan}\left(a+bf^{c+dx}\right)-\frac{b\left(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)}{2\sqrt{-b^2}\,d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] x*ArcTan[a + b*f^(c + d*x)] - (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a*
b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyLog
[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c + d*x
))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

Maple [A]
time = 0.14, size = 162, normalized size = 0.83

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\arctan\left(a+b\,f^{dx+c}\right)-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i+b\,f^{dx\cdots}}{i+\cdots}\right.}{2}\cdots}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\arctan\left(a+b\,f^{dx+c}\right)-\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i+b\,f^{dx+c}+a}{i+a}\right)}{2}+\frac{i\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{i-b\,f^{dx+c}-a}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{i+b\,f^{dx\cdots}}{i+\cdots}\right.}{2}\cdots}{d\ln(f)}$
risch	$-\frac{ix\ln\left(1+i\left(a+b\,f^{dx+c}\right)\right)}{2}+\frac{i\,\mathrm{dilog}\left(\frac{b\,f^{dx}\,f^{c}+a-i}{a-i}\right)}{2d\ln(f)}+\frac{i\ln\left(\frac{b\,f^{dx}\,f^{c}+a-i}{a-i}\right)x}{2}+\frac{i\ln\left(\frac{b\,f^{dx}\,f^{c}+a-i}{a-i}\right)c}{2d}-\frac{ic\ln\left(if^{dx}\,f\cdots\right.}{2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arctan(a+b*f^(d*x+c))-1/2*I*ln(-b*f^(d*x+c))*ln
((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-a)
)-1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-a))
)
```

Maxima [A]
time = 0.54, size = 189, normalized size = 0.96

$$\frac{(dx+c)\arctan\left(bf^{dx+c}+a\right)}{d}-\frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log(f)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\arctan\left(bf^{dx+c}+a\right)\log\left(\frac{b^2f^{2\,dx+2\,c}}{a^2+1}\right)+i\,\mathrm{Li}_2\left(\frac{i\,bf^{dx+c}+i\,a+1}{i\,a+1}\right)-i\,\mathrm{Li}_2\left(\frac{i\,bf^{dx+c}+i\,a-1}{i\,a-1}\right)}{2\,d\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arctan(b*f^(d*x + c) + a)/d - 1/2*(2*(d*x + c)*arctan((b^2*f^(d*x
 + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a*b
```

```
*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2*c)
/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog((I*b
*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

Fricas [A]
time = 2.31, size = 212, normalized size = 1.08

$$\frac{2\,dx\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(i\,dx+i\,c\right)\log\left(f\right)\log\left(\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}\right)+\left(-i\,dx-i\,c\right)\log\left(f\right)\log\left(\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}\right)+i\,\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)-i\,\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*d*x*arctan(b*f^(d*x + c) + a)*log(f) + I*c*log(b*f^(d*x + c) + a + I
)*log(f) - I*c*log(b*f^(d*x + c) + a - I)*log(f) + (I*d*x + I*c)*log(f)*log
((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I*d*x - I*c)*log(f)*log
((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + I*dilog(-(a^2 + (a*b + I*
b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) - I*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c
) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(a + bf^{c+dx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a+b*f**(d*x+c)),x)
[Out] Integral(atan(a + b*f**(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a+b*f^(d*x+c)),x, algorithm="giac")
[Out] integrate(arctan(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atan}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*f^(c + d*x)),x)
[Out] int(atan(a + b*f^(c + d*x)), x)
```

3.117 $\int x\textbf{ArcTan}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=232

$$\frac{1}{2}x^2\text{ArcTan}\left(a+bf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)+\frac{1}{4}ix^2\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)-\frac{ix\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix\text{Po}}{}$$

```
[Out] 1/2*x^2*arctan(a+b*f^(d*x+c))-1/4*I*x^2*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/4*I*x
^2*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d/l
n(f)+1/2*I*x*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+1/2*I*polylog(3,I*b*
f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-1/2*I*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2
```

Rubi [A]
time = 0.12, antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5251, 2612, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(\frac{bf^{c+dx}}{i-a}\right)}{2d^2\log^2(f)}+\frac{i\text{Li}_3\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d^2\log^2(f)}+\frac{ix\text{Li}_2\left(\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix\text{Li}_2\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}+\frac{1}{4}ix^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{4}ix^2\log\left(ia+ibf^{c+dx}+1\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)-\frac{1}{4}ix^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/4)*x^2*Log[1 - I*a - I*b*f^(c + d*x)] - (I/4)*x^2*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/4)*x^2*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/4)*x^2*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*
Log[f]) - ((I/2)*x*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) - ((I
/2)*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + ((I/2)*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tan^{-1}\left(a+bf^{c+dx}\right)\,dx=\frac{1}{2}i\int x\log\left(1-ia-ibf^{c+dx}\right)\,dx-\frac{1}{2}i\int x\log\left(1+ia+ibf^{c+dx}\right)\,dx$$

$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+}}{i-}\right.$$

$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+}}{i-}\right.$$

$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+}}{i-}\right.$$

$$=\frac{1}{4}ix^2\log\left(1-ia-ibf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1+ia+ibf^{c+dx}\right)+\frac{1}{4}ix^2\log\left(1-\frac{bf^{c+}}{i-}\right.$$

Mathematica [A]
time = 0.06, size = 236, normalized size = 1.02

$$\frac{i\left(d^2x^2\log^2(f)\log\left(1-ia-ibf^{c+dx}\right)-d^2x^2\log^2(f)\log\left(1+ia+ibf^{c+dx}\right)-d^2x^2\log^2(f)\log\left(\frac{i+a+bf^{c+dx}}{i+a}\right)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-i+a}\right)+2dx\log(f)\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{i+a}\right)-2\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{i+a}\right)\right)}{4d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] ((I/4)*(d^2*x^2*Log[f]^2*Log[1 - I*a - I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*
Log[1 + I*a + I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*Log[(I + a + b*f^(c + d*x
))/(I + a)] + d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(-I + a)] + 2*d*x*Lo
g[f]*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 2*d*x*Log[f]*PolyLog[2, -((b*f^(
c + d*x))/(I + a))] - 2*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 2*PolyLog[3,
-((b*f^(c + d*x))/(I + a))]))/(d^2*Log[f]^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(200) = 400$.
time = 0.06, size = 672, normalized size = 2.90

method	result
risch	$-\frac{i \ln\left(1-\frac{i b\, f^{d x} f^{c}}{-i a+1}\right) x^{2}}{4}-\frac{i x^{2} \ln\left(1+i\left(a+b\, f^{d x+c}\right)\right)}{4}+\frac{i c\, \mathrm{dilog}\left(\frac{b\, f^{d x} f^{c}+i+a}{i+a}\right)}{2 d^{2} \ln\left(f\right)}+\frac{i \ln\left(1-\frac{i b\, f^{d x} f^{c}}{-i a-1}\right) c^{2}}{4 d^{2}}+\frac{i\, \mathrm{polylog}\left(2, \frac{i b\, f^{d x} f^{c}}{-i a-1}\right.}{2 d \ln\left(f\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^2-1/4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))+
1/2*I/d^2/ln(f)*c*dilog((b*f^(d*x)*f^c+I+a)/(I+a))+1/4*I/d^2*ln(1-I*b/(-I*a
-1)*f^(d*x)*f^c)*c^2-1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2-1/2*I/d*c*
ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+I+a)/(I+a))
-1/2*I/d^2/ln(f)*c*dilog((b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/d/ln(f)*polylog(2
,I*b/(-I*a-1)*f^(d*x)*f^c)*x+1/2*I/d^2/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)
*f^c)*c+1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))+1/2*I/d*ln(1-I*b/(-I*a-1)*f^(d*x)
*f^c)*x*c+1/4*I/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)+1/2*I/d*c*ln((b*f^(d*x)*f
^c+I+a)/(I+a))*x-1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)-1/2*I/d*ln(1-I*b/(
1-I*a)*f^(d*x)*f^c)*x*c-1/2*I/d^2/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*
c+1/4*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2+1/2*I/d^2/ln(f)^2*polylog(3,I*b/
(1-I*a)*f^(d*x)*f^c)-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))-1/2*I/d^2/
ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)-1/2*I/d/ln(f)*polylog(2,I*b/(1-
I*a)*f^(d*x)*f^c)*x
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(b*f^(d*x)*f^c + a)
```

Fricas [A]
time = 3.17, size = 304, normalized size = 1.31

$$\frac{2\,d^2x^2\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^2-i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2+i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2+2i\,dx\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)-2i\,dx\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)+\left(i\,d^2x^2-i\,c^2\right)\log\left(f\right)^2\log\left(\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}\right)+\left(-i\,d^2x^2+i\,c^2\right)\log\left(f\right)^2\log\left(\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}\right)-2i\,\mathrm{polylog}\left(3,-\frac{(ab+i\,b)f^{dx+c}}{a^2+1}\right)+2i\,\mathrm{polylog}\left(3,-\frac{(ab-i\,b)f^{dx+c}}{a^2+1}\right)}{4\,d^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*x^2*arctan(b*f^(d*x + c) + a)*log(f)^2 - I*c^2*log(b*f^(d*x + c)
 + a + I)*log(f)^2 + I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 + 2*I*d*x*di
log(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) - 2*I*d*x*di
log(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (I*d^2*x^2
 - I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I
*d^2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1
)) - 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 2*I*polylog(3, -(
a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atan}\left(a + b f^{c} f^{dx}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x*atan(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atan}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a + b*f^(c + d*x)),x)
```

```
[Out] int(x*atan(a + b*f^(c + d*x)), x)
```

3.118 $\int x^2 \mathbf{ArcTan}\left(a + bf^{c+dx}\right) \, dx$

Optimal. Leaf size=302

$$\frac{1}{3}x^3\text{ArcTan}\left(a+bf^{c+dx}\right)-\frac{1}{6}ix^3\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)+\frac{1}{6}ix^3\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)-\frac{ix^2\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}+\frac{ix^2\text{P}}{}$$

```
[Out] 1/3*x^3*arctan(a+b*f^(d*x+c))-1/6*I*x^3*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/6*I*x
^3*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x^2*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d
/ln(f)+1/2*I*x^2*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+I*x*polylog(3,I*
b*f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-I*x*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/d^2/
ln(f)^2-I*polylog(4,I*b*f^(d*x+c)/(1-I*a))/d^3/ln(f)^3+I*polylog(4,-I*b*f^(
d*x+c)/(1+I*a))/d^3/ln(f)^3
```

Rubi [A]
time = 0.16, antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5251, 2612, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)}-\frac{i\text{Li}_4\left(-\frac{bf^{c+dx}}{a+i}\right)}{d^3\log^3(f)}-\frac{ix\text{Li}_3\left(\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}+\frac{ix\text{Li}_3\left(-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}+\frac{ix^2\text{Li}_2\left(\frac{bf^{c+dx}}{i-a}\right)}{2d\log(f)}-\frac{ix^2\text{Li}_2\left(-\frac{bf^{c+dx}}{a+i}\right)}{2d\log(f)}+\frac{1}{6}ix^3\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{6}ix^3\log\left(ia+ibf^{c+dx}+1\right)+\frac{1}{6}ix^3\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)-\frac{1}{6}ix^3\log\left(1+\frac{bf^{c+dx}}{a+i}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] + (I/6)*x^3*Log[1 - (b*f^(c + d*x))/(I - a)] - (I/6)*x^3*Log[1 + (
b*f^(c + d*x))/(I + a)] + ((I/2)*x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(
d*Log[f]) - ((I/2)*x^2*PolyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) -
 (I*x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d^2*Log[f]^2) + (I*x*PolyLog[3,
 -((b*f^(c + d*x))/(I + a))])/(d^2*Log[f]^2) + (I*PolyLog[4, (b*f^(c + d*x)
)/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(c + d*x))/(I + a))])/(d^
3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 5251

```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Dist[I/2, Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Dist[I/2, Int[
x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IntegerQ[m] && m > 0
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tan^{-1}\left(a+bf^{c+dx}\right)\,dx = \frac{1}{2}i\int x^2\log\left(1-ia-ibf^{c+dx}\right)\,dx - \frac{1}{2}i\int x^2\log\left(1+ia+ibf^{c+dx}\right)\,dx$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf}{i}\right.$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf}{i}\right.$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf}{i}\right.$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf}{i}\right.$$

$$= \frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) + \frac{1}{6}ix^3\log\left(1-\frac{bf}{i}\right.$$

Mathematica [A]
time = 0.01, size = 334, normalized size = 1.11

$$\frac{1}{6}ix^3\log\left(1-ia-ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1+ia+ibf^{c+dx}\right) - \frac{1}{6}ix^3\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right) + \frac{1}{6}ix^3\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right) - \frac{ix^2\text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)} + \frac{ix^2\text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d\log(f)} + \frac{ix\text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{d^2\log^2(f)} - \frac{ix\text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^2\log^2(f)} + \frac{i\text{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^3\log^3(f)} - \frac{i\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{i+a}\right)}{d^3\log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

```
[Out] (I/6)*x^3*Log[1 - I*a - I*b*f^(c + d*x)] - (I/6)*x^3*Log[1 + I*a + I*b*f^(c
 + d*x)] - (I/6)*x^3*Log[1 - (I*b*f^(c + d*x))/(1 - I*a)] + (I/6)*x^3*Log[1
 + (I*b*f^(c + d*x))/(1 + I*a)] - ((I/2)*x^2*PolyLog[2, (I*b*f^(c + d*x))/(
1 - I*a)])/(d*Log[f]) + ((I/2)*x^2*PolyLog[2, ((-I)*b*f^(c + d*x))/(1 + I*a
)])/(d*Log[f]) + (I*x*PolyLog[3, (I*b*f^(c + d*x))/(1 - I*a)])/(d^2*Log[f]^
2) - (I*x*PolyLog[3, ((-I)*b*f^(c + d*x))/(1 + I*a)])/(d^2*Log[f]^2) + (I*P
olyLog[4, (b*f^(c + d*x))/(I - a)])/(d^3*Log[f]^3) - (I*PolyLog[4, -((b*f^(
c + d*x))/(I + a))])/(d^3*Log[f]^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(268) = 536$.
time = 0.06, size = 758, normalized size = 2.51

method	result
risch	$\frac{ic^2\ln\left(\frac{b\,f^{dx}f^c+a-i}{a-i}\right)x}{2d^2} - \frac{i\ln\left(1-\frac{ib\,f^{dx}f^c}{-ia-1}\right)c^3}{3d^3} + \frac{ix^3\ln\left(1-i\left(a+b\,f^{dx+c}\right)\right)}{6} - \frac{i\,\mathrm{polylog}\left(3,\frac{ib\,f^{dx}f^c}{-ia-1}\right)x}{d^2\ln(f)^2} - \frac{ic^2\,\mathrm{dilog}\left(\frac{b\,f^{dx}f^c+i+}{i+a}\right.}{2d^3\ln(f)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x-1/3*I/d^3*ln(1-I*b/(-I*a-1)*f
^(d*x)*f^c)*c^3+1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))+1/2*I/d/ln(f)*polylog(2,I
*b/(-I*a-1)*f^(d*x)*f^c)*x^2+I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)*f^(d*x)*f^
c)*x-1/2*I/d/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/2*I/d^3*c^3*ln(
(b*f^(d*x)*f^c+a-I)/(a-I))-1/6*I*x^3*ln(1+I*(a+b*f^(d*x+c)))+1/2*I/d^3/ln(f
)*c^2*dilog((b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f
^c)*x*c^2-1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^3-1/6*I/d^3*c^3*ln(I*f^(d*x
)*f^c*b+I*a+1)-I/d^2/ln(f)^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)*x-1/2*I/d^
2*c^2*ln((b*f^(d*x)*f^c+I+a)/(I+a))*x-1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*
f^c+I+a)/(I+a))-1/2*I/d^3/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2+I/d
^3/ln(f)^3*polylog(4,I*b/(-I*a-1)*f^(d*x)*f^c)+1/6*I/d^3*c^3*ln(1-I*a-I*f^(
d*x)*f^c*b)+1/6*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^3-I/d^3/ln(f)^3*polylog(
4,I*b/(1-I*a)*f^(d*x)*f^c)-1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x*c^2+1
/2*I/d^3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2-1/2*I/d^3*c^3*ln((b*f
^(d*x)*f^c+I+a)/(I+a))+1/3*I/d^3*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*f
^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(b*f^(d*x)*f^c + a)
```

Fricas [A]
time = 2.99, size = 378, normalized size = 1.25

$$\frac{2\,d^3x^3\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^3+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2+i\,c^3\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^3-i\,c^3\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^3+\left(i\,d^3x^3+i\,c^3\right)\log\left(f\right)^3\log\left(\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}\right)+\left(-i\,d^3x^3-i\,c^3\right)\log\left(f\right)^3\log\left(\frac{a^2+(ab-i\,b)f^{dx+c}+1}{a^2+1}\right)-6i\,dx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{(ab+i\,b)f^{dx+c}}{a^2+1}\right)+6i\,dx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{(ab-i\,b)f^{dx+c}}{a^2+1}\right)+6i\,\mathrm{polylog}\left(4,-\frac{(ab+i\,b)f^{dx+c}}{a^2+1}\right)-6i\,\mathrm{polylog}\left(4,-\frac{(ab-i\,b)f^{dx+c}}{a^2+1}\right)}{6\,d^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*d^3*x^3*arctan(b*f^(d*x + c) + a)*log(f)^3 + 3*I*d^2*x^2*dilog(-(a^2
 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - 3*I*d^2*x^2*dilog
(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + I*c^3*log(b
*f^(d*x + c) + a + I)*log(f)^3 - I*c^3*log(b*f^(d*x + c) + a - I)*log(f)^3
+ (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2
```

```
 + 1)) + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x + c) +
 1)/(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 +
1)) + 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)) + 6*I*p
olylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*polylog(4, -(a*b - I*b)
*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atan}\left(a + bf^c f^{dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x**2*atan(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atan}\left(a + b\, f^{c+d\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a + b*f^(c + d*x)),x)
```

```
[Out] int(x^2*atan(a + b*f^(c + d*x)), x)
```

3.119 $\int e^{-x}\text{ArcTan}(e^x)\,dx$

Optimal. Leaf size=25

$$x - e^{-x}\text{ArcTan}(e^x) - \frac{1}{2}\log\left(1 + e^{2x}\right)$$

```
[Out] x-arctan(exp(x))/exp(x)-1/2*ln(1+exp(2*x))
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2225, 5315, 2320, 36, 29, 31}

$$-e^{-x}\text{ArcTan}(e^x) + x - \frac{1}{2}\log\left(e^{2x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[E^x]/E^x,x]
```

```
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{-x}\tan^{-1}\left(e^x\right)\,dx &= -e^{-x}\tan^{-1}\left(e^x\right)+\int \frac{1}{1+e^{2x}}\,dx\\
&= -e^{-x}\tan^{-1}\left(e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\
&= -e^{-x}\tan^{-1}\left(e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\
&= x-e^{-x}\tan^{-1}\left(e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$x-e^{-x}\text{ArcTan}(e^x)-\frac{1}{2}\log\left(1+e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[E^x]/E^x,x]
[Out] x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$-\arctan\left(\mathrm{e}^{x}\right)\mathrm{e}^{-x}-\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}+\ln\left(\mathrm{e}^{x}\right)$	23
default	$-\arctan\left(\mathrm{e}^{x}\right)\mathrm{e}^{-x}-\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}+\ln\left(\mathrm{e}^{x}\right)$	23
risch	$\frac{i\mathrm{e}^{-x}\ln\left(1+i\mathrm{e}^{x}\right)}{2}-\frac{i\left(-i\ln\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{x}+2ix\,\mathrm{e}^{x}+\ln\left(1-i\mathrm{e}^{x}\right)\right)\mathrm{e}^{-x}}{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(exp(x))/exp(x)-1/2*ln(exp(x)^2+1)+ln(exp(x))
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.76

$$-\arctan\left(e^{x}\right)e^{(-x)}-\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -arctan(e^x)*e^(-x) - 1/2*log(e^(-2*x) + 1)
```

Fricas [A]
time = 2.91, size = 28, normalized size = 1.12

$$\frac{1}{2}\left(2\,xe^{x}-e^{x}\log\left(e^{(2x)}+1\right)-2\arctan\left(e^{x}\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) - 2*arctan(e^x))*e^(-x)
```

Sympy [A]
time = 1.47, size = 19, normalized size = 0.76

$$x-\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\operatorname{atan}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(exp(x))/exp(x),x)
```

```
[Out] x - log(exp(2*x) + 1)/2 - exp(-x)*atan(exp(x))
```

Giac [A]
time = 0.43, size = 20, normalized size = 0.80

$$-\arctan\left(e^{x}\right)e^{(-x)}+x-\frac{1}{2}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^x)*e^(-x) + x - 1/2*log(e^(2*x) + 1)
```

Mupad [B]
time = 0.09, size = 20, normalized size = 0.80

$$x-\frac{\ln\left({\mathrm{e}}^{2\,x}+1\right)}{2}-\mathrm{atan}\left({\mathrm{e}}^x\right)\,{\mathrm{e}}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(x))*exp(-x),x)
[Out] x - log(exp(2*x) + 1)/2 - atan(exp(x))*exp(-x)
```

3.120 $\int \frac{\text{ArcTan}(x)}{(-1+x)^3} \, dx$

Optimal. Leaf size=45

$$\frac{1}{4(1-x)}-\frac{\text{ArcTan}(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{8}\log\left(1+x^2\right)$$

```
[Out] 1/4/(1-x)-1/2*arctan(x)/(1-x)^2-1/4*ln(1-x)+1/8*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4972, 724, 815, 266}

$$-\frac{\text{ArcTan}(x)}{2(1-x)^2}+\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] 1/(4*(1 - x)) - ArcTan[x]/(2*(1 - x)^2) - Log[1 - x]/4 + Log[1 + x^2]/8
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(x)}{(-1+x)^3}\,dx &= -\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{2}\int \frac{1}{(-1+x)^2\,(1+x^2)}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{4}\int \frac{-1-x}{(-1+x)\,(1+x^2)}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}+\frac{1}{4}\int \left(\frac{1}{1-x}+\frac{x}{1+x^2}\right)dx\\
&= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{4}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{\tan^{-1}(x)}{2(1-x)^2}-\frac{1}{4}\log(1-x)+\frac{1}{8}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.78

$$\frac{1}{8}\left(-\frac{2}{-1+x}-\frac{4\text{ArcTan}(x)}{(-1+x)^2}-2\log(1-x)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(-1 + x)^3,x]
```

```
[Out] (-2/(-1 + x) - (4*ArcTan[x])/(-1 + x)^2 - 2*Log[1 - x] + Log[1 + x^2])/8
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.71

method	result	siz
default	$-\frac{\arctan(x)}{2(x-1)^2}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}+\frac{\ln(x^2+1)}{8}$	32
risch	$\frac{i\ln(ix+1)}{4(x-1)^2}-\frac{i\left(-2i\ln(x-1)x^2+i\ln(x^2+1)x^2+4i\ln(x-1)x-2i\ln(x^2+1)x-2i\ln(x-1)+i\ln(x^2+1)-2ix+2i+2\ln(-ix+1)\right)}{8(x-1)^2}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/(x-1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x-1)^2*arctan(x)-1/4/(x-1)-1/4*ln(x-1)+1/8*ln(x^2+1)
```

Maxima [A]
time = 0.49, size = 31, normalized size = 0.69

$$-\frac{1}{4\,(x-1)}-\frac{\arctan\left(x\right)}{2\,(x-1)^2}+\frac{1}{8}\,\log\left(x^2+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(x - 1)
```

Fricas [A]
time = 2.33, size = 50, normalized size = 1.11

$$\frac{(x^2 - 2\,x + 1)\log\left(x^2 + 1\right) - 2\,(x^2 - 2\,x + 1)\log\left(x - 1\right) - 2\,x - 4\,\arctan\left(x\right) + 2}{8\,(x^2 - 2\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((x^2 - 2*x + 1)*log(x^2 + 1) - 2*(x^2 - 2*x + 1)*log(x - 1) - 2*x - 4*
arctan(x) + 2)/(x^2 - 2*x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(31) = 62$.
time = 0.24, size = 153, normalized size = 3.40

$$-\frac{2x^2\log\left(x-1\right)}{8x^2-16x+8}+\frac{x^2\log\left(x^2+1\right)}{8x^2-16x+8}+\frac{4x\log\left(x-1\right)}{8x^2-16x+8}-\frac{2x\log\left(x^2+1\right)}{8x^2-16x+8}-\frac{2x}{8x^2-16x+8}-\frac{2\log\left(x-1\right)}{8x^2-16x+8}+\frac{\log\left(x^2+1\right)}{8x^2-16x+8}-\frac{4\operatorname{atan}\left(x\right)}{8x^2-16x+8}+\frac{2}{8x^2-16x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/(-1+x)**3,x)
```

```
[Out] -2*x**2*log(x - 1)/(8*x**2 - 16*x + 8) + x**2*log(x**2 + 1)/(8*x**2 - 16*x
+ 8) + 4*x*log(x - 1)/(8*x**2 - 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 - 16*
x + 8) - 2*x/(8*x**2 - 16*x + 8) - 2*log(x - 1)/(8*x**2 - 16*x + 8) + log(x
**2 + 1)/(8*x**2 - 16*x + 8) - 4*atan(x)/(8*x**2 - 16*x + 8) + 2/(8*x**2 -
16*x + 8)
```

Giac [A]
time = 0.45, size = 32, normalized size = 0.71

$$-\frac{1}{4\,(x-1)} - \frac{\arctan\left(x\right)}{2\,(x-1)^2} + \frac{1}{8}\,\log\left(x^2+1\right) - \frac{1}{4}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(abs(x -
 1))
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.69

$$\frac{\ln\left(x^2+1\right)}{8} - \frac{\ln\left(x-1\right)}{4} - \frac{\frac{x}{4} + \frac{\operatorname{atan}(x)}{2} - \frac{1}{4}}{(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x - 1)^3,x)
[Out] log(x^2 + 1)/8 - log(x - 1)/4 - (x/4 + atan(x)/2 - 1/4)/(x - 1)^2
```

3.121 $\int \frac{\text{ArcTan}(1+2x)}{(4+3x)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{1}{34(4+3x)}+\frac{8}{867}\text{ArcTan}(1+2x)-\frac{\text{ArcTan}(1+2x)}{6(4+3x)^2}+\frac{5}{289}\log(4+3x)-\frac{5}{578}\log\left(1+2x+2x^2\right)$$

```
[Out] -1/34/(4+3*x)+8/867*arctan(1+2*x)-1/6*arctan(1+2*x)/(4+3*x)^2+5/289*ln(4+3*
x)-5/578*ln(2*x^2+2*x+1)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5153, 2007, 723, 814, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}(2x+1)}{6(3x+4)^2}+\frac{8}{867}\text{ArcTan}(2x+1)-\frac{5}{578}\log\left(2x^2+2x+1\right)-\frac{1}{34(3x+4)}+\frac{5}{289}\log(3x+4)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] -1/34*1/(4 + 3*x) + (8*ArcTan[1 + 2*x])/867 - ArcTan[1 + 2*x]/(6*(4 + 3*x)^
2) + (5*Log[4 + 3*x])/289 - (5*Log[1 + 2*x + 2*x^2])/578
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(1+2x)}{(4+3x)^3}\,dx = -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{1}{3}\int \frac{1}{(4+3x)^2\left(1+(1+2x)^2\right)}\,dx$$
$$= -\frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{1}{3}\int \frac{1}{(4+3x)^2\left(2+4x+4x^2\right)}\,dx$$
$$= -\frac{1}{34(4+3x)} - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{1}{102}\int \frac{4-12x}{(4+3x)\left(2+4x+4x^2\right)}\,dx$$
$$= -\frac{1}{34(4+3x)} - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{1}{102}\int \left(\frac{90}{17(4+3x)} - \frac{2(7+30x)}{17\left(1+2x+2x^2\right)}\right)\,dx$$
$$= -\frac{1}{34(4+3x)} - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{1}{867}\int \frac{7+30x}{1+2x+2x^2}\,dx$$
$$= -\frac{1}{34(4+3x)} - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{5}{578}\int \frac{2+4x}{1+2x+2x^2}\,dx + \frac{8}{867}\int$$
$$= -\frac{1}{34(4+3x)} - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{5}{578}\log\left(1+2x+2x^2\right) - \frac{8}{867}\mathrm{Su}$$
$$= -\frac{1}{34(4+3x)} + \frac{8}{867}\tan^{-1}(1+2x) - \frac{\tan^{-1}(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{5}{578}\log\left(1-\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 81, normalized size = 1.27

$$\frac{-289\mathrm{ArcTan}(1+2x)+(4+3x)(-51-(15-8i)(4+3x)\log(i+(1+i)x)-(15+8i)(4+3x)\log(1+(1+i)x)+120\log(4+3x)+90x\log(4+3x))}{1734(4+3x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

```
[Out] (-289*ArcTan[1 + 2*x] + (4 + 3*x)*(-51 - (15 - 8*I)*(4 + 3*x)*Log[I + (1 +
I)*x] - (15 + 8*I)*(4 + 3*x)*Log[1 + (1 + I)*x] + 120*Log[4 + 3*x] + 90*x*L
og[4 + 3*x]))/(1734*(4 + 3*x)^2)
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.84

method	result
derivativedivides	$-\frac{2\arctan(1+2x)}{3(8+6x)^2}-\frac{5\ln\left((1+2x)^2+1\right)}{578}+\frac{8\arctan(1+2x)}{867}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}$
default	$-\frac{2\arctan(1+2x)}{3(8+6x)^2}-\frac{5\ln\left((1+2x)^2+1\right)}{578}+\frac{8\arctan(1+2x)}{867}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}$
risch	$\frac{i\ln(1+i(1+2x))}{12(4+3x)^2}-\frac{i(-480i\ln(2x+1+i)-480i\ln(2x+1-i)-720i\ln(2x+1+i)x-720i\ln(2x+1-i)x+540i\ln(4+3x)x^2+1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1+2*x)/(4+3*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(8+6*x)^2*arctan(1+2*x)-5/578*ln((1+2*x)^2+1)+8/867*arctan(1+2*x)-1/17
/(8+6*x)+5/289*ln(8+6*x)
```

Maxima [A]
time = 0.48, size = 54, normalized size = 0.84

$$-\frac{1}{34\,(3\,x+4)}-\frac{\arctan\left(2\,x+1\right)}{6\,(3\,x+4)^2}+\frac{8}{867}\,\arctan\left(2\,x+1\right)-\frac{5}{578}\,\log\left(2\,x^2+2\,x+1\right)+\frac{5}{289}\,\log\left(3\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1) -
 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(3*x + 4)
```

Fricas [A]
time = 1.49, size = 77, normalized size = 1.20

$$\frac{\left(48\,x^2+128\,x-11\right)\arctan\left(2\,x+1\right)-5\,\left(9\,x^2+24\,x+16\right)\log\left(2\,x^2+2\,x+1\right)+10\,\left(9\,x^2+24\,x+16\right)\log\left(3\,x+4\right)-51\,x-68}{578\,\left(9\,x^2+24\,x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="fricas")
```

```
[Out] 1/578*((48*x^2 + 128*x - 11)*arctan(2*x + 1) - 5*(9*x^2 + 24*x + 16)*log(2*
x^2 + 2*x + 1) + 10*(9*x^2 + 24*x + 16)*log(3*x + 4) - 51*x - 68)/(9*x^2 +
24*x + 16)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(56) = 112$.
time = 0.26, size = 223, normalized size = 3.48

$$\frac{90x^2\log\left(3x+4\right)}{5202x^2+13872x+9248}-\frac{45x^2\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248}+\frac{48x^2\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248}+\frac{240x\log\left(3x+4\right)}{5202x^2+13872x+9248}-\frac{120x\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248}+\frac{128x\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248}-\frac{51x}{5202x^2+13872x+9248}+\frac{160\log\left(3x+4\right)}{5202x^2+13872x+9248}-\frac{80\log\left(2x^2+2x+1\right)}{5202x^2+13872x+9248}-\frac{11\operatorname{atan}\left(2x+1\right)}{5202x^2+13872x+9248}-\frac{68}{5202x^2+13872x+9248}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+2*x)/(4+3*x)**3,x)
```

```
[Out] 90*x**2*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 45*x**2*log(2*x**2 + 2*
x + 1)/(5202*x**2 + 13872*x + 9248) + 48*x**2*atan(2*x + 1)/(5202*x**2 + 13
872*x + 9248) + 240*x*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 120*x*log
(2*x**2 + 2*x + 1)/(5202*x**2 + 13872*x + 9248) + 128*x*atan(2*x + 1)/(5202
*x**2 + 13872*x + 9248) - 51*x/(5202*x**2 + 13872*x + 9248) + 160*log(3*x +
```

```
 4)/(5202*x**2 + 13872*x + 9248) - 80*log(2*x**2 + 2*x + 1)/(5202*x**2 + 13
872*x + 9248) - 11*atan(2*x + 1)/(5202*x**2 + 13872*x + 9248) - 68/(5202*x*
*2 + 13872*x + 9248)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.70, size = 46, normalized size = 0.72

$$\frac{5\,\ln\left(x+\frac{4}{3}\right)}{289}-\frac{5\,\ln\left(x^2+x+\frac{1}{2}\right)}{578}+\frac{8\,\mathrm{atan}(2\,x+1)}{867}-\frac{\frac{3\,x}{34}+\frac{\mathrm{atan}(2\,x+1)}{6}+\frac{2}{17}}{\left(3\,x+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(2*x + 1)/(3*x + 4)^3,x)
```

```
[Out] (5*log(x + 4/3))/289 - (5*log(x + x^2 + 1/2))/578 + (8*atan(2*x + 1))/867 -
 ((3*x)/34 + atan(2*x + 1)/6 + 2/17)/(3*x + 4)^2
```

3.122 $\int \text{ArcTan}\left(\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=30

$$-\sqrt{1+x}+2\text{ArcTan}\left(\sqrt{1+x}\right)+x\text{ArcTan}\left(\sqrt{1+x}\right)$$

```
[Out] 2*arctan((1+x)^(1/2))+x*arctan((1+x)^(1/2))-(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5311, 81, 65, 209}

$$x\text{ArcTan}\left(\sqrt{x+1}\right)+2\text{ArcTan}\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[1 + x]],x]
```

```
[Out] -Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\sqrt{1+x}\right)\,dx &= x\tan^{-1}\left(\sqrt{1+x}\right)-\int \frac{x}{\sqrt{1+x}\,(4+2x)}\,dx\\ &= -\sqrt{1+x}+x\tan^{-1}\left(\sqrt{1+x}\right)+2\int \frac{1}{\sqrt{1+x}\,(4+2x)}\,dx\\ &= -\sqrt{1+x}+x\tan^{-1}\left(\sqrt{1+x}\right)+4\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx,x,\sqrt{1+x}\right)\\ &= -\sqrt{1+x}+2\tan^{-1}\left(\sqrt{1+x}\right)+x\tan^{-1}\left(\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.73

$$-\sqrt{1+x}+(2+x)\text{ArcTan}\left(\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[1 + x]],x]
[Out] -Sqrt[1 + x] + (2 + x)*ArcTan[Sqrt[1 + x]]
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
derivativedivides	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25
default	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan((1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] (1+x)*arctan((1+x)^(1/2))-(1+x)^(1/2)+arctan((1+x)^(1/2))
```

Maxima [A]
time = 0.48, size = 24, normalized size = 0.80

$$(x+1)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}+\arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

Fricas [A]
time = 1.93, size = 18, normalized size = 0.60

$$(x+2)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 2)*arctan(sqrt(x + 1)) - sqrt(x + 1)
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.87

$$x\,\mathrm{atan}\left(\sqrt{x+1}\right)-\sqrt{x+1}+2\,\mathrm{atan}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan((1+x)**(1/2)),x)
[Out] x*atan(sqrt(x + 1)) - sqrt(x + 1) + 2*atan(sqrt(x + 1))
```

Giac [A]
time = 0.41, size = 24, normalized size = 0.80

$$(x+1)\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}+\arctan\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((1+x)^(1/2)),x, algorithm="giac")
[Out] (x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

Mupad [B]
time = 0.08, size = 24, normalized size = 0.80

$$\mathrm{atan}\left(\sqrt{x+1}\right)-\sqrt{x+1}+\mathrm{atan}\left(\sqrt{x+1}\right)\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2)),x)
[Out] atan((x + 1)^(1/2)) - (x + 1)^(1/2) + atan((x + 1)^(1/2))*(x + 1)
```

3.123 $\int \frac{1}{(1+x^2)(2+\mathbf{ArcTan}(x))}\,dx$

Optimal. Leaf size=5

$$\log(2+\mathrm{ArcTan}(x))$$

```
[Out] ln(2+arctan(x))
```

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5002}

$$\log(\mathrm{ArcTan}(x)+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

```
[Out] Log[2 + ArcTan[x]]
```

Rule 5002

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{(1+x^2)\,(2+\tan^{-1}(x))}\,dx = \log\left(2+\tan^{-1}(x)\right)$$

Mathematica [A]
time = 0.02, size = 5, normalized size = 1.00

$$\log(2+\mathrm{ArcTan}(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

```
[Out] Log[2 + ArcTan[x]]
```

Maple [A]
time = 0.09, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\ln(2+\arctan(x))$	6
default	$\ln(2+\arctan(x))$	6
risch	$\ln(-\ln(-ix+1)+\ln(ix+1)+4i)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(2+arctan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(2+arctan(x))
```

Maxima [A]
time = 0.27, size = 5, normalized size = 1.00

$$\log(\arctan(x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="maxima")
```

```
[Out] log(arctan(x) + 2)
```

Fricas [A]
time = 2.34, size = 5, normalized size = 1.00

$$\log(\arctan(x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="fricas")
```

```
[Out] log(arctan(x) + 2)
```

Sympy [A]
time = 0.13, size = 5, normalized size = 1.00

$$\log(\operatorname{atan}(x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(2+atan(x)),x)
```

```
[Out] log(atan(x) + 2)
```

Giac [A]
time = 0.42, size = 5, normalized size = 1.00

$$\log(\arctan(x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="giac")
```

```
[Out] log(arctan(x) + 2)
```

Mupad [B]
time = 0.22, size = 5, normalized size = 1.00

$$\ln\left(\operatorname{atan}(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(atan(x) + 2)),x)
```

```
[Out] log(atan(x) + 2)
```

3.124 $\int \frac{1}{(a+ax^2)(b-2b\mathbf{ArcTan}(x))}\,dx$

Optimal. Leaf size=17

$$-\frac{\log(1-2\text{ArcTan}(x))}{2ab}$$

```
[Out] -1/2*ln(1-2*arctan(x))/a/b
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{5002\}$

$$-\frac{\log(1-2\text{ArcTan}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

```
[Out] -1/2*Log[1 - 2*ArcTan[x]]/(a*b)
```

Rule 5002

```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rubi steps

$$\int \frac{1}{(a+ax^2)\left(b-2b\tan^{-1}(x)\right)}\,dx = -\frac{\log\left(1-2\tan^{-1}(x)\right)}{2ab}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\log(-1+2\text{ArcTan}(x))}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

```
[Out] -1/2*Log[-1 + 2*ArcTan[x]]/(a*b)
```

Maple [A]
time = 0.08, size = 19, normalized size = 1.12

method	result	size
default	$-\frac{\ln(2b\arctan(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-i-\ln(-ix+1)+\ln(ix+1))}{2ab}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+a)/(b-2*b*arctan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*ln(2*b*arctan(x)-b)/b
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.94

$$-\frac{\log\left(|2\arctan\left(x\right)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

Fricas [A]
time = 2.15, size = 15, normalized size = 0.88

$$-\frac{\log\left(2\arctan\left(x\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*arctan(x) - 1)/(a*b)
```

Sympy [A]
time = 0.24, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{atan}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+a)/(b-2*b*atan(x)),x)
```

```
[Out] -log(atan(x) - 1/2)/(2*a*b)
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.94

$$-\frac{\log\left(|2\arctan\left(x\right)-1|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

Mupad [B]
time = 0.13, size = 15, normalized size = 0.88

$$-\frac{\ln\left(2\,\mathrm{atan}(x)-1\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x^2)*(b - 2*b*atan(x))),x)
```

```
[Out] -log(2*atan(x) - 1)/(2*a*b)
```

3.125 $\int \frac{x+x^3+(1+x)^2\mathbf{ArcTan}(x)}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{1+x}+\frac{\text{ArcTan}(x)^2}{2}+\log(1+x)$$

[Out] 1/(1+x)+1/2*arctan(x)^2+ln(1+x)

Rubi [A]
time = 0.11, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6857, 45, 5004}

$$\frac{\text{ArcTan}(x)^2}{2}+\frac{1}{x+1}+\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]

[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x+x^3+(1+x)^2\tan^{-1}(x)}{(1+x)^2(1+x^2)}\,dx = \int \left(\frac{x}{(1+x)^2}+\frac{\tan^{-1}(x)}{1+x^2}\right)dx$$
$$= \int \frac{x}{(1+x)^2}\,dx + \int \frac{\tan^{-1}(x)}{1+x^2}\,dx$$
$$= \frac{1}{2}\tan^{-1}(x)^2 + \int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)dx$$
$$= \frac{1}{1+x}+\frac{1}{2}\tan^{-1}(x)^2+\log(1+x)$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{1}{1+x}+\frac{\text{ArcTan}(x)^2}{2}+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
[Out] (1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

Maple [A]
time = 0.12, size = 17, normalized size = 0.94

method	result	size
default	$\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\ln(1+x)$	17
risch	$-\frac{\ln(ix+1)^2}{8}+\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{-\ln(-ix+1)^2x+8\ln(1+x)x-\ln(-ix+1)^2+8\ln(1+x)+8}{8+8x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/(1+x)+1/2*arctan(x)^2+ln(1+x)
```

Maxima [A]
time = 0.57, size = 16, normalized size = 0.89

$$\frac{1}{2}\arctan(x)^2+\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x)^2 + 1/(x + 1) + log(x + 1)
```

Fricas [A]
time = 2.58, size = 26, normalized size = 1.44

$$\frac{(x+1)\arctan{(x)}^{2}+2\,(x+1)\log{(x+1)}+2}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x)^2 + 2*(x + 1)*log(x + 1) + 2)/(x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.28, size = 53, normalized size = 2.94

$$\frac{2x\log{(x+1)}}{2x+2}+\frac{x\operatorname{atan}^{2}{(x)}}{2x+2}+\frac{2\log{(x+1)}}{2x+2}+\frac{\operatorname{atan}^{2}{(x)}}{2x+2}+\frac{2}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**3+(1+x)**2*atan(x))/(1+x)**2/(x**2+1),x)
```

```
[Out] 2*x*log(x + 1)/(2*x + 2) + x*atan(x)**2/(2*x + 2) + 2*log(x + 1)/(2*x + 2)
+ atan(x)**2/(2*x + 2) + 2/(2*x + 2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(16) = 32$.
time = 0.41, size = 104, normalized size = 5.78

$$\frac{(x+1)\left(\frac{1}{x+1}-1\right)\arctan\left((x+1)\left(\frac{1}{x+1}-1\right)\right)^{2}+2\,(x+1)\left(\frac{1}{x+1}-1\right)\log\left(-(x+1)\left(\frac{1}{x+1}-1\right)+1\right)-\arctan\left((x+1)\left(\frac{1}{x+1}-1\right)\right)^{2}-2\,\log\left(-(x+1)\left(\frac{1}{x+1}-1\right)+1\right)-2}{2\left((x+1)\left(\frac{1}{x+1}-1\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*((x + 1)*(1/(x + 1) - 1)*arctan((x + 1)*(1/(x + 1) - 1))^2 + 2*(x + 1)*
(1/(x + 1) - 1)*log(-(x + 1)*(1/(x + 1) - 1) + 1) - arctan((x + 1)*(1/(x +
1) - 1))^2 - 2*log(-(x + 1)*(1/(x + 1) - 1) + 1) - 2)/((x + 1)*(1/(x + 1) -
 1) - 1)
```

Mupad [B]
time = 0.13, size = 16, normalized size = 0.89

$$\ln{(x+1)}+\frac{1}{x+1}+\frac{\operatorname{atan}(x)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + atan(x)*(x + 1)^2 + x^3)/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] log(x + 1) + 1/(x + 1) + atan(x)^2/2
```

3.126 $\int -x^3 \mathbf{ArcTan}\left(\sqrt{x} - \sqrt{1+x}\right) dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{\operatorname{ArcTan}\left(\sqrt{x}\right)}{8}-\frac{1}{8}x^4\operatorname{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] 1/24*x^(3/2)-1/40*x^(5/2)+1/56*x^(7/2)+1/16*Pi*x^4+1/8*arctan(x^(1/2))-1/8*
x^4*arctan(x^(1/2))-1/8*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5267, 30, 4946, 52, 65, 209}

$$-\frac{1}{8}x^4\operatorname{ArcTan}\left(\sqrt{x}\right)+\frac{\operatorname{ArcTan}\left(\sqrt{x}\right)}{8}+\frac{x^{7/2}}{56}-\frac{x^{5/2}}{40}+\frac{x^{3/2}}{24}+\frac{\pi x^4}{16}-\frac{\sqrt{x}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -1/8*Sqrt[x] + x^(3/2)/24 - x^(5/2)/40 + x^(7/2)/56 + (Pi*x^4)/16 + ArcTan[
Sqrt[x]]/8 - (x^4*ArcTan[Sqrt[x]])/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -x^3 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right) dx &= -\left(\frac{1}{2}\int x^3 \tan^{-1}\left(\sqrt{x}\right) dx\right)+\frac{1}{4}\pi\int x^3\,dx \\
&= \frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int\frac{x^{7/2}}{1+x}\,dx \\
&= \frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int\frac{x^{5/2}}{1+x}\,dx \\
&= -\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int\frac{x^{3/2}}{1+x}\,dx \\
&= \frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{16}\int\frac{\sqrt{x}}{1+x}\,dx \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{16}\int\frac{}{\sqrt{x}\,(} \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\mathrm{Subst}\Bigg(\int \\
&= -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{1}{8}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}x^4\tan^{-1}\left(\sqrt{\right.\right.
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 0.85

$$
\frac{\mathrm{ArcTan}\left(\sqrt{x}\right)}{8}-\frac{1}{840}\sqrt{x}\left(105-35x+21x^2-15x^3+210x^{7/2}\mathrm{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] ArcTan[Sqrt[x]]/8 - (Sqrt[x]*(105 - 35*x + 21*x^2 - 15*x^3 + 210*x^(7/2)*Ar
cTan[Sqrt[x] - Sqrt[1 + x]]))/840
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.66

method	result	size
default	$-\frac{x^{4} \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{4}+\frac{x^{\frac{7}{2}}}{56}-\frac{x^{\frac{5}{2}}}{40}+\frac{x^{\frac{3}{2}}}{24}-\frac{\sqrt{x}}{8}+\frac{\arctan\left(\sqrt{x}\right)}{8}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4*arctan(x^(1/2)-(1+x)^(1/2))+1/56*x^(7/2)-1/40*x^(5/2)+1/24*x^(3/2)
-1/8*x^(1/2)+1/8*arctan(x^(1/2))
```

Maxima [A]
time = 0.56, size = 44, normalized size = 0.65

$$\frac{1}{4}\,x^{4} \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{56}\,x^{\frac{7}{2}}-\frac{1}{40}\,x^{\frac{5}{2}}+\frac{1}{24}\,x^{\frac{3}{2}}-\frac{1}{8}\,\sqrt{x}+\frac{1}{8}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(sqrt(x + 1) - sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/24*
x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

Fricas [A]
time = 2.73, size = 40, normalized size = 0.59

$$\frac{1}{4}\,\left(x^{4}-1\right) \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{840}\,\left(15\,x^{3}-21\,x^{2}+35\,x-105\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/840*(15*x^3 - 21*x^2 + 35*x
 - 105)*sqrt(x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 44, normalized size = 0.65

$$-\frac{1}{4}x^4\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{56}x^{\frac{7}{2}}-\frac{1}{40}x^{\frac{5}{2}}+\frac{1}{24}x^{\frac{3}{2}}-\frac{1}{8}\sqrt{x}+\frac{1}{8}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^4*arctan(-sqrt(x + 1) + sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/2
4*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

Mupad [B]
time = 1.65, size = 72, normalized size = 1.06

$$\frac{x^{3/2}}{24}-\frac{\sqrt{x}}{8}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{x^5}{2}+\frac{x^4}{2}\right)}{2\,x+2}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\;1\mathrm{i}\right)^2}{x+1}\right)\;1\mathrm{i}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/16 - x^(1/2)/8 + x^(3/2)/24 - x^(5/2)/
40 + x^(7/2)/56 + (atan((x + 1)^(1/2) - x^(1/2))*(x^4/2 + x^5/2))/(2*x + 2)
```

3.127 $\int -x^2 \mathbf{ArcTan}\left(\sqrt{x} - \sqrt{1+x}\right) dx$

Optimal. Leaf size=59

$$\frac{\sqrt{x}}{6} - \frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\pi x^3}{12} - \frac{\operatorname{ArcTan}\left(\sqrt{x}\right)}{6} - \frac{1}{6}x^3\operatorname{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] -1/18*x^(3/2)+1/30*x^(5/2)+1/12*Pi*x^3-1/6*arctan(x^(1/2))-1/6*x^3*arctan(x
^(1/2))+1/6*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5267, 30, 4946, 52, 65, 209}

$$-\frac{1}{6}x^3\operatorname{ArcTan}\left(\sqrt{x}\right) - \frac{\operatorname{ArcTan}\left(\sqrt{x}\right)}{6} + \frac{x^{5/2}}{30} - \frac{x^{3/2}}{18} + \frac{\pi x^3}{12} + \frac{\sqrt{x}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] Sqrt[x]/6 - x^(3/2)/18 + x^(5/2)/30 + (Pi*x^3)/12 - ArcTan[Sqrt[x]]/6 - (x^
3*ArcTan[Sqrt[x]])/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -x^2 \tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -\left(\frac{1}{2}\int x^2\tan^{-1}\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int x^2\,dx\\
&= \frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int\frac{x^{5/2}}{1+x}\,dx\\
&= \frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int\frac{x^{3/2}}{1+x}\,dx\\
&= -\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{12}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{12}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\text{Subst}\left(\int\frac{1}{1+x^2}\,d\right.\\
&= \frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}x^3\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.90

$$\frac{1}{90}\left(-15\text{ArcTan}\left(\sqrt{x}\right)-\sqrt{x}\left(-15+5x-3x^2+30x^{5/2}\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (-15*ArcTan[Sqrt[x]] - Sqrt[x]*(-15 + 5*x - 3*x^2 + 30*x^(5/2)*ArcTan[Sqrt[
x] - Sqrt[1 + x]]))/90
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.68

method	result	size
default	$-\frac{x^{3} \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3}+\frac{x^{\frac{5}{2}}}{30}-\frac{x^{\frac{3}{2}}}{18}+\frac{\sqrt{x}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{6}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*arctan(x^(1/2)-(1+x)^(1/2))+1/30*x^(5/2)-1/18*x^(3/2)+1/6*x^(1/2)-
1/6*arctan(x^(1/2))
```

Maxima [A]
time = 0.54, size = 39, normalized size = 0.66

$$\frac{1}{3}\,x^{3} \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{30}\,x^{\frac{5}{2}}-\frac{1}{18}\,x^{\frac{3}{2}}+\frac{1}{6}\,\sqrt{x}-\frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x + 1) - sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6*s
qrt(x) - 1/6*arctan(sqrt(x))
```

Fricas [A]
time = 3.38, size = 35, normalized size = 0.59

$$\frac{1}{3}\,\left(x^{3}+1\right) \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{90}\,\left(3\,x^{2}-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/90*(3*x^2 - 5*x + 15)*sqrt(
x)
```

Sympy [A]
time = 163.51, size = 46, normalized size = 0.78

$$\frac{x^{\frac{5}{2}}}{30}-\frac{x^{\frac{3}{2}}}{18}+\frac{\sqrt{x}}{6}-\frac{x^{3} \operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] x**(5/2)/30 - x**(3/2)/18 + sqrt(x)/6 - x**3*atan(sqrt(x) - sqrt(x + 1))/3
- atan(sqrt(x))/6
```

Giac [A]
time = 0.44, size = 39, normalized size = 0.66

$$-\frac{1}{3}x^3\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{30}x^{\frac{5}{2}}-\frac{1}{18}x^{\frac{3}{2}}+\frac{1}{6}\sqrt{x}-\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arctan(-sqrt(x + 1) + sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6
*sqrt(x) - 1/6*arctan(sqrt(x))
```

Mupad [B]
time = 0.94, size = 65, normalized size = 1.10

$$\frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{2\,x^4}{3}+\frac{2\,x^3}{3}\right)}{2\,x+2}+\frac{\ln\left(\frac{{\left(\sqrt{x}-\mathrm{i}\right)}^2}{x+1}\right)\,1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/12 + x^(1/2)/6 - x^(3/2)/18 + x^(5/2)/30
 + (atan((x + 1)^(1/2) - x^(1/2))*((2*x^3)/3 + (2*x^4)/3))/(2*x + 2)
```

3.128 $\int -x\textbf{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4}-\frac{1}{4}x^2\text{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] 1/12*x^(3/2)+1/8*Pi*x^2+1/4*arctan(x^(1/2))-1/4*x^2*arctan(x^(1/2))-1/4*x^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5267, 30, 4946, 52, 65, 209}

$$-\frac{1}{4}x^2\text{ArcTan}\left(\sqrt{x}\right)+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{\sqrt{x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] -1/4*Sqrt[x] + x^(3/2)/12 + (Pi*x^2)/8 + ArcTan[Sqrt[x]]/4 - (x^2*ArcTan[Sq
rt[x]])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -x\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)dx &= -\left(\frac{1}{2}\int x\tan^{-1}\left(\sqrt{x}\right)dx\right)+\frac{1}{4}\pi\int x\,dx\\
&= \frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{x^{3/2}}{1+x}\,dx\\
&= \frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{8}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{8}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{}\right.\\
&= -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}x^2\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.96

$$\frac{1}{12}\left(3\mathrm{ArcTan}\left(\sqrt{x}\right)-\sqrt{x}\left(3-x+6x^{3/2}\mathrm{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

```
[Out] (3*ArcTan[Sqrt[x]] - Sqrt[x]*(3 - x + 6*x^(3/2)*ArcTan[Sqrt[x] - Sqrt[1 + x
]]))/12
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.70

method	result	size
default	$-\frac{x^{2} \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*arctan(x^(1/2)-(1+x)^(1/2))+1/12*x^(3/2)-1/4*x^(1/2)+1/4*arctan(x^
(1/2))
```

Maxima [A]
time = 0.54, size = 34, normalized size = 0.68

$$\frac{1}{2} x^{2} \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12} x^{\frac{3}{2}}-\frac{1}{4} \sqrt{x}+\frac{1}{4} \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*ar
ctan(sqrt(x))
```

Fricas [A]
time = 2.65, size = 28, normalized size = 0.56

$$\frac{1}{2} \left(x^{2}-1\right) \arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12} (x-3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*(x - 3)*sqrt(x)
```

Sympy [A]
time = 96.43, size = 39, normalized size = 0.78

$$\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}-\frac{x^{2} \operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{2}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] x**(3/2)/12 - sqrt(x)/4 - x**2*atan(sqrt(x) - sqrt(x + 1))/2 + atan(sqrt(x)
)/4
```

Giac [A]
time = 0.42, size = 34, normalized size = 0.68

$$-\frac{1}{2}\,x^{2}\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{12}\,x^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{x}+\frac{1}{4}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x^2*arctan(-sqrt(x + 1) + sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*
arctan(sqrt(x))
```

Mupad [B]
time = 0.85, size = 58, normalized size = 1.16

$$\frac{x^{3/2}}{12}-\frac{\sqrt{x}}{4}+\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(x^3+x^2\right)}{2\,x+2}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/8 - x^(1/2)/4 + x^(3/2)/12 + (atan((x
+ 1)^(1/2) - x^(1/2))*(x^2 + x^3))/(2*x + 2)
```

3.129 $\int -\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}-\frac{1}{2}x\text{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] 1/4*Pi*x-1/2*arctan(x^(1/2))-1/2*x*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5267, 8, 4930, 52, 65, 209}

$$-\frac{1}{2}x\text{ArcTan}\left(\sqrt{x}\right)-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{\pi x}{4}+\frac{\sqrt{x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -\left(\frac{1}{2}\int \tan^{-1}\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int 1\,dx\\
&= \frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 31, normalized size = 0.84

$$
\frac{\sqrt{x}}{2}-(1+x)\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
[Out] Sqrt[x]/2 - (1 + x)*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

Maple [A]
time = 0.01, size = 28, normalized size = 0.76

method	result	size
default	$-x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*arctan(x^(1/2)-(1+x)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

Maxima [A]
time = 0.60, size = 26, normalized size = 0.70

$$x\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 4.93, size = 22, normalized size = 0.59

$$(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

Sympy [A]
time = 43.08, size = 29, normalized size = 0.78

$$\frac{\sqrt{x}}{2}-x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.73

$$-x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B]
time = 0.89, size = 40, normalized size = 1.08

$$x\,\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] x*atan((x + 1)^(1/2) - x^(1/2)) - (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 +
x^(1/2)/2
```

3.130 $\int -\frac{\mathbf{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=42

$$\frac{1}{4}\pi\log(x)-\frac{1}{2}i\mathrm{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\mathrm{PolyLog}\left(2,i\sqrt{x}\right)$$

```
[Out] 1/4*Pi*ln(x)-1/2*I*polylog(2,-I*x^(1/2))+1/2*I*polylog(2,I*x^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5267, 29, 4944, 4940, 2438}

$$-\frac{1}{2}i\mathrm{Li}_2\left(-i\sqrt{x}\right)+\frac{1}{2}i\mathrm{Li}_2\left(i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] (Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x]
]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x}\,dx\\
&= \frac{1}{4}\pi\log(x)-\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \sqrt{x}\right)+\frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+}{x}\right.\\
&= \frac{1}{4}\pi\log(x)-\frac{1}{2}i\text{Li}_2\left(-i\sqrt{x}\right)+\frac{1}{2}i\text{Li}_2\left(i\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 84, normalized size = 2.00

$$-\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)\log(x)+\frac{1}{4}i\left(\left(\log\left(1-i\sqrt{x}\right)-\log\left(1+i\sqrt{x}\right)\right)\log(x)-2\text{PolyLog}\left(2,-i\sqrt{x}\right)+2\text{PolyLog}\left(2,i\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

```
[Out] -(ArcTan[Sqrt[x] - Sqrt[1 + x]]*Log[x]) + (I/4)*((Log[1 - I*Sqrt[x]] - Log[
1 + I*Sqrt[x]])*Log[x] - 2*PolyLog[2, (-I)*Sqrt[x]] + 2*PolyLog[2, I*Sqrt[x
]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(28) = 56.
time = 0.20, size = 194, normalized size = 4.62

method	result
default	$2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\ln\left(1-\frac{\left(1+i\left(\sqrt{x}-\sqrt{1+x}\right)\right)^4}{\left(\left(\sqrt{x}-\sqrt{1+x}\right)^2+1\right)^2}\right)-2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\ln\left(1\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1-(1+I*(x^(1/2)-(1+x)^(1/2)))^4/((x^(1/2)-
(1+x)^(1/2))^2+1)^2)-2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)
^(1/2)))^4/((x^(1/2)-(1+x)^(1/2))^2+1)^2)+1/2*I*dilog(1+(1+I*(x^(1/2)-(1+x)
^(1/2)))^4/((x^(1/2)-(1+x)^(1/2))^2+1)^2)-1/2*I*dilog(1-(1+I*(x^(1/2)-(1+x)
^(1/2)))^4/((x^(1/2)-(1+x)^(1/2))^2+1)^2)
```

Maxima [A]
time = 0.48, size = 43, normalized size = 1.02

$$\frac{1}{4}\,\pi\log\left(x+1\right)+\arctan\left(\sqrt{x+1}-\sqrt{x}\right)\log\left(x\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(i\,\sqrt{x}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-i\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*log(x + 1) + arctan(sqrt(x + 1) - sqrt(x))*log(x) + 1/2*I*dilog(I*sq
rt(x) + 1) - 1/2*I*dilog(-I*sqrt(x) + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x + 1) - sqrt(x))/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctan(-sqrt(x + 1) + sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x,x)
[Out] int(atan((x + 1)^(1/2) - x^(1/2))/x, x)
```

3.131 $\int -\frac{\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=41

$$-\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2x}$$

```
[Out] -1/4*Pi/x+1/2*arctan(x^(1/2))+1/2*arctan(x^(1/2))/x+1/2/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2x}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{1}{2\sqrt{x}}-\frac{\pi}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] -1/4*Pi/x + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x]]/(2*x)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^2}\,dx\\
&= -\frac{\pi}{4x}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.98

$$
\frac{1}{2\sqrt{x}}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

```
[Out] 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x] - Sqrt[1 + x]]/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.
time = 0.02, size = 57, normalized size = 1.39

method	result
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{1+x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\ln\left(\sqrt{1+x}-1\right)}{4}-\frac{\ln\left(1+\sqrt{1+\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/2)-(1+x)^(1/2))/x+1/2/x^(1/2)+1/2*arctanh((1+x)^(1/2))+1/2*arct
an(x^(1/2))+1/4*ln((1+x)^(1/2)-1)-1/4*ln(1+(1+x)^(1/2))
```

Maxima [A]
time = 0.53, size = 29, normalized size = 0.71

$$-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(x + 1) - sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 4.70, size = 28, normalized size = 0.68

$$-\frac{2(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\sqrt{x}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) - sqrt(x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(31) = 62$.
time = 33.70, size = 537, normalized size = 13.10

$$-\frac{2x^{\frac{5}{2}}\sqrt{x+1}\,\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}+\frac{x^{\frac{5}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}-\frac{4x^{\frac{3}{2}}\sqrt{x+1}\,\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}+\frac{x^{\frac{3}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}-\frac{2\sqrt{x}\sqrt{x+1}\,\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}+\frac{2x^{3}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}-\frac{x^{2}\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}+\frac{4x^{2}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}-\frac{x\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}+\frac{2x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^{3}+2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**2,x)
```

```
[Out] -2*x**(5/2)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1
) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + x**(5/2)/(-2*x**(5/2)*sqrt(
x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) - 4*x**(3/2)*sqrt(x + 1)
*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x +
 1) + 2*x**3 + 2*x**2) + x**(3/2)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqr
t(x + 1) + 2*x**3 + 2*x**2) - 2*sqrt(x)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) +
 2*x**3*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*s
qrt(x + 1) + 2*x**3 + 2*x**2) - x**2*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) -
 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + 4*x**2*atan(sqrt(x) - sqrt(x +
 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) -
 x*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 +
 2*x**2) + 2*x*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**
(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2)
```

Giac [A]
time = 0.41, size = 28, normalized size = 0.68

$$\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] arctan(-sqrt(x + 1) + sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

Mupad [B]
time = 1.41, size = 44, normalized size = 1.07

$$-\frac{\operatorname{atan}\left(\sqrt{x+1}-\sqrt{x}\right)-\frac{\sqrt{x}}{2}}{x}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^2,x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 - (atan((x + 1)^(1/2) - x^(1/2)) - x
^(1/2)/2)/x
```

3.132 $\int -\frac{\mathbf{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$-\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4x^2}$$

```
[Out] -1/8*Pi/x^2+1/12/x^(3/2)-1/4*arctan(x^(1/2))+1/4*arctan(x^(1/2))/x^2-1/4/x^
(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4x^2}-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{4}+\frac{1}{12x^{3/2}}-\frac{\pi}{8x^2}-\frac{1}{4\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -1/8*Pi/x^2 + 1/(12*x^(3/2)) - 1/(4*Sqrt[x]) - ArcTan[Sqrt[x]]/4 + ArcTan[S
qrt[x]]/(4*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx &= -\left(\frac{1}{2}\int\frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx\right)+\frac{1}{4}\pi\int\frac{1}{x^3}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int\frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}+\frac{1}{8}\int\frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{8}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}-\frac{1}{4}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right.\\
&= -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{1}{4}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{4x^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.96

$$-\frac{\sqrt{x}\,(-1+3x)+3x^2\text{ArcTan}\left(\sqrt{x}\right)-6\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

```
[Out] -1/12*(Sqrt[x]*(-1 + 3*x) + 3*x^2*ArcTan[Sqrt[x]] - 6*ArcTan[Sqrt[x] - Sqrt
[1 + x]])/x^2
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.70

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2x^{2}}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x^(1/2)-(1+x)^(1/2))/x^2+1/12/x^(3/2)-1/4/x^(1/2)-1/4*arctan(x^(
1/2))
```

Maxima [A]
time = 0.54, size = 34, normalized size = 0.68

$$-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{2\,x^{2}}+\frac{1}{12\,x^{\frac{3}{2}}}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4/sqrt(x) - 1/2*arctan(sqrt(x + 1) - sqrt(x))/x^2 + 1/12/x^(3/2) - 1/4*a
rctan(sqrt(x))
```

Fricas [A]
time = 5.29, size = 35, normalized size = 0.70

$$\frac{6\,(x^{2}-1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{12\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(42) = 84$.
time = 129.50, size = 755, normalized size = 15.10

$$\frac{6x^{\frac{7}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{3x^{\frac{7}{2}}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{6x^{\frac{5}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{2x^{\frac{5}{2}}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{6x^{\frac{3}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{x^{\frac{3}{2}}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{6\sqrt{x}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{6x^{4}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{3x^{3}\sqrt{x+1}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{6x^{3}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{2x^{2}\sqrt{x+1}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{6x^{2}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}-\frac{x\sqrt{x+1}}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}+\frac{6x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-12x^{\frac{7}{2}}\sqrt{x+1}-12x^{\frac{5}{2}}\sqrt{x+1}+12x^{4}+12x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**3,x)
```

```
[Out] 6*x**(7/2)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1
) - 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) - 3*x**(7/2)/(-12*x**(7/2)
*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) + 6*x**(5/2)*sq
rt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/
2)*sqrt(x + 1) + 12*x**4 + 12*x**3) - 2*x**(5/2)/(-12*x**(7/2)*sqrt(x + 1)
- 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) - 6*x**(3/2)*sqrt(x + 1)*ata
n(sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1
) + 12*x**4 + 12*x**3) + x**(3/2)/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*s
qrt(x + 1) + 12*x**4 + 12*x**3) - 6*sqrt(x)*sqrt(x + 1)*atan(sqrt(x) - sqrt
(x + 1))/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12
*x**3) - 6*x**4*atan(sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1) - 12*
x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) + 3*x**3*sqrt(x + 1)/(-12*x**(7/2
)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) - 6*x**3*atan(
sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1)
+ 12*x**4 + 12*x**3) + 2*x**2*sqrt(x + 1)/(-12*x**(7/2)*sqrt(x + 1) - 12*x*
*(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3) + 6*x**2*atan(sqrt(x) - sqrt(x + 1)
)/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3)
- x*sqrt(x + 1)/(-12*x**(7/2)*sqrt(x + 1) - 12*x**(5/2)*sqrt(x + 1) + 12*x*
*4 + 12*x**3) + 6*x*atan(sqrt(x) - sqrt(x + 1))/(-12*x**(7/2)*sqrt(x + 1) -
 12*x**(5/2)*sqrt(x + 1) + 12*x**4 + 12*x**3)
```

Giac [A]
time = 0.43, size = 34, normalized size = 0.68

$$-\frac{3x-1}{12x^{\frac{3}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{2x^2}-\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*x - 1)/x^(3/2) + 1/2*arctan(-sqrt(x + 1) + sqrt(x))/x^2 - 1/4*arct
an(sqrt(x))
```

Mupad [B]
time = 1.36, size = 49, normalized size = 0.98

$$-\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{12}+\frac{x^{3/2}}{4}}{x^2}+\frac{\ln\left(\frac{\left(\sqrt{x}-\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^3,x)
```

```
[Out] (log((x^(1/2) - 1i)^2/(x + 1))*1i)/8 - (atan((x + 1)^(1/2) - x^(1/2))/2 - x
^(1/2)/12 + x^(3/2)/4)/x^2
```

3.133 $\int -\frac{\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx$

Optimal. Leaf size=59

$$-\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{6}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{6x^3}$$

```
[Out] -1/12*Pi/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6*arctan(x^(1/2))+1/6*arctan(x^(1/
2))/x^3+1/6/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5267, 30, 4946, 53, 65, 209}

$$\frac{\text{ArcTan}\left(\sqrt{x}\right)}{6x^3}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{6}-\frac{1}{18x^{3/2}}+\frac{1}{30x^{5/2}}-\frac{\pi}{12x^3}+\frac{1}{6\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] -1/12*Pi/x^3 + 1/(30*x^(5/2)) - 1/(18*x^(3/2)) + 1/(6*Sqrt[x]) + ArcTan[Sqr
t[x]]/6 + ArcTan[Sqrt[x]]/(6*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx &= -\left(\frac{1}{2}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^4}\,dx\right)+\frac{1}{4}\pi\int \frac{1}{x^4}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{7/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}-\frac{1}{12}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{12}\int \frac{1}{\sqrt{x}\,(1+x}\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}+\frac{1}{6}\text{Subst}\Bigg(\int \frac{1}{1+}\\
&= -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{1}{6}\tan^{-1}\left(\sqrt{x}\right)+\frac{\tan^{-1}\left(\sqrt{x}\right)}{6x^3}
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 51, normalized size = 0.86

$$\frac{1}{90}\left(-\frac{-3+5x-15x^2}{x^{5/2}}+15\text{ArcTan}\left(\sqrt{x}\right)+\frac{30\text{ArcTan}\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

```
[Out] (-((-3 + 5*x - 15*x^2)/x^(5/2)) + 15*ArcTan[Sqrt[x]] + (30*ArcTan[Sqrt[x] -
 Sqrt[1 + x]])/x^3)/90
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.68

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(x^(1/2)-(1+x)^(1/2))/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6/x^(1/2)+1
/6*arctan(x^(1/2))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 0.66

$$\frac{1}{6\sqrt{x}}-\frac{1}{18x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}+\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6/sqrt(x) - 1/18/x^(3/2) - 1/3*arctan(sqrt(x + 1) - sqrt(x))/x^3 + 1/30/x
^(5/2) + 1/6*arctan(sqrt(x))
```

Fricas [A]
time = 9.51, size = 40, normalized size = 0.68

$$-\frac{30\left(x^3+1\right)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\left(15x^2-5x+3\right)\sqrt{x}}{90x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/90*(30*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) - (15*x^2 - 5*x + 3)*sqrt
(x))/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 39, normalized size = 0.66

$$\frac{15\,x^2-5\,x+3}{90\,x^{\frac{5}{2}}}+\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3}+\frac{1}{6}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/90*(15*x^2 - 5*x + 3)/x^(5/2) + 1/3*arctan(-sqrt(x + 1) + sqrt(x))/x^3 +
1/6*arctan(sqrt(x))
```

Mupad [B]
time = 0.94, size = 56, normalized size = 0.95

$$-\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{30}+\frac{x^{3/2}}{18}-\frac{x^{5/2}}{6}}{x^3}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2))/x^4,x)
```

```
[Out] (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/12 - (atan((x + 1)^(1/2) - x^(1/2))/3
- x^(1/2)/30 + x^(3/2)/18 - x^(5/2)/6)/x^3
```

3.134 $\int \frac{\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)*(-c^2*x^2+a)^(1/2)/c/(1+m)/(d-c^2*d*x^
2/a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5265, 5263}

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$
$$= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [**A**]
time = 0.05, size = 63, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqrt
[d - (c^2*d*x^2)/a])
```

Maple [**A**]
time = 0.27, size = 59, normalized size = 0.94

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^{1+m}\sqrt{-c^2x^2+a}}{(1+m)\sqrt{\frac{d(-c^2x^2+a)}{a}}\;c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)/(1+m)*(-c^2*x^2+a)^(1/2)/(d*(-c^2*x^2+
a)/a)^(1/2)/c
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(57) = 114$.
time = 9.77, size = 126, normalized size = 2.00

$$-\frac{\sqrt{-c^2x^2+a}\,a\left(-\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)^m\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}{acdm+acd-(c^3dm+c^3d)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*(-arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))^m*sqr
t(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))/(a*c*d
*m + a*c*d - (c^3*d*m + c^3*d)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}^m\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**m/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**m/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^m/sqrt(-c^2*d*x^2/a + d), x)
```

<u>**Mupad**</u> [B]
time = 0.73, size = 57, normalized size = 0.90

$$\frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^{m+1}\,\sqrt{a-c^2\,x^2}}{c\,\left(m+1\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^m/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] (atan((c*x)/(a - c^2*x^2)^(1/2))^(m + 1)*(a - c^2*x^2)^(1/2))/(c*(m + 1)*(d
 - (c^2*d*x^2)/a)^(1/2))
```

3.135 $\int \frac{\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/3*arctan(c*x/(-c^2*x^2+a)^(1/2))^3*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5265, 5263}

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d(-c^2x^2+a)}{a}}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3 a}{3\sqrt{-c^2x^2+a}\,dc}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/3/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a)
^(1/2))^3*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x
^2 - a))^2/(c^2*d*x^2 - a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}\left(\frac{cx}{\sqrt{a - c^{2}x^{2}}}\right)}{\sqrt{-d\left(-1 + \frac{c^{2}x^{2}}{a}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))**2/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2),x)
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.136 $\int \frac{\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] 1/2*arctan(c*x/(-c^2*x^2+a)^(1/2))^2*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {5265, 5263}

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{a-c^2x^2}}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$
$$= \frac{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [**A**]
time = 0.02, size = 59, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

```
[Out] (Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d*
x^2)/a])
```

Maple [**A**]
time = 0.09, size = 57, normalized size = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d(-c^2x^2+a)}{a}}\,\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2 a}{2\sqrt{-c^2x^2+a}\,dc}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a)
^(1/2))^2*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^
2 - a))/(c^2*d*x^2 - a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(atan(c*x/sqrt(a - c**2*x**2))/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2),x)
```

```
[Out] int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\,\mathbf{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{a-c^2x^2}\,\log\left(\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

```
[Out] ln(arctan(c*x/(-c^2*x^2+a)^(1/2)))*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(1/
2)
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5265, 5261}

$$\frac{\sqrt{a-c^2x^2}\,\log\left(\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Rule 5261

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= \frac{\sqrt{a-c^2x^2}\log\left(\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Mathematica [**A**]
time = 0.04, size = 55, normalized size = 1.00

$$\frac{\sqrt{a-c^2x^2}\log\left(\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

```
[Out] (Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*d
*x^2)/a])
```

Maple [**A**]
time = 0.62, size = 55, normalized size = 1.00

method	result	size
default	$\frac{\sqrt{\frac{d(-c^2x^2+a)}{a}}\ \ln\left(\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)\right)a}{\sqrt{-c^2x^2+a}\ dc}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*ln(arctan(c*x/(-c^2*x^2+a
)^(1/2)))*a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

Fricas [A]
time = 3.05, size = 83, normalized size = 1.51

$$-\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}\,\log\left(2\,\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)}{c^3dx^2-acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)*log(2*arctan(sqrt(-c^2*x^2
 + a)*c*x/(c^2*x^2 - a)))/(c^3*d*x^2 - a*c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\,\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

Mupad [B]
time = 0.60, size = 49, normalized size = 0.89

$$\frac{\ln\left(\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\right)\,\sqrt{a-c^2\,x^2}}{c\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] (log(atan((c*x)/(a - c^2*x^2)^(1/2)))*(a - c^2*x^2)^(1/2))/(c*(d - (c^2*d*x
^2)/a)^(1/2))
```

3.138 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\,\mathbf{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

[Out] -(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5265, 5263}

$$-\frac{\sqrt{a-c^2x^2}}{c\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]

[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]))

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$= -\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 1.00

$$-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^
2]]))
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.00

method	result	size
default	$-\frac{\sqrt{\frac{d(-c^2x^2+a)}{a}}\,a}{\sqrt{-c^2x^2+a}\,dc\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^2+a
)^(1/2))
```

Maxima [A]
time = 0.41, size = 29, normalized size = 0.51

$$-\frac{\sqrt{a}}{c\sqrt{d}\arctan\left(cx, \sqrt{-c^2x^2+a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a)))
```

Fricas [A]
time = 4.17, size = 82, normalized size = 1.44

$$-\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{(c^3dx^2-acd)\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*arcta
n(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^2), x)
```

Mupad [B]
time = 0.60, size = 51, normalized size = 0.89

$$-\frac{\sqrt{a-c^2\,x^2}}{c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(c*atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)
^(1/2))
```

3.139 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\,\mathbf{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

```
[Out] -1/2*(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {5265, 5263}

$$-\frac{\sqrt{a-c^2x^2}}{2c\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2\sqrt{d-\frac{c^2dx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2
*x^2]]^2)
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$
$$= -\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\tan^{-1}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 1.00

$$-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\,\text{ArcTan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
[Out] -1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2
*x^2]]^2)
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{d(-c^2x^2+a)}{a}}\,a}{2\sqrt{-c^2x^2+a}\,dc\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x,method=_RETU
RNVERBOSE)
[Out] -1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^2
+a)^(1/2))^2
```

Maxima [A]
time = 0.44, size = 29, normalized size = 0.49

$$-\frac{\sqrt{a}}{2\,c\sqrt{d}\,\arctan\left(cx,\sqrt{-c^2x^2+a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a))^2)
```

Fricas [A]
time = 4.19, size = 82, normalized size = 1.39

$$\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{2\,(c^3dx^2-acd)\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*ar
ctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\,\operatorname{atan}^3\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**3/(d-c**2*d*x**2/a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**3),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^3), x)
```

Mupad [B]
time = 0.61, size = 51, normalized size = 0.86

$$-\frac{\sqrt{a-c^2\,x^2}}{2\,c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^3*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

```
[Out] -(a - c^2*x^2)^(1/2)/(2*c*atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2
)/a)^(1/2))
```

3.140 $\int \frac{\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

```
[Out] arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^(1+m)*(-a*e^2/b-e^2*x^2)^(1/2)/e/(1+m)
/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5265, 5263}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
```

```
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.10, size = 66, normalized size = 0.92

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]

[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^(1
 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^m}{\sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [A]
time = 3.15, size = 111, normalized size = 1.54

$$-\frac{\sqrt{bx^2+a}\left(-\arctan\left(\frac{bx\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{bx^2+a}\right)\right)^m\sqrt{-\frac{(bx^2+a)e^2}{b}}\arctan\left(\frac{bx\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{bx^2+a}\right)e^{(-1)}}{(bm+b)x^2+am+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*(-arctan(b*x*sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(b*x^2 + a)))
^m*sqrt(-(b*x^2 + a)*e^2/b)*arctan(b*x*sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(b*x
^2 + a))*e^(-1)/((b*m + b)*x^2 + a*m + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atan}^m\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**m/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**m/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algorit
hm="giac")
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^m/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^m}{\sqrt{b\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2),x)
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2), x)
```

3.141 $\int \frac{\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

```
[Out] 1/3*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5265, 5263}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
```

```
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$
$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.06, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^3)
/(3*e*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^2}{\sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(arctan(b*x*sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(b*x^2 + a))^2/sqrt(b*x
^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{2}\left(\frac{ex}{\sqrt{-\frac{ae^{2}}{b} - e^{2}x^{2}}}\right)}{\sqrt{a + bx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2/sqrt(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2),x)
```

```
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2), x)
```

3.142 $\int \frac{\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}$$

```
[Out] 1/2*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^
2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5265, 5263}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^2)
/(2*e*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)}{\sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

```
[Out] int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-arctan(b*x*sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(b*x^2 + a))/sqrt(b*x^
2 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b} - e^2x^2}}\right)}{\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))/sqrt(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))/sqrt(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.143 $\int \frac{1}{\sqrt{a+bx^2}\,\mathbf{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\,\log\left(\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

```
[Out] ln(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2)))*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^2+
a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5265, 5261}

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\,\log\left(\text{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

```
[Out] (Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]
]])/(e*Sqrt[a + b*x^2])
```

Rule 5261

```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}}{\sqrt{a+bx^2}}$$

$$= \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.91

$$\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\log\left(\mathrm{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
[Out] (Sqrt[-((e^2*(a + b*x^2))/b)]*Log[ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]
]])/(e*Sqrt[a + b*x^2])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)\sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [A]
time = 1.92, size = 62, normalized size = 0.97

$$\frac{\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}\log\left(2\arctan\left(\frac{bx\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{bx^2+a}\right)\right)}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)*log(2*arctan(b*x*sqrt(-(b*x^2 + a)*e^2/b)*e
^(-1)/(b*x^2 + a)))/sqrt(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorit
hm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)),x)
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)), x)
```

3.144 $\int \frac{1}{\sqrt{a+bx^2}\,\mathbf{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

```
[Out] -(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5265, 5263}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),x]
```

```
[Out] -(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^
2)/b) - e^2*x^2]]))
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
```

```
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 0.91

$$\frac{e\sqrt{a+bx^2}}{b\sqrt{-\frac{e^2(a+bx^2)}{b}}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),
x]
```

```
[Out] (e*Sqrt[a + b*x^2])/(b*Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^
2*(a + b*x^2))/b)]])
```

Maple [**F**]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^2\sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [**A**]
time = 2.40, size = 61, normalized size = 0.92

$$\frac{\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{\sqrt{bx^2+a}\,\arctan\left(\frac{bx\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{bx^2+a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(sqrt(b*x^2 + a)*arctan(b*x*sqrt(-(b*x^2 +
a)*e^2/b)*e^(-1)/(b*x^2 + a)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\,\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algor
ithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)),x)
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)), x)
```

3.145 $\int \frac{1}{\sqrt{a+bx^2}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

```
[Out] -1/2*(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x
^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5265, 5263}

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),x]
```

```
[Out] -1/2*Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a
*e^2)/b) - e^2*x^2]]^2)
```

Rule 5263

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

Rule 5265

```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Dist[Sqrt[a + b*x^2]/Sqrt[d + e*x^2], Int[ArcTan[c*
```

```
(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\int \frac{1}{\sqrt{-\frac{ae^2}{b}-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\tan^{-1}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.91

$$-\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}}{2e\sqrt{a+bx^2}\,\text{ArcTan}\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),
x]
```

```
[Out] -1/2*Sqrt[-((e^2*(a + b*x^2))/b)]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((e
^2*(a + b*x^2))/b)]]^2)
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^3 \sqrt{x^2b+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

```
[Out] int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^
2)-_SAGE_VAR_a)
```

Fricas [**A**]
time = 2.58, size = 62, normalized size = 0.91

$$-\frac{\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{2\sqrt{bx^2+a}\,\arctan\left(\frac{bx\sqrt{-\frac{(bx^2+a)e^2}{b}}\,e^{(-1)}}{bx^2+a}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/2*sqrt(-(b*x^2 + a)*e^2/b)*e^(-1)/(sqrt(b*x^2 + a)*arctan(b*x*sqrt(-(b*x
^2 + a)*e^2/b)*e^(-1)/(b*x^2 + a))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^2}\operatorname{atan}^3\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**3/(b*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algor
ithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)}^3\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)),x)
[Out] int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)), x)
```

3.146 $\int \frac{\text{ArcTan}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

Optimal. Leaf size=101

$$\frac{i\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{PolyLog}(2,ic(a+bx))}{2b}-\frac{i\text{PolyLog}(3,-ic(a+bx))}{2b}$$

```
[Out] 1/2*I*ln(d*(b*x+a))*polylog(2,-I*c*(b*x+a))/b-1/2*I*ln(d*(b*x+a))*polylog(2
,I*c*(b*x+a))/b-1/2*I*polylog(3,-I*c*(b*x+a))/b+1/2*I*polylog(3,I*c*(b*x+a)
)/b
```

Rubi [A]
time = 0.20, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4940, 2438, 5317, 2494, 2481, 2421, 6724}

$$\frac{i\text{Li}_2(-ic(a+bx))\log(d(a+bx))}{2b}-\frac{i\text{Li}_2(ic(a+bx))\log(d(a+bx))}{2b}-\frac{i\text{Li}_3(-ic(a+bx))}{2b}+\frac{i\text{Li}_3(ic(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] ((I/2)*Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)])/b - ((I/2)*Log[d*(a +
 b*x)]*PolyLog[2, I*c*(a + b*x)])/b - ((I/2)*PolyLog[3, (-I)*c*(a + b*x)])/
b + ((I/2)*PolyLog[3, I*c*(a + b*x)])/b
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5317

```
Int[(ArcTan[v_]*Log[w_])/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*v]*(Log[w]/(a + b*x)), x], x] - Dist[I/2, Int[Log[1 + I*v]*(Log[w]
/(a + b*x)), x], x] /; FreeQ[{a, b}, x] && LinearQ[v, x] && LinearQ[w, x] &
& EqQ[Simplify[D[v/(a + b*x), x]], 0] && EqQ[Simplify[D[w/(a + b*x), x]], 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(c(a+bx))\log(d(a+bx))}{a+bx}\,dx &= \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-ic(a+bx))}{a+bx}\,dx-\frac{1}{2}i\int \frac{\log(d(a+bx))\\
&= \frac{1}{2}i\int \frac{\log(d(a+bx))\log(1-iac-ibcx)}{a+bx}\,dx-\frac{1}{2}i\int \frac{\log(d(a+bx))\\
&= \frac{i\text{Subst}\left(\int \frac{\log(dx)\log\left(\frac{iabc+b(1-iac)}{b}-icx\right)}{x}\,dx,x,a+bx\right)}{2b}-\frac{i\text{Subst}\left(\int \frac{\log(\\
&= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))\\
&= \frac{i\log(d(a+bx))\text{Li}_2(-ic(a+bx))}{2b}-\frac{i\log(d(a+bx))\text{Li}_2(ic(a+bx))\end{aligned}$$

Mathematica [A]

time = 2.09, size = 166, normalized size = 1.64

$$\frac{2\text{ArcTan}(c(a+bx))\log(a+bx)\log(d(a+bx))+i(\log(a+bx)\log(d(a+bx))\log(1+iac+ibcx)-\log(a+bx)\log(d(a+bx))\log(1-ic(a+bx))+\log(d(a+bx))\text{PolyLog}(2,-ic(a+bx))-\log(d(a+bx))\text{PolyLog}(2,ic(a+bx))-\text{PolyLog}(3,-ic(a+bx))+\text{PolyLog}(3,ic(a+bx)))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

```
[Out] (2*ArcTan[c*(a + b*x)]*Log[a + b*x]*Log[d*(a + b*x)] + I*(Log[a + b*x]*Log[
d*(a + b*x)]*Log[1 + I*a*c + I*b*c*x] - Log[a + b*x]*Log[d*(a + b*x)]*Log[1
 - I*c*(a + b*x)] + Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)] - Log[d*(
a + b*x)]*PolyLog[2, I*c*(a + b*x)] - PolyLog[3, (-I)*c*(a + b*x)] + PolyLo
g[3, I*c*(a + b*x)]))/(2*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 1087, normalized size = 10.76

method	result	size
risch	Expression too large to display	1087

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d*(b*x+a))^3-1/4/b*Pi*csgn(I*d*(b*x+a)
)^3*dilog(-I*(c*(b*x+a)+I))-1/4*I*(-I*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*(b*x+a)
)*csgn(I*d*(b*x+a))+I*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2+I*ln(b*x+a
)*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2-I*ln(b*x+a)*Pi*csgn(I*d*(b*x+a))^3
+2*ln(d)*ln(b*x+a)+ln(b*x+a)^2)/b*ln(1+I*c*(b*x+a))-1/2*I*polylog(3,-I*c*(b
*x+a))/b+1/4/b*ln(-I*(-c*(b*x+a)+I))*ln(b*x+a)*Pi*csgn(I*d*(b*x+a))^3-1/4/b
*ln(-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d*(b*x+a))^3+1/4/b*dilog(
-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2+1/4/b*dilog(-I*c*(b*x+a))*Pi
*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2+1/4/b*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x
+a))^2*dilog(-I*(c*(b*x+a)+I))+1/4/b*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2*dilog
(-I*(c*(b*x+a)+I))-1/4/b*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))*dil
og(-I*(c*(b*x+a)+I))+1/4/b*ln(-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I
*(b*x+a))*csgn(I*d*(b*x+a))^2+1/2*I/b*ln(-I*(-c*(b*x+a)+I))*ln(b*x+a)*ln(d)
-1/2*I/b*ln(-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*ln(d)-1/4/b*ln(-I*(-c*(b*x+
a)+I))*ln(b*x+a)*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2+1/4/b*ln(-I*(-c*(b*
x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2-1/4/b*ln(-I*(-c*
(b*x+a)+I))*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2-1/4/b*dilog(-I*c*(b*
x+a))*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))+1/2*I*polylog(3,I*c*(b
*x+a))/b+1/4*I/b*ln(b*x+a)^2*ln(1+I*c*(b*x+a))+1/2*I/b*ln(b*x+a)*polylog(2,
-I*c*(b*x+a))-1/2*I/b*ln(d)*dilog(-I*(c*(b*x+a)+I))+1/4*I/b*ln(-I*(c*(b*x+a
)+I))*ln(b*x+a)^2-1/2*I/b*ln(b*x+a)*polylog(2,I*c*(b*x+a))-1/4*I/b*ln(b*x+a
)^2*ln(1-I*c*(b*x+a))-1/2*I/b*dilog(-I*c*(b*x+a))*ln(d)+1/4/b*ln(-I*(-c*(b*
x+a)+I))*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))-1/4/b*ln(
```

```
-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(
b*x+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="fricas")
[Out] integral(arctan(b*c*x + a*c)*log(b*d*x + a*d)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log (ad+bdx) \operatorname{atan}(ac+bcx)}{a+bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
[Out] Integral(log(a*d + b*d*x)*atan(a*c + b*c*x)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="giac")
[Out] integrate(arctan((b*x + a)*c)*log((b*x + a)*d)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atan}(c\,(a+b\,x))\,\ln \left(d\,(a+b\,x)\right)}{a+b\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x),x)
[Out] int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x), x)
```

3.147 $\int e^{c(a+bx)}\textbf{ArcTan}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=48

$$\frac{e^{ac+bcx}\text{ArcTan}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sinh(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 5315, 2320, 12, 266}

$$\frac{e^{ac+bcx}\text{ArcTan}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sinh[c*(a + b*x)]])/(b*c) - Log[1 + E^(2*c*(a + b*x
))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tan^{-1}(\sinh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tan^{-1}(\sinh(x))\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\text{sech}(x)\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\tan^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 1.27

$$-\frac{e^{c(a+bx)}\text{ArcTan}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
[Out] -((E^(c*(a + b*x))*ArcTan[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] + Log[
1 + E^(2*c*(a + b*x))])/(b*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.27, size = 1299, normalized size = 27.06

method	result	size
risch	Expression too large to display	1299

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
[Out] 1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+
a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a)
```

```
)-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c
*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(
c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b
*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/
b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x
+a))+2*a/b-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^
2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))
)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2
*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*csg
n(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*
(exp(c*(b*x+a))-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/2/b/c*P
i*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(
-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(
b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a)
)-I)^2)*exp(c*(b*x+a))+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-ln(1+exp(2
*c*(b*x+a)))/b/c+1/2/b/c*exp(c*(b*x+a))*Pi-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)
)*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(e
xp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2
)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(
c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x
+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))-I/b/c*exp(c*(
b*x+a))*ln(exp(c*(b*x+a))-I)
```

Maxima [A]
time = 0.52, size = 48, normalized size = 1.00

$$\frac{\arctan\left(\sinh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}-\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctan(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [A]
time = 3.44, size = 75, normalized size = 1.56

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\sinh\left(bcx+ac\right)\right)-\log\left(\frac{2\,\cosh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2*
cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atan}\left(\sinh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sinh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(sinh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.44, size = 65, normalized size = 1.35

$$\frac{\left(\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}-\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.77, size = 66, normalized size = 1.38

$$\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}\right)}{b\,c}-\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(sinh(a*c + b*c*x)),x)
```

```
[Out] (exp(b*c*x)*exp(a*c)*atan((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))
/2))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

3.148 $\int e^{c(a+bx)}\mathbf{ArcTan}(\cosh(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{e^{ac+bcx}\text{ArcTan}(\cosh(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(cosh(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 5315, 2320, 12, 1261, 646, 31}

$$\frac{e^{ac+bcx}\text{ArcTan}(\cosh(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}-\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Cosh[c*(a + b*x)]])/(b*c) - ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) - ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tan^{-1}(\cosh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tan^{-1}(\cosh(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int\frac{e^x\sinh(x)}{1+\cosh^2(x)}\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int\frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{2\text{Subst}\left(\int\frac{x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int\frac{-1+x}{1+6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{3-2\sqrt{2}+x}\,dx,x\right.}{2bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\cosh(c(a+bx)))}{bc}-\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2bc}\right.}{2bc}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 146, normalized size = 1.42

$$\frac{-4c(a+bx)+2e^{c(a+bx)}\text{ArcTan}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)+7ac\#1^2+7bcx\#1^2-7\log\left(e^{c(a+bx)}-\#1\right)\#1^2}{1+3\#1^2}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

```
[Out] (-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(2*E^(c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b
*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^2)/
(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 1375, normalized size = 13.35

method	result	size
risch	Expression too large to display	1375

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))-1/4/b/c
*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*
(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I
*exp(-c*(b*x+a)))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I*ex
p(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b
/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp
(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(
2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(
-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(
exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*
(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*
(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(
b*x+a)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I*exp(-c*(b*
x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*
x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(
b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b
*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp
(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(exp(-
c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))-1/4/b/
c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c
```

```
*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(
b*x+a))))*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp
(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(
c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))
*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp
(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(-exp(2*c*(b
*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/2/b/c*exp(c*(b*x+a))*Pi-1/
2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)+1/2/b/c*2^(1/2)*ln(exp(2*c
*(b*x+a))+(2^(1/2)-1)^2)+2*a/b+1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))
+1+2*I*exp(c*(b*x+a)))-1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)-1/2/b/c*l
n(exp(2*c*(b*x+a))+(2^(1/2)-1)^2)
```

Maxima [A]
time = 0.49, size = 131, normalized size = 1.27

$$\frac{\arctan\left(\cosh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{\left(-2\,bcx-2\,ac\right)}-3}{2\sqrt{2}+e^{\left(-2\,bcx-2\,ac\right)}+3}\right)}{2\,bc}-\frac{2\left(bcx+ac\right)}{bc}-\frac{\log\left(6\,e^{\left(-2\,bcx-2\,ac\right)}+e^{\left(-4\,bcx-4\,ac\right)}+1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*log(-(2*sqrt(
2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b*c
) - 2*(b*c*x + a*c)/(b*c) - 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x -
4*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.
time = 2.43, size = 221, normalized size = 2.15

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}-3\right)\sinh(bcx+ac)^2+2\sqrt{2}-3}{\cosh(bcx+ac)^2+\sinh(bcx+ac)^2+3}\right)-\log\left(\frac{2\left(\cosh(bcx+ac)^2+\sinh(bcx+ac)^2+3\right)}{\cosh(bcx+ac)^2-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*cos
h(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2 +
2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2*(co
sh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(
b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atan}\left(\cosh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(cosh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(cosh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.44, size = 154, normalized size = 1.50

$$\frac{\left(\sqrt{2}\,e^{(-ac)}\log\left(-\frac{2\sqrt{2}\,e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}\,e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}+\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*e
^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*ar
ctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.31, size = 133, normalized size = 1.29

$$\frac{\ln\left(-8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}-1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\left(\sqrt{2}+1\right)}{2\,b\,c}+\frac{e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(cosh(a*c + b*c*x)),x)
```

```
[Out] (log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^
(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) + 6*2^(1/2)*exp
(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) + (exp(a*c + b*c*x)*atan((exp(b*c*x
)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.149 $\int e^{c(a+bx)}\mathbf{ArcTan}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=180

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{e^{ac+bcx}\text{ArcTan}(\tanh(c(a+bx)))}{bc}-\frac{\log\left(1+e^{2c(a+bx)}-\right.}{2\sqrt{2}\,b}$$

[Out] exp(b*c*x+a*c)*arctan(tanh(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2225, 5315, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\text{ArcTan}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}+\frac{e^{ac+bcx}\text{ArcTan}(\tanh(c(a+bx)))}{bc}-\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}+\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c) - ArcTan[1 + Sqrt[2]*E^(a
*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Tanh[c*(a + b*x)]])/(b
*c) - Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
+ Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
```

```
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\tanh(ac+bcx))\,dx=\frac{\text{Subst}\left(\int e^x\tan^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{2e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}-\frac{2\text{Subst}\left(\int \frac{e^{3x}}{1+e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}-\frac{2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}-\text{Sub}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,e^{ac+bcx}\right)}{2bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\log\left(1-\sqrt{2}\,e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}\,bc}+$$

$$=\frac{\tan^{-1}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}-\frac{\tan^{-1}\left(1+\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{e^{ac+bcx}\tan^{-1}(\text{ta}}{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.07, size = 89, normalized size = 0.49

$$\frac{2e^{c(a+bx)}\text{ArcTan}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
[Out] (2*E^(c*(a + b*x))*ArcTan[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])/(
2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 1355, normalized size = 7.53

method	result	size
risch	Expression too large to display	1355

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp
(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*cs
gn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi
*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x
+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c
*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+
a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+
exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/
(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))
^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a)
)))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))-1/
4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(
2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/2*I/b/c*exp(c*(b*x+a
))*ln(exp(2*c*(b*x+a))+I)-1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(
1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-
1/4/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp
(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))+1/
2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*cs
gn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a)))
)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b
*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))
+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+
a))+1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^3*e
xp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a)
)))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*
(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+e
xp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x
+a))-I)+1/4*I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4*
I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp
(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*exp(c*(b*x+a))*Pi
```

Maxima [A]
time = 0.50, size = 167, normalized size = 0.93

$$\frac{\arctan\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{(bcx+ac)}\right)\right)}{2bc}-\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{(bcx+ac)}\right)\right)}{2bc}+\frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}-\frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2bcx+2ac)}+1\right)}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="maxima")

[Out] arctan(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(153) = 306$.
time = 3.07, size = 431, normalized size = 2.39

$$\frac{4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)+4\arctan\left(\frac{e^{(2\,bcx+2\,ac)}-1}{e^{(2\,bcx+2\,ac)}+1}\right)e^{(bcx+ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="fricas")

[Out] 1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c)) + 4*arctan((e^(2*b*c*x + 2*a*c) - 1)/(e^(2*b*c*x + 2*a*c) + 1))*e^(b*c*x + a*c))/(b*c)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\,\mathrm{atan}\left(\tanh\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c*(b*x+a))*atan(tanh(b*c*x+a*c)),x)

[Out] exp(a*c)*Integral(exp(b*c*x)*atan(tanh(a*c + b*c*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.42, size = 164, normalized size = 0.91

$$\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4\mathrm{i}\right)+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4\mathrm{i}\right)-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4-4\mathrm{i}\right)-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4+4\mathrm{i}\right)+\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,\left(1+1\mathrm{i}\right)}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(tanh(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(exp
(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.150 $\int e^{c(a+bx)}\textbf{ArcTan}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=180

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\text{ArcTan}\left(1+\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{e^{ac+bcx}\text{ArcTan}(\coth(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right.}{2\sqrt{2}}$$

```
[Out] exp(b*c*x+a*c)*arctan(coth(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^(
1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1
+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(b*x
+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {2225, 5315, 12, 2281, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,e^{ac+bcx}\right)}{\sqrt{2}\,bc}+\frac{\text{ArcTan}\left(\sqrt{2}\,e^{ac+bcx}+1\right)}{\sqrt{2}\,bc}+\frac{e^{ac+bcx}\text{ArcTan}(\coth(c(a+bx)))}{bc}+\frac{\log\left(e^{2c(a+bx)}-\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}-\frac{\log\left(e^{2c(a+bx)}+\sqrt{2}\,e^{ac+bcx}+1\right)}{2\sqrt{2}\,bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*E^(a*c + b*c*x)]/(Sqrt[2]*b*c)) + ArcTan[1 + Sqrt[2]*E
^(a*c + b*c*x)]/(Sqrt[2]*b*c) + (E^(a*c + b*c*x)*ArcTan[Coth[c*(a + b*x)]])
/(b*c) + Log[1 + E^(2*c*(a + b*x)) - Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*
c) - Log[1 + E^(2*c*(a + b*x)) + Sqrt[2]*E^(a*c + b*c*x)]/(2*Sqrt[2]*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
```

```
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\coth(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tan^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2e^{3x}}{-1-e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{e^{3x}}{-1-e^{4x}}\,dx,x,ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1-x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1-x^2}{-1-x^4}\,dx,x,e^{ac+bcx}\right)}{bc} - \frac{\text{Sub}}{}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,e^{ac+bcx}\right)}{2bc}$$

$$= \frac{e^{ac+bcx}\tan^{-1}(\coth(c(a+bx)))}{bc} + \frac{\log\left(1-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}\right)}{2\sqrt{2}bc} -$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{\tan^{-1}\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\tan^{-1}(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 89, normalized size = 0.49

$$\frac{2e^{c(a+bx)}\text{ArcTan}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
[Out] (2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))]
 + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 & ]
)/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 1355, normalized size = 7.53

method	result	size
risch	Expression too large to display	1355

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+I))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a
))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+
a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-
1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*
c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp
(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*e
xp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*
csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/
b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b
*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*
(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn
(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-
I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))
-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(
b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*
c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1+I)*
(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn
((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*
Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1
/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-I)-1/4*I/b/c*ln(exp(c*(b*x+a))-
1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)+
1/2*I*2^(1/2))*2^(1/2)-1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/2
))*2^(1/2)+1/4*I/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)+1
/4/b/c*ln(exp(c*(b*x+a))-1/2*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)+1/4/b/c*ln(exp(
c*(b*x+a))-1/2*2^(1/2)+1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2
*2^(1/2)-1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)+1/2*I
*2^(1/2))*2^(1/2)-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-
1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/
2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+I)+1/4/b/c*exp(c*(b*x+a))*Pi
```

Maxima [A]
time = 0.49, size = 167, normalized size = 0.93

$$\frac{\arctan\left(\coth\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{\left(bcx+ac\right)}\right)\right)}{2bc}+\frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{\left(bcx+ac\right)}\right)\right)}{2bc}-\frac{\sqrt{2}\log\left(\sqrt{2}e^{\left(bcx+ac\right)}+e^{\left(2bcx+2ac\right)}+1\right)}{4bc}+\frac{\sqrt{2}\log\left(-\sqrt{2}e^{\left(bcx+ac\right)}+e^{\left(2bcx+2ac\right)}+1\right)}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*x +
 a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*c*x
+ a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(153) = 306$.
time = 3.41, size = 431, normalized size = 2.39

$$-\frac{4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}e^{(bcx+ac)}+\sqrt{2}\sqrt{-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)-\sqrt{2}\,bc\left(\frac{1}{b^4c^4}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^3c^3\left(\frac{1}{b^4c^4}\right)^{\frac{3}{4}}e^{(bcx+ac)}+b^2c^2\sqrt{\frac{1}{b^4c^4}}+e^{(2\,bcx+2\,ac)}\right)-4\arctan\left(\frac{e^{(2\,bcx+2\,ac)}+1}{e^{(2\,bcx+2\,ac)}-1}\right)e^{(bcx+ac)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(1/(b^4*c^4))^(
1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(sqrt(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(
b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*c))*b*c*(1/(b^4
*c^4))^(1/4) - 1) + 4*sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*arctan(-sqrt(2)*b*c*(
1/(b^4*c^4))^(1/4)*e^(b*c*x + a*c) + sqrt(2)*sqrt(-sqrt(2)*b^3*c^3*(1/(b^4*
c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*b*c*x + 2*a*
c))*b*c*(1/(b^4*c^4))^(1/4) + 1) + sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(sqrt
(2)*b^3*c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4))
 + e^(2*b*c*x + 2*a*c)) - sqrt(2)*b*c*(1/(b^4*c^4))^(1/4)*log(-sqrt(2)*b^3*
c^3*(1/(b^4*c^4))^(3/4)*e^(b*c*x + a*c) + b^2*c^2*sqrt(1/(b^4*c^4)) + e^(2*
b*c*x + 2*a*c)) - 4*arctan((e^(2*b*c*x + 2*a*c) + 1)/(e^(2*b*c*x + 2*a*c) -
 1))*e^(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\,\mathrm{atan}\,(\coth\,(ac+bcx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(coth(a*c + b*c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.66, size = 164, normalized size = 0.91

$$\frac{4\,e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{e^{2\,b\,c\,x}\,e^{2\,a\,c}+1}{e^{2\,b\,c\,x}\,e^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4-4\mathrm{i}\right)-e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(-4+4\mathrm{i}\right)+e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,\left(-1+1\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4-4\mathrm{i}\right)+e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,\left(1-\mathrm{i}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,\left(4+4\mathrm{i}\right)-e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,\left(1+1\mathrm{i}\right)}{4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atan(coth(a*c + b*c*x)),x)
```

```
[Out] (2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*
log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2^(
1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(4 +
 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(2*b*
c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.151 $\int e^{c(a+bx)}\mathbf{ArcTan}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=103

$$\frac{e^{ac+bcx}\text{ArcTan}(\text{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arctan(sech(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(1
/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 5315, 2320, 12, 1261, 646, 31}

$$\frac{e^{ac+bcx}\text{ArcTan}(\text{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTan[Sech[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
2*Sqrt[2] + E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
+ E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tan^{-1}(\operatorname{sech}(ac+bcx))\,dx=\frac{\text{Subst}\left(\int e^x\tan^{-1}(\operatorname{sech}(x))\,dx,x,ac+bcx\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{e^x\operatorname{sech}(x)\tanh(x)}{1+\operatorname{sech}^2(x)}\,dx,x,ac+\right.}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{2x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int\frac{x(-1+x^2)}{1+6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{-1+x}{1+6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{3-2\sqrt{2}+x}\,dx,\right.}{2bc}$$

$$=\frac{e^{ac+bcx}\tan^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}+e^{2ac+2b}\right.}{2bc}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 145, normalized size = 1.41

$$\frac{4c(a+bx)+2e^{c(a+bx)}\text{ArcTan}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)-7ac\#1^2-7bcx\#1^2+7\log\left(e^{c(a+bx)}-\#1\right)\#1^2}{1+3\#1^2}\&\right]}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

```
[Out] (4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(
a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a +
 b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#1^2
)/(1 + 3*#1^2) & ])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 842, normalized size = 8.17

method	result
risch	$-\frac{i e^{c(bx+a)}\ln\left(e^{2c(bx+a)}+1+2i e^{c(bx+a)}\right)}{2bc}+\frac{\pi\,\text{csgn}\left(i\left(-e^{2c(bx+a)}-1+2i e^{c(bx+a)}\right)\right)\text{csgn}\left(\frac{i}{1+e^{2c(bx+a)}}\right)\text{csgn}\left(\frac{i\left(-e^{2c(bx+a)}-1+2i e^{c(b}{1+e^{2c(bx+a)}}\right.}{4bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))+1/4/b/c
*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x
+a))))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a)))
)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))
)*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*e
xp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x
+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*P
i*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(I/(1+exp(2*c*(b*x+a)
)))*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*ex
p(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a
))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*
csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp
(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn
(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b
*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c
*(b*x+a))))^3*exp(c*(b*x+a))-1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(2^(1/2)-1
)^2)+1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)-2*a/b+1/2*I/b/c*exp
(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/2/b/c*ln(exp(2*c*(b
*x+a))+(2^(1/2)-1)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)
```

Maxima [A]
time = 0.53, size = 169, normalized size = 1.64

$$\frac{\arctan\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2bcx+2ac)}-3}{2\sqrt{2}+e^{(2bcx+2ac)}+3}\right)}{8bc}+\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2bcx-2ac)}-3}{2\sqrt{2}+e^{(-2bcx-2ac)}+3}\right)}{8bc}+\frac{\log\left(e^{(4bcx+4ac)}+6e^{(2bcx+2ac)}+1\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 3/8*sqrt(2)*log(-(2*sqrt(
2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c)
+ 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-
2*b*c*x - 2*a*c) + 3))/(b*c) + 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x +
 2*a*c) + 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.
time = 3.33, size = 276, normalized size = 2.68

$$\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}+3\right)\sinh(bcx+ac)^2+2\sqrt{2}+3}{\cosh(bcx+ac)^2+\sinh(bcx+ac)^2+3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^2+\sinh(bcx+ac)^2+3\right)}{\cosh(bcx+ac)^2-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
+ sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x
+ a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*
c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2
*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^2 + s
inh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2
 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b
*c*x + a*c)^2)))/(b*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 154, normalized size = 1.50

$$-\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2ac)}-e^{(2bcx+4ac)}-3e^{(2ac)}}{2\sqrt{2}e^{(2ac)}+e^{(2bcx+4ac)}+3e^{(2ac)}}\right)-2\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4bcx+4ac)}+6e^{(2bcx+2ac)}+1\right)\right)e^{(ac)}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2*a
rctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e^(4
*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.82, size = 135, normalized size = 1.31

$$\frac{e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{b\,c}+\frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}-\frac{\ln\left(8\,e^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*atan(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))
/2)))/(b*c) + (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a
+ b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2) + 6
*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.152 $\int e^{c(a+bx)} \mathbf{ArcTan}(\mathbf{csch}(ac+bcx))\,dx$

Optimal. Leaf size=47

$$\frac{e^{ac+bcx}\mathrm{ArcTan}(\mathrm{csch}(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

[Out] exp(b*c*x+a*c)*arctan(csch(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2225, 5315, 2320, 12, 266}

$$\frac{e^{ac+bcx}\mathrm{ArcTan}(\mathrm{csch}(c(a+bx)))}{bc}+\frac{\log\left(e^{2c(a+bx)}+1\right)}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]

[Out] (E^(a*c + b*c*x)*ArcTan[Csch[c*(a + b*x)]])/(b*c) + Log[1 + E^(2*c*(a + b*x))]/(b*c)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tan^{-1}(\operatorname{csch}(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^x\tan^{-1}(\operatorname{csch}(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int e^x\operatorname{sech}(x)\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int\frac{2x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{2\operatorname{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\tan^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 1.21

$$\frac{e^{c(a+bx)}\operatorname{ArcTan}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] + Log
[1 + E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 885, normalized size = 18.83

method	result	size
risch	Expression too large to display	885

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I))^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(
```

```
c*(b*x+a))-I))*csgn(I*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csg
n(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a)
)-I)^2)*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I
*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(
I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/4/b/c*Pi*cs
gn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1)
)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))
+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^2*exp(c
*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*(exp(c*(b*x+a))+I)
^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))
+I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b
*x+a))-1))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(e
xp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2/(
exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1
))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-2*a/b
+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+ln(1+exp(2*c*(b*x+a)))/b/c
```

Maxima [A]
time = 0.55, size = 47, normalized size = 1.00

$$\frac{\arctan\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctan(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c) +
 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.
time = 2.87, size = 131, normalized size = 2.79

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)}{\cosh\left(bcx+ac\right)^{2}+2\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)^{2}-1}\right)+\log\left(\frac{2\,\cosh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] ((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sinh
(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
 + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) -
 sinh(b*c*x + a*c))))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atan(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atan(csch(a*c + b*c*x)), x)
```

Giac [A]
time = 0.45, size = 66, normalized size = 1.40

$$\frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e^
(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.70, size = 67, normalized size = 1.43

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{atan}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*atan(1/((exp(
b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2)))/(b*c)
```

3.153 $\int \frac{\left(a+b\mathbf{ArcTan}(cx^n)\right)(d+e\log(fx^m))}{x}\,dx$

Optimal. Leaf size=163

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\text{PolyLog}(2,-icx^n)}{2n}+\frac{ibe\log(fx^m)\,\text{PolyLog}(2,-icx^n)}{2n}-\frac{ibd\text{PolyLog}(2,icx^n)}{2n}-$$

[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*I*b*d*polylog(2,-I*c*x^n)/n+1/2*I*b*e*ln(f*x^m)*polylog(2,-I*c*x^n)/n-1/2*I*b*d*polylog(2,I*c*x^n)/n-1/2*I*b*e*ln(f*x^m)*polylog(2,I*c*x^n)/n-1/2*I*b*e*m*polylog(3,-I*c*x^n)/n^2+1/2*I*b*e*m*polylog(3,I*c*x^n)/n^2

Rubi [A]
time = 0.43, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$,
Rules used = {2338, 6874, 4944, 4940, 2438, 5127, 5125, 2421, 6724}

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\text{Li}_2(-icx^n)}{2n}-\frac{ibd\text{Li}_2(icx^n)}{2n}+\frac{ibe\text{Li}_2(-icx^n)\log(fx^m)}{2n}-\frac{ibe\text{Li}_2(icx^n)\log(fx^m)}{2n}-\frac{ibem\text{Li}_3(-icx^n)}{2n^2}+\frac{ibem\text{Li}_3(icx^n)}{2n^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]

[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])/n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e*m*PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5125

```
Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[Log[d*x^m]*(Log[1 - I*c*x^n]/x), x], x] - Dist[I/2, Int[Log[d*x^
m]*(Log[1 + I*c*x^n]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 5127

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTan[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_), x
_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcT
an[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx = \int \left(\frac{d(a+b\tan^{-1}(cx^n))}{x} + \frac{e(a+b\tan^{-1}(cx^n))\log(fx^m)}{x}\right)$$

$$= d\int \frac{a+b\tan^{-1}(cx^n)}{x}\,dx + e\int \frac{(a+b\tan^{-1}(cx^n))\log(fx^m)}{x}$$

$$= (ae)\int \frac{\log(fx^m)}{x}\,dx + (be)\int \frac{\tan^{-1}(cx^n)\log(fx^m)}{x}\,dx + \frac{d\mathrm{S}}{}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} + \frac{1}{2}(ibe)\int \frac{\log(fx^m)\log(1-icx^n)}{x}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} + \frac{ibd\mathrm{Li}_2(-icx^n)}{2n} + \frac{ibe\log(fx^m)\mathrm{Li}}{2n}$$

$$= ad\log(x) + \frac{ae\log^2(fx^m)}{2m} + \frac{ibd\mathrm{Li}_2(-icx^n)}{2n} + \frac{ibe\log(fx^m)\mathrm{Li}}{2n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.19, size = 116, normalized size = 0.71

$$-\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2} + \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)(d+e\log(fx^m))}{n} + \frac{1}{2}a\log(x)(2d-em\log(x)+2e\log(fx^m))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^2
*x^(2*n))])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, -(
c^2*x^(2*n))]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log
[f*x^m]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 896, normalized size = 5.50

method	result	size
risch	Expression too large to display	896

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4/n*Pi*dilog(1+
I*c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/n*dilog(1+I*c*x^n)*b*d-1/2*I/n
*dilog(1-I*c*x^n)*b*d+1/4/n*Pi*dilog(1-I*c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)
```

```
^2-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n*ln(f)
*dilog(1-I*c*x^n)*b*e-1/2*I*b*e*m*polylog(3,-I*c*x^n)/n^2-1/4/n*Pi*dilog(1-
I*c*x^n)*b*e*csgn(I*f*x^m)^3+1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csgn(I*f*x^m)^3-
1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)+1/2*I/n*ln(f)*dilog(1+I*c*x^n)*b*e-1/2*
I*e*b*ln(-I*(-c*x^n+I))*ln(x)^2*m+1/2*I*e*b*ln(-I*(-c*x^n+I))*ln(x^m)*ln(x)
+1/2*I*e*b*ln(1+I*c*x^n)*m*ln(x)^2-1/2*I*e*b*ln(1+I*c*x^n)*ln(x^m)*ln(x)+1/
2*I*e*b*ln(-I*(c*x^n+I))*ln(x)^2*m-1/2*I*e*b*ln(-I*(c*x^n+I))*ln(x^m)*ln(x)
-1/2*I*e*b*ln(1-I*c*x^n)*m*ln(x)^2+1/2*I*e*b*ln(x^m)*ln(1-I*c*x^n)*ln(x)-1/
2*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c
*x^n)*m*ln(x)+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4/n*Pi*d
ilog(1-I*c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4/n*Pi*dilog(1+I*
c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/2*I/n*Pi*ln(x^n)*a*e*csgn(
I*f)*csgn(I*f*x^m)^2+1/2*I*e*b/n*dilog(-I*(c*x^n+I))*m*ln(x)+1/2*I*e*b*m/n*
ln(x)*polylog(2,-I*c*x^n)-1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*ln(x^m
)+1/2*I*e*b/n*dilog(-I*c*x^n)*m*ln(x)-1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n
)+1/2*e*a/m*ln(x^m)^2-1/4/n*Pi*dilog(1+I*c*x^n)*b*e*csgn(I*x^m)*csgn(I*f*x^
m)^2-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f*x^m)^3+1/n*ln(f)*ln(x^n)*a*e+1/n*ln(x^
n)*a*d+1/2*I*b*e*m*polylog(3,I*c*x^n)/n^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*m*e*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(c*x^n) - integrate(-1/2*(b*c*m
*n*e^(n*log(x) + 1)*log(x)^2 - 2*b*c*n*e^(n*log(x) + 1)*log(x)*log(x^m) - 2
*(b*c*e*log(f) + b*c*d)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(126) = 252.
time = 3.32, size = 264, normalized size = 1.62

$$\frac{2\,amn^2e\log(x)^2 + 2i\,bme\,\mathrm{polylog}(3, i\,cx^n) - 2i\,bme\,\mathrm{polylog}(3, -i\,cx^n) + 2\left(bmn^2e\log(x)^2 + 2\left(bn^2e\log(f) + bdn^2\right)\log(x)\right)\arctan(cx^n) - 2\left(i\,bmne\log(x) + i\,bne\log(f) + i\,bdn\right)\mathrm{Li}_2(i\,cx^n) - 2\left(-i\,bmne\log(x) - i\,bne\log(f) - i\,bdn\right)\mathrm{Li}_2(-i\,cx^n) + \left(i\,bmn^2e\log(x)^2 - 2\left(-i\,bn^2e\log(f) - i\,bdn^2\right)\log(x)\right)\log(i\,cx^n + 1) + \left(-i\,bmn^2e\log(x)^2 - 2\left(i\,bn^2e\log(f) + i\,bdn^2\right)\log(x)\right)\log(-i\,cx^n + 1) + 4\left(an^2e\log(f) + adn^2\right)\log(x)}{4\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*m*n^2*e*log(x)^2 + 2*I*b*m*e*polylog(3, I*c*x^n) - 2*I*b*m*e*polyl
og(3, -I*c*x^n) + 2*(b*m*n^2*e*log(x)^2 + 2*(b*n^2*e*log(f) + b*d*n^2)*log(
x))*arctan(c*x^n) - 2*(I*b*m*n*e*log(x) + I*b*n*e*log(f) + I*b*d*n)*dilog(I
*c*x^n) - 2*(-I*b*m*n*e*log(x) - I*b*n*e*log(f) - I*b*d*n)*dilog(-I*c*x^n)
+ (I*b*m*n^2*e*log(x)^2 - 2*(-I*b*n^2*e*log(f) - I*b*d*n^2)*log(x))*log(I*c
```

```
*x^n + 1) + (-I*b*m*n^2*e*log(x)^2 - 2*(I*b*n^2*e*log(f) + I*b*d*n^2)*log(x
))*log(-I*c*x^n + 1) + 4*(a*n^2*e*log(f) + a*d*n^2)*log(x))/n^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**n))*(d+e*ln(f*x**m))/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{atan}(c\,x^n))\,\left(d+e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x,x)
[Out] int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```