Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.2-Inverse-cosine/147-5.2.5-Inverse-cosine-functions

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [118]. This is test number [147].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (118)	0.00 (0)
Mathematica	96.61 (114)	3.39 (4)
Maple	66.10 (78)	33.90 (40)
Fricas	43.22 (51)	56.78 (67)
Giac	43.22 (51)	56.78 (67)
Sympy	28.81 (34)	71.19 (84)
Maxima	25.42 (30)	74.58 (88)
Mupad	18.64 (22)	81.36 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

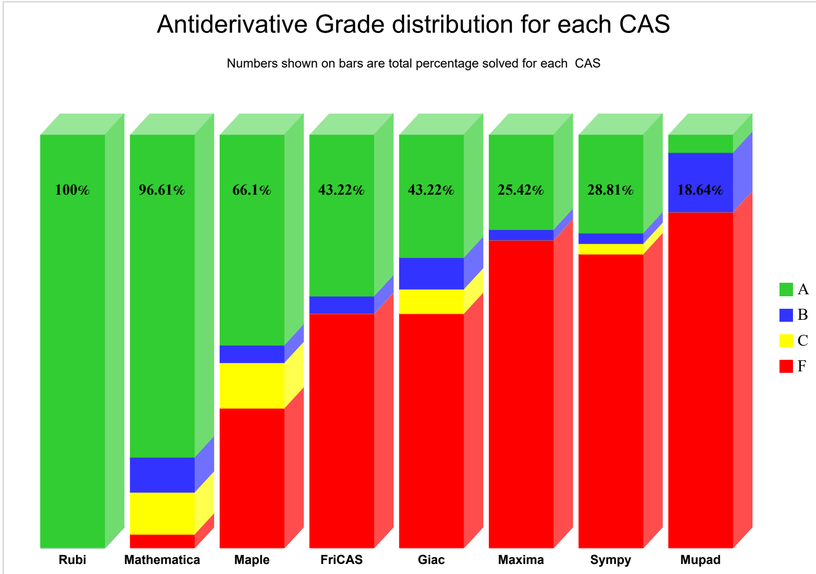
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

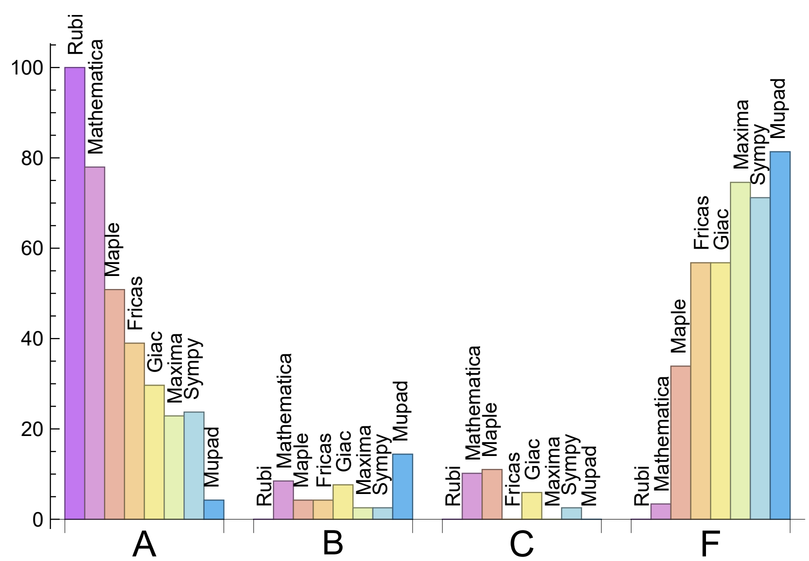
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	77.97	8.47	10.17	3.39
Maple	50.85	4.24	11.02	33.90
Fricas	38.98	4.24	0.00	56.78
Giac	29.66	7.63	5.93	56.78
Sympy	23.73	2.54	2.54	71.19
Maxima	22.88	2.54	0.00	74.58
Mupad	N/A	14.41	0.00	81.36

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	40	100.00 %	0.00 %	0.00 %
Fricas	67	62.69 %	0.00 %	37.31 %
Giac	67	74.63 %	2.99 %	22.39 %
Maxima	88	65.91 %	0.00 %	34.09 %
Sympy	84	86.90 %	8.33 %	4.76 %
Mupad	96	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

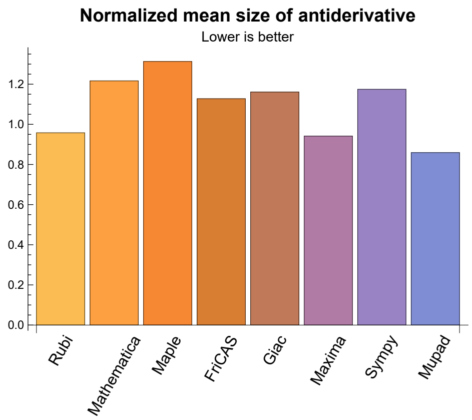
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

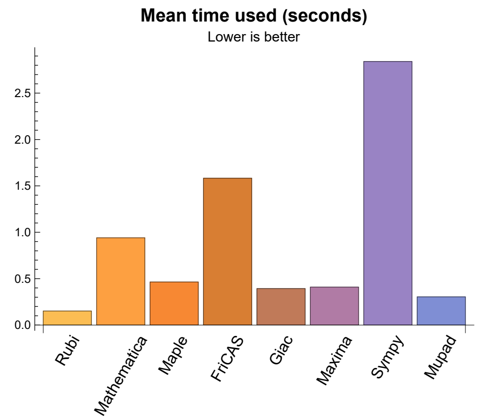
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	186.74	0.96	86.50	1.00
Mathematica	0.94	283.41	1.22	90.00	0.97
Maple	0.46	418.88	1.31	74.00	1.19
Maxima	0.41	58.43	0.94	40.50	0.85
Fricas	1.58	78.41	1.13	42.00	0.87
Sympy	2.84	66.53	1.17	55.50	1.12
Giac	0.39	76.63	1.16	52.00	1.03
Mupad	0.30	36.09	0.86	32.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



Normalized mean size of antiderivative
Lower is better
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad



Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{19, 23, 101, 105, 106\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{4, 5, 9, 13, 17, 18, 20, 21, 78, 79, 85, 86\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

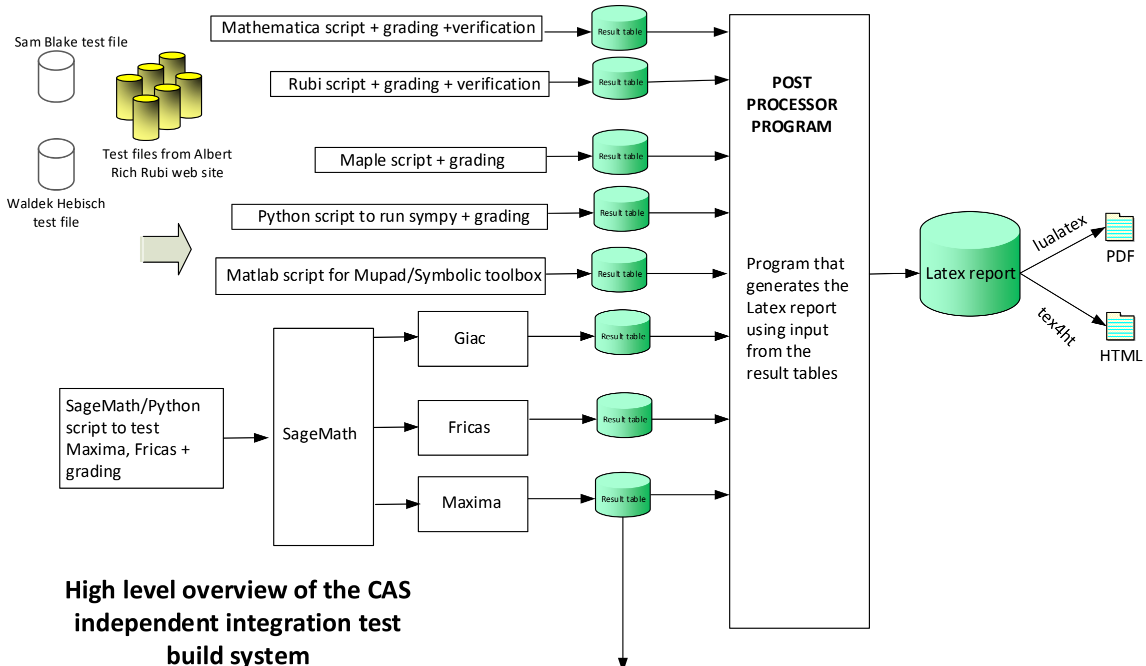
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 19, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 118 }

B grade: { 9, 13, 17, 18, 20, 21, 27, 55, 69, 114 }

C grade: { 37, 38, 39, 40, 41, 42, 43, 44, 48, 50, 52, 115 }

F grade: { 102, 103, 104, 107 }

2.1.3 Maple

A grade: { 4, 17, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 76, 83, 101, 103, 104, 105, 106, 107, 114, 115, 116 }

B grade: { 9, 13, 18, 52, 102 }

C grade: { 1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16 }

F grade: { 20, 21, 22, 51, 70, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 117, 118 }

2.1.4 Maxima

A grade: { 16, 23, 27, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 76, 83, 101, 105, 106 }

B grade: { 24, 25, 26 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

2.1.5 FriCAS

A grade: { 19, 23, 24, 25, 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 53, 54, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 101, 105, 106, 108, 109, 110, 111, 117, 118 }

B grade: { 29, 30, 31, 55, 114 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 56, 63, 69, 70, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 115, 116 }

2.1.6 Sympy

A grade: { 23, 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 54, 55, 57, 59, 60, 61, 62, 65, 66, 67, 68, 71, 105, 108, 109, 110, 111 }

B grade: { 24, 25, 72 }

C grade: { 53, 58, 64 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 56, 63, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 112, 113, 114, 115, 116, 117, 118 }

2.1.7 Giac

A grade: { 19, 23, 25, 26, 27, 29, 32, 33, 34, 35, 36, 46, 47, 49, 53, 57, 58, 59, 60, 61, 62, 64, 68, 71, 72, 76, 83, 101, 105, 106, 108, 109, 110, 111, 116 }

B grade: { 24, 30, 31, 54, 55, 65, 66, 67, 114 }

C grade: { 37, 38, 39, 40, 43, 44, 115 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 41, 42, 45, 48, 50, 51, 52, 56, 63, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 117, 118 }

2.1.8 Mupad

A grade: { 19, 23, 101, 105, 106 }

B grade: { 27, 32, 33, 47, 49, 54, 55, 57, 58, 62, 68, 71, 72, 76, 83, 114, 118 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 56, 59, 60, 61, 63, 64, 65, 66, 67, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	670	670	442	1408	0	0	0	0	-1
N.S.	1	1.00	0.66	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	0.682	1.730	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	320	981	0	0	0	0	-1
N.S.	1	1.00	0.71	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.793	1.088	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	219	628	0	0	0	0	-1
N.S.	1	1.00	0.92	2.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.954	1.102	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	725	725	1095	1209	0	0	0	0	-1
N.S.	1	1.00	1.51	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.322	2.222	0.914	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	1116	1573	0	0	0	0	-1
N.S.	1	1.00	1.31	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.917	6.940	1.302	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	959	959	910	2168	0	0	0	0	-1
N.S.	1	1.00	0.95	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	2.679	1.342	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	591	1550	0	0	0	0	-1
N.S.	1	1.00	0.87	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	1.368	1.016	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	337	1012	0	0	0	0	-1
N.S.	1	1.00	0.91	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	1.012	1.058	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	3034	2132	0	0	0	0	-1
N.S.	1	1.00	2.85	2.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.549	9.947	0.899	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1281	1281	1144	3051	0	0	0	0	-1
N.S.	1	1.00	0.89	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	4.890	1.434	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	940	940	794	2230	0	0	0	0	-1
N.S.	1	1.00	0.84	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	2.897	1.109	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	526	1419	0	0	0	0	-1
N.S.	1	1.00	1.02	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	1.969	1.086	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1637	1637	6216	4692	0	0	0	0	-1
N.S.	1	1.00	3.80	2.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.880	14.515	0.994	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	342	861	0	0	0	0	-1
N.S.	1	1.00	0.76	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.669	1.398	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	266	507	0	0	0	0	-1
N.S.	1	1.00	0.99	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.439	0.946	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	172	247	108	0	0	0	-1
N.S.	1	1.00	1.35	1.94	0.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.237	0.683	0.488	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	930	487	0	0	0	0	-1
N.S.	1	1.00	2.51	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	1.163	0.579	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	1108	1622	0	0	0	0	-1
N.S.	1	1.00	2.23	3.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	3.280	1.342	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.125	0.110	2.351	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	2330	0	0	0	0	0	-1
N.S.	1	1.00	4.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.549	24.056	3.494	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	1248	0	0	0	0	0	-1
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	3.658	3.331	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.018	0.020	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.135	0.145	2.768	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	104	235	333	94	255	242	-1
N.S.	1	1.00	0.76	1.72	2.43	0.69	1.86	1.77	-0.01
time (sec)	N/A	0.139	0.078	0.047	0.486	2.445	0.300	0.409	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	83	161	220	75	170	156	-1
N.S.	1	1.00	0.88	1.71	2.34	0.80	1.81	1.66	-0.01
time (sec)	N/A	0.084	0.058	0.006	0.470	2.670	0.174	0.413	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	78	153	59	104	88	-1
N.S.	1	1.00	0.86	0.98	1.91	0.74	1.30	1.10	-0.01
time (sec)	N/A	0.054	0.033	0.007	0.468	2.826	0.111	0.417	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	154	33	32	41	46	32	88
N.S.	1	1.00	4.28	0.92	0.89	1.14	1.28	0.89	2.44
time (sec)	N/A	0.013	0.065	0.007	0.468	3.380	0.073	0.411	0.543

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	228	199	0	0	0	0	-1
N.S.	1	1.00	1.29	1.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.143	0.829	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	81	0	360	0	79	-1
N.S.	1	1.00	1.25	1.29	0.00	5.71	0.00	1.25	-0.02
time (sec)	N/A	0.055	0.040	0.007	0.000	5.122	0.000	0.410	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	126	124	0	482	0	242	-1
N.S.	1	1.00	1.22	1.20	0.00	4.68	0.00	2.35	-0.01
time (sec)	N/A	0.081	0.125	0.006	0.000	2.306	0.000	0.451	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	168	240	0	580	0	557	-1
N.S.	1	1.00	1.17	1.67	0.00	4.03	0.00	3.87	-0.01
time (sec)	N/A	0.121	0.131	0.005	0.000	1.696	0.000	0.441	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	74	71	0	66	109	78	60
N.S.	1	1.00	0.90	0.87	0.00	0.80	1.33	0.95	0.73
time (sec)	N/A	0.057	0.028	0.098	0.000	1.734	0.151	0.421	0.281

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	48	0	53	63	52	44
N.S.	1	1.00	1.04	1.02	0.00	1.13	1.34	1.11	0.94
time (sec)	N/A	0.037	0.018	0.083	0.000	1.047	0.100	0.418	0.253

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	12	-1
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	1.00	-0.08
time (sec)	N/A	0.014	0.024	0.080	0.000	0.000	0.000	0.430	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	0	0	38	-1
N.S.	1	1.00	1.00	0.92	0.00	0.00	0.00	0.95	-0.02
time (sec)	N/A	0.052	0.041	0.082	0.000	0.000	0.000	0.417	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	53	0	0	0	57	-1
N.S.	1	1.00	1.00	0.82	0.00	0.00	0.00	0.88	-0.02
time (sec)	N/A	0.054	0.038	0.083	0.000	0.000	0.000	0.441	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	140	0	0	0	183	-1
N.S.	1	1.00	0.81	1.26	0.00	0.00	0.00	1.65	-0.01
time (sec)	N/A	0.098	0.037	0.389	0.000	0.000	0.000	0.530	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	105	0	0	0	139	-1
N.S.	1	1.00	0.85	1.18	0.00	0.00	0.00	1.56	-0.01
time (sec)	N/A	0.063	0.029	0.174	0.000	0.000	0.000	0.523	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	90	66	0	0	0	95	-1
N.S.	1	1.00	1.64	1.20	0.00	0.00	0.00	1.73	-0.02
time (sec)	N/A	0.053	0.029	0.169	0.000	0.000	0.000	0.442	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	78	28	0	0	0	51	-1
N.S.	1	1.00	2.36	0.85	0.00	0.00	0.00	1.55	-0.03
time (sec)	N/A	0.024	0.035	0.098	0.000	0.000	0.000	0.458	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	97	84	0	0	0	0	-1
N.S.	1	1.00	1.52	1.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.038	0.174	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	139	120	0	0	0	0	-1
N.S.	1	1.00	1.54	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.209	0.182	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	128	93	0	0	0	167	-1
N.S.	1	1.00	1.21	0.88	0.00	0.00	0.00	1.58	-0.01
time (sec)	N/A	0.114	0.037	0.513	0.000	0.000	0.000	0.519	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	133	95	0	0	0	171	-1
N.S.	1	1.00	1.23	0.88	0.00	0.00	0.00	1.58	-0.01
time (sec)	N/A	0.084	0.075	0.305	0.000	0.000	0.000	0.514	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	92	0	0	0	0	-1
N.S.	1	1.00	0.87	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.030	0.316	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	39	27	-1
N.S.	1	1.00	0.88	0.97	0.88	0.76	1.15	0.79	-0.03
time (sec)	N/A	0.021	0.012	0.437	0.480	1.206	1.067	0.421	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	69	79	41	48	46	42
N.S.	1	1.00	0.94	1.35	1.55	0.80	0.94	0.90	0.82
time (sec)	N/A	0.028	0.021	0.079	0.474	1.236	0.189	0.417	0.302

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	63	79	0	33	48	0	-1
N.S.	1	1.00	1.15	1.44	0.00	0.60	0.87	0.00	-0.02
time (sec)	N/A	0.020	0.110	0.007	0.000	0.293	0.750	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	29
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.83
time (sec)	N/A	0.018	0.011	0.017	0.478	1.535	0.080	0.446	0.328

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	65	0	33	44	0	-1
N.S.	1	1.00	0.79	1.51	0.00	0.77	1.02	0.00	-0.02
time (sec)	N/A	0.023	10.009	0.006	0.000	0.490	0.621	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.024	0.054	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	57	0	11	44	0	-1
N.S.	1	1.00	1.38	1.97	0.00	0.38	1.52	0.00	-0.03
time (sec)	N/A	0.011	0.029	0.006	0.000	0.686	0.660	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	61	56	72	93	95	77	-1
N.S.	1	1.00	1.05	0.97	1.24	1.60	1.64	1.33	-0.02
time (sec)	N/A	0.027	0.035	0.112	0.472	1.543	1.886	0.441	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	39	28	32	48	64	28
N.S.	1	1.00	0.97	1.15	0.82	0.94	1.41	1.88	0.82
time (sec)	N/A	0.013	0.016	0.007	0.468	1.374	0.936	0.438	0.239

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	84	30	45	65	27	55	28
N.S.	1	1.00	3.11	1.11	1.67	2.41	1.00	2.04	1.04
time (sec)	N/A	0.012	0.070	0.007	0.476	1.559	1.065	0.412	0.567

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	77	0	0	0	0	-1
N.S.	1	1.00	1.00	1.28	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.016	0.525	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	31	36	26	31	28
N.S.	1	1.00	1.00	1.07	1.03	1.20	0.87	1.03	0.93
time (sec)	N/A	0.017	0.014	0.004	0.465	2.228	0.424	0.421	0.028

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	47	77	47	100	44	42
N.S.	1	1.00	0.98	0.92	1.51	0.92	1.96	0.86	0.82
time (sec)	N/A	0.025	0.019	0.009	0.468	1.410	1.981	0.439	0.285

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	55	49	49	100	52	-1
N.S.	1	1.00	0.84	0.98	0.88	0.88	1.79	0.93	-0.02
time (sec)	N/A	0.029	0.023	0.009	0.463	3.129	1.909	0.435	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	46	53	52	36	82	52	-1
N.S.	1	1.00	0.59	0.68	0.67	0.46	1.05	0.67	-0.01
time (sec)	N/A	0.020	0.027	0.007	0.475	1.468	4.861	0.406	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	41	41	40	31	66	40	-1
N.S.	1	1.00	0.68	0.68	0.67	0.52	1.10	0.67	-0.02
time (sec)	N/A	0.015	0.021	0.007	0.468	1.639	2.141	0.437	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	43	26	25	24	29	25	35
N.S.	1	1.00	1.16	0.70	0.68	0.65	0.78	0.68	0.95
time (sec)	N/A	0.008	0.013	0.007	0.471	3.441	0.098	0.412	0.640

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	59	0	0	0	0	-1
N.S.	1	1.00	0.96	1.05	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.026	0.375	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	21	22	44	40	-1
N.S.	1	1.00	0.89	0.81	0.78	0.81	1.63	1.48	-0.04
time (sec)	N/A	0.010	0.015	0.004	0.471	0.959	1.862	0.413	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	35	34	28	53	74	-1
N.S.	1	1.00	0.86	0.70	0.68	0.56	1.06	1.48	-0.02
time (sec)	N/A	0.013	0.023	0.013	0.470	1.494	4.662	0.446	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	37	47	46	33	68	106	-1
N.S.	1	1.00	0.54	0.69	0.68	0.49	1.00	1.56	-0.01
time (sec)	N/A	0.016	0.029	0.005	0.478	1.115	11.751	0.433	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	42	59	58	38	80	138	-1
N.S.	1	1.00	0.49	0.69	0.67	0.44	0.93	1.60	-0.01
time (sec)	N/A	0.019	0.031	0.009	0.465	1.970	32.013	0.428	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	20	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.80	0.76	0.76
time (sec)	N/A	0.013	0.007	0.003	0.462	1.241	0.101	0.423	0.396

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	141	84	0	0	0	0	-1
N.S.	1	1.00	2.07	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.101	0.555	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.026	0.044	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	40	39	48	61	39	99
N.S.	1	1.00	0.91	0.85	0.83	1.02	1.30	0.83	2.11
time (sec)	N/A	0.040	0.021	0.022	0.478	1.508	0.203	0.425	0.646

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	0	41	59	76	41	111
N.S.	1	1.00	0.90	0.00	0.85	1.23	1.58	0.85	2.31
time (sec)	N/A	0.040	0.031	0.029	0.482	2.571	25.479	0.406	0.354

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	207	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.153	0.070	0.000	1.011	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	144	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.083	0.056	0.000	0.937	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	91	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	1.44	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.043	0.054	0.000	0.985	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	45	48	0	55	39
N.S.	1	1.00	0.95	1.05	1.05	1.12	0.00	1.28	0.91
time (sec)	N/A	0.027	0.019	0.011	0.471	1.150	0.000	0.435	0.465

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.082	0.056	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	133	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.259	0.054	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	147	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.192	0.055	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	207	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.155	0.054	0.000	0.968	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	162	0	0	144	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.087	0.053	0.000	0.732	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	91	0	0	-1
N.S.	1	1.00	1.56	0.00	0.00	1.44	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.041	0.054	0.000	0.859	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	45	48	0	50	39
N.S.	1	1.00	0.95	1.05	1.05	1.12	0.00	1.16	0.91
time (sec)	N/A	0.026	0.020	0.012	0.467	0.669	0.000	0.418	0.435

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.078	0.056	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	131	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.255	0.053	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	149	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.172	0.058	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.072	1.779	0.056	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.049	0.472	0.058	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.067	0.055	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	114	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.121	0.058	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	177	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.278	0.056	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	234	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.038	0.518	0.056	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	308	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.045	0.379	0.055	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	256	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.044	1.472	0.057	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	200	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.034	0.422	0.062	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	157	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.081	0.055	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	115	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.120	0.056	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	161	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.236	0.065	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	233	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.031	0.416	0.054	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	309	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.039	0.384	0.053	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.066	1.026	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	707	0	0	0	0	-1
N.S.	1	1.00	0.00	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.217	1.329	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	0	401	0	0	0	0	-1
N.S.	1	1.00	0.00	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.358	0.174	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	0	171	0	0	0	0	-1
N.S.	1	1.00	0.00	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.272	0.151	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.073	0.389	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	1.185	0.358	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	102	0	0	0	0	-1
N.S.	1	1.00	0.00	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.543	0.557	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	0	0	55	105	82	-1
N.S.	1	1.00	0.62	0.00	0.00	0.68	1.30	1.01	-0.01
time (sec)	N/A	0.049	0.107	0.016	0.000	1.137	0.484	0.432	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	50	0	0	46	85	69	-1
N.S.	1	1.00	0.61	0.00	0.00	0.56	1.04	0.84	-0.01
time (sec)	N/A	0.045	0.090	0.005	0.000	1.397	0.243	0.434	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	0	0	36	58	44	-1
N.S.	1	1.00	0.73	0.00	0.00	0.88	1.41	1.07	-0.02
time (sec)	N/A	0.025	0.027	0.004	0.000	1.416	0.140	0.415	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	32	0	0	28	37	31	-1
N.S.	1	1.00	0.82	0.00	0.00	0.72	0.95	0.79	-0.03
time (sec)	N/A	0.011	0.026	0.003	0.000	1.370	0.077	0.431	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.042	0.004	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	55	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.047	0.005	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	345	45	0	140	0	95	43
N.S.	1	1.00	7.19	0.94	0.00	2.92	0.00	1.98	0.90
time (sec)	N/A	0.024	0.252	0.208	0.000	4.185	0.000	0.454	0.629

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	56	21	0	0	0	37	-1
N.S.	1	1.00	2.15	0.81	0.00	0.00	0.00	1.42	-0.04
time (sec)	N/A	0.048	0.064	0.678	0.000	0.000	0.000	0.433	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	0	0	0	5	-1
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	1.00	-0.20
time (sec)	N/A	0.041	0.032	0.275	0.000	0.000	0.000	0.441	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	42	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.08	0.00	0.00	-0.03
time (sec)	N/A	0.046	0.034	0.279	0.000	2.727	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	0	0	27	0	0	25
N.S.	1	1.00	0.81	0.00	0.00	0.87	0.00	0.00	0.81
time (sec)	N/A	0.041	0.021	0.293	0.000	3.869	0.000	0.000	0.298

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [102] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	16	12	1.00	31	0.387
2	A	13	8	1.00	31	0.258
3	A	8	6	1.00	29	0.207
4	A	22	19	1.00	31	0.613
5	A	35	22	1.00	31	0.710
6	A	24	17	1.00	31	0.548
7	A	20	12	1.00	31	0.387
8	A	12	9	1.00	29	0.310
9	A	29	23	1.00	31	0.742
10	A	30	18	1.00	31	0.581
11	A	26	15	1.00	31	0.484
12	A	14	10	1.00	29	0.345
13	A	37	28	1.00	31	0.903
14	A	13	7	1.00	31	0.226
15	A	9	7	1.00	31	0.226
16	A	6	5	1.00	29	0.172
17	A	10	7	1.00	31	0.226
18	A	13	10	1.00	31	0.323
19	A	0	0	0.00	0	0.000
20	A	13	9	1.00	35	0.257
21	A	11	8	1.00	33	0.242
22	A	9	7	1.00	25	0.280
23	A	0	0	0.00	0	0.000
24	A	6	6	1.00	10	0.600
25	A	5	5	1.00	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	1.00	8	0.625
27	A	3	3	1.00	6	0.500
28	A	9	6	1.00	10	0.600
29	A	4	4	1.00	10	0.400
30	A	5	5	1.00	10	0.500
31	A	6	6	1.00	10	0.600
32	A	5	4	1.00	8	0.500
33	A	4	4	1.00	8	0.500
34	A	3	3	1.00	8	0.375
35	A	4	4	1.00	8	0.500
36	A	5	5	1.00	8	0.625
37	A	7	6	1.00	10	0.600
38	A	6	6	1.00	10	0.600
39	A	5	5	1.00	10	0.500
40	A	4	4	1.00	10	0.400
41	A	5	5	1.00	10	0.500
42	A	6	6	1.00	10	0.600
43	A	7	7	1.00	14	0.500
44	A	7	7	1.00	15	0.467
45	A	7	7	1.00	19	0.368
46	A	3	3	1.00	14	0.214
47	A	5	5	1.00	10	0.500
48	A	4	4	1.00	10	0.400
49	A	3	3	1.00	8	0.375
50	A	6	6	1.00	6	1.000
51	A	5	5	1.00	10	0.500
52	A	3	3	1.00	10	0.300
53	A	6	6	1.00	10	0.600
54	A	3	3	1.00	8	0.375
55	A	5	5	1.00	6	0.833
56	A	5	5	1.00	10	0.500
57	A	3	3	1.00	10	0.300
58	A	5	5	1.00	10	0.500
59	A	5	4	1.00	10	0.400
60	A	8	6	1.00	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	8	0.750
62	A	6	6	1.00	6	1.000
63	A	5	5	1.00	10	0.500
64	A	3	3	1.00	10	0.300
65	A	4	4	1.00	10	0.400
66	A	5	4	1.00	10	0.400
67	A	6	4	1.00	10	0.400
68	A	3	3	1.00	12	0.250
69	A	5	5	1.00	10	0.500
70	A	5	5	1.00	10	0.500
71	A	4	4	1.00	12	0.333
72	A	4	4	1.00	14	0.286
73	A	3	2	1.00	14	0.143
74	A	5	4	1.00	14	0.286
75	A	2	2	1.00	14	0.143
76	A	4	3	1.00	12	0.250
77	A	1	1	1.00	14	0.071
78	A	1	1	1.00	14	0.071
79	A	2	2	1.00	14	0.143
80	A	3	2	1.00	14	0.143
81	A	5	4	1.00	14	0.286
82	A	2	2	1.00	14	0.143
83	A	4	3	1.00	12	0.250
84	A	1	1	1.00	14	0.071
85	A	1	1	1.00	14	0.071
86	A	2	2	1.00	14	0.143
87	A	2	2	1.00	16	0.125
88	A	2	2	1.00	16	0.125
89	A	1	1	1.00	16	0.062
90	A	1	1	1.00	16	0.062
91	A	1	1	1.00	16	0.062
92	A	2	2	1.00	16	0.125
93	A	2	2	1.00	16	0.125
94	A	2	2	1.00	16	0.125
95	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	1	1.00	16	0.062
97	A	1	1	1.00	16	0.062
98	A	1	1	1.00	16	0.062
99	A	2	2	1.00	16	0.125
100	A	2	2	1.00	16	0.125
101	A	0	0	0.00	0	0.000
102	A	8	8	1.00	40	0.200
103	A	7	7	1.00	40	0.175
104	A	6	7	1.00	38	0.184
105	A	0	0	0.00	0	0.000
106	A	0	0	0.00	0	0.000
107	A	6	6	1.00	10	0.600
108	A	6	4	1.00	10	0.400
109	A	6	4	1.00	10	0.400
110	A	5	4	1.00	8	0.500
111	A	2	2	1.00	6	0.333
112	A	6	5	1.00	10	0.500
113	A	6	4	1.00	10	0.400
114	A	6	6	1.00	10	0.600
115	A	3	3	1.00	19	0.158
116	A	2	2	1.00	17	0.118
117	A	2	2	1.00	26	0.077
118	A	2	2	1.00	26	0.077

Chapter 3

Listing of integrals

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3.1 $\int (f+gx)^3 \sqrt{d-c^2dx^2} (a+b\mathbf{ArcCos}(cx)) \, dx$

Optimal. Leaf size=670

$$-\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}-\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{3bfg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}+\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

```
[Out] 1/2*f^3*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/8*f*g^2*x*(a+b*arccos(c*
x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)
^(1/2)-f^2*g*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/3*g^
3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(-c^2*x^2
+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(-c^2*d*x^2+d)^(
1/2)/c/(-c^2*x^2+1)^(1/2)-2/15*b*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1
)^(1/2)+1/4*b*c*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16*b*f*g^
2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+1/3*b*c*f^2*g*x^3*(-c^2*d*x
^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*
x^2+1)^(1/2)+3/16*b*c*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2
5*b*c*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f^3*(a+b*arccos(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-3/16*f*g^2*(a+b*arccos(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {4862, 4848, 4742, 4738, 30, 4768, 4784, 4796, 272, 45, 4780, 12}

$$\frac{1}{2}f^3x\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))-\frac{f^3\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{4bc\sqrt{1-c^2x^2}}-\frac{f^2g(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{c^2}-\frac{3fg^2x\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{8c^2}+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))+\frac{g^3(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{5c^4}-\frac{g^3(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{3c^4}-\frac{3fg^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{16bc^3\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}+\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}-\frac{3bfg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}+\frac{3bcfg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{bcg^3x^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}-\frac{bg^3x^3\sqrt{d-c^2dx^2}}{45c\sqrt{1-c^2x^2}}-\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2])) - (2*b*g^3*x*Sqrt[
d - c^2*d*x^2])/(15*c^3*Sqrt[1 - c^2*x^2]) + (b*c*f^3*x^2*Sqrt[d - c^2*d*x^
2])/(4*Sqrt[1 - c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[
1 - c^2*x^2]) + (b*c*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(3*Sqrt[1 - c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[1 - c^2*x^2]) + (3*b*c*f*g^2*x^
4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (b*c*g^3*x^5*Sqrt[d - c^2*d
*x^2])/(25*Sqrt[1 - c^2*x^2]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*c^2) + (3*f
*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^3\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)+3f^2gx\sqrt{\quad}\right.}{}$$

$$= \frac{\left(f^3\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}} + \frac{\left(3\right.}{}$$

$$= \frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}$$

$$= -\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{bcf^2gx^3\sqrt{d}}{3\sqrt{1-c}}$$

$$= -\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}-\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-}}{4\sqrt{1-c^2}}$$

Mathematica [A]
time = 0.68, size = 442, normalized size = 0.66

$$\frac{240a\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(-16g^3-c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g^3x^3)\right)-3600ac\sqrt{d}\,f(4c^2f^2+3g^2)\sqrt{1-c^2x^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)-2400bc^2f^2g\sqrt{d-c^2dx^2}\left(9cx+12(1-c^2x^2)^{3/2}\,\mathrm{ArcCos}(cx)-\cos(3\,\mathrm{ArcCos}(cx))\right)+3600bc^3f^3\sqrt{d-c^2dx^2}\left(\cos(2\,\mathrm{ArcCos}(cx))+2\,\mathrm{ArcCos}(cx)(-\mathrm{ArcCos}(cx)+\sin(2\,\mathrm{ArcCos}(cx)))\right)+675bcfg^2\sqrt{d-c^2dx^2}\left(-8\,\mathrm{ArcCos}(cx)^2+\cos(4\,\mathrm{ArcCos}(cx))+4\,\mathrm{ArcCos}(cx)\sin(4\,\mathrm{ArcCos}(cx))\right)-8bg^3\sqrt{d-c^2dx^2}\left(16cx(30+5c^2x^2-9c^4x^4)+15\,\mathrm{ArcCos}(cx)\left(30\sqrt{1-c^2x^2}-5\sin(3\,\mathrm{ArcCos}(cx))-3\sin(5\,\mathrm{ArcCos}(cx))\right)\right)}{28800c^4\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (240*a*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(-16*g^3 - c^2*g*(120*f^2 + 45
*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x
^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2400*b*c^2*f^2*g*Sqrt[d -
 c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] - Cos[3*ArcCos[c*x]
]) + 3600*b*c^3*f^3*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]
*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 675*b*c*f*g^2*Sqrt[d - c^2*d*x^2]*(
-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*ArcCos[c*x]]) -
 8*b*g^3*Sqrt[d - c^2*d*x^2]*(16*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4) + 15*ArcC
os[c*x]*(30*Sqrt[1 - c^2*x^2] - 5*Sin[3*ArcCos[c*x]] - 3*Sin[5*ArcCos[c*x]]
)))/(28800*c^4*Sqrt[1 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.73, size = 1408, normalized size = 2.10

method	result	size
default	Expression too large to display	1408

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/5*a*g^3*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15*a*g^3/d/c^4*(-c^2*d*x^2+d)^(
3/2)-3/4*a*f*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^2*x*(-c^2*d*x^2
+d)^(1/2)+3/8*a*f*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^
2+d)^(1/2))-a*f^2*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f^3*x*(-c^2*d*x^2+d)^(
1/2)+1/2*a*f^3*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
+b*(1/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c
*x)^2*f*(4*c^2*f^2+3*g^2)+1/800*(-d*(c^2*x^2-1))^(1/2)*(16*I*(-c^2*x^2+1)^(
1/2)*x^5*c^5+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^
2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*g^3*(I+5*arccos(c*x))/c^4/(c^2*x^2-1)+3/25
6*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^
2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*g^2*(I+4*ar
ccos(c*x))/c^3/(c^2*x^2-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(4*I*(-c^2*x^2+1)^(
1/2)*x^3*c^3+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*g*(12*I*f^2*
c^2+36*arccos(c*x)*c^2*f^2+I*g^2+3*arccos(c*x)*g^2)/c^4/(c^2*x^2-1)+1/16*(-
d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+
1)^(1/2)-2*c*x)*f^3*(I+2*arccos(c*x))/c/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(
1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(6*I*f^2*c^2+6*arccos(c*x)*c^2*
f^2+I*g^2+arccos(c*x)*g^2)/c^4/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(c^2
*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-6*I*f^2*c^2+6*arccos(c*x)*c^2*f^2-I*g^
2+arccos(c*x)*g^2)/c^4/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*
x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f^3*(-I+2*arccos
(c*x))/c/(c^2*x^2-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*
(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-12*I*f^2*c^2+3
6*arccos(c*x)*c^2*f^2-I*g^2+3*arccos(c*x)*g^2)/c^4/(c^2*x^2-1)+3/256*(-d*(c
^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*g^2*(-I+4*arccos(c
*x))/c^3/(c^2*x^2-1)+1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4-16
*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2+20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-5*
I*(-c^2*x^2+1)^(1/2)*x*c-1)*g^3*(-I+5*arccos(c*x))/c^4/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^3 - 1/15*a*g^3*(3*
(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 3/
8*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d)
+ sqrt(d)*arcsin(c*x)/c^3) - (-c^2*d*x^2 + d)^(3/2)*a*f^2*g/(c^2*d) + sqrt(
d)*integrate((b*g^3*x^3 + 3*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acos}(cx))\,(f+gx)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.2 $\int (f+gx)^2\sqrt{d-c^2dx^2}\,(a+b\mathbf{ArcCos}(cx))\,dx$

Optimal. Leaf size=450

$$-\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}+\frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}+\frac{bcg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

```
[Out] 1/2*f^2*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/8*g^2*x*(a+b*arccos(c*x)
)*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/
2)-2/3*f*g*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/3*b*f*
g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+1/4*b*c*f^2*x^2*(-c^2*d*x^2+d
)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+
1)^(1/2)+2/9*b*c*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/16*b*c*g
^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f^2*(a+b*arccos(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arccos(c*x))^2*(-
c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4862, 4848, 4742, 4738, 30, 4768, 4784, 4796}

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{f^2\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))^2}{4bc\sqrt{1-c^2x^2}}-\frac{2fg(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))}{3c^2}-\frac{g^2x\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))}{8c^2}+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{g^2\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))^2}{16bc^3\sqrt{1-c^2x^2}}+\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}-\frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}+\frac{bcg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[1 - c^2*x^2]) + (b*c*f^2*x^2*Sqr
t[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) - (b*g^2*x^2*Sqrt[d - c^2*d*x^2])/(
16*c*Sqrt[1 - c^2*x^2]) + (2*b*c*f*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[1 - c
^2*x^2]) + (b*c*g^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (f^2*
x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x]))/4 - (2*f*g*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c
^2) - (f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x
^2]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c^3*Sqrt[1 - c
^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
```

```
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
```

```
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^2\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)+2fgx\sqrt{1\right.}{\sqrt{}}$$

$$= \frac{\left(f^2\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}} + \frac{(2}{}$$

$$= \frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\left(a\right.$$

$$= -\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{2bcfgx^3\sqrt{d}}{9\sqrt{1-}}$$

$$= -\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{bg^2x^2\sqrt{d-}}{16c\sqrt{1-}}$$

Mathematica [A]
time = 0.79, size = 320, normalized size = 0.71

$$\frac{48ac\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\left(12c^2f^2x+16fg(-1+c^2x^2)+3g^2x(-1+2c^2x^2)\right)-144a\sqrt{d}\left(4c^2f^2+g^2\right)\sqrt{1-c^2x^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)-64bcfg\sqrt{d-c^2dx^2}\left(9cx+12(1-c^2x^2)^{3/2}\mathrm{ArcCos}(cx)-\cos(3\mathrm{ArcCos}(cx))\right)+144bc^2f^2\sqrt{d-c^2dx^2}\left(\cos(2\mathrm{ArcCos}(cx))+2\mathrm{ArcCos}(cx)(-\mathrm{ArcCos}(cx)+\sin(2\mathrm{ArcCos}(cx)))\right)+9bg^2\sqrt{d-c^2dx^2}\left(-8\mathrm{ArcCos}(cx)^2+\cos(4\mathrm{ArcCos}(cx))+4\mathrm{ArcCos}(cx)\sin(4\mathrm{ArcCos}(cx))\right)}{1152c^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(12*c^2*f^2*x + 16*f*g*(-1 +
c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*c^2*f^2 + g^2)*Sqrt
[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] -
64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x]
```

```
- Cos[3*ArcCos[c*x]]) + 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x
]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 9*b*g^2*Sqrt[d -
c^2*d*x^2]*(-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*Arc
Cos[c*x]]))/(1152*c^3*Sqrt[1 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.09, size = 981, normalized size = 2.18

method	result
default	$-\frac{a\,g^2 x\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{4c^2 d}+\frac{a\,g^2 x\sqrt{-c^2 d\,x^2+d}}{8c^2}+\frac{a\,g^2 d\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8c^2\sqrt{c^2 d}}-\frac{2afg\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3c^2 d}+\frac{a\,f^2 x\sqrt{-\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4*a*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2
)+1/8*a*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)-2/3*a*f*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(-c^2*d*x^2+d)^(1/2)+1/2
*a*f^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/16
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*(4
*c^2*f^2+g^2)+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+
8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*
c*x)*g^2*(I+4*arccos(c*x))/c^3/(c^2*x^2-1)+1/36*(-d*(c^2*x^2-1))^(1/2)*(4*I
*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+
1)*f*g*(I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*(
-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*f^2*(I+2*ar
ccos(c*x))/c/(c^2*x^2-1)-1/4*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x
*c+c^2*x^2-1)*f*g*(arccos(c*x)+I)/c^2/(c^2*x^2-1)-1/4*(-d*(c^2*x^2-1))^(1/2
)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*g*(arccos(c*x)-I)/c^2/(c^2*x^2-1)+
1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-
c^2*x^2+1)^(1/2)-2*c*x)*f^2*(-I+2*arccos(c*x))/c/(c^2*x^2-1)+1/36*(-d*(c^2*
x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2
*x^2+1)^(1/2)*x*c+1)*f*g*(-I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/256*(-d*(c^2*
x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(
1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*g^2*(-I+4*arccos(c*x))/
c^3/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^2 + 1/8*a*g^2*(sqr
t(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcs
in(c*x)/c^3) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*f*g/(c^2*d) + sqrt(d)*integrate
((b*g^2*x^2 + 2*b*f*g*x + b*f^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccos(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acos}{(cx)})\,(f+gx)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {(f+g\,x)}^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.3 $\int (f+gx)\sqrt{d-c^2dx^2}\,(a+b\mathbf{ArcCos}(cx))\,dx$

Optimal. Leaf size=238

$$-\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}+\frac{1}{2}fx\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{g(1-c^2x}{}$$

```
[Out] 1/2*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/3*g*(-c^2*x^2+1)*(a+b*arcc
os(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^
2+1)^(1/2)+1/4*b*c*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/9*b*c*g*
x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f*(a+b*arccos(c*x))^2*(-c^2
*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4862, 4848, 4742, 4738, 30, 4768}

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{f\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))^2}{4bc\sqrt{1-c^2x^2}}-\frac{g(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))}{3c^2}+\frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/3*(b*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2]) + (b*c*f*x^2*Sqrt[d
- c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (b*c*g*x^3*Sqrt[d - c^2*d*x^2])/(9*Sq
rt[1 - c^2*x^2]) + (f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1
- c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
```

```
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int (f+gx)\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))+gx\sqrt{1-c^2x}\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(f\sqrt{d-c^2dx^2}\right)\int \sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\,dx}{\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{d}\right.}{}$$

$$= \frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{g(1-c^2x^2)\sqrt{d-c^2dx^2}}{3c^2}$$

$$= -\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}} + \frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} + \frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.95, size = 219, normalized size = 0.92

$$\frac{12a\sqrt{d-c^2dx^2}\,(3c^2fx+2g(-1+c^2x^2))-36ac\sqrt{d}\,f\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+\frac{2bg\sqrt{d-c^2dx^2}\,\left(-9cx-12(1-c^2x^2)^{3/2}\mathrm{ArcCos}(cx)+\cos(3\mathrm{ArcCos}(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{9bcf\sqrt{d-c^2dx^2}\,\left(-2\mathrm{ArcCos}(cx)^2+\cos(2\mathrm{ArcCos}(cx))+2\mathrm{ArcCos}(cx)\sin(2\mathrm{ArcCos}(cx))\right)}{\sqrt{1-c^2x^2}}}{72c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d]
*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sqrt
[d - c^2*d*x^2]*(-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*ArcCos
[c*x]]))/Sqrt[1 - c^2*x^2] + (9*b*c*f*Sqrt[d - c^2*d*x^2]*(-2*ArcCos[c*x]^2
 + Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/Sqrt[1 - c^2*x^2
])/(72*c^2)
```

Maple [C] Result contains complex when optimal does not.
time = 1.10, size = 628, normalized size = 2.64

method	result
default	$-\frac{ag\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{3c^2d}+\frac{afx\sqrt{-c^2dx^2+d}}{2}+\frac{afd\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{-c^2x}}{4c(c^2x^2-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*
d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f+1/72*(-d*(
c^2*x^2-1))^(1/2)*(4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4-3*I*(-c^2*x^2+1
)^(1/2)*x*c-5*c^2*x^2+1)*g*(I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/16*(-d*(c^2*
x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2
)-2*c*x)*f*(I+2*arccos(c*x))/c/(c^2*x^2-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-
c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(arccos(c*x)+I)/c^2/(c^2*x^2-1)-1/8*(-d*(
c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)/c^
2/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+
2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))/c/(c^2*x^2-1)+1/
72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c
^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))/c^2/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f + sqrt(d)*integrat
e((b*g*x + b*f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x), x) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a*g/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acos}(cx))\,(f+gx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\operatorname{acos}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.4 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\mathbf{ArcCos}(cx))}{f+gx}\,dx$

Optimal. Leaf size=725

$$\frac{a\sqrt{d-c^2dx^2}}{g}+\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}+\frac{b\sqrt{d-c^2dx^2}\,\mathrm{ArcCos}(cx)}{g}-\frac{cx\sqrt{d-c^2dx^2}\,(a+b\mathrm{ArcCos}(cx))^2}{2bg\sqrt{1-c^2x^2}}+\frac{\left(1-\frac{c^2f^2}{g^2}\right.}{}$$

```
[Out] a*(-c^2*d*x^2+d)^(1/2)/g+b*arccos(c*x)*(-c^2*d*x^2+d)^(1/2)/g+b*c*x*(-c^2*d
*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arccos(c*x))^2*(-c^2*d*x^2+
d)^(1/2)/b/g/(-c^2*x^2+1)^(1/2)+1/2*(1-c^2*f^2/g^2)*(a+b*arccos(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/b/c/(g*x+f)/(-c^2*x^2+1)^(1/2)-a*arctan((c^2*f*x+g)/(c^2*
f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)
/g^2/(-c^2*x^2+1)^(1/2)-I*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(
c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^
2*x^2+1)^(1/2)+I*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*
f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^
(1/2)-b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*
(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+b*polylog(2
,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/
2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)-1/2*(a+b*arccos(c*x))^2*(-c^
2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)
```

Rubi [A]
time = 1.32, antiderivative size = 725, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {4862, 4850, 697, 4842, 6874, 739, 210, 1668, 12, 4884, 4882, 4768, 8, 4858, 3402, 2296, 2221, 2317, 2438}

$$\frac{\sqrt{d-c^2dx^2}\left(1-\frac{c^2f^2}{g^2}\right)(a+b\mathrm{ArcCos}(cx))^2}{2bc\sqrt{1-c^2x^2}\,(f+gx)}-\frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(a+b\mathrm{ArcCos}(cx))^2}{2bc(f+gx)}-\frac{cx\sqrt{d-c^2dx^2}\,(a+b\mathrm{ArcCos}(cx))^2}{2bg\sqrt{1-c^2x^2}}-\frac{a\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\mathrm{ArcTan}\left(\frac{c^2fx+g}{\sqrt{1-c^2x^2}\,\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{a\sqrt{d-c^2dx^2}}{g}-\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\mathrm{Li}_2\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\mathrm{Li}_2\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}-\frac{ib\mathrm{ArcCos}(cx)\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\log\left(1+\frac{ge^{i\mathrm{ArcCos}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{ib\mathrm{ArcCos}(cx)\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\log\left(1+\frac{ge^{i\mathrm{ArcCos}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{1-c^2x^2}}+\frac{b\mathrm{ArcCos}(cx)\sqrt{d-c^2dx^2}}{g}+\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] (a*Sqrt[d - c^2*d*x^2])/g + (b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[1 - c^2*x^2
]) + (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/(2*b*g*Sqrt[1 - c^2*x^2]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 - c^2*x^2]) - (S
qrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f + g*x
)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[
c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (I*b*Sqrt[c^2
*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) + (I*b*Sqrt[c^2*f^2 -
g^2]*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + S
```

```
qrt[c^2*f^2 - g^2])])/(g^2*Sqrt[1 - c^2*x^2]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt
[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^
2]))])/(g^2*Sqrt[1 - c^2*x^2]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[1 - c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4842

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
```

```
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 4850

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e
, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4882

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4884

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))}{f+gx}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc(f+gx)} + \frac{\sqrt{d-c^2dx^2}\,\int \frac{(-g-2c \cdots}{\cdots}}{2bc\sqrt{\cdots}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+b \cdots}{2bc(f+gx)\sqrt{1-c^2 \cdots}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+b \cdots}{2bc(f+gx)\sqrt{1-c^2 \cdots}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+b \cdots}{2bc(f+gx)\sqrt{1-c^2 \cdots}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d \cdots}}{2bc(f \cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} - \frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bg\sqrt{1-c^2x^2}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d \cdots}}{2bc(f \cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\,(a+b\cos \cdots}{2bg\sqrt{1-c^2x^2}}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{\cdots}}{\cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{\cdots}}{\cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{\cdots}}{\cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{\cdots}}{\cdots}$$

$$= \frac{a\sqrt{d-c^2dx^2}}{g} + \frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}} + \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g} - \frac{cx\sqrt{\cdots}}{\cdots}$$

Mathematica [A]
time = 2.22, size = 1095, normalized size = 1.51

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] -1/2*(-2*a*g*Sqrt[d - c^2*d*x^2] + 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2
*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log
[f + g*x] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d
]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((-2*
c*g*x)/Sqrt[1 - c^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c
^2*x^2] + (2*(-(c*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[A
rcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)]
- (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2
*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[
Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g
*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2]
)/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[
-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sq
rt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-
(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I
)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f +
g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2
*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[
((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))
/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2,
 ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[Ar
cCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] -
 PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) +
g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c
*x]/2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2])))/g^2
```

Maple [A]
time = 0.91, size = 1209, normalized size = 1.67

method	result
default	$\frac{a\sqrt{-c^{2}d\left(x+\frac{f}{g}\right)^{2}+\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^{2}f^{2}-g^{2}\right)}{g^{2}}}}{g}+\frac{a\,c^{2}df\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\left(x+\frac{f}{g}\right)^{2}+\frac{2c^{2}df\left(x+\frac{f}{g}\right)}{g}-\frac{d(c^{2}f^{2}}{g}}}\right.}{g^{2}\sqrt{c^{2}d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] a/g*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*
c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln
((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+1/2*b*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*c/g^2-b*(-d*(c^2*x^2-1))^(1
/2)/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c+b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-
1)/g*arccos(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/g*arccos(c*x)
+I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2
-1)/g^2*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1
/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^
(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2
-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(-1/
(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^
2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))-b*(-d*(c^2*
x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(
(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)
^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x), x)
```

3.5 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\textbf{ArcCos}(cx))}{(f+gx)^2}\,dx$

Optimal. Leaf size=851

$$-\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)}-\frac{b\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)}{g(f+gx)}+\frac{bc^3f^2\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)^2}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}-\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\,(a-}{2bc\left(c^2f^2-g^2\right)(f+gx)^2}$$

```
[Out] -a*(-c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arccos(c*x)*(-c^2*d*x^2+d)^(1/2)/g/(g*x
+f)+1/2*b*c^3*f^2*arccos(c*x)^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(-c^
2*x^2+1)^(1/2)-1/2*(c^2*f*x+g)^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b
/c/(c^2*f^2-g^2)/(g*x+f)^2/(-c^2*x^2+1)^(1/2)-a*c^3*f^2*arcsin(c*x)*(-c^2*d
*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(-c^2*x^2+1)^(1/2)-b*c*ln(g*x+f)*(-c^2*d*x^
2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+a*c^2*f*arctan((c^2*f*x+g)/(c^2*f^2-g^2)^
(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^
2*x^2+1)^(1/2)+I*b*c^2*f*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f
-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x
^2+1)^(1/2)-I*b*c^2*f*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c
^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+
1)^(1/2)+b*c^2*f*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)
^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2)-b*
c^2*f*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-
c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2)-1/2*(a+b*arcc
os(c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2
```

Rubi [A]
time = 1.92, antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.710$, Rules used = {4862, 4850, 37, 4840, 12, 1665, 858, 222, 739, 210, 4884, 4882, 4738, 4858, 3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{bf^2\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}-\frac{af^2\sqrt{d-c^2dx^2}\,\text{ArcSin}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{1-c^2x^2}}+\frac{af\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{fxc^2+g}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}+\frac{ibf\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)\log\left(\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}-\frac{ibf\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)\log\left(\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}+1\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}+\frac{bf\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}-\frac{bf\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}-\frac{b\sqrt{d-c^2dx^2}\log(f+gx)c}{g^2\sqrt{1-c^2x^2}}-\frac{b\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)}{g(f+gx)}-\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)}-\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{2b(f+gx)^2c}-\frac{\left(fxc^2+g\right)^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{2b\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d - c^2*d*x^2]*ArcCos[c*
x])/(g*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*g^2*(c
^2*f^2 - g^2)*Sqrt[1 - c^2*x^2]) - ((g + c^2*f*x)^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*(f + g*x)^2*Sqrt[1 - c^2*x^2]) -
 (Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*(f +
g*x)^2) - (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcSin[c*x])/(g^2*(c^2*f^2 - g^2)*
Sqrt[1 - c^2*x^2]) + (a*c^2*f*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqr
t[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2
```

```
*x^2]) + (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*
x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*
x^2]) - (I*b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x
^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 - c^2*x^2]) + (b*
c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^
2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2]) - (b*c^2*f*Sqr
t[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g
^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4840

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.)
 + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)^
m, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*n, Int[SimplifyIntegra
nd[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && LtQ[
m + p + 1, 0]
```

Rule 4850

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e
, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
```

```
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4882

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4884

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))}{(f+gx)^2}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))}{(f+gx)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc(f+gx)^2} + \frac{\sqrt{d-c^2dx^2}\,\int \frac{(-2g-2c\cdots}{}}{2bc\sqrt{1-\cdots}}$$

$$= -\frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2d\cdots}}{2bc(f\cdots}$$

$$= -\frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2d\cdots}}{2bc(f\cdots}$$

$$= -\frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2d\cdots}}{2bc(f\cdots}$$

$$= -\frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2d\cdots}}{2bc(f\cdots}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1\cdots}}{}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{(g+c^2fx)^2\,\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))^2}{2bc\,(c^2f^2-g^2)\,(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1\cdots}}{}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)^2}{2g^2\,(c^2f^2-g^2)\,\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\,\sqrt{d-\cdots}}{2bc\,(c^2f^2-g^2)\cdots}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\,\cos^{-\cdots}}{2g^2\,(c^2f^2-g^2)\,\sqrt{1-\cdots}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\,\cos^{-\cdots}}{2g^2\,(c^2f^2-g^2)\,\sqrt{1-\cdots}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\,\cos^{-\cdots}}{2g^2\,(c^2f^2-g^2)\,\sqrt{1-\cdots}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\,\cos^{-1}(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\,\cos^{-\cdots}}{2g^2\,(c^2f^2-g^2)\,\sqrt{1-\cdots}}$$

Mathematica [A]
time = 6.94, size = 1116, normalized size = 1.31

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

```
[Out] -1/2*((2*a*g*Sqrt[d - c^2*d*x^2])/(f + g*x) - 2*a*c*Sqrt[d]*ArcTan[(c*x*Sqr
t[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (2*a*c^2*Sqrt[d]*f*Log[f + g*
x])/Sqrt[-(c^2*f^2) + g^2] + (2*a*c^2*Sqrt[d]*f*Log[d*(g + c^2*f*x) + Sqrt[
d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[-(c^2*f^2) + g^2] + b*
c*Sqrt[d - c^2*d*x^2]*((2*g*ArcCos[c*x])/(c*f + c*g*x) - ArcCos[c*x]^2/Sqrt
[1 - c^2*x^2] + (2*Log[1 + (g*x)/f])/Sqrt[1 - c^2*x^2] + (2*c*f*(2*ArcCos[c
*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcC
os[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) +
g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/S
qrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos
[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[(
(c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) +
g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])
*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((
c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2)
+ g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan
[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))
] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/S
qrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2
])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCo
s[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - S
qrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) +
g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(
c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(
c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[1 - c
^2*x^2])))/g^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 1573, normalized size = 1.85

method	result	size
default	Expression too large to display	1573

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2-g^2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g^2*c^4*f^2/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))-a/g^3*c^4*f^3/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln(
(-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+
a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/(c^2*f^2-g^
2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+a*c^2
/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*c/g^2-(-d*(c^2*x^2-1))^(1/2)*
(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*arccos(c*x)*(c^2*f*x+g+I*(-c^2*x^2+1)^
(1/2)*c*f)/(c^2*x^2-1)/g^2/(g*x+f)-c*(I*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c
*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x)*(c^2*f^2-g^
2)^(1/2)*c*f-I*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c
*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x)*(c^2*f^2-g^2)^(1/2)*c*f+dilog(-1/(-c*f
+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1
/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1
/2)*c*f-dilog((c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f
+(c^2*f^2-g^2)^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))*
(c^2*f^2-g^2)^(1/2)*c*f-ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^
2*x^2+1)^(1/2))+g)*c^2*f^2+ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(
-c^2*x^2+1)^(1/2))+g)*g^2-2*Im(arccos(c*x))*c^2*f^2+2*Im(arccos(c*x))*g^2+2
*ln(exp(I*Re(arccos(c*x))))*c^2*f^2-2*ln(exp(I*Re(arccos(c*x))))*g^2)*(-d*(
c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)/(c^2*x^2-1)/g^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acos}(cx))}{(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{(f+g\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

```
[Out] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

3.6 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\mathbf{ArcCos}(cx))\,dx$

Optimal. Leaf size=959

$$-\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}-\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{3bdfg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{2bcdf^2gx^3\sqrt{d-}}{5\sqrt{1-c^2}}$$

```
[Out] 3/8*d*f^3*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/16*d*f*g^2*x*(a+b*arcc
os(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d
*x^2+d)^(1/2)+1/4*d*f^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/
2)+1/2*d*f*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/5*
d*f^2*g*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/5*d*g^3
*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*(-c^2*
x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*(-c^2*d
*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2/35*b*d*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/
(-c^2*x^2+1)^(1/2)+5/16*b*c*d*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/
2)-3/32*b*d*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+2/5*b*c*d*f
^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x^3*(-c^2*d*
x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(-c^2*d*x^2+d)^(1/2)
/(-c^2*x^2+1)^(1/2)+7/32*b*c*d*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^
(1/2)-3/25*b*c^3*d*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+8/175*
b*c*d*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/12*b*c^3*d*f*g^2*x^
6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*x^7*(-c^2*d*x^2+
d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16*d*f^3*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/b/c/(-c^2*x^2+1)^(1/2)-3/32*d*f*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)
^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 959, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 4768, 200, 4788, 4784, 4796, 272, 45, 4780, 12, 380}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) - (2*b*d*g^3*x
*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^3*x^2*Sqrt[d
- c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2]
)/(32*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqr
t[1 - c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[1 - c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (7*b*c*
d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d*f^2*g*
```

```
x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) + (8*b*c*d*g^3*x^5*Sqrt[d -
 c^2*d*x^2])/(175*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^
2])/(12*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt
[1 - c^2*x^2]) + (3*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3
*d*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (3*d*f*g^2*x
^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f^3*x*(1 - c^2*x^2)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*f*g^2*x^3*(1 - c^2*x^2)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (3*d*f^2*g*(1 - c^2*x^2)^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (d*g^3*(1 - c^2*x^2)^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^4) + (d*g^3*(1 - c^2*x^2)^3*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) - (3*d*f^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
```

```
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4788

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2
*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)^3 \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f^3\left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right)+3\right.}{}$$

$$= \frac{\left(df^3\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}} +$$

$$= \frac{1}{4}df^3x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{2}dfg^2x$$

$$= \frac{3}{8}df^3x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d-c^2dx}$$

$$= -\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{2bcdf^2}{5}$$

$$= -\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}-\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2}{16\sqrt{}}$$

Mathematica [A]
time = 2.68, size = 910, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-88200*b*c*d*f*(2*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 17640
0*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(352800*b*c^3*f
^2*g*x + 44100*b*c*g^3*x + 564480*a*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 53760*a*g
^3*Sqrt[1 - c^2*x^2] - 588000*a*c^4*f^3*x*Sqrt[1 - c^2*x^2] + 176400*a*c^2*
f*g^2*x*Sqrt[1 - c^2*x^2] - 1128960*a*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] + 268
80*a*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 235200*a*c^6*f^3*x^3*Sqrt[1 - c^2*x^2]
 - 823200*a*c^4*f*g^2*x^3*Sqrt[1 - c^2*x^2] + 564480*a*c^6*f^2*g*x^4*Sqrt[1
```

```
 - c^2*x^2] - 215040*a*c^4*g^3*x^4*Sqrt[1 - c^2*x^2] + 470400*a*c^6*f*g^2*x
^5*Sqrt[1 - c^2*x^2] + 134400*a*c^6*g^3*x^6*Sqrt[1 - c^2*x^2] - 7350*b*c*f*
(16*c^2*f^2 + 3*g^2)*Cos[2*ArcCos[c*x]] - 4900*b*g*(12*c^2*f^2 + g^2)*Cos[3
*ArcCos[c*x]] + 7350*b*c^3*f^3*Cos[4*ArcCos[c*x]] - 11025*b*c*f*g^2*Cos[4*A
rcCos[c*x]] + 7056*b*c^2*f^2*g*Cos[5*ArcCos[c*x]] - 588*b*g^3*Cos[5*ArcCos[
c*x]] + 2450*b*c*f*g^2*Cos[6*ArcCos[c*x]] + 300*b*g^3*Cos[7*ArcCos[c*x]]) +
 140*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-4200*c^2*f^2*g*Sqrt[1 - c^2*x^2]
 + 416*g^3*Sqrt[1 - c^2*x^2] + 6720*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] - 1256*
c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 864*g^3*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*
x]] + 120*g^3*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 1680*c^3*f^3*Sin[2*A
rcCos[c*x]] + 315*c*f*g^2*Sin[2*ArcCos[c*x]] - 420*c^2*f^2*g*Sin[3*ArcCos[c
*x]] + 140*g^3*Sin[3*ArcCos[c*x]] - 210*c^3*f^3*Sin[4*ArcCos[c*x]] + 315*c*
f*g^2*Sin[4*ArcCos[c*x]] - 252*c^2*f^2*g*Sin[5*ArcCos[c*x]] + 84*g^3*Sin[5*
ArcCos[c*x]] - 105*c*f*g^2*Sin[6*ArcCos[c*x]]))/(940800*c^4*Sqrt[1 - c^2*x^
2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.34, size = 2168, normalized size = 2.26

method	result	size
default	Expression too large to display	2168

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/7*a*g^3*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35*a*g^3/d/c^4*(-c^2*d*x^2+d)^(
5/2)-1/2*a*f*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/8*a*f*g^2/c^2*x*(-c^2*d*x^2
+d)^(3/2)+3/16*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/16*a*f*g^2/c^2*d^2/(c
^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/5*a*f^2*g/c^2/d*
(-c^2*d*x^2+d)^(5/2)+1/4*a*f^3*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f^3*d*x*(-c^2*d
*x^2+d)^(1/2)+3/8*a*f^3*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^
2+d)^(1/2))+b*(3/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-
1)*arccos(c*x)^2*f*(2*c^2*f^2+g^2)*d-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*I*(-
c^2*x^2+1)^(1/2)*x^7*c^7+64*c^8*x^8-112*I*(-c^2*x^2+1)^(1/2)*x^5*c^5-144*c^
6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+104*c^4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*
c-25*c^2*x^2+1)*g^3*(I+7*arccos(c*x))*d/(c^2*x^2-1)/c^4-1/768*(-d*(c^2*x^2-
1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/
2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^
2+1)^(1/2)-6*c*x)*f*g^2*(I+6*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/3200*(-d*(c^2
*x^2-1))^(1/2)*(16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+16*c^6*x^6-20*I*(-c^2*x^2+1
)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*g*(12*I
*f^2*c^2+60*arccos(c*x)*c^2*f^2-I*g^2-5*arccos(c*x)*g^2)*d/(c^2*x^2-1)/c^4-
1/512*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*(2*I*f^
```

```
2*c^2+8*arccos(c*x)*c^2*f^2-3*I*g^2-12*arccos(c*x)*g^2)*d/c^3/(c^2*x^2-1)+1
/384*(-d*(c^2*x^2-1))^(1/2)*(4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4-3*I*(
-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*g*(12*I*f^2*c^2+36*arccos(c*x)*c^2*f^2+I
*g^2+3*arccos(c*x)*g^2)*d/(c^2*x^2-1)/c^4-3/128*(-d*(c^2*x^2-1))^(1/2)*(I*(
-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(8*I*f^2*c^2+8*arccos(c*x)*c^2*f^2+I*g^2
+arccos(c*x)*g^2)*d/(c^2*x^2-1)/c^4-3/128*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I
*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-8*I*f^2*c^2+8*arccos(c*x)*c^2*f^2-I*g^2+arcc
os(c*x)*g^2)*d/(c^2*x^2-1)/c^4+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-16*I*f^2*c^2+32
*arccos(c*x)*c^2*f^2-3*I*g^2+6*arccos(c*x)*g^2)*d/c^3/(c^2*x^2-1)+1/384*(-d
*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I
*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-12*I*f^2*c^2+36*arccos(c*x)*c^2*f^2-I*g^2+3*
arccos(c*x)*g^2)*d/(c^2*x^2-1)/c^4-1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^
6-28*c^4*x^4-16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2+20*I*(-c^2*x^2+1)^(
1/2)*x^3*c^3-5*I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-12*I*f^2*c^2+60*arccos(c*x)*
c^2*f^2+I*g^2-5*arccos(c*x)*g^2)*d/(c^2*x^2-1)/c^4-1/768*(-d*(c^2*x^2-1))^(
1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x
^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)
^(1/2)-6*c*x)*f*g^2*(-I+6*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/6272*(-d*(c^2*x^
2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6-64*I*(-c^2*x^2+1)^(1/2)*x^7*c^7+104*c^4
*x^4+112*I*(-c^2*x^2+1)^(1/2)*x^5*c^5-25*c^2*x^2-56*I*(-c^2*x^2+1)^(1/2)*x^
3*c^3+7*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g^3*(-I+7*arccos(c*x))*d/(c^2*x^2-1)/c^
4-3/512*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*(10*I
*f^2*c^2+24*arccos(c*x)*c^2*f^2+3*I*g^2)*cos(3*arccos(c*x))*d/c^3/(c^2*x^2-
1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*(34*
I*f^2*c^2+56*arccos(c*x)*c^2*f^2+3*I*g^2+24*arccos(c*x)*g^2)*sin(3*arccos(c
*x))*d/c^3/(c^2*x^2-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^3 - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*g^3 + 1/16*a*f*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c
^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 +
3*d^(3/2)*arcsin(c*x)/c^3) - 3/5*(-c^2*d*x^2 + d)^(5/2)*a*f^2*g/(c^2*d) + s
qrt(d)*integrate(-(b*c^2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x -
b*d*f^3 + (3*b*c^2*d*f^2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
+ (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*c^
2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d*f^
2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccos(c*x))*sqrt(-c^
2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)\left(f+gx\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))*(f + g*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.7 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\mathbf{ArcCos}(cx)) \, dx$

Optimal. Leaf size=680

$$-\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}-\frac{bc^3df^2x^4\sqrt{d-c}}{16\sqrt{1-c^2x}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/16*d*g^2*x*(a+b*arccos
(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2
+d)^(1/2)+1/4*d*f^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1
/6*d*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-2/5*d*f*g*
(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*x*(-c
^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+5/16*b*c*d*f^2*x^2*(-c^2*d*x^2+d)^(1
/2)/(-c^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)
^(1/2)+4/15*b*c*d*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*c^
3*d*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+7/96*b*c*d*g^2*x^4*(-c^
2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(-c^2*d*x^2+d)^(1/
2)/(-c^2*x^2+1)^(1/2)-1/36*b*c^3*d*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1
)^(1/2)-3/16*d*f^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1
)^(1/2)-1/32*d*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2
+1)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 4768, 200, 4788, 4784, 4796}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))+\frac{1}{4}df^2x(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))-\frac{3df^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{16bc\sqrt{1-c^2x^2}}-\frac{2dfg(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{5c^2}-\frac{dg^2x\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))}{16c^2}+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))+\frac{1}{6}dg^2x^3(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))-\frac{dg^2\sqrt{d-c^2dx^2}(a+b\text{ArcCos}(cx))^2}{32bc^3\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}-\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{7bcdg^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{bc^3df^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{2bc^3dfgx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}-\frac{bc^3dg^2x^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f^2*x
^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d - c^2*
d*x^2])/(32*c*Sqrt[1 - c^2*x^2]) + (4*b*c*d*f*g*x^3*Sqrt[d - c^2*d*x^2])/(1
5*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c
^2*x^2]) + (7*b*c*d*g^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (3*d*f^2*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 +
(d*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 + (d*g^2*
x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (2*d*f*g*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f^2*Sq
```

```
rt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2]) - (d*g^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c^3*Sqrt[1 - c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
```

```
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4788

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2
*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
```

```
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)^2 \left(1-c^2x^2\right)^{3/2} \left(a+b\cos^{-1}(c\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f^2(1-c^2x^2)^{3/2}\left(a+b\cos^{-1}(cx)\right)+\right.}{}$$

$$= \frac{\left(df^2\sqrt{d-c^2dx^2}\right)\int (1-c^2x^2)^{3/2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{1}{4}df^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{6}dg^2x$$

$$= \frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d-c^2d}$$

$$= -\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{4bcdfg}{15}$$

$$= -\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bdg^2x}{32c}$$

Mathematica [A]
time = 1.37, size = 591, normalized size = 0.87

$$\frac{-1800bd(6c^2f^2+g^2)\sqrt{d-c^2dx^2}\,\mathrm{ArcCos}(cx)^2-3600ad^{3/2}(6c^2f^2+g^2)\sqrt{1-c^2x^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-d\sqrt{d-c^2dx^2}\left(14400bc^2fgx+23040acfg\sqrt{1-c^2x^2}-36000ac^3f^2x\sqrt{1-c^2x^2}+3600acg^2x\sqrt{1-c^2x^2}-46080ac^3fgx^2\sqrt{1-c^2x^2}+14400ac^5f^2x^3\sqrt{1-c^2x^2}-16800ac^3g^2x^3\sqrt{1-c^2x^2}+23040ac^5fgx^4\sqrt{1-c^2x^2}+9600ac^5g^2x^5\sqrt{1-c^2x^2}-450b(16c^2f^2+g^2)\cos(2\mathrm{ArcCos}(cx))-2400bcfg\cos(3\mathrm{ArcCos}(cx))+450bc^2f^2\cos(4\mathrm{ArcCos}(cx))-225bg^2\cos(4\mathrm{ArcCos}(cx))+288bcfg\cos(5\mathrm{ArcCos}(cx))+50bg^2\cos(6\mathrm{ArcCos}(cx))\right)+60bd\sqrt{d-c^2dx^2}\,\mathrm{ArcCos}(cx)\left(-400cfg\sqrt{1-c^2x^2}+640c^3fgx^2\sqrt{1-c^2x^2}+15(16c^2f^2+g^2)\sin(2\mathrm{ArcCos}(cx))-40cfg\sin(3\mathrm{ArcCos}(cx))-30c^2f^2\sin(4\mathrm{ArcCos}(cx))+15g^2\sin(4\mathrm{ArcCos}(cx))-24cfg\sin(5\mathrm{ArcCos}(cx))-5g^2\sin(6\mathrm{ArcCos}(cx))\right)}{57600c^3\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-1800*b*d*(6*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*d^(
3/2)*(6*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(14400*b*c^2*f*g*x + 23040
*a*c*f*g*Sqrt[1 - c^2*x^2] - 36000*a*c^3*f^2*x*Sqrt[1 - c^2*x^2] + 3600*a*c
*g^2*x*Sqrt[1 - c^2*x^2] - 46080*a*c^3*f*g*x^2*Sqrt[1 - c^2*x^2] + 14400*a*
c^5*f^2*x^3*Sqrt[1 - c^2*x^2] - 16800*a*c^3*g^2*x^3*Sqrt[1 - c^2*x^2] + 230
40*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 9600*a*c^5*g^2*x^5*Sqrt[1 - c^2*x^2] -
```

```
 450*b*(16*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] - 2400*b*c*f*g*Cos[3*ArcCos[c*
x]] + 450*b*c^2*f^2*Cos[4*ArcCos[c*x]] - 225*b*g^2*Cos[4*ArcCos[c*x]] + 288
*b*c*f*g*Cos[5*ArcCos[c*x]] + 50*b*g^2*Cos[6*ArcCos[c*x]]) + 60*b*d*Sqrt[d
- c^2*d*x^2]*ArcCos[c*x]*(-400*c*f*g*Sqrt[1 - c^2*x^2] + 640*c^3*f*g*x^2*Sq
rt[1 - c^2*x^2] + 15*(16*c^2*f^2 + g^2)*Sin[2*ArcCos[c*x]] - 40*c*f*g*Sin[3
*ArcCos[c*x]] - 30*c^2*f^2*Sin[4*ArcCos[c*x]] + 15*g^2*Sin[4*ArcCos[c*x]] -
 24*c*f*g*Sin[5*ArcCos[c*x]] - 5*g^2*Sin[6*ArcCos[c*x]]))/(57600*c^3*Sqrt[1
 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.02, size = 1550, normalized size = 2.28

method	result	size
default	Expression too large to display	1550

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/6*a*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^(3/
2)+1/16*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a*g^2/c^2*d^2/(c^2*d)^(1/2)
*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/5*a*f*g/c^2/d*(-c^2*d*x^2+d
)^(5/2)+1/4*a*f^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f^2*d*x*(-c^2*d*x^2+d)^(1/2)
+3/8*a*f^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b
*(1/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x
)^2*(6*c^2*f^2+g^2)*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2
)*x^6*c^6+32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*
x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*g^2*(I+6*arccos
(c*x))*d/c^3/(c^2*x^2-1)-1/400*(-d*(c^2*x^2-1))^(1/2)*(16*I*(-c^2*x^2+1)^(1
/2)*x^5*c^5+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^2
*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*f*g*(I+5*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/5
12*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c
^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(8*arccos(c*
x)*c^2*f^2+2*I*f^2*c^2-4*arccos(c*x)*g^2-I*g^2)*d/c^3/(c^2*x^2-1)-1/8*(-d*(
c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*f*g*(arccos(c*x)+I)*
d/c^2/(c^2*x^2-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*
x*c-1)*f*g*(arccos(c*x)-I)*d/c^2/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(
-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-16*
I*f^2*c^2+32*arccos(c*x)*c^2*f^2-I*g^2+2*arccos(c*x)*g^2)*d/c^3/(c^2*x^2-1)
+1/48*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^
3*c^3+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-I+3*arccos(c*x))*d/c^2/(c^2*x^2-1
)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^
7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^
2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*g^2*(-I+6*arccos(c*x))*d/c^3/(c^2*
x^2-1)-1/600*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*
```

```
g*(11*I+45*arccos(c*x))*cos(4*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/300*(-d*(c^2
*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*g*(7*I+15*arccos(c*x)
)*sin(4*arccos(c*x))*d/c^2/(c^2*x^2-1)-3/512*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^
2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*(10*I*f^2*c^2+24*arccos(c*x)*c^2*f^2+I*g^2)*c
os(3*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2
+c*x*(-c^2*x^2+1)^(1/2)-I)*(34*I*f^2*c^2+56*arccos(c*x)*c^2*f^2+I*g^2+8*arc
cos(c*x)*g^2)*sin(3*arccos(c*x))*d/c^3/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^2 + 1/48*a*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d
*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcs
in(c*x)/c^3) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*f*g/(c^2*d) + sqrt(d)*integrate
(-(b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*g*x - b*d*f^2 + (b*c^2*d*f
^2 - b*d*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (a
*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*
g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 +
 d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\,(cx+1)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))*(f + g*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.8 $\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\mathbf{ArcCos}(cx))\,dx$

Optimal. Leaf size=370

$$-\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}-\frac{bc^3dfx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bc^3dgx^5\sqrt{d-c^2d}}{25\sqrt{1-c^2x^2}}$$

```
[Out] 3/8*d*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(-c^2*x^2+1)*(a+
b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/5*d*g*(-c^2*x^2+1)^2*(a+b*arccos(c*x)
)*(-c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^
(1/2)+5/16*b*c*d*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+2/15*b*c*d*g
*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(-c^2*d*x^2
+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*
x^2+1)^(1/2)-3/16*d*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^
2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 4768, 200}

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))+\frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{3df\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))^2}{16bc\sqrt{1-c^2x^2}}-\frac{dg\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))}{5c^2}+\frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}+\frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}-\frac{bc^3dfx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bc^3dgx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/5*(b*d*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2]) + (5*b*c*d*f*x^2*S
qrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (2*b*c*d*g*x^3*Sqrt[d - c^2*d*
x^2])/(15*Sqrt[1 - c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt
[1 - c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[1 - c^2*x^2])
 + (3*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 + (d*f*x*(1 - c^2*x^
2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/4 - (d*g*(1 - c^2*x^2)^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(16*b*c*Sqrt[1 - c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int (f+gx)\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f\left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)+g\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(df\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{3/2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}} +$$

$$= \frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{dg(1-}{}$$

$$= \frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) + \frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{}$$

$$= -\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}} + \frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} + \frac{2bcdgx^3\sqrt{}}{15\sqrt{1-}}$$

Mathematica [A]
time = 1.01, size = 337, normalized size = 0.91

$$\frac{-1800bcdf\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)^2-3600acd^{3/2}f\sqrt{1-c^2x^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)-d\sqrt{d-c^2dx^2}\left(-1200bcf\cos(2\text{ArcCos}(cx))-200bg\cos(3\text{ArcCos}(cx))+3\left(400bcgx+80a\sqrt{1-c^2x^2}\left(8g(-1+c^2x^2)^2+5c^2fx(-5+2c^2x^2)\right)+25bcf\cos(4\text{ArcCos}(cx))+8bg\cos(5\text{ArcCos}(cx))\right)\right)+20bd\sqrt{d-c^2dx^2}\,\text{ArcCos}(cx)\left(-100g\sqrt{1-c^2x^2}+160c^2gx^2\sqrt{1-c^2x^2}+120cf\sin(2\text{ArcCos}(cx))-10g\sin(3\text{ArcCos}(cx))-15cf\sin(4\text{ArcCos}(cx))-6g\sin(5\text{ArcCos}(cx))\right)}{9600c^2\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-1800*b*c*d*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*c*d^(3/2)*f*Sqrt[
1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d
*Sqrt[d - c^2*d*x^2]*(-1200*b*c*f*Cos[2*ArcCos[c*x]] - 200*b*g*Cos[3*ArcCos
[c*x]] + 3*(400*b*c*g*x + 80*a*Sqrt[1 - c^2*x^2]*(8*g*(-1 + c^2*x^2)^2 + 5*
c^2*f*x*(-5 + 2*c^2*x^2)) + 25*b*c*f*Cos[4*ArcCos[c*x]] + 8*b*g*Cos[5*ArcCo
s[c*x]])) + 20*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-100*g*Sqrt[1 - c^2*x^2
] + 160*c^2*g*x^2*Sqrt[1 - c^2*x^2] + 120*c*f*Sin[2*ArcCos[c*x]] - 10*g*Sin
```

```
[3*ArcCos[c*x]] - 15*c*f*Sin[4*ArcCos[c*x]] - 6*g*Sin[5*ArcCos[c*x]]))/(960
0*c^2*Sqrt[1 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.06, size = 1012, normalized size = 2.74

method	result
default	$-\frac{ag(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+\frac{afx(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{-c^2dx^2+d}}{8}+\frac{3afd^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{-}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a*g/c^2/d*(-c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*
d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))+b*(3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c
^2*x^2-1)*arccos(c*x)^2*f*d-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*I*(-c^2*x^2+1)
^(1/2)*x^5*c^5+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-
c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*g*(I+5*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/
256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-
c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*(I+4*arcc
os(c*x))*d/c/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*g*(arccos(c*x)+I)*d/c^2/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1
/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)*d/c^2/(c^2*x^2-1
)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*
(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))*d/c/(c^2*x^2-1)+1/96*(-d*(c^
2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c
^2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/1200*(-d*(c
^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(11*I+45*arccos(c*x
))*cos(4*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/600*(-d*(c^2*x^2-1))^(1/2)*(I*x^2
*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*g*(7*I+15*arccos(c*x))*sin(4*arccos(c*x))*d/
c^2/(c^2*x^2-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*
x*c-1)*f*(5*I+12*arccos(c*x))*cos(3*arccos(c*x))*d/c/(c^2*x^2-1)-1/256*(-d*
(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*(17*I+28*arccos(c
*x))*sin(3*arccos(c*x))*d/c/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxi
ma")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f - 1/5*(-c^2*d*x^2 + d)^(5/2)*a*g/(c^2*d) + sqrt(d)*integra
te(-(b*c^2*d*g*x^3 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*sqrt(c*x + 1)*sqrt(-c
*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fric
as")
```

```
[Out] integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-d(cx-1)(cx+1))^{\frac{3}{2}} (a+b\operatorname{acos}(cx))(f+gx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\operatorname{acos}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.9 $\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\textbf{ArcCos}(cx))}{f+gx}\,dx$

Optimal. Leaf size=1064

$$-\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3}+\frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}}-\frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{1-c^2x^2}}+\frac{bc^3dfx^2\sqrt{d-c^2d}}{4g^2\sqrt{1-c^2x^2}}$$

```
[Out] -a*d*(c*f-g)*(c*f+g)*(-c^2*d*x^2+d)^(1/2)/g^3-b*d*(c*f-g)*(c*f+g)*arccos(c*
x)*(-c^2*d*x^2+d)^(1/2)/g^3+1/2*c^2*d*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^
(1/2)/g^2+1/3*d*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/g+1/3*b
*c*d*x*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-b*c*d*(c*f-g)*(c*f+g)*x*(-
c^2*d*x^2+d)^(1/2)/g^3/(-c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(-c^2*d*x^2+d)^
(1/2)/g^2/(-c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x
^2+1)^(1/2)-1/4*c*d*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2/(-c^2*
x^2+1)^(1/2)+1/2*c*d*(c*f-g)*(c*f+g)*x*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/b/g^3/(-c^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2-g^2)^2*(a+b*arccos(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(-c^2*x^2+1)^(1/2)+a*d*(c^2*f^2-g^2)^(3/2
)*arctan((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*d*x^2+d)
^(1/2)/g^4/(-c^2*x^2+1)^(1/2)+I*b*d*(c^2*f^2-g^2)^(3/2)*arccos(c*x)*ln(1+(c
*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/
g^4/(-c^2*x^2+1)^(1/2)-I*b*d*(c^2*f^2-g^2)^(3/2)*arccos(c*x)*ln(1+(c*x+I*(-
c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^4/(-c
^2*x^2+1)^(1/2)+b*d*(c^2*f^2-g^2)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2
))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)
-b*d*(c^2*f^2-g^2)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*
f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)+1/2*d*(c*f-g)*
(c*f+g)*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/g^2
/(g*x+f)
```

Rubi [A]
time = 1.55, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 23, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.742$, Rules used = {4862, 4852, 4742, 4738, 30, 4768, 4850, 697, 4842, 6874, 739, 210, 1668, 12, 4884, 4882, 8, 4858, 3402, 2296, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] -((a*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2])/g^3) + (b*c*d*x*Sqrt[d - c^
2*d*x^2])/(3*g*Sqrt[1 - c^2*x^2]) - (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c
^2*d*x^2])/(g^3*Sqrt[1 - c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d - c^2*d*x^2])/(4
```

```
*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d - c^2*d*x^2])/(9*g*Sqrt[1 - c
^2*x^2]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/g^3 +
(c^2*d*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + (d*(1 - c^2*x
^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (c*d*f*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x])^2)/(4*b*g^2*Sqrt[1 - c^2*x^2]) + (c*d*(c*f - g)*(c
*f + g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*g^3*Sqrt[1 - c^2*
x^2]) + (d*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*
b*c*g^4*(f + g*x)*Sqrt[1 - c^2*x^2]) + (d*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*
x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*
d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*
f^2 - g^2]*Sqrt[1 - c^2*x^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (I*b*d*(c^2*f^2 -
 g^2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(
c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) - (I*b*d*(c^2*f^2 - g^
2)^(3/2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 + Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[1 - c^2*x^2]) + (b*d*(c^2*f^2 - g^2)^(3
/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*
f^2 - g^2]))])/(g^4*Sqrt[1 - c^2*x^2]) - (b*d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])
)])/(g^4*Sqrt[1 - c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4842

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 4850

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[1/(b*c*Sqrt[d]*(n +
```

```
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e
, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4852

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4882

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4884

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\cos^{-1}(cx))}{f+gx}\,dx = \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(1-c^2x^2)^{3/2}(a+b\cos^{-1}(cx))}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(\frac{c^2f\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))}{g^2} - \frac{c^2x\sqrt{1-c^2x^2}\,(a+}{g}\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\right)\int \frac{\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))}{f+gx}\,dx}{\sqrt{1-c^2x^2}} + \frac{\left(c^2df\sqrt{\right.}{}$$

$$= \frac{c^2dfx\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))}{2g^2} + \frac{d(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+}{3g}$$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + $$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + $$

$$= \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}} - \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}} + $$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{}}{4g^2\sqrt{1}}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{}}{4g^2\sqrt{1}}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bc^3dfx^2\sqrt{}}{4g^2\sqrt{1}}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\right.}{g}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\right.}{g}$$

$$= -\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3} + \frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}} + \frac{bcd\left(1-\right.}{g}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3034 vs. 2(1064) = 2128.
time = 9.95, size = 3034, normalized size = 2.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f*
x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*ArcT
an[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) + (a*
d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*f^2)
 + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d
*(-1 + c^2*x^2))]])/g^4 - (b*d*Sqrt[d*(1 - c^2*x^2)]*((-2*c*g*x)/Sqrt[1 - c
^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] + (2*(-(c
*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcC
os[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[(
(c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) +
 g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[
-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]
]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqr
t[c*f + c*g*x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[Ar
cCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqr
t[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^
2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c
*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f
 - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sq
rt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-
(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g
*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f
 + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c
*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/(Sqr
t[-(c^2*f^2) + g^2]*Sqrt[1 - c^2*x^2])))/(2*g^2) + (b*d*Sqrt[d*(1 - c^2*x^2
)]*((9*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Co
t[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)]
+ (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - A
rcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
```

```
(I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*
x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2]
)/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2)
 + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Ta
n[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqr
t[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2
) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) +
g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g +
Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(
c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*
(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/Sqrt[-(c^2*f^2)
+ g^2] + (18*c*g*(-4*c^2*f^2 + g^2)*x + 18*g*(-4*c^2*f^2 + g^2)*Sqrt[1 - c^
2*x^2]*ArcCos[c*x] + 18*c*f*(2*c^2*f^2 - g^2)*ArcCos[c*x]^2 + 9*c*f*g^2*Cos
[2*ArcCos[c*x]] - 2*g^3*Cos[3*ArcCos[c*x]] - (9*(8*c^4*f^4 - 8*c^2*f^2*g^2
+ g^4)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Co
t[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)]
+ (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - A
rcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
(I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*
x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2]
)/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2)
 + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Ta
n[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan
[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqr
t[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2
) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2131 vs. 2(998) = 1996.
time = 0.90, size = 2132, normalized size = 2.00

method	result	size
default	Expression too large to display	2132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -a/g^3*d*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c
^2*f^2-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g*arccos(c*x)+1/3*a/g*(-c
^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)-b*(c^2*f^2-g^
```

```
2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog(
-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2
-g^2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))+b*(c^2*f
^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*d
ilog((c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2
-g^2)^(1/2))*c*f+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))+1/8*b*(-d
*(c^2*x^2-1))^(1/2)*f*d*c/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)+1/9*b*(-d*(c^2
*x^2-1))^(1/2)*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^3*c^3-4/3*b*(-d*(c^2*x^
2-1))^(1/2)*d/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c-1/3*b*(-d*(c^2*x^2-1))^(
1/2)*d/(c^2*x^2-1)/g*arccos(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^
2*x^2-1)/g*arccos(c*x)*x^2*c^2+b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g^3*a
rccos(c*x)*c^2*f^2-a/g*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g
^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^
2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-a/g^5*d^2/(-d*(c
^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d
*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4+2*a/g^3*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/
2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g
))*c^2*f^2+a/g*d*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)
^(1/2)+1/2*a/g^2*c^2*d*f*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g
^2)/g^2)^(1/2)*x+3/2*a/g^2*c^2*d^2*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-a/g^4*d^2*
c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+I*b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*ln(((c*x+I*(-c^2*x^2+1)^(1/2))
*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x)-I*b*(c^2
*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4
*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-
g^2)^(1/2)))*arccos(c*x)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^4/(c^2*x^2-1)/g
^2*arccos(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^2/(c^2*x^2-1)/g^2*arc
cos(c*x)*x-b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g^3*arccos(c*x)*x^2*c^4*f
^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^
2*f^3*d*c^3/g^4+3/4*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)
*arccos(c*x)^2*f*d*c/g^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d*c^3/(c^2*x^2-1)/g
^2*(-c^2*x^2+1)^(1/2)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)/g^3*(-c^2*
x^2+1)^(1/2)*x*c^3*f^2
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*d
*x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x), x)
```

3.10 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\mathbf{ArcCos}(cx))\,dx$

Optimal. Leaf size=1281

$$-\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd^2f^3x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{15bd^2fg^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}+\frac{3bcd^2}{}$$

[Out] 5/16*d^2*f^3*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-2/63*b*d^2*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+25/96*b*c*d^2*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/189*b*d^2*g^3*x^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^3*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/21*b*c*d^2*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*g^3*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/81*b*c^5*d^2*g^3*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+3/64*b*c^5*d^2*f*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/7*b*d^2*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-15/256*b*d^2*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+3/7*b*c*d^2*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+59/256*b*c*d^2*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-9/35*b*c^3*d^2*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-17/96*b*c^3*d^2*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+3/49*b*c^5*d^2*f^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/36*b*d^2*f^3*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+15/64*d^2*f*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f^3*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/7*d^2*g^3*(-c^2*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+1/9*d^2*g^3*(-c^2*x^2+1)^4*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-15/256*d^2*f*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)-5/32*d^2*f^3*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-15/128*d^2*f*g^2*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/7*d^2*f^2*g*(-c^2*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2

Rubi [A]

time = 0.78, antiderivative size = 1281, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 267, 4768, 200, 4788, 4784, 4796, 272, 45, 4780, 12, 380}

Antiderivative was successfully verified.

[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) - (2*b*d^2*g
^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f^3*x^2*
Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d -
c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d
*x^2])/(7*Sqrt[1 - c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*S
qrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 -
c^2*x^2]) + (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[1 - c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^2*x^2]) +
(b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[1 - c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d - c^2*d*x^2])/(441*Sqrt[1 - c^2*x^2]) + (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*d*x
^2])/(81*Sqrt[1 - c^2*x^2]) - (b*d^2*f^3*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 -
(15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(128*c^2) + (15*d^
2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/64 + (5*d^2*f^3*x*(1 -
 c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (5*d^2*f*g^2*x^3*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (d^2*f^3*x*(1 - c
^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6 + (3*d^2*f*g^2*x^3*(1
- c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8 - (3*d^2*f^2*g*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^2) - (d^2*g^3*(1
- c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^4) + (d^2*g^3*(1
 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(9*c^4) - (5*d^2*f^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2]) - (1
5*d^2*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c^3*Sqrt[1 -
c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
```

```
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
```

```
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4788

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2
*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
```

```
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)^3 \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(c\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^3(1-c^2x^2)^{5/2} \left(a+b\cos^{-1}(cx)\right)+\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d-c^2dx^2}\right)\int (1-c^2x^2)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{1}{6}d^2f^3x(1-c^2x^2)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{3}{8}d^2f$$

$$= -\frac{bd^2f^3(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{24}d^2f^3x(1-c^2x^2)$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{9bc}{}$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd}{9}$$

$$= -\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd}{9}$$

Mathematica [A]
time = 4.89, size = 1144, normalized size = 0.89

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-3175200*b*c*f*(8*c^2*f^2 + 3*g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2
- 6350400*a*c*Sqrt[d]*f*(8*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*S
qrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-38102
400*b*c^3*f^2*g*x - 3810240*b*c*g^3*x - 69672960*a*c^2*f^2*g*Sqrt[1 - c^2*x
^2] - 5160960*a*g^3*Sqrt[1 - c^2*x^2] + 111767040*a*c^4*f^3*x*Sqrt[1 - c^2*
x^2] - 19051200*a*c^2*f*g^2*x*Sqrt[1 - c^2*x^2] + 209018880*a*c^4*f^2*g*x^2
*Sqrt[1 - c^2*x^2] - 2580480*a*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] - 88058880*a*c
^6*f^3*x^3*Sqrt[1 - c^2*x^2] + 149869440*a*c^4*f*g^2*x^3*Sqrt[1 - c^2*x^2]
- 209018880*a*c^6*f^2*g*x^4*Sqrt[1 - c^2*x^2] + 38707200*a*c^4*g^3*x^4*Sqrt
[1 - c^2*x^2] + 27095040*a*c^8*f^3*x^5*Sqrt[1 - c^2*x^2] - 172730880*a*c^6*
```

f*g^2*x^5*Sqrt[1 - c^2*x^2] + 69672960*a*c^8*f^2*g*x^6*Sqrt[1 - c^2*x^2] - 49029120*a*c^6*g^3*x^6*Sqrt[1 - c^2*x^2] + 60963840*a*c^8*f*g^2*x^7*Sqrt[1 - c^2*x^2] + 18063360*a*c^8*g^3*x^8*Sqrt[1 - c^2*x^2] + 3810240*b*c*f*(5*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] + 282240*b*g*(27*c^2*f^2 + 2*g^2)*Cos[3*ArcCos[c*x]] - 1905120*b*c^3*f^3*Cos[4*ArcCos[c*x]] + 952560*b*c*f*g^2*Cos[4*ArcCos[c*x]] - 1524096*b*c^2*f^2*g*Cos[5*ArcCos[c*x]] + 141120*b*c^3*f^3*Cos[6*ArcCos[c*x]] - 423360*b*c*f*g^2*Cos[6*ArcCos[c*x]] + 155520*b*c^2*f^2*g*Cos[7*ArcCos[c*x]] - 38880*b*g^3*Cos[7*ArcCos[c*x]] + 59535*b*c*f*g^2*Cos[8*ArcCos[c*x]] + 7840*b*g^3*Cos[9*ArcCos[c*x]]) + 504*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-261504*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 62616*g^3*Sqrt[1 - c^2*x^2] + 503424*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2] - 120576*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] - 41472*g*(3*c^2*f^2 - 2*g^2)*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 5760*g*(3*c^2*f^2 - 2*g^2)*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 75600*c^3*f^3*Sin[2*ArcCos[c*x]] + 15120*c*f*g^2*Sin[2*ArcCos[c*x]] - 40320*c^2*f^2*g*Sin[3*ArcCos[c*x]] + 6720*g^3*Sin[3*ArcCos[c*x]] - 15120*c^3*f^3*Sin[4*ArcCos[c*x]] + 7560*c*f*g^2*Sin[4*ArcCos[c*x]] - 24192*c^2*f^2*g*Sin[5*ArcCos[c*x]] + 6048*g^3*Sin[5*ArcCos[c*x]] + 1680*c^3*f^3*Sin[6*ArcCos[c*x]] - 5040*c*f*g^2*Sin[6*ArcCos[c*x]] + 900*g^3*Sin[7*ArcCos[c*x]] + 945*c*f*g^2*Sin[8*ArcCos[c*x]] + 140*g^3*Sin[9*ArcCos[c*x]])))/(162570240*c^4*Sqrt[1 - c^2*x^2])

Maple [C] Result contains complex when optimal does not.
time = 1.43, size = 3051, normalized size = 2.38

method	result	size
default	Expression too large to display	3051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a*g^3*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63*a*g^3/d/c^4*(-c^2*d*x^2+d)^(7/2)-3/8*a*f*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/16*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+15/128*a*f*g^2/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+15/128*a*f*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/7*a*f^2*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+1/6*a*f^3*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f^3*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f^3*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-3/1024*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*(22*I*f^2*c^2+32*arccos(c*x)*c^2*f^2+3*I*g^2+12*arccos(c*x)*g^2)*sin(3*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*I*(-c^2*x^2+1)^(1/2)*x^9*c^9+256*c^10*x^10-576*I*(-c^2*x^2+1)^(1/2)*x^7*c^7-704*c^8*x^8+432*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+688*c^6*x^6-120*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-280*c^4*x^4+9*I*(-c^2*x^2+1)^(1/2)*x*c+41*c^2*x^2-1)*g^3*(I+9*arccos(c*x))*d^2/c^4/(c^2*x^2-1)+3/16384*(-d*(c^2*x^2-1))^
```

(1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9-256*I*(-c^2*x^2+1)^(1/2
)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5-32*I*(-c
^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+8*c*x)*f*g^2*(I+8*a
rccos(c*x))*d^2/c^3/(c^2*x^2-1)+3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*I*(-c^2*
x^2+1)^(1/2)*x^7*c^7+64*c^8*x^8-112*I*(-c^2*x^2+1)^(1/2)*x^5*c^5-144*c^6*x^
6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+104*c^4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*c-25
*c^2*x^2+1)*g*(4*I*f^2*c^2+28*arccos(c*x)*c^2*f^2-I*g^2-7*arccos(c*x)*g^2)*
d^2/c^4/(c^2*x^2-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*
x^6*c^6+32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^
2+1)^(1/2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*f*(I*f^2*c^2+6*ar
ccos(c*x)*c^2*f^2-3*I*g^2-18*arccos(c*x)*g^2)*d^2/c^3/(c^2*x^2-1)-3/640*(-d
*(c^2*x^2-1))^(1/2)*(16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+16*c^6*x^6-20*I*(-c^2*
x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*f^
2*g*(I+5*arccos(c*x))*d^2/(c^2*x^2-1)/c^2-3/1024*(-d*(c^2*x^2-1))^(1/2)*(8*
I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^
3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*(8*arccos(c*x)*c^2*f^2+2*I*f^2*c^2-4*ar
ccos(c*x)*g^2-I*g^2)*d^2/c^3/(c^2*x^2-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*I
*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+
1)*g*(27*I*f^2*c^2+81*arccos(c*x)*c^2*f^2+2*I*g^2+6*arccos(c*x)*g^2)*d^2/c^
4/(c^2*x^2-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^
2-1)*g*(10*I*f^2*c^2+10*arccos(c*x)*c^2*f^2+I*g^2+arccos(c*x)*g^2)*d^2/c^4/
(c^2*x^2-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-
1)*g*(-10*I*f^2*c^2+10*arccos(c*x)*c^2*f^2-I*g^2+arccos(c*x)*g^2)*d^2/c^4/(
c^2*x^2-1)+3/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*
c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-5*I*f^2*c^2+10*arccos(c*x)*c^2*f^2-
I*g^2+2*arccos(c*x)*g^2)*d^2/c^3/(c^2*x^2-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2)*
x*c+1)*g*(-27*I*f^2*c^2+81*arccos(c*x)*c^2*f^2-2*I*g^2+6*arccos(c*x)*g^2)*d
^2/c^4/(c^2*x^2-1)-3/640*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4-16*I
*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2+20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-5*I*
(-c^2*x^2+1)^(1/2)*x*c-1)*f^2*g*(-I+5*arccos(c*x))*d^2/(c^2*x^2-1)/c^2+1/23
04*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I
*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c
^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*f*(-I*f^2*c^2+6*arccos(c*x)*c^2*f^2+3*I*
g^2-18*arccos(c*x)*g^2)*d^2/c^3/(c^2*x^2-1)+3/25088*(-d*(c^2*x^2-1))^(1/2)*
(64*c^8*x^8-144*c^6*x^6-64*I*(-c^2*x^2+1)^(1/2)*x^7*c^7+104*c^4*x^4+112*I*(
-c^2*x^2+1)^(1/2)*x^5*c^5-25*c^2*x^2-56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+7*I*(-
c^2*x^2+1)^(1/2)*x*c+1)*g*(-4*I*f^2*c^2+28*arccos(c*x)*c^2*f^2+I*g^2-7*arcc
os(c*x)*g^2)*d^2/c^4/(c^2*x^2-1)+3/16384*(-d*(c^2*x^2-1))^(1/2)*(-128*I*(-c
^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^
7*x^7-160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*
x^2*c^2-88*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+8*c*x)*f*g^2*(-I+8*arccos(c*x))*d^2
/c^3/(c^2*x^2-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8-
256*I*(-c^2*x^2+1)^(1/2)*x^9*c^9+688*c^6*x^6+576*I*(-c^2*x^2+1)^(1/2)*x^7*c
^7-280*c^4*x^4-432*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+41*c^2*x^2+120*I*(-c^2*x^2+

```
1)^(1/2)*x^3*c^3-9*I*(-c^2*x^2+1)^(1/2)*x*c-1)*g^3*(-I+9*arccos(c*x))*d^2/c
^4/(c^2*x^2-1)-3/1024*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*
x*c-1)*f*(18*I*f^2*c^2+48*arccos(c*x)*c^2*f^2+5*I*g^2+4*arccos(c*x)*g^2)*co
s(3*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+5/256*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*(8*c^2*f^2+3*g^2)*d^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^3 + 1/128*(8*(-c^2*d*
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*f*g^2 - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*g^3 - 3/7*(-c^2*d*x^2 + d)^(7/2)*a*f^2*g/(c^2*d)
 + sqrt(d)*integrate((b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2*f
^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d^2
*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*b*
c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a*c
^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f^3
- 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*
b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.11 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\mathbf{ArcCos}(cx)) \, dx$

Optimal. Leaf size=940

$$-\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}+\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{5bc^3d^2\ldots}{96\ldots}$$

```
[Out] -1/36*b*d^2*f^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(a
+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-5/128*d^2*g^2*x*(a+b*arccos(c*x))*(-c^
2*d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2
)+5/24*d^2*f^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+5/48*d
^2*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*
x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(-c
^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-2/7*d^2*f*g*(-c^2*x^2+1)
^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(-c^2*d*x^2+d
)^(1/2)/c/(-c^2*x^2+1)^(1/2)+25/96*b*c*d^2*f^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c
^2*x^2+1)^(1/2)-5/256*b*d^2*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/
2)+2/7*b*c*d^2*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/96*b*c^3*d
^2*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+59/768*b*c*d^2*g^2*x^4*(
-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*(-c^2*d*x^2+d
)^(1/2)/(-c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)^(1/2)+2/49*b*c^5*d^2*f*g*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(
1/2)+1/64*b*c^5*d^2*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/32*d^
2*f^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-5/256
*d^2*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 940, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 267, 4768, 200, 4788, 4784, 4796, 272, 45}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*
f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d - c^2*d*x^2])/(256*c*Sqrt[1 - c^2*x^2]) + (2*b*c*d^2*f*g*x^3*Sqrt[d - c^
2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])
/(96*Sqrt[1 - c^2*x^2]) + (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqr
t[1 - c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1 - c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[1 - c^2*x^2]
) + (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2]) + (b*c
```

```
^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[1 - c^2*x^2]) - (b*d^2*f^2*(1
- c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCos[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]
))/64 + (5*d^2*f^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/24 + (5*d^2*g^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/48 + (d^2*f^2*x*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/6
 + (d^2*g^2*x^3*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/8
- (2*d^2*f*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(7*c^
2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(32*b*c*Sqrt[1 -
 c^2*x^2]) - (5*d^2*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(256*b*c
^3*Sqrt[1 - c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4788

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
```

```
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c*(n/(f*(m + 2
*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right)\, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)^2 \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^2\left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right)+\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{5/2} \left(a+b\cos^{-1}(cx)\right) dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{1}{6}d^2f^2x\left(1-c^2x^2\right)^2 \sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right)+\frac{1}{8}d^2$$

$$= -\frac{bd^2f^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{24}d^2f^2x\left(1-c^2x^2\right)$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{6bc}{}$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{2b}{}$$

$$= -\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{5b}{}$$

Mathematica [**A**]
time = 2.90, size = 794, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-352800*b*(8*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 70560
0*a*Sqrt[d]*(8*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-2822400*b*c^2*f*g*x
 - 5160960*a*c*f*g*Sqrt[1 - c^2*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 - c^2*x^
2] - 705600*a*c*g^2*x*Sqrt[1 - c^2*x^2] + 15482880*a*c^3*f*g*x^2*Sqrt[1 - c
^2*x^2] - 9784320*a*c^5*f^2*x^3*Sqrt[1 - c^2*x^2] + 5550720*a*c^3*g^2*x^3*S
qrt[1 - c^2*x^2] - 15482880*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 3010560*a*c^7
*f^2*x^5*Sqrt[1 - c^2*x^2] - 6397440*a*c^5*g^2*x^5*Sqrt[1 - c^2*x^2] + 5160
960*a*c^7*f*g*x^6*Sqrt[1 - c^2*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1 - c^2*x^
2] + 141120*b*(15*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] + 564480*b*c*f*g*Cos[3*
```

```
ArcCos[c*x]] - 211680*b*c^2*f^2*Cos[4*ArcCos[c*x]] + 35280*b*g^2*Cos[4*ArcC
os[c*x]] - 112896*b*c*f*g*Cos[5*ArcCos[c*x]] + 15680*b*c^2*f^2*Cos[6*ArcCos
[c*x]] - 15680*b*g^2*Cos[6*ArcCos[c*x]] + 11520*b*c*f*g*Cos[7*ArcCos[c*x]]
+ 2205*b*g^2*Cos[8*ArcCos[c*x]]) + 168*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-
58112*c*f*g*Sqrt[1 - c^2*x^2] + 111872*c^3*f*g*x^2*Sqrt[1 - c^2*x^2] - 2764
8*c*f*g*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 3840*c*f*g*(1 - c^2*x^2)^(
3/2)*Cos[4*ArcCos[c*x]] + 25200*c^2*f^2*Sin[2*ArcCos[c*x]] + 1680*g^2*Sin[2
*ArcCos[c*x]] - 8960*c*f*g*Sin[3*ArcCos[c*x]] - 5040*c^2*f^2*Sin[4*ArcCos[c
*x]] + 840*g^2*Sin[4*ArcCos[c*x]] - 5376*c*f*g*Sin[5*ArcCos[c*x]] + 560*c^2
*f^2*Sin[6*ArcCos[c*x]] - 560*g^2*Sin[6*ArcCos[c*x]] + 105*g^2*Sin[8*ArcCos
[c*x]])))/(18063360*c^3*Sqrt[1 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.11, size = 2230, normalized size = 2.37

method	result	size
default	Expression too large to display	2230

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/8*a*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^(5/
2)+5/192*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a*g^2/c^2*d^2*x*(-c^2*d*x
^2+d)^(1/2)+5/128*a*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*
d*x^2+d)^(1/2))-2/7*a*f*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+1/6*a*f^2*x*(-c^2*d*x^
2+d)^(5/2)+5/24*a*f^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(-c^2*d*x^2
+d)^(1/2)+5/16*a*f^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
)^(1/2))+b*(-1/160*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)
-I)*f*g*(3*I+5*arccos(c*x))*sin(4*arccos(c*x))*d^2/(c^2*x^2-1)/c^2+1/16384*
(-d*(c^2*x^2-1))^(1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9-256*I*
(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272
*c^5*x^5-32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+8*
c*x)*g^2*(I+8*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+1/3136*(-d*(c^2*x^2-1))^(1/2
)*(64*I*(-c^2*x^2+1)^(1/2)*x^7*c^7+64*c^8*x^8-112*I*(-c^2*x^2+1)^(1/2)*x^5*
c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+104*c^4*x^4-7*I*(-c^2*x^2+1
)^(1/2)*x*c-25*c^2*x^2+1)*f*g*(I+7*arccos(c*x))*d^2/(c^2*x^2-1)/c^2+1/2304*
(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7-48*I*(-c
^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x
^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*(I*f^2*c^2+6*arccos(c*x)*c^2*f^2-I*g^2-6*arc
cos(c*x)*g^2)*d^2/c^3/(c^2*x^2-1)-1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*
x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*
(-c^2*x^2+1)^(1/2)+4*c*x)*(6*I*f^2*c^2+24*arccos(c*x)*c^2*f^2-I*g^2-4*arcco
s(c*x)*g^2)*d^2/c^3/(c^2*x^2-1)-5/64*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)
^(1/2)*x*c+c^2*x^2-1)*f*g*(arccos(c*x)+I)*d^2/(c^2*x^2-1)/c^2-5/64*(-d*(c^2
```

```
*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*g*(arccos(c*x)-I)*d^2
/(c^2*x^2-1)/c^2+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*
c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(30*arccos(c*x)*c^2*f^2+2*arccos(
c*x)*g^2-15*I*f^2*c^2-I*g^2)*d^2/c^3/(c^2*x^2-1)+1/64*(-d*(c^2*x^2-1))^(1/2
)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^2*x^2+1)^(1/2
)*x*c+1)*f*g*(-I+3*arccos(c*x))*d^2/(c^2*x^2-1)/c^2+1/2304*(-d*(c^2*x^2-1))
^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)
*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+
1)^(1/2)-6*c*x)*(-I*f^2*c^2+6*arccos(c*x)*c^2*f^2+I*g^2-6*arccos(c*x)*g^2)*
d^2/c^3/(c^2*x^2-1)+1/16384*(-d*(c^2*x^2-1))^(1/2)*(-128*I*(-c^2*x^2+1)^(1/
2)*x^8*c^8+128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7-160*I*(
-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^
3*x^3-I*(-c^2*x^2+1)^(1/2)+8*c*x)*g^2*(-I+8*arccos(c*x))*d^2/c^3/(c^2*x^2-1
)+1/3920*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*g*(1
1*I+70*arccos(c*x))*cos(6*arccos(c*x))*d^2/(c^2*x^2-1)/c^2-3/1024*(-d*(c^2*
x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(22*I*f^2*c^2+32*arccos(
c*x)*c^2*f^2+I*g^2+4*arccos(c*x)*g^2)*sin(3*arccos(c*x))*d^2/c^3/(c^2*x^2-1
)-1/80*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*g*(I+5
*arccos(c*x))*cos(4*arccos(c*x))*d^2/(c^2*x^2-1)/c^2+5/256*(-d*(c^2*x^2-1))
^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*(8*c^2*f^2+g^2)*d^2
-1/1024*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*(54*I*f
^2*c^2+144*arccos(c*x)*c^2*f^2+5*I*g^2+4*arccos(c*x)*g^2)*cos(3*arccos(c*x)
)*d^2/c^3/(c^2*x^2-1)+3/7840*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^
2+1)^(1/2)-I)*f*g*(9*I+35*arccos(c*x))*sin(6*arccos(c*x))*d^2/(c^2*x^2-1)/c
^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^2 + 1/384*(8*(-c^2*d*
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*g^2 - 2/7*(-c^2*d*x^2 + d)^(7/2)*a*f*g/(c^2*d) + sqrt(d)*integ
rate((b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*c^2*d^2*f*g*x^3 + 2*b*d
^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*g^2)*x^4 - (2*b*c^2*d^2
*f^2 - b*d^2*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2
*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*
g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2
+ d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.12 $\int (f+gx) \left(d-c^2dx^2\right)^{5/2} (a+b\textbf{ArcCos}(cx)) \, dx$

Optimal. Leaf size=517

$$-\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{5bc^3d^2fx^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{}}{35\sqrt{1-}}$$

```
[Out] -1/36*b*d^2*f*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*a
rccos(c*x))*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(-c^2*x^2+1)*(a+b*arccos(c*x)
)*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d
*x^2+d)^(1/2)-1/7*d^2*g*(-c^2*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/
2)/c^2-1/7*b*d^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+25/96*b*c*d^
2*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/7*b*c*d^2*g*x^3*(-c^2*d*x
^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)^(1/2)-3/35*b*c^3*d^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/
2)+1/49*b*c^5*d^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/32*d^2*f*
(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {4862, 4848, 4744, 4742, 4738, 30, 14, 267, 4768, 200}

$$\frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))+\frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))-\frac{5d^2f\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))^2}{32bc\sqrt{1-c^2x^2}}-\frac{d^2g\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\,(a+b\text{ArcCos}(cx))}{7c^2}+\frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{bd^2f\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}-\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}+\frac{bc^5d^2gx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}-\frac{5bc^3d^2fx^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/7*(b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[1 - c^2*x^2]) + (25*b*c*d^2*f*
x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[1 - c^2*x^2]) + (b*c*d^2*g*x^3*Sqrt[d - c
^2*d*x^2])/(7*Sqrt[1 - c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d - c^2*d*x^2])/
(96*Sqrt[1 - c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[1
 - c^2*x^2]) + (b*c^5*d^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[1 - c^2*x^2])
 - (b*d^2*f*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/16 + (5*d^2*f*x*(1 - c^2*x^2)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCos[c*x]))/24 + (d^2*f*x*(1 - c^2*x^2)^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x]))/6 - (d^2*g*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCos[c*x]))/(7*c^2) - (5*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
s[c*x])^2)/(32*b*c*Sqrt[1 - c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4744

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (D
ist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[x*(1 -
c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (f+gx)\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f\left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)+g\right.}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2f\sqrt{d-c^2dx^2}\right)\int \left(1-c^2x^2\right)^{5/2}\left(a+b\cos^{-1}(cx)\right)\,dx}{\sqrt{1-c^2x^2}} -$$

$$= \frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cos^{-1}(cx)\right) - \frac{d^2g(1}{}$$

$$= -\frac{bd^2f\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c} + \frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{}$$

$$= -\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}} - \frac{3bc^3d^2gx^5}{35\sqrt{}}$$

$$= -\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}} + \frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}} + \frac{bcd^2gx}{7\sqrt{}}$$

Mathematica [A]
time = 1.97, size = 526, normalized size = 1.02

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (d^2*(-88200*b*c*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 176400*a*c*Sqrt[d]*f
*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] + Sqrt[d - c^2*d*x^2]*(-44100*b*c*g*x - 80640*a*g*Sqrt[1 - c^2*x^2] + 38
8080*a*c^2*f*x*Sqrt[1 - c^2*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 - c^2*x^2] - 3
05760*a*c^4*f*x^3*Sqrt[1 - c^2*x^2] - 241920*a*c^4*g*x^4*Sqrt[1 - c^2*x^2]
+ 94080*a*c^6*f*x^5*Sqrt[1 - c^2*x^2] + 80640*a*c^6*g*x^6*Sqrt[1 - c^2*x^2]
 + 66150*b*c*f*Cos[2*ArcCos[c*x]] + 8820*b*g*Cos[3*ArcCos[c*x]] - 6615*b*c*
f*Cos[4*ArcCos[c*x]] - 1764*b*g*Cos[5*ArcCos[c*x]] + 490*b*c*f*Cos[6*ArcCos
[c*x]] + 180*b*g*Cos[7*ArcCos[c*x]]) + 84*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]
*(-1816*g*Sqrt[1 - c^2*x^2] + 3496*c^2*g*x^2*Sqrt[1 - c^2*x^2] - 864*g*(1 -
 c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 120*g*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos
[c*x]] + 1575*c*f*Sin[2*ArcCos[c*x]] - 280*g*Sin[3*ArcCos[c*x]] - 315*c*f*S
in[4*ArcCos[c*x]] - 168*g*Sin[5*ArcCos[c*x]] + 35*c*f*Sin[6*ArcCos[c*x]])))
/(564480*c^2*Sqrt[1 - c^2*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.09, size = 1419, normalized size = 2.74

method	result	size
default	Expression too large to display	1419

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f
*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(5/32*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f*d^2+1/6272*
(-d*(c^2*x^2-1))^(1/2)*(64*I*(-c^2*x^2+1)^(1/2)*x^7*c^7+64*c^8*x^8-112*I*(-
c^2*x^2+1)^(1/2)*x^5*c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+104*c^
4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*c-25*c^2*x^2+1)*g*(I+7*arccos(c*x))*d^2/c^2/
(c^2*x^2-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+
32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^2+1)^(1/
2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*f*(I+6*arccos(c*x))*d^2/c
/(c^2*x^2-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2
-1)*g*(arccos(c*x)+I)*d^2/c^2/(c^2*x^2-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*(c^2
*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)*d^2/c^2/(c^2*x^2-1)+15/2
56*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^
2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))*d^2/c/(c^2*x^2-1)+1/128*(-d*(c^2
*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+3*I*(-c^
2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))*d^2/c^2/(c^2*x^2-1)+1/7840*(-d*(
c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(11*I+70*arccos(c*
```

```
x))*cos(6*arccos(c*x))*d^2/c^2/(c^2*x^2-1)+3/15680*(-d*(c^2*x^2-1))^(1/2)*(
I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*g*(9*I+35*arccos(c*x))*sin(6*arccos(c*x
))*d^2/c^2/(c^2*x^2-1)+5/4608*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1
)^(1/2)*x*c-1)*f*(5*I+24*arccos(c*x))*cos(5*arccos(c*x))*d^2/c/(c^2*x^2-1)+
1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*(29*I+
96*arccos(c*x))*sin(5*arccos(c*x))*d^2/c/(c^2*x^2-1)-1/160*(-d*(c^2*x^2-1))
^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(I+5*arccos(c*x))*cos(4*arcco
s(c*x))*d^2/c^2/(c^2*x^2-1)-1/320*(-d*(c^2*x^2-1))^(1/2)*(I*x^2*c^2+c*x*(-c
^2*x^2+1)^(1/2)-I)*g*(3*I+5*arccos(c*x))*sin(4*arccos(c*x))*d^2/c^2/(c^2*x^
2-1)-9/512*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*(3
*I+8*arccos(c*x))*cos(3*arccos(c*x))*d^2/c/(c^2*x^2-1)-3/512*(-d*(c^2*x^2-1
))^(1/2)*(I*x^2*c^2+c*x*(-c^2*x^2+1)^(1/2)-I)*f*(11*I+16*arccos(c*x))*sin(3
*arccos(c*x))*d^2/c/(c^2*x^2-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f - 1/7*(-c^2*d*x^2 + d
)^(7/2)*a*g/(c^2*d) + sqrt(d)*integrate((b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4
- 2*b*c^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c
^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccos(c*x))*sqrt(-
c^2*d*x^2 + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))*(f + g*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.13 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\mathbf{ArcCos}(cx))}{f+gx}\,dx$

Optimal. Leaf size=1637

$$\frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5}-\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}}-\frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}}+\frac{bcd^2(c^2f^2-g^2)^2\,x\sqrt{d}}{g^5\sqrt{1-c^2x^2}}$$

```
[Out] -2/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-1/45*b*c^3*d^2*x^
3*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)+1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+
d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-a*d^2*(c^2*f^2-g^2)^(5/2)*arctan((c^2*f*x+g)/
(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+
1)^(1/2)+a*d^2*(c^2*f^2-g^2)^2*(-c^2*d*x^2+d)^(1/2)/g^5+1/8*c^2*d^2*f*x*(a+
b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*x^3*(a+b*arccos(c*x))
*(-c^2*d*x^2+d)^(1/2)/g^2-b*d^2*(c^2*f^2-g^2)^(5/2)*polylog(2,-(c*x+I*(-c^2
*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*
x^2+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^(5/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2)
)*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)-
1/3*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/g+1/5*d^2*(-c^2
*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/g-1/4*b*c^3*d^2*f*(c^2*f^2
-2*g^2)*x^2*(-c^2*d*x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)+1/4*c*d^2*f*(c^2*f^
2-2*g^2)*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^4/(-c^2*x^2+1)^(1/2)-
1/2*c*d^2*(c^2*f^2-g^2)^2*x*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^5/
(-c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2-g^2)^3*(a+b*arccos(c*x))^2*(-c^2*d*x^2+
d)^(1/2)/b/c/g^6/(g*x+f)/(-c^2*x^2+1)^(1/2)+I*b*d^2*(c^2*f^2-g^2)^(5/2)*arc
cos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2
*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2-g^2)^2*(a+b*arccos(
c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)-I*b*d^2*(c^
2*f^2-g^2)^(5/2)*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^
2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)-1/3*b*c*d^2*(c^2
*f^2-2*g^2)*x*(-c^2*d*x^2+d)^(1/2)/g^3/(-c^2*x^2+1)^(1/2)+b*c*d^2*(c^2*f^2-
g^2)^2*x*(-c^2*d*x^2+d)^(1/2)/g^5/(-c^2*x^2+1)^(1/2)+1/16*b*c^3*d^2*f*x^2*(
-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+1/9*b*c^3*d^2*(c^2*f^2-2*g^2)*x^
3*(-c^2*d*x^2+d)^(1/2)/g^3/(-c^2*x^2+1)^(1/2)-1/16*b*c^5*d^2*f*x^4*(-c^2*d*
x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^2*arccos(c*x)*(-c^2
*d*x^2+d)^(1/2)/g^5-1/3*d^2*(c^2*f^2-2*g^2)*(-c^2*x^2+1)*(a+b*arccos(c*x))*
(-c^2*d*x^2+d)^(1/2)/g^3-1/2*c^2*d^2*f*(c^2*f^2-2*g^2)*x*(a+b*arccos(c*x))*
(-c^2*d*x^2+d)^(1/2)/g^4+1/16*c*d^2*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1
/2)/b/g^2/(-c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 1.88, antiderivative size = 1637, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 28, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.903$, Rules used = {4862, 4852, 4742, 4738, 30, 4768, 4784, 4796, 272, 45, 4780, 12, 4850, 697, 4842, 6874, 739, 210, 1668, 4884, 4882, 8, 4858, 3402, 2296, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2])/g^5 - (2*b*c*d^2*x*Sqrt[d - c
^2*d*x^2])/(15*g*Sqrt[1 - c^2*x^2]) - (b*c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d -
 c^2*d*x^2])/(3*g^3*Sqrt[1 - c^2*x^2]) + (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[
d - c^2*d*x^2])/(g^5*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x
^2])/(16*g^2*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d
 - c^2*d*x^2])/(4*g^4*Sqrt[1 - c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^
2])/(45*g*Sqrt[1 - c^2*x^2]) + (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x^3*Sqrt[d - c^
2*d*x^2])/(9*g^3*Sqrt[1 - c^2*x^2]) - (b*c^5*d^2*f*x^4*Sqrt[d - c^2*d*x^2])
/(16*g^2*Sqrt[1 - c^2*x^2]) + (b*c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqr
t[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])
/g^5 + (c^2*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(8*g^2) - (c^2
*d^2*f*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*g^4)
 - (c^4*d^2*f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - (d^2*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g) - (d^2*(c^2*f^2
 - 2*g^2)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(3*g^3) +
(d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(5*g) + (c*d^
2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(16*b*g^2*Sqrt[1 - c^2*x^2])
 + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4
*b*g^4*Sqrt[1 - c^2*x^2]) - (c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x])^2)/(2*b*g^5*Sqrt[1 - c^2*x^2]) - (d^2*(c^2*f^2 - g^2)^3
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*g^6*(f + g*x)*Sqrt[1 - c
^2*x^2]) - (d^2*(c^2*f^2 - g^2)^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 - g^2)^(5/2)*Sq
rt[d - c^2*d*x^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^
2])])/(g^6*Sqrt[1 - c^2*x^2]) - (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2
*d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2
])])/(g^6*Sqrt[1 - c^2*x^2]) + (I*b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*
d*x^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]
)])/(g^6*Sqrt[1 - c^2*x^2]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^6*
Sqrt[1 - c^2*x^2]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyL
og[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[1 -
c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
```

```
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4780

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[
c*x], u, x] + Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[Simplif
yIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && E
qQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2
, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 4784

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]], Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]
 + Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(f
*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
```

```
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4842

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCos[c*x])^n, u, x] + Dist[b*c*
n, Int[SimplifyIntegrand[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[p,
 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 4850

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCo
s[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e
, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4852

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rule 4882

```
Int[ArcCos[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :
> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcCos[c*x]^n, RFx, x]}, Int[u, x
] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ[n
, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 4884

```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\cos^{-1}(cx))}{f+gx}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(1-c^2x^2)^{5/2}(a+b\cos^{-1}(cx))}{f+gx}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(-\frac{c^2f(c^2f^2-2g^2)\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))}{g^4} + \frac{c^2(c^2f^2}{}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d-c^2dx^2}\right)\int x^2\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\;dx}{g^2\sqrt{1-c^2x^2}} + \frac{\left(c^4d\right.}{}$$

$$= -\frac{c^2d^2f(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}\,(a+b\cos^{-1}(cx))}{2g^4} - \frac{c^4d^2fx^3\sqrt{d-}}{}$$

$$= -\frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}} - \frac{bc^3d^2f(c^2f^2-2g^2)\,x^2\sqrt{d-c^2d}}{4g^4\sqrt{1-c^2x^2}}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}} + \frac{bc^3d^2fx^2}{16g^2\sqrt{}}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}} + \frac{bc^3d^2fx^2}{16g^2\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

$$= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} - \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-}{3g^3\sqrt{}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6216 vs. 2(1637) = 3274.
time = 14.51, size = 6216, normalized size = 3.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4691 vs. 2(1519) = 3038.
time = 0.99, size = 4692, normalized size = 2.87

method	result	size
default	Expression too large to display	4692

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*
x^2-1)/g^6*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^
(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4+2*I*b*d^2*(-d*(c^2*x^2-1))^(1/2)
*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*arccos(c*x)*ln(((c*
x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)
))*c^2*f^2-I*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^
(1/2)/(c^2*x^2-1)/g^6*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2
*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-2*I*b*d^2*(-d*(c^2*x^2-
1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*arccos(c*x
)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2
-g^2)^(1/2)))*c^2*f^2+1/4*a/g^2*c^2*d*f*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*x+7/8*a/g^2*c^2*d^2*f*(-c^2*d*(x+f/g)^2+2*c^2
*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+15/8*a/g^2*c^2*d^3*f/(c^2*d)^(1
/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2
-g^2)/g^2)^(1/2))-1/2*a/g^4*d^2*c^4*f^3*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x-5/2*a/g^4*d^3*c^4*f^3/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^
(1/2))+a/g^6*d^3*c^6*f^5/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*(x+f/
g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^7*d^3/(-d*(c^2*f^2
-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*
f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g
^2)^(1/2))/(x+f/g))*c^6*f^6-3*a/g^5*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c
^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4
```

```
*f^4+3*a/g^3*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*
c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2*d*
f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2-1/3*a/g^3*d*(-c^2*
d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*c^2*f^2+a/g^5*d^
2*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^4*f^4-
2*a/g^3*d^2*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2
)*c^2*f^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^6/(c^2*x^2-1)/g^2*arccos(c*x
)*x^5+11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^4/(c^2*x^2-1)/g^2*arccos(c*x)*x
^3-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*c^2/(c^2*x^2-1)/g^2*arccos(c*x)*x+1/3
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c*x)*x^4*c^6*f^2-8/3*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c*x)*x^2*c^4*f^2-1/2*b*(
-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^6/(c^2*x^2-1)/g^4*arccos(c*x)*x^3+1/2*b*(-d
*(c^2*x^2-1))^(1/2)*f^3*d^2*c^4/(c^2*x^2-1)/g^4*arccos(c*x)*x+b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(c^2*x^2-1)/g^5*arccos(c*x)*x^2*c^6*f^4+1/5*a/g*(-c^2*d*(x+
f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(5/2)-23/15*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(c^2*x^2-1)/g*arccos(c*x)+1/3*a/g*d*(-c^2*d*(x+f/g)^2+2*c^2*d
*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+a/g*d^2*(-c^2*d*(x+f/g)^2+2*c^2*d*f
/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2
*x^2-1)/g*arccos(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1
)/g*arccos(c*x)*x^4*c^4+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g*ar
ccos(c*x)*x^2*c^2+7/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)/g^3*arccos(c
*x)*c^2*f^2-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2
+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2
*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(c^2*x^2-1)/g^5*arccos(c*x)*c^4*f^4+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2
*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog(-1/(-c*f+(c^2*f^2-
g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1
/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))-b*d^2*(-d*(c^2*x^2-1))^(1/
2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*dilog((c*x+I*(-c^
2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))+1/(c*f+(c^2*f^2-g^2)^(1/2))*c*f
+1/(c*f+(c^2*f^2-g^2)^(1/2))*(c^2*f^2-g^2)^(1/2))+33/128*b*(-d*(c^2*x^2-1))
^(1/2)*f*d^2*c/(c^2*x^2-1)/g^2*(-c^2*x^2+1)^(1/2)-1/8*b*(-d*(c^2*x^2-1))^(1
/2)*f^3*d^2*c^3/(c^2*x^2-1)/g^4*(-c^2*x^2+1)^(1/2)-1/25*b*(-d*(c^2*x^2-1))^
(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^5*c^5+11/45*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x^3*c^3-23/15*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(c^2*x^2-1)/g*(-c^2*x^2+1)^(1/2)*x*c+I*b*d^2*(-d*(c^2*x^2-1))^(
1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^2*arccos(c*x)*ln(
(-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)
^(1/2)))-2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1
/2)/(c^2*x^2-1)/g^4*dilog(-1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c*x+I*(-c^2*x^2+1)
^(1/2))*g-1/(-c*f+(c^2*f^2-g^2)^(1/2))*c*f+1/(-c*f+(c^2*f^2-g^2)^(1/2))*(c^
2*f^2-g^2)^(1/2))*c^2*f^2+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/g^4*dilog((c*x...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="giac
")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x),x)

[Out] int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x), x)

3.14 $\int \frac{(f+gx)^3(a+b\text{ArcCos}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=450

$$-\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{2bg^3x\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}}-\frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}-\frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}-\frac{3f^2g(1-c^2x^2)\,(a+b\text{Ar}}{c^2\sqrt{d-c^2dx^2}}$$

```
[Out] -3*f^2*g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2/3*g^3*(-
c^2*x^2+1)*(a+b*arccos(c*x))/c^4/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*x*(-c^2*x^2
+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/3*g^3*x^2*(-c^2*x^2+1)*(a+
b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(-c^2*x^2+1)^(1/2)/c/(-
c^2*d*x^2+d)^(1/2)-2/3*b*g^3*x*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-
3/4*b*f*g^2*x^2*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(-c
^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/2*f^3*(a+b*arccos(c*x))^2*(-c^2*x^
2+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arccos(c*x))^2*(-c^2*x^2
+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4862, 4848, 4738, 4768, 8, 4796, 30}

$$-\frac{f^3\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{2bc\sqrt{d-c^2dx^2}}-\frac{3f^2g(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{c^2\sqrt{d-c^2dx^2}}-\frac{3fg^2x(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{2c^2\sqrt{d-c^2dx^2}}-\frac{g^3x^2(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{3c^2\sqrt{d-c^2dx^2}}-\frac{2g^3(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{3c^4\sqrt{d-c^2dx^2}}-\frac{3fg^2\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}-\frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}-\frac{2bg^3x\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*g^3*x*Sqrt[
1 - c^2*x^2])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 - c^2*x^2
])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 - c^2*x^2])/(9*c*Sqrt[d -
c^2*d*x^2]) - (3*f^2*g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2
*d*x^2]) - (2*g^3*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^4*Sqrt[d - c^2*d*
x^2]) - (3*f*g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(2*c^2*Sqrt[d - c^2*d
*x^2]) - (g^3*x^2*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/(3*c^2*Sqrt[d - c^2*d*
x^2]) - (f^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2]) - (3*f*g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c^3*Sqrt[d
- c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^3(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f^3(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}} + \frac{3f^2gx(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}} + \frac{3fg^2x^2(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3f^2g\sqrt{1-c^2x^2}\right)\int \frac{x(a+b\cos^{-1}(cx}{\sqrt{1-c^2x^2}}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{3f^2g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}} - 3f^2$$

$$= -\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{2bg^3x\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3}{9c}$$

Mathematica [A]
time = 0.67, size = 342, normalized size = 0.76

$$\frac{18bc\sqrt{d}\,f(2c^2f^2+3g^2)(-1+c^2x^2)\operatorname{ArcCos}(cx)^2 - 36acf(2c^2f^2+3g^2)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\operatorname{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \sqrt{d}\,g(-1+c^2x^2)\left(8bcx(6g^2+c^2(27f^2+g^2x^2)) + 12a\sqrt{1-c^2x^2}(4g^2+c^2(18f^2+9fgx+2g^2x^2)) + 27bcfg\cos(2\operatorname{ArcCos}(cx))\right) + 6b\sqrt{d}\,g(-1+c^2x^2)\operatorname{ArcCos}(cx)\left(4\sqrt{1-c^2x^2}(2g^2+c^2(9f^2+g^2x^2)) + 9cfg\sin(2\operatorname{ArcCos}(cx))\right)}{72c^4\sqrt{d}\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (18*b*c*Sqrt[d]*f*(2*c^2*f^2 + 3*g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 36*a*c
*f*(2*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sq
rt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(8*
b*c*x*(6*g^2 + c^2*(27*f^2 + g^2*x^2)) + 12*a*Sqrt[1 - c^2*x^2]*(4*g^2 + c^
2*(18*f^2 + 9*f*g*x + 2*g^2*x^2)) + 27*b*c*f*g*Cos[2*ArcCos[c*x]]) + 6*b*Sq
rt[d]*g*(-1 + c^2*x^2)*ArcCos[c*x]*(4*Sqrt[1 - c^2*x^2]*(2*g^2 + c^2*(9*f^2
 + g^2*x^2)) + 9*c*f*g*Sin[2*ArcCos[c*x]]))/(72*c^4*Sqrt[d]*Sqrt[1 - c^2*x^
2]*Sqrt[d - c^2*d*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 1.40, size = 861, normalized size = 1.91

method	result

default	$-\frac{a\,g^{3}x^{2}\sqrt{-c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2a\,g^{3}\sqrt{-c^{2}d\,x^{2}+d}}{3d\,c^{4}}-\frac{3af\,g^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{2c^{2}d}+\frac{3af\,g^{2}\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{2}\sqrt{c^{2}d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3*a*g^3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(-c^2*d*x^2+d)^(1
/2)-3/2*a*f*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+3/2*a*f*g^2/c^2/(c^2*d)^(1/2)*
arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3*a*f^2*g/c^2/d*(-c^2*d*x^2+d)
^(1/2)+a*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(
1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)
^2*f*(2*c^2*f^2+3*g^2)+1/144*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)
*x*c+2*c^2*x^2-1)*g^3*(I+3*arccos(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-
1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(4*I*f^2*c^2+4*arccos(c*x)
*c^2*f^2+I*g^2+arccos(c*x)*g^2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2
)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-4*I*f^2*c^2+4*arccos(c*x)*c^2*f^
2-I*g^2+arccos(c*x)*g^2)/c^4/d/(c^2*x^2-1)+1/144*(-d*(c^2*x^2-1))^(1/2)*(2*
c^2*x^2-2*I*(-c^2*x^2+1)^(1/2)*x*c-1)*g^3*(-I+3*arccos(c*x))/c^4/d/(c^2*x^2
-1)+3/8*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*f*g^2*arccos(c*x)*x-3/16*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2-1/24*(-d*(
c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*arccos(c*x)*g^3*cos(4*arccos(c*x))+1/72
*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*g^3*sin(4*arccos(c*x))-3/8*(-d*(c
^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2*arccos(c*x)*cos(3*arccos(c*x))+3/1
6*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2*sin(3*arccos(c*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/3*a*g^3*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*
d)) - 3/2*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d
))) + b*f^3*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f^3*arcsin(c*x)^2/(
c*sqrt(d)) - 3*b*f^2*g*x/(c*sqrt(d)) + a*f^3*arcsin(c*x)/(c*sqrt(d)) - 3*sq
rt(-c^2*d*x^2 + d)*b*f^2*g*arccos(c*x)/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d)*a*f
^2*g/(c^2*d) - sqrt(d)*integrate((b*g^3*x^3 + 3*b*f*g^2*x^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*d*x^2 - d),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(c^2*
d*x^2 - d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.15 $\int \frac{(f+gx)^2(a+b\text{ArcCos}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=270

$$-\frac{2bfgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{bg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}-\frac{2fg(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2x(1-c^2x^2)\,(a+b\text{ArcCos}(cx)}{2c^2\sqrt{d-c^2dx^2}}$$

```
[Out] -2*f*g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/2*g^2*x*(-
c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(-c^2*x^2+1
)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x
^2+d)^(1/2)-1/2*f^2*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/b/c/(-c^2*d*x^2+
d)^(1/2)-1/4*g^2*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/b/c^3/(-c^2*d*x^2+d
)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4862, 4848, 4738, 4768, 8, 4796, 30}

$$-\frac{f^2\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2x(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{2c^2\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{2bfgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{bg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
 - c^2*x^2])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c^2*x^2)*(a + b*ArcCos[
c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])
)/(2*c^2*Sqrt[d - c^2*d*x^2]) - (f^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^
2)/(2*b*c*Sqrt[d - c^2*d*x^2]) - (g^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
```

```
2*d + e, 0] && NeQ[n, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^2(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f^2(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}} + \frac{2fgx(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}} + \frac{g^2x^2(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}}\right) d \cdots}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(2fg\sqrt{1-c^2x^2}\right)\int \frac{x(a+b\cos^{-1}(cx))}{\sqrt{1-c^2x^2}} \cdots}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{2fg(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} - \cdots$$

$$= -\frac{2bfgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg(1-c^2x^2)\left(a+b\cos^{-1}(c \cdots\right.}{c^2\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.44, size = 266, normalized size = 0.99

$$\frac{2b\sqrt{d}\,(2c^2f^2+g^2)(-1+c^2x^2)\,\text{ArcCos}(cx)^2 - 4a(2c^2f^2+g^2)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \sqrt{d}\,g(-1+c^2x^2)\left(4c\left(4bcfx+a(4f+gx)\sqrt{1-c^2x^2}\right)+bg\cos(2\text{ArcCos}(cx))\right) + 2b\sqrt{d}\,g(-1+c^2x^2)\,\text{ArcCos}(cx)\left(8cf\sqrt{1-c^2x^2}+g\sin(2\text{ArcCos}(cx))\right)}{8c^3\sqrt{d}\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*b*Sqrt[d]*(2*c^2*f^2 + g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 4*a*(2*c^2*f^
2 + g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(4*c*(4*b*c*f*x +
 a*(4*f + g*x)*Sqrt[1 - c^2*x^2]) + b*g*Cos[2*ArcCos[c*x]]) + 2*b*Sqrt[d]*g
*(-1 + c^2*x^2)*ArcCos[c*x]*(8*c*f*Sqrt[1 - c^2*x^2] + g*Sin[2*ArcCos[c*x]]
))/(8*c^3*Sqrt[d]*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.95, size = 507, normalized size = 1.88

method	result
default	$-\frac{a\,g^2 x\sqrt{-c^2d\,x^2+d}}{2c^2d} + \frac{a\,g^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}} - \frac{2afg\sqrt{-c^2d\,x^2+d}}{c^2d} + \frac{a\,f^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+ \cdots}}\right)}{\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*
f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(
c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2*(2*c^2
*f^2+g^2)-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*f*g*(
arccos(c*x)+I)/c^2/d/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^
2+1)^(1/2)*x*c-1)*f*g*(arccos(c*x)-I)/c^2/d/(c^2*x^2-1)+1/8*(-d*(c^2*x^2-1)
)^(1/2)/c^2/d/(c^2*x^2-1)*arccos(c*x)*g^2*x-1/16*(-d*(c^2*x^2-1))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*g^2-1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2
*x^2-1)*arccos(c*x)*g^2*cos(3*arccos(c*x))+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/
d/(c^2*x^2-1)*g^2*sin(3*arccos(c*x)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*a*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b
*f^2*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f^2*arcsin(c*x)^2/(c*sqrt(
d)) + b*g^2*integrate(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sqrt(
c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d) - 2*b*f*g*x/(c*sqrt(d)) + a*f^2*arcsin
(c*x)/(c*sqrt(d)) - 2*sqrt(-c^2*d*x^2 + d)*b*f*g*arccos(c*x)/(c^2*d) - 2*sq
rt(-c^2*d*x^2 + d)*a*f*g/(c^2*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
+ 2*b*f*g*x + b*f^2)*arccos(c*x))/(c^2*d*x^2 - d), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.16 $\int \frac{(f+gx)(a+b\mathbf{ArcCos}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=127

$$-\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{g(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{c^2\sqrt{d-c^2dx^2}}-\frac{f\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

```
[Out] -g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-b*g*x*(-c^2*x^2+
1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/2*f*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2
)/b/c/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4862, 4848, 4738, 4768, 8}

$$-\frac{f\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c^2*x^2)*(a
 + b*ArcCos[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (f*Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4848

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{1-c^2x^2}\int \left(\frac{f\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}+\frac{gx\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f\sqrt{1-c^2x^2}\right)\int \frac{a+b\cos^{-1}(cx)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\int \frac{x\left(a+b\cos^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}} - \frac{(bg\sqrt{\ldots}}{}$$

$$= -\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{f\sqrt{1-c^2x^2}\,(a+b\,c\ldots}{2bc\sqrt{d-c^2d\ldots}}$$

Mathematica [A]
time = 0.24, size = 172, normalized size = 1.35

$$\frac{-2\sqrt{d}\,g\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}\right)+2b\sqrt{d}\,g(-1+c^2x^2)\,\text{ArcCos}(cx)-bc\sqrt{d}\,f\sqrt{1-c^2x^2}\,\text{ArcCos}(cx)^2-2acf\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)}{2c^2\sqrt{d}\,\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*Sqrt[d]*g*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2]) + 2*b*Sqrt[d]*g*(-1
 + c^2*x^2)*ArcCos[c*x] - b*c*Sqrt[d]*f*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 - 2
*a*c*f*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
c^2*x^2))])/(2*c^2*Sqrt[d]*Sqrt[d - c^2*d*x^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.68, size = 247, normalized size = 1.94

method	result
default	$-\frac{ag\sqrt{-c^2 d\,x^2+d}}{c^2 d}+\frac{af\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2 f}{2cd(c^2x^2-1)}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c
^2*x^2-1)*arccos(c*x)^2*f-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*g*(arccos(c*x)+I)/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/
2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)/c^2/d/(c^2*x^2-1)
)
```

Maxima [A]
time = 0.49, size = 108, normalized size = 0.85

$$\frac{bf\arccos(cx)\arcsin(cx)}{c\sqrt{d}}+\frac{bf\arcsin(cx)^2}{2\,c\sqrt{d}}-\frac{bgx}{c\sqrt{d}}+\frac{af\arcsin(cx)}{c\sqrt{d}}-\frac{\sqrt{-c^2dx^2+d}\,bg\arccos(cx)}{c^2d}-\frac{\sqrt{-c^2dx^2+d}\,ag}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] b*f*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f*arcsin(c*x)^2/(c*sqrt(d))
 - b*g*x/(c*sqrt(d)) + a*f*arcsin(c*x)/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*b
*g*arccos(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a*g/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x))/(c
^2*d*x^2 - d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((g*x + f)*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f+g\,x)\,\left(a+b\,\mathrm{acos}(c\,x)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.17 $\int \frac{a+b\text{ArcCos}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=370

$$\frac{i\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}}-\frac{i\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{e^{iA}}{cf+\sqrt{}}\right.}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}}$$

```
[Out] I*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1
/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*(a+b*ar
ccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c
^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)+b*polylog(2,-(c*x+
I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*
f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))
*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*
d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4862, 4858, 3402, 2296, 2221, 2317, 2438}

$$\frac{i\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{i\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}+\frac{b\sqrt{1-c^2x^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{1-c^2x^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (I*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
 - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (I*Sq
rt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sq
rt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1
- c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*PolyLog[
2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g
^2]*Sqrt[d - c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{a+b\cos^{-1}(cx)}{(f+gx)\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{a+bx}{cf+g\cos(x)}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(2\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{e^{ix}(a+bx)}{2ce^{ix}f+g+e^{2ix}g}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(2g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{e^{ix}(a+bx)}{2cf+2e^{ix}g-2\sqrt{c^2f^2-g^2}}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} + \frac{\left(2g\right.}{}$$

$$= \frac{i\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\,(a+}{}$$

$$= \frac{i\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\,(a+}{}$$

$$= \frac{i\sqrt{1-c^2x^2}\,(a+b\cos^{-1}(cx))\log\left(1+\frac{e^{i\cos^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}\,(a+}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 930 vs. $2(370) = 740$.
time = 1.16, size = 930, normalized size = 2.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2
) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[1 - c^2*x^2]*(2*ArcCos[c*x
]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos
[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^
2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqr
t[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c
```

```
*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c
*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)
*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*S
qrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*
f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) +
g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[A
rcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))]
- (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqr
t[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])
*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[
c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqr
t[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^
2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*
f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^
2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))])))/Sqrt[d - c^2*d*x^2])/Sqrt[-(c^2*f^2)
 + g^2]
```

Maple [A]
time = 0.58, size = 487, normalized size = 1.32

method	result
default	$-\frac{a\ln\left(\frac{-\frac{2d\left(c^2f^2-g^2\right)}{g^2}+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}\sqrt{-c^2d\left(x+\frac{f}{g}\right)^2+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}-\frac{b\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*f^2-g^2)
^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-
c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-I*arccos(c*x)*ln(((c*x
+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2))
)+dilog((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*
f^2-g^2)^(1/2)))-dilog(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2
))/(c*f+(c^2*f^2-g^2)^(1/2))))/d/(c^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x
^2 - d*g*x - d*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acos}(cx)}{\sqrt{-d(cx-1)(cx+1)}\,(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acos}(c\,x)}{(f+g\,x)\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acos(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)), x)
```

3.18 $\int \frac{a+b\text{ArcCos}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=496

$$\frac{g(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}}+\frac{ic^2f\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}-\frac{ic^2f\sqrt{1}}{}$$

```
[Out] g*(-c^2*x^2+1)*(a+b*arccos(c*x))/(c^2*f^2-g^2)/(g*x+f)/(-c^2*d*x^2+d)^(1/2)
+b*c*ln(g*x+f)*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)/(-c^2*d*x^2+d)^(1/2)+I*c^2*
f*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1
/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-I*c^2*f*(
a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)
))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+b*c^2*f*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)
^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-(c*x+I*(
-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2
-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {4862, 4858, 3405, 3402, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{g(1-c^2x^2)\,(a+b\text{ArcCos}(cx))}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)\,(f+gx)}+\frac{ic^2f\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}-\frac{ic^2f\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}+\frac{bc^2f\sqrt{1-c^2x^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}-\frac{bc^2f\sqrt{1-c^2x^2}\,\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\,\log(f+gx)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (g*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c
^2*d*x^2]) + (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(I*A
rcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d
- c^2*d*x^2]) - (I*c^2*f*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Log[1 + (E^(
I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt
[d - c^2*d*x^2]) + (b*c*Sqrt[1 - c^2*x^2]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c
*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*
d*x^2]) - (b*c^2*f*Sqrt[1 - c^2*x^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*
f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
```

```
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4858

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[-(c^(m + 1)*Sqrt[d])^(-1), Subst
[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b
, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &&
 (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 4862

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p], Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Integer
Q[p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{a+b\cos^{-1}(cx)}{(f+gx)^2\sqrt{1-c^2x^2}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(c\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\cos(x))^2}\,dx, x, \cos^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} - \frac{\left(c^2f\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\cos(x)}\,dx, \right.}{(c^2f^2-g^2)\sqrt{d-c^2dx^2}}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{cf+x}\,dx, x, cgx\right.}{(c^2f^2-g^2)\sqrt{d-c^2dx^2}}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{1-c^2x^2}\log(f+gx)}{(c^2f^2-g^2)\sqrt{d-c^2dx^2}} - \frac{\left(2c^2fg\sqrt{1-\right.}{}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1-\right.}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2d}}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1-\right.}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2d}}$$

$$= \frac{g(1-c^2x^2)\left(a+b\cos^{-1}(cx)\right)}{(c^2f^2-g^2)(f+gx)\sqrt{d-c^2dx^2}} + \frac{ic^2f\sqrt{1-c^2x^2}\left(a+b\cos^{-1}(cx)\right)\log\left(1-\right.}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2d}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1108 vs. $2(496) = 992$.
time = 3.28, size = 1108, normalized size = 2.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Lo
g[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*
x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g
)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[1 - c^2*x^2]*(-((g*Sqrt[1 -
```

```
 c^2*x^2]*ArcCos[c*x])/((c*f - g)*(c*f + g)*(c*f + c*g*x))) - Log[1 + (g*x)
/f]/(c^2*f^2 - g^2) - (c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*
Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*A
rcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTa
nh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^
2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] +
(ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(
c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2
) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqr
t[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g
)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I
*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[
-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTa
nh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g
)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f
+ g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f -
I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]
/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[
2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[
ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))]
)))/(-(c^2*f^2) + g^2)^(3/2)))/Sqrt[d - c^2*d*x^2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1621 vs. 2(492) = 984.
time = 1.34, size = 1622, normalized size = 3.27

method	result	size
default	Expression too large to display	1622

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-c^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-c
^2*d*(x+f/g)^2+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+b*(
-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2
*x^2+1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2
-g^2)/(g*x+f)*x^3*c^4*f-I*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1
)/(c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+1)^(1/2)*x*c*g+b*(-d*(c^2*x^2-1))^(1/2)*a
rccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-I*b*(-d*(c^2*x^2-1
))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^2*x^2+1)^(1/2)
*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+
```

```
f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)
/(g*x+f)*g+I*b*c^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/
(c^2*f^2-g^2)^(3/2)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2
))/(c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x)*f-I*b*c^2*(-d*(c^2*x^2-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*ln((-(c*x+I*(-c^2*x^2+
1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*arccos(c*x
)*f-b*c^3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-
g^2)^2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)^(1/2))+g
)*f^2+2*b*c^3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*
f^2-g^2)^2*ln(c*x+I*(-c^2*x^2+1)^(1/2))*f^2-b*c^2*(-d*(c^2*x^2-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*dilog((-(c*x+I*(-c^2*x^2
+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*f+b*c^2*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*
dilog(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-
g^2)^(1/2)))*f+b*c*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)/
(c^2*f^2-g^2)^2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+I*(-c^2*x^2+1)
^(1/2))+g)*g^2-2*b*c*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1
)/(c^2*f^2-g^2)^2*ln(c*x+I*(-c^2*x^2+1)^(1/2))*g^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d
*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acos}{\left(cx \right)}}{\sqrt{-d \left(cx - 1\right) \left(cx + 1\right)} \left(f + gx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acos}\left(c\,x\right)}{{\left(f+g\,x\right)}^2\,\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acos(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.19 $\int \frac{(a+b\mathbf{ArcCos}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{(a+b\text{ArcCos}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\text{ArcCos}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
[Out] Defer[Int][((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\left(a+b\cos^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Mathematica [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcCos}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [A]
time = 2.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arccos(cx))^n \ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/(c^2*
x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.20 $\int \frac{(a+b\text{ArcCos}(cx))^2 \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} \, dx$

Optimal. Leaf size=496

$$-\frac{im(a+b\text{ArcCos}(cx))^4}{12b^2c}+\frac{m(a+b\text{ArcCos}(cx))^3\log\left(1+\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m(a+b\text{ArcCos}(cx))^3\log\left(1\right.}{3bc}$$

```
[Out] -1/12*I*m*(a+b*arccos(c*x))^4/b^2/c-1/3*(a+b*arccos(c*x))^3*ln(h*(g*x+f)^m)
/b/c+1/3*m*(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*
f^2-g^2)^(1/2)))/b/c+1/3*m*(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^(1/
2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/b/c-I*m*(a+b*arccos(c*x))^2*polylog(2,-(c*
x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-I*m*(a+b*arccos(c*x)
)^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+2*
b*m*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
-g^2)^(1/2)))/c+2*b*m*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+2*I*b^2*m*polylog(4,-(c*x+I*(-c^2*x^2+1)^
(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+2*I*b^2*m*polylog(4,-(c*x+I*(-c^2*x^2
+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

Rubi [A]
time = 0.55, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4738, 4864, 4826, 4616, 2221, 2611, 6744, 2320, 6724}

$$-\frac{im(a+b\text{ArcCos}(cx))^4}{12b^2c}-\frac{im(a+b\text{ArcCos}(cx))^2\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im(a+b\text{ArcCos}(cx))^2\text{Li}_2\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{2bm(a+b\text{ArcCos}(cx))\text{Li}_3\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{2bm(a+b\text{ArcCos}(cx))\text{Li}_3\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{m(a+b\text{ArcCos}(cx))^3\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc}+\frac{m(a+b\text{ArcCos}(cx))^3\log\left(1+\frac{ge^{i\text{ArcCos}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{3bc}-\frac{(a+b\text{ArcCos}(cx))^3\log\left(h(f+gx)^m\right)}{3bc}+\frac{2ib^2m\text{Li}_4\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{2ib^2m\text{Li}_4\left(-\frac{e^{i\text{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-1/12*I)*m*(a + b*ArcCos[c*x])^4)/(b^2*c) + (m*(a + b*ArcCos[c*x])^3*Log[
1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c) + (m*(a + b
*ArcCos[c*x])^3*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])
/(3*b*c) - ((a + b*ArcCos[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (I*m*(a + b
*ArcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^
2]))])/c - (I*m*(a + b*ArcCos[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c
*f + Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos[c*x])*PolyLog[3, -((E
^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/c + (2*b*m*(a + b*ArcCos
[c*x])*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/c
+ ((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2
]))])/c + ((2*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f
^2 - g^2]))])/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4864

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-Log[h*(f + g*x)^m]
)*((a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[g*(m/(b*c*
Sqrt[d]*(n + 1))), Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGt
Q[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(a+b\cos^{-1}(cx))^2 \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = -\frac{(a+b\cos^{-1}(cx))^3 \log(h(f+gx)^m)}{3bc} + \frac{(gm)\int \frac{(a+b\cos^{-1}(cx))^3}{f+gx}\,dx}{3bc}$$

$$= -\frac{(a+b\cos^{-1}(cx))^3 \log(h(f+gx)^m)}{3bc} - \frac{(gm)\text{Subst}\left(\int \frac{(a+bx)^3 \sin}{cf+g\cos(}\right.}{3bc}$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} - \frac{(a+b\cos^{-1}(cx))^3 \log(h(f+gx)^m)}{3bc} +$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} + \frac{m(a+b\cos^{-1}(cx))^3 \log\left(1+\frac{e^{i\cos^-}}{cf-\sqrt{c^2}}\right.}{3bc}$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} + \frac{m(a+b\cos^{-1}(cx))^3 \log\left(1+\frac{e^{i\cos^-}}{cf-\sqrt{c^2}}\right.}{3bc}$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} + \frac{m(a+b\cos^{-1}(cx))^3 \log\left(1+\frac{e^{i\cos^-}}{cf-\sqrt{c^2}}\right.}{3bc}$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} + \frac{m(a+b\cos^{-1}(cx))^3 \log\left(1+\frac{e^{i\cos^-}}{cf-\sqrt{c^2}}\right.}{3bc}$$

$$= -\frac{im(a+b\cos^{-1}(cx))^4}{12b^2c} + \frac{m(a+b\cos^{-1}(cx))^3 \log\left(1+\frac{e^{i\cos^-}}{cf-\sqrt{c^2}}\right.}{3bc}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2330 vs. $2(496) = 992$.
time = 24.06, size = 2330, normalized size = 4.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-6*I)*a^2*m*ArcCos[c*x]^2 - (4*I)*a*b*m*ArcCos[c*x]^3 - I*b^2*m*ArcCos[c*
x]^4 + (48*I)*a^2*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*ArcTan[((c*f - g)*Tan
[ArcCos[c*x]/2])/Sqrt[c^2*f^2 - g^2]] + 12*a*b*m*ArcCos[c*x]^2*Log[1 + (E^(
I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])] + 8*b^2*m*ArcCos[c*x]^3*Log[
1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])] + 12*a^2*m*ArcCos[c*
```

x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 12*a*b*m*Ar
cCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 4*
b^2*m*ArcCos[c*x]^3*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))
/g] + 24*a^2*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])
*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 48*a*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*
f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] +
 24*b^2*m*ArcCos[c*x]^2*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*Arc
Cos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 12*a*b*m*ArcCos[c*x]^2*Log[1 +
(E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])] + 8*b^2*m*ArcCos[c*x]^3*
Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])] + 12*a^2*m*ArcCo
s[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] + 12*a*b*
m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g]
 + 4*b^2*m*ArcCos[c*x]^3*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^
2]))/g] - 24*a^2*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c
x])(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 48*a*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 +
 (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/
g] - 24*b^2*m*ArcCos[c*x]^2*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I
*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 12*a^2*m*ArcCos[c*x]*Log[f
 + g*x] - 12*a^2*m*ArcSin[c*x]*Log[f + g*x] - 12*a*b*ArcCos[c*x]^2*Log[h*(f
 + g*x)^m] - 4*b^2*ArcCos[c*x]^3*Log[h*(f + g*x)^m] + 12*a^2*ArcSin[c*x]*Log
[h*(f + g*x)^m] - 4*b^2*m*ArcCos[c*x]^3*Log[1 + (g*(c*x + I*Sqrt[1 - c^2*x^
2]))/(c*f - Sqrt[c^2*f^2 - g^2])] - 12*a*b*m*ArcCos[c*x]^2*Log[1 + ((c*f -
Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - 4*b^2*m*ArcCos[c*x]^
3*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - 48
*a*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + ((c*f - Sqrt[c
^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - 24*b^2*m*ArcCos[c*x]^2*Arc
Sin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x +
I*Sqrt[1 - c^2*x^2]))/g] - 4*b^2*m*ArcCos[c*x]^3*Log[1 + (g*(c*x + I*Sqrt[1
 - c^2*x^2]))/(c*f + Sqrt[c^2*f^2 - g^2])] - 12*a*b*m*ArcCos[c*x]^2*Log[1 +
 ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - 4*b^2*m*Arc
Cos[c*x]^3*Log[1 + ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2])
)/g] + 48*a*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + ((c*f
 + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] + 24*b^2*m*ArcCos[c
*x]^2*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + ((c*f + Sqrt[c^2*f^2 - g^2]
)*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - (12*I)*b*m*ArcCos[c*x]*(2*a + b*ArcCos[
c*x])*PolyLog[2, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2])] - (1
2*I)*a^2*m*PolyLog[2, (E^(I*ArcCos[c*x])*(-(c*f) + Sqrt[c^2*f^2 - g^2]))/g]
 - (24*I)*a*b*m*ArcCos[c*x]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[
c^2*f^2 - g^2]))] - (12*I)*b^2*m*ArcCos[c*x]^2*PolyLog[2, -((E^(I*ArcCos[c*
x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))] - (12*I)*a^2*m*PolyLog[2, -((E^(I*ArcC
os[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g)] + 24*a*b*m*PolyLog[3, (E^(I*ArcCo
s[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2])] + 24*b^2*m*ArcCos[c*x]*PolyLog[3
, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2])] + 24*a*b*m*PolyLog[
3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))] + 24*b^2*m*ArcCos[
c*x]*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))] + (24

```
*I)*b^2*m*PolyLog[4, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2])]
+ (24*I)*b^2*m*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2
]))])/(12*c)
```

Maple [F]
time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arccos(cx))^2\ln(h(gx+f)^m)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (b^2*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/(sqrt(c*x + 1
)*sqrt(-c*x + 1)), x)*log(h) + 2*a*b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x)*log(h) + b^2*c*integrat
e(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2*log((g*x + f)^m)/(sqrt(c*x +
 1)*sqrt(-c*x + 1)), x) + 2*a*b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x)*log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a^2*c*int
egrate(log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a^2*arctan2(c*
x, sqrt(-c^2*x^2 + 1))*log(h))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*
log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acos}(cx))^2 \log(h(f + gx)^m)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(h\,(f + g\,x)^m)\,(a + b\operatorname{acos}(c\,x))^2}{\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^2)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^2)/(1 - c^2*x^2)^(1/2), x)
```

3.21 $\int \frac{(a+b\mathbf{ArcCos}(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=374

$$-\frac{im(a+b\mathrm{ArcCos}(cx))^3}{6b^2c}+\frac{m(a+b\mathrm{ArcCos}(cx))^2\log\left(1+\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m(a+b\mathrm{ArcCos}(cx))^2\log\left(1+\right.}{2bc}$$

```
[Out] -1/6*I*m*(a+b*arccos(c*x))^3/b^2/c-1/2*(a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/
b/c+1/2*m*(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f
^2-g^2)^(1/2)))/b/c+1/2*m*(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/b/c-I*m*(a+b*arccos(c*x))*polylog(2,-(c*x+I
*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-I*m*(a+b*arccos(c*x))*p
olylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+b*m*pol
ylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+b*m*polyl
og(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

Rubi [A]
time = 0.42, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4738, 4864, 4826, 4616, 2221, 2611, 2320, 6724}

$$-\frac{im(a+b\mathrm{ArcCos}(cx))^3}{6b^2c}-\frac{im(a+b\mathrm{ArcCos}(cx))\mathrm{Li}_2\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{im(a+b\mathrm{ArcCos}(cx))\mathrm{Li}_2\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{m(a+b\mathrm{ArcCos}(cx))^2\log\left(1+\frac{ge^{i\mathrm{ArcCos}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}+\frac{m(a+b\mathrm{ArcCos}(cx))^2\log\left(1+\frac{ge^{i\mathrm{ArcCos}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{2bc}-\frac{(a+b\mathrm{ArcCos}(cx))^2\log\left(h(f+gx)^m\right)}{2bc}+\frac{bm\mathrm{Li}_3\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{bm\mathrm{Li}_3\left(-\frac{e^{i\mathrm{ArcCos}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-1/6*I)*m*(a + b*ArcCos[c*x])^3)/(b^2*c) + (m*(a + b*ArcCos[c*x])^2*Log[1
 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c) + (m*(a + b*
ArcCos[c*x])^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/
(2*b*c) - ((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (I*m*(a + b*
ArcCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])
)])/c - (I*m*(a + b*ArcCos[c*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f +
Sqrt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f -
Sqrt[c^2*f^2 - g^2]))])/c + (b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f +
Sqrt[c^2*f^2 - g^2]))])/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4616

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] + b
*E^(I*(c + d*x)))), x], x] - Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(a +
Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4864

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-Log[h*(f + g*x)^m]
)*((a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Dist[g*(m/(b*c*
Sqrt[d]*(n + 1))), Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IGt
Q[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\cos^{-1}(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = -\frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} + \frac{(gm)\int \frac{\left(a+b\cos^{-1}(cx)\right)^2}{f+gx}\,dx}{2bc}$$

$$= -\frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\text{Subst}\left(\int \frac{(a+bx)^2\sin(x)}{cf+g\cos(x)}\right.}{2bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c} - \frac{\left(a+b\cos^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} +$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c} + \frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}}}{cf-\sqrt{c^2f}}\right.}{2bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c} + \frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}}}{cf-\sqrt{c^2f}}\right.}{2bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c} + \frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}}}{cf-\sqrt{c^2f}}\right.}{2bc}$$

$$= -\frac{im\left(a+b\cos^{-1}(cx)\right)^3}{6b^2c} + \frac{m\left(a+b\cos^{-1}(cx)\right)^2\log\left(1+\frac{e^{i\cos^{-1}}}{cf-\sqrt{c^2f}}\right.}{2bc}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1248 vs. 2(374) = 748.
time = 3.66, size = 1248, normalized size = 3.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((-3*I)*a*m*ArcCos[c*x]^2 - I*b*m*ArcCos[c*x]^3 + (24*I)*a*m*ArcSin[Sqrt[1
+ (c*f)/g]/Sqrt[2]]*ArcTan[((c*f - g)*Tan[ArcCos[c*x]/2])/Sqrt[c^2*f^2 - g^
```

```
2]] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2
 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^
2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[
c^2*f^2 - g^2]))/g] + 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(
I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 12*b*m*ArcCos[c*x]*ArcSin[
Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 -
 g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[
c^2*f^2 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt
[c^2*f^2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f
+ Sqrt[c^2*f^2 - g^2]))/g] - 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1
 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 12*b*m*ArcCos[c*x]*
ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^
2*f^2 - g^2]))/g] - 6*a*m*ArcCos[c*x]*Log[f + g*x] - 6*a*m*ArcSin[c*x]*Log[
f + g*x] - 3*b*ArcCos[c*x]^2*Log[h*(f + g*x)^m] + 6*a*ArcSin[c*x]*Log[h*(f
+ g*x)^m] - 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x +
 I*Sqrt[1 - c^2*x^2]))/g] - 12*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqr
t[2]]*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g]
- 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1
- c^2*x^2]))/g] + 12*b*m*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[
1 + ((c*f + Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - (6*I)*b*
m*ArcCos[c*x]*PolyLog[2, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sqrt[c^2*f^2 - g^2
])] - (6*I)*a*m*PolyLog[2, (E^(I*ArcCos[c*x])*(-(c*f) + Sqrt[c^2*f^2 - g^2]
))/g] - (6*I)*b*m*ArcCos[c*x]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqr
t[c^2*f^2 - g^2]))] - (6*I)*a*m*PolyLog[2, -((E^(I*ArcCos[c*x])*(c*f + Sqrt
[c^2*f^2 - g^2]))/g)] + 6*b*m*PolyLog[3, (E^(I*ArcCos[c*x])*g)/(-(c*f) + Sq
rt[c^2*f^2 - g^2])] + 6*b*m*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[
c^2*f^2 - g^2]))]])/(6*c)
```

Maple [F]
time = 3.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \arccos\left(cx\right)\right) \ln\left(h(gx+f)^{m}\right)}{\sqrt{-c^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] (b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sqrt(c*x + 1)*sq
rt(-c*x + 1)), x)*log(h) + b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)*log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a*c*integrat
e(log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a*arctan2(c*x, sqrt
(-c^2*x^2 + 1))*log(h))/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*log((g*x + f)^m*h)/(c^2*x^
2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acos}{(cx)}) \log{(h(f + gx)^m)}}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acos(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}{\sqrt{1-c^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acos(c*x)))/(1 - c^2*x^2)^(1/2),x)
[Out] int((log(h*(f + g*x)^m)*(a + b*acos(c*x)))/(1 - c^2*x^2)^(1/2), x)
```

3.22 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=237

$$\frac{im\text{ArcSin}(cx)^2}{2c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{\text{ArcS}}{}$$

```
[Out] 1/2*I*m*arcsin(c*x)^2/c+arcsin(c*x)*ln(h*(g*x+f)^m)/c-m*arcsin(c*x)*ln(1-I*
(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-m*arcsin(c*x)*ln(
1-I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2
,I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2,
I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

Rubi [A]

time = 0.24, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {222, 2451, 4825, 4615, 2221, 2317, 2438}

$$\frac{im\text{Li}_2\left(\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{Li}_2\left(\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ige^{i\text{ArcSin}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ige^{i\text{ArcSin}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{\text{ArcSin}(cx)\log\left(h(f+gx)^m\right)}{c}+\frac{im\text{ArcSin}(cx)^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sin^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{x\cos(x)}{c^2f+cg\sin(x)}\,dx, x, \sin^{-1}(cx)\right)$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{e^{ix}x}{c^2f - ice^{ix}g - c\sqrt{c}\ldots}\right.$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

Mathematica [A]
time = 0.02, size = 246, normalized size = 1.04

$$\frac{im\text{ArcSin}(cx)^2}{2c} - \frac{m\text{ArcSin}(cx)\log\left(1-\frac{ice^{i\text{ArcSin}(cx)}g}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\text{ArcSin}(cx)\log\left(1-\frac{ice^{i\text{ArcSin}(cx)}g}{c^2f+c\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\text{ArcSin}(cx)\log\left(h(f+gx)^m\right)}{c} + \frac{im\text{PolyLog}\left(2,\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\text{PolyLog}\left(2,\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*g
)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])]])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*Ar
cSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])]])/c + (ArcSin[c*x]*Log[h*(f +
 g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 -
 g^2])]])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2])]])/c
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.23 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}\,(a+b\mathbf{ArcCos}(cx))}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))},x\right)$$

```
[Out] Unintegrable(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\,\left(a+b\cos^{-1}(cx)\right)}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\,\left(a+b\cos^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\,(a+b\text{ArcCos}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Maple [A]
time = 2.77, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^m\right)}{(a+b\arccos(cx))\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b
)*arccos(c*x) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*acos(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)))
, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.24 $\int x^3 \mathbf{ArcCos}(a+bx)\,dx$

Optimal. Leaf size=137

$$\frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2}-\frac{x^3\sqrt{1-(a+bx)^2}}{16b}+\frac{(4a(16+19a^2)-(9+26a^2)(a+bx))\sqrt{1-(a+bx)^2}}{96b^4}+\frac{1}{4}x^4\mathrm{A}$$

```
[Out] 1/4*x^4*arccos(b*x+a)+1/32*(8*a^4+24*a^2+3)*arcsin(b*x+a)/b^4+7/48*a*x^2*(1
-(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1-(b*x+a)^2)^(1/2)/b+1/96*(4*a*(19*a^2+16)-
(26*a^2+9)*(b*x+a))*(1-(b*x+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4890, 4828, 757, 847, 794, 222}

$$\frac{(4a(19a^2+16)-(26a^2+9)(a+bx))\sqrt{1-(a+bx)^2}}{96b^4}+\frac{(8a^4+24a^2+3)\,\mathrm{ArcSin}(a+bx)}{32b^4}+\frac{1}{4}x^4\mathrm{ArcCos}(a+bx)+\frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2}-\frac{x^3\sqrt{1-(a+bx)^2}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[1 - (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 - (a + b*x)^2])/(16*
b) + ((4*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x))*Sqrt[1 - (a + b*x)^2])/(
96*b^4) + (x^4*ArcCos[a + b*x])/4 + ((3 + 24*a^2 + 8*a^4)*ArcSin[a + b*x])/
(32*b^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \cos^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \cos^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4\cos^{-1}(a+bx) + \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1-x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4}x^4\cos^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3+4a^2}{b^2}+\frac{7ax}{b^2}\right)\left(-\frac{a}{b}+\right.}{\sqrt{1-x^2}}\right.$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{1}{4}x^4\cos^{-1}(a+bx) + \frac{1}{48}\text{Subst}\left(\int\right.$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{(4a(16+19a^2)-(9+26a^2)(a+bx}{96b^4}$$

$$= \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{(4a(16+19a^2)-(9+26a^2)(a+bx}{96b^4}$$

Mathematica [A]
time = 0.08, size = 104, normalized size = 0.76

$$\frac{\sqrt{1-a^2-2abx-b^2x^2}\,(55a+50a^3-9bx-26a^2bx+14ab^2x^2-6b^3x^3)+24b^4x^4\text{ArcCos}(a+bx)+3(3+24a^2+8a^4)\,\text{ArcSin}(a+bx)}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a + b*x],x]
```

```
[Out] (Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x + 14
*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCos[a + b*x] + 3*(3 + 24*a^2 + 8*a^
4)*ArcSin[a + b*x])/(96*b^4)
```

Maple [A]
time = 0.05, size = 235, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{\arccos(bx+a)a^4}{4} - \arccos(bx+a)a^3(bx+a) + \frac{3\arccos(bx+a)a^2(bx+a)^2}{2} - \arccos(bx+a)a(bx+a)^3 + \frac{\arccos(bx+a)(bx+a)^4}{4} + \frac{\arcsin(b}{}}{}$

default	$\frac{\frac{\arccos(bx+a)a^4}{4} - \arccos(bx+a)a^3(bx+a) + \frac{3\arccos(bx+a)a^2(bx+a)^2}{2} - \arccos(bx+a)a(bx+a)^3 + \frac{\arccos(bx+a)(bx+a)^4}{4} + \text{arcsi}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arccos(b*x+a)*a^4-arccos(b*x+a)*a^3*(b*x+a)+3/2*arccos(b*x+a)*a^
2*(b*x+a)^2-arccos(b*x+a)*a*(b*x+a)^3+1/4*arccos(b*x+a)*(b*x+a)^4+1/4*arcsi
n(b*x+a)*a^4+a^3*(1-(b*x+a)^2)^(1/2)+3/2*a^2*(-1/2*(b*x+a)*(1-(b*x+a)^2)^(1
/2)+1/2*arcsin(b*x+a))-a*(-1/3*(b*x+a)^2*(1-(b*x+a)^2)^(1/2)-2/3*(1-(b*x+a)
^2)^(1/2))-1/16*(b*x+a)^3*(1-(b*x+a)^2)^(1/2)-3/32*(b*x+a)*(1-(b*x+a)^2)^(1
/2)+3/32*arcsin(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(120) = 240$.
time = 0.49, size = 333, normalized size = 2.43

$$\frac{1}{4}x^4\arccos(bx+a) - \frac{1}{96}\left(\frac{6\sqrt{-b^2x^2-2abx-a^2+1}\,x^3}{b^2} - \frac{14\sqrt{-b^2x^2-2abx-a^2+1}\,ax^2}{b^3} + \frac{105a^4\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^5} + \frac{35\sqrt{-b^2x^2-2abx-a^2+1}\,a^2x}{b^4} - \frac{90(a^2-1)a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^5} - \frac{105\sqrt{-b^2x^2-2abx-a^2+1}\,a^3}{b^5} - \frac{9\sqrt{-b^2x^2-2abx-a^2+1}\,(a^2-1)x}{b^4} + \frac{9(a^2-1)^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^5} + \frac{55\sqrt{-b^2x^2-2abx-a^2+1}\,(a^2-1)a}{b^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccos(b*x + a) - 1/96*(6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^3/b^
2 - 14*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsin(-(b^2*
x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 + 35*sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*a^2*x/b^4 - 90*(a^2 - 1)*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 -
(a^2 - 1)*b^2))/b^5 - 105*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/b^5 - 9*sq
rt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*x/b^4 + 9*(a^2 - 1)^2*arcsin(-(b
^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 + 55*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*(a^2 - 1)*a/b^5)*b
```

Fricas [A]
time = 2.44, size = 94, normalized size = 0.69

$$\frac{3\left(8b^4x^4 - 8a^4 - 24a^2 - 3\right)\arccos(bx+a) - \left(6b^3x^3 - 14ab^2x^2 - 50a^3 + (26a^2+9)bx - 55a\right)\sqrt{-b^2x^2 - 2abx - a^2 + 1}}{96b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*arccos(b*x + a) - (6*b^3*x^3 - 14*
a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55*a)*sqrt(-b^2*x^2 - 2*a*b*x - a^2
 + 1))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.
time = 0.30, size = 255, normalized size = 1.86

$$\begin{cases} -\frac{a^4 \operatorname{acos}(a+bx)}{4b^4} + \frac{25a^3\sqrt{-a^2-2abx-b^2x^2+1}}{48b^4} - \frac{13a^2x\sqrt{-a^2-2abx-b^2x^2+1}}{48b^3} - \frac{3a^2 \operatorname{acos}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{-a^2-2abx-b^2x^2+1}}{48b^2} + \frac{55a\sqrt{-a^2-2abx-b^2x^2+1}}{96b^4} + \frac{x^4 \operatorname{acos}(a+bx)}{4} - \frac{x^3\sqrt{-a^2-2abx-b^2x^2+1}}{16b} - \frac{3x\sqrt{-a^2-2abx-b^2x^2+1}}{32b^3} - \frac{3\operatorname{acos}(a+bx)}{32b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acos}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acos(a + b*x)/(4*b**4) + 25*a**3*sqrt(-a**2 - 2*a*b*x - b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(4
8*b**3) - 3*a**2*acos(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(-a**2 - 2*a*b*x - b
**2*x**2 + 1)/(48*b**2) + 55*a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(96*b*
*4) + x**4*acos(a + b*x)/4 - x**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(16
*b) - 3*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(32*b**3) - 3*acos(a + b*x)
/(32*b**4), Ne(b, 0)), (x**4*acos(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(120) = 240$.
time = 0.41, size = 242, normalized size = 1.77

$$\frac{(bx+a)^4 \arccos(bx+a)}{4b^4} - \frac{(bx+a)^3 a \arccos(bx+a)}{b^4} + \frac{3(bx+a)^2 a^2 \arccos(bx+a)}{2b^4} - \frac{(bx+a)a^3 \arccos(bx+a)}{b^4} - \frac{\sqrt{-(bx+a)^2+1}\,(bx+a)^3}{16b^4} + \frac{\sqrt{-(bx+a)^2+1}\,(bx+a)^2 a}{3b^4} - \frac{3\sqrt{-(bx+a)^2+1}\,(bx+a)a^2}{4b^4} + \frac{\sqrt{-(bx+a)^2+1}\,a^3}{b^4} - \frac{3a^2 \arccos(bx+a)}{4b^4} - \frac{3\sqrt{-(bx+a)^2+1}\,(bx+a)}{32b^4} + \frac{2\sqrt{-(bx+a)^2+1}\,a}{3b^4} - \frac{3\arccos(bx+a)}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4*arccos(b*x + a)/b^4 - (b*x + a)^3*a*arccos(b*x + a)/b^4 + 3
/2*(b*x + a)^2*a^2*arccos(b*x + a)/b^4 - (b*x + a)*a^3*arccos(b*x + a)/b^4
- 1/16*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^3/b^4 + 1/3*sqrt(-(b*x + a)^2 + 1)*
(b*x + a)^2*a/b^4 - 3/4*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a^2/b^4 + sqrt(-(b
*x + a)^2 + 1)*a^3/b^4 - 3/4*a^2*arccos(b*x + a)/b^4 - 3/32*sqrt(-(b*x + a)
^2 + 1)*(b*x + a)/b^4 + 2/3*sqrt(-(b*x + a)^2 + 1)*a/b^4 - 3/32*arccos(b*x
+ a)/b^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acos}(a + b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(a + b*x),x)
```

```
[Out] int(x^3*acos(a + b*x), x)
```

3.25 $\int x^2 \textbf{ArcCos}(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}-\frac{(4+11a^2-5abx)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\text{ArcCos}(a+bx)-\frac{a(3+2a^2)\,\text{ArcSin}(a+bx}{6b^3}$$

```
[Out] 1/3*x^3*arccos(b*x+a)-1/6*a*(2*a^2+3)*arcsin(b*x+a)/b^3-1/9*x^2*(1-(b*x+a)^
2)^(1/2)/b-1/18*(-5*a*b*x+11*a^2+4)*(1-(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4890, 4828, 757, 794, 222}

$$-\frac{a(2a^2+3)\,\text{ArcSin}(a+bx)}{6b^3}-\frac{(11a^2-5abx+4)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\text{ArcCos}(a+bx)-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a + b*x],x]
```

```
[Out] -1/9*(x^2*Sqrt[1 - (a + b*x)^2])/b - ((4 + 11*a^2 - 5*a*b*x)*Sqrt[1 - (a +
b*x)^2])/(18*b^3) + (x^3*ArcCos[a + b*x])/3 - (a*(3 + 2*a^2)*ArcSin[a + b*x
])/(6*b^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\cos^{-1}(a+bx)\,dx=\frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cos^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{3}x^3\cos^{-1}(a+bx)+\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1-x^2}}\,dx,x,a+bx\right)$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}+\frac{1}{3}x^3\cos^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(-\frac{2+3a^2}{b^2}+\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\frac{x}{b}\right.}{\sqrt{1-x^2}}\right.$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}-\frac{(4+11a^2-5abx)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\cos^{-1}(a+bx)-$$

$$=-\frac{x^2\sqrt{1-(a+bx)^2}}{9b}-\frac{(4+11a^2-5abx)\sqrt{1-(a+bx)^2}}{18b^3}+\frac{1}{3}x^3\cos^{-1}(a+bx)-$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.88

$$-\frac{\sqrt{1-a^2-2abx-b^2x^2}\,(4+11a^2-5abx+2b^2x^2)-6b^3x^3\text{ArcCos}(a+bx)+3a(3+2a^2)\,\text{ArcSin}(a+bx)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a + b*x],x]
```

```
[Out] -1/18*(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)
 - 6*b^3*x^3*ArcCos[a + b*x] + 3*a*(3 + 2*a^2)*ArcSin[a + b*x])/b^3
```

Maple [A]
time = 0.01, size = 161, normalized size = 1.71

method	result
derivativedivides	$-\frac{\arccos(bx+a)a^3}{3}+\arccos(bx+a)a^2(bx+a)-\arccos(bx+a)a(bx+a)^2+\frac{\arccos(bx+a)(bx+a)^3}{3}-\frac{\arcsin(bx+a)a^3}{3}-a^2\sqrt{1-(}$
default	$-\frac{\arccos(bx+a)a^3}{3}+\arccos(bx+a)a^2(bx+a)-\arccos(bx+a)a(bx+a)^2+\frac{\arccos(bx+a)(bx+a)^3}{3}-\frac{\arcsin(bx+a)a^3}{3}-a^2\sqrt{1-(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arccos(b*x+a)*a^3+arccos(b*x+a)*a^2*(b*x+a)-arccos(b*x+a)*a*(b*
x+a)^2+1/3*arccos(b*x+a)*(b*x+a)^3-1/3*arcsin(b*x+a)*a^3-a^2*(1-(b*x+a)^2)^
(1/2)-a*(-1/2*(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a))-1/9*(b*x+a)^2*
(1-(b*x+a)^2)^(1/2)-2/9*(1-(b*x+a)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(82) = 164.
time = 0.47, size = 220, normalized size = 2.34

$$\frac{1}{3}x^3\arccos(bx+a)-\frac{1}{18}b\left(\frac{2\sqrt{-b^2x^2-2abx-a^2+1}\,x^2}{b^2}-\frac{15a^3\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^4}-\frac{5\sqrt{-b^2x^2-2abx-a^2+1}\,ax}{b^3}+\frac{9(a^2-1)a\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^4}+\frac{15\sqrt{-b^2x^2-2abx-a^2+1}\,a^2}{b^4}-\frac{4\sqrt{-b^2x^2-2abx-a^2+1}\,(a^2-1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(b*x + a) - 1/18*b*(2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^2/
b^2 - 15*a^3*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 - 5*s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^3 + 9*(a^2 - 1)*a*arcsin(-(b^2*x +
a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + 15*sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*a^2/b^4 - 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)/b^4)
```

Fricas [A]
time = 2.67, size = 75, normalized size = 0.80

$$\frac{3\left(2b^3x^3+2a^3+3a\right)\arccos(bx+a)-\left(2b^2x^2-5abx+11a^2+4\right)\sqrt{-b^2x^2-2abx-a^2+1}}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*arccos(b*x + a) - (2*b^2*x^2 - 5*a*b*x +
11*a^2 + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(83) = 166$.
time = 0.17, size = 170, normalized size = 1.81

$$\begin{cases} \frac{a^3 \operatorname{acos}(a+bx)}{3b^3} - \frac{11a^2\sqrt{-a^2-2abx-b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{-a^2-2abx-b^2x^2+1}}{18b^2} + \frac{a \operatorname{acos}(a+bx)}{2b^3} + \frac{x^3 \operatorname{acos}(a+bx)}{3} - \frac{x^2\sqrt{-a^2-2abx-b^2x^2+1}}{9b} - \frac{2\sqrt{-a^2-2abx-b^2x^2+1}}{9b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acos}(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(b*x+a),x)
```

```
[Out] Piecewise((a**3*acos(a + b*x)/(3*b**3) - 11*a**2*sqrt(-a**2 - 2*a*b*x - b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(18*b**
2) + a*acos(a + b*x)/(2*b**3) + x**3*acos(a + b*x)/3 - x**2*sqrt(-a**2 - 2*
a*b*x - b**2*x**2 + 1)/(9*b) - 2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(9*b
**3), Ne(b, 0)), (x**3*acos(a)/3, True))
```

Giac [A]
time = 0.41, size = 156, normalized size = 1.66

$$\frac{(bx+a)^3 \arccos(bx+a)}{3\,b^3} - \frac{(bx+a)^2 a \arccos(bx+a)}{b^3} + \frac{(bx+a)a^2 \arccos(bx+a)}{b^3} - \frac{\sqrt{-(bx+a)^2+1}\,(bx+a)^2}{9\,b^3} + \frac{\sqrt{-(bx+a)^2+1}\,(bx+a)a}{2\,b^3} - \frac{\sqrt{-(bx+a)^2+1}\,a^2}{b^3} + \frac{a \arccos(bx+a)}{2\,b^3} - \frac{2\,\sqrt{-(bx+a)^2+1}}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*arccos(b*x + a)/b^3 - (b*x + a)^2*a*arccos(b*x + a)/b^3 + (
b*x + a)*a^2*arccos(b*x + a)/b^3 - 1/9*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^2/b
^3 + 1/2*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a/b^3 - sqrt(-(b*x + a)^2 + 1)*a^
2/b^3 + 1/2*a*arccos(b*x + a)/b^3 - 2/9*sqrt(-(b*x + a)^2 + 1)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acos}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(a + b*x),x)
```

```
[Out] int(x^2*acos(a + b*x), x)
```

3.26 $\int x\mathbf{ArcCos}(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\text{ArcCos}(a+bx)+\frac{(1+2a^2)\,\text{ArcSin}(a+bx)}{4b^2}$$

```
[Out] 1/2*x^2*arccos(b*x+a)+1/4*(2*a^2+1)*arcsin(b*x+a)/b^2+3/4*a*(1-(b*x+a)^2)^(
1/2)/b^2-1/4*x*(1-(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.625$, Rules used = {4890, 4828, 757, 655, 222}

$$\frac{(2a^2+1)\,\text{ArcSin}(a+bx)}{4b^2}+\frac{1}{2}x^2\text{ArcCos}(a+bx)+\frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 - (a + b*x)^2])/(4*b) + (x^
2*ArcCos[a + b*x])/2 + ((1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
```

```
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos^{-1}(a+bx)\,dx &= \frac{\mathrm{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\cos^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{1}{2}\mathrm{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1-x^2}}\,dx,x,a+bx\right)\\ &= -\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)-\frac{1}{4}\mathrm{Subst}\left(\int\frac{-\frac{1+2a^2}{b^2}+\frac{3ax}{b^2}}{\sqrt{1-x^2}}\,dx,x,a+bx\right)\\ &= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{(1+2a^2)\,\mathrm{Subst}\left(\int\right.}{}\\ &= \frac{3a\sqrt{1-(a+bx)^2}}{4b^2}-\frac{x\sqrt{1-(a+bx)^2}}{4b}+\frac{1}{2}x^2\cos^{-1}(a+bx)+\frac{(1+2a^2)\sin^{-1}(a+}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.86

$$\frac{(3a-bx)\sqrt{1-a^2-2abx-b^2x^2}+2b^2x^2\mathrm{ArcCos}(a+bx)+(1+2a^2)\,\mathrm{ArcSin}(a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*b^2*x^2*ArcCos[a + b*x]
+ (1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

Maple [A]
time = 0.01, size = 78, normalized size = 0.98

method	result	s

derivativedivides	$\frac{\frac{\arccos(bx+a)(bx+a)^2}{2}-\arccos(bx+a)a(bx+a)+a\sqrt{1-(bx+a)^2}-\frac{(bx+a)\sqrt{1-(bx+a)^2}}{4}+\frac{\arcsin(bx+a)}{4}}{b^2}$
default	$\frac{\frac{\arccos(bx+a)(bx+a)^2}{2}-\arccos(bx+a)a(bx+a)+a\sqrt{1-(bx+a)^2}-\frac{(bx+a)\sqrt{1-(bx+a)^2}}{4}+\frac{\arcsin(bx+a)}{4}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arccos(b*x+a)*(b*x+a)^2-arccos(b*x+a)*a*(b*x+a)+a*(1-(b*x+a)^2)^
(1/2)-1/4*(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/4*arcsin(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.
time = 0.47, size = 153, normalized size = 1.91

$$\frac{1}{2}x^2\arccos(bx+a)-\frac{1}{4}b\left(\frac{3a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,x}{b^2}-\frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^3}-\frac{3\sqrt{-b^2x^2-2abx-a^2+1}\,a}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(b*x + a) - 1/4*b*(3*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 -
 (a^2 - 1)*b^2))/b^3 + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 - (a^2 - 1)
*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 3*sqrt(-b^2*x^2
 - 2*a*b*x - a^2 + 1)*a/b^3)
```

Fricas [A]
time = 2.83, size = 59, normalized size = 0.74

$$\frac{(2b^2x^2-2a^2-1)\arccos(bx+a)-\sqrt{-b^2x^2-2abx-a^2+1}\,(bx-3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 - 1)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*(b*x - 3*a))/b^2
```

Sympy [A]
time = 0.11, size = 104, normalized size = 1.30

$$\begin{cases} -\frac{a^2\operatorname{acos}(a+bx)}{2b^2}+\frac{3a\sqrt{-a^2-2abx-b^2x^2+1}}{4b^2}+\frac{x^2\operatorname{acos}(a+bx)}{2}-\frac{x\sqrt{-a^2-2abx-b^2x^2+1}}{4b}-\frac{\operatorname{acos}(a+bx)}{4b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{acos}(a)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acos(a + b*x)/(2*b**2) + 3*a*sqrt(-a**2 - 2*a*b*x - b**2*x
**2 + 1)/(4*b**2) + x**2*acos(a + b*x)/2 - x*sqrt(-a**2 - 2*a*b*x - b**2*x*
*2 + 1)/(4*b) - acos(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acos(a)/2, True))
```

Giac [A]
time = 0.42, size = 88, normalized size = 1.10

$$\frac{(bx+a)^2 \arccos(bx+a)}{2\,b^2} - \frac{(bx+a)a \arccos(bx+a)}{b^2} - \frac{\sqrt{-(bx+a)^2+1}\,(bx+a)}{4\,b^2} + \frac{\sqrt{-(bx+a)^2+1}\,a}{b^2} - \frac{\arccos(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x + a)^2*arccos(b*x + a)/b^2 - (b*x + a)*a*arccos(b*x + a)/b^2 - 1/4
*sqrt(-(b*x + a)^2 + 1)*(b*x + a)/b^2 + sqrt(-(b*x + a)^2 + 1)*a/b^2 - 1/4*
arccos(b*x + a)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acos}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(a + b*x),x)
```

```
[Out] int(x*acos(a + b*x), x)
```

3.27 $\int \mathbf{ArcCos}(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{1-(a+bx)^2}}{b}+\frac{(a+bx)\text{ArcCos}(a+bx)}{b}$$

```
[Out] (b*x+a)*arccos(b*x+a)/b-(1-(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4888, 4716, 267}

$$\frac{(a+bx)\text{ArcCos}(a+bx)}{b}-\frac{\sqrt{1-(a+bx)^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x],x]
```

```
[Out] -(Sqrt[1 - (a + b*x)^2]/b) + ((a + b*x)*ArcCos[a + b*x])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cos^{-1}(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{1-(a+bx)^2}}{b} + \frac{(a+bx)\cos^{-1}(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(36) = 72.
time = 0.07, size = 154, normalized size = 4.28

$$x\text{ArcCos}(a+bx) - \frac{2b\sqrt{1-a^2-2abx-b^2x^2} + 2ab\text{ArcTan}\left(\frac{\sqrt{-b^2}\,x - \sqrt{1-a^2-2abx-b^2x^2}}{a}\right) + a\sqrt{-b^2}\,\log\left(-1+2abx+2b^2x^2+2\sqrt{-b^2}\,x\sqrt{1-a^2-2abx-b^2x^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x],x]
```

```
[Out] x*ArcCos[a + b*x] - (2*b*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*a*b*ArcTan[(
Sqrt[-b^2]*x - Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + a*Sqrt[-b^2]*Log[-1
+ 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]])/
(2*b^2)
```

Maple [A]
time = 0.01, size = 33, normalized size = 0.92

method	result	size
derivativedivides	$\frac{(bx+a)\arccos(bx+a)-\sqrt{1-(bx+a)^2}}{b}$	33
default	$\frac{(bx+a)\arccos(bx+a)-\sqrt{1-(bx+a)^2}}{b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arccos(b*x+a)-(1-(b*x+a)^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 32, normalized size = 0.89

$$\frac{(bx+a)\arccos(bx+a) - \sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="maxima")
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

Fricas [A]
time = 3.38, size = 41, normalized size = 1.14

$$\frac{(bx+a)\arccos(bx+a) - \sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="fricas")
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b
```

Sympy [A]
time = 0.07, size = 46, normalized size = 1.28

$$\begin{cases} \frac{a\operatorname{acos}(a+bx)}{b} + x\operatorname{acos}(a+bx) - \frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}}{b} & \text{for } b \neq 0 \\ x\operatorname{acos}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a),x)
[Out] Piecewise((a*acos(a + b*x)/b + x*acos(a + b*x) - sqrt(-a**2 - 2*a*b*x - b**
2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a), True))
```

Giac [A]
time = 0.41, size = 32, normalized size = 0.89

$$\frac{(bx+a)\arccos(bx+a) - \sqrt{-(bx+a)^2 + 1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a),x, algorithm="giac")
[Out] ((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

Mupad [B]
time = 0.54, size = 88, normalized size = 2.44

$$x\operatorname{acos}(a+b\,x) - \frac{\sqrt{-a^2 - 2\,a\,b\,x - b^2\,x^2 + 1}}{b} - \frac{a\,\ln\left(\sqrt{-a^2 - 2\,a\,b\,x - b^2\,x^2 + 1} - \frac{x\,b^2 + a\,b}{\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x),x)
```

```
[Out] x*acos(a + b*x) - (1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2)/b - (a*log((1 - b^2*x
^2 - 2*a*b*x - a^2)^(1/2) - (a*b + b^2*x)/(-b^2)^(1/2)))/(-b^2)^(1/2)
```

3.28 $\int \frac{\textbf{ArcCos}(a+bx)}{x}\,dx$

Optimal. Leaf size=177

$$-\frac{1}{2}i\text{ArcCos}(a+bx)^2+\text{ArcCos}(a+bx)\log\left(1-\frac{e^{i\text{ArcCos}(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\text{ArcCos}(a+bx)\log\left(1-\frac{e^{i\text{ArcCos}(a+bx)}}{a+i\sqrt{1-a^2}}\right)-$$

```
[Out] -1/2*I*arccos(b*x+a)^2+arccos(b*x+a)*ln(1-(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a-
I*(-a^2+1)^(1/2)))+arccos(b*x+a)*ln(1-(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a+I*(-
a^2+1)^(1/2)))-I*polylog(2,(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a-I*(-a^2+1)^(1/2
)))-I*polylog(2,(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a+I*(-a^2+1)^(1/2)))
```

Rubi [A]

time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4890, 4826, 4618, 2221, 2317, 2438}

$$-i\text{Li}_2\left(\frac{e^{i\text{ArcCos}(a+bx)}}{a-i\sqrt{1-a^2}}\right)-i\text{Li}_2\left(\frac{e^{i\text{ArcCos}(a+bx)}}{a+i\sqrt{1-a^2}}\right)+\text{ArcCos}(a+bx)\log\left(1-\frac{e^{i\text{ArcCos}(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\text{ArcCos}(a+bx)\log\left(1-\frac{e^{i\text{ArcCos}(a+bx)}}{a+i\sqrt{1-a^2}}\right)-\frac{1}{2}i\text{ArcCos}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*ArcCos[a + b*x]^2 + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/
(a - I*Sqrt[1 - a^2])] + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a +
 I*Sqrt[1 - a^2])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a - I*Sqrt[1 - a^2
])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a + I*Sqrt[1 - a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4618

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

Rule 4826

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\sin(x)}{-\frac{a}{b}+\frac{\cos(x)}{b}}\,dx, x, \cos^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{e^{ix}x}{-\frac{ia}{b}-\frac{\sqrt{1-a^2}}{b}+\frac{ie^{ix}}{b}}\,dx, x, \cos^{-1}(a+bx)\right)}{b} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(\right.$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(\right.$$

$$= -\frac{1}{2}i\cos^{-1}(a+bx)^2 + \cos^{-1}(a+bx)\log\left(1-\frac{e^{i\cos^{-1}(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \cos^{-1}(a+bx)\log\left(\right.$$

Mathematica [A]
time = 0.14, size = 228, normalized size = 1.29

$$-\frac{1}{2}i\text{ArcCos}(a+bx)^2 - 4i\text{ArcSin}\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\text{ArcTan}\left(\frac{(1+a)\tan\left(\frac{1}{2}\text{ArcCos}(a+bx)\right)}{\sqrt{-1+a^2}}\right) + \left(\text{ArcCos}(a+bx) - 2\text{ArcSin}\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\right)\log\left(1+\left(-a+\sqrt{-1+a^2}\right)e^{i\text{ArcCos}(a+bx)}\right) + \left(\text{ArcCos}(a+bx) + 2\text{ArcSin}\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\right)\log\left(1-\left(a+\sqrt{-1+a^2}\right)e^{i\text{ArcCos}(a+bx)}\right) - i\left(\text{PolyLog}\left(2, \left(a-\sqrt{-1+a^2}\right)e^{i\text{ArcCos}(a+bx)}\right) + \text{PolyLog}\left(2, \left(a+\sqrt{-1+a^2}\right)e^{i\text{ArcCos}(a+bx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*ArcCos[a + b*x]^2 - (4*I)*ArcSin[Sqrt[1 - a]/Sqrt[2]]*ArcTan[((1 +
 a)*Tan[ArcCos[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a + b*x] - 2*ArcSin[S
qrt[1 - a]/Sqrt[2]])*Log[1 + (-a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])] +
 (ArcCos[a + b*x] + 2*ArcSin[Sqrt[1 - a]/Sqrt[2]])*Log[1 - (a + Sqrt[-1 + a
^2])*E^(I*ArcCos[a + b*x])] - I*(PolyLog[2, (a - Sqrt[-1 + a^2])*E^(I*ArcCo
s[a + b*x])] + PolyLog[2, (a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])])
```

Maple [A]
time = 0.83, size = 199, normalized size = 1.12

method	result
derivativedivides	$-\frac{i\arccos(bx+a)^2}{2}+\arccos(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-i\sqrt{1-(bx+a)^2}}{a+\sqrt{a^2-1}}\right)+\arccos(bx+a)$
default	$-\frac{i\arccos(bx+a)^2}{2}+\arccos(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-i\sqrt{1-(bx+a)^2}}{a+\sqrt{a^2-1}}\right)+\arccos(bx+a)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*arccos(b*x+a)^2+arccos(b*x+a)*ln(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)^2)^
(1/2))/(a+(a^2-1)^(1/2)))+arccos(b*x+a)*ln(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a)^
2)^(1/2))/(-a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a)^2)^(1
/2))/(-a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)^2)^(1/2))/
(a+(a^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(b*x + a)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x,x)
```

```
[Out] Integral(acos(a + b*x)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)/x,x)
```

```
[Out] int(acos(a + b*x)/x, x)
```

3.29 $\int \frac{\mathrm{ArcCos}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=63

$$-\frac{\mathrm{ArcCos}(a+bx)}{x}+\frac{b\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}$$

```
[Out] -arccos(b*x+a)/x+b*arctanh((1-a*(b*x+a))/(-a^2+1)^(1/2)/(1-(b*x+a)^2)^(1/2)
)/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4890, 4828, 739, 212}

$$\frac{b\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}-\frac{\mathrm{ArcCos}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^2,x]
```

```
[Out] -(ArcCos[a + b*x]/x) + (b*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 -
 (a + b*x)^2])])/Sqrt[1 - a^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(a+bx)}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}\\ &= -\frac{\cos^{-1}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1-x^2}}\,dx, x, a+bx\right)\\ &= -\frac{\cos^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\frac{1}{b^2}-\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}-\frac{a(a+bx)}{b}}{\sqrt{1-(a+bx)^2}}\right)\\ &= -\frac{\cos^{-1}(a+bx)}{x} + \frac{b\tanh^{-1}\left(\frac{b\left(\frac{1}{b}-\frac{a(a+bx)}{b}\right)}{\sqrt{1-a^2}\,\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.25

$$-\frac{\text{ArcCos}(a+bx)}{x} + \frac{b\left(-\log(x)+\log\left(1-a^2-abx+\sqrt{1-a^2}\,\sqrt{1-a^2-2abx-b^2x^2}\right)\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^2,x]
```

```
[Out] -(ArcCos[a + b*x]/x) + (b*(-Log[x] + Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sq
rt[1 - a^2 - 2*a*b*x - b^2*x^2]]))/Sqrt[1 - a^2]
```

Maple [A]
time = 0.01, size = 81, normalized size = 1.29

method	result	size
derivativedivides	$b\left(-\frac{\arccos(bx+a)}{bx} + \frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{\sqrt{-a^2+1}}\right)$	81

default	$b\left(-\frac{\arccos(bx+a)}{bx}+\frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{\sqrt{-a^2+1}}\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arccos(b*x+a)/b/x+1/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2
)*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/b/x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(57) = 114$.
time = 5.12, size = 360, normalized size = 5.71

$$\left[-\frac{\sqrt{-a^2+1}\,bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2(a^2-1)x\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+2(a^2-(a^2-1)x-1)\arccos(bx+a)}{2(a^2-1)x},\ -\frac{\sqrt{a^2-1}\,bx\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{a^2-1}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+(a^2-1)x\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+(a^2-(a^2-1)x-1)\arccos(bx+a)}{(a^2-1)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b*
x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1) -
 4*a^2 + 2)/x^2) + 2*(a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*
(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^2 - (a^2 - 1)*x - 1)*arccos
(b*x + a))/((a^2 - 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2*a*b*
x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(
a^3 - a)*b*x - 2*a^2 + 1)) + (a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (a^2 - (a^2 - 1)*x - 1)*
arccos(b*x + a))/((a^2 - 1)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**2,x)
```

```
[Out] Integral(acos(a + b*x)/x**2, x)
```

Giac [A]
time = 0.41, size = 79, normalized size = 1.25

$$-\frac{2\,b^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}\,|b|}-\frac{\arccos\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -2*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*abs(b)) - arccos(b*x + a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)/x^2,x)
```

```
[Out] int(acos(a + b*x)/x^2, x)
```

3.30 $\int \frac{\text{ArcCos}(a+bx)}{x^3} dx$

Optimal. Leaf size=103

$$\frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x}-\frac{\text{ArcCos}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}}$$

```
[Out] -1/2*arccos(b*x+a)/x^2+1/2*a*b^2*arctanh((1-a*(b*x+a))/(-a^2+1)^(1/2)/(1-(b
*x+a)^2)^(1/2))/(-a^2+1)^(3/2)+1/2*b*(1-(b*x+a)^2)^(1/2)/(-a^2+1)/x
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4890, 4828, 745, 739, 212}

$$\frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}}+\frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x}-\frac{\text{ArcCos}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(2*(1 - a^2)*x) - ArcCos[a + b*x]/(2*x^2) + (a*b^
2*ArcTanh[(1 - a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(2*(1 -
 a^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cos^{-1}(a+bx)}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1-x^2}}\,dx, x, a+bx\right)\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} - \frac{(ab)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2(1-a^2)}\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} + \frac{(ab)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}-\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}-\frac{a(a+bx)}{b}}{\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)}\\
&= \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)x} - \frac{\cos^{-1}(a+bx)}{2x^2} + \frac{ab^2\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 126, normalized size = 1.22

$$-\frac{\text{ArcCos}(a+bx) - \frac{bx\left(\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2}-abx\log(x)+abx\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2}\right)\right)}{(1-a^2)^{3/2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^3,x]
```

```
[Out] -1/2*(ArcCos[a + b*x] - (b*x*(Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^
2] - a*b*x*Log[x] + a*b*x*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2
- 2*a*b*x - b^2*x^2]]))/(1 - a^2)^(3/2))/x^2
```

Maple [A]
time = 0.01, size = 124, normalized size = 1.20

method	result
derivativedivides	$b^2\left(-\frac{\arccos(bx+a)}{2b^2x^2}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{2(-a^2+1)bx}+\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2}}{bx}\right.}{2(-a^2+1)^{\frac{3}{2}}}\right.$
default	$b^2\left(-\frac{\arccos(bx+a)}{2b^2x^2}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{2(-a^2+1)bx}+\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2}}{bx}\right.}{2(-a^2+1)^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arccos(b*x+a)/b^2/x^2+1/2/(-a^2+1)/b/x*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+1/2*a/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2
*a*b*x-a^2+1)^(1/2))/b/x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(87) = 174.
time = 2.31, size = 482, normalized size = 4.68

$$\left[-\frac{\sqrt{-a^2+1}\,ab^2x^2\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx+2\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2(a^4-2a^2+1)x^2\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+2\sqrt{-b^2x^2-2abx-a^2+1}\,(a^2-1)bx+2(a^4-(a^4-2a^2+1)x^2-2a^2+1)\arccos(bx+a)}{4(a^4-2a^2+1)x^2},\ \frac{\sqrt{a^2-1}\,ab^2x^2\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{a^2-1}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)-(a^4-2a^2+1)x^2\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)-\sqrt{-b^2x^2-2abx-a^2+1}\,(a^2-1)bx-(a^4-(a^4-2a^2+1)x^2-2a^2+1)\arccos(bx+a)}{2(a^4-2a^2+1)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 + 1)*a*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 -
 a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2
+ 1) - 4*a^2 + 2)/x^2) + 2*(a^4 - 2*a^2 + 1)*x^2*arctan(sqrt(-b^2*x^2 - 2*a
*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*sqrt(-b^2*x^2
- 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x + 2*(a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2
 + 1)*arccos(b*x + a))/((a^4 - 2*a^2 + 1)*x^2), 1/2*(sqrt(a^2 - 1)*a*b^2*x^
2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)
/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (a^4 - 2*a^2 +
1)*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x - (a^4 -
(a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*arccos(b*x + a))/((a^4 - 2*a^2 + 1)*x^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**3,x)
```

```
[Out] Integral(acos(a + b*x)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(87) = 174.
time = 0.45, size = 242, normalized size = 2.35

$$\left(\frac{ab^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\left(a^2|b|-|b|\right)\sqrt{a^2-1}}-\frac{ab^2-\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)b^2}{b^2x+ab}}{\left(a^3|b|-a|b|\right)\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)^2a}{\left(b^2x+ab\right)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2\,abx-a^2+1}\,|b|+b\right)}{b^2x+ab}\right)}\right)b-\frac{\arccos\left(bx+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] (a*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x + a
*b) - 1)/sqrt(a^2 - 1))/((a^2*abs(b) - abs(b))*sqrt(a^2 - 1)) - (a*b^2 - (s
qrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^3*abs(
b) - a*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2*x
+ a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x + a
*b))))*b - 1/2*arccos(b*x + a)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)/x^3,x)
```

```
[Out] int(acos(a + b*x)/x^3, x)
```

3.31 $\int \frac{\text{ArcCos}(a+bx)}{x^4} dx$

Optimal. Leaf size=144

$$\frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2}+\frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2x}-\frac{\text{ArcCos}(a+bx)}{3x^3}+\frac{(1+2a^2)b^3\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}}$$

```
[Out] -1/3*arccos(b*x+a)/x^3+1/6*(2*a^2+1)*b^3*arctanh((1-a*(b*x+a))/(-a^2+1)^(1/
2)/(1-(b*x+a)^2)^(1/2))/(-a^2+1)^(5/2)+1/6*b*(1-(b*x+a)^2)^(1/2)/(-a^2+1)/x
^2+1/2*a*b^2*(1-(b*x+a)^2)^(1/2)/(-a^2+1)^2/x
```

Rubi [A]
time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4890, 4828, 759, 821, 739, 212}

$$\frac{(2a^2+1)b^3\tanh^{-1}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}}+\frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2x}+\frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2}-\frac{\text{ArcCos}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[1 - (a + b*x)^2])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[1 - (a + b*x)^2])
/(2*(1 - a^2)^2*x) - ArcCos[a + b*x]/(3*x^3) + ((1 + 2*a^2)*b^3*ArcTanh[(1
- a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(6*(1 - a^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/((((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 4828

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{x^4}\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\cos^{-1}(a+bx)}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1-x^2}}\,dx, x, a+bx\right) \\
&= \frac{b\sqrt{1-(a+bx)^2}}{6\,(1-a^2)\,x^2} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{b^2\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1-x^2}}\,dx, x, a+bx\right)}{6\,(1-a^2)} \\
&= \frac{b\sqrt{1-(a+bx)^2}}{6\,(1-a^2)\,x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\,(1-a^2)^2\,x} - \frac{\cos^{-1}(a+bx)}{3x^3} - \frac{((1+2a^2)\,b^2)\,\text{Subst}\Big(\int}{} \\
&= \frac{b\sqrt{1-(a+bx)^2}}{6\,(1-a^2)\,x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\,(1-a^2)^2\,x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{((1+2a^2)\,b^2)\,\text{Subst}\Big(\int}{} \\
&= \frac{b\sqrt{1-(a+bx)^2}}{6\,(1-a^2)\,x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2\,(1-a^2)^2\,x} - \frac{\cos^{-1}(a+bx)}{3x^3} + \frac{(1+2a^2)\,b^3\tanh^{-1}\Big(}{6\,(1}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 168, normalized size = 1.17

$$
\frac{\sqrt{1-a^2}\,bx(1-a^2+3abx)\sqrt{1-a^2-2abx-b^2x^2} - 2(1-a^2)^{5/2}\,\text{ArcCos}(a+bx) - (1+2a^2)\,b^3x^3\log(x) + (1+2a^2)\,b^3x^3\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2}\right)}{6\,(1-a^2)^{5/2}\,x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/x^4,x]
```

```
[Out] (Sqrt[1 - a^2]*b*x*(1 - a^2 + 3*a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] -
2*(1 - a^2)^(5/2)*ArcCos[a + b*x] - (1 + 2*a^2)*b^3*x^3*Log[x] + (1 + 2*a^2
)*b^3*x^3*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*
x^2]])/(6*(1 - a^2)^(5/2)*x^3)
```

Maple [A]
time = 0.00, size = 240, normalized size = 1.67

method	result

derivativedivides	$b^3\left(-\frac{\arccos(bx+a)}{3b^3x^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{6(-a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{(-a^2+1)bx}-\frac{a\ln\left(\frac{-2a^2}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$
default	$b^3\left(-\frac{\arccos(bx+a)}{3b^3x^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{6(-a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{(-a^2+1)bx}-\frac{a\ln\left(\frac{-2a^2}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*arccos(b*x+a)/b^3/x^3+1/6/(-a^2+1)/b^2/x^2*(-b^2*x^2-2*a*b*x-a^2+
1)^(1/2)-1/2*a/(-a^2+1)*(-1/(-a^2+1)/b/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a/(
-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2))/b/x))+1/6/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b
^2*x^2-2*a*b*x-a^2+1)^(1/2))/b/x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(122) = 244$.
time = 1.70, size = 580, normalized size = 4.03

$$\left[-\frac{(2a^2+1)\sqrt{-a^2+1}\,b^3x^3\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+4(a^6-3a^4+3a^2-1)x^3\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+4(a^6-3a^4-(a^6-3a^4+3a^2-1)x^3+3a^2-1)\arccos(bx+a)-2(3(a^3-a)b^2x^2-(a^4-2a^2+1)bx)\sqrt{-b^2x^2-2abx-a^2+1}}{12(a^6-3a^4+3a^2-1)x^3},\ -\frac{(2a^2+1)\sqrt{a^2-1}\,b^3x^3\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(abx+a^2-1)\sqrt{a^2-1}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+2(a^6-3a^4+3a^2-1)x^3\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}\,(bx+a)}{b^2x^2+2abx+a^2-1}\right)+2(a^6-3a^4-(a^6-3a^4+3a^2-1)x^3+3a^2-1)\arccos(bx+a)-(3(a^3-a)b^2x^2-(a^4-2a^2+1)bx)\sqrt{-b^2x^2-2abx-a^2+1}}{6(a^6-3a^4+3a^2-1)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/12*((2*a^2 + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4
 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)
*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 4*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*arctan(
sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1))
 + 4*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*arccos(b*x +
 a) - 2*(3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(-b^2*x^2 - 2*a*b
*x - a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), -1/6*((2*a^2 + 1)*sqrt(a^2
- 1)*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sq
rt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + 2*(a
^6 - 3*a^4 + 3*a^2 - 1)*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x
+ a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2
 - 1)*x^3 + 3*a^2 - 1)*arccos(b*x + a) - (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^
2 + 1)*b*x)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*
x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/x**4,x)
```

```
[Out] Integral(acos(a + b*x)/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(122) = 244.
time = 0.44, size = 557, normalized size = 3.87

$$-\frac{1}{3}b\left(\frac{(2a^2b^3+b^3)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{(a^4|b|-2a^2|b|+|b|)\sqrt{a^2-1}}-\frac{\frac{4\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^2a^4b^3}{(b^2x+ab)^2}+4a^4b^3-\frac{11\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)a^3b^3}{b^2x+ab}-\frac{5\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^3a^3b^3}{(b^2x+ab)^3}+\frac{7\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^2a^2b^3}{(b^2x+ab)^2}-a^2b^3+\frac{2\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)ab^3}{b^2x+ab}+\frac{2\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^3ab^3}{(b^2x+ab)^3}-\frac{2\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^2b^3}{(b^2x+ab)^2}}{(a^6|b|-2a^4|b|+a^2|b|)\left(\frac{\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)^2a}{(b^2x+ab)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2abx-a^2+1}|b|+b\right)}{b^2x+ab}\right)^2}\right)-\frac{\arccos(bx+a)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*((2*a^2*b^3 + b^3)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b
) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^4*abs(b) - 2*a^2*abs(b) + ab
s(b))*sqrt(a^2 - 1)) - (4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2
*a^4*b^3/(b^2*x + a*b)^2 + 4*a^4*b^3 - 11*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 7*(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - a^2*b^3 + 2*(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) + 2*(sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-b^2*x^2 - 2*a
```

```
*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^6*abs(b) - 2*a^4*abs
(b) + a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^2
*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*x
+ a*b))^2)) - 1/3*arccos(b*x + a)/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}(a+b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)/x^4,x)
```

```
[Out] int(acos(a + b*x)/x^4, x)
```

3.32 $\int \mathbf{ArcCos}(a+bx)^3\,dx$

Optimal. Leaf size=82

$$\frac{6\sqrt{1-(a+bx)^2}}{b}-\frac{6(a+bx)\text{ArcCos}(a+bx)}{b}-\frac{3\sqrt{1-(a+bx)^2}\,\text{ArcCos}(a+bx)^2}{b}+\frac{(a+bx)\text{ArcCos}(a+b}{b}$$

```
[Out] -6*(b*x+a)*arccos(b*x+a)/b+(b*x+a)*arccos(b*x+a)^3/b+6*(1-(b*x+a)^2)^(1/2)/
b-3*arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4888, 4716, 4768, 267}

$$\frac{(a+bx)\text{ArcCos}(a+bx)^3}{b}-\frac{3\sqrt{1-(a+bx)^2}\,\text{ArcCos}(a+bx)^2}{b}-\frac{6(a+bx)\text{ArcCos}(a+bx)}{b}+\frac{6\sqrt{1-(a+bx)^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^3,x]
```

```
[Out] (6*Sqrt[1 - (a + b*x)^2])/b - (6*(a + b*x)*ArcCos[a + b*x])/b - (3*Sqrt[1 -
 (a + b*x)^2]*ArcCos[a + b*x]^2)/b + ((a + b*x)*ArcCos[a + b*x]^3)/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
```

```
}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\cos^{-1}(a+bx)^3}{b} + \frac{3\text{Subst}\left(\int \frac{x\cos^{-1}(x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^2}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^3}{b} - \frac{6\text{Subst}\left(\int \cos^{-1}(x)\,d\right.}{b}$$

$$= -\frac{6(a+bx)\cos^{-1}(a+bx)}{b} - \frac{3\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^2}{b} + \frac{(a+bx)\cos^{-1}(a+}{b}$$

$$= \frac{6\sqrt{1-(a+bx)^2}}{b} - \frac{6(a+bx)\cos^{-1}(a+bx)}{b} - \frac{3\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^2}{b} +$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.90

$$\frac{6\sqrt{1-(a+bx)^2} - 6(a+bx)\text{ArcCos}(a+bx) - 3\sqrt{1-(a+bx)^2}\,\text{ArcCos}(a+bx)^2 + (a+bx)\text{ArcCos}(a+bx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^3,x]
[Out] (6*Sqrt[1 - (a + b*x)^2] - 6*(a + b*x)*ArcCos[a + b*x] - 3*Sqrt[1 - (a + b*
x)^2]*ArcCos[a + b*x]^2 + (a + b*x)*ArcCos[a + b*x]^3)/b
```

Maple [A]
time = 0.10, size = 71, normalized size = 0.87

method	result	siz
derivativedivides	$\frac{\arccos(bx+a)^3(bx+a)-3\arccos(bx+a)^2\sqrt{1-(bx+a)^2}+6\sqrt{1-(bx+a)^2}-6(bx+a)\arccos(bx+a)}{b}$	71
default	$\frac{\arccos(bx+a)^3(bx+a)-3\arccos(bx+a)^2\sqrt{1-(bx+a)^2}+6\sqrt{1-(bx+a)^2}-6(bx+a)\arccos(bx+a)}{b}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/b*(arccos(b*x+a)^3*(b*x+a)-3*arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)+6*(1-(b*x+a)^2)^(1/2)-6*(b*x+a)*arccos(b*x+a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^3 - 3*b*integrate(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 - 1), x)
```

Fricas [A]
time = 1.73, size = 66, normalized size = 0.80

$$\frac{(bx+a)\arccos(bx+a)^3 - 6\,(bx+a)\arccos(bx+a) - 3\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}\left(\arccos(bx+a)^2 - 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*arccos(b*x + a)^3 - 6*(b*x + a)*arccos(b*x + a) - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(arccos(b*x + a)^2 - 2))/b
```

Sympy [A]
time = 0.15, size = 109, normalized size = 1.33

$$\begin{cases} \frac{a\operatorname{acos}^{3}(a+bx)}{b} - \frac{6a\operatorname{acos}(a+bx)}{b} + x\operatorname{acos}^{3}(a+bx) - 6x\operatorname{acos}(a+bx) - \frac{3\sqrt{-a^2-2abx-b^2x^2+1}\operatorname{acos}^{2}(a+bx)}{b} + \frac{6\sqrt{-a^2-2abx-b^2x^2+1}}{b} & \text{for } b \neq 0 \\ x\operatorname{acos}^{3}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**3,x)
```

```
[Out] Piecewise((a*acos(a + b*x)**3/b - 6*a*acos(a + b*x)/b + x*acos(a + b*x)**3 - 6*x*acos(a + b*x) - 3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)*acos(a + b*x)**2/b + 6*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a)**3, True))
```

Giac [A]
time = 0.42, size = 78, normalized size = 0.95

$$\frac{(bx+a)\arccos(bx+a)^3}{b} - \frac{3\sqrt{-(bx+a)^2+1}\arccos(bx+a)^2}{b} - \frac{6\,(bx+a)\arccos(bx+a)}{b} + \frac{6\sqrt{-(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b*x + a)*arccos(b*x + a)^3/b - 3*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)^2/
b - 6*(b*x + a)*arccos(b*x + a)/b + 6*sqrt(-(b*x + a)^2 + 1)/b
```

Mupad [B]
time = 0.28, size = 60, normalized size = 0.73

$$-\frac{\left(3\,\mathrm{acos}(a+b\,x)^2-6\right)\,\sqrt{1-{(a+b\,x)}^2}}{b}-\frac{\left(6\,\mathrm{acos}(a+b\,x)-\mathrm{acos}(a+b\,x)^3\right)\,(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)^3,x)
```

```
[Out] - ((3*acos(a + b*x)^2 - 6)*(1 - (a + b*x)^2)^(1/2))/b - ((6*acos(a + b*x) -
 acos(a + b*x)^3)*(a + b*x))/b
```

3.33 $\int \mathbf{ArcCos}(a+bx)^2\,dx$

Optimal. Leaf size=47

$$-2x-\frac{2\sqrt{1-(a+bx)^2}\,\mathrm{ArcCos}(a+bx)}{b}+\frac{(a+bx)\mathrm{ArcCos}(a+bx)^2}{b}$$

```
[Out] -2*x+(b*x+a)*arccos(b*x+a)^2/b-2*arccos(b*x+a)*(1-(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4888, 4716, 4768, 8}

$$\frac{(a+bx)\mathrm{ArcCos}(a+bx)^2}{b}-\frac{2\sqrt{1-(a+bx)^2}\,\mathrm{ArcCos}(a+bx)}{b}-2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^2,x]
```

```
[Out] -2*x - (2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x])/b + ((a + b*x)*ArcCos[a + 
b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - 
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p + 
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, 
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cos^{-1}(a+bx)^2}{b} + \frac{2\text{Subst}\left(\int \frac{x\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}(\int 1\,dx, x, a+}{b}$$
$$= -2x - \frac{2\sqrt{1-(a+bx)^2}\cos^{-1}(a+bx)}{b} + \frac{(a+bx)\cos^{-1}(a+bx)^2}{b}$$

Mathematica [**A**]
time = 0.02, size = 49, normalized size = 1.04

$$\frac{-2(a+bx) - 2\sqrt{1-(a+bx)^2}\,\text{ArcCos}(a+bx) + (a+bx)\text{ArcCos}(a+bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^2,x]
[Out] (-2*(a + b*x) - 2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x] + (a + b*x)*ArcCos[
a + b*x]^2)/b
```

Maple [**A**]
time = 0.08, size = 48, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\arccos(bx+a)^2(bx+a)-2bx-2a-2\arccos(bx+a)\sqrt{1-(bx+a)^2}}{b}$	48
default	$\frac{\arccos(bx+a)^2(bx+a)-2bx-2a-2\arccos(bx+a)\sqrt{1-(bx+a)^2}}{b}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(arccos(b*x+a)^2*(b*x+a)-2*b*x-2*a-2*arccos(b*x+a)*(1-(b*x+a)^2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^2 - 2*b*integrate(
sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x
- a + 1), b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1), x)
```

Fricas [A]
time = 1.05, size = 53, normalized size = 1.13

$$\frac{(bx+a)\arccos{(bx+a)}^2-2\,bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,\arccos{(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*arccos(b*x + a)^2 - 2*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)
*arccos(b*x + a))/b
```

Sympy [A]
time = 0.10, size = 63, normalized size = 1.34

$$\begin{cases}\frac{a\operatorname{acos}^2{(a+bx)}}{b}+x\operatorname{acos}^2{(a+bx)}-2x-\frac{2\sqrt{-a^2-2abx-b^2x^2+1}\operatorname{acos}{(a+bx)}}{b} & \text{for } b\neq 0\\ x\operatorname{acos}^2{(a)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**2,x)
```

```
[Out] Piecewise((a*acos(a + b*x)**2/b + x*acos(a + b*x)**2 - 2*x - 2*sqrt(-a**2 -
 2*a*b*x - b**2*x**2 + 1)*acos(a + b*x)/b, Ne(b, 0)), (x*acos(a)**2, True))
```

Giac [A]
time = 0.42, size = 52, normalized size = 1.11

$$\frac{(bx+a)\arccos{(bx+a)}^2}{b}-\frac{2\,\sqrt{-(bx+a)^2+1}\,\arccos{(bx+a)}}{b}-\frac{2\,(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*arccos(b*x + a)^2/b - 2*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)/b
- 2*(b*x + a)/b
```

Mupad [B]
time = 0.25, size = 44, normalized size = 0.94

$$\frac{\left(\operatorname{acos}(a+b\,x)^2-2\right)\,\left(a+b\,x\right)}{b}-\frac{2\operatorname{acos}(a+b\,x)\,\sqrt{1-{(a+b\,x)}^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)^2,x)
```

```
[Out] ((acos(a + b*x)^2 - 2)*(a + b*x))/b - (2*acos(a + b*x)*(1 - (a + b*x)^2)^(1
/2))/b
```

3.34 $\int \frac{1}{\text{ArcCos}(a+bx)} \, dx$

Optimal. Leaf size=12

$$-\frac{\text{Si}(\text{ArcCos}(a+bx))}{b}$$

```
[Out] -Si(arccos(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4888, 4720, 3380}

$$-\frac{\text{Si}(\text{ArcCos}(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-1),x]
```

```
[Out] -(SinIntegral[ArcCos[a + b*x]]/b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{b}$$
$$= -\frac{\text{Si}(\cos^{-1}(a+bx))}{b}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 1.00

$$-\frac{\text{Si}(\text{ArcCos}(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-1),x]
[Out] -(SinIntegral[ArcCos[a + b*x]]/b)
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\text{sinIntegral}(\arccos(bx+a))}{b}$	13
default	$-\frac{\text{sinIntegral}(\arccos(bx+a))}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a),x,method=_RETURNVERBOSE)
[Out] -Si(arccos(b*x+a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="maxima")
[Out] integrate(1/arccos(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="fricas")
[Out] integral(1/arccos(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acos}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a),x)
[Out] Integral(1/acos(a + b*x), x)
```

Giac [A]
time = 0.43, size = 12, normalized size = 1.00

$$-\frac{\operatorname{Si}(\arccos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a),x, algorithm="giac")
[Out] -sin_integral(arccos(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\operatorname{acos}(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x),x)
[Out] int(1/acos(a + b*x), x)
```

3.35 $\int \frac{1}{\mathbf{ArcCos}(a+bx)^2} dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-(a+bx)^2}}{b\text{ArcCos}(a+bx)}-\frac{\text{CosIntegral}(\text{ArcCos}(a+bx))}{b}$$

[Out] -Ci(arccos(b*x+a))/b+(1-(b*x+a)^2)^(1/2)/b/arccos(b*x+a)

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4888, 4718, 4810, 3383}

$$\frac{\sqrt{1-(a+bx)^2}}{b\text{ArcCos}(a+bx)}-\frac{\text{CosIntegral}(\text{ArcCos}(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b*x]^(-2),x]

[Out] Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
```

```
}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{b}$$
$$= \frac{\sqrt{1-(a+bx)^2}}{b\cos^{-1}(a+bx)} - \frac{\text{Ci}(\cos^{-1}(a+bx))}{b}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 1.00

$$\frac{\sqrt{1-(a+bx)^2}}{b\text{ArcCos}(a+bx)} - \frac{\text{CosIntegral}(\text{ArcCos}(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-2),x]
[Out] Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\sqrt{1-(bx+a)^2}}{\arccos(bx+a)} - \text{cosineIntegral}(\arccos(bx+a))}{b}$	37
default	$\frac{\frac{\sqrt{1-(bx+a)^2}}{\arccos(bx+a)} - \text{cosineIntegral}(\arccos(bx+a))}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/arccos(b*x+a)*(1-(b*x+a)^2)^(1/2)-Ci(arccos(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)*integrate(sqrt(b
*x + a + 1)*(b*x + a)*sqrt(-b*x - a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*arc
tan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)), x) - sqrt(b*x + a + 1
)*sqrt(-b*x - a + 1))/(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x
+ a))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)^(-2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acos}^{2}(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**2,x)
```

```
[Out] Integral(acos(a + b*x)**(-2), x)
```

Giac [A]
time = 0.42, size = 38, normalized size = 0.95

$$-\frac{\operatorname{Ci}(\arccos(bx + a))}{b} + \frac{\sqrt{-(bx + a)^{2} + 1}}{b \arccos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -cos_integral(arccos(b*x + a))/b + sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x + a
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x)^2,x)
```

```
[Out] int(1/acos(a + b*x)^2, x)
```

3.36 $\int \frac{1}{\mathbf{ArcCos}(a+bx)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1-(a+bx)^2}}{2b\text{ArcCos}(a+bx)^2}+\frac{a+bx}{2b\text{ArcCos}(a+bx)}+\frac{\text{Si}(\text{ArcCos}(a+bx))}{2b}$$

```
[Out] 1/2*(b*x+a)/b/arccos(b*x+a)+1/2*Si(arccos(b*x+a))/b+1/2*(1-(b*x+a)^2)^(1/2)
/b/arccos(b*x+a)^2
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4888, 4718, 4808, 4720, 3380}

$$\frac{\text{Si}(\text{ArcCos}(a+bx))}{2b}+\frac{a+bx}{2b\text{ArcCos}(a+bx)}+\frac{\sqrt{1-(a+bx)^2}}{2b\text{ArcCos}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-3),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a + b
*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)^2}\,dx, x, a+bx\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)}\,dx, x, a+bx\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \cos^{-1}(a+bx)\right)}{2b}\\
&= \frac{\sqrt{1-(a+bx)^2}}{2b\cos^{-1}(a+bx)^2} + \frac{a+bx}{2b\cos^{-1}(a+bx)} + \frac{\text{Si}(\cos^{-1}(a+bx))}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 1.00

$$\frac{\sqrt{1-(a+bx)^2}}{2b\text{ArcCos}(a+bx)^2} + \frac{a+bx}{2b\text{ArcCos}(a+bx)} + \frac{\text{Si}(\text{ArcCos}(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-3),x]
```

```
[Out] Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a + b
*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

Maple [A]
time = 0.08, size = 53, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{\sqrt{1-(bx+a)^2}}{2\arccos(bx+a)^2}+\frac{bx+a}{2\arccos(bx+a)}+\frac{\text{sinIntegral}(\arccos(bx+a))}{2}}{b}$	53
default	$\frac{\frac{\sqrt{1-(bx+a)^2}}{2\arccos(bx+a)^2}+\frac{bx+a}{2\arccos(bx+a)}+\frac{\text{sinIntegral}(\arccos(bx+a))}{2}}{b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2/arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)+1/2/arccos(b*x+a)*(b*x+a)+1/2*
Si(arccos(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^2*integrate(
1/arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a), x) - (b*x + a)*ar
ctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a) - sqrt(b*x + a + 1)*sq
rt(-b*x - a + 1))/(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)
^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(b*x + a)^(-3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acos}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**3,x)
```

```
[Out] Integral(acos(a + b*x)**(-3), x)
```

Giac [A]
time = 0.44, size = 57, normalized size = 0.88

$$\frac{\mathrm{Si}\left(\arccos\left(bx+a\right)\right)}{2\,b}+\frac{bx+a}{2\,b\arccos\left(bx+a\right)}+\frac{\sqrt{-\left(bx+a\right)^{2}+1}}{2\,b\arccos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(arccos(b*x + a))/b + 1/2*(b*x + a)/(b*arccos(b*x + a)) + 1
/2*sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x + a)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x)^3,x)
```

```
[Out] int(1/acos(a + b*x)^3, x)
```

3.37 $\int \operatorname{ArcCos}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=111

$$-\frac{15(a+bx)\sqrt{\operatorname{ArcCos}(a+bx)}}{4b}-\frac{5\sqrt{1-(a+bx)^2}\operatorname{ArcCos}(a+bx)^{3/2}}{2b}+\frac{(a+bx)\operatorname{ArcCos}(a+bx)^{5/2}}{b}+\frac{15\sqrt{\frac{\pi}{2}}}{}$$

```
[Out] (b*x+a)*arccos(b*x+a)^(5/2)/b+15/8*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^
(1/2))*2^(1/2)*Pi^(1/2)/b-5/2*arccos(b*x+a)^(3/2)*(1-(b*x+a)^2)^(1/2)/b-15/
4*(b*x+a)*arccos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4888, 4716, 4768, 4810, 3385, 3433}

$$\frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\operatorname{ArcCos}(a+bx)}\right)}{4b}+\frac{(a+bx)\operatorname{ArcCos}(a+bx)^{5/2}}{b}-\frac{5\sqrt{1-(a+bx)^2}\operatorname{ArcCos}(a+bx)^{3/2}}{2b}-\frac{15(a+bx)\sqrt{\operatorname{ArcCos}(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(5/2),x]
```

```
[Out] (-15*(a + b*x)*Sqrt[ArcCos[a + b*x]])/(4*b) - (5*Sqrt[1 - (a + b*x)^2]*ArcC
os[a + b*x]^(3/2))/(2*b) + ((a + b*x)*ArcCos[a + b*x]^(5/2))/b + (15*Sqrt[P
i/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/(4*b)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^{5/2}\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^{5/2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} + \frac{5\text{Subst}\left(\int \frac{x\cos^{-1}(x)^{3/2}}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b}$$

$$= -\frac{5\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{5/2}}{b} - \frac{15\text{Subst}\left(\int \sqrt{\right.}}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\,\text{co}}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\,\text{co}}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\,\text{co}}{}$$

$$= -\frac{15(a+bx)\sqrt{\cos^{-1}(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\,\cos^{-1}(a+bx)^{3/2}}{2b} + \frac{(a+bx)\,\text{co}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 90, normalized size = 0.81

$$-\frac{\frac{\sqrt{\operatorname{ArcCos}(a+bx)}\,\operatorname{Gamma}\left(\frac{7}{2},-i\operatorname{ArcCos}(a+bx)\right)}{2\sqrt{-i\operatorname{ArcCos}(a+bx)}}+\frac{\sqrt{\operatorname{ArcCos}(a+bx)}\,\operatorname{Gamma}\left(\frac{7}{2},i\operatorname{ArcCos}(a+bx)\right)}{2\sqrt{i\operatorname{ArcCos}(a+bx)}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(5/2),x]
```

```
[Out] -(((Sqrt[ArcCos[a + b*x]]*Gamma[7/2, (-I)*ArcCos[a + b*x]])/(2*Sqrt[(-I)*Ar
cCos[a + b*x]]) + (Sqrt[ArcCos[a + b*x]]*Gamma[7/2, I*ArcCos[a + b*x]])/(2*
Sqrt[I*ArcCos[a + b*x]]))/b)
```

Maple [A]
time = 0.39, size = 140, normalized size = 1.26

method	result
default	$-\frac{\sqrt{2}\,\left(-4\arccos(bx+a)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,bx-4\arccos(bx+a)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,a+10\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{-b^2x^2-2abx-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*2^(1/2)*(-4*arccos(b*x+a)^(5/2)*2^(1/2)*Pi^(1/2)*b*x-4*arccos(b*x+a)
^(5/2)*2^(1/2)*Pi^(1/2)*a+10*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2)+15*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*b*x+15*2^(1/2
)*arccos(b*x+a)^(1/2)*Pi^(1/2)*a-15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x
+a)^(1/2)))/Pi^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acos}^{\frac{5}{2}}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(5/2),x)
```

```
[Out] Integral(acos(a + b*x)**(5/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.53, size = 183, normalized size = 1.65

$$\frac{\arccos(bx+a)^{\frac{5}{2}} e^{(i \arccos(bx+a))}}{2b} + \frac{\arccos(bx+a)^{\frac{5}{2}} e^{(-i \arccos(bx+a))}}{2b} + \frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{(i \arccos(bx+a))}}{4b} - \frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{(-i \arccos(bx+a))}}{4b} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{32b} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{32b} - \frac{15\sqrt{\arccos(bx+a)}\, e^{(i \arccos(bx+a))}}{8b} - \frac{15\sqrt{\arccos(bx+a)}\, e^{(-i \arccos(bx+a))}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*arccos(b*x + a)^(5/2)*e^(I*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(5/
2)*e^(-I*arccos(b*x + a))/b + 5/4*I*arccos(b*x + a)^(3/2)*e^(I*arccos(b*x +
 a))/b - 5/4*I*arccos(b*x + a)^(3/2)*e^(-I*arccos(b*x + a))/b - (15/32*I +
15/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x + a)))/b
+ (15/32*I - 15/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos
(b*x + a)))/b - 15/8*sqrt(arccos(b*x + a))*e^(I*arccos(b*x + a))/b - 15/8*s
qrt(arccos(b*x + a))*e^(-I*arccos(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{acos}\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)^(5/2),x)
```

```
[Out] int(acos(a + b*x)^(5/2), x)
```

3.38 $\int \mathbf{ArcCos}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\operatorname{ArcCos}(a+bx)}}{2b}+\frac{(a+bx)\operatorname{ArcCos}(a+bx)^{3/2}}{b}+\frac{3\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\operatorname{ArcCos}(a+bx)}\right)}{2b}$$

```
[Out] (b*x+a)*arccos(b*x+a)^(3/2)/b+3/4*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(
1/2))*2^(1/2)*Pi^(1/2)/b-3/2*(1-(b*x+a)^2)^(1/2)*arccos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4888, 4716, 4768, 4720, 3386, 3432}

$$\frac{3\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\operatorname{ArcCos}(a+bx)}\right)}{2b}+\frac{(a+bx)\operatorname{ArcCos}(a+bx)^{3/2}}{b}-\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\operatorname{ArcCos}(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 - (a + b*x)^2]*Sqrt[ArcCos[a + b*x]])/(2*b) + ((a + b*x)*ArcCos[
a + b*x]^(3/2))/b + (3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]
])/(2*b)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \cos^{-1}(a+bx)^{3/2}\,dx = \frac{\text{Subst}\left(\int \cos^{-1}(x)^{3/2}\,dx,x,a+bx\right)}{b}$$

$$= \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\text{Subst}\left(\int \frac{x\sqrt{\cos^{-1}(x)}}{\sqrt{1-x^2}}\,dx,x,a+bx\right)}{2b}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} - \frac{3\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\text{Subst}\left(\int \frac{\sin}{\sqrt{}}\right.}{}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\text{Subst}\left(\int \sin\right.}{}$$

$$= -\frac{3\sqrt{1-(a+bx)^2}\,\sqrt{\cos^{-1}(a+bx)}}{2b} + \frac{(a+bx)\cos^{-1}(a+bx)^{3/2}}{b} + \frac{3\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 76, normalized size = 0.85

$$-\frac{\sqrt{-i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{5}{2},-i\text{ArcCos}(a+bx)\right)+\sqrt{i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{5}{2},i\text{ArcCos}(a+bx)\right)}{2b\sqrt{\text{ArcCos}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(3/2),x]
```

```
[Out] -1/2*(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[5/2, (-I)*ArcCos[a + b*x]] + Sqrt[I*
ArcCos[a + b*x]]*Gamma[5/2, I*ArcCos[a + b*x]])/(b*Sqrt[ArcCos[a + b*x]])
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.18

method	result
default	$-\frac{\sqrt{2}\,\left(-2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,bx-2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a+3\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}\,\sqrt{\pi}\,\sqrt{-b^{2}x^{2}-2}\right.}{4b\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*2^(1/2)*(-2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*b*x-2*arccos(b*x+a)
^(3/2)*2^(1/2)*Pi^(1/2)*a+3*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2)-3*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2)))/P
i^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acos}^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(3/2),x)
```

```
[Out] Integral(acos(a + b*x)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.52, size = 139, normalized size = 1.56

$$\frac{\arccos(bx+a)^{\frac{3}{2}}e^{(i\arccos(bx+a))}}{2b}+\frac{\arccos(bx+a)^{\frac{3}{2}}e^{(-i\arccos(bx+a))}}{2b}+\frac{(3i-3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{16b}-\frac{(3i+3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{16b}+\frac{3i\sqrt{\arccos(bx+a)}e^{(i\arccos(bx+a))}}{4b}-\frac{3i\sqrt{\arccos(bx+a)}e^{(-i\arccos(bx+a))}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*arccos(b*x + a)^(3/2)*e^(I*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(3/
2)*e^(-I*arccos(b*x + a))/b + (3/16*I - 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I -
 1/2)*sqrt(2)*sqrt(arccos(b*x + a)))/b - (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*e
rf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(b*x + a)))/b + 3/4*I*sqrt(arccos(b*x
+ a))*e^(I*arccos(b*x + a))/b - 3/4*I*sqrt(arccos(b*x + a))*e^(-I*arccos(b*
x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)^(3/2),x)
```

```
[Out] int(acos(a + b*x)^(3/2), x)
```

3.39 $\int \sqrt{\text{ArcCos}(a+bx)}\, dx$

Optimal. Leaf size=55

$$\frac{(a+bx)\sqrt{\text{ArcCos}(a+bx)}}{b}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(a+bx)}\right)}{b}$$

```
[Out] -1/2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+(b*x
+a)*arccos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4888, 4716, 4810, 3385, 3433}

$$\frac{(a+bx)\sqrt{\text{ArcCos}(a+bx)}}{b}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] ((a + b*x)*Sqrt[ArcCos[a + b*x]])/b - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[
ArcCos[a + b*x]]])/b
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
```

```
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{\cos^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{2b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{2b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b}\\
&= \frac{(a+bx)\sqrt{\cos^{-1}(a+bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 90, normalized size = 1.64

$$-\frac{-\frac{\sqrt{\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{3}{2},-i\text{ArcCos}(a+bx)\right)}{2\sqrt{-i\text{ArcCos}(a+bx)}}-\frac{\sqrt{\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{3}{2},i\text{ArcCos}(a+bx)\right)}{2\sqrt{i\text{ArcCos}(a+bx)}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] -((-1/2*(Sqrt[ArcCos[a + b*x]]*Gamma[3/2, (-I)*ArcCos[a + b*x]])/Sqrt[(-I)*
ArcCos[a + b*x]] - (Sqrt[ArcCos[a + b*x]]*Gamma[3/2, I*ArcCos[a + b*x]])/(2
*Sqrt[I*ArcCos[a + b*x]]))/b)
```

Maple [A]
time = 0.17, size = 66, normalized size = 1.20

method	result	size
default	$\frac{-\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}\,\sqrt{\pi}+2\arccos(bx+a)bx+2\arccos(bx+a)a}{2b\sqrt{\arccos\left(bx+a\right)}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/arccos(b*x+a)^(1/2)*(-FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*
2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)+2*arccos(b*x+a)*b*x+2*arccos(b*x+a)*a)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{acos}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(acos(a + b*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 95, normalized size = 1.73

$$\frac{(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{8b}-\frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{8b}+\frac{\sqrt{\arccos(bx+a)}\,e^{(i\arccos(bx+a))}}{2b}+\frac{\sqrt{\arccos(bx+a)}\,e^{(-i\arccos(bx+a))}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] (1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x +
a)))/b - (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arc
cos(b*x + a)))/b + 1/2*sqrt(arccos(b*x + a))*e^(I*arccos(b*x + a))/b + 1/2*
sqrt(arccos(b*x + a))*e^(-I*arccos(b*x + a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\operatorname{acos}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)^(1/2),x)
```

```
[Out] int(acos(a + b*x)^(1/2), x)
```

3.40 $\int \frac{1}{\sqrt{\mathbf{ArcCos}(a+bx)}} \, dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{2\pi}\, S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcCos}(a+bx)}\right)}{b}$$

```
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4888, 4720, 3386, 3432}

$$-\frac{\sqrt{2\pi}\, S\left(\sqrt{\frac{2}{\pi}}\sqrt{\mathrm{ArcCos}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcCos[a + b*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
```

```
}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos^{-1}(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{b}\\
&= -\frac{2\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b}\\
&= -\frac{\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 78, normalized size = 2.36

$$-\frac{-\sqrt{-i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{1}{2},-i\text{ArcCos}(a+bx)\right)-\sqrt{i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{1}{2},i\text{ArcCos}(a+bx)\right)}{2b\sqrt{\text{ArcCos}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[ArcCos[a + b*x]],x]
[Out] -1/2*(-(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*ArcCos[a + b*x]]) - Sqrt
[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])/(b*Sqrt[ArcCos[a + b*x]]
)
```

Maple [A]
time = 0.10, size = 28, normalized size = 0.85

method	result	size
default	$-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{b}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{acos}\left(a + bx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acos(a + b*x)), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.46, size = 51, normalized size = 1.55

$$-\frac{(i-1)\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}\right)}{4\,b}+\frac{(i+1)\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x +
 a)))/b + (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(ar
ccos(b*x + a)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\operatorname{acos}(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x)^(1/2),x)
[Out] int(1/acos(a + b*x)^(1/2), x)
```

3.41 $\int \frac{1}{\mathbf{ArcCos}(a+bx)^{3/2}} dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\text{ArcCos}(a+bx)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcCos}(a+bx)}\right)}{b}$$

[Out] -2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+2*(1-(b*x+a)^2)^(1/2)/b/arccos(b*x+a)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4888, 4718, 4810, 3385, 3433}

$$\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\text{ArcCos}(a+bx)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\text{ArcCos}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a + b*x]^(-3/2),x]

[Out] (2*Sqrt[1 - (a + b*x)^2])/(b*Sqrt[ArcCos[a + b*x]]) - (2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{-1}(a+bx)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{3/2}}\,dx, x, a+bx\right)}{b} \\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\,\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{b} \\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{b} \\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{4\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{b} \\
&= \frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\cos^{-1}(a+bx)}} - \frac{2\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\cos^{-1}(a+bx)}\right)}{b}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 97, normalized size = 1.52

$$-\frac{-2\sqrt{1-(a+bx)^2} - i\sqrt{-i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{1}{2}, -i\text{ArcCos}(a+bx)\right) + i\sqrt{i\text{ArcCos}(a+bx)}\,\text{Gamma}\left(\frac{1}{2}, i\text{ArcCos}(a+bx)\right)}{b\sqrt{\text{ArcCos}(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-3/2),x]
[Out] -((-2*Sqrt[1 - (a + b*x)^2] - I*Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*
ArcCos[a + b*x]] + I*Sqrt[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])
/(b*Sqrt[ArcCos[a + b*x]]))
```

Maple [A]
time = 0.17, size = 84, normalized size = 1.31

method	result
default	$-\frac{\sqrt{2}\left(2\arccos(bx+a)\pi\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)-\sqrt{2}\,\sqrt{\arccos(bx+a)}\,\sqrt{\pi}\,\sqrt{-b^2x^2-2abx}}{b\sqrt{\pi}\,\arccos(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*2^(1/2)/Pi^(1/2)/arccos(b*x+a)*(2*arccos(b*x+a)*Pi*FresnelC(2^(1/2)/Pi
^(1/2)*arccos(b*x+a)^(1/2))-2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^2-
2*a*b*x-a^2+1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(3/2),x)
```

```
[Out] Integral(acos(a + b*x)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(arccos(b*x + a)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x)^(3/2),x)
[Out] int(1/acos(a + b*x)^(3/2), x)
```

3.42 $\int \frac{1}{\mathbf{ArcCos}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-(a+bx)^2}}{3b\text{ArcCos}(a+bx)^{3/2}}+\frac{4(a+bx)}{3b\sqrt{\text{ArcCos}(a+bx)}}+\frac{4\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(a+bx)}\right)}{3b}$$

```
[Out] 4/3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+2/3*(
1-(b*x+a)^2)^(1/2)/b/arccos(b*x+a)^(3/2)+4/3*(b*x+a)/b/arccos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4888, 4718, 4808, 4720, 3386, 3432}

$$\frac{4\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(a+bx)}\right)}{3b}+\frac{4(a+bx)}{3b\sqrt{\text{ArcCos}(a+bx)}}+\frac{2\sqrt{1-(a+bx)^2}}{3b\text{ArcCos}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - (a + b*x)^2])/(3*b*ArcCos[a + b*x]^(3/2)) + (4*(a + b*x))/(3*b*
Sqrt[ArcCos[a + b*x]]) + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a +
b*x]]])/(3*b)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{-1}(a+bx)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{-1}(x)^{5/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)^{3/2}}\,dx, x, a+bx\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{\cos^{-1}(x)}}\,dx, x, a+bx\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(a+bx)\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{8\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(a+bx)}\right)}{3b}$$

$$= \frac{2\sqrt{1-(a+bx)^2}}{3b\cos^{-1}(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\cos^{-1}(a+bx)}} + \frac{4\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(a+bx)}\right)}{3b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 139, normalized size = 1.54

$$-\frac{2\left(-\sqrt{1-(a+bx)^2}-e^{-i\text{ArcCos}(a+bx)}\text{ArcCos}(a+bx)-e^{i\text{ArcCos}(a+bx)}\text{ArcCos}(a+bx)+i(-i\text{ArcCos}(a+bx))^{3/2}\text{Gamma}\left(\frac{1}{2},-i\text{ArcCos}(a+bx)\right)-i(i\text{ArcCos}(a+bx))^{3/2}\text{Gamma}\left(\frac{1}{2},i\text{ArcCos}(a+bx)\right)\right)}{3b\text{ArcCos}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(-Sqrt[1 - (a + b*x)^2] - ArcCos[a + b*x]/E^(I*ArcCos[a + b*x]) - E^(I*
ArcCos[a + b*x])*ArcCos[a + b*x] + I*((-I)*ArcCos[a + b*x])^(3/2)*Gamma[1/2
, (-I)*ArcCos[a + b*x]] - I*(I*ArcCos[a + b*x])^(3/2)*Gamma[1/2, I*ArcCos[a
 + b*x]]))/(3*b*ArcCos[a + b*x]^(3/2))
```

Maple [A]
time = 0.18, size = 120, normalized size = 1.33

method	result
default	$\frac{\sqrt{2}\left(4\arccos(bx+a)^2\pi\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)+2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,bx+2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,a+\sqrt{}\right.}{3b\sqrt{\pi}\arccos(bx+a)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*2^(1/2)/Pi^(1/2)*(4*arccos(b*x+a)^2*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arcc
os(b*x+a)^(1/2))+2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*b*x+2*arccos(b*x+a)
^(3/2)*2^(1/2)*Pi^(1/2)*a+2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^2-2*
a*b*x-a^2+1)^(1/2))/arccos(b*x+a)^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(b*x+a)**(5/2),x)
```

```
[Out] Integral(acos(a + b*x)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acos}\left(a+b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acos(a + b*x)^(5/2),x)
```

```
[Out] int(1/acos(a + b*x)^(5/2), x)
```

3.43 $\int \frac{1}{\sqrt{a+b\text{ArcCos}(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}\,d}$$

```
[Out] -cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(d*x+c))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/d/b^(1/2)+FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(d*x+c))^(1/2)/
b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4888, 4720, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[c + d*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cos^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cos^{-1}(c+dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\right.}{}$$

$$= -\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\cos^{-1}(c+dx)}\right)}{bd} + \frac{\left(2\sin\left(\frac{a}{b}\right)\right)}{}$$

$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d} + \frac{\sqrt{2\pi}\,C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\cos^{-1}(c+dx)}}{\sqrt{b}}\right.}{\sqrt{b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 128, normalized size = 1.21

$$\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\text{ArcCos}(c+dx))}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{i(a+b\text{ArcCos}(c+dx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\text{ArcCos}(c+dx))}{b}}\,\text{Gamma}\left(\frac{1}{2},\frac{i(a+b\text{ArcCos}(c+dx))}{b}\right)\right)}{2d\sqrt{a+b\text{ArcCos}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[c + d*x]],x]
[Out] (Sqrt[((-I)*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c + 
d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2, 
(I*(a + b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a + b*ArcCos[c + d*x]
])
```

Maple [A]
time = 0.51, size = 93, normalized size = 0.88

method	result
default	$\frac{\sqrt{2}\sqrt{\pi}\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\text{S}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(dx+c)}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(dx+c)}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1
/2)*(a+b*arccos(d*x+c))^(1/2)/b)+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(d*x+c))^(1/2)/b))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\operatorname{acos}{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(c + d*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.52, size = 167, normalized size = 1.58

$$\frac{i\sqrt{\pi}\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos\left(dx+c\right)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos\left(dx+c\right)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{d\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}-\frac{i\sqrt{\pi}\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos\left(dx+c\right)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos\left(dx+c\right)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{d\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(d*(I*sqrt(
2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2)*s
qrt(b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(d*x + c
) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(d*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*s
qrt(abs(b))))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acos(c + d*x))^(1/2), x)
```

3.44 $\int \frac{1}{\sqrt{a-b\mathbf{ArcCos}(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}+\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)\sin\left(\right.}{\sqrt{b}\,d}$$

```
[Out] -cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a-b*arccos(d*x+c))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/d/b^(1/2)+FresnelC(2^(1/2)/Pi^(1/2)*(a-b*arccos(d*x+c))^(1/2)/
b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4888, 4720, 3387, 3386, 3432, 3385, 3433}

$$\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\text{ArcCos}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c + d*x]])/Sq
rt[b]])/(Sqrt[b]*d)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c
 + d*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*d)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\cos^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b\cos^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a-b\cos^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a-b\cos^{-1}(c+dx)\right)}{bd} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos}{\sqrt{}}\right.}{}$$

$$= -\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a-b\cos^{-1}(c+dx)}\right)}{bd} + \frac{\left(2\sin\left(\frac{a}{b}\right)\right.}{}$$

$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)S\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\cos^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,d} + \frac{\sqrt{2\pi}\,C\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 133, normalized size = 1.23

$$\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a-b\text{ArcCos}(c+dx))}{b}}\,\text{Gamma}\left(\frac{1}{2},-\frac{i(a-b\text{ArcCos}(c+dx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a-b\text{ArcCos}(c+dx))}{b}}\,\text{Gamma}\left(\frac{1}{2},\frac{i(a-b\text{ArcCos}(c+dx))}{b}\right)\right)}{2d\sqrt{a-b\text{ArcCos}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```

```
[Out] (Sqrt[((-I)*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a - b*ArcCos[c +
d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2,
(I*(a - b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a - b*ArcCos[c + d*x]
])
```

Maple [A]
time = 0.30, size = 95, normalized size = 0.88

method	result
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{a-b\arccos\left(dx+c\right)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a-b\arccos\left(dx+\right.}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*arccos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1
/2)*(a-b*arccos(d*x+c))^(1/2)/b)+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a-b*arccos(d*x+c))^(1/2)/b))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-b*arccos(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - b\operatorname{acos}{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*acos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - b*acos(c + d*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.51, size = 171, normalized size = 1.58

$$\frac{i\sqrt{\pi}\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{-b\arccos\left(dx+c\right)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos\left(dx+c\right)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{d\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}-\frac{i\sqrt{\pi}\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{-b\arccos\left(dx+c\right)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos\left(dx+c\right)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{d\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(d*(I*sqr
t(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2)
*sqrt(-b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(-b*arccos(d*x
 + c) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(d*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(
2)*sqrt(abs(b))))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-b\,\mathrm{acos}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*acos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - b*acos(c + d*x))^(1/2), x)
```

3.45 $\int \frac{\text{ArcCos}(a+bx)}{\frac{ad}{b}+dx} dx$

Optimal. Leaf size=68

$$-\frac{i\text{ArcCos}(a+bx)^2}{2d}+\frac{\text{ArcCos}(a+bx)\log\left(1+e^{2i\text{ArcCos}(a+bx)}\right)}{d}-\frac{i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(a+bx)}\right)}{2d}$$

```
[Out] -1/2*I*arccos(b*x+a)^2/d+arccos(b*x+a)*ln(1+(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2
)/d-1/2*I*polylog(2,-(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {4890, 12, 4722, 3800, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-e^{2i\text{ArcCos}(a+bx)}\right)}{2d}-\frac{i\text{ArcCos}(a+bx)^2}{2d}+\frac{\text{ArcCos}(a+bx)\log\left(1+e^{2i\text{ArcCos}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((-1/2*I)*ArcCos[a + b*x]^2)/d + (ArcCos[a + b*x]*Log[1 + E^((2*I)*ArcCos[a
 + b*x])])/d - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a + b*x])])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4890

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\cos^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(a+bx)\right)}{d}\\
&= -\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(a+bx)\right)}{d}\\
&= -\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d}-\frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\right.}{d}\\
&= -\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d}+\frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.}{2d}\\
&= -\frac{i\cos^{-1}(a+bx)^2}{2d}+\frac{\cos^{-1}(a+bx)\log\left(1+e^{2i\cos^{-1}(a+bx)}\right)}{d}-\frac{i\text{Li}_2\left(-e^{2i\cos^{-1}(a+bx)}\right)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.87

$$-\frac{i\left(\operatorname{ArcCos}(a+bx)\left(\operatorname{ArcCos}(a+bx)+2i\log\left(1+e^{2i\operatorname{ArcCos}(a+bx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcCos}(a+bx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a + b*x]/((a*d)/b + d*x),x]
[Out] ((-1/2*I)*(ArcCos[a + b*x]*(ArcCos[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcCos
[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcCos[a + b*x])]))/d
```

Maple [A]
time = 0.32, size = 92, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{ib\arccos(bx+a)^2}{2d}+\frac{b\arccos(bx+a)\ln\left(1+\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{d}-\frac{ib\operatorname{polylog}\left(2,-\left(bx+a+i\sqrt{1-(bx+a)^2}\right.\right.}{2d}}{b}$
default	$\frac{-\frac{ib\arccos(bx+a)^2}{2d}+\frac{b\arccos(bx+a)\ln\left(1+\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{d}-\frac{ib\operatorname{polylog}\left(2,-\left(bx+a+i\sqrt{1-(bx+a)^2}\right.\right.}{2d}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2*I*b/d*arccos(b*x+a)^2+b/d*arccos(b*x+a)*ln(1+(b*x+a+I*(1-(b*x+a)^
2)^(1/2))^2)-1/2*I*b/d*polylog(2,-(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
[Out] integrate(arccos(b*x + a)/(d*x + a*d/b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccos(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{acos}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acos(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccos(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{acos}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(acos(a + b*x)/(d*x + (a*d)/b), x)
```

3.46 $\int \sqrt{1-x^2} \operatorname{ArcCos}(x) \, dx$

Optimal. Leaf size=34

$$\frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2}\operatorname{ArcCos}(x)-\frac{\operatorname{ArcCos}(x)^2}{4}$$

[Out] 1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4742, 4738, 30}

$$\frac{1}{2}\sqrt{1-x^2}\,x\operatorname{ArcCos}(x)-\frac{\operatorname{ArcCos}(x)^2}{4}+\frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]*ArcCos[x],x]

[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4742

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Dist[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(a + b*ArcCos[c*x])^n/Sqrt[1
 - c^2*x^2], x], x] + Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]
], Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{1-x^2} \cos^{-1}(x)\,dx = \frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x) + \frac{\int x\,dx}{2} + \frac{1}{2}\int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$= \frac{x^2}{4} + \frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x) - \frac{1}{4}\cos^{-1}(x)^2$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(x^2 + 2x\sqrt{1-x^2}\,\mathrm{ArcCos}(x) - \mathrm{ArcCos}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Maple [A]
time = 0.44, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-\sqrt{-x^2+1}\,x+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arccos(x)*(-(-x^2+1)^(1/2)*x+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Maxima [A]
time = 0.48, size = 30, normalized size = 0.88

$$\frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}\,x + \arcsin(x)\right)\arccos(x) + \frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Fricas [A]
time = 1.21, size = 26, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A]
time = 1.07, size = 39, normalized size = 1.15

$$\left(\begin{cases} \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}\right)\operatorname{acos}(x) + \begin{cases} \text{NaN} & \text{for}\ x < -1 \\ \frac{x^2}{4} + \frac{\operatorname{asin}^2(x)}{4} & \text{for}\ x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4, x < 1), (nan, True))
```

Giac [A]
time = 0.42, size = 27, normalized size = 0.79

$$\frac{1}{2}\sqrt{-x^2+1}\,x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{acos}(x)\,\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x)*(1 - x^2)^(1/2),x)
```

```
[Out] int(acos(x)*(1 - x^2)^(1/2), x)
```

3.47 $\int x^3 \mathbf{ArcCos}(ax^2)\, dx$

Optimal. Leaf size=51

$$-\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{1}{4}x^4\text{ArcCos}\left(ax^2\right)+\frac{\text{ArcSin}(ax^2)}{8a^2}$$

```
[Out] 1/4*x^4*arccos(a*x^2)+1/8*arcsin(a*x^2)/a^2-1/8*x^2*(-a^2*x^4+1)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4927, 12, 281, 327, 222}

$$\frac{\text{ArcSin}(ax^2)}{8a^2}-\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{1}{4}x^4\text{ArcCos}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a*x^2],x]
```

```
[Out] -1/8*(x^2*Sqrt[1 - a^2*x^4])/a + (x^4*ArcCos[a*x^2])/4 + ArcSin[a*x^2]/(8*a
^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos^{-1}\left(ax^2\right)\,dx &= \frac{1}{4}x^4\cos^{-1}\left(ax^2\right)+\frac{1}{4}\int \frac{2ax^5}{\sqrt{1-a^2x^4}}\,dx\\
&= \frac{1}{4}x^4\cos^{-1}\left(ax^2\right)+\frac{1}{2}a\int \frac{x^5}{\sqrt{1-a^2x^4}}\,dx\\
&= \frac{1}{4}x^4\cos^{-1}\left(ax^2\right)+\frac{1}{4}a\text{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{1}{4}x^4\cos^{-1}\left(ax^2\right)+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\,dx,x,x^2\right)}{8a}\\
&= -\frac{x^2\sqrt{1-a^2x^4}}{8a}+\frac{1}{4}x^4\cos^{-1}\left(ax^2\right)+\frac{\sin^{-1}\left(ax^2\right)}{8a^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.94

$$\frac{-ax^2\sqrt{1-a^2x^4}+2a^2x^4\text{ArcCos}(ax^2)+\text{ArcSin}(ax^2)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a*x^2],x]
```

```
[Out] (-(a*x^2*Sqrt[1 - a^2*x^4]) + 2*a^2*x^4*ArcCos[a*x^2] + ArcSin[a*x^2])/(8*a
^2)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.35

method	result	size

default	$\frac{x^{4} \arccos\left(a\,x^{2}\right)}{4}+\frac{a\left(-\frac{x^{2} \sqrt{-a^{2} x^{4}+1}}{4 a^{2}}+\frac{\arctan\left(\frac{\sqrt{a^{2}}\,x^{2}}{\sqrt{-a^{2} x^{4}+1}}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arccos(a*x^2)+1/2*a*(-1/4*x^2*(-a^2*x^4+1)^(1/2)/a^2+1/4/a^2/(a^2)^
(1/2)*arctan((a^2)^(1/2)*x^2/(-a^2*x^4+1)^(1/2)))
```

Maxima [A]
time = 0.47, size = 79, normalized size = 1.55

$$\frac{1}{4}\,x^{4} \arccos\left(ax^{2}\right)-\frac{1}{8}\,a\left(\frac{\arctan\left(\frac{\sqrt{-a^{2}x^{4}+1}}{ax^{2}}\right)}{a^{3}}+\frac{\sqrt{-a^{2}x^{4}+1}}{\left(a^{4}-\frac{(a^{2}x^{4}-1)a^{2}}{x^{4}}\right)x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccos(a*x^2) - 1/8*a*(arctan(sqrt(-a^2*x^4 + 1)/(a*x^2))/a^3 + sqr
t(-a^2*x^4 + 1)/((a^4 - (a^2*x^4 - 1)*a^2/x^4)*x^2))
```

Fricas [A]
time = 1.24, size = 41, normalized size = 0.80

$$-\frac{\sqrt{-a^{2}x^{4}+1}\,ax^{2}-\left(2\,a^{2}x^{4}-1\right)\arccos\left(ax^{2}\right)}{8\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(-a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 - 1)*arccos(a*x^2))/a^2
```

Sympy [A]
time = 0.19, size = 48, normalized size = 0.94

$$\begin{cases} \frac{x^{4} \operatorname{acos}\left(ax^{2}\right)}{4}-\frac{x^{2}\sqrt{-a^{2}x^{4}+1}}{8a}-\frac{\operatorname{acos}\left(ax^{2}\right)}{8a^{2}} & \text{for}\: a\neq 0 \\ \frac{\pi x^{4}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(a*x**2),x)
```

```
[Out] Piecewise((x**4*acos(a*x**2)/4 - x**2*sqrt(-a**2*x**4 + 1)/(8*a) - acos(a*x
**2)/(8*a**2), Ne(a, 0)), (pi*x**4/8, True))
```

Giac [A]
time = 0.42, size = 46, normalized size = 0.90

$$\frac{2\,a^2x^4\arccos\left(ax^2\right)-\sqrt{-a^2x^4+1}\,ax^2-\arccos\left(ax^2\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a^2*x^4*arccos(a*x^2) - sqrt(-a^2*x^4 + 1)*a*x^2 - arccos(a*x^2))/a^
2
```

Mupad [B]
time = 0.30, size = 42, normalized size = 0.82

$$\frac{\mathrm{acos}(a\,x^2)\,\left(2\,a^2\,x^4-1\right)}{8\,a^2}-\frac{x^2\,\sqrt{1-a^2\,x^4}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acos(a*x^2),x)
```

```
[Out] (acos(a*x^2)*(2*a^2*x^4 - 1))/(8*a^2) - (x^2*(1 - a^2*x^4)^(1/2))/(8*a)
```

3.48 $\int x^2 \mathbf{ArcCos}(ax^2)\,dx$

Optimal. Leaf size=55

$$-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{1}{3}x^3\text{ArcCos}(ax^2)+\frac{2F\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{9a^{3/2}}$$

```
[Out] 1/3*x^3*arccos(a*x^2)+2/9*EllipticF(x*a^(1/2),I)/a^(3/2)-2/9*x*(-a^2*x^4+1)
^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4927, 12, 327, 227}

$$\frac{2F\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{9a^{3/2}}-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{1}{3}x^3\text{ArcCos}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 - a^2*x^4])/(9*a) + (x^3*ArcCos[a*x^2])/3 + (2*EllipticF[ArcSi
n[Sqrt[a]*x], -1])/(9*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
```

```
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^{-1}\left(ax^2\right)\,dx&=\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)+\frac{1}{3}\int\frac{2ax^4}{\sqrt{1-a^2x^4}}\,dx\\&=\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)+\frac{1}{3}(2a)\int\frac{x^4}{\sqrt{1-a^2x^4}}\,dx\\&=-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)+\frac{2\int\frac{1}{\sqrt{1-a^2x^4}}\,dx}{9a}\\&=-\frac{2x\sqrt{1-a^2x^4}}{9a}+\frac{1}{3}x^3\cos^{-1}\left(ax^2\right)+\frac{2F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{9a^{3/2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 63, normalized size = 1.15

$$\frac{1}{9}\left(-\frac{2x\sqrt{1-a^2x^4}}{a}+3x^3\text{ArcCos}\left(ax^2\right)+\frac{2iF\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{(-a)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a*x^2],x]
[Out] ((-2*x*Sqrt[1 - a^2*x^4])/a + 3*x^3*ArcCos[a*x^2] + ((2*I)*EllipticF[I*ArcS
inh[Sqrt[-a]*x], -1])/(-a)^(3/2))/9
```

Maple [A]
time = 0.01, size = 79, normalized size = 1.44

method	result	size
default	$\frac{x^3\arccos(a\,x^2)}{3}+\frac{2a\left(-\frac{x\sqrt{-a^2x^4+1}}{3a^2}+\frac{\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,\text{EllipticF}\left(x\sqrt{a},i\right)}{3a^{\frac{5}{2}}\sqrt{-a^2x^4+1}}\right)}{3}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccos(a*x^2)+2/3*a*(-1/3/a^2*x*(-a^2*x^4+1)^(1/2)+1/3/a^(5/2)*(-a*
x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/2)*EllipticF(x*a^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan2(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2) - 2*a*integrate(1/
3*x^4*e^(1/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a^2*x^4 + (a^
2*x^4 - 1)*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x)
```

Fricas [A]
time = 0.29, size = 33, normalized size = 0.60

$$\frac{3\,ax^3\arccos\left(ax^2\right) - 2\,\sqrt{-a^2x^4+1}\,x}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*x^3*arccos(a*x^2) - 2*sqrt(-a^2*x^4 + 1)*x)/a
```

Sympy [A]
time = 0.75, size = 48, normalized size = 0.87

$$\frac{ax^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|a^2x^4e^{2i\pi}\right)}{6\Gamma\left(\frac{9}{4}\right)} + \frac{x^3\operatorname{acos}\left(ax^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a*x**2),x)
```

```
[Out] a*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), a**2*x**4*exp_polar(2*I*pi))/(6
*gamma(9/4)) + x**3*acos(a*x**2)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccos(a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{acos}(a x^2) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(a*x^2),x)
[Out] int(x^2*acos(a*x^2), x)
```

3.49 $\int x\text{ArcCos}(ax^2)\,dx$

Optimal. Leaf size=35

$$-\frac{\sqrt{1-a^2x^4}}{2a}+\frac{1}{2}x^2\text{ArcCos}(ax^2)$$

```
[Out] 1/2*x^2*arccos(a*x^2)-1/2*(-a^2*x^4+1)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6847, 4716, 267}

$$\frac{1}{2}x^2\text{ArcCos}(ax^2)-\frac{\sqrt{1-a^2x^4}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a*x^2],x]
```

```
[Out] -1/2*Sqrt[1 - a^2*x^4]/a + (x^2*ArcCos[a*x^2])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x\cos^{-1}\left(ax^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cos^{-1}(ax)\,dx, x, x^2\right) \\
&= \frac{1}{2}x^2\cos^{-1}\left(ax^2\right) + \frac{1}{2}a\text{Subst}\left(\int \frac{x}{\sqrt{1-a^2x^2}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^4}}{2a} + \frac{1}{2}x^2\cos^{-1}\left(ax^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$
-\frac{\sqrt{1-a^2x^4}}{2a} + \frac{1}{2}x^2\text{ArcCos}\left(ax^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a*x^2],x]
[Out] -1/2*Sqrt[1 - a^2*x^4]/a + (x^2*ArcCos[a*x^2])/2
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.91

method	result	size
derivativedivides	$\frac{a\,x^2\arccos(a\,x^2)-\sqrt{-a^2x^4+1}}{2a}$	32
default	$\frac{a\,x^2\arccos(a\,x^2)-\sqrt{-a^2x^4+1}}{2a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(a*x^2*arccos(a*x^2)-(-a^2*x^4+1)^(1/2))
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.89

$$
\frac{ax^2\arccos\left(ax^2\right) - \sqrt{-a^2x^4+1}}{2\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="maxima")
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

Fricas [A]
time = 1.54, size = 31, normalized size = 0.89

$$\frac{ax^2 \arccos\left(ax^2\right) - \sqrt{-a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

Sympy [A]
time = 0.08, size = 32, normalized size = 0.91

$$\begin{cases} \frac{x^2 \operatorname{acos}\left(ax^2\right)}{2} - \frac{\sqrt{-a^2x^4 + 1}}{2a} & \text{for}\ a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a*x**2),x)
```

```
[Out] Piecewise((x**2*acos(a*x**2)/2 - sqrt(-a**2*x**4 + 1)/(2*a), Ne(a, 0)), (pi
*x**2/4, True))
```

Giac [A]
time = 0.45, size = 31, normalized size = 0.89

$$\frac{ax^2 \arccos\left(ax^2\right) - \sqrt{-a^2x^4 + 1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

Mupad [B]
time = 0.33, size = 29, normalized size = 0.83

$$\frac{x^2 \operatorname{acos}(a\,x^2)}{2} - \frac{\sqrt{1 - a^2\,x^4}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(a*x^2),x)
```

```
[Out] (x^2*acos(a*x^2))/2 - (1 - a^2*x^4)^(1/2)/(2*a)
```

3.50 $\int \textbf{ArcCos}(ax^2)\,dx$

Optimal. Leaf size=43

$$x\text{ArcCos}(ax^2) + \frac{2E\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{\sqrt{a}} - \frac{2F\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{\sqrt{a}}$$

```
[Out] x*arccos(a*x^2)+2*EllipticE(x*a^(1/2),I)/a^(1/2)-2*EllipticF(x*a^(1/2),I)/a
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4925, 12, 313, 227, 1213, 435}

$$x\text{ArcCos}(ax^2) - \frac{2F\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{\sqrt{a}} + \frac{2E\left(\text{ArcSin}(\sqrt{a}\,x)\middle|-1\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2],x]
```

```
[Out] x*ArcCos[a*x^2] + (2*EllipticE[ArcSin[Sqrt[a]*x], -1])/Sqrt[a] - (2*Ellipti
cF[ArcSin[Sqrt[a]*x], -1])/Sqrt[a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(ax^2\right)\,dx &= x\cos^{-1}\left(ax^2\right)+\int \frac{2ax^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)+(2a)\int \frac{x^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)-2\int \frac{1}{\sqrt{1-a^2x^4}}\,dx+2\int \frac{1+ax^2}{\sqrt{1-a^2x^4}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)-\frac{2F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}+2\int \frac{\sqrt{1+ax^2}}{\sqrt{1-ax^2}}\,dx\\
&= x\cos^{-1}\left(ax^2\right)+\frac{2E\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}-\frac{2F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 34, normalized size = 0.79

$$x\text{ArcCos}\left(ax^2\right)+\frac{2}{3}ax^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};a^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2],x]
```

```
[Out] x*ArcCos[a*x^2] + (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, a^2*x^4])/3
```

Maple [A]
time = 0.01, size = 65, normalized size = 1.51

method	result	size

default	$x \arccos\left(a\,x^{2}\right)-\frac{2\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{a}\,,i\right)-\operatorname{EllipticE}\left(x\sqrt{a}\,,i\right)\right)}{\sqrt{a}\,\sqrt{-a^{2}x^{4}+1}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccos(a*x^2)-2/a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/
2)*(EllipticF(x*a^(1/2),I)-EllipticE(x*a^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="maxima")
```

```
[Out] x*arctan2(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2) - 2*a*integrate(x^2*e^(1
/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a^2*x^4 + (a^2*x^4 - 1)
*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x)
```

Fricas [A]
time = 0.49, size = 33, normalized size = 0.77

$$\frac{ax^{2}\arccos\left(ax^{2}\right)-2\sqrt{-a^{2}x^{4}+1}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="fricas")
```

```
[Out] (a*x^2*arccos(a*x^2) - 2*sqrt(-a^2*x^4 + 1))/(a*x)
```

Sympy [A]
time = 0.62, size = 44, normalized size = 1.02

$$\frac{ax^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|a^{2}x^{4}e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}+x\operatorname{acos}\left(ax^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2),x)
```

```
[Out] a*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), a**2*x**4*exp_polar(2*I*pi))/(2
*gamma(7/4)) + x*acos(a*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2),x, algorithm="giac")
[Out] integrate(arccos(a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{acos}\left(a\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a*x^2),x)
[Out] int(acos(a*x^2), x)
```

3.51 $\int \frac{\text{ArcCos}(ax^2)}{x} dx$

Optimal. Leaf size=62

$$-\frac{1}{4}i\text{ArcCos}(ax^2)^2 + \frac{1}{2}\text{ArcCos}(ax^2)\log\left(1+e^{2i\text{ArcCos}(ax^2)}\right) - \frac{1}{4}i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(ax^2)}\right)$$

```
[Out] -1/4*I*arccos(a*x^2)^2+1/2*arccos(a*x^2)*ln(1+(a*x^2+I*(-a^2*x^4+1)^(1/2))^
2)-1/4*I*polylog(2,-(a*x^2+I*(-a^2*x^4+1)^(1/2))^2)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3800, 2221, 2317, 2438}

$$-\frac{1}{4}i\text{Li}_2\left(-e^{2i\text{ArcCos}(ax^2)}\right) - \frac{1}{4}i\text{ArcCos}(ax^2)^2 + \frac{1}{2}\text{ArcCos}(ax^2)\log\left(1+e^{2i\text{ArcCos}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2]/x,x]
```

```
[Out] (-1/4*I)*ArcCos[a*x^2]^2 + (ArcCos[a*x^2]*Log[1 + E^((2*I)*ArcCos[a*x^2])])
/2 - (I/4)*PolyLog[2, -E^((2*I)*ArcCos[a*x^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4915

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[-p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax^2)}{x}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax^2)\right)\right)\\ &= -\frac{1}{4}i\cos^{-1}(ax^2)^2 + i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax^2)\right)\\ &= -\frac{1}{4}i\cos^{-1}(ax^2)^2 + \frac{1}{2}\cos^{-1}(ax^2)\log\left(1+e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{2}\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx\right.\\ &= -\frac{1}{4}i\cos^{-1}(ax^2)^2 + \frac{1}{2}\cos^{-1}(ax^2)\log\left(1+e^{2i\cos^{-1}(ax^2)}\right) + \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,\right.\\ &= -\frac{1}{4}i\cos^{-1}(ax^2)^2 + \frac{1}{2}\cos^{-1}(ax^2)\log\left(1+e^{2i\cos^{-1}(ax^2)}\right) - \frac{1}{4}i\text{Li}_2\left(-e^{2i\cos^{-1}(ax^2)}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.90

$$-\frac{1}{4}i\left(\text{ArcCos}(ax^2)\left(\text{ArcCos}(ax^2)+2i\log\left(1+e^{2i\text{ArcCos}(ax^2)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(ax^2)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2]/x,x]
```

```
[Out] (-1/4*I)*(ArcCos[a*x^2]*(ArcCos[a*x^2] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*x^
2])]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^2])])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\arccos(a\,x^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2)/x,x)
```

```
[Out] int(arccos(a*x^2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="maxima")
[Out] integrate(arccos(a*x^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="fricas")
[Out] integral(arccos(a*x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}(ax^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2)/x,x)
[Out] Integral(acos(a*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x,x, algorithm="giac")
[Out] integrate(arccos(a*x^2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}(a\,x^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a*x^2)/x,x)
[Out] int(acos(a*x^2)/x, x)
```

3.52 $\int \frac{\text{ArcCos}(ax^2)}{x^2} dx$

Optimal. Leaf size=29

$$-\frac{\text{ArcCos}(ax^2)}{x} - 2\sqrt{a}\, F\left(\text{ArcSin}\left(\sqrt{a}\, x\right)\middle| -1\right)$$

```
[Out] -arccos(a*x^2)/x-2*EllipticF(x*a^(1/2),I)*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4927, 12, 227}

$$-\frac{\text{ArcCos}(ax^2)}{x} - 2\sqrt{a}\, F\left(\text{ArcSin}\left(\sqrt{a}\, x\right)\middle| -1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^2]/x^2,x]
```

```
[Out] -(ArcCos[a*x^2]/x) - 2*Sqrt[a]*EllipticF[ArcSin[Sqrt[a]*x], -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}(ax^2)}{x^2}\,dx &= -\frac{\cos^{-1}(ax^2)}{x} - \int \frac{2a}{\sqrt{1-a^2x^4}}\,dx \\ &= -\frac{\cos^{-1}(ax^2)}{x} - (2a)\int \frac{1}{\sqrt{1-a^2x^4}}\,dx \\ &= -\frac{\cos^{-1}(ax^2)}{x} - 2\sqrt{a}\,F\left(\sin^{-1}\left(\sqrt{a}\,x\right)\middle|-1\right)\end{aligned}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.03, size = 40, normalized size = 1.38

$$-\frac{\text{ArcCos}(ax^2) + 2i\sqrt{-a}\,xF\left(i\sinh^{-1}\left(\sqrt{-a}\,x\right)\middle|-1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^2]/x^2,x]
```

```
[Out] -((ArcCos[a*x^2] + (2*I)*Sqrt[-a]*x*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/x
)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.
time = 0.01, size = 57, normalized size = 1.97

method	result	size
default	$-\frac{\arccos(a\,x^2)}{x} - \frac{2\sqrt{a}\,\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,\text{EllipticF}\left(x\sqrt{a}\,,i\right)}{\sqrt{-a^2x^4+1}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccos(a*x^2)/x-2*a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1
/2)*EllipticF(x*a^(1/2),I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] (2*a*x*integrate(e^(1/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a^
2*x^4 + (a^2*x^4 - 1)*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x) - arctan2(s
qrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2))/x
```

Fricas [A]
time = 0.69, size = 11, normalized size = 0.38

$$-\frac{\arccos(ax^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="fricas")
[Out] -arccos(a*x^2)/x
```

Sympy [A]
time = 0.66, size = 44, normalized size = 1.52

$$-\frac{ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}-\frac{\operatorname{acos}(ax^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**2)/x**2,x)
[Out] -a*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**4*exp_polar(2*I*pi))/(2*g
amma(5/4)) - acos(a*x**2)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^2)/x^2,x, algorithm="giac")
[Out] integrate(arccos(a*x^2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acos}(a\,x^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a*x^2)/x^2,x)
[Out] int(acos(a*x^2)/x^2, x)
```

3.53 $\int x^2 \mathbf{ArcCos}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=58

$$-\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\sec^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

```
[Out] 1/3*x^3*arcsec(x/a)-1/6*a^3*arctanh((1-a^2/x^2)^(1/2))-1/6*a*x^2*(1-a^2/x^2
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4917, 5328, 272, 44, 65, 214}

$$-\frac{1}{6}ax^2\sqrt{1-\frac{a^2}{x^2}}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)+\frac{1}{3}x^3\sec^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[a/x],x]
```

```
[Out] -1/6*(a*Sqrt[1 - a^2/x^2]*x^2) + (x^3*ArcSec[x/a])/3 - (a^3*ArcTanh[Sqrt[1
- a^2/x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^{-1}\left(\frac{a}{x}\right) dx &= \int x^2 \sec^{-1}\left(\frac{x}{a}\right) dx \\
&= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{3}a\int \frac{x}{\sqrt{1-\frac{a^2}{x^2}}}\, dx \\
&= \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\, dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}\, x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{12}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\, dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}\, x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\, dx, x, \sqrt{1-\frac{a^2}{x^2}}\right) \\
&= -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}\, x^2 + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3 \tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.05

$$\frac{1}{3}x^3\text{ArcCos}\left(\frac{a}{x}\right) - \frac{1}{6}a\left(\sqrt{1-\frac{a^2}{x^2}}\, x^2 + a^2\log\left(\left(1+\sqrt{1-\frac{a^2}{x^2}}\right)x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[a/x],x]
```

```
[Out] (x^3*ArcCos[a/x])/3 - (a*(Sqrt[1 - a^2/x^2]*x^2 + a^2*Log[(1 + Sqrt[1 - a^2
/x^2])*x]))/6
```

Maple [A]
time = 0.11, size = 56, normalized size = 0.97

method	result	size
derivativedivides	$-a^{3}\left(-\frac{x^{3}\arccos\left(\frac{a}{x}\right)}{3a^{3}}+\frac{x^{2}\sqrt{1-\frac{a^{2}}{x^{2}}}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^{2}}{x^{2}}}}\right)}{6}\right)$	56
default	$-a^{3}\left(-\frac{x^{3}\arccos\left(\frac{a}{x}\right)}{3a^{3}}+\frac{x^{2}\sqrt{1-\frac{a^{2}}{x^{2}}}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^{2}}{x^{2}}}}\right)}{6}\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-1/3/a^3*x^3*arccos(a/x)+1/6/a^2*x^2*(1-a^2/x^2)^(1/2)+1/6*arctanh(1/
(1-a^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.47, size = 72, normalized size = 1.24

$$\frac{1}{3}\,x^{3}\arccos\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^{2}\log\left(\sqrt{-\frac{a^{2}}{x^{2}}+1}+1\right)-a^{2}\log\left(\sqrt{-\frac{a^{2}}{x^{2}}+1}-1\right)+2\,x^{2}\sqrt{-\frac{a^{2}}{x^{2}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a/x) - 1/12*(a^2*log(sqrt(-a^2/x^2 + 1) + 1) - a^2*log(sqrt(
-a^2/x^2 + 1) - 1) + 2*x^2*sqrt(-a^2/x^2 + 1))*a
```

Fricas [A]
time = 1.54, size = 93, normalized size = 1.60

$$\frac{1}{6}\,a^{3}\log\left(x\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}-x\right)-\frac{1}{6}\,ax^{2}\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}+\frac{1}{3}\left(x^{3}-1\right)\arccos\left(\frac{a}{x}\right)+\frac{2}{3}\,\arctan\left(\frac{x\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}-x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt(-(a^2 - x^2)/x^2) - x) - 1/6*a*x^2*sqrt(-(a^2 - x^2)/x^2
) + 1/3*(x^3 - 1)*arccos(a/x) + 2/3*arctan((x*sqrt(-(a^2 - x^2)/x^2) - x)/a
)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.89, size = 95, normalized size = 1.64

$$-\frac{a\left(\begin{cases}\frac{a^2\operatorname{acosh}\left(\frac{x}{a}\right)}{2}-\frac{ax}{2\sqrt{-1+\frac{x^2}{a^2}}}+\frac{x^3}{2a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right|>1\\ -\frac{ia^2\operatorname{asin}\left(\frac{x}{a}\right)}{2}+\frac{iax\sqrt{1-\frac{x^2}{a^2}}}{2} & \text{otherwise}\end{cases}\right)}{3}+\frac{x^3\operatorname{acos}\left(\frac{a}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(a/x),x)
```

```
[Out] -a*Piecewise((a**2*acosh(x/a)/2 - a*x/(2*sqrt(-1 + x**2/a**2)) + x**3/(2*a*
sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (-I*a**2*asin(x/a)/2 + I*a*x*sq
rt(1 - x**2/a**2)/2, True))/3 + x**3*acos(a/x)/3
```

Giac [A]
time = 0.44, size = 77, normalized size = 1.33

$$-\frac{a^4\left(\frac{2x^2\sqrt{-\frac{a^2}{x^2}+1}}{a^2}+\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{a^2}{x^2}+1}+1\right)\right)-4\,ax^3\arccos\left(\frac{a}{x}\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(a/x),x, algorithm="giac")
```

```
[Out] -1/12*(a^4*(2*x^2*sqrt(-a^2/x^2 + 1)/a^2 + log(sqrt(-a^2/x^2 + 1) + 1) - lo
g(-sqrt(-a^2/x^2 + 1) + 1)) - 4*a*x^3*arccos(a/x))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\operatorname{acos}\left(\frac{a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(a/x),x)
```

```
[Out] int(x^2*acos(a/x), x)
```

3.54 $\int x\mathbf{ArcCos}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2}a\sqrt{1-\frac{a^2}{x^2}}\,x+\frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arcsec(x/a)-1/2*a*x*(1-a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4917, 5328, 197}

$$\frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)-\frac{1}{2}ax\sqrt{1-\frac{a^2}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[a/x],x]
```

```
[Out] -1/2*(a*Sqrt[1 - a^2/x^2]*x) + (x^2*ArcSec[x/a])/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\cos^{-1}\left(\frac{a}{x}\right)\,dx=\int x\sec^{-1}\left(\frac{x}{a}\right)\,dx$$
$$=\frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)-\frac{1}{2}a\int\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\,dx$$
$$=-\frac{1}{2}a\sqrt{1-\frac{a^2}{x^2}}\,x+\frac{1}{2}x^2\sec^{-1}\left(\frac{x}{a}\right)$$

Mathematica [**A**]
time = 0.02, size = 33, normalized size = 0.97

$$\frac{1}{2}\left(-a\sqrt{1-\frac{a^2}{x^2}}\,x+x^2\text{ArcCos}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[a/x],x]
[Out] (-(a*Sqrt[1 - a^2/x^2]*x) + x^2*ArcCos[a/x])/2
```

Maple [**A**]
time = 0.01, size = 39, normalized size = 1.15

method	result	size
derivativedivides	$-a^2\left(-\frac{x^2\arccos\left(\frac{a}{x}\right)}{2a^2}+\frac{x\sqrt{1-\frac{a^2}{x^2}}}{2a}\right)$	39
default	$-a^2\left(-\frac{x^2\arccos\left(\frac{a}{x}\right)}{2a^2}+\frac{x\sqrt{1-\frac{a^2}{x^2}}}{2a}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(a/x),x,method=_RETURNVERBOSE)
[Out] -a^2*(-1/2/a^2*x^2*arccos(a/x)+1/2/a*x*(1-a^2/x^2)^(1/2))
```

Maxima [**A**]
time = 0.47, size = 28, normalized size = 0.82

$$\frac{1}{2}\,x^2\arccos\left(\frac{a}{x}\right)-\frac{1}{2}\,ax\sqrt{-\frac{a^2}{x^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-a^2/x^2 + 1)
```

Fricas [A]
time = 1.37, size = 32, normalized size = 0.94

$$\frac{1}{2}\,x^{2}\arccos\left(\frac{a}{x}\right)-\frac{1}{2}\,ax\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-(a^2 - x^2)/x^2)
```

Sympy [A]
time = 0.94, size = 48, normalized size = 1.41

$$-\frac{a\left(\begin{cases} a\sqrt{-1+\frac{x^{2}}{a^{2}}} & \text{for}\ \left|\frac{x^{2}}{a^{2}}\right|>1\\ ia\sqrt{1-\frac{x^{2}}{a^{2}}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x^{2}\operatorname{acos}\left(\frac{a}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(a/x),x)
```

```
[Out] -a*Piecewise((a*sqrt(-1 + x**2/a**2), Abs(x**2/a**2) > 1), (I*a*sqrt(1 - x*
*2/a**2), True))/2 + x**2*acos(a/x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.
time = 0.44, size = 64, normalized size = 1.88

$$-\frac{a^{3}\left(\frac{x\left(\sqrt{-\frac{a^{2}}{x^{2}}+1}-1\right)}{a}-\frac{a}{x\left(\sqrt{-\frac{a^{2}}{x^{2}}+1}-1\right)}\right)-2\,ax^{2}\arccos\left(\frac{a}{x}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(a/x),x, algorithm="giac")
```

```
[Out] -1/4*(a^3*(x*(sqrt(-a^2/x^2 + 1) - 1)/a - a/(x*(sqrt(-a^2/x^2 + 1) - 1))) -
 2*a*x^2*arccos(a/x))/a
```

Mupad [B]
time = 0.24, size = 28, normalized size = 0.82

$$\frac{x^2 \operatorname{acos}\left(\frac{a}{x}\right)}{2} - \frac{a\,x\,\sqrt{1-\frac{a^2}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(a/x),x)
[Out] (x^2*acos(a/x))/2 - (a*x*(1 - a^2/x^2)^(1/2))/2
```

3.55 $\int \text{ArcCos}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=27

$$x\sec^{-1}\left(\frac{x}{a}\right)-a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

```
[Out] x*arcsec(x/a)-a*arctanh((1-a^2/x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.833$, Rules used = {4917, 5322, 272, 65, 214}

$$x\sec^{-1}\left(\frac{x}{a}\right)-a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x],x]
```

```
[Out] x*ArcSec[x/a] - a*ArcTanh[Sqrt[1 - a^2/x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5322

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\frac{a}{x}\right)\,dx &= \int \sec^{-1}\left(\frac{x}{a}\right)\,dx \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}\,x}\,dx \\
&= x\sec^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-\frac{a^2}{x^2}}\right)}{a} \\
&= x\sec^{-1}\left(\frac{x}{a}\right) - a\tanh^{-1}\left(\sqrt{1-\frac{a^2}{x^2}}\right)
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.
time = 0.07, size = 84, normalized size = 3.11

$$x\text{ArcCos}\left(\frac{a}{x}\right) - \frac{a\sqrt{-a^2+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-a^2+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-a^2+x^2}}\right)\right)}{2\sqrt{1-\frac{a^2}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x],x]
[Out] x*ArcCos[a/x] - (a*Sqrt[-a^2 + x^2]*(-Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1 +
 x/Sqrt[-a^2 + x^2]]))/(2*Sqrt[1 - a^2/x^2]*x)
```

Maple [**A**]
time = 0.01, size = 30, normalized size = 1.11

method	result	size

derivativedivides	$-a\left(-\frac{x\arccos\left(\frac{a}{x}\right)}{a}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$	30
default	$-a\left(-\frac{x\arccos\left(\frac{a}{x}\right)}{a}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(-1/a*x*arccos(a/x)+arctanh(1/(1-a^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.48, size = 45, normalized size = 1.67

$$-\frac{1}{2}a\left(\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)\right)+x\arccos\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(sqrt(-a^2/x^2 + 1) + 1) - log(sqrt(-a^2/x^2 + 1) - 1)) + x*arcc
os(a/x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.
time = 1.56, size = 65, normalized size = 2.41

$$(x-1)\arccos\left(\frac{a}{x}\right)+a\log\left(x\sqrt{-\frac{a^2-x^2}{x^2}}-x\right)+2\arctan\left(\frac{x\sqrt{-\frac{a^2-x^2}{x^2}}-x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="fricas")
```

```
[Out] (x - 1)*arccos(a/x) + a*log(x*sqrt(-(a^2 - x^2)/x^2) - x) + 2*arctan((x*sqr
t(-(a^2 - x^2)/x^2) - x)/a)
```

Sympy [A]
time = 1.07, size = 27, normalized size = 1.00

$$-a\left(\begin{cases}\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right|>1\\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise}\end{cases}\right)+x\operatorname{acos}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x),x)
```

```
[Out] -a*Piecewise((acosh(x/a), Abs(x**2/a**2) > 1), (-I*asin(x/a), True)) + x*ac
os(a/x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(25) = 50$.
time = 0.41, size = 55, normalized size = 2.04

$$-\frac{a^2\left(\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{a^2}{x^2}+1}+1\right)\right)-2\,ax\arccos\left(\frac{a}{x}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x),x, algorithm="giac")
```

```
[Out] -1/2*(a^2*(log(sqrt(-a^2/x^2 + 1) + 1) - log(-sqrt(-a^2/x^2 + 1) + 1)) - 2*
a*x*arccos(a/x))/a
```

Mupad [B]
time = 0.57, size = 28, normalized size = 1.04

$$x\,\mathrm{acos}\left(\frac{a}{x}\right)-a\,\mathrm{sign}(x)\,\ln\left(x+\sqrt{x^2-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a/x),x)
```

```
[Out] x*acos(a/x) - a*sign(x)*log(x + (x^2 - a^2)^(1/2))
```

3.56 $\int \frac{\text{ArcCos}\left(\frac{a}{x}\right)}{x}\,dx$

Optimal. Leaf size=60

$$\frac{1}{2}i\text{ArcCos}\left(\frac{a}{x}\right)^2-\text{ArcCos}\left(\frac{a}{x}\right)\log\left(1+e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right)$$

```
[Out] 1/2*I*arccos(a/x)^2-arccos(a/x)*ln(1+(a/x+I*(1-a^2/x^2)^(1/2))^2)+1/2*I*pol
ylog(2,-(a/x+I*(1-a^2/x^2)^(1/2))^2)
```

Rubi [A]

time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3800, 2221, 2317, 2438}

$$\frac{1}{2}i\text{Li}_2\left(-e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right)+\frac{1}{2}i\text{ArcCos}\left(\frac{a}{x}\right)^2-\text{ArcCos}\left(\frac{a}{x}\right)\log\left(1+e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x,x]
```

```
[Out] (I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*Po
lyLog[2, -E^((2*I)*ArcCos[a/x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4915

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[-p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x}\,dx &= \text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - 2i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \cos^{-1}\right.\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}}\right.\\
&= \frac{1}{2}i\cos^{-1}\left(\frac{a}{x}\right)^2 - \cos^{-1}\left(\frac{a}{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\frac{a}{x}\right)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.00

$$\frac{1}{2}i\text{ArcCos}\left(\frac{a}{x}\right)^2 - \text{ArcCos}\left(\frac{a}{x}\right)\log\left(1+e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\text{PolyLog}\left(2, -e^{2i\text{ArcCos}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x,x]
```

```
[Out] (I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*Po
lyLog[2, -E^((2*I)*ArcCos[a/x])]
```

Maple [A]
time = 0.52, size = 77, normalized size = 1.28

method	result
derivativedivides	$\frac{i\arccos\left(\frac{a}{x}\right)^2}{2} - \arccos\left(\frac{a}{x}\right)\ln\left(1+\left(\frac{a}{x}+i\sqrt{1-\frac{a^2}{x^2}}\right)^2\right) + \frac{i\,\text{polylog}\left(2,-\left(\frac{a}{x}+i\sqrt{1-\frac{a^2}{x^2}}\right)^2\right)}{2}$

default	$\frac{i \arccos\left(\frac{a}{x}\right)^{2}}{2}-\arccos\left(\frac{a}{x}\right) \ln\left(1+\left(\frac{a}{x}+i\sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)+\frac{i \operatorname{polylog}\left(2,-\left(\frac{a}{x}+i\sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)}{2}$	7

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccos(a/x)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*I*arccos(a/x)^2-arccos(a/x)*ln(1+(a/x+I*(1-a^2/x^2)^(1/2))^2)+1/2*I*polylog(2,-(a/x+I*(1-a^2/x^2)^(1/2))^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x,x, algorithm="maxima")

[Out] -I*a^2*integrate(-log(x)/(a^2*x - x^3), x) - a*integrate(-sqrt(a + x)*sqrt(-a + x)*log(x)/(a^2*x - x^3), x) + arctan(sqrt(a + x)*sqrt(-a + x)/a)*log(x) - 1/2*I*log(x^2)*log(x) + 1/2*I*log(x)^2 + 1/2*I*log(x)*log((a + x)/a) + 1/2*I*log(x)*log((a - x)/a) + 1/2*I*dilog(x/a) + 1/2*I*dilog(-x/a)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccos(a/x)/x,x, algorithm="fricas")

[Out] integral(arccos(a/x)/x, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acos(a/x)/x,x)

[Out] Integral(acos(a/x)/x, x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]Undef/
Unsigned
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a/x)/x,x)
```

```
[Out] int(acos(a/x)/x, x)
```

3.57 $\int \frac{\text{ArcCos}\left(\frac{a}{x}\right)}{x^2} dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

[Out] -arcsec(x/a)/x+(1-a^2/x^2)^(1/2)/a

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4917, 5328, 267}

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[a/x]/x^2,x]

[Out] Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx$$
$$= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x} + a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}\,x^3}\,dx$$
$$= \frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\text{ArcCos}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^2,x]
[Out] Sqrt[1 - a^2/x^2]/a - ArcCos[a/x]/x
```

Maple [A]
time = 0.00, size = 32, normalized size = 1.07

method	result	size
derivativedivides	$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x}-\sqrt{1-\frac{a^2}{x^2}}}{a}$	32
default	$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x}-\sqrt{1-\frac{a^2}{x^2}}}{a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/a*(a/x*arccos(a/x)-(1-a^2/x^2)^(1/2))
```

Maxima [A]
time = 0.47, size = 31, normalized size = 1.03

$$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x}-\sqrt{-\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

Fricas [A]
time = 2.23, size = 36, normalized size = 1.20

$$-\frac{a\arccos\left(\frac{a}{x}\right)-x\sqrt{-\frac{a^2-x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*arccos(a/x) - x*sqrt(-(a^2 - x^2)/x^2))/(a*x)
```

Sympy [A]
time = 0.42, size = 26, normalized size = 0.87

$$\begin{cases} -\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}+\frac{\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{for}\ a\neq 0\\ -\frac{\pi}{2x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**2,x)
```

```
[Out] Piecewise((-acos(a/x)/x + sqrt(-a**2/x**2 + 1)/a, Ne(a, 0)), (-pi/(2*x), Tr
ue))
```

Giac [A]
time = 0.42, size = 31, normalized size = 1.03

$$-\frac{\frac{a\arccos\left(\frac{a}{x}\right)}{x}-\sqrt{-\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^2,x, algorithm="giac")
```

```
[Out] -(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

Mupad [B]
time = 0.03, size = 28, normalized size = 0.93

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a/x)/x^2,x)
```

```
[Out] (1 - a^2/x^2)^(1/2)/a - acos(a/x)/x
```

3.58 $\int \frac{\text{ArcCos}\left(\frac{a}{x}\right)}{x^3} dx$

Optimal. Leaf size=51

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax}-\frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/4*arccsc(x/a)/a^2-1/2*arcsec(x/a)/x^2+1/4*(1-a^2/x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4917, 5328, 342, 327, 222}

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax}-\frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x^3,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/(4*a*x) - ArcCsc[x/a]/(4*a^2) - ArcSec[x/a]/(2*x^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}\,x^4}\,dx \\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-a^2x^2}}\,dx, x, \frac{1}{x}\right)}{4a} \\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax} - \frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.98

$$\frac{a\sqrt{1-\frac{a^2}{x^2}}\,x - 2a^2\mathrm{ArcCos}\left(\frac{a}{x}\right) - x^2\mathrm{ArcSin}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^3,x]
[Out] (a*Sqrt[1 - a^2/x^2]*x - 2*a^2*ArcCos[a/x] - x^2*ArcSin[a/x])/(4*a^2*x^2)
```

Maple [A]
time = 0.01, size = 47, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{\frac{a^2\arccos\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{1-\frac{a^2}{x^2}}}{4x}+\frac{\arcsin\left(\frac{a}{x}\right)}{4}}{a^2}$	47

default	$-\frac{\frac{a^{2}\arccos\left(\frac{a}{x}\right)}{2x^{2}}-\frac{a\sqrt{1-\frac{a^{2}}{x^{2}}}}{4x}+\frac{\arcsin\left(\frac{a}{x}\right)}{4}}{a^{2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/a^2*(1/2*a^2/x^2*arccos(a/x)-1/4*a/x*(1-a^2/x^2)^(1/2)+1/4*arcsin(a/x))
```

Maxima [A]
time = 0.47, size = 77, normalized size = 1.51

$$-\frac{1}{4}\,a\left(\frac{x\sqrt{-\frac{a^{2}}{x^{2}}+1}}{a^{2}x^{2}\left(\frac{a^{2}}{x^{2}}-1\right)-a^{4}}-\frac{\arctan\left(\frac{x\sqrt{-\frac{a^{2}}{x^{2}}+1}}{a}\right)}{a^{3}}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="maxima")
[Out] -1/4*a*(x*sqrt(-a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 - 1) - a^4) - arctan(x*sqrt(
-a^2/x^2 + 1)/a)/a^3) - 1/2*arccos(a/x)/x^2
```

Fricas [A]
time = 1.41, size = 47, normalized size = 0.92

$$\frac{ax\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}-\left(2\,a^{2}-x^{2}\right)\arccos\left(\frac{a}{x}\right)}{4\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="fricas")
[Out] 1/4*(a*x*sqrt(-(a^2 - x^2)/x^2) - (2*a^2 - x^2)*arccos(a/x))/(a^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 100, normalized size = 1.96

$$\frac{a\left(\begin{cases}\frac{i\sqrt{\frac{a^{2}}{x^{2}}-1}}{2a^{2}x}+\frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^{3}} & \text{for}\ \left|\frac{a^{2}}{x^{2}}\right|>1\\-\frac{1}{2x^{3}\sqrt{-\frac{a^{2}}{x^{2}}+1}}+\frac{1}{2a^{2}x\sqrt{-\frac{a^{2}}{x^{2}}+1}}-\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{2a^{3}} & \text{otherwise}\end{cases}\right)}{2}-\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**3,x)
```

```
[Out] a*Piecewise((I*sqrt(a**2/x**2 - 1)/(2*a**2*x) + I*acosh(a/x)/(2*a**3), Abs(
a**2/x**2) > 1), (-1/(2*x**3*sqrt(-a**2/x**2 + 1)) + 1/(2*a**2*x*sqrt(-a**2
/x**2 + 1)) - asin(a/x)/(2*a**3), True))/2 - acos(a/x)/(2*x**2)
```

Giac [A]
time = 0.44, size = 44, normalized size = 0.86

$$\frac{\frac{\arccos\left(\frac{a}{x}\right)}{a} - \frac{2\,a\arccos\left(\frac{a}{x}\right)}{x^2} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{x}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(arccos(a/x)/a - 2*a*arccos(a/x)/x^2 + sqrt(-a^2/x^2 + 1)/x)/a
```

Mupad [B]
time = 0.28, size = 42, normalized size = 0.82

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{4\,a\,x} - \frac{\mathrm{acos}\left(\frac{a}{x}\right)\,\left(\frac{2\,a^2}{x^2}-1\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a/x)/x^3,x)
```

```
[Out] (1 - a^2/x^2)^(1/2)/(4*a*x) - (acos(a/x)*((2*a^2)/x^2 - 1))/(4*a^2)
```

3.59 $\int \frac{\text{ArcCos}\left(\frac{a}{x}\right)}{x^4} dx$

Optimal. Leaf size=56

$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3}-\frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] -1/9*(1-a^2/x^2)^(3/2)/a^3-1/3*arcsec(x/a)/x^3+1/3*(1-a^2/x^2)^(1/2)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4917, 5328, 272, 45}

$$-\frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3}+\frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a/x]/x^4,x]
```

```
[Out] Sqrt[1 - a^2/x^2]/(3*a^3) - (1 - a^2/x^2)^(3/2)/(9*a^3) - ArcSec[x/a]/(3*x^
3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5328

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Dist[b*(d/(c*(m + 1
```

```
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx &= \int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}+\frac{1}{3}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}\,x^5}\,dx\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}-\frac{1}{6}a\text{Subst}\left(\int \frac{x}{\sqrt{1-a^2x}}\,dx,x,\frac{1}{x^2}\right)\\
&= -\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}-\frac{1}{6}a\text{Subst}\left(\int \left(\frac{1}{a^2\sqrt{1-a^2x}}-\frac{\sqrt{1-a^2x}}{a^2}\right)dx,x,\frac{1}{x^2}\right)\\
&= \frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3}-\frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.84

$$\frac{\sqrt{1-\frac{a^2}{x^2}}\,x(a^2+2x^2)-3a^3\text{ArcCos}\left(\frac{a}{x}\right)}{9a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a/x]/x^4,x]
[Out] (Sqrt[1 - a^2/x^2]*x*(a^2 + 2*x^2) - 3*a^3*ArcCos[a/x])/(9*a^3*x^3)
```

Maple [A]
time = 0.01, size = 55, normalized size = 0.98

method	result	size
derivativedivides	$-\frac{\frac{a^3\arccos\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1-\frac{a^2}{x^2}}}{9x^2}-\frac{2\sqrt{1-\frac{a^2}{x^2}}}{9}}{a^3}$	55
default	$-\frac{\frac{a^3\arccos\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1-\frac{a^2}{x^2}}}{9x^2}-\frac{2\sqrt{1-\frac{a^2}{x^2}}}{9}}{a^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(1/3*a^3/x^3*arccos(a/x)-1/9*a^2/x^2*(1-a^2/x^2)^(1/2)-2/9*(1-a^2/x^
2)^(1/2))
```

Maxima [A]
time = 0.46, size = 49, normalized size = 0.88

$$-\frac{1}{9}\,a\left(\frac{\left(-\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\sqrt{-\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\arccos\left(\frac{a}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/9*a*((-a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(-a^2/x^2 + 1)/a^4) - 1/3*arccos(a
/x)/x^3
```

Fricas [A]
time = 3.13, size = 49, normalized size = 0.88

$$-\frac{3\,a^3\arccos\left(\frac{a}{x}\right)-\left(a^2x+2\,x^3\right)\sqrt{-\frac{a^2-x^2}{x^2}}}{9\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*a^3*arccos(a/x) - (a^2*x + 2*x^3)*sqrt(-(a^2 - x^2)/x^2))/(a^3*x^3)
```

Sympy [A]
time = 1.91, size = 100, normalized size = 1.79

$$\frac{a\left(\begin{cases}\frac{\sqrt{-1+\frac{x^2}{a^2}}}{3ax^3}+\frac{2\sqrt{-1+\frac{x^2}{a^2}}}{3a^3x} & \text{for}\ \left|\frac{x^2}{a^2}\right|>1\\ \frac{i\sqrt{1-\frac{x^2}{a^2}}}{3ax^3}+\frac{2i\sqrt{1-\frac{x^2}{a^2}}}{3a^3x} & \text{otherwise}\end{cases}\right)}{3}-\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a/x)/x**4,x)
```

```
[Out] a*Piecewise((sqrt(-1 + x**2/a**2)/(3*a*x**3) + 2*sqrt(-1 + x**2/a**2)/(3*a*
*3*x), Abs(x**2/a**2) > 1), (I*sqrt(1 - x**2/a**2)/(3*a*x**3) + 2*I*sqrt(1
- x**2/a**2)/(3*a**3*x), True))/3 - acos(a/x)/(3*x**3)
```

Giac [A]
time = 0.44, size = 52, normalized size = 0.93

$$-\frac{\frac{3\,a\arccos\left(\frac{a}{x}\right)}{x^3}-\frac{2\sqrt{-\frac{a^2}{x^2}+1}}{a^2}-\frac{\sqrt{-\frac{a^2}{x^2}+1}}{x^2}}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a/x)/x^4,x, algorithm="giac")
[Out] -1/9*(3*a*arccos(a/x)/x^3 - 2*sqrt(-a^2/x^2 + 1)/a^2 - sqrt(-a^2/x^2 + 1)/x
^2)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a/x)/x^4,x)
[Out] int(acos(a/x)/x^4, x)
```

3.60 $\int x^2\textbf{ArcCos}(\sqrt{x})\,dx$

Optimal. Leaf size=78

$$-\frac{5}{48}\sqrt{1-x}\,\sqrt{x}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\text{ArcCos}(\sqrt{x})-\frac{5}{96}\text{ArcSin}(1-2x)$$

```
[Out] 1/3*x^3*arccos(x^(1/2))+5/96*arcsin(-1+2*x)-5/72*x^(3/2)*(1-x)^(1/2)-1/18*x
^(5/2)*(1-x)^(1/2)-5/48*(1-x)^(1/2)*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4927, 12, 52, 55, 633, 222}

$$\frac{1}{3}x^3\text{ArcCos}(\sqrt{x})-\frac{5}{96}\text{ArcSin}(1-2x)-\frac{1}{18}\sqrt{1-x}\,x^{5/2}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{5}{48}\sqrt{1-x}\,\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[x])/48 - (5*Sqrt[1 - x]*x^(3/2))/72 - (Sqrt[1 - x]*x^(
5/2))/18 + (x^3*ArcCos[Sqrt[x]])/3 - (5*ArcSin[1 - 2*x])/96
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1-x}}\,dx\\
&= \frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{36}\int \frac{x^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{48}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{48}\sqrt{1-x}\,\sqrt{x}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{96}\int\\
&= -\frac{5}{48}\sqrt{1-x}\,\sqrt{x}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)+\frac{5}{96}\int\\
&= -\frac{5}{48}\sqrt{1-x}\,\sqrt{x}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{96}\text{Sub}\\
&= -\frac{5}{48}\sqrt{1-x}\,\sqrt{x}-\frac{5}{72}\sqrt{1-x}\,x^{3/2}-\frac{1}{18}\sqrt{1-x}\,x^{5/2}+\frac{1}{3}x^3\cos^{-1}\left(\sqrt{x}\right)-\frac{5}{96}\sin
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.59

$$\frac{1}{144}\left(-\sqrt{-((-1+x)x)}\left(15+10x+8x^2\right)+48x^3\text{ArcCos}\left(\sqrt{x}\right)+15\text{ArcSin}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-((-1 + x)*x)]*(15 + 10*x + 8*x^2)) + 48*x^3*ArcCos[Sqrt[x]] + 15*A
rcSin[Sqrt[x]])/144
```

Maple [A]
time = 0.01, size = 53, normalized size = 0.68

method	result	size
derivativedivides	$\frac{x^{3} \arccos\left(\sqrt{x}\right)}{3}-\frac{x^{\frac{5}{2}} \sqrt{1-x}}{18}-\frac{5 x^{\frac{3}{2}} \sqrt{1-x}}{72}-\frac{5 \sqrt{1-x}\, \sqrt{x}}{48}+\frac{5 \arcsin\left(\sqrt{x}\right)}{48}$	53
default	$\frac{x^{3} \arccos\left(\sqrt{x}\right)}{3}-\frac{x^{\frac{5}{2}} \sqrt{1-x}}{18}-\frac{5 x^{\frac{3}{2}} \sqrt{1-x}}{72}-\frac{5 \sqrt{1-x}\, \sqrt{x}}{48}+\frac{5 \arcsin\left(\sqrt{x}\right)}{48}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccos(x^(1/2))-1/18*x^(5/2)*(1-x)^(1/2)-5/72*x^(3/2)*(1-x)^(1/2)-5
/48*(1-x)^(1/2)*x^(1/2)+5/48*arcsin(x^(1/2))
```

Maxima [A]
time = 0.48, size = 52, normalized size = 0.67

$$\frac{1}{3} x^{3} \arccos\left(\sqrt{x}\right) - \frac{1}{18} x^{\frac{5}{2}} \sqrt{-x+1} - \frac{5}{72} x^{\frac{3}{2}} \sqrt{-x+1} - \frac{5}{48} \sqrt{x} \sqrt{-x+1} + \frac{5}{48} \arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
+ 1) - 5/48*sqrt(x)*sqrt(-x + 1) + 5/48*arcsin(sqrt(x))
```

Fricas [A]
time = 1.47, size = 36, normalized size = 0.46

$$-\frac{1}{144} \left(8 x^{2} + 10 x + 15\right) \sqrt{x} \sqrt{-x+1} + \frac{1}{48} \left(16 x^{3} - 5\right) \arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(-x + 1) + 1/48*(16*x^3 - 5)*arccos(
sqrt(x))
```

Sympy [A]
time = 4.86, size = 82, normalized size = 1.05

$$\frac{x^{3} \operatorname{acos}\left(\sqrt{x}\right)}{3} + \frac{\begin{cases} \frac{x^{\frac{3}{2}} (1-x)^{\frac{3}{2}}}{6} + \frac{3 \sqrt{x}\,(1-2x) \sqrt{1-x}}{16} - \frac{\sqrt{x} \sqrt{1-x}}{2} + \frac{5 \operatorname{asin}\left(\sqrt{x}\right)}{16} & \text{for}\ \sqrt{x} > -1 \wedge \sqrt{x} < 1 \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(x**(1/2)),x)
```

```
[Out] x**3*acos(sqrt(x))/3 + Piecewise((x**(3/2)*(1 - x)**(3/2)/6 + 3*sqrt(x)*(1
- 2*x)*sqrt(1 - x)/16 - sqrt(x)*sqrt(1 - x)/2 + 5*asin(sqrt(x))/16, (sqrt(x
) > -1) & (sqrt(x) < 1)))/3
```

Giac [A]
time = 0.41, size = 52, normalized size = 0.67

$$\frac{1}{3}\,x^3\arccos\left(\sqrt{x}\right)-\frac{1}{18}\,x^{\frac{5}{2}}\sqrt{-x+1}-\frac{5}{72}\,x^{\frac{3}{2}}\sqrt{-x+1}-\frac{5}{48}\,\sqrt{x}\,\sqrt{-x+1}-\frac{5}{48}\,\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
+ 1) - 5/48*sqrt(x)*sqrt(-x + 1) - 5/48*arccos(sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acos}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(x^(1/2)),x)
```

```
[Out] int(x^2*acos(x^(1/2)), x)
```

3.61 $\int x\text{ArcCos}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=60

$$-\frac{3}{16}\sqrt{1-x}\,\sqrt{x}-\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\text{ArcCos}\left(\sqrt{x}\right)-\frac{3}{32}\text{ArcSin}(1-2x)$$

```
[Out] 1/2*x^2*arccos(x^(1/2))+3/32*arcsin(-1+2*x)-1/8*x^(3/2)*(1-x)^(1/2)-3/16*(1
-x)^(1/2)*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4927, 12, 52, 55, 633, 222}

$$\frac{1}{2}x^2\text{ArcCos}\left(\sqrt{x}\right)-\frac{3}{32}\text{ArcSin}(1-2x)-\frac{1}{8}\sqrt{1-x}\,x^{3/2}-\frac{3}{16}\sqrt{1-x}\,\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCos[Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[x])/16 - (Sqrt[1 - x]*x^(3/2))/8 + (x^2*ArcCos[Sqrt[x]
])/2 - (3*ArcSin[1 - 2*x])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int x\cos^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{1-x}}\,dx\\
&= \frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{1-x}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\,\sqrt{x}-\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{32}\int\frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\,\sqrt{x}-\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)+\frac{3}{32}\int\frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\frac{3}{16}\sqrt{1-x}\,\sqrt{x}-\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,\right.\\
&= -\frac{3}{16}\sqrt{1-x}\,\sqrt{x}-\frac{1}{8}\sqrt{1-x}\,x^{3/2}+\frac{1}{2}x^2\cos^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.68

$$\frac{1}{16}\left(-\sqrt{-((-1+x)x)}\,(3+2x)+8x^2\text{ArcCos}\left(\sqrt{x}\right)+3\text{ArcSin}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCos[Sqrt[x]],x]
[Out] (-(Sqrt[-((-1 + x)*x)]*(3 + 2*x)) + 8*x^2*ArcCos[Sqrt[x]] + 3*ArcSin[Sqrt[x
]])/16
```

Maple [A]
time = 0.01, size = 41, normalized size = 0.68

method	result	size
derivativedivides	$\frac{x^2 \arccos\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1-x}}{8}-\frac{3\sqrt{1-x}\,\sqrt{x}}{16}+\frac{3\arcsin\left(\sqrt{x}\right)}{16}$	41
default	$\frac{x^2 \arccos\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1-x}}{8}-\frac{3\sqrt{1-x}\,\sqrt{x}}{16}+\frac{3\arcsin\left(\sqrt{x}\right)}{16}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccos(x^(1/2))-1/8*x^(3/2)*(1-x)^(1/2)-3/16*(1-x)^(1/2)*x^(1/2)+3/
16*arcsin(x^(1/2))
```

Maxima [A]
time = 0.47, size = 40, normalized size = 0.67

$$\frac{1}{2}\,x^2\arccos\left(\sqrt{x}\right)-\frac{1}{8}\,x^{\frac{3}{2}}\sqrt{-x+1}-\frac{3}{16}\,\sqrt{x}\,\sqrt{-x+1}+\frac{3}{16}\,\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x +
 1) + 3/16*arcsin(sqrt(x))
```

Fricas [A]
time = 1.64, size = 31, normalized size = 0.52

$$-\frac{1}{16}\,(2\,x+3)\sqrt{x}\,\sqrt{-x+1}+\frac{1}{16}\,\left(8\,x^2-3\right)\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x + 3)*sqrt(x)*sqrt(-x + 1) + 1/16*(8*x^2 - 3)*arccos(sqrt(x))
```

Sympy [A]
time = 2.14, size = 66, normalized size = 1.10

$$\frac{x^2\operatorname{acos}\left(\sqrt{x}\right)}{2}+\frac{\begin{cases}\frac{\sqrt{x}\,(1-2x)\sqrt{1-x}}{8}-\frac{\sqrt{x}\,\sqrt{1-x}}{2}+\frac{3\operatorname{asin}\left(\sqrt{x}\right)}{8} & \text{for}\ \sqrt{x}>-1\wedge\sqrt{x}<1\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x**(1/2)),x)
```

```
[Out] x**2*acos(sqrt(x))/2 + Piecewise((sqrt(x)*(1 - 2*x)*sqrt(1 - x)/8 - sqrt(x)
*sqrt(1 - x)/2 + 3*asin(sqrt(x))/8, (sqrt(x) > -1) & (sqrt(x) < 1)))/2
```

Giac [A]
time = 0.44, size = 40, normalized size = 0.67

$$\frac{1}{2}\,x^{2}\arccos\left(\sqrt{x}\right)-\frac{1}{8}\,x^{\frac{3}{2}}\sqrt{-x+1}-\frac{3}{16}\,\sqrt{x}\,\sqrt{-x+1}-\frac{3}{16}\,\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x +
 1) - 3/16*arccos(sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{acos}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acos(x^(1/2)),x)
```

```
[Out] int(x*acos(x^(1/2)), x)
```

3.62 $\int \text{ArcCos}(\sqrt{x})\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{1-x}\,\sqrt{x}+x\text{ArcCos}(\sqrt{x})-\frac{1}{4}\text{ArcSin}(1-2x)$$

```
[Out] x*arccos(x^(1/2))+1/4*arcsin(-1+2*x)-1/2*(1-x)^(1/2)*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4925, 12, 52, 55, 633, 222}

$$x\text{ArcCos}(\sqrt{x})-\frac{1}{4}\text{ArcSin}(1-2x)-\frac{1}{2}\sqrt{1-x}\,\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]],x]
```

```
[Out] -1/2*(Sqrt[1 - x]*Sqrt[x]) + x*ArcCos[Sqrt[x]] - ArcSin[1 - 2*x]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\sqrt{x}\right)\,dx &= x\cos^{-1}\left(\sqrt{x}\right)+\int \frac{\sqrt{x}}{2\sqrt{1-x}}\,dx\\
&= x\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx\\
&= -\frac{1}{2}\sqrt{1-x}\,\sqrt{x}+x\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\
&= -\frac{1}{2}\sqrt{1-x}\,\sqrt{x}+x\cos^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\frac{1}{2}\sqrt{1-x}\,\sqrt{x}+x\cos^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\
&= -\frac{1}{2}\sqrt{1-x}\,\sqrt{x}+x\cos^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.16

$$-\frac{1}{2}\sqrt{-((-1+x)x)}+x\text{ArcCos}\left(\sqrt{x}\right)+\text{ArcTan}\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]],x]
```

```
[Out] -1/2*Sqrt[-((-1 + x)*x)] + x*ArcCos[Sqrt[x]] + ArcTan[Sqrt[x]/(-1 + Sqrt[1
- x])]
```

Maple [A]
time = 0.01, size = 26, normalized size = 0.70

method	result	size
derivativedivides	$x \arccos\left(\sqrt{x}\right)-\frac{\sqrt{1-x}\,\sqrt{x}}{2}+\frac{\arcsin\left(\sqrt{x}\right)}{2}$	26
default	$x \arccos\left(\sqrt{x}\right)-\frac{\sqrt{1-x}\,\sqrt{x}}{2}+\frac{\arcsin\left(\sqrt{x}\right)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccos(x^(1/2))-1/2*(1-x)^(1/2)*x^(1/2)+1/2*arcsin(x^(1/2))
```

Maxima [A]
time = 0.47, size = 25, normalized size = 0.68

$$x \arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\,\sqrt{-x+1}+\frac{1}{2}\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="maxima")
[Out] x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) + 1/2*arcsin(sqrt(x))
```

Fricas [A]
time = 3.44, size = 24, normalized size = 0.65

$$\frac{1}{2}(2x-1)\arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x - 1)*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1)
```

Sympy [A]
time = 0.10, size = 29, normalized size = 0.78

$$-\frac{\sqrt{x}\,\sqrt{1-x}}{2}+x \operatorname{acos}\left(\sqrt{x}\right)-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2)),x)
[Out] -sqrt(x)*sqrt(1 - x)/2 + x*acos(sqrt(x)) - acos(sqrt(x))/2
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.68

$$x \arccos\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x}\,\sqrt{-x+1}-\frac{1}{2}\arccos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) - 1/2*arccos(sqrt(x))
```

Mupad [B]
time = 0.64, size = 35, normalized size = 0.95

$$\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)+x\operatorname{acos}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2)),x)
```

```
[Out] atan(x^(1/2)/((1 - x)^(1/2) - 1)) + x*acos(x^(1/2)) - (x^(1/2)*(1 - x)^(1/2
))/2
```

3.63 $\int \frac{\mathbf{ArcCos}(\sqrt{x})}{x} \, dx$

Optimal. Leaf size=56

$$-i\text{ArcCos}(\sqrt{x})^2 + 2\text{ArcCos}(\sqrt{x}) \log\left(1 + e^{2i\text{ArcCos}(\sqrt{x})}\right) - i\text{PolyLog}\left(2, -e^{2i\text{ArcCos}(\sqrt{x})}\right)$$

```
[Out] -I*arccos(x^(1/2))^2+2*arccos(x^(1/2))*ln(1+(x^(1/2)+I*(1-x)^(1/2))^2)-I*po
lylog(2,-(x^(1/2)+I*(1-x)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3800, 2221, 2317, 2438}

$$-i\text{Li}_2\left(-e^{2i\text{ArcCos}(\sqrt{x})}\right) - i\text{ArcCos}(\sqrt{x})^2 + 2\text{ArcCos}(\sqrt{x}) \log\left(1 + e^{2i\text{ArcCos}(\sqrt{x})}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x,x]
```

```
[Out] (-I)*ArcCos[Sqrt[x]]^2 + 2*ArcCos[Sqrt[x]]*Log[1 + E^((2*I)*ArcCos[Sqrt[x]]
)] - I*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4915

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[-p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x}\,dx &= -\left(2\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(\sqrt{x}\right)\right)\right)\\ &= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 4i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\sqrt{x}\right)\right)\\ &= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) - 2\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,d\right.\\ &= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) + i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.\\ &= -i\cos^{-1}\left(\sqrt{x}\right)^2 + 2\cos^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right) - i\text{Li}_2\left(-e^{2i\cos^{-1}\left(\sqrt{x}\right)}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.96

$$-i\left(\text{ArcCos}\left(\sqrt{x}\right)\left(\text{ArcCos}\left(\sqrt{x}\right)+2i\log\left(1+e^{2i\text{ArcCos}\left(\sqrt{x}\right)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\text{ArcCos}\left(\sqrt{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x,x]
```

```
[Out] (-I)*(ArcCos[Sqrt[x]]*(ArcCos[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcCos[Sqrt
[x]])]) + PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])])
```

Maple [A]
time = 0.38, size = 59, normalized size = 1.05

method	result
derivativedivides	$-i\arccos\left(\sqrt{x}\right)^2+2\arccos\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+i\sqrt{1-x}\right)^2\right)-i\,\text{polylog}\left(2,-\left(\sqrt{x}\right.\right.$
default	$-i\arccos\left(\sqrt{x}\right)^2+2\arccos\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+i\sqrt{1-x}\right)^2\right)-i\,\text{polylog}\left(2,-\left(\sqrt{x}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -I*arccos(x^(1/2))^2+2*arccos(x^(1/2))*ln(1+(x^(1/2)+I*(1-x)^(1/2))^2)-I*po
lylog(2,-(x^(1/2)+I*(1-x)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(sqrt(x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x,x)
```

```
[Out] Integral(acos(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(sqrt(x))/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x,x)
```

```
[Out] int(acos(x^(1/2))/x, x)
```

3.64 $\int \frac{\mathbf{ArcCos}(\sqrt{x})}{x^2} \, dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\text{ArcCos}(\sqrt{x})}{x}$$

```
[Out] -arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4927, 12, 37}

$$\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\text{ArcCos}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^2,x]
```

```
[Out] Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{x} - \int \frac{1}{2\sqrt{1-x}\,x^{3/2}}\,dx\\ &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx\\ &= \frac{\sqrt{1-x}}{\sqrt{x}} - \frac{\cos^{-1}\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 24, normalized size = 0.89

$$\frac{\sqrt{x-x^2} - \text{ArcCos}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^2,x]
[Out] (Sqrt[x - x^2] - ArcCos[Sqrt[x]])/x
```

Maple [**A**]
time = 0.00, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1-x}}{\sqrt{x}}$	22
default	$-\frac{\arccos\left(\sqrt{x}\right)}{x}+\frac{\sqrt{1-x}}{\sqrt{x}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)
```

Maxima [**A**]
time = 0.47, size = 21, normalized size = 0.78

$$\frac{\sqrt{-x+1}}{\sqrt{x}} - \frac{\arccos\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x + 1)/sqrt(x) - arccos(sqrt(x))/x
```

Fricas [A]
time = 0.96, size = 22, normalized size = 0.81

$$\frac{\sqrt{x}\,\sqrt{-x+1}-\arccos\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="fricas")
[Out] (sqrt(x)*sqrt(-x + 1) - arccos(sqrt(x)))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 1.86, size = 44, normalized size = 1.63

$$-\frac{\begin{cases} -\frac{2i\sqrt{x-1}}{\sqrt{x}} & \text{for}\ |x|>1 \\ -\frac{2\sqrt{1-x}}{\sqrt{x}} & \text{otherwise} \end{cases}}{2}-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**2,x)
[Out] -Piecewise((-2*I*sqrt(x - 1)/sqrt(x), Abs(x) > 1), (-2*sqrt(1 - x)/sqrt(x),
 True))/2 - acos(sqrt(x))/x
```

Giac [A]
time = 0.41, size = 40, normalized size = 1.48

$$\frac{\sqrt{-x+1}-1}{2\sqrt{x}}-\frac{\arccos\left(\sqrt{x}\right)}{x}-\frac{\sqrt{x}}{2\left(\sqrt{-x+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^2,x, algorithm="giac")
[Out] 1/2*(sqrt(-x + 1) - 1)/sqrt(x) - arccos(sqrt(x))/x - 1/2*sqrt(x)/(sqrt(-x +
 1) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x^2,x)
[Out] int(acos(x^(1/2))/x^2, x)
```

3.65 $\int \frac{\text{ArcCos}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{1-x}}{6x^{3/2}}+\frac{\sqrt{1-x}}{3\sqrt{x}}-\frac{\text{ArcCos}\left(\sqrt{x}\right)}{2x^2}$$

```
[Out] -1/2*arccos(x^(1/2))/x^2+1/6*(1-x)^(1/2)/x^(3/2)+1/3*(1-x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4927, 12, 47, 37}

$$-\frac{\text{ArcCos}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{1-x}}{6x^{3/2}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^3,x]
```

```
[Out] Sqrt[1 - x]/(6*x^(3/2)) + Sqrt[1 - x]/(3*Sqrt[x]) - ArcCos[Sqrt[x]]/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2}\int \frac{1}{2\sqrt{1-x}\,x^{5/2}}\,dx\\
&= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx\\
&= \frac{\sqrt{1-x}}{6x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{6}\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx\\
&= \frac{\sqrt{1-x}}{6x^{3/2}}+\frac{\sqrt{1-x}}{3\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 0.86

$$\left(\frac{1}{6x^{3/2}}+\frac{1}{3\sqrt{x}}\right)\sqrt{1-x}-\frac{\text{ArcCos}\left(\sqrt{x}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^3,x]
[Out] (1/(6*x^(3/2)) + 1/(3*Sqrt[x]))*Sqrt[1 - x] - ArcCos[Sqrt[x]]/(2*x^2)
```

Maple [**A**]
time = 0.01, size = 35, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{1-x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$	35
default	$-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{1-x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccos(x^(1/2))/x^2+1/6*(1-x)^(1/2)/x^(3/2)+1/3*(1-x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.47, size = 34, normalized size = 0.68

$$\frac{\sqrt{-x+1}}{3\sqrt{x}}+\frac{\sqrt{-x+1}}{6\,x^{\frac{3}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x + 1)/sqrt(x) + 1/6*sqrt(-x + 1)/x^(3/2) - 1/2*arccos(sqrt(x))/x
^2
```

Fricas [A]
time = 1.49, size = 28, normalized size = 0.56

$$\frac{(2\,x+1)\sqrt{x}\,\sqrt{-x+1}-3\arccos\left(\sqrt{x}\right)}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*((2*x + 1)*sqrt(x)*sqrt(-x + 1) - 3*arccos(sqrt(x)))/x^2
```

Sympy [A]
time = 4.66, size = 53, normalized size = 1.06

$$-\frac{\begin{cases}-\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} & \text{for}\ \sqrt{x}>-1\wedge\sqrt{x}<1\end{cases}}{2}-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**3,x)
```

```
[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)**(3/2)/(3*x**(3/2)), (sqrt(x) >
-1) & (sqrt(x) < 1)))/2 - acos(sqrt(x))/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.45, size = 74, normalized size = 1.48

$$\frac{\left(\sqrt{-x+1}-1\right)^{3}}{48\,x^{\frac{3}{2}}}+\frac{3\left(\sqrt{-x+1}-1\right)}{16\sqrt{x}}-\frac{x^{\frac{3}{2}}\left(\frac{9\left(\sqrt{-x+1}-1\right)^{2}}{x}+1\right)}{48\left(\sqrt{-x+1}-1\right)^{3}}-\frac{\arccos\left(\sqrt{x}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(-x + 1) - 1)^3/x^(3/2) + 3/16*(sqrt(-x + 1) - 1)/sqrt(x) - 1/48*
x^(3/2)*(9*(sqrt(-x + 1) - 1)^2/x + 1)/(sqrt(-x + 1) - 1)^3 - 1/2*arccos(sq
rt(x))/x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x^3,x)
```

```
[Out] int(acos(x^(1/2))/x^3, x)
```

3.66 $\int \frac{\text{ArcCos}(\sqrt{x})}{x^4} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{1-x}}{15x^{5/2}}+\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}-\frac{\text{ArcCos}(\sqrt{x})}{3x^3}$$

```
[Out] -1/3*arccos(x^(1/2))/x^3+1/15*(1-x)^(1/2)/x^(5/2)+4/45*(1-x)^(1/2)/x^(3/2)+
8/45*(1-x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4927, 12, 47, 37}

$$-\frac{\text{ArcCos}(\sqrt{x})}{3x^3}+\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{\sqrt{1-x}}{15x^{5/2}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^4,x]
```

```
[Out] Sqrt[1 - x]/(15*x^(5/2)) + (4*Sqrt[1 - x])/(45*x^(3/2)) + (8*Sqrt[1 - x])/(
45*Sqrt[x]) - ArcCos[Sqrt[x]]/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{1}{3}\int \frac{1}{2\sqrt{1-x}\,x^{7/2}}\,dx \\
&= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{1}{6}\int \frac{1}{\sqrt{1-x}\,x^{7/2}}\,dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{2}{15}\int \frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}}+\frac{4\sqrt{1-x}}{45x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{4}{45}\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx \\
&= \frac{\sqrt{1-x}}{15x^{5/2}}+\frac{4\sqrt{1-x}}{45x^{3/2}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.54

$$\frac{\sqrt{-((-1+x)x)}\,(3+4x+8x^2)-15\text{ArcCos}\left(\sqrt{x}\right)}{45x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^4,x]
[Out] (Sqrt[-((-1 + x)*x)]*(3 + 4*x + 8*x^2) - 15*ArcCos[Sqrt[x]])/(45*x^3)
```

Maple [A]
time = 0.00, size = 47, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}+\frac{\sqrt{1-x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1-x}}{45x^{\frac{3}{2}}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$	47

default	$-\frac{\arccos\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1-x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1-x}}{45x^{\frac{3}{2}}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccos(x^(1/2))/x^3+1/15*(1-x)^(1/2)/x^(5/2)+4/45*(1-x)^(1/2)/x^(3/2)+
8/45*(1-x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.48, size = 46, normalized size = 0.68

$$\frac{8\sqrt{-x+1}}{45\sqrt{x}}+\frac{4\sqrt{-x+1}}{45\,x^{\frac{3}{2}}}+\frac{\sqrt{-x+1}}{15\,x^{\frac{5}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] 8/45*sqrt(-x + 1)/sqrt(x) + 4/45*sqrt(-x + 1)/x^(3/2) + 1/15*sqrt(-x + 1)/x
^(5/2) - 1/3*arccos(sqrt(x))/x^3
```

Fricas [A]
time = 1.11, size = 33, normalized size = 0.49

$$\frac{(8\,x^{2}+4\,x+3)\sqrt{x}\,\sqrt{-x+1}\,-15\,\arccos\left(\sqrt{x}\right)}{45\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/45*((8*x^2 + 4*x + 3)*sqrt(x)*sqrt(-x + 1) - 15*arccos(sqrt(x)))/x^3
```

Sympy [A]
time = 11.75, size = 68, normalized size = 1.00

$$-\frac{\begin{cases}-\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{2(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}-\frac{(1-x)^{\frac{5}{2}}}{5x^{\frac{5}{2}}} & \text{for}\ \sqrt{x}>-1\wedge\sqrt{x}<1\end{cases}}{3}-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**4,x)
```

```
[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - 2*(1 - x)**(3/2)/(3*x**(3/2)) - (1 - x)*
*(5/2)/(5*x**(5/2)), (sqrt(x) > -1) & (sqrt(x) < 1)))/3 - acos(sqrt(x))/(3*
x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.
time = 0.43, size = 106, normalized size = 1.56

$$\frac{\left(\sqrt{-x+1}-1\right)^{5}}{480\,x^{\frac{5}{2}}}+\frac{5\left(\sqrt{-x+1}-1\right)^{3}}{288\,x^{\frac{3}{2}}}+\frac{5\left(\sqrt{-x+1}-1\right)}{48\,\sqrt{x}}-\frac{\left(\frac{150\left(\sqrt{-x+1}-1\right)^{4}}{x^{2}}+\frac{25\left(\sqrt{-x+1}-1\right)^{2}}{x}+3\right)x^{\frac{5}{2}}}{1440\left(\sqrt{-x+1}-1\right)^{5}}-\frac{\arccos\left(\sqrt{x}\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] 1/480*(sqrt(-x + 1) - 1)^5/x^(5/2) + 5/288*(sqrt(-x + 1) - 1)^3/x^(3/2) + 5
/48*(sqrt(-x + 1) - 1)/sqrt(x) - 1/1440*(150*(sqrt(-x + 1) - 1)^4/x^2 + 25*
(sqrt(-x + 1) - 1)^2/x + 3)*x^(5/2)/(sqrt(-x + 1) - 1)^5 - 1/3*arccos(sqrt(
x))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x^4,x)
```

```
[Out] int(acos(x^(1/2))/x^4, x)
```

3.67 $\int \frac{\mathbf{ArcCos}\left(\sqrt{x}\right)}{x^5}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}-\frac{\operatorname{ArcCos}\left(\sqrt{x}\right)}{4x^4}$$

```
[Out] -1/4*arccos(x^(1/2))/x^4+1/28*(1-x)^(1/2)/x^(7/2)+3/70*(1-x)^(1/2)/x^(5/2)+
2/35*(1-x)^(1/2)/x^(3/2)+4/35*(1-x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4927, 12, 47, 37}

$$-\frac{\operatorname{ArcCos}\left(\sqrt{x}\right)}{4x^4}+\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{\sqrt{1-x}}{28x^{7/2}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x]]/x^5,x]
```

```
[Out] Sqrt[1 - x]/(28*x^(7/2)) + (3*Sqrt[1 - x])/(70*x^(5/2)) + (2*Sqrt[1 - x])/(
35*x^(3/2)) + (4*Sqrt[1 - x])/(35*Sqrt[x]) - ArcCos[Sqrt[x]]/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 4927

```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{x^5}\,dx &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{1}{4}\int \frac{1}{2\sqrt{1-x}\,x^{9/2}}\,dx\\ &= -\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{1}{8}\int \frac{1}{\sqrt{1-x}\,x^{9/2}}\,dx\\ &= \frac{\sqrt{1-x}}{28x^{7/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{28}\int \frac{1}{\sqrt{1-x}\,x^{7/2}}\,dx\\ &= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{35}\int \frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx\\ &= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{2\sqrt{1-x}}{35x^{3/2}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{2}{35}\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx\\ &= \frac{\sqrt{1-x}}{28x^{7/2}}+\frac{3\sqrt{1-x}}{70x^{5/2}}+\frac{2\sqrt{1-x}}{35x^{3/2}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}-\frac{\cos^{-1}\left(\sqrt{x}\right)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.49

$$\frac{\sqrt{-((-1+x)x)}\,(5+6x+8x^2+16x^3)-35\text{ArcCos}\left(\sqrt{x}\right)}{140x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[-((-1 + x)*x)]*(5 + 6*x + 8*x^2 + 16*x^3) - 35*ArcCos[Sqrt[x]])/(140*
x^4)
```

Maple [A]
time = 0.01, size = 59, normalized size = 0.69

method	result	size

derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}+\frac{\sqrt{1-x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1-x}}{70x^{\frac{5}{2}}}+\frac{2\sqrt{1-x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$	59
default	$-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}+\frac{\sqrt{1-x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1-x}}{70x^{\frac{5}{2}}}+\frac{2\sqrt{1-x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arccos(x^(1/2))/x^4+1/28*(1-x)^(1/2)/x^(7/2)+3/70*(1-x)^(1/2)/x^(5/2)+
2/35*(1-x)^(1/2)/x^(3/2)+4/35*(1-x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.47, size = 58, normalized size = 0.67

$$\frac{4\,\sqrt{-x+1}}{35\,\sqrt{x}}+\frac{2\,\sqrt{-x+1}}{35\,x^{\frac{3}{2}}}+\frac{3\,\sqrt{-x+1}}{70\,x^{\frac{5}{2}}}+\frac{\sqrt{-x+1}}{28\,x^{\frac{7}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 4/35*sqrt(-x + 1)/sqrt(x) + 2/35*sqrt(-x + 1)/x^(3/2) + 3/70*sqrt(-x + 1)/x
^(5/2) + 1/28*sqrt(-x + 1)/x^(7/2) - 1/4*arccos(sqrt(x))/x^4
```

Fricas [A]
time = 1.97, size = 38, normalized size = 0.44

$$\frac{\left(16\,x^3+8\,x^2+6\,x+5\right)\sqrt{x}\,\sqrt{-x+1}\,-35\,\arccos\left(\sqrt{x}\right)}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 + 8*x^2 + 6*x + 5)*sqrt(x)*sqrt(-x + 1) - 35*arccos(sqrt(x))
)/x^4
```

Sympy [A]
time = 32.01, size = 80, normalized size = 0.93

$$-\frac{\begin{cases}-\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{(1-x)^{\frac{3}{2}}}{x^{\frac{3}{2}}}-\frac{3(1-x)^{\frac{5}{2}}}{5x^{\frac{5}{2}}}-\frac{(1-x)^{\frac{7}{2}}}{7x^{\frac{7}{2}}} & \text{for}\ \sqrt{x}>-1\wedge\sqrt{x}<1\end{cases}}{4}-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**5,x)
```

```
[Out] -Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)**(3/2)/x**(3/2) - 3*(1 - x)**(5/
2)/(5*x**(5/2)) - (1 - x)**(7/2)/(7*x**(7/2)), (sqrt(x) > -1) & (sqrt(x) <
1)))/4 - acos(sqrt(x))/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.
time = 0.43, size = 138, normalized size = 1.60

$$\frac{\left(\sqrt{-x+1}-1\right)^{7}}{3584\,x^{\frac{7}{2}}}+\frac{7\left(\sqrt{-x+1}-1\right)^{5}}{2560\,x^{\frac{5}{2}}}+\frac{7\left(\sqrt{-x+1}-1\right)^{3}}{512\,x^{\frac{3}{2}}}+\frac{35\left(\sqrt{-x+1}-1\right)}{512\,\sqrt{x}}-\frac{\left(\frac{1225\left(\sqrt{-x+1}-1\right)^{6}}{x^{3}}+\frac{245\left(\sqrt{-x+1}-1\right)^{4}}{x^{2}}+\frac{49\left(\sqrt{-x+1}-1\right)^{2}}{x}+5\right)x^{\frac{7}{2}}}{17920\left(\sqrt{-x+1}-1\right)^{7}}-\frac{\arccos\left(\sqrt{x}\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/3584*(sqrt(-x + 1) - 1)^7/x^(7/2) + 7/2560*(sqrt(-x + 1) - 1)^5/x^(5/2) +
 7/512*(sqrt(-x + 1) - 1)^3/x^(3/2) + 35/512*(sqrt(-x + 1) - 1)/sqrt(x) - 1
/17920*(1225*(sqrt(-x + 1) - 1)^6/x^3 + 245*(sqrt(-x + 1) - 1)^4/x^2 + 49*(
sqrt(-x + 1) - 1)^2/x + 5)*x^(7/2)/(sqrt(-x + 1) - 1)^7 - 1/4*arccos(sqrt(x
))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x^5,x)
```

```
[Out] int(acos(x^(1/2))/x^5, x)
```

3.68 $\int \frac{\text{ArcCos}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx$

Optimal. Leaf size=25

$$-2\sqrt{1-x}+2\sqrt{x}\,\text{ArcCos}\left(\sqrt{x}\right)$$

[Out] -2*(1-x)^(1/2)+2*arccos(x^(1/2))*x^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6847, 4716, 267}

$$2\sqrt{x}\,\text{ArcCos}\left(\sqrt{x}\right)-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[ArcCos[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx &= 2\text{Subst}\left(\int \cos^{-1}(x) \, dx, x, \sqrt{x}\right)\\
&= 2\sqrt{x}\,\cos^{-1}\left(\sqrt{x}\right)+2\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}} \, dx, x, \sqrt{x}\right)\\
&= -2\sqrt{1-x}+2\sqrt{x}\,\cos^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-2\sqrt{1-x}+2\sqrt{x}\,\text{ArcCos}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x]]/Sqrt[x],x]
[Out] -2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]
```

Maple [A]
time = 0.00, size = 20, normalized size = 0.80

method	result	size
derivativedivides	$-2\sqrt{1-x}+2\arccos\left(\sqrt{x}\right)\sqrt{x}$	20
default	$-2\sqrt{1-x}+2\arccos\left(\sqrt{x}\right)\sqrt{x}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(1-x)^(1/2)+2*arccos(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 0.46, size = 19, normalized size = 0.76

$$2\sqrt{x}\arccos\left(\sqrt{x}\right)-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

Fricas [A]
time = 1.24, size = 19, normalized size = 0.76

$$2\sqrt{x}\arccos\left(\sqrt{x}\right)-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

Sympy [A]
time = 0.10, size = 20, normalized size = 0.80

$$2\sqrt{x}\,\text{acos}\left(\sqrt{x}\right)-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*acos(sqrt(x)) - 2*sqrt(1 - x)
```

Giac [A]
time = 0.42, size = 19, normalized size = 0.76

$$2\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

Mupad [B]
time = 0.40, size = 19, normalized size = 0.76

$$2\sqrt{x}\,\mathrm{acos}\left(\sqrt{x}\right) - 2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*x^(1/2)*acos(x^(1/2)) - 2*(1 - x)^(1/2)
```

3.69 $\int \frac{\mathbf{ArcCos}(ax^n)}{x}\,dx$

Optimal. Leaf size=68

$$-\frac{i\text{ArcCos}(ax^n)^2}{2n}+\frac{\text{ArcCos}(ax^n)\log\left(1+e^{2i\text{ArcCos}(ax^n)}\right)}{n}-\frac{i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(ax^n)}\right)}{2n}$$

```
[Out] -1/2*I*arccos(a*x^n)^2/n+arccos(a*x^n)*ln(1+(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))
^2)/n-1/2*I*polylog(2,-(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))^2)/n
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3800, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-e^{2i\text{ArcCos}(ax^n)}\right)}{2n}-\frac{i\text{ArcCos}(ax^n)^2}{2n}+\frac{\text{ArcCos}(ax^n)\log\left(1+e^{2i\text{ArcCos}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^n]/x,x]
```

```
[Out] ((-1/2*I)*ArcCos[a*x^n]^2)/n + (ArcCos[a*x^n]*Log[1 + E^((2*I)*ArcCos[a*x^n
])])/n - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[a*x^n])])/n
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4915

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[-p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(ax^n)}{x}\,dx &= -\frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax^n)\right)}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax^n)\right)}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \text{co}\right.}{n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} + \frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\cos^{-1}}\right.}{2n}\\
&= -\frac{i\cos^{-1}(ax^n)^2}{2n} + \frac{\cos^{-1}(ax^n)\log\left(1+e^{2i\cos^{-1}(ax^n)}\right)}{n} - \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}(ax^n)}\right)}{2n}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(68) = 136.
time = 0.10, size = 141, normalized size = 2.07

$$\text{ArcCos}(ax^n)\log(x) + \frac{a\left(-\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)^2 - 2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)\log\left(1-e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right) + 2n\log(x)\log\left(\sqrt{-a^2}\,x^n+\sqrt{1-a^2x^{2n}}\right) + \text{PolyLog}\left(2, e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right)\right)}{2\sqrt{-a^2}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^n]/x,x]
```

```
[Out] ArcCos[a*x^n]*Log[x] + (a*(-ArcSinh[Sqrt[-a^2]*x^n]^2 - 2*ArcSinh[Sqrt[-a^2
]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] + 2*n*Log[x]*Log[Sqrt[-a^2]*
x^n + Sqrt[1 - a^2*x^(2*n)]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-a^2]*x^n])]))
/(2*Sqrt[-a^2]*n)
```

Maple [A]
time = 0.56, size = 84, normalized size = 1.24

method	result	si

derivativedivides	$\frac{-\frac{i \arccos(a x^n)^2}{2}+\arccos(a x^n) \ln\left(1+\left(a x^n+i\sqrt{1-a^2 x^{2n}}\right)^2\right)-\frac{i \,\mathrm{polylog}\left(2,-\left(a x^n+i\sqrt{1-a^2 x^{2n}}\right)^2\right)}{2}}{n}$	84
default	$\frac{-\frac{i \arccos(a x^n)^2}{2}+\arccos(a x^n) \ln\left(1+\left(a x^n+i\sqrt{1-a^2 x^{2n}}\right)^2\right)-\frac{i \,\mathrm{polylog}\left(2,-\left(a x^n+i\sqrt{1-a^2 x^{2n}}\right)^2\right)}{2}}{n}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/2*I*arccos(a*x^n)^2+arccos(a*x^n)*ln(1+(a*x^n+I*(1-a^2*(x^n)^2)^(1/
2))^2)-1/2*I*polylog(2,-(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)*x^n*log(x)/(a^2*x*x^(2*n) -
 x), x) + arctan(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)/(a*x^n))*log(x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}\left(ax^{n}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**n)/x,x)
```

```
[Out] Integral(acos(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^n)/x,x, algorithm="giac")
[Out] integrate(arccos(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acos}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a*x^n)/x,x)
[Out] int(acos(a*x^n)/x, x)
```

3.70 $\int \frac{\text{ArcCos}(ax^5)}{x} \, dx$

Optimal. Leaf size=62

$$-\frac{1}{10}i\text{ArcCos}(ax^5)^2+\frac{1}{5}\text{ArcCos}(ax^5)\log\left(1+e^{2i\text{ArcCos}(ax^5)}\right)-\frac{1}{10}i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(ax^5)}\right)$$

```
[Out] -1/10*I*arccos(a*x^5)^2+1/5*arccos(a*x^5)*ln(1+(a*x^5+I*(-a^2*x^10+1)^(1/2)
)^2)-1/10*I*polylog(2,-(a*x^5+I*(-a^2*x^10+1)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3800, 2221, 2317, 2438}

$$-\frac{1}{10}i\text{Li}_2\left(-e^{2i\text{ArcCos}(ax^5)}\right)-\frac{1}{10}i\text{ArcCos}(ax^5)^2+\frac{1}{5}\text{ArcCos}(ax^5)\log\left(1+e^{2i\text{ArcCos}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[a*x^5]/x,x]
```

```
[Out] (-1/10*I)*ArcCos[a*x^5]^2 + (ArcCos[a*x^5]*Log[1 + E^((2*I)*ArcCos[a*x^5])]
)/5 - (I/10)*PolyLog[2, -E^((2*I)*ArcCos[a*x^5])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
```

```
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4915

```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[-p^(-1), Subst[In
t[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{-1}(ax^5)}{x}\,dx &= -\left(\frac{1}{5}\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}(ax^5)\right)\right)\\
&= -\frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{2}{5}i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax^5)\right)\\
&= -\frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{1}{5}\cos^{-1}(ax^5)\log\left(1+e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{5}\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\right.\\
&= -\frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{1}{5}\cos^{-1}(ax^5)\log\left(1+e^{2i\cos^{-1}(ax^5)}\right) + \frac{1}{10}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\
&= -\frac{1}{10}i\cos^{-1}(ax^5)^2 + \frac{1}{5}\cos^{-1}(ax^5)\log\left(1+e^{2i\cos^{-1}(ax^5)}\right) - \frac{1}{10}i\text{Li}_2\left(-e^{2i\cos^{-1}(ax^5)}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.90

$$-\frac{1}{10}i\left(\text{ArcCos}(ax^5)\left(\text{ArcCos}(ax^5)+2i\log\left(1+e^{2i\text{ArcCos}(ax^5)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\text{ArcCos}(ax^5)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[a*x^5]/x,x]
```

```
[Out] (-1/10*I)*(ArcCos[a*x^5]*(ArcCos[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*x
^5])]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^5])])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\arccos(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(a*x^5)/x,x)
```

```
[Out] int(arccos(a*x^5)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arccos(a*x^5)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arccos(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acos}\left(ax^{5}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(a*x**5)/x,x)
[Out] Integral(acos(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arccos(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acos}(a\,x^5)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(a*x^5)/x,x)
[Out] int(acos(a*x^5)/x, x)
```

3.71 $\int x^3 \textbf{ArcCos}(a+bx^4)\, dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{1-(a+bx^4)^2}}{4b}+\frac{(a+bx^4)\operatorname{ArcCos}(a+bx^4)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arccos(b*x^4+a)/b-1/4*(1-(b*x^4+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 4888, 4716, 267}

$$\frac{(a+bx^4)\operatorname{ArcCos}(a+bx^4)}{4b}-\frac{\sqrt{1-(a+bx^4)^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCos[a + b*x^4],x]
```

```
[Out] -1/4*Sqrt[1 - (a + b*x^4)^2]/b + ((a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^3 \cos^{-1}\left(a+bx^4\right)\,dx = \frac{1}{4}\text{Subst}\left(\int \cos^{-1}(a+bx)\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \cos^{-1}(x)\,dx, x, a+bx^4\right)}{4b}$$
$$= \frac{(a+bx^4)\cos^{-1}\left(a+bx^4\right)}{4b} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx, x, a+bx^4\right)}{4b}$$
$$= -\frac{\sqrt{1-(a+bx^4)^2}}{4b} + \frac{(a+bx^4)\cos^{-1}\left(a+bx^4\right)}{4b}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.91

$$\frac{-\sqrt{1-(a+bx^4)^2}+(a+bx^4)\,\text{ArcCos}(a+bx^4)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCos[a + b*x^4],x]
[Out] (-Sqrt[1 - (a + b*x^4)^2] + (a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.85

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\arccos(b\,x^4+a)-\sqrt{1-(b\,x^4+a)^2}}{4b}$	40
default	$\frac{(b\,x^4+a)\arccos(b\,x^4+a)-\sqrt{1-(b\,x^4+a)^2}}{4b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccos(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arccos(b*x^4+a)-(1-(b*x^4+a)^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 39, normalized size = 0.83

$$\frac{(bx^4+a)\arccos\left(bx^4+a\right)-\sqrt{-(bx^4+a)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

Fricas [A]
time = 1.51, size = 48, normalized size = 1.02

$$\frac{(bx^4+a)\arccos\left(bx^4+a\right)-\sqrt{-b^2x^8-2\,abx^4-a^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-b^2*x^8 - 2*a*b*x^4 - a^2 + 1))/
b
```

Sympy [A]
time = 0.20, size = 61, normalized size = 1.30

$$\begin{cases} \frac{a\operatorname{acos}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acos}\left(a+bx^4\right)}{4}-\frac{\sqrt{-a^2-2abx^4-b^2x^8+1}}{4b} & \text{for}\: b\neq 0 \\ \frac{x^4\operatorname{acos}\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acos(b*x**4+a),x)
```

```
[Out] Piecewise((a*acos(a + b*x**4)/(4*b) + x**4*acos(a + b*x**4)/4 - sqrt(-a**2
- 2*a*b*x**4 - b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*acos(a)/4, True))
```

Giac [A]
time = 0.42, size = 39, normalized size = 0.83

$$\frac{(bx^4+a)\arccos\left(bx^4+a\right)-\sqrt{-\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccos(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

Mupad [B]
time = 0.65, size = 99, normalized size = 2.11

$$\frac{x^4\operatorname{acos}(b\,x^4+a)}{4}-\frac{\sqrt{-a^2-2\,a\,b\,x^4-b^2\,x^8+1}}{4\,b}-\frac{a\,\ln\left(\sqrt{-a^2-2\,a\,b\,x^4-b^2\,x^8+1}-\frac{b^2\,x^4+a\,b}{\sqrt{-b^2}}\right)}{4\,\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*acos(a + b*x^4),x)

[Out] (x^4*acos(a + b*x^4))/4 - (1 - b^2*x^8 - 2*a*b*x^4 - a^2)^(1/2)/(4*b) - (a*log((1 - b^2*x^8 - 2*a*b*x^4 - a^2)^(1/2) - (a*b + b^2*x^4)/(-b^2)^(1/2)))/(4*(-b^2)^(1/2))

3.72 $\int x^{-1+n}\mathbf{ArcCos}(a+bx^n)\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{1-(a+bx^n)^2}}{bn}+\frac{(a+bx^n)\operatorname{ArcCos}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arccos(a+b*x^n)/b/n-(1-(a+b*x^n)^2)^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6847, 4888, 4716, 267}

$$\frac{(a+bx^n)\operatorname{ArcCos}(a+bx^n)}{bn}-\frac{\sqrt{1-(a+bx^n)^2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCos[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 - (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4888

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, n
}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}\cos^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \cos^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \cos^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\ &= \frac{(a+bx^n)\cos^{-1}(a+bx^n)}{bn}+\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx,x,a+bx^n\right)}{bn}\\ &= -\frac{\sqrt{1-(a+bx^n)^2}}{bn}+\frac{(a+bx^n)\cos^{-1}(a+bx^n)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.90

$$\frac{-\sqrt{1-(a+bx^n)^2}+(a+bx^n)\,\text{ArcCos}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCos[a + b*x^n],x]
[Out] (-Sqrt[1 - (a + b*x^n)^2] + (a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{n-1}\arccos\left(a+b\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*arccos(a+b*x^n),x)
[Out] int(x^(n-1)*arccos(a+b*x^n),x)
```

Maxima [A]
time = 0.48, size = 41, normalized size = 0.85

$$\frac{(bx^n+a)\arccos(bx^n+a)-\sqrt{-(bx^n+a)^2+1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [A]
time = 2.57, size = 59, normalized size = 1.23

$$\frac{bx^n \arccos\left(bx^n + a\right) + a\arccos\left(bx^n + a\right) - \sqrt{-b^2x^{2n} - 2\,abx^n - a^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] (b*x^n*arccos(b*x^n + a) + a*arccos(b*x^n + a) - sqrt(-b^2*x^(2*n) - 2*a*b*
x^n - a^2 + 1))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 25.48, size = 76, normalized size = 1.58

$$\begin{cases} \log\left(x\right)\operatorname{acos}\left(a\right) & \text{for}\: b = 0 \wedge n = 0 \\ \log\left(x\right)\operatorname{acos}\left(a + b\right) & \text{for}\: n = 0 \\ \frac{x^n \operatorname{acos}\left(a\right)}{n} & \text{for}\: b = 0 \\ \frac{a\operatorname{acos}\left(a + bx^n\right)}{bn} + \frac{x^n\operatorname{acos}\left(a + bx^n\right)}{n} - \frac{\sqrt{-a^2 - 2abx^n - b^2x^{2n} + 1}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acos(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*acos(a), Eq(b, 0) & Eq(n, 0)), (log(x)*acos(a + b), Eq(n,
 0)), (x**n*acos(a)/n, Eq(b, 0)), (a*acos(a + b*x**n)/(b*n) + x**n*acos(a +
 b*x**n)/n - sqrt(-a**2 - 2*a*b*x**n - b**2*x**(2*n) + 1)/(b*n), True))
```

Giac [A]
time = 0.41, size = 41, normalized size = 0.85

$$\frac{\left(bx^n + a\right)\arccos\left(bx^n + a\right) - \sqrt{-\left(bx^n + a\right)^2 + 1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="giac")
```

```
[Out] ((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

Mupad [B]
time = 0.35, size = 111, normalized size = 2.31

$$\frac{x^n\,\mathrm{acos}(a + b\,x^n)}{n} - \frac{\sqrt{1 - b^2\,x^{2\,n} - 2\,a\,b\,x^n - a^2}}{b\,n} - \frac{a\,\ln\left(\sqrt{1 - b^2\,x^{2\,n} - 2\,a\,b\,x^n - a^2} - \frac{a\,b + b^2\,x^n}{\sqrt{-b^2}}\right)}{n\,\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acos(a + b*x^n),x)
```

```
[Out] (x^n*acos(a + b*x^n))/n - (1 - b^2*x^(2*n) - 2*a*b*x^n - a^2)^(1/2)/(b*n) -
 (a*log((1 - b^2*x^(2*n) - 2*a*b*x^n - a^2)^(1/2) - (a*b + b^2*x^n)/(-b^2)^
(1/2)))/(n*(-b^2)^(1/2))
```

3.73 $\int (a+b\textbf{ArcCos}(1+dx^2))^4 \, dx$

Optimal. Leaf size=127

$$384b^4x+\frac{192b^3\sqrt{-2dx^2-d^2x^4}\,(a+b\text{ArcCos}(1+dx^2))}{dx}-48b^2x(a+b\text{ArcCos}(1+dx^2))^2-\frac{8b\sqrt{-2dx^2-d^2x^4}}{}$$

[Out] 384*b^4*x-48*b^2*x*(a+b*arccos(d*x^2+1))^2+x*(a+b*arccos(d*x^2+1))^4+192*b^3*(a+b*arccos(d*x^2+1))*(-d^2*x^4-2*d*x^2)^(1/2)/d/x-8*b*(a+b*arccos(d*x^2+1))^3*(-d^2*x^4-2*d*x^2)^(1/2)/d/x

Rubi [A]
time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4899, 8}

$$\frac{192b^3\sqrt{-d^2x^4-2dx^2}\,(a+b\text{ArcCos}(dx^2+1))}{dx}-48b^2x(a+b\text{ArcCos}(dx^2+1))^2-\frac{8b\sqrt{-d^2x^4-2dx^2}\,(a+b\text{ArcCos}(dx^2+1))^3}{dx}+x(a+b\text{ArcCos}(dx^2+1))^4+384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) - 48*b^2*x*(a + b*ArcCos[1 + d*x^2])^2 - (8*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos^{-1}(1+dx^2))^4\,dx &= -\frac{8b\sqrt{-2dx^2-d^2x^4}\,(a+b\cos^{-1}(1+dx^2))^3}{dx}+x(a+b\cos^{-1}(1+dx^2))^4\\ &= \frac{192b^3\sqrt{-2dx^2-d^2x^4}\,(a+b\cos^{-1}(1+dx^2))}{dx}-48b^2x(a+b\cos^{-1}(1+d\\ &= 384b^4x+\frac{192b^3\sqrt{-2dx^2-d^2x^4}\,(a+b\cos^{-1}(1+dx^2))}{dx}-48b^2x(a+b\cos\end{aligned}$$

Mathematica [A]
time = 0.15, size = 249, normalized size = 1.96

$$\frac{(a^4-48a^2b^2+384b^4)\,dx^2-8ab(a^2-24b^2)\sqrt{-dx^2(2+dx^2)}+4b\left(a^3dx^2-24ab^2dx^2-6a^2b\sqrt{-dx^2(2+dx^2)}+48b^3\sqrt{-dx^2(2+dx^2)}\right)\mathrm{ArcCos}(1+dx^2)+6b^2\left(a^2dx^2-8b^2dx^2-4ab\sqrt{-dx^2(2+dx^2)}\right)\mathrm{ArcCos}(1+dx^2)^2+4b^3\left(adx^2-2b\sqrt{-dx^2(2+dx^2)}\right)\mathrm{ArcCos}(1+dx^2)^3+b^4dx^2\mathrm{ArcCos}(1+dx^2)^4}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[-(d*x^2*(2
+ d*x^2))] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(2 + d*
x^2))] + 48*b^3*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2] + 6*b^2*(a^2*
d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^2
 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^3 + b
^4*d*x^2*ArcCos[1 + d*x^2]^4)/(d*x)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [A]
time = 1.01, size = 207, normalized size = 1.63

$$\frac{b^4dx^2\arccos\left(dx^2+1\right)^4+4\,ab^3dx^2\arccos\left(dx^2+1\right)^3+6\,(a^2b^2-8\,b^4)dx^2\arccos\left(dx^2+1\right)^2+4\,(a^3b-24\,ab^3)dx^2\arccos\left(dx^2+1\right)+(a^4-48\,a^2b^2+384\,b^4)dx^2-8\left(b^4\arccos\left(dx^2+1\right)^3+3\,ab^3\arccos\left(dx^2+1\right)^2+a^3b-24\,ab^3+3\,(a^2b^2-8\,b^4)\arccos\left(dx^2+1\right)\right)\sqrt{-d^2x^4-2\,dx^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*arccos(d*x^2 + 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 + 1)^3 + 6*(a^2
*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 + 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arccos
(d*x^2 + 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 + 1)
^3 + 3*a*b^3*arccos(d*x^2 + 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^4)*a
rccos(d*x^2 + 1))*sqrt(-d^2*x^4 - 2*d*x^2))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acos}\left(dx^2 + 1\right)\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**4,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \operatorname{acos}\left(d\,x^2 + 1\right)\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^4,x)
```

```
[Out] int((a + b*acos(d*x^2 + 1))^4, x)
```

3.74 $\int (a+b\text{ArcCos}(1+dx^2))^3\,dx$

Optimal. Leaf size=110

$$-24ab^2x+\frac{48b^3\sqrt{-2dx^2-d^2x^4}}{dx}-24b^3x\text{ArcCos}(1+dx^2)-\frac{6b\sqrt{-2dx^2-d^2x^4}\,(a+b\text{ArcCos}(1+dx^2))^2}{dx}+x(a$$

```
[Out] -24*a*b^2*x-24*b^3*x*arccos(d*x^2+1)+x*(a+b*arccos(d*x^2+1))^3+48*b^3*(-d^2
*x^4-2*d*x^2)^(1/2)/d/x-6*b*(a+b*arccos(d*x^2+1))^2*(-d^2*x^4-2*d*x^2)^(1/2
)/d/x
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4899, 4925, 12, 1602}

$$-\frac{6b\sqrt{-d^2x^4-2dx^2}\,(a+b\text{ArcCos}(dx^2+1))^2}{dx}+x(a+b\text{ArcCos}(dx^2+1))^3-24ab^2x-24b^3x\text{ArcCos}(dx^2+1)+\frac{48b^3\sqrt{-d^2x^4-2dx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^3,x]
```

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[1 +
 d*x^2] - (6*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3\,dx = -\frac{6b\sqrt{-2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3$$
$$= -24ab^2x-\frac{6b\sqrt{-2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(1\right.\right.$$
$$= -24ab^2x-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(1+\right.\right.}{dx}$$
$$= -24ab^2x-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(1+\right.\right.}{dx}$$
$$= -24ab^2x+\frac{48b^3\sqrt{-2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-}}{}$$

Mathematica [A]
time = 0.08, size = 162, normalized size = 1.47

$$\frac{a(a^2-24b^2)\,dx^2-6b(a^2-8b^2)\,\sqrt{-dx^2\,(2+dx^2)}+3b\left(a^2dx^2-8b^2dx^2-4ab\sqrt{-dx^2\,(2+dx^2)}\right)\mathrm{ArcCos}(1+dx^2)+3b^2\left(adx^2-2b\sqrt{-dx^2\,(2+dx^2)}\right)\mathrm{ArcCos}(1+dx^2)^2+b^3dx^2\mathrm{ArcCos}(1+dx^2)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[-(d*x^2*(2 + d*x^2))] + 3*
b*(a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d
*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^
2 + b^3*d*x^2*ArcCos[1 + d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2+1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^3,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [A]
time = 0.94, size = 144, normalized size = 1.31

$$\frac{b^3dx^2\arccos\left(dx^2+1\right)^3+3\,ab^2dx^2\arccos\left(dx^2+1\right)^2+3\left(a^2b-8\,b^3\right)dx^2\arccos\left(dx^2+1\right)+\left(a^3-24\,ab^2\right)dx^2-6\,\sqrt{-d^2x^4-2\,dx^2}\left(b^3\arccos\left(dx^2+1\right)^2+2\,ab^2\arccos\left(dx^2+1\right)+a^2b-8\,b^3\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*arccos(d*x^2 + 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 + 1)^2 + 3*(a^2
*b - 8*b^3)*d*x^2*arccos(d*x^2 + 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^2*
x^4 - 2*d*x^2)*(b^3*arccos(d*x^2 + 1)^2 + 2*a*b^2*arccos(d*x^2 + 1) + a^2*b
 - 8*b^3))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{acos}\left(dx^2+1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^3,x)
```

```
[Out] int((a + b*acos(d*x^2 + 1))^3, x)
```

3.75 $\int \left(a+b\mathbf{ArcCos}(1+dx^2)\right)^2\,dx$

Optimal. Leaf size=63

$$-8b^2x-\frac{4b\sqrt{-2dx^2-d^2x^4}\,(a+b\text{ArcCos}(1+dx^2))}{dx}+x\big(a+b\text{ArcCos}\big(1+dx^2\big)\big)^2$$

```
[Out] -8*b^2*x+x*(a+b*arccos(d*x^2+1))^2-4*b*(a+b*arccos(d*x^2+1))*(-d^2*x^4-2*d*
x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4899, 8}

$$-\frac{4b\sqrt{-d^2x^4-2dx^2}\,(a+b\text{ArcCos}(dx^2+1))}{dx}+x\big(a+b\text{ArcCos}\big(dx^2+1\big)\big)^2-8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^2,x]
```

```
[Out] -8*b^2*x - (4*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2\,dx&=-\frac{4b\sqrt{-2dx^2-d^2x^4}\,(a+b\cos^{-1}(1+dx^2))}{dx}+x\big(a+b\cos^{-1}\left(1+dx^2\right)\big)^2-\\&=-8b^2x-\frac{4b\sqrt{-2dx^2-d^2x^4}\,(a+b\cos^{-1}(1+dx^2))}{dx}+x\big(a+b\cos^{-1}\left(1+d\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.56

$$(a^2-8b^2)\,x-\frac{4ab\sqrt{-dx^2\,(2+dx^2)}}{dx}+\frac{2b\Big(adx^2-2b\sqrt{-dx^2\,(2+dx^2)}\Big)\,\text{ArcCos}(1+dx^2)}{dx}+b^2x\text{ArcCos}\big(1+dx^2\big)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^2,x]
```

```
[Out] (a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + (2*b*(a*d*x^2
- 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2])/(d*x) + b^2*x*ArcCos[1
 + d*x^2]^2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^2,x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [A]
time = 0.99, size = 91, normalized size = 1.44

$$\frac{b^2dx^2\arccos\left(dx^2+1\right)^2+2\,abdx^2\arccos\left(dx^2+1\right)+(a^2-8\,b^2)dx^2-4\,\sqrt{-d^2x^4-2\,dx^2}\,(b^2\arccos\left(dx^2+1\right)+ab)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*arccos(d*x^2 + 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 + 1) + (a^2 - 8*b
^2)*d*x^2 - 4*sqrt(-d^2*x^4 - 2*d*x^2)*(b^2*arccos(d*x^2 + 1) + a*b))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{acos}\left(dx^2 + 1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**2,x)
[Out] Integral((a + b*acos(d*x**2 + 1))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 + 1) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a + b\,\text{acos}\left(d\,x^2 + 1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^2,x)
[Out] int((a + b*acos(d*x^2 + 1))^2, x)
```

3.76 $\int (a + b\mathbf{ArcCos}(1 + dx^2))\, dx$

Optimal. Leaf size=43

$$ax - \frac{2b\sqrt{-2dx^2 - d^2x^4}}{dx} + bx\mathrm{ArcCos}\left(1 + dx^2\right)$$

```
[Out] a*x+b*x*arccos(d*x^2+1)-2*b*(-d^2*x^4-2*d*x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4925, 12, 1602}

$$ax + bx\mathrm{ArcCos}\left(dx^2 + 1\right) - \frac{2b\sqrt{-d^2x^4 - 2dx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCos[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)\,dx = ax+b\int \cos^{-1}\left(1+dx^2\right)\,dx$$
$$= ax+bx\cos^{-1}\left(1+dx^2\right)+b\int \frac{2dx^2}{\sqrt{-2dx^2-d^2x^4}}\,dx$$
$$= ax+bx\cos^{-1}\left(1+dx^2\right)+(2bd)\int \frac{x^2}{\sqrt{-2dx^2-d^2x^4}}\,dx$$
$$= ax-\frac{2b\sqrt{-2dx^2-d^2x^4}}{dx}+bx\cos^{-1}\left(1+dx^2\right)$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.95

$$ax-\frac{2b\sqrt{-dx^2\left(2+dx^2\right)}}{dx}+bx\text{ArcCos}\left(1+dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCos[1 + d*x^2],x]
[Out] a*x - (2*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

Maple [A]
time = 0.01, size = 45, normalized size = 1.05

method	result	size
default	$ax+b\left(x\arccos\left(d\,x^2+1\right)+\frac{2x\left(d\,x^2+2\right)}{\sqrt{-d^2x^4-2d\,x^2}}\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccos(d*x^2+1),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*arccos(d*x^2+1)+2/(-d^2*x^4-2*d*x^2)^(1/2)*x*(d*x^2+2))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 1.05

$$\left(x\arccos\left(dx^2+1\right)+\frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{-dx^2-2}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="maxima")
```

```
[Out] (x*arccos(d*x^2 + 1) + 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(-d*x^2 - 2)*d))*b
+ a*x
```

Fricas [A]
time = 1.15, size = 48, normalized size = 1.12

$$\frac{bdx^2 \arccos\left(dx^2+1\right)+adx^2-2\sqrt{-d^2x^4-2\,dx^2}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*arccos(d*x^2 + 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 - 2*d*x^2)*b)/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{acos}\left(dx^2+1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acos(d*x**2+1),x)
```

```
[Out] Integral(a + b*acos(d*x**2 + 1), x)
```

Giac [A]
time = 0.43, size = 55, normalized size = 1.28

$$\left(x\arccos\left(dx^2+1\right)+\frac{2\sqrt{2}\sqrt{-d}\,\operatorname{sgn}(x)}{d}-\frac{2\sqrt{-d^2x^2-2\,d}}{d\operatorname{sgn}\left(x\right)}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2+1),x, algorithm="giac")
```

```
[Out] (x*arccos(d*x^2 + 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d - 2*sqrt(-d^2*x^2 - 2*d)
/(d*sgn(x)))*b + a*x
```

Mupad [B]
time = 0.47, size = 39, normalized size = 0.91

$$a\,x+b\,x\operatorname{acos}\left(d\,x^2+1\right)-\frac{2\,b\,\sqrt{1-{\left(d\,x^2+1\right)}^2}}{d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acos(d*x^2 + 1),x)
```

```
[Out] a*x + b*x*acos(d*x^2 + 1) - (2*b*(1 - (d*x^2 + 1)^2)^(1/2))/(d*x)
```

3.77 $\int \frac{1}{a+b\text{ArcCos}(1+dx^2)}\,dx$

Optimal. Leaf size=99

$$\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}$$

```
[Out] 1/2*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*cos(1/2*a/b)/b*2^(1/2)/(-d*x^2)^(1/2)
+1/2*x*Si(1/2*(a+b*arccos(d*x^2+1))/b)*sin(1/2*a/b)/b*2^(1/2)/(-d*x^2)^(1/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4901}

$$\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sq
rt[-(d*x^2)]) + (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)
])/(Sqrt[2]*b*Sqrt[-(d*x^2)])
```

Rule 4901

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[(-d)
*x^2])), x] + Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[(-d)*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cos^{-1}\left(1+dx^2\right)}\,dx=\frac{x\cos\left(\frac{a}{2b}\right)\text{Ci}\left(\frac{a+b\cos^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{-dx^2}}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.86

$$-\frac{\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)\right)}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

```
[Out] -((Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x
^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))/(
b*d*x))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\arccos\left(d\,x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1)),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccos(d*x^2 + 1) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\operatorname{acos}\left(dx^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1)),x)
```

```
[Out] Integral(1/(a + b*acos(d*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccos(d*x^2 + 1) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{acos}\left(d\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1)),x)
```

```
[Out] int(1/(a + b*acos(d*x^2 + 1)), x)
```

3.78 $\int \frac{1}{\left(a+b\text{ArcCos}(1+dx^2)\right)^2}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{-2dx^2-d^2x^4}}{2bdx\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)}+\frac{x\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{-dx^2}}$$

```
[Out] -1/4*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x^2+1))/b)/b^2*2^(1/2)/(-d*x^2)^(1/2)+1/4*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*sin(1/2*a/b)/b^2*2^(1/2)/(-d*x^2)^(1/2)+1/2*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))
```

Rubi [A]
time = 0.02, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4910}

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{-dx^2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{2bdx\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])) + (x*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[-(d*x^2)]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[-(d*x^2)])
```

Rule 4910

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[-2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])), x] + (Simp[x*Sin[a/(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2])), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2}\,dx=\frac{\sqrt{-2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)}+\frac{x\text{Ci}\left(\frac{a+b\cos^{-1}(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}}{2\sqrt{2}}$$

Mathematica [A]
time = 0.26, size = 133, normalized size = 0.88

$$\frac{\sqrt{-dx^2(2+dx^2)}\left(\frac{b}{a+b\text{ArcCos}(1+dx^2)}-\frac{\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)\right)}{2+dx^2}\right)}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

```
[Out] (Sqrt[-(d*x^2*(2 + d*x^2))]*(b/(a + b*ArcCos[1 + d*x^2]) - (Cos[ArcCos[1 +
d*x^2]/2]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Cos[
a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))/(2 + d*x^2)))/(2*b^
2*d*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^2,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccos(d*x^2 + 1)^2 + 2*a*b*arccos(d*x^2 + 1) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**2,x)
[Out] Integral((a + b*acos(d*x**2 + 1))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^2,x)
[Out] int(1/(a + b*acos(d*x^2 + 1))^2, x)
```

3.79 $\int \frac{1}{\left(a+b\mathbf{ArcCos}(1+dx^2)\right)^3} dx$

Optimal. Leaf size=173

$$\frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)}-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(1+dx^2)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{-dx^2}}-\frac{x}{}$$

```
[Out] 1/8*x/b^2/(a+b*arccos(d*x^2+1))-1/16*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*cos(
1/2*a/b)/b^3*2^(1/2)/(-d*x^2)^(1/2)-1/16*x*Si(1/2*(a+b*arccos(d*x^2+1))/b)*
sin(1/2*a/b)/b^3*2^(1/2)/(-d*x^2)^(1/2)+1/4*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/
(a+b*arccos(d*x^2+1))^2
```

Rubi [A]

time = 0.03, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4913, 4901}

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{-dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(dx^2+1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{-dx^2}}+\frac{x}{8b^2\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)}+\frac{\sqrt{-d^2x^4-2dx^2}}{4bdx\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[1 + d*x^2])) - (x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d
*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)]) - (x*Sin[a/(2*b)]*SinIntegral
[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[-(d*x^2)])
```

Rule 4901

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[(-d)
*x^2])), x] + Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[(-d)*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^3}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2} + \frac{x}{8b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)} - \frac{\int \frac{1}{a+b\cos^{-1}(1+}}{8b^2}$$

$$= \frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^2} + \frac{x}{8b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)} - \frac{x\cos\left(\frac{a}{2b}\right)\mathrm{Ci}}{8\sqrt{2}}$$

Mathematica [A]
time = 0.19, size = 147, normalized size = 0.85

$$\frac{\frac{2b^2\sqrt{-dx^2\left(2+dx^2\right)}}{d\left(a+b\mathrm{ArcCos}\left(1+dx^2\right)\right)^2}+\frac{bx^2}{a+b\mathrm{ArcCos}\left(1+dx^2\right)}+\frac{\sin\left(\frac{1}{2}\mathrm{ArcCos}\left(1+dx^2\right)\right)\left(\cos\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\mathrm{ArcCos}\left(1+dx^2\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\mathrm{ArcCos}\left(1+dx^2\right)}{2b}\right)\right)}{d}}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

```
[Out] ((2*b^2*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*(a + b*ArcCos[1 + d*x^2])^2) + (b*x^
2)/(a + b*ArcCos[1 + d*x^2]) + (Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*CosI
ntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a + b*
ArcCos[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^3,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^3,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccos(d*x^2 + 1)^3 + 3*a*b^2*arccos(d*x^2 + 1)^2 + 3*a^2*b
*arccos(d*x^2 + 1) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{acos}(dx^2 + 1))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\operatorname{acos}(d\,x^2 + 1))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^3,x)
```

```
[Out] int(1/(a + b*acos(d*x^2 + 1))^3, x)
```

3.80 $\int \left(a+b\mathbf{ArcCos}(-1+dx^2)\right)^4 \, dx$

Optimal. Leaf size=127

$$384b^4x+\frac{192b^3\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(-1+dx^2))}{dx}-48b^2x\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^2-\frac{8b\sqrt{2dx^2-d^2x^4}}{}$$

```
[Out] 384*b^4*x-48*b^2*x*(a+b*arccos(d*x^2-1))^2+x*(a+b*arccos(d*x^2-1))^4+192*b^
3*(a+b*arccos(d*x^2-1))*(-d^2*x^4+2*d*x^2)^(1/2)/d/x-8*b*(a+b*arccos(d*x^2-
1))^3*(-d^2*x^4+2*d*x^2)^(1/2)/d/x
```

Rubi [A]

time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4899, 8}

$$\frac{192b^3\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(dx^2-1))}{dx}-48b^2x\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^2-\frac{8b\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(dx^2-1))^3}{dx}+x\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^4+384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d
*x) - 48*b^2*x*(a + b*ArcCos[-1 + d*x^2])^2 - (8*b*Sqrt[2*d*x^2 - d^2*x^4]*
(a + b*ArcCos[-1 + d*x^2])^3)/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^4\,dx &= -\frac{8b\sqrt{2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3}{dx}+x\left(a+b\cos^{-1}\left(-1+dx\right.\right.\\ &= \frac{192b^3\sqrt{2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx}-48b^2x\left(a+b\cos^{-1}\left(-1\right.\right.\\ &= 384b^4x+\frac{192b^3\sqrt{2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx}-48b^2x\left(a+b\,\mathrm{c}\right.\end{aligned}$$

Mathematica [A]
time = 0.16, size = 249, normalized size = 1.96

$$\frac{(a^4-48a^2b^2+384b^4)\,dx^2-8ab(a^2-24b^2)\,\sqrt{dx^2\,(2-dx^2)}+4b\Big(a^3dx^2-24ab^2dx^2-6a^2b\sqrt{-dx^2\,(-2+dx^2)}+48b^3\sqrt{-dx^2\,(-2+dx^2)}\Big)\,\mathrm{ArcCos}(-1+dx^2)+6b^2\Big(a^2dx^2-8b^2dx^2-4ab\sqrt{-dx^2\,(-2+dx^2)}\Big)\,\mathrm{ArcCos}(-1+dx^2)^2+4b^3\Big(adx^2-2b\sqrt{-dx^2\,(-2+dx^2)}\Big)\,\mathrm{ArcCos}(-1+dx^2)^3+b^4dx^2\mathrm{ArcCos}(-1+dx^2)^4}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

```
[Out] ((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[d*x^2*(2 -
d*x^2)] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(-2 + d*x^
2))] + 48*b^3*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2] + 6*b^2*(a^2*
d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]
^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]^3
 + b^4*d*x^2*ArcCos[-1 + d*x^2]^4)/(d*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a +b \arccos \left(d \,x^{2}-1\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^4,x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^4 + 4*(x*arccos(d*x^2
- 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*a^3*b + a^4*x - in
tegrate(2*(4*sqrt(-d*x^2 + 2)*b^4*sqrt(d)*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d
)*x, d*x^2 - 1)^3 - 2*(a*b^3*d*x^2 - 2*a*b^3)*arctan2(sqrt(-d*x^2 + 2)*sqrt
(d)*x, d*x^2 - 1)^3 - 3*(a^2*b^2*d*x^2 - 2*a^2*b^2)*arctan2(sqrt(-d*x^2 + 2
)*sqrt(d)*x, d*x^2 - 1)^2)/(d*x^2 - 2), x)
```

Fricas [A]
time = 0.97, size = 207, normalized size = 1.63

$$\frac{b^4dx^2\arccos\left(dx^2-1\right)^4+4\,ab^3dx^2\arccos\left(dx^2-1\right)^3+6\,(a^2b^2-8\,b^4)dx^2\arccos\left(dx^2-1\right)^2+4\,(a^3b-24\,ab^3)dx^2\arccos\left(dx^2-1\right)+(a^4-48\,a^2b^2+384\,b^4)dx^2-8\left(b^4\arccos\left(dx^2-1\right)^3+3\,ab^3\arccos\left(dx^2-1\right)^2+a^3b-24\,ab^3+3\,(a^2b^2-8\,b^4)\arccos\left(dx^2-1\right)\right)\sqrt{-d^2x^4+2\,dx^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*arccos(d*x^2 - 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 - 1)^3 + 6*(a^2
*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 - 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arccos
(d*x^2 - 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 - 1)
^3 + 3*a*b^3*arccos(d*x^2 - 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^4)*a
rccos(d*x^2 - 1))*sqrt(-d^2*x^4 + 2*d*x^2))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acos}\left(dx^{2} - 1\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**4,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^4,x)
```

```
[Out] int((a + b*acos(d*x^2 - 1))^4, x)
```

3.81 $\int (a + b\mathbf{ArcCos}(-1 + dx^2))^3\, dx$

Optimal. Leaf size=110

$$-24ab^2x+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}-24b^3x\text{ArcCos}(-1+dx^2)-\frac{6b\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(-1+dx^2))^2}{dx}+x(a$$

```
[Out] -24*a*b^2*x-24*b^3*x*arccos(d*x^2-1)+x*(a+b*arccos(d*x^2-1))^3+48*b^3*(-d^2
*x^4+2*d*x^2)^(1/2)/d/x-6*b*(a+b*arccos(d*x^2-1))^2*(-d^2*x^4+2*d*x^2)^(1/2
)/d/x
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4899, 4925, 12, 1602}

$$-\frac{6b\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(dx^2-1))^2}{dx}+x(a+b\text{ArcCos}(dx^2-1))^3-24ab^2x-24b^3x\text{ArcCos}(dx^2-1)+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^3,x]
```

```
[Out] -24*a*b^2*x + (48*b^3*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) - 24*b^3*x*ArcCos[-1 +
 d*x^2] - (6*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^2)/(d*x)
+ x*(a + b*ArcCos[-1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3\,dx = -\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\left(-1+dx^2\right.\right.$$

$$= -24ab^2x-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}{dx}+x\left(a+b\cos^{-1}\right.$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1\right.\right.}{dx}$$

$$= -24ab^2x-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1\right.\right.}{dx}$$

$$= -24ab^2x+\frac{48b^3\sqrt{2dx^2-d^2x^4}}{dx}-24b^3x\cos^{-1}\left(-1+dx^2\right)-\frac{6b\sqrt{2dx^2-}}{}$$

Mathematica [A]
time = 0.09, size = 162, normalized size = 1.47

$$\frac{a(a^2-24b^2)dx^2-6b(a^2-8b^2)\sqrt{dx^2(2-dx^2)}+3b\left(a^2dx^2-8b^2dx^2-4ab\sqrt{-dx^2(-2+dx^2)}\right)\text{ArcCos}(-1+dx^2)+3b^2\left(adx^2-2b\sqrt{-dx^2(-2+dx^2)}\right)\text{ArcCos}(-1+dx^2)^2+b^3dx^2\text{ArcCos}(-1+dx^2)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^3,x]
[Out] (a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[d*x^2*(2 - d*x^2)] + 3*b*(
a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*
x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2]
^2 + b^3*d*x^2*ArcCos[-1 + d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^3,x)
[Out] int((a+b*arccos(d*x^2-1))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^3 + 3*(x*arccos(d*x^2
- 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*a^2*b + a^3*x - in
tegrate(3*(2*sqrt(-d*x^2 + 2)*b^3*sqrt(d)*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d
)*x, d*x^2 - 1)^2 - (a*b^2*d*x^2 - 2*a*b^2)*arctan2(sqrt(-d*x^2 + 2)*sqrt(d
)*x, d*x^2 - 1)^2)/(d*x^2 - 2), x)
```

Fricas [A]
time = 0.73, size = 144, normalized size = 1.31

$$\frac{b^3dx^2\arccos\left(dx^2-1\right)^3+3\,ab^2dx^2\arccos\left(dx^2-1\right)^2+3\left(a^2b-8\,b^3\right)dx^2\arccos\left(dx^2-1\right)+\left(a^3-24\,ab^2\right)dx^2-6\,\sqrt{-d^2x^4+2\,dx^2}\left(b^3\arccos\left(dx^2-1\right)^2+2\,ab^2\arccos\left(dx^2-1\right)+a^2b-8\,b^3\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*arccos(d*x^2 - 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 - 1)^2 + 3*(a^2
*b - 8*b^3)*d*x^2*arccos(d*x^2 - 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^2*
x^4 + 2*d*x^2)*(b^3*arccos(d*x^2 - 1)^2 + 2*a*b^2*arccos(d*x^2 - 1) + a^2*b
 - 8*b^3))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{acos}\left(dx^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{acos}\left(d\,x^2 - 1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^3,x)
[Out] int((a + b*acos(d*x^2 - 1))^3, x)
```

3.82 $\int (a+b\text{ArcCos}(-1+dx^2))^2\, dx$

Optimal. Leaf size=63

$$-8b^2x-\frac{4b\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(-1+dx^2))}{dx}+x\big(a+b\text{ArcCos}\big(-1+dx^2\big)\big)^2$$

```
[Out] -8*b^2*x+x*(a+b*arccos(d*x^2-1))^2-4*b*(a+b*arccos(d*x^2-1))*(-d^2*x^4+2*d*
x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4899, 8}

$$-\frac{4b\sqrt{2dx^2-d^2x^4}\,(a+b\text{ArcCos}(dx^2-1))}{dx}+x\big(a+b\text{ArcCos}\big(dx^2-1\big)\big)^2-8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

```
[Out] -8*b^2*x - (4*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x) +
 x*(a + b*ArcCos[-1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2\,dx&=-\frac{4b\sqrt{2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx}+x\big(a+b\cos^{-1}\left(-1+dx^2\right)\big)\\&=-8b^2x-\frac{4b\sqrt{2dx^2-d^2x^4}\,\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}{dx}+x\big(a+b\cos^{-1}\left(-1\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.56

$$(a^2-8b^2)x-\frac{4ab\sqrt{-dx^2(-2+dx^2)}}{dx}+\frac{2b\left(adx^2-2b\sqrt{-dx^2(-2+dx^2)}\right)\text{ArcCos}(-1+dx^2)}{dx}+b^2x\text{ArcCos}(-1+dx^2)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

```
[Out] (a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2])/(d*x) + b^2*x*ArcCo
s[-1 + d*x^2]^2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^2,x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arccos(d*x^2 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*
a*b + (x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^2 - 4*sqrt(d)*integ
rate(sqrt(-d*x^2 + 2)*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)/(d*x
^2 - 2), x))*b^2 + a^2*x
```

Fricas [A]
time = 0.86, size = 91, normalized size = 1.44

$$\frac{b^2dx^2\arccos\left(dx^2-1\right)^2+2\,abdx^2\arccos\left(dx^2-1\right)+(a^2-8\,b^2)dx^2-4\,\sqrt{-d^2x^4+2\,dx^2}\,(b^2\arccos\left(dx^2-1\right)+ab)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*arccos(d*x^2 - 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 - 1) + (a^2 - 8*b
^2)*d*x^2 - 4*sqrt(-d^2*x^4 + 2*d*x^2)*(b^2*arccos(d*x^2 - 1) + a*b))/(d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{acos}\left(dx^2 - 1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^2,x)
```

```
[Out] int((a + b*acos(d*x^2 - 1))^2, x)
```

3.83 $\int (a+b\text{ArcCos}(-1+dx^2))\, dx$

Optimal. Leaf size=43

$$ax-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}+bx\text{ArcCos}\left(-1+dx^2\right)$$

```
[Out] a*x+b*x*arccos(d*x^2-1)-2*b*(-d^2*x^4+2*d*x^2)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4925, 12, 1602}

$$ax+bx\text{ArcCos}\left(dx^2-1\right)-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCos[-1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4925

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)\,dx = ax+b\int \cos^{-1}\left(-1+dx^2\right)\,dx$$
$$= ax+bx\cos^{-1}\left(-1+dx^2\right)+b\int \frac{2dx^2}{\sqrt{2dx^2-d^2x^4}}\,dx$$
$$= ax+bx\cos^{-1}\left(-1+dx^2\right)+(2bd)\int \frac{x^2}{\sqrt{2dx^2-d^2x^4}}\,dx$$
$$= ax-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}+bx\cos^{-1}\left(-1+dx^2\right)$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.95

$$ax-\frac{2b\sqrt{-dx^2\left(-2+dx^2\right)}}{dx}+bx\text{ArcCos}\left(-1+dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCos[-1 + d*x^2],x]
[Out] a*x - (2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

Maple [A]
time = 0.01, size = 45, normalized size = 1.05

method	result	size
default	$ax+b\left(x\arccos\left(d\,x^2-1\right)+\frac{2x\left(d\,x^2-2\right)}{\sqrt{-d^2x^4+2d\,x^2}}\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccos(d*x^2-1),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*arccos(d*x^2-1)+2/(-d^2*x^4+2*d*x^2)^(1/2)*x*(d*x^2-2))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 1.05

$$\left(x\arccos\left(dx^2-1\right)+\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{-dx^2+2}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="maxima")
```

[Out] (x*arccos(d*x^2 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*b + a*x

Fricas [A]
time = 0.67, size = 48, normalized size = 1.12

$$\frac{bdx^2 \arccos(dx^2-1) + adx^2 - 2\sqrt{-d^2x^4 + 2\,dx^2}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="fricas")

[Out] (b*d*x^2*arccos(d*x^2 - 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 + 2*d*x^2)*b)/(d*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{acos}\left(dx^2-1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*acos(d*x**2-1),x)

[Out] Integral(a + b*acos(d*x**2 - 1), x)

Giac [A]
time = 0.42, size = 50, normalized size = 1.16

$$\left(x\arccos\left(dx^2-1\right) + \frac{2\sqrt{2}\,\mathrm{sgn}(x)}{\sqrt{d}} - \frac{2\sqrt{-d^2x^2 + 2\,d}}{d\mathrm{sgn}\,(x)}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arccos(d*x^2-1),x, algorithm="giac")

[Out] (x*arccos(d*x^2 - 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*sqrt(-d^2*x^2 + 2*d)/(d*sgn(x)))*b + a*x

Mupad [B]
time = 0.44, size = 39, normalized size = 0.91

$$a\,x + b\,x\,\operatorname{acos}\left(d\,x^2-1\right) - \frac{2\,b\,\sqrt{1-{\left(d\,x^2-1\right)}^2}}{d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b*acos(d*x^2 - 1),x)

[Out] a*x + b*x*acos(d*x^2 - 1) - (2*b*(1 - (d*x^2 - 1)^2)^(1/2))/(d*x)

3.84 $\int \frac{1}{a+b\mathbf{ArcCos}(-1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

```
[Out] -1/2*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x^2-1))/b)/b*2^(1/2)/(d*x^2)^(1/2)
+1/2*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)*sin(1/2*a/b)/b*2^(1/2)/(d*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4902}

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(dx^2-1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(dx^2-1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

```
[Out] (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]
)/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 4902

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Sin
[a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*
x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cos^{-1}(-1+dx^2)}\,dx=\frac{x\text{Ci}\left(\frac{a+b\cos^{-1}(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\cos^{-1}(-1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.87

$$\frac{\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)\right)}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

```
[Out] (Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*S
in[a/(2*b)] - Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/
(b*d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\arccos \left(d\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1)),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccos(d*x^2 - 1) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\operatorname{acos}\left(dx^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1)),x)
```

```
[Out] Integral(1/(a + b*acos(d*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="giac")
[Out] integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{acos}\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1)),x)
[Out] int(1/(a + b*acos(d*x^2 - 1)), x)
```

3.85 $\int \frac{1}{\left(a+b\mathbf{ArcCos}(-1+dx^2)\right)^2}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\mathrm{ArcCos}\left(-1+dx^2\right)\right)}-\frac{x\cos\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\mathrm{ArcCos}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\mathrm{ArcCos}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}$$

```
[Out] -1/4*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)*cos(1/2*a/b)/b^2*2^(1/2)/(d*x^2)^(1/
2)-1/4*x*Si(1/2*(a+b*arccos(d*x^2-1))/b)*sin(1/2*a/b)/b^2*2^(1/2)/(d*x^2)^(
1/2)+1/2*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2-1))
```

Rubi [A]
time = 0.01, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4911}

$$-\frac{x\cos\left(\frac{a}{2b}\right)\mathrm{CosIntegral}\left(\frac{a+b\mathrm{ArcCos}(dx^2-1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\mathrm{ArcCos}(dx^2-1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\mathrm{ArcCos}\left(dx^2-1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])) - (x*Cos[a/(2*
b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^
2]) - (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqr
t[2]*b^2*Sqrt[d*x^2])
```

Rule 4911

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[
2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])), x] + (-Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt
[d*x^2])), x] - Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])
/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}\,dx=\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}-\frac{x\cos\left(\frac{a}{2b}\right)\mathrm{Ci}\left(\frac{a+b\cos^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sin$$

Mathematica [A]
time = 0.26, size = 131, normalized size = 0.88

$$\frac{\sqrt{-dx^2(-2+dx^2)}\left(\frac{b}{a+b\text{ArcCos}(-1+dx^2)}+\frac{\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)\right)}{-2+dx^2}\right)}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

```
[Out] (Sqrt[-(d*x^2*(-2 + d*x^2))]*(b/(a + b*ArcCos[-1 + d*x^2]) + (Sin[ArcCos[-1
 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)] +
Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/(-2 + d*x^2)))
/(2*b^2*d*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^2,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)*sqrt
(d)*integrate(1/2*sqrt(-d*x^2 + 2)*x/(a*b*d*x^2 - 2*a*b + (b^2*d*x^2 - 2*b^
2)*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)), x) - sqrt(-d*x^2 + 2)*s
qrt(d))/(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccos(d*x^2 - 1)^2 + 2*a*b*arccos(d*x^2 - 1) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(dx^{2} - 1\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**2,x)
[Out] Integral((a + b*acos(d*x**2 - 1))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^2,x)
[Out] int(1/(a + b*acos(d*x^2 - 1))^2, x)
```

3.86 $\int \frac{1}{\left(a+b\mathbf{ArcCos}(-1+dx^2)\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)}-\frac{x\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right.}{8\sqrt{2}\,b^3\sqrt{dx^2}}$$

```
[Out] 1/8*x/b^2/(a+b*arccos(d*x^2-1))+1/16*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x^
2-1))/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)*s
in(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a
+b*arccos(d*x^2-1))^2
```

Rubi [A]
time = 0.02, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4913, 4902}

$$-\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\text{ArcCos}(dx^2-1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}+\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\text{ArcCos}(dx^2-1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}+\frac{x}{8b^2\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)}+\frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[-1 + d*x^2])^2) + x/(8*b^2*(
a + b*ArcCos[-1 + d*x^2])) - (x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b
)]*Sin[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*Cos[a/(2*b)]*SinIntegral[
(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2])
```

Rule 4902

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Sin
[a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*
x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^3}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}-\frac{\int \frac{}{a+b\,\mathrm{co}}}{}$$

$$= \frac{\sqrt{2dx^2-d^2x^4}}{4bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^2}+\frac{x}{8b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)}-\frac{x\mathrm{Ci}\left(\frac{a}{}\right.}{}$$

Mathematica [A]
time = 0.17, size = 149, normalized size = 0.87

$$\frac{\frac{2b^2\sqrt{-dx^2\left(-2+dx^2\right)}}{d\left(a+b\mathrm{ArcCos}(-1+dx^2)\right)^2}+\frac{bx^2}{a+b\mathrm{ArcCos}(-1+dx^2)}-\frac{\cos\left(\frac{1}{2}\mathrm{ArcCos}(-1+dx^2)\right)\left(\mathrm{CosIntegral}\left(\frac{a+b\mathrm{ArcCos}(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\mathrm{Si}\left(\frac{a+b\mathrm{ArcCos}(-1+dx^2)}{2b}\right)\right)}{d}}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```

```
[Out] ((2*b^2*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*(a + b*ArcCos[-1 + d*x^2])^2) + (b*
x^2)/(a + b*ArcCos[-1 + d*x^2]) - (Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral[(
a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Cos[a/(2*b)]*SinIntegral[(a
 + b*ArcCos[-1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^3,x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*d*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*d*x + 2*sqrt(
-d*x^2 + 2)*b*sqrt(d) - 8*(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2
- 1)^2 + 2*a*b^3*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*b^2
```

```
*d)*integrate(1/8/(b^3*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b
^2), x))/(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^2 + 2*a*b^3*
d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*b^2*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*arccos(d*x^2 - 1)^3 + 3*a*b^2*arccos(d*x^2 - 1)^2 + 3*a^2*b
*arccos(d*x^2 - 1) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acos}{\left(dx^{2} - 1 \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**3,x)
[Out] Integral((a + b*acos(d*x**2 - 1))**(-3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^3,x)
[Out] int(1/(a + b*acos(d*x^2 - 1))^3, x)
```

3.87 $\int \left(a+b\mathbf{ArcCos}(1+dx^2)\right)^{5/2}\,dx$

Optimal. Leaf size=249

$$-\frac{5b\sqrt{-2dx^2-d^2x^4}\left(a+b\text{ArcCos}(1+dx^2)\right)^{3/2}}{dx}+x\left(a+b\text{ArcCos}(1+dx^2)\right)^{5/2}-\frac{30\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{}}{}\right.}{}$$

```
[Out] x*(a+b*arccos(d*x^2+1))^(5/2)-30*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*arc
cos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(5/2)
/d/x+30*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*
a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(5/2)/d/x-5*b*(a+b*arccos(d*x^
2+1))^(3/2)*(-d^2*x^4-2*d*x^2)^(1/2)/d/x+30*b^2*sin(1/2*arccos(d*x^2+1))^2*
(a+b*arccos(d*x^2+1))^(1/2)/d/x
```

Rubi [A]
time = 0.07, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4899, 4896}

$$\frac{30b^2\sin^2\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{dx}-\frac{5b\sqrt{-d^2x^4-2dx^2}\left(a+b\text{ArcCos}(dx^2+1)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\,\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}dx}-\frac{30\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}dx}+x\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[1 + d*x^2])^(5/2) - (30*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b
^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(
(b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos
[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(5
/2)*d*x) + (30*b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2
)/(d*x)
```

Rule 4896

```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sqrt
[a + b*ArcCos[1 + d*x^2]]*(Sin[ArcCos[1 + d*x^2]/2]^2/(d*x)), x] + (-Simp[2
*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] + Simp[2*Sqrt[Pi]*Cos[a/(
2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx=-\frac{5b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}{dx}+x\big(a+b\cos^{-1}\left(1+dx^2\right)$$

$$=-\frac{5b\sqrt{-2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}{dx}+x\big(a+b\cos^{-1}\left(1+dx^2\right)$$

Mathematica [A]
time = 1.78, size = 256, normalized size = 1.03

$$-\frac{2\sin\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)\left(\frac{15\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\mathrm{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}}-\frac{15\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\mathrm{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{\left(\frac{1}{b}\right)^{5/2}}+\sqrt{a+b\mathrm{ArcCos}\left(1+dx^2\right)}\left(5ab\cos\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)+(a^2-15b^2)\sin\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)+b^2\mathrm{ArcCos}(1+dx^2)^2\sin\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)+b\mathrm{ArcCos}(1+dx^2)\left(5b\cos\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)+2a\sin\left(\frac{1}{2}\mathrm{ArcCos}(1+dx^2)\right)\right)\right)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*((15*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) - (15*Sqrt[Pi]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b
)])/(b^(-1))^(5/2) + Sqrt[a + b*ArcCos[1 + d*x^2]]*(5*a*b*Cos[ArcCos[1 + d*
x^2]/2] + (a^2 - 15*b^2)*Sin[ArcCos[1 + d*x^2]/2] + b^2*ArcCos[1 + d*x^2]^2
*Sin[ArcCos[1 + d*x^2]/2] + b*ArcCos[1 + d*x^2]*(5*b*Cos[ArcCos[1 + d*x^2]/
2] + 2*a*Sin[ArcCos[1 + d*x^2]/2]))))/(d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2+1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(5/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acos}\left(dx^{2} + 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(5/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b\,\mathrm{acos}\left(d\,x^2 + 1\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^(5/2),x)
```

```
[Out] int((a + b*acos(d*x^2 + 1))^(5/2), x)
```

3.88 $\int (a+b\textbf{ArcCos}(1+dx^2))^{3/2}\,dx$

Optimal. Leaf size=207

$$-\frac{3b\sqrt{-2dx^2-d^2x^4}\,\sqrt{a+b\text{ArcCos}\,(1+dx^2)}}{dx}+x\big(a+b\text{ArcCos}\big(1+dx^2\big)\big)^{3/2}+\frac{6\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\text{FresnelC}\Bigg(\sqrt{\cdots}}{\cdots}$$

```
[Out] x*(a+b*arccos(d*x^2+1))^(3/2)+6*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arcc
os(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(3/2)/
d/x+6*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/
b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(3/2)/d/x-3*b*(-d^2*x^4-2*d*x^2)
^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/d/x
```

Rubi [A]
time = 0.05, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4899, 4904}

$$-\frac{3b\sqrt{-d^2x^4-2dx^2}\,\sqrt{a+b\text{ArcCos}\,(dx^2+1)}}{dx}+\frac{6\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\,(dx^2+1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\sqrt{\pi}\,\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\,(dx^2+1)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+x\big(a+b\text{ArcCos}\big(dx^2+1\big)\big)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/((b^
(-1))^(3/2)*d*x) + (6*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
 d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/((b^(-1))^(3/2)*
d*x)
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4904

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
```

```
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx=-\frac{3b\sqrt{-2dx^2-d^2x^4}\,\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)$$

$$=-\frac{3b\sqrt{-2dx^2-d^2x^4}\,\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(1+dx^2\right)\right)$$

Mathematica [A]
time = 0.47, size = 200, normalized size = 0.97

$$-\frac{2\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(-3\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)-3\sqrt{\pi}S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+\left(\frac{1}{b}\right)^{3/2}\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}\left(3b\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+a\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+b\text{ArcCos}(1+dx^2)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\right)\right)}{\left(\frac{1}{b}\right)^{3/2}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(-3*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1
)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]] - 3*Sqrt[Pi]*FresnelS[(Sqrt[b^(
-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + (b^(-1))^(3/2)
*Sqrt[a + b*ArcCos[1 + d*x^2]]*(3*b*Cos[ArcCos[1 + d*x^2]/2] + a*Sin[ArcCos
[1 + d*x^2]/2] + b*ArcCos[1 + d*x^2]*Sin[ArcCos[1 + d*x^2]/2])))/((b^(-1))^
(3/2)*d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2+1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(3/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acos}{\left(d x^{2} + 1 \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^(3/2),x)
[Out] int((a + b*acos(d*x^2 + 1))^(3/2), x)
```

3.89 $\int \sqrt{a+b\mathbf{ArcCos}\left(1+dx^2\right)}\,dx$

Optimal. Leaf size=184

$$\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)}{\sqrt{\frac{1}{b}}\,dx}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b}\cdots}{\cdots}\right.}{\cdots}$$

```
[Out] 2*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*s
in(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x/(1/b)^(1/2)-2*FresnelC((1/b)^(1/2)*(a+
b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))*Pi
^(1/2)/d/x/(1/b)^(1/2)-2*sin(1/2*arccos(d*x^2+1))^2*(a+b*arccos(d*x^2+1))^(
1/2)/d/x
```

Rubi [A]
time = 0.02, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4896}

$$-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}\,dx}+\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}\,dx}-\frac{2\sin^2\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2
]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Fr
esnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*
Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[a + b*ArcCos[1 + d*x
^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x)
```

Rule 4896

```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sqrt
[a + b*ArcCos[1 + d*x^2]]*(Sin[ArcCos[1 + d*x^2]/2]^2/(d*x)), x] + (-Simp[2
*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] + Simp[2*Sqrt[Pi]*Cos[a/(
2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cos^{-1}(1+dx^2)}\,dx = \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\cos^{-1}(1+dx^2)\right)}{\sqrt{\frac{1}{b}}\,dx} -$$

Mathematica [A]
time = 0.07, size = 157, normalized size = 0.85

$$-\frac{2\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(-\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\right)}{\sqrt{\frac{1}{b}}\,dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(-(Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]) + Sqrt[Pi]*FresnelC[(Sqrt[b^(-1
)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + Sqrt[b^(-1)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]))/(Sqrt[b^(-1)]*d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\arccos(d\,x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2+1))^(1/2),x)
```

```
[Out] int((a+b*arccos(d*x^2+1))^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acos}(dx^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(d*x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 + 1) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{acos}(d\,x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 + 1))^(1/2),x)
```

```
[Out] int((a + b*acos(d*x^2 + 1))^(1/2), x)
```

3.90 $\int \frac{1}{\sqrt{a+b\mathbf{ArcCos}\left(1+dx^2\right)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)}{dx}-\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\,S\left(\frac{\sqrt{\ldots}}{\ldots}\right.}{\ldots}$$

```
[Out] -2*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*
sin(1/2*arccos(d*x^2+1))*(1/b)^(1/2)*Pi^(1/2)/d/x-2*FresnelS((1/b)^(1/2)*(a
+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))*(
1/b)^(1/2)*Pi^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4904}

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*Ar
cCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(-1
)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]
*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Rule 4904

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(1+dx^2)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\,\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)C\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\cos^{-1}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\cos^{-1}(1+\right.}{dx}$$

Mathematica [A]
time = 0.12, size = 114, normalized size = 0.79

$$-\frac{2\sqrt{\frac{1}{b}}\,\sqrt{\pi}\left(\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)+S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*(Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[1 + d*x^2]])/Sqrt[Pi]] + FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
 d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\arccos(d\,x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}{\left(dx^{2} + 1 \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(d*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 + 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(d\,x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acos(d*x^2 + 1))^(1/2), x)
```

3.91 $\int \frac{1}{\left(a+b\mathbf{ArcCos}(1+dx^2)\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt{-2dx^2-d^2x^4}}{bdx\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+\right.}{dx}$$

```
[Out] 2*(1/b)^(3/2)*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)
/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x-2*(1/b)^(3/2)*FresnelC((1/
b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(
d*x^2+1))*Pi^(1/2)/d/x+(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4907}

$$\frac{\sqrt{-d^2x^4-2dx^2}}{bdx\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 +
d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*(b^(-1))^(3/2)*Sqrt
[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/
(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

Rule 4907

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[2*(
1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] + Simp[2*(1/b)^(3/2)*Sqrt
[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx=\frac{\sqrt{-2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\cos^{-1}\cdots}}{\sqrt{\pi}}\right.\cdots}{dx\cdots}$$

Mathematica [A]
time = 0.28, size = 177, normalized size = 0.93

$$\frac{\frac{\sqrt{-dx^2(2+dx^2)}}{\sqrt{a+b\text{ArcCos}(1+dx^2)}}+2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)-2\sqrt{\frac{1}{b}}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

```
[Out] (Sqrt[-(d*x^2*(2 + d*x^2))]/Sqrt[a + b*ArcCos[1 + d*x^2]] + 2*Sqrt[b^(-1)]*
Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])
/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2] - 2*Sqrt[b^(-1)]*Sqrt[Pi]*FresnelC[(Sqr
t[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[
1 + d*x^2]/2])/(b*d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acos}(dx^2 + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{acos}(d\, x^2 + 1))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acos(d*x^2 + 1))^(3/2), x)
```

3.92 $\int \frac{1}{\left(a+b\text{ArcCos}(1+dx^2)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+}}{}\right.}{}$$

[Out] 2/3*(1/b)^(5/2)*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x+2/3*(1/b)^(5/2)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x+1/3*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))^(3/2)+1/3*x/b^2/(a+b*arccos(d*x^2+1))^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4913, 4904}

$$\frac{x}{3b^2\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}+\frac{\sqrt{-d^2x^4-2dx^2}}{3bdx\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)^{3/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{3dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]
*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos
[1 + d*x^2]/2])/(3*d*x) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2
]/2])/(3*d*x)
```

Rule 4904

```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx=\frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}-\frac{\int \frac{}{\sqrt{a+}}}{}$$

$$=\frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{5/2}}{}$$

Mathematica [A]
time = 0.52, size = 234, normalized size = 1.06

$$\frac{2\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(b\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\,(a+b\text{ArcCos}(1+dx^2))^{3/2}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\,(a+b\text{ArcCos}(1+dx^2))^{3/2}S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)-a\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)-b\text{ArcCos}(1+dx^2)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\right)}{3b^2dx\left(a+b\text{ArcCos}(1+dx^2)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*Sin[ArcCos[1 + d*x^2]/2]*(b*Cos[ArcCos[1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt
[Pi]*(a + b*ArcCos[1 + d*x^2])^(3/2)*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sq
rt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCo
s[1 + d*x^2])^(3/2)*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/S
qrt[Pi]]*Sin[a/(2*b)] - a*Sin[ArcCos[1 + d*x^2]/2] - b*ArcCos[1 + d*x^2]*Si
n[ArcCos[1 + d*x^2]/2]))/(3*b^2*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{acos}(dx^2 + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(5/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\operatorname{acos}(d\,x^2 + 1))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^(5/2),x)
[Out] int(1/(a + b*acos(d*x^2 + 1))^(5/2), x)
```

3.93 $\int \frac{1}{\left(a+b\text{ArcCos}(1+dx^2)\right)^{7/2}} \, dx$

Optimal. Leaf size=269

$$\frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\text{ArcCos}\left(1+dx^2\right)\right)^{3/2}}-\frac{\sqrt{-2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\text{ArcCos}\left(1+dx^2\right)}}-\frac{2\left(\frac{1}{b}\right)}{}$$

```
[Out] 1/15*x/b^2/(a+b*arccos(d*x^2+1))^(3/2)-2/15*(1/b)^(7/2)*cos(1/2*a/b)*Fresne
lS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1
))*Pi^(1/2)/d/x+2/15*(1/b)^(7/2)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))
^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x+1/5*(-d
^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))^(5/2)-1/15*(-d^2*x^4-2*d*
x^2)^(1/2)/b^3/d/x/(a+b*arccos(d*x^2+1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4913, 4907}

$$-\frac{\sqrt{-d^2x^4-2dx^2}}{15b^3dx\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}+\frac{x}{15b^2\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)^{3/2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{5bdx\left(a+b\text{ArcCos}\left(dx^2+1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{15dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{ArcCos}(dx^2+1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2+1\right)}}{\sqrt{\pi}}\right)}{15dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```

```
[Out] Sqrt[-2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[1 + d*x^2])^(3/2)) - Sqrt[-2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[1 + d*x^2]]) - (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*F
resnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1
 + d*x^2]/2])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*
Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]
/2])/(15*d*x)
```

Rule 4907

```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[2*(
1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] + Simp[2*(1/b)^(3/2)*Sqrt
[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{\int \frac{}{(a+}}{}$$

$$= \frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{}{15b^3c}$$

Mathematica [A]
time = 0.38, size = 308, normalized size = 1.14

$$-\frac{2\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\left(a^2\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)-3b^2\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+2ab\text{ArcCos}(1+dx^2)\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+b^2\text{ArcCos}(1+dx^2)^2\cos\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(1+dx^2)\right)^{5/2}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)-\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(1+dx^2)\right)^{5/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+ab\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)+b^2\text{ArcCos}(1+dx^2)\sin\left(\frac{1}{2}\text{ArcCos}(1+dx^2)\right)\right)}{15b^3dx\left(a+b\text{ArcCos}(1+dx^2)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```

```
[Out] (-2*Sin[ArcCos[1 + d*x^2]/2]*(a^2*Cos[ArcCos[1 + d*x^2]/2] - 3*b^2*Cos[ArcC
os[1 + d*x^2]/2] + 2*a*b*ArcCos[1 + d*x^2]*Cos[ArcCos[1 + d*x^2]/2] + b^2*A
rcCos[1 + d*x^2]^2*Cos[ArcCos[1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*
ArcCos[1 + d*x^2])^(5/2)*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*Arc
Cos[1 + d*x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[1 + d*x^2]
)^(5/2)*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)] + a*b*Sin[ArcCos[1 + d*x^2]/2] + b^2*ArcCos[1 + d*x^2]*Sin[ArcCos
[1 + d*x^2]/2]))/(15*b^3*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2
)-2)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acos}(dx^2 + 1))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2+1))**(7/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 + 1))**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 + 1) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(d x^{2} + 1\right)\right)^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 + 1))^(7/2),x)
[Out] int(1/(a + b*acos(d*x^2 + 1))^(7/2), x)
```

3.94 $\int \left(a+b\text{ArcCos}(-1+dx^2)\right)^{5/2}\,dx$

Optimal. Leaf size=249

$$-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\text{ArcCos}(-1+dx^2)\right)^{3/2}}{dx}+x\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{5/2}-\frac{30b^2\sqrt{a+b\text{ArcCos}\left(-1+\right.}}{}$$

```
[Out] x*(a+b*arccos(d*x^2-1))^(5/2)+30*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*Fres
nelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/(1/b)^(5/2)
/d/x+30*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))
^(1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/(1/b)^(5/2)/d/x-5*b*(a+b*arccos(d*x^
2-1))^(3/2)*(-d^2*x^4+2*d*x^2)^(1/2)/d/x-30*b^2*cos(1/2*arccos(d*x^2-1))^2*
(a+b*arccos(d*x^2-1))^(1/2)/d/x
```

Rubi [A]
time = 0.04, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4899, 4897}

$$-\frac{30b^2\cos^2\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{dx}-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\text{ArcCos}(dx^2-1)\right)^{3/2}}{dx}+\frac{30\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}dx}+\frac{30\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}dx}+x\left(a+b\text{ArcCos}(dx^2-1)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*(
a + b*ArcCos[-1 + d*x^2])^(5/2) - (30*b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Co
s[ArcCos[-1 + d*x^2]/2]^2)/(d*x) + (30*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1
+ d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]
])/((b^(-1))^(5/2)*d*x) + (30*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-
1))^(5/2)*d*x)
```

Rule 4897

```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCos[-1 + d*x^2]]*(Cos[(1/2)*ArcCos[-1 + d*x^2]]^2/(d*x)), x] + (-
Simp[2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] - Simp[2*Sqrt[Pi]
*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx=-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}{dx}+x\big(a+b\cos^{-1}\big(-1+$$

$$=-\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}{dx}+x\big(a+b\cos^{-1}\big(-1+$$

Mathematica [A]
time = 1.47, size = 256, normalized size = 1.03

$$\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(\frac{15\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}}+\frac{15\sqrt{\pi}\,S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{\left(\frac{1}{b}\right)^{5/2}}+\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}\left(\left(a^2-15b^2\right)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+b^2\text{ArcCos}(-1+dx^2)^2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)-5ab\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+b\text{ArcCos}(-1+dx^2)\left(2a\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)-5b\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\right)\right)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*((15*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(b^(-1))^(5/2) + (15*Sqrt[Pi
]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2
*b)])/(b^(-1))^(5/2) + Sqrt[a + b*ArcCos[-1 + d*x^2]]*((a^2 - 15*b^2)*Cos[A
rcCos[-1 + d*x^2]/2] + b^2*ArcCos[-1 + d*x^2]^2*Cos[ArcCos[-1 + d*x^2]/2] -
 5*a*b*Sin[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1 + d*x^2]*(2*a*Cos[ArcCos[-1
+ d*x^2]/2] - 5*b*Sin[ArcCos[-1 + d*x^2]/2]))))/(d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2-1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(5/2),x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acos}\left(dx^{2} - 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(5/2),x)
[Out] Integral((a + b*acos(d*x**2 - 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^(5/2),x)
[Out] int((a + b*acos(d*x^2 - 1))^(5/2), x)
```

3.95 $\int \left(a+b\text{ArcCos}(-1+dx^2)\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$-\frac{3b\sqrt{2dx^2-d^2x^4}\,\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{dx}+x\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{3/2}+\frac{6\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcC}\right.}{}$$

```
[Out] x*(a+b*arccos(d*x^2-1))^(3/2)+6*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*Fresn
elS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/(1/b)^(3/2)/
d/x-6*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(
1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/(1/b)^(3/2)/d/x-3*b*(-d^2*x^4+2*d*x^2)
^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4899, 4905}

$$-\frac{3b\sqrt{2dx^2-d^2x^4}\,\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{dx}-\frac{6\sqrt{\pi}\,\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+\frac{6\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}+x\left(a+b\text{ArcCos}(dx^2-1)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/(d*x) + x*(a
+ b*ArcCos[-1 + d*x^2])^(3/2) + (6*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*
x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(
(b^(-1))^(3/2)*d*x) - (6*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[
b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/((b^(-1))^(
3/2)*d*x)
```

Rule 4899

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos[
c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4905

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
```

```
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx = -\frac{3b\sqrt{2dx^2-d^2x^4}\,\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx\right.\right.$$

$$= -\frac{3b\sqrt{2dx^2-d^2x^4}\,\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}{dx}+x\left(a+b\cos^{-1}\left(-1+dx\right.\right.$$

Mathematica [A]
time = 0.42, size = 200, normalized size = 0.97

$$\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(3\sqrt{\pi}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)-3\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+\left(\frac{1}{b}\right)^{3/2}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}\left(a\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+b\text{ArcCos}(-1+dx^2)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)-3b\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\right)\right)}{\left(\frac{1}{b}\right)^{3/2}dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(3*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] - 3*Sqrt[Pi]*FresnelC[(Sqrt[b^(
-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)] + (b^(-1))^(3/2
)*Sqrt[a + b*ArcCos[-1 + d*x^2]]*(a*Cos[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1
 + d*x^2]*Cos[ArcCos[-1 + d*x^2]/2] - 3*b*Sin[ArcCos[-1 + d*x^2]/2])))/((b^
(-1))^(3/2)*d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(a+b\arccos\left(d\,x^2-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(3/2),x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{acos}\left(dx^{2} - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(3/2),x)
[Out] Integral((a + b*acos(d*x**2 - 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^(3/2),x)
[Out] int((a + b*acos(d*x^2 - 1))^(3/2), x)
```

3.96 $\int \sqrt{a+b\textbf{ArcCos}\left(-1+dx^2\right)}\,dx$

Optimal. Leaf size=184

$$\frac{2\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}\cos^2\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)}{dx}-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\text{FresnelC}}{\sqrt{\frac{1}{b}}\,dx}$$

```
[Out] -2*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d
*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x/(1/b)^(1/2)-2*cos(1/2*arccos(d*x^2-1)
)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*P
i^(1/2)/d/x/(1/b)^(1/2)+2*cos(1/2*arccos(d*x^2-1))^2*(a+b*arccos(d*x^2-1))^
(1/2)/d/x
```

Rubi [A]
time = 0.02, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4897}

$$-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}\,dx}-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}\,dx}+\frac{2\cos^2\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos[ArcCos[-1 + d*x^2]/2]^2)/(d*x) - (2*S
qrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[
a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Cos[
ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]
)/Sqrt[Pi]]*Sin[a/(2*b)])/(Sqrt[b^(-1)]*d*x)
```

Rule 4897

```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCos[-1 + d*x^2]]*(Cos[(1/2)*ArcCos[-1 + d*x^2]]^2/(d*x)), x] + (-
Simp[2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] - Simp[2*Sqrt[Pi]
*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}\cos^2\left(\frac{1}{2}\cos^{-1}\left(-1+dx^2\right)\right)}{dx} - \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{c}}{}$$

Mathematica [A]
time = 0.08, size = 157, normalized size = 0.85

$$-\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(-\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)+\sqrt{\pi}\,S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{1}{b}}\,dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (-2*Cos[ArcCos[-1 + d*x^2]/2]*(-(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]
]*Cos[ArcCos[-1 + d*x^2]/2]) + Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] + Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]))/(Sqrt[b^(-1)]*d*
x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\arccos\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos(d*x^2-1))^(1/2),x)
```

```
[Out] int((a+b*arccos(d*x^2-1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acos}{\left(dx^{2} - 1 \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(d*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\mathrm{acos}\left(d\,x^2-1\right)}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acos(d*x^2 - 1))^(1/2),x)
```

```
[Out] int((a + b*acos(d*x^2 - 1))^(1/2), x)
```

3.97 $\int \frac{1}{\sqrt{a+b\textbf{ArcCos}(-1+dx^2)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx}+\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{1}{2}\text{A}\right.}{}$$

```
[Out] -2*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d
*x^2-1))^(1/2)/Pi^(1/2))*(1/b)^(1/2)*Pi^(1/2)/d/x+2*cos(1/2*arccos(d*x^2-1)
)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*(
1/b)^(1/2)*Pi^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4905}

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

```
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(
Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b^(
-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

Rule 4905

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^{-1}(-1+dx^2)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\,\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(-1+dx^2)\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\cos^{-1}(-1}{\sqrt{\pi}}\right.}{dx}$$

Mathematica [A]
time = 0.12, size = 115, normalized size = 0.79

$$-\frac{2\sqrt{\frac{1}{b}}\,\sqrt{\pi}\,\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)-\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\,\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
[Out] (-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*(Cos[a/(2*b)]*FresnelS[
(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]] - FresnelC[(Sqrt[b^
(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]))/(d*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\arccos\left(d\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
[Out] int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}\left(dx^2 - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acos(d*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{acos}\left(d\, x^2 - 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acos(d*x^2 - 1))^(1/2), x)
```

3.98 $\int \frac{1}{\left(a+b\text{ArcCos}(-1+dx^2)\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}-\frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcC}}}{\sqrt{}}\right.}{dx}$$

```
[Out] -2*(1/b)^(3/2)*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(
a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x-2*(1/b)^(3/2)*cos(1/2*arc
cos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*si
n(1/2*a/b)*Pi^(1/2)/d/x+(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2-1)
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4908}

$$\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]) - (2*(b^(-1)
)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-
1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) - (2*(b^(-1))^(3/2)*Sq
rt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-
1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

Rule 4908

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[2*
(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*(1/b)^(3/2)*S
qrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos^{-1}(-1+dx^2))^{3/2}}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\cos^{-1}(-1+dx^2)}} - \frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\,\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\cos^{-1}(-1}{}$$

Mathematica [A]
time = 0.24, size = 161, normalized size = 0.85

$$\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(-\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)-\sqrt{\frac{1}{b}}\sqrt{\pi}\,S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+\frac{\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)}{\sqrt{a+b\text{ArcCos}(-1+dx^2)}}\right)}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(-(Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC
[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]) - Sqrt[b^(-1)]*Sq
rt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)] + Sin[ArcCos[-1 + d*x^2]/2]/Sqrt[a + b*ArcCos[-1 + d*x^2]]))/(b*d
*x)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\arccos(d\,x^2-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
[Out] int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acos}(dx^2 - 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acos(d*x^2 - 1))^(3/2), x)
```

3.99 $\int \frac{1}{\left(a+b\mathbf{ArcCos}(-1+dx^2)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}+\frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(-$$

```
[Out] 2/3*(1/b)^(5/2)*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)*
(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x-2/3*(1/b)^(5/2)*cos(1/2*
arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))
*sin(1/2*a/b)*Pi^(1/2)/d/x+1/3*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d
*x^2-1))^(3/2)+1/3*x/b^2/(a+b*arccos(d*x^2-1))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4913, 4905}

$$\frac{x}{3b^2\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}+\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^{3/2}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{3dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{3dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/2)) + x/(3*b
^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[a/(2*b)
]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d
*x^2]])/Sqrt[Pi]])/(3*d*x) - (2*(b^(-1))^(5/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x
^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin
[a/(2*b)])/(3*d*x)
```

Rule 4905

```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx=\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}-\frac{\int -}{\sqrt{}}$$

$$2\left(\frac{1}{b}\right.$$

$$=\frac{\sqrt{2dx^2-d^2x^4}}{3bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}+\frac{x}{3b^2\sqrt{a+b\cos^{-1}\left(-1+dx^2\right)}}+-$$

Mathematica [A]
time = 0.42, size = 233, normalized size = 1.05

$$\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(a\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+b\text{ArcCos}(-1+dx^2)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(-1+dx^2)\right)^{3/2}\cos\left(\frac{a}{2b}\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)-\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(-1+dx^2)\right)^{3/2}\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)+b\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\right)}{3b^2dx\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(a*Cos[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1 + d
*x^2]*Cos[ArcCos[-1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 +
d*x^2])^(3/2)*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]])/Sqrt[Pi]] - Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(3/2)*F
resnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)
] + b*Sin[ArcCos[-1 + d*x^2]/2]))/(3*b^2*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/
2))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(d x^{2} - 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(5/2),x)
[Out] Integral((a + b*acos(d*x**2 - 1))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^(5/2),x)
[Out] int(1/(a + b*acos(d*x^2 - 1))^(5/2), x)
```

3.100 $\int \frac{1}{\left(a+b\textbf{ArcCos}\left(-1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\text{ArcCos}\left(-1+dx^2\right)\right)^{3/2}}-\frac{\sqrt{2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\text{ArcCos}\left(-1+dx^2\right)}}+$$

```
[Out] 1/15*x/b^2/(a+b*arccos(d*x^2-1))^(3/2)+2/15*(1/b)^(7/2)*cos(1/2*a/b)*cos(1/
2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2
))*Pi^(1/2)/d/x+2/15*(1/b)^(7/2)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1
/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/d/x+1/5*(-d
^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2-1))^(5/2)-1/15*(-d^2*x^4+2*d*
x^2)^(1/2)/b^3/d/x/(a+b*arccos(d*x^2-1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4913, 4908}

$$-\frac{\sqrt{2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}+\frac{x}{15b^2\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^{3/2}}+\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\text{ArcCos}\left(dx^2-1\right)\right)^{5/2}}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{15dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{ArcCos}(dx^2-1)\right)S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}\left(dx^2-1\right)}}{\sqrt{\pi}}\right)}{15dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```

```
[Out] Sqrt[2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2)) + x/(15*
b^2*(a + b*ArcCos[-1 + d*x^2])^(3/2)) - Sqrt[2*d*x^2 - d^2*x^4]/(15*b^3*d*x
*Sqrt[a + b*ArcCos[-1 + d*x^2]]) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[a/(2*b)]*
Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x
^2]])/Sqrt[Pi]])/(15*d*x) + (2*(b^(-1))^(7/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^
2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[
a/(2*b)])/(15*d*x)
```

Rule 4908

```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[2*
(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*(1/b)^(3/2)*S
qrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rule 4913

```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCos[c + d*x^2])^(n + 2), x], x] - Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx=\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}-$$

$$=\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{5/2}}+\frac{x}{15b^2\left(a+b\cos^{-1}\left(-1+dx^2\right)\right)^{3/2}}-$$

Mathematica [A]
time = 0.38, size = 309, normalized size = 1.15

$$\frac{2\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\left(ab\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+b^2\text{ArcCos}(-1+dx^2)\cos\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(-1+dx^2)\right)^{5/2}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)+\sqrt{\frac{1}{b}}\sqrt{\pi}\left(a+b\text{ArcCos}(-1+dx^2)\right)^{5/2}S\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ArcCos}(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)-a^2\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)+3b^2\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)-2ab\text{ArcCos}(-1+dx^2)\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)-b^2\text{ArcCos}(-1+dx^2)^2\sin\left(\frac{1}{2}\text{ArcCos}(-1+dx^2)\right)\right)}{15b^3dx\left(a+b\text{ArcCos}(-1+dx^2)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (2*Cos[ArcCos[-1 + d*x^2]/2]*(a*b*Cos[ArcCos[-1 + d*x^2]/2] + b^2*ArcCos[-1
 + d*x^2]*Cos[ArcCos[-1 + d*x^2]/2] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-
1 + d*x^2])^(5/2)*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1
+ d*x^2]])/Sqrt[Pi]] + Sqrt[b^(-1)]*Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(5/
2)*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(
2*b)] - a^2*Sin[ArcCos[-1 + d*x^2]/2] + 3*b^2*Sin[ArcCos[-1 + d*x^2]/2] - 2
*a*b*ArcCos[-1 + d*x^2]*Sin[ArcCos[-1 + d*x^2]/2] - b^2*ArcCos[-1 + d*x^2]^
2*Sin[ArcCos[-1 + d*x^2]/2]))/(15*b^3*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^2-1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acos}\left(dx^{2} - 1\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acos(d*x**2-1))**(7/2),x)
```

```
[Out] Integral((a + b*acos(d*x**2 - 1))**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\operatorname{acos}\left(d\,x^2-1\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acos(d*x^2 - 1))^(7/2),x)
[Out] int(1/(a + b*acos(d*x^2 - 1))^(7/2), x)
```

3.101 $\int \frac{\left(a+b\mathbf{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]

time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [**A**]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \arccos \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)\right)^{n}}{-c^{2} x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] -integrate((b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^n}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.102 $\int \frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=279

$$\frac{i\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}-\frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{3ib\left(a+b\text{A}\right.}{}$$

```
[Out] 1/4*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arccos((-c*x+1)
^(1/2)/(c*x+1)^(1/2)))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(
c*x+1))^(1/2))^2)/c+3/2*I*b*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*po
lylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c-3
/2*b^2*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c-3/4*I*b^3*polylog(4,-((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A]

time = 0.15, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6813, 4722, 3800, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3b^2\text{Li}_3\left(-e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3ib\text{Li}_2\left(-e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}+\frac{i\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}-\frac{\log\left(1+e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}-\frac{3ib^3\text{Li}_4\left(-e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] ((I/4)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (((3*I)/2)*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
^2*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (3*b^2*(
a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^((2*I)*ArcCos[Sqrt
[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) - (((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*Arc
Cos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cos^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^3\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}}\,dx, x, \cos^{-1}\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

Mathematica [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]

[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(308) = 616.
time = 1.33, size = 707, normalized size = 2.53

method	result
default	$-\frac{a^3\ln(cx-1)}{2c}+\frac{a^3\ln(cx+1)}{2c}+\frac{ib^3\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c}-\frac{b^3\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1}\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)+1/4*I*b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^4-b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-3/2*b^3/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-3/4*I*b^3*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c+I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3*I*a*b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-3/2*a*b^2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*a^2*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2*I*a^2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorithm="maxima")

[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^3*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1))^3 + 3*a*b^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1))^2 + 3*a^2*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)))/(c^2*x^2 - 1), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^3*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccos(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))
+ a^3)/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.103 $\int \frac{\left(a+b\mathbf{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=207

$$\frac{i\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{ib\Big(a+b\text{ArcC}}{}$$

```
[Out] 1/3*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arccos((-c*x+1)
^(1/2)/(c*x+1)^(1/2)))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(
c*x+1))^(1/2))^2)/c+I*b*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(
2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c-1/2*b^2
*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/
c
```

Rubi [A]
time = 0.12, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 4722, 3800, 2221, 2611, 2320, 6724}

$$\frac{ib\text{Li}_2\left(-e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{i\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log\left(1+e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}-\frac{b^2\text{Li}_3\left(-e^{2i\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] ((I/3)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])])/c + (I*b*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog
[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c - (b^2*PolyLog[3, -E
^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cos^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^2\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\right.\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1\right.}{c}$$

Mathematica [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.17, size = 401, normalized size = 1.94

method	result

default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}+\frac{ib^2\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}-\frac{b^2\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+1/3*I*b^2/c*arccos((-c*x+1)^(1/2)/
(c*x+1)^(1/2))^3-b^2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+I*b^2/c*arccos((-c*x
+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*
x+1)/(c*x+1))^(1/2))^2)-1/2*b^2*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*
(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c+I*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2
))^2-2*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+I*a*b/c*polylog(2,-((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^2*arctan2(sqrt(2)*
sqrt(c)*sqrt(x), sqrt(-c*x + 1))^2 + 2*a*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x),
 sqrt(-c*x + 1)))/(c^2*x^2 - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="fricas")
```

```
[Out] integral(-(b^2*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccos(sqrt(-
c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algo
rithm="giac")
```

```
[Out] integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2}{c^2\,x^2-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.104 $\int \frac{a+b\,\mathbf{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=141

$$\frac{i\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}-\frac{\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2i\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{ib\operatorname{PolyLog}\left(2\right.}{}$$

```
[Out] 1/2*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arccos((-c*x+1)
^(1/2)/(c*x+1)^(1/2)))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*
x+1))^(1/2))^2)/c+1/2*I*b*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c
*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {212, 6813, 4722, 3800, 2221, 2317, 2438}

$$\frac{i\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1+e^{2i\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{ib\operatorname{Li}_2\left(-e^{2i\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((I/2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)/(b*c) - ((a + b*ArcCo
s[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])])/c + ((I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\cos^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= \frac{\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e\right.}{c}$$

$$= \frac{i\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e\right.}{c}$$

Mathematica [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.15, size = 171, normalized size = 1.21

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{ib\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}-\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1-\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/2*I*b/c*arccos((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2-b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c
*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+1/2*I*b*polylog(2,-((-c*x+1)^(
1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="maxima")
```

```
[Out] 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) - b*integrate(arctan2(sqrt(2)*sqrt(
c)*sqrt(x), sqrt(-c*x + 1))/(c^2*x^2 - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algori
thm="fricas")
```

```
[Out] integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm="giac")

[Out] integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)

[Out] int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)

3.105 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\text{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Maple [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \arccos \left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="maxima")
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="fricas")
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ac^{2}x^{2} - a + bc^{2}x^{2}\operatorname{acos}{\left(\frac{\sqrt{-cx + 1}}{\sqrt{cx + 1}} \right)} - b\operatorname{acos}{\left(\frac{\sqrt{-cx + 1}}{\sqrt{cx + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acos(sqrt(-c*x + 1)/sqrt(c*x + 1
)) - b*acos(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algo
rithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.106 $\int \frac{1}{(1-c^2x^2)\left(a+b\mathbf{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cos^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\operatorname{ArcCos}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x
]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \arccos \left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="maxima")
```

```
[Out] ((sqrt(2)*b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + sqrt(2)*
a*b*c - (sqrt(2)*b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) +
 sqrt(2)*a*b*c^2)*x)*sqrt(c)*integrate(1/2*sqrt(-c*x + 1)*sqrt(x)/((b^2*c^3
*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^3)*x^3 - 2*(b^2*c
^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^2)*x^2 + (b^2*c
*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c)*x), x) - sqrt(2)
*sqrt(-c*x + 1)*sqrt(c)*sqrt(x))/(b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sq
rt(-c*x + 1)) + a*b*c - (b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x
 + 1)) + a*b*c^2)*x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(sqrt(-c*x + 1)/sqrt(c
*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1
))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, al
gorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2)
, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.107 $\int \text{ArcCos}\left(ce^{a+bx}\right)\,dx$

Optimal. Leaf size=84

$$-\frac{i\text{ArcCos}\left(ce^{a+bx}\right)^2}{2b}+\frac{\text{ArcCos}\left(ce^{a+bx}\right)\log\left(1+e^{2i\text{ArcCos}\left(ce^{a+bx}\right)}\right)}{b}-\frac{i\text{PolyLog}\left(2,-e^{2i\text{ArcCos}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] -1/2*I*arccos(c*exp(b*x+a))^2/b+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+I*(
1-c^2*exp(b*x+a)^2)^(1/2))^2)/b-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*exp
(b*x+a)^2)^(1/2))^2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2320, 4722, 3800, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-e^{2i\text{ArcCos}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{i\text{ArcCos}\left(ce^{a+bx}\right)^2}{2b}+\frac{\text{ArcCos}\left(ce^{a+bx}\right)\log\left(1+e^{2i\text{ArcCos}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[c*E^(a + b*x)],x]
```

```
[Out] ((-1/2*I)*ArcCos[c*E^(a + b*x)]^2)/b + (ArcCos[c*E^(a + b*x)]*Log[1 + E^((2
*I)*ArcCos[c*E^(a + b*x)])])/b - ((I/2)*PolyLog[2, -E^((2*I)*ArcCos[c*E^(a
+ b*x)])])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(ce^{a+bx}\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\cos^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b}\\
&= -\frac{\text{Subst}\left(\int x\tan(x)\,dx, x, \cos^{-1}\left(ce^{a+bx}\right)\right)}{b}\\
&= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \cos^{-1}\left(ce^{a+bx}\right)\right)}{b}\\
&= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1+e^{2i\right.}\right.}{}\\
&= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.}{2b}\\
&= -\frac{i\cos^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cos^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2i\cos^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{i\text{Li}_2\left(-e^{2i\cos^{-1}\left(ce^{a+bx}\right.}\right.}{2b}
\end{aligned}$$

Mathematica [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int \text{ArcCos}\left(ce^{a+bx}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCos[c*E^(a + b*x)],x]
```

```
[Out] Integrate[ArcCos[c*E^(a + b*x)], x]
```

Maple [A]
time = 0.56, size = 102, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{i \arccos\left(c\,\mathrm{e}^{bx+a}\right)^{2}}{2}+\arccos\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+\left(c\,\mathrm{e}^{bx+a}+i\sqrt{1-c^{2}\mathrm{e}^{2bx+2a}}\right)^{2}\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(c\,\mathrm{e}^{bx+a}+i\sqrt{1-c^{2}\mathrm{e}\ldots}\right.\right.}{2}}{b}$
default	$\frac{-\frac{i \arccos\left(c\,\mathrm{e}^{bx+a}\right)^{2}}{2}+\arccos\left(c\,\mathrm{e}^{bx+a}\right)\ln\left(1+\left(c\,\mathrm{e}^{bx+a}+i\sqrt{1-c^{2}\mathrm{e}^{2bx+2a}}\right)^{2}\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(c\,\mathrm{e}^{bx+a}+i\sqrt{1-c^{2}\mathrm{e}\ldots}\right.\right.}{2}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(c*exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*I*arccos(c*exp(b*x+a))^2+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+
I*(1-c^2*exp(b*x+a)^2)^(1/2))^2)-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*ex
p(b*x+a)^2)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b^2*c^2*integrate(x*e^(2*b*x + 2*a)/(c^4*e^(4*b*x + 4*a) - c^2*e^
(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(log(c*e^(b*x + a) + 1) + log(-
c*e^(b*x + a) + 1))), x) + 2*b^2*c*integrate(x*e^(b*x + a + 1/2*log(c*e^(b*
x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1))/(c^4*e^(4*b*x + 4*a) - c^2*e^(2*
b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(log(c*e^(b*x + a) + 1) + log(-c*e
^(b*x + a) + 1))), x) - 2*b*x*arctan(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*e^(b*x
 + a) + 1)*e^(-b*x - a)/c) - I*b*x*log(c*e^(b*x + a) + 1) - I*b*x*log(-c*e^
(b*x + a) + 1) - I*dilog(c*e^(b*x + a)) - I*dilog(-c*e^(b*x + a)))/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acos}\left(ce^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(c*exp(b*x+a)),x)
```

```
[Out] Integral(acos(c*exp(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccos(c*e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acos}\left(c\,\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(c*exp(a + b*x)),x)
```

```
[Out] int(acos(c*exp(a + b*x)), x)
```

3.108 $\int e^{\text{ArcCos}(ax)} x^3 \, dx$

Optimal. Leaf size=81

$$\frac{e^{\text{ArcCos}(ax)} \cos(2\text{ArcCos}(ax))}{10a^4} + \frac{e^{\text{ArcCos}(ax)} \cos(4\text{ArcCos}(ax))}{34a^4} - \frac{e^{\text{ArcCos}(ax)} \sin(2\text{ArcCos}(ax))}{20a^4} - \frac{e^{\text{ArcCos}(ax)} \sin(4\text{A}}{136a^4}$$

```
[Out] 1/10*exp(arccos(a*x))*cos(2*arccos(a*x))/a^4+1/34*exp(arccos(a*x))*cos(4*ar
ccos(a*x))/a^4-1/20*exp(arccos(a*x))*sin(2*arccos(a*x))/a^4-1/136*exp(arcco
s(a*x))*sin(4*arccos(a*x))/a^4
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4921, 12, 4557, 4517}

$$-\frac{e^{\text{ArcCos}(ax)} \sin(2\text{ArcCos}(ax))}{20a^4} - \frac{e^{\text{ArcCos}(ax)} \sin(4\text{ArcCos}(ax))}{136a^4} + \frac{e^{\text{ArcCos}(ax)} \cos(2\text{ArcCos}(ax))}{10a^4} + \frac{e^{\text{ArcCos}(ax)} \cos(4\text{ArcCos}(ax))}{34a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x^3,x]
```

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(10*a^4) + (E^ArcCos[a*x]*Cos[4*ArcCos[a
*x]])/(34*a^4) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*x]])/(20*a^4) - (E^ArcCos[a*
x]*Sin[4*ArcCos[a*x]])/(136*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\cos^{-1}(ax)}x^3\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x\cos^3(x)\sin(x)}{a^3}\,dx,x,\cos^{-1}(ax)\right)}{a}\\ &= -\frac{\text{Subst}\left(\int e^x\cos^3(x)\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{a^4}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(2x)+\frac{1}{8}e^x\sin(4x)\right)\,dx,x,\cos^{-1}(ax)\right)}{a^4}\\ &= -\frac{\text{Subst}\left(\int e^x\sin(4x)\,dx,x,\cos^{-1}(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int e^x\sin(2x)\,dx,x,\cos^{-1}(ax)\right)}{4a^4}\\ &= \frac{e^{\cos^{-1}(ax)}\cos\left(2\cos^{-1}(ax)\right)}{10a^4}+\frac{e^{\cos^{-1}(ax)}\cos\left(4\cos^{-1}(ax)\right)}{34a^4}-\frac{e^{\cos^{-1}(ax)}\sin\left(2\cos^{-1}(ax)\right)}{20a^4}-\end{aligned}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 0.62

$$-\frac{e^{\text{ArcCos}(ax)}(-68\cos(2\text{ArcCos}(ax))-20\cos(4\text{ArcCos}(ax))+34\sin(2\text{ArcCos}(ax))+5\sin(4\text{ArcCos}(ax)))}{680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x^3,x]
```

```
[Out] -1/680*(E^ArcCos[a*x]*(-68*Cos[2*ArcCos[a*x]] - 20*Cos[4*ArcCos[a*x]] + 34*
Sin[2*ArcCos[a*x]] + 5*Sin[4*ArcCos[a*x]]))/a^4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{\arccos(ax)}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x^3,x)
```

```
[Out] int(exp(arccos(a*x))*x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arccos(a*x)), x)
```

Fricas [A]
time = 1.14, size = 55, normalized size = 0.68

$$\frac{\left(20\,a^{4}x^{4} - 3\,a^{2}x^{2} - (5\,a^{3}x^{3} + 6\,ax)\sqrt{-a^{2}x^{2} + 1} - 6\right)e^{(\arccos(ax))}}{85\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="fricas")
```

```
[Out] 1/85*(20*a^4*x^4 - 3*a^2*x^2 - (5*a^3*x^3 + 6*a*x)*sqrt(-a^2*x^2 + 1) - 6)*
e^(arccos(a*x))/a^4
```

Sympy [A]
time = 0.48, size = 105, normalized size = 1.30

$$\begin{cases} \frac{4x^{4}e^{\operatorname{acos}(ax)}}{17} - \frac{x^{3}\sqrt{-a^{2}x^{2}+1}\,e^{\operatorname{acos}(ax)}}{17a} - \frac{3x^{2}e^{\operatorname{acos}(ax)}}{85a^{2}} - \frac{6x\sqrt{-a^{2}x^{2}+1}\,e^{\operatorname{acos}(ax)}}{85a^{3}} - \frac{6e^{\operatorname{acos}(ax)}}{85a^{4}} & \text{for}\ a \neq 0 \\ \frac{x^{4}e^{\frac{\pi}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x**3,x)
```

```
[Out] Piecewise((4*x**4*exp(acos(a*x))/17 - x**3*sqrt(-a**2*x**2 + 1)*exp(acos(a*
x))/(17*a) - 3*x**2*exp(acos(a*x))/(85*a**2) - 6*x*sqrt(-a**2*x**2 + 1)*exp
(acos(a*x))/(85*a**3) - 6*exp(acos(a*x))/(85*a**4), Ne(a, 0)), (x**4*exp(pi
/2)/4, True))
```

Giac [A]
time = 0.43, size = 82, normalized size = 1.01

$$\frac{4}{17}\,x^{4}e^{(\arccos(ax))} - \frac{\sqrt{-a^{2}x^{2}+1}\,x^{3}e^{(\arccos(ax))}}{17\,a} - \frac{3\,x^{2}e^{(\arccos(ax))}}{85\,a^{2}} - \frac{6\,\sqrt{-a^{2}x^{2}+1}\,xe^{(\arccos(ax))}}{85\,a^{3}} - \frac{6\,e^{(\arccos(ax))}}{85\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^3,x, algorithm="giac")
```

```
[Out] 4/17*x^4*e^(arccos(a*x)) - 1/17*sqrt(-a^2*x^2 + 1)*x^3*e^(arccos(a*x))/a -
3/85*x^2*e^(arccos(a*x))/a^2 - 6/85*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/a^
3 - 6/85*e^(arccos(a*x))/a^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{e}^{\mathrm{acos}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(acos(a*x)),x)
```

```
[Out] int(x^3*exp(acos(a*x)), x)
```

3.109 $\int e^{\text{ArcCos}(ax)}x^2\,dx$

Optimal. Leaf size=82

$$\frac{e^{\text{ArcCos}(ax)}x}{8a^2}-\frac{e^{\text{ArcCos}(ax)}\sqrt{1-a^2x^2}}{8a^3}+\frac{3e^{\text{ArcCos}(ax)}\cos(3\text{ArcCos}(ax))}{40a^3}-\frac{e^{\text{ArcCos}(ax)}\sin(3\text{ArcCos}(ax))}{40a^3}$$

```
[Out] 1/8*exp(arccos(a*x))*x/a^2+3/40*exp(arccos(a*x))*cos(3*arccos(a*x))/a^3-1/4
0*exp(arccos(a*x))*sin(3*arccos(a*x))/a^3-1/8*exp(arccos(a*x))*(-a^2*x^2+1)
^(1/2)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4921, 12, 4557, 4517}

$$-\frac{e^{\text{ArcCos}(ax)}\sin(3\text{ArcCos}(ax))}{40a^3}+\frac{3e^{\text{ArcCos}(ax)}\cos(3\text{ArcCos}(ax))}{40a^3}+\frac{xe^{\text{ArcCos}(ax)}}{8a^2}-\frac{\sqrt{1-a^2x^2}\,e^{\text{ArcCos}(ax)}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x^2,x]
```

```
[Out] (E^ArcCos[a*x]*x)/(8*a^2) - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(8*a^3) + (3*
E^ArcCos[a*x]*Cos[3*ArcCos[a*x]])/(40*a^3) - (E^ArcCos[a*x]*Sin[3*ArcCos[a*
x]])/(40*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\cos^{-1}(ax)}x^2\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x\cos^2(x)\sin(x)}{a^2}\,dx,x,\cos^{-1}(ax)\right)}{a}\\ &= -\frac{\text{Subst}\left(\int e^x\cos^2(x)\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}e^x\sin(x)+\frac{1}{4}e^x\sin(3x)\right)\,dx,x,\cos^{-1}(ax)\right)}{a^3}\\ &= -\frac{\text{Subst}\left(\int e^x\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{4a^3}-\frac{\text{Subst}\left(\int e^x\sin(3x)\,dx,x,\cos^{-1}(ax)\right)}{4a^3}\\ &= \frac{e^{\cos^{-1}(ax)}x}{8a^2}-\frac{e^{\cos^{-1}(ax)}\sqrt{1-a^2x^2}}{8a^3}+\frac{3e^{\cos^{-1}(ax)}\cos\left(3\cos^{-1}(ax)\right)}{40a^3}-\frac{e^{\cos^{-1}(ax)}\sin\left(3\cos^{-1}\right.}{40a^3}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 50, normalized size = 0.61

$$-\frac{e^{\text{ArcCos}(ax)}\left(-5ax+5\sqrt{1-a^2x^2}-3\cos(3\text{ArcCos}(ax))+\sin(3\text{ArcCos}(ax))\right)}{40a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x^2,x]
[Out] -1/40*(E^ArcCos[a*x]*(-5*a*x + 5*Sqrt[1 - a^2*x^2] - 3*Cos[3*ArcCos[a*x]] +
 Sin[3*ArcCos[a*x]]))/a^3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\arccos(ax)}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x^2,x)
[Out] int(exp(arccos(a*x))*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arccos(a*x)), x)
```

Fricas [A]
time = 1.40, size = 46, normalized size = 0.56

$$\frac{\left(3\,a^3x^3 - ax - (a^2x^2+1)\sqrt{-a^2x^2+1}\right)e^{(\arccos(ax))}}{10\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] 1/10*(3*a^3*x^3 - a*x - (a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/a
^3
```

Sympy [A]
time = 0.24, size = 85, normalized size = 1.04

$$\begin{cases} \frac{3x^3e^{\operatorname{acos}(ax)}}{10} - \frac{x^2\sqrt{-a^2x^2+1}\,e^{\operatorname{acos}(ax)}}{10a} - \frac{xe^{\operatorname{acos}(ax)}}{10a^2} - \frac{\sqrt{-a^2x^2+1}\,e^{\operatorname{acos}(ax)}}{10a^3} & \text{for } a \neq 0 \\ \frac{x^3e^{\frac{\pi}{2}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x**2,x)
```

```
[Out] Piecewise((3*x**3*exp(acos(a*x))/10 - x**2*sqrt(-a**2*x**2 + 1)*exp(acos(a*
x))/(10*a) - x*exp(acos(a*x))/(10*a**2) - sqrt(-a**2*x**2 + 1)*exp(acos(a*x
))/(10*a**3), Ne(a, 0)), (x**3*exp(pi/2)/3, True))
```

Giac [A]
time = 0.43, size = 69, normalized size = 0.84

$$\frac{3}{10}\,x^3e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}\,x^2e^{(\arccos(ax))}}{10\,a} - \frac{xe^{(\arccos(ax))}}{10\,a^2} - \frac{\sqrt{-a^2x^2+1}\,e^{(\arccos(ax))}}{10\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x^2,x, algorithm="giac")
```

```
[Out] 3/10*x^3*e^(arccos(a*x)) - 1/10*sqrt(-a^2*x^2 + 1)*x^2*e^(arccos(a*x))/a -
1/10*x*e^(arccos(a*x))/a^2 - 1/10*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{\operatorname{acos}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(acos(a*x)),x)
[Out] int(x^2*exp(acos(a*x)), x)
```

3.110 $\int e^{\mathbf{ArcCos}(ax)} x\,dx$

Optimal. Leaf size=41

$$\frac{e^{\text{ArcCos}(ax)}\cos(2\text{ArcCos}(ax))}{5a^2}-\frac{e^{\text{ArcCos}(ax)}\sin(2\text{ArcCos}(ax))}{10a^2}$$

```
[Out] 1/5*exp(arccos(a*x))*cos(2*arccos(a*x))/a^2-1/10*exp(arccos(a*x))*sin(2*arc
cos(a*x))/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4921, 12, 4557, 4517}

$$\frac{e^{\text{ArcCos}(ax)}\cos(2\text{ArcCos}(ax))}{5a^2}-\frac{e^{\text{ArcCos}(ax)}\sin(2\text{ArcCos}(ax))}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]*x,x]
```

```
[Out] (E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/(5*a^2) - (E^ArcCos[a*x]*Sin[2*ArcCos[a*
x]])/(10*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
```

```
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\cos^{-1}(ax)} x\,dx &= -\frac{\text{Subst}\left(\int \frac{e^x \cos(x)\sin(x)}{a}\,dx, x, \cos^{-1}(ax)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int e^x \cos(x)\sin(x)\,dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{2}e^x \sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int e^x \sin(2x)\,dx, x, \cos^{-1}(ax)\right)}{2a^2} \\
&= \frac{e^{\cos^{-1}(ax)} \cos\left(2\cos^{-1}(ax)\right)}{5a^2} - \frac{e^{\cos^{-1}(ax)} \sin\left(2\cos^{-1}(ax)\right)}{10a^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.73

$$-\frac{e^{\text{ArcCos}(ax)}(-2\cos(2\text{ArcCos}(ax)) + \sin(2\text{ArcCos}(ax)))}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]*x,x]
[Out] -1/10*(E^ArcCos[a*x]*(-2*Cos[2*ArcCos[a*x]] + Sin[2*ArcCos[a*x]]))/a^2
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\arccos(ax)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))*x,x)
[Out] int(exp(arccos(a*x))*x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*e^(arccos(a*x)), x)
```

Fricas [A]
time = 1.42, size = 36, normalized size = 0.88

$$\frac{\left(2\,a^{2}x^{2}-\sqrt{-a^{2}x^{2}+1}\,ax-1\right)e^{\left(\arccos\left(ax\right)\right)}}{5\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="fricas")
[Out] 1/5*(2*a^2*x^2 - sqrt(-a^2*x^2 + 1)*a*x - 1)*e^(arccos(a*x))/a^2
```

Sympy [A]
time = 0.14, size = 58, normalized size = 1.41

$$\begin{cases}\frac{2x^{2}e^{\operatorname{acos}\left(ax\right)}}{5}-\frac{x\sqrt{-a^{2}x^{2}+1}\,e^{\operatorname{acos}\left(ax\right)}}{5a}-\frac{e^{\operatorname{acos}\left(ax\right)}}{5a^{2}} & \text{for}\: a\neq 0\\\frac{x^{2}e^{\frac{\pi}{2}}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))*x,x)
[Out] Piecewise((2*x**2*exp(acos(a*x))/5 - x*sqrt(-a**2*x**2 + 1)*exp(acos(a*x))/
(5*a) - exp(acos(a*x))/(5*a**2), Ne(a, 0)), (x**2*exp(pi/2)/2, True))
```

Giac [A]
time = 0.41, size = 44, normalized size = 1.07

$$\frac{2}{5}\,x^{2}e^{\left(\arccos\left(ax\right)\right)}-\frac{\sqrt{-a^{2}x^{2}+1}\,xe^{\left(\arccos\left(ax\right)\right)}}{5\,a}-\frac{e^{\left(\arccos\left(ax\right)\right)}}{5\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))*x,x, algorithm="giac")
[Out] 2/5*x^2*e^(arccos(a*x)) - 1/5*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/a - 1/5*
e^(arccos(a*x))/a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{e}^{\mathrm{acos}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(acos(a*x)),x)
[Out] int(x*exp(acos(a*x)), x)
```

3.111 $\int e^{\text{ArcCos}(ax)}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}e^{\text{ArcCos}(ax)}x-\frac{e^{\text{ArcCos}(ax)}\sqrt{1-a^2x^2}}{2a}$$

```
[Out] 1/2*exp(arccos(a*x))*x-1/2*exp(arccos(a*x))*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4921, 4517}

$$\frac{1}{2}xe^{\text{ArcCos}(ax)}-\frac{\sqrt{1-a^2x^2}\,e^{\text{ArcCos}(ax)}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x],x]
```

```
[Out] (E^ArcCos[a*x]*x)/2 - (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/(2*a)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\cos^{-1}(ax)}\,dx&=-\frac{\text{Subst}\left(\int e^x\sin(x)\,dx,x,\cos^{-1}(ax)\right)}{a}\\&=\frac{1}{2}e^{\cos^{-1}(ax)}x-\frac{e^{\cos^{-1}(ax)}\sqrt{1-a^2x^2}}{2a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.82

$$-\frac{e^{\operatorname{ArcCos}(ax)}\left(-ax+\sqrt{1-a^2x^2}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x],x]
[Out] -1/2*(E^ArcCos[a*x]*(-(a*x) + Sqrt[1 - a^2*x^2]))/a
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\arccos(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x)),x)
[Out] int(exp(arccos(a*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="maxima")
[Out] integrate(e^(arccos(a*x)), x)
```

Fricas [A]
time = 1.37, size = 28, normalized size = 0.72

$$\frac{\left(ax-\sqrt{-a^2x^2+1}\right)e^{(\arccos(ax))}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="fricas")
[Out] 1/2*(a*x - sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/a
```

Sympy [A]
time = 0.08, size = 37, normalized size = 0.95

$$\begin{cases} \frac{xe^{\operatorname{acos}(ax)}}{2}-\frac{\sqrt{-a^2x^2+1}\,e^{\operatorname{acos}(ax)}}{2a} & \text{for } a\neq 0\\ xe^{\frac{\pi}{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x)),x)
```

```
[Out] Piecewise((x*exp(acos(a*x))/2 - sqrt(-a**2*x**2 + 1)*exp(acos(a*x))/(2*a),
Ne(a, 0)), (x*exp(pi/2), True))
```

Giac [A]
time = 0.43, size = 31, normalized size = 0.79

$$\frac{1}{2} \, x e^{\left(\arccos\left(a x\right)\right)} - \frac{\sqrt{-a^{2} x^{2} + 1} \, e^{\left(\arccos\left(a x\right)\right)}}{2 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x)),x, algorithm="giac")
```

```
[Out] 1/2*x*e^(arccos(a*x)) - 1/2*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{e}^{\mathrm{acos}\left(a\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(a*x)),x)
```

```
[Out] int(exp(acos(a*x)), x)
```

3.112 $\int \frac{e^{\text{ArcCos}(ax)}}{x} dx$

Optimal. Leaf size=45

$$ie^{\text{ArcCos}(ax)} - 2ie^{\text{ArcCos}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\text{ArcCos}(ax)}\right)$$

```
[Out] I*exp(arccos(a*x))-2*I*exp(arccos(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],-(a
*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4921, 12, 4527, 2225, 2283}

$$ie^{\text{ArcCos}(ax)} - 2ie^{\text{ArcCos}(ax)} {}_2F_1\left(-\frac{i}{2}, 1; 1 - \frac{i}{2}; -e^{2i\text{ArcCos}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]/x,x]
```

```
[Out] I*E^ArcCos[a*x] - (2*I)*E^ArcCos[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2,
 -E^((2*I)*ArcCos[a*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4527

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x)
```

```
))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\cos^{-1}(ax)}}{x}\,dx &= -\frac{\text{Subst}\left(\int ae^x\tan(x)\,dx, x, \cos^{-1}(ax)\right)}{a}\\
&= -\text{Subst}\left(\int e^x\tan(x)\,dx, x, \cos^{-1}(ax)\right)\\
&= -\left(i\text{Subst}\left(\int \left(-e^x+\frac{2e^x}{1+e^{2ix}}\right)\,dx, x, \cos^{-1}(ax)\right)\right)\\
&= i\text{Subst}\left(\int e^x\,dx, x, \cos^{-1}(ax)\right) - 2i\text{Subst}\left(\int \frac{e^x}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right)\\
&= ie^{\cos^{-1}(ax)} - 2ie^{\cos^{-1}(ax)}\,{}_2F_1\left(-\frac{i}{2}, 1; 1-\frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.76

$$i\left(-e^{\text{ArcCos}(ax)}\,{}_2F_1\left(-\frac{i}{2}, 1; 1-\frac{i}{2}; -e^{2i\text{ArcCos}(ax)}\right)+\left(\frac{1}{5}-\frac{2i}{5}\right)e^{(1+2i)\text{ArcCos}(ax)}\,{}_2F_1\left(1, 1-\frac{i}{2}; 2-\frac{i}{2}; -e^{2i\text{ArcCos}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]/x,x]
```

```
[Out] I*(-(E^ArcCos[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcCos[a
*x])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcCos[a*x])*Hypergeometric2F1[1, 1 -
 I/2, 2 - I/2, -E^((2*I)*ArcCos[a*x])])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\arccos(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))/x,x)
[Out] int(exp(arccos(a*x))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="maxima")
[Out] integrate(e^(arccos(a*x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="fricas")
[Out] integral(e^(arccos(a*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acos}\,(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))/x,x)
[Out] Integral(exp(acos(a*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x,x, algorithm="giac")
[Out] integrate(e^(arccos(a*x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{e}^{\mathrm{acos}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(a*x))/x,x)
```

```
[Out] int(exp(acos(a*x))/x, x)
```

3.113 $\int \frac{e^{\mathbf{ArcCos}(ax)}}{x^2}\,dx$

Optimal. Leaf size=87

$$(1+i)ae^{(1+i)\text{ArcCos}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\text{ArcCos}(ax)}\right)-(2+2i)ae^{(1+i)\text{ArcCos}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\text{ArcCos}(ax)}\right)$$

```
[Out] (1+I)*a*exp((1+I)*arccos(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],-(a*x+I
*(-a^2*x^2+1)^(1/2))^2)-(2+2*I)*a*exp((1+I)*arccos(a*x))*hypergeom([2, 1/2-
1/2*I],[3/2-1/2*I],-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4921, 12, 4559, 2283}

$$(1+i)ae^{(1+i)\text{ArcCos}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},1;\frac{3}{2}-\frac{i}{2};-e^{2i\text{ArcCos}(ax)}\right)-(2+2i)ae^{(1+i)\text{ArcCos}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2},2;\frac{3}{2}-\frac{i}{2};-e^{2i\text{ArcCos}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCos[a*x]/x^2,x]
```

```
[Out] (1 + I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2 - I/2
, -E^((2*I)*ArcCos[a*x])] - (2 + 2*I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeomet
ric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4559

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

Rule 4921

```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
-b^(-1), Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Arc
Cos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\cos^{-1}(ax)}}{x^2}\,dx &= -\frac{\text{Subst}\left(\int a^2 e^x \sec(x)\tan(x)\,dx, x, \cos^{-1}(ax)\right)}{a}\\ &= -\left(a\text{Subst}\left(\int e^x \sec(x)\tan(x)\,dx, x, \cos^{-1}(ax)\right)\right)\\ &= -\left(a\text{Subst}\left(\int \left(\frac{4ie^{(1+i)x}}{(1+e^{2ix})^2} - \frac{2ie^{(1+i)x}}{1+e^{2ix}}\right)dx, x, \cos^{-1}(ax)\right)\right)\\ &= (2ia)\text{Subst}\left(\int \frac{e^{(1+i)x}}{1+e^{2ix}}\,dx, x, \cos^{-1}(ax)\right) - (4ia)\text{Subst}\left(\int \frac{e^{(1+i)x}}{(1+e^{2ix})^2}\,dx, x, \cos^{-1}(ax)\right)\\ &= (1+i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; -e^{2i\cos^{-1}(ax)}\right) - (2+2i)ae^{(1+i)\cos^{-1}(ax)}\,{}_2F_1\left(\frac{1}{2}\right.\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.63

$$-\frac{e^{\text{ArcCos}(ax)}}{x} + (1-i)ae^{(1+i)\text{ArcCos}(ax)}\,{}_2F_1\left(\frac{1}{2}-\frac{i}{2}, 1; \frac{3}{2}-\frac{i}{2}; -e^{2i\text{ArcCos}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCos[a*x]/x^2,x]
[Out] -(E^ArcCos[a*x]/x) + (1 - I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/
2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccos(a*x))/x^2,x)
[Out] int(exp(arccos(a*x))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccos(a*x))/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arccos(a*x))/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acos}{(ax)}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acos(a*x))/x**2,x)
```

```
[Out] Integral(exp(acos(a*x))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccos(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arccos(a*x))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{\operatorname{acos}(a\,x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acos(a*x))/x^2,x)
```

```
[Out] int(exp(acos(a*x))/x^2, x)
```

3.114 $\int \text{ArcCos}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=48

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arcsec(a/c+b*x/c)/b-c*arctanh((1-c^2/(b*x+a)^2)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4917, 5358, 379, 272, 65, 212}

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcSec[a/c + (b*x)/c])/b - (c*ArcTanh[Sqrt[1 - c^2/(a + b*x)^2]]
)/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 4917

```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 5358

```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{-1}\left(\frac{c}{a+bx}\right) dx &= \int \sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx \\
&= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1-\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}} dx \\
&= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x} dx, x, \frac{a}{c}+\frac{bx}{c}\right)}{b} \\
&= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x} dx, x, \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b} \\
&= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b} \\
&= \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\tanh^{-1}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 345 vs. 2(48) = 96.

time = 0.25, size = 345, normalized size = 7.19

$$x\text{ArcCos}\left(\frac{c}{a+bx}\right)-\frac{(a+bx)\sqrt{\frac{a^2-c^2+2abx+b^2x^2}{(a+bx)^2}}\left(2a\left(b+\sqrt{b^2}\right)\text{ArcTan}\left(\frac{a+\sqrt{b^2}\,x-\sqrt{a^2-c^2+2abx+b^2x^2}}{c}\right)+2a\left(-b+\sqrt{b^2}\right)\text{ArcTan}\left(\frac{a-\sqrt{b^2}\,x+\sqrt{a^2-c^2+2abx+b^2x^2}}{c}\right)-c\left(\sqrt{b^2}\log\left(-a-\sqrt{b^2}\,x+\sqrt{a^2-c^2+2abx+b^2x^2}\right)+\left(-b+\sqrt{b^2}\right)\log\left(a-\sqrt{b^2}\,x+\sqrt{a^2-c^2+2abx+b^2x^2}\right)+b\log\left(ab^2+(b^2)^{3/2}x-b^2\sqrt{a^2-c^2+2abx+b^2x^2}\right)\right)\right)}{2b^2\sqrt{a^2-c^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[c/(a + b*x)],x]
```

```
[Out] x*ArcCos[c/(a + b*x)] - ((a + b*x)*Sqrt[(a^2 - c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*(2*a*(b + Sqrt[b^2])*ArcTan[(a + Sqrt[b^2]*x - Sqrt[a^2 - c^2 + 2
*a*b*x + b^2*x^2])/c] + 2*a*(-b + Sqrt[b^2])*ArcTan[(a - Sqrt[b^2]*x + Sqrt
[a^2 - c^2 + 2*a*b*x + b^2*x^2])/c] - c*(Sqrt[b^2]*Log[-a - Sqrt[b^2]*x + S
qrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]] + (-b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x
 + Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]] + b*Log[a*b^2 + (b^2)^(3/2)*x - b^2*
Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]])))/(2*b^2*Sqrt[a^2 - c^2 + 2*a*b*x + b
^2*x^2])
```

Maple [A]
time = 0.21, size = 45, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\arccos\left(\frac{c}{bx+a}\right)}{c}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$	45
default	$-\frac{c\left(-\frac{(bx+a)\arccos\left(\frac{c}{bx+a}\right)}{c}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*c*(-1/c*(b*x+a)*arccos(c/(b*x+a))+arctanh(1/(1-c^2/(b*x+a)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(b*x + a + c)*sqrt(b*x + a - c)/c) - integrate((b^2*c*x^2 + a*
b*c*x)*e^(1/2*log(b*x + a + c) + 1/2*log(b*x + a - c))/(b^2*c^2*x^2 + 2*a*b
*c^2*x + a^2*c^2 - c^4 + (b^2*x^2 + 2*a*b*x + a^2 - c^2)*e^(log(b*x + a + c
) + log(b*x + a - c))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.
time = 4.19, size = 140, normalized size = 2.92

$$\frac{bx\arccos\left(\frac{c}{bx+a}\right)+2\,a\arctan\left(-\frac{bx-(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+a}{c}\right)+c\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*arccos(c/(b*x + a)) + 2*a*arctan(-(b*x - (b*x + a)*sqrt((b^2*x^2 + 2*a
*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/c) + c*log(-b*x + (b*x +
a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acos}\left(\frac{c}{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(c/(b*x+a)),x)
```

```
[Out] Integral(acos(c/(a + b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.
time = 0.45, size = 95, normalized size = 1.98

$$-\frac{b\left(\frac{c^2\left(\log\left(\sqrt{-\frac{c^2}{(bx+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{c^2}{(bx+a)^2}+1}+1\right)\right)}{b^2}-\frac{2\,(bx+a)c\arccos\left(-\frac{c}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*(c^2*(log(sqrt(-c^2/(b*x + a)^2 + 1) + 1) - log(-sqrt(-c^2/(b*x + a)
^2 + 1) + 1))/b^2 - 2*(b*x + a)*c*arccos(-c/((b*x + a)*(a/(b*x + a) - 1) -
a))/b^2)/c
```

Mupad [B]
time = 0.63, size = 43, normalized size = 0.90

$$\frac{\mathrm{acos}\left(\frac{c}{a+b\,x}\right)\,\left(a+b\,x\right)}{b}-\frac{c\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{{\left(a+b\,x\right)}^2}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(c/(a + b*x)),x)
```

```
[Out] (acos(c/(a + b*x))*(a + b*x))/b - (c*atanh(1/(1 - c^2/(a + b*x)^2)^(1/2)))/
b
```

3.115 $\int \frac{x}{\sqrt{1-x^2}\,\sqrt{\mathbf{ArcCos}(x)}}\,dx$

Optimal. Leaf size=26

$$-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(x)}\right)$$

```
[Out] -FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))*2^(1/2)*Pi^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4810, 3385, 3433}

$$-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,\sqrt{\text{ArcCos}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

```
[Out] -(Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[x]]])
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\cos^{-1}(x)}}\,dx = -\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \cos^{-1}(x)\right)$$
$$= -\left(2\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\cos^{-1}(x)}\right)\right)$$
$$= -\sqrt{2\pi}\,C\left(\sqrt{\frac{2}{\pi}}\sqrt{\cos^{-1}(x)}\right)$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.06, size = 56, normalized size = 2.15

$$\frac{i\left(\sqrt{-i\text{ArcCos}(x)}\,\text{Gamma}\left(\frac{1}{2}, -i\text{ArcCos}(x)\right) - \sqrt{i\text{ArcCos}(x)}\,\text{Gamma}\left(\frac{1}{2}, i\text{ArcCos}(x)\right)\right)}{2\sqrt{\text{ArcCos}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
[Out] ((I/2)*(Sqrt[(-I)*ArcCos[x]]*Gamma[1/2, (-I)*ArcCos[x]] - Sqrt[I*ArcCos[x]]
*Gamma[1/2, I*ArcCos[x]]))/Sqrt[ArcCos[x]]
```

Maple [**A**]
time = 0.68, size = 21, normalized size = 0.81

method	result	size
default	$-\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(x)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))*2^(1/2)*Pi^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acos}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2)/acos(x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acos(x))), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 37, normalized size = 1.42

$$\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right)-\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="giac")
```

```
[Out] (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(x))) -
 (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x}{\sqrt{\operatorname{acos}(x)}\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)),x)
```

```
[Out] int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

3.116 $\int \frac{x}{\sqrt{1-x^2}\,\mathbf{ArcCos}(x)}\,dx$

Optimal. Leaf size=5

$$-\text{CosIntegral}(\text{ArcCos}(x))$$

```
[Out] -Ci(arccos(x))
```

Rubi [A]
time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4810, 3383}

$$-\text{CosIntegral}(\text{ArcCos}(x))$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*ArcCos[x]),x]
```

```
[Out] -CosIntegral[ArcCos[x]]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{1-x^2}\cos^{-1}(x)}\,dx &= -\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \cos^{-1}(x)\right) \\ &= -\text{Ci}\left(\cos^{-1}(x)\right) \end{aligned}$$

Mathematica [A]
time = 0.03, size = 5, normalized size = 1.00

$$-\text{CosIntegral}(\text{ArcCos}(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^2]*ArcCos[x]),x]
```

```
[Out] -CosIntegral[ArcCos[x]]
```

Maple [A]
time = 0.28, size = 6, normalized size = 1.20

method	result	size
default	$-\operatorname{cosineIntegral}\left(\arccos\left(x\right)\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccos(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -Ci(arccos(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^2 + 1)*arccos(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + 1)*x/((x^2 - 1)*arccos(x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x-1\right)\left(x+1\right)}\operatorname{acos}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acos(x)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*acos(x)), x)
```

Giac [**A**]
time = 0.44, size = 5, normalized size = 1.00

$$-\operatorname{Ci}\left(\arccos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] -cos_integral(arccos(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.20

$$\int \frac{x}{\operatorname{acos}\left(x\right)\,\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acos(x)*(1 - x^2)^(1/2)),x)
[Out] int(x/(acos(x)*(1 - x^2)^(1/2)), x)
```

3.117 $\int \frac{\mathbf{ArcCos}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{-bx^2}\,\text{ArcCos}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

```
[Out] -arccos((b*x^2+1)^(1/2))^(1+n)*(-b*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A]

time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4919, 4738}

$$-\frac{\sqrt{-bx^2}\,\text{ArcCos}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] -((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4919

```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[(-b)*x^2]/(b*x), Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx &= \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)^n}{\sqrt{1-x^2}}\,dx, x, \sqrt{1+bx^2}\right)}{bx}\\ &= -\frac{\sqrt{-bx^2}\,\cos^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$-\frac{\sqrt{-bx^2}\,\text{ArcCos}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
[Out] -((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\arccos\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
[Out] int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_b)
```

Fricas [A]
time = 2.73, size = 42, normalized size = 1.08

$$-\frac{\sqrt{-bx^2}\arccos\left(\sqrt{bx^2+1}\right)^n\arccos\left(\sqrt{bx^2+1}\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2)*arccos(sqrt(b*x^2 + 1))^n*arccos(sqrt(b*x^2 + 1))/((b*n + b)*
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty}x & \text{for } b = 0 \wedge n = -1 \\ 0^n x & \text{for } b = 0 \\ \int \frac{1}{\sqrt{bx^2+1}\operatorname{acos}\left(\sqrt{bx^2+1}\right)}\,dx & \text{for } n = -1 \\ -\frac{\sqrt{-bx^2}\operatorname{acos}\left(\sqrt{bx^2+1}\right)\operatorname{acos}^n\left(\sqrt{bx^2+1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (Integral(1/(s
qrt(b*x**2 + 1)*acos(sqrt(b*x**2 + 1))), x), Eq(n, -1)), (-sqrt(-b*x**2)*ac
os(sqrt(b*x**2 + 1))*acos(sqrt(b*x**2 + 1))**n/(b*n*x + b*x), True))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acos}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2),x)
```

```
[Out] int(acos((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2), x)
```

3.118 $\int \frac{1}{\sqrt{1+bx^2}\,\mathbf{ArcCos}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{-bx^2}\,\log\left(\mathrm{ArcCos}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

[Out] -ln(arccos((b*x^2+1)^(1/2)))*(-b*x^2)^(1/2)/b/x

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4919, 4736}

$$-\frac{\sqrt{-bx^2}\,\log\left(\mathrm{ArcCos}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]

[Out] -((Sqrt[-(b*x^2)]*Log[ArcCos[Sqrt[1 + b*x^2]]])/(b*x))

Rule 4736

```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(-(b*c)^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(Log[a +
 b*ArcCos[c*x]]/(b*c*Sqrt[d])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0]
```

Rule 4919

```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[(-b)*x^2]/(b*x), Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx^2}\,\cos^{-1}\left(\sqrt{1+bx^2}\right)}\,dx &= \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\cos^{-1}(x)}\,dx, x, \sqrt{1+bx^2}\right)}{bx}\\ &= -\frac{\sqrt{-bx^2}\,\log\left(\cos^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.81

$$\frac{x\log\left(\text{ArcCos}\left(\sqrt{1+bx^2}\right)\right)}{\sqrt{-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]
[Out] (x*Log[ArcCos[Sqrt[1 + b*x^2]]])/Sqrt[-(b*x^2)]
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\arccos\left(\sqrt{b\,x^2+1}\right)\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
[Out] int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found sqrt(-_SAGE_VAR_b)
```

Fricas [A]
time = 3.87, size = 27, normalized size = 0.87

$$-\frac{\sqrt{-bx^2}\,\log\left(\arccos\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x^2)*log(arccos(sqrt(b*x^2 + 1)))/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+1}\,\mathrm{acos}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(b*x**2 + 1)*acos(sqrt(b*x**2 + 1))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 0.30, size = 25, normalized size = 0.81

$$\frac{\ln\left(\mathrm{acos}\left(\sqrt{b\,x^2+1}\right)\right)\,\sqrt{x^2}}{\sqrt{-b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acos((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)),x)
[Out] (log(acos((b*x^2 + 1)^(1/2)))*(x^2)^(1/2))/((-b)^(1/2)*x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 560
4.2 Listing of Grading functions 560

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
            fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
            gamma,loggamma,digamma,zeta,polylog,LambertW,
            elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
      print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
     return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
  try:
     if expn.parent() is SR:
        return expn.operator() is None
     if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
     if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

     return False

  except AttributeError as error:
     print("Exception,AttributeError in  is_atom")
     print ("cought exception" , type(error).___name___ )
     return False


def expnType(expn):

  if debug:
     print (">>>>>Enter expnType, expn=", expn)
     print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
     return 1
  elif type(expn)==list:   #isinstance(expn,list):
     return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
     if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
        return 1
     else:
        return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
     if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
     elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```